Transmission Channels of Fiscal Policy at ZLB

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Abstract

In the era of crisis (e.g. COVID-19), zero lower bound (ZLB) on nominal interest rates tend to bind, and monetary policy cannot provide appropriate stimulus. Fiscal policy is an appropriate stabilization tool at the ZLB. I build an analytical TANK model to understand how the fiscal policy tool will be transmitted to the agents. Consumption taxes and labor taxes replicate the effects of monetary policy through the intertemporal substitution channel. Debt-financed lumpsum transfers and a permanent increase in the government debt level replicate the effects of monetary policy through the redistribution channel. And I explore how to replicate similar results in the mechanism in the HANK framework.

1 Introduction

Yet, recent research on Heterogeneous Agent New Keynesian (HANK) models shows that household heterogeneity matters for the transmission mechanism of monetary and fiscal policy. In particular, redistribution dynamics are crucial. As monetary and fiscal policy differently redistribute among households, this raises the question: can fiscal policy replicate the outcomes of monetary policy in a model with household heterogeneity? In this paper, I show through the lens of a standard one-asset HANK model that 4-Instrument Unconventional Fiscal Policy (4I-UFP) achieves the same outcomes as hypothetically unconstrained monetary policy at the ZLB. 4I-UFP replicates both macroeconomic aggregates and welfare that would prevail if monetary policy could set negative nominal interest rates with four instruments: consumption taxes, labor taxes, debt-financed lump-sum transfers, and a permanently higher government debt level. The

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intuition for my result is that with these four instruments, 4I-UFP replicates the effects of monetary policy through both the intertemporal substitution channel and the redistribution channel. In line with Correia et al. [2013], 4I-UFP replicates the effects through the intertemporal substitution channel with tax policies: pre-announced paths for higher future consumption taxes influence the intertemporal consumption decision of households in the same way as a decrease in the real interest rate. Yet, higher consumption taxes incentivize households to reduce their labor supply. Lower labor taxes offset this incentive. Since HANK models feature heterogeneity in marginal propensities to consume (MPC) and a precautionary savings motive of households, redistribution dynamics additionally matter for the transmission of monetary policy. In particular, expansionary monetary policy reduces the return on savings from which asset-rich households disproportionately suffer. As Kaplan and Violante [2018], I assume that the government passes on its reduced interest rate payments to all households via lump-sum transfers to keep government debt constant. This way, expansionary monetary policy redistributes towards low-asset households who have higher marginal propensities to consume (MPCs). I first highlight the importance of these redistribution dynamics in a simple Two Agent New Keynesian (TANK) model, in which there is MPC heterogeneity, but households do not have a precautionary savings motive. Expansionary monetary policy redistributes from Ricardian households, who hold all assets, to Hand-to-Mouth (HtM) households. With one additional instrument - lump-sum transfers financed by a tax on Ricardian households - fiscal policy can replicate these redistribution dynamics of monetary policy. I show analytically that these lump-sum transfers together with consumption taxes and labor taxes are sufficient for UFP to be a perfect substitute for monetary policy in TANK. The intuition is that, this way, UFP replicates the effects through both the intertemporal substitution channel and the redistribution channel. Subsequently, I show that in HANK, 4I-UFP replicates the effects of unconstrained monetary policy through the redistribution channel with debt-financed lump-sum transfers and a higher government debt level in the long-run. The intuition for the higher lump-sum transfers is the same as in TANK: 4I-UFP needs to provide additional resources to households with high MPCs in the same periods as unconstrained monetary policy does. Unlike unconstrained monetary policy, 4I-UFP induces a permanent increase in precautionary savings which puts downward pressure on the real interest rate in the long-run. The reason is that permanently higher consumption taxes and permanently lower labor taxes redistribute from low- to high-productivity households such that the income risk of households permanently increases. To offset this downward pressure on the real interest rate, 4I-UFP increases the asset supply by permanently increasing the government debt level. The increase in precautionary savings is strong enough to fully absorb the newly issued debt due to the higher transfers while the ZLB binds.

Given the equivalence in macroeconomic aggregates and in welfare, I conclude that 4I-UFP is a perfect substitute for unconstrained monetary policy at the ZLB. I stress the importance of the two instruments targeting the redistribution channel. First, I illustrate that when fiscal policy uses only taxes and transfer policies but does not increase the government debt level in the long-run, the economy converges to a new teady state with lower output. Without the fourth instrument, fiscal policy cannot satisfy the higher precautionary savings demand induced by higher consumption taxes and lower labor taxes. The resulting lower interest rate in the new steady state worsens insurance possibilities which reduces aggregate labor supply. Second, I highlight the importance of debt-financed transfers for the short-run stimulus. When UFP purely consists of tax policies, it does not provide enough resources to constrained households while the ZLB is binding. Consequently, it fails to replicate macroeconomic aggregates, even in the short-run.

Related literature. Feldstein [2002] and Hall [2011] propose to increase future consumption taxes when monetary policy is constrained by the ZLB. Correia et al. [2013] show that, by replicating its effects through the intertemporal substitution channel, a combination of consumption taxes and labor taxes is a perfect substitute for monetary policy in RANK. I revisit their seminal result through the lens of a simple TANK model and show that their result relies on the fact that monetary policy is non-redistributive in RANK. I build my analysis in HANK on this insight. There is a large literature on the transmission mechanism of monetary policy in HANK (see among many others Farhi and Werning [2016], McKay et al. [2016] , Kaplan et al. [2018], Auclert and Rognlie [2018], de Ferra et al. [2020], Auclert et al. [2018]). The transmission mechanism of monetary policy in my quantitative HANK model is in line with these findings. I design 4I-UFP accordingly. Recently, the HANK literature has also studied fiscal policy. Auclert and Rognlie [2018] and Hagedorn et al. [2019] analyze fiscal multipliers in HANK models. Unlike my paper, they do not study whether fiscal policy can replicate macroeconomic outcomes of unconstrained monetary policy. Bhandari et al. [2019] examine the optimal interaction between monetary policy and distortionary taxes in a HANK model. In contrast to my paper, they abstract from consumption taxes and analyze how idiosyncratic insurance options shape optimal monetary and fiscal policy. Thus, my equivalence result is more interesting in models with heterogeneity since I also obtain equivalence in distributional outcomes.

2 A Simple TANK Model

I build a simple TANK model to illustrate this idea. There are two types of agents: Ricardian and Hand-to-Mouth. All households are infinitely lived and have the same CRRA preferences with separable disutility from labor:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(c_t^i \right) - v \frac{\left(n_t^i \right)^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

The decision problem of Ricardian households is to maximize their utility subject to equation (2) by choosing c_t^R, n_t^R, and b_t. The decision problem of Hand-to-Mouth households is to maximize their utility subject to equation (3) by choosing c_t^H and n_t^H. Where $b_{R,t}$ denotes her holdings of one-period riskless bonds, d_t denotes dividend payments from firms, Tr_t denotes lump-sum transfers τ_t^R denotes a non-distortionary tax on Ricardian households, and ϕ denotes a constant transfer between both types of households. I set $\tau^{\bar{R}} = 0$ and $\phi = \left(1 - \frac{1}{1+\bar{\mu}}\right) \bar{B}$ such that in steady state, both types receive the same income. Ricardian's Euler equation subject to (2) is equation (4)

$$\left(1+\tau_t^c\right) c_t^R + b_t = \left(1-\tau_t^n\right) n_t^R w_t + \frac{d_t}{1-\lambda} + r_{t-1} b_{t-1} - \frac{\lambda}{1-\lambda} \phi + Tr_t - \tau_t^R \tag{2}$$

$$\left(1+\tau_t^c\right) c_t^H = \left(1-\tau_t^n\right) n_t^H w_t + Tr_t + \phi \tag{3}$$

$$\beta \frac{u'\left(c_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)} r_t = \frac{u'\left(c_t\right)}{\left(1+\tau_t^c\right)} \tag{4}$$

And leisure-labor condition:

$$n_t^{\varphi} = c_t^{-1} w_t \frac{1-\tau_t^L}{1+\tau_t^C} \tag{5}$$

Final good firms produce in a perfectly competitive market using intermediate goods as inputs. Their decision problem is

$$\max_{y_{j,t}} \left\{ P_t Y_t - \int_0^1 p_{j,t} y_{j,t} dj \right\} \tag{6}$$

subject to a CES production technology

$$Y_t = \left(\int_0^1 y_{j,t}^{1/\mu} dj \right)^{\mu} \tag{7}$$

where $y_{j,t}$ denotes the intermediate good produced by firm j and $p_{j,t}$ is the corresponding price. Y_t is the final consumption good, P_t is the overall price index and μ measures the degree of substitution between the input factors. The aggregate price index is given by:

$$P_t = \left(\int_0^1 p_{j,t}^{1/(1-\mu)} dj \right)^{1-\mu} \tag{8}$$

Solving the maximization problem yields the demand function of the final firm for the intermediate good j :

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{\frac{\mu}{1-\mu}} Y_t \tag{9}$$

These intermediates are produced by monopolistically competitive firms. Their pricing decisions are subject to quadratic adjustment costs Rotemberg, parameterized by ξ, giving rise to nominal rigidity and a conventional New Keynesian Phillips curve. As in Bilbiie 2019 the government levies an optimal subsidy τ^S, financed through lump-sum taxes on all firms, that induces marginal cost pricing in steady state.

$$\begin{aligned} E_t \sum_{k=0}^{\infty} Q_{t,t+k} \Big\{ & \left(1+\tau^s\right) P_{t+k}(i) Y_{t+k}(i) - w_{t+k} n_{t+k}(i) \\ & - \xi \left(\frac{P_{t+k}(i)}{P_{t+k-1}(i)} - 1\right)^2 Y_{t+k}(i) - mc_{t+k}\left(Y_{t+k}(i) - n_{t+k}(i)\right) \Big\} \end{aligned} \tag{10}$$

By adjusting $P_{t+k-1}(i), n_{t+k}(i)$, I can get following Philips Curve

$$(1-\eta) + \eta mc_t - \xi\left(\Pi_t - 1\right)\Pi_t + \beta E_t \xi \left(\frac{c^U_{t+1}}{c^U_t}\right)^{-\sigma} \left(\Pi_{t+1} - 1\right)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0 \tag{11}$$

Aggregate labor supply, consumption, and savings are:

$$\begin{aligned} N_t &= \lambda n^H_t + (1-\lambda) n^R_t \\ C_t &= \lambda c^H_t + (1-\lambda) c^R_t \end{aligned} \tag{12}$$

and

$$B^d_t = (1-\lambda) b_{R,t} \tag{13}$$

respectively. Given these aggregates, labor market clearing requires:

$$L_t = N_t \tag{14}$$

the bond market clears when:

$$\bar{B} = B^d_t \tag{15}$$

and the goods market clears when:

$$Y_t = C_t \tag{16}$$

Dividend payments are given by:

$$D_t = Y_t - w_t L_t \tag{17}$$

Now I consider how to set equivalent fiscal conditions when the economy is tracked into ZLB. Equivalence in the first-order conditions of households is straightforward: any path of consumption, $\{c_t^{R*}\}_{t=0}^{\infty}$, that satisfies the sequence of the Ricardian's Euler equations with $\{r_t^{MP}\}_{t=0}^{\infty}$ and steady state tax rates, also satisfies the sequence of the Ricardian's Euler equations with interest rates in steady state and $\left\{\tau_t^{C,UFP}\right\}_{t=0}^{\infty}$ which satisfies condition (4). In addition, any paths of consumption and labor, $\{c_t^{R*}, n_t^{R*}\}_{t=0}^{\infty}$, (any paths of $\left\{c_{H,t}^{*}, n_{H,t}^{*}\right\}_{t=0}^{\infty}$) that satisfy the sequence of labor-leisure equations of the Ricardian household (the labor-leisure condition of the HtM household) with steady state taxes, satisfy the sequence of labor-leisure conditions (18) of the Ricardian household

$$
\begin{aligned}
(1+\bar{r})\frac{1+\tau_t^{C,UFP}}{1+\tau_{t+1}^{C,UFP}} &= 1+r_t^{MP}, \\
\frac{1-\tau_t^{L,UFP}}{1+\tau_t^{C,UFP}} &= \frac{1-\bar{\tau}^{L}}{1+\tau^{C}},
\end{aligned} \tag{18}
$$

As aforementioned, the $\{c_t^{R*}, n_t^{R*}\}_{t=0}^{\infty}$ sequence will be same between these two types

$$
\begin{aligned}
c_t^{H,MP} &= \frac{(1-\overline{\tau^{n}})}{(1+\overline{\tau^{c}})} n_t w_t + \frac{Tr_t^{MP}}{(1+\overline{\tau^{c}})} + \frac{\phi}{(1+\bar{\tau}^{c})} \\
c_t^{H,UFP} &= \frac{(1-\tau_t^{n})}{(1+\tau_t^{c})} n_t w_t + \frac{Tr_t^{UFP}}{(1+\tau_t^{c})} + \frac{\phi}{(1+\tau_t^{c})}
\end{aligned}
$$

If $c_t^{H,UFP} = c_t^{H,MP}$

$$
Tr_t^{UFP} = \frac{Tr_t^{MP}\left(1+\tau_t^{c}\right)}{(1+\bar{\tau}^{c})} + \frac{\phi\left(1+\tau_t^{c}\right)}{(1+\bar{\tau}^{c})} - \phi \tag{19}
$$

For Richardian household

$$
\begin{aligned}
c_t^{R,UFP} = {} & \frac{(1-\tau_t^{n})}{(1+\tau_t^{c})} n_t w_t + \frac{d_t}{1-\lambda}\frac{1}{(1+\tau_t^{c})} + \frac{1}{(1+\tau_t^{c})} r_{t-1} b_{t-1} - \frac{1}{(1+\tau_t^{c})}\frac{\lambda}{1-\lambda}\phi + \frac{1}{(1+\tau_t^{c})} Tr_t \\
& -\tau_t^{R}\frac{1}{(1+\tau_t^{c})} - b_t \frac{1}{(1+\tau_t^{c})} \\
c_t^{R,UFP} = {} & \frac{(1-\overline{\tau^{n}})}{(1+\bar{\tau}^{c})} n_t w_t + \frac{d_t}{1-\lambda}\frac{1}{(1+\bar{\tau}^{c})} + \frac{1}{(1+\bar{\tau}^{c})} r_{t-1} b_{t-1} - \frac{1}{(1+\bar{\tau}^{c})}\frac{\lambda}{1-\lambda}\phi + \frac{1}{(1+\bar{\tau}^{c})} Tr_t \\
& -\tau_t^{R}\frac{1}{(1+\bar{\tau}^{c})} - b_t \frac{1}{(1+\bar{\tau}^{c})}
\end{aligned}
$$

If $c_t^{R,UFP} = c_t^{R,MP}$

$$
\tau_t^{R,UFP} = \frac{1}{1-\lambda}\frac{1+\tau_t^{C,UFP}}{1+\tau^{C}}\bar{B}\left[\frac{1}{1+r_t} - \frac{1}{1+\bar{r}}\right] \tag{20}
$$

Satisfying the government's budget constraint. I now show, that $\tau_t^{L,UFP}$, $\tau_t^{C,UFP}$, and Tr_t^{UFP} set according to (18) as well as $\tau_t^{R,UFP}$ set according to equation (20) satisfy the government's budget constraint. From the government's budget constraint in the monetary policy experiment, I obtain:

$$Tr_t^{MP} = \bar{B}\left(\frac{1}{1+r_t^{MP}} - 1\right) + T_t^{MP}$$

where $T_t^{MP} = \tau^{\bar{C}} C^* + \tau^L w^* L^*$. Inserting Tr_t^{UFP} set according to equation (18) (and using Tr_t^{MP}) as well as τ_t^R set according to condition (20) into the government budget constraint yields:

$$\begin{aligned} d_t^* \left(\tfrac{1+\tau_t^C}{1+\tau^C} - 1\right) + \left(\tfrac{1+\tau_t^C}{1+\tau^C}\right) \bar{B}\left(\tfrac{1}{1+r_t^{MP}} - 1\right) + \tfrac{1+\tau_t^C}{1+\tau^C} T_t^{MP} \\ + \bar{B}\left(1 - \tfrac{1}{1+\bar{r}}\right)\left(\tfrac{1+\tau_t^C}{1+\tau^C} - 1\right) = \bar{B}\left(\tfrac{1}{1+\bar{r}} - 1\right) + T_t^{UFP} \\ + \left(\tfrac{1+\tau_t^C}{1+\tau^{\bar{C}}} - 1\right) \bar{B}\left(\tfrac{1}{1+r_t^{MP}} - \tfrac{1}{1+\bar{r}}\right). \end{aligned}$$

Collecting terms gives:

$$d_t^* \left(\frac{1+\tau_t^C}{1+\tau^C} - 1\right) + \frac{1+\tau_t^C}{1+\tau^C} T_t^{MP} - T_t^{UFP} = 0$$

Using the goods market clearing condition, $Y_t = C_t$, and the profit equation $d_t = Y_t - w_t L_t$, it now holds that (dropping the superscript UFP for the sake of readability):

$$\begin{aligned} d_t^* \left(\frac{1+\tau_t^C}{1+\tau^C} - 1\right) * \frac{1+\tau_t^C}{1+\tau^C}\left(\tau^C C^* + \tau^L w_t^* L_t^*\right) = \tau_t^C C_t^* + \tau_t^L w_t^* L_t^* \\ \iff d_t^*\left(\tau_t^C - \tau^C\right) + C_t^*\left(\tau^C - \tau_t^C\right) = \left(\tau_t^L + \tau^C \tau_t^L - \tau^L - \tau_t^C \tau^L\right)\left(C_t^* - d_t^*\right) \\ \iff \left(\tau^{\bar{C}} - \tau_t^C - \tau_t^L - \tau^C \tau_t^L + \tau^L + \tau_t^C \tau^L\right) C_t^* = \left(\tau^C - \tau_t^C - \tau_t^L - \tau^{\bar{C}} \tau_t^L + \tau^L + \tau_t^C \tau^L\right) d_t^* \end{aligned}$$

Thus, the government budget constraint is satisfied, if:

$$\begin{aligned} \tau^{\bar{C}} - \tau_t^C - \tau_t^L - \tau^C \tau_t^L + \tau^{\bar{L}} + \tau_t^C \tau^{\bar{L}} = 0 \\ \iff \tau^{\bar{C}} - \tau_t^C - \left(1 + \overline{\tau^C}\right)\tau_t^L + \left(1 + \tau_t^C\right)\overline{\tau^L} = 0 \\ \iff 1 + \tau^C - \left(1 + \tau_t^C\right) + \left(1 + \tau_t^C\right)\tau^{\bar{L}} = \left(1 + \tau^{\bar{C}}\right)\tau_t^L \\ \iff -\left(1 - \tau^L\right)\left(1 + \tau_t^C\right) = \left(\tau_t^C - 1\right)\left(1 + \tau^C\right) \\ \iff \frac{1 - \tau_t^L}{1 + \tau_t^C} = \frac{1 - \bar{\tau}^L}{1 + \tau^C}. \end{aligned}$$

Which holds given that τ_t^C and τ_t^L are set according to condition (4) and (5). Consistency with optimal behavior of firms. Given the same households' behavior $\left\{c_{R,t}^*, n_{R,t}^*, c_{H,t}^*, n_{H,t}^*\right\}_{t=0}^{\infty}$ in both policy cases, the firms also face the same demand for goods and the same supply of labor. Hence, if $\{w_t^*, d_t^*, \pi_t^*, \tilde{p}_t^*/P_t^*\}_{t=0}^{\infty}$, are equilibrium paths in the monetary policy experiment, they are also equilibrium paths in the UFP case.

Market clearing conditions. Given that households and firms behave exactly the same as in the monetary policy experiment and government debt is constant in both cases, X^{UFP} clears all markets if X^{MP} clears all markets.

Thus, I have proven that UFP set according to Proposition 1 leads to the same allocation as the monetary policy experiment which implies that UFP and monetary policy are perfect substitutes in TANK.

3 iMPC of UFP

Matching the empirical pattern of intertemporal marginal propensities to consume (iMPCs) is important for understanding the aggregate effects of macroeconomic policy such as fiscal stimulus measures. In particular, Auclert and Rognlie [2018] demonstrate that in a number of important theoretical benchmark cases - with no capital, fully demand-determined labor, and passive monetary policy - iMPCs are sufficient statistics for the general equilibrium effects of demand shocks such as government purchases. To fix ideas, suppose household behavior can be summarized by an aggregate consumption function $c_t\left(\{y_s - t_s\}\right)$, so that consumption in any period t depends only on the path of post-tax income in every time period s, where y is pre-tax income and t are net taxes. Then the goods market clearing condition $y_t = c_t\left(\{y_s - t_s\}\right) + g_t$ implies a fixed point in the path of output. And the impulse response of output to a change in fiscal policy crucially depends on the iMPC matrix $\mathbf{M}$ of partial derivatives of aggregate consumption with respect to after-tax income x_s at date s, a typical element being $M_{t,c} = \partial c_t / \partial x_s$. Specifically and to first order, total differentiation yields the intertemporal Keynesian cross equation: $dy_t = dg_t + \sum_{s=0}^{\infty} M_{t,s}\left(dy_s - dt_s\right)$. Intuitively, the iMPCs fully characterize the interaction of the household block with the rest of the economy. Consequently, within the benchmark environment of Auclert et al. [2018], matching the iMPC moments produced by heterogeneous-agent models, in particular, is sufficient to replicate their predictions for the aggregate impact of public demand shocks.

The solution offered in this paper is to model precisely such an intermediately-constrained household, thereby generalizing the stark form of ”limited asset market participation” characteristic of the traditional two-agent model. To this end, I substitute hand-to-mouth households for ”workers” who likewise do not own any firm equity but who can participate in financial markets subject to convex bond portfolio adjustment costs (”PACs,” for short).

To develop this argument as transparently as possible, consider the partial equilibrium consumption-savings problem facing a worker household. Given processes for per-period income and the real interest rate, $\left\{x_t^W, r_t\right\}$, where $1 + r_{t-1} = R_{t-1}/\Pi_t$, she maximizes the present discounted value of lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^W\right)$ As in Section

(2), the period utility function is of log form. Unlike a hand-to-mouth household, the worker is able to borrow and save in bonds. The asset value carried into period t is b_{t-1}^W. Different from the unconstrained type of household, however, workers' savings choices are subject to a cost. The household is penalized when their holdings deviate from some benchmark level; the strength of this financial friction is indexed by ψ^W. The adjustment cost takes a simple quadratic form and the target-level is equal to the steady-state value b^W. The per-period budget constraint accordingly reads

$$b_t^W + \frac{\psi^W}{2}\frac{\left(b_t^W - b^W\right)^2}{\chi^W} = \chi_t^W + (1+r_{t-1})\, b_{t-1}^W + Tr_t - (1+\tau_t^c)c_t^W, \quad t = 0,1,2,\ldots$$

$$\chi_t^W = (1-\tau_t^n)n_t^R w_t - \tau_t^R - \lambda/(1-\lambda)\phi + d_t/(1-\lambda)$$

Thus, the cost of increasing bond holdings by one unit is greater than unity because it includes the marginal cost of adjusting the size of the portfolio. To rule out any wealth effects, the costs are rebated to the workers as a lump-sum, Tr_t, without this being taken into account by workers when making savings decisions. Scaling the adjustment cost by steadystate income χ^W ensures comparability across different model specifications.

Solving the household's problem for the optimal choice of a process for consumption and bond holdings $\{c_t^i, b_t^i\}_{t=0}^{\infty}$ yields the Euler equation

$$u'\left(c_t^R\right) = \beta E_t \frac{1+\tau_{t+1}^c}{1+\tau_t^c} u'\left(c_{t+1}^R\right)\frac{(1+r_t)}{1+\frac{\psi^R}{\chi^R}\left(b_t^R - \overline{b^R}\right)} \tag{21}$$

Equation (21) thus features an endogenous, multiplicative wedge $\left(1+\left(\psi^W/x^W\right)\left(b_t^W - b^W\right)\right)^{-1}$ that is not present in the 'standard' Euler equation of an unconstrained household. That latter case is nested for $\psi^W = 0$, whereas if $\psi^W \to \infty$ the worker household behaves in hand-to-mouth fashion.

It proves instructive to consider a log-linear approximation of the household's optimal consumption-savings behavior. As before, the Taylor approximation is done around a steady-state with zero inflation, no debt, and total income normalized

$$\chi_t^R = (1-\tau_t^n)\, n_t^R w_t - \tau_t^R - \frac{\lambda}{1-\lambda}\phi + \frac{d_t}{1-\lambda} \tag{22}$$

$$b_t^R + \frac{\psi^R}{2}\frac{\left(b_t^R - \overline{b^R}\right)^2}{\overline{\chi^R}} = \chi_t^R + Tr_t + (1+r_{t-1}) \tag{23}$$

$$b_{t-1}^R - (1+\tau_t^c)\, c_t^R u'\left(c_t^R\right) = \beta E_t \frac{1+\tau_{t+1}^c}{1+\tau_t^c} u'\left(c_{t+1}^R\right)\frac{(1+r_t)}{1+\frac{\psi^R}{\chi^R}\left(b_t^R - \overline{b^R}\right)} \tag{24}$$

It proves instructive to consider a log-linear approximation of the household's optimal consumption-savings behavior. As before, the Taylor approximation is done around a steady-state with zero inflation, no debt, and total income normalized to unity. The Euler equation and budget constraint (upon canceling out adjustment costs and rebate) can then be written as

$$\widehat{c_t^R} = E_t\left[\widehat{c_t^R} - \widehat{r_t}\right] + \psi^W \widetilde{b_t^R} - \tau_{t+1}^{\widehat{c}} + \widehat{\tau_t^c} \tag{25}$$

$$\widetilde{b_t^R} = \widehat{\chi_t^R} + \frac{b_{t-1}^R}{\beta} - \widehat{c_t^R} - \widehat{\tau_t^c} \tag{26}$$

$$\widetilde{b_t} = \mu_1 \widetilde{b_{t-1}^W} + \sum_{l=0}^{\infty} \mu_2^{-(l+1)} E_t\left[\left(\chi_{t+l}^{\widehat{R}} - \chi_{t+l+1}^R\right) + \widehat{r_{t+l}}\right]$$

And then I can get IRF equation according to (25) and (26)

$$\widehat{c_t^R} = \widehat{\chi_t^R} + \frac{\widetilde{b_{t-1}^R}}{\beta} - \widehat{\tau_t^c} - \mu_1 \widetilde{b_{t-1}^W} - \sum_{l=0}^{\infty} \mu_2^{-(l+1)} E_t\left[\left(\chi_{t+l}^{\widehat{R}} - \chi_{t+l+1}^R\right) + \widehat{r_{t+l}}\right]$$

It's a similar specification compared to Auclert et al. [2018].

4 HANK Model

This part I introduce a HANK model to prove that if I set UFP based on rules given in Section 2 I will also get the similar impulse response in the case UFP and MP.

4.1 Demographic

Time is discrete and runs from $t = 0$ to ∞. The economy is populated by a unit mass of agents, or households, who face no aggregate uncertainty, but may face idiosyncratic uncertainty. Agents vary in their idiosyncratic ability state e, which follows a Markov process with fixed transition matrix Π. I assume that the mass of agents in idiosyncratic state e is always equal to $\pi(e)$, the probability of e in the stationary distribution of Π. The average ability level is normalized to be one, so that $\sum_e \pi(e)e = 1$. If agents are permanently different, Π is the identity matrix and π the initial distribution over e.

4.2 Earnings

The family faces certain budget constraints. Every household's income comes from wages, capital returns. Wage income is regarding with ability e_t, wage w_t, and government proportion tax τ_t. Liquid asset a_t can get interest rate r_t^a. In addition, each term can borrow capital ≥ 0, which is a constraint for households.

As a result, the budget constraint can be represented by:

$$(1+\tau_t^c)c_t + a_t + \frac{b_t}{(1+r_t)^{R-1}} = (1-\tau_t^n)\, w_t n_t e_t + (1+r_t^a)\, a_{t-1} + \frac{(1+r_t^b)\, b_{t-1}}{(1+r_t)^R} + Tr_t - \Psi(b_t, a_t) \tag{27}$$

As Kaplan, Moll, and Violante 2018, the adjustment cost function is specified as

$$\Psi(b_t,) = \chi_0 |b_t| + \chi_1 \left| \frac{b_t}{\max\{a_t, \underline{a_t}\}} \right|^{\chi_2} \max\{a_t, \underline{a}\}, \quad \chi_2 > 1, \underline{a} > 0 \tag{28}$$

In order to describe the illiquid cost between the transferring between liquid assets and Assets I set the function $\Psi(b_t,)$. The transaction cost has two components that play different roles: the linear component generates an inaction region in households' optimal deposit policies due to some households the marginal gain from depositing or withdrawing the first dollar is smaller than the marginal cost of transacting χ_0. The convex component ensures that deposit rates are finite, $|dt| < \infty$ and hence household's holdings of assets never jump. Finally, scaling the convex term by illiquid assets a above some threshold a delivers the desirable property that marginal costs $\Psi(b_t,)$ are homogeneous of degree zero in the deposit rate d/a so that the marginal cost of transacting depends on the fraction of illiquid assets transacted, rather than the raw size of the transaction. The threshold > 0 guarantees that costs remain finite for individuals with $= 0$.

4.3 Preferences

In period t, agent i enjoys the consumption of a generic consumption good c_{it} and gets disutility from working n_{it} hours, leading to a time- 0 utility of

$$\mathbb{E}\left[\sum_{t\geq 0} \beta^t \left\{u\left(c_{it}\right) - v\left(n_{it}\right)\right\}\right]$$

The utility function is separable felicity function; agents enjoy the consumption of a generic consumption good c_t and get disutility from working hours n_t.

$$u\left(c_t, n_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{n_t^{1+v}}{1+v} \tag{29}$$

Therefore, the dynamic programming problem for agents is

$$V_t\left(e_t, b_{t-1}, a_{t-1}\right) = \max_{c_t, b_t, a_t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{n_t^{1+v}}{1+v} + \beta E_t V_{t+1}\left(e_{t+1}, b_t, a_t\right) \right\} \tag{30}$$

This specification has three state variables e_t, b_{t-1}, a_{t-1}, where e_t is individual productivity following a Markov process as in the other models. All in all, the household block takes as inputs the sequences of interest rates $\{r_s^a, r_s^b\}$, wage per efficiency units $\{w_s\}$, labor tax rate $\{\tau_s\}$ and labor demand $\{N_s\}$ as inputs. The relevant outputs are illiquid asset demand, liquid asset demand, productivity-weighted marginal utility, consumption, and portfolio adjustment costs. Similar to İmrohoroğlu and Zhao 2020, the state transition follows a Markov chain, which represents the transition between 3 states, namely health and disease.

$$\Pi(\eta, \eta^{'}) = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \tag{31}$$

p_{11} denotes the transition of state from health to health p_{12} denotes from health to sick, and so on. In real life, this shows that the individual will decline in work efficiency due to certain factors, such as diseases.

As discrete variable, e_t will not improve the complexity of this problem. The only thing I should focus is the continuous variables a_t and b_t. I will explain the algorithm to solve this double state variable dynamic programming problem.

4.4 Production

In order to describe the sticky prices, I now assume a standard two-tier production structure. Intermediate goods are produced by a mass one of identical monopolistically competitive firms, whose shares are traded and owned by households. All firms have the same production technology, now assumed to be Cobb-Douglas in labor and capital.

$$F(k_{t-1}, n_t) = k_{t-1}^{\alpha} n_t^{1-\alpha} \tag{32}$$

Final goods firms aggregate intermediate goods with a constant elasticity of substitution $\mu/(\mu-1) > 1$. Capital is subject to quadratic capital adjustment costs, so that the costs arising from choosing capital stocks k_{t-1} and k_t in any period t are given by $\phi(x)$,

$$\phi(x) = x - (1-\delta) + \frac{1}{2\delta\epsilon_I}(x-1)^2 \tag{33}$$

where $\delta > 0$ denotes depreciation and $\epsilon_I > 0$ is the sensitivity of net investment to Tobin's Q. Finally, any firm chooses a price p_t in period t subject to Rotemberg [1982] adjustment costs $\xi(p_t, p_{t-1})$ where $\kappa_p > 0.24$.

$$\xi(p_t, p_{t-1}) = \frac{1}{2\kappa^p}\left(\frac{p_t - p_{t-1}}{p_{t-1}}\right)^2 \tag{34}$$

Intermediate goods producer maximize the value by adjust the sequence p_t, k_t, n_t. Subject to the final goods market requirement:

$$F(k_{t-1}, n_t) = Y_t \left(\frac{p_t}{P_t}\right)^{\frac{-\mu}{\mu-1}} \tag{35}$$

Since all these firms are identical, then I have

$$k_t = K_t \quad n_t = N_t \quad p_t = P_t \tag{36}$$

in equilibrium.

4.5 Labor Market

I assume that each household provides a continuous differentiated labor service, each represented by a union. Unions set hours and wages to maximize the average utility of their members and make their consumption-savings including other identical union decisions. Changing the nominal wage incurs quadratic adjustment costs. The programming problem of union k is

Pretax labor income is subject to a log-linear retention function as in Heathcote et al. [2017]. This retention function is indexed to real wages, so that if P_t is the nominal price of consumption goods, W_t is the nominal wage per unit of ability, and e_{it} is the agent's current ability, real after-tax income is

$$z_{it} \equiv \tau_t \left(\frac{W_t}{P_t} e_{it} n_{it}\right)^{1-\lambda}$$

$$\begin{aligned} U_t\left(w_{kt-1}\right) = \max_{n_{kt}, w_{kt}} \int \left[u\left(c_{it}\right) - v\left(n_{kt}\right)\right] dD_t - \frac{\mu_w}{1-\mu_w} \frac{1}{2\kappa_w} \left[\log\left(1+\pi_{kt}^w\right)\right]^2 N_t + \beta U_{t+1}\left(w_{kt}\right) \\ \text{s.t. } n_{kt} = \left(\frac{w_{kt}}{w_t}\right)^{-\frac{\mu w}{\mu w-1}} N_t \end{aligned} \tag{37}$$

where π_{kt}^w is wage inflation

$$\pi_{kt}^w = \left(1+\pi_t\right) \frac{w_{kt}}{w_{kt-1}} - 1 \tag{38}$$

Same as the setup of that in Auclert et al. [2018] and, as shown there, leads to a wage Phillips curve of the form

$$\log\left(1+\pi_t^w\right) = \kappa_w \left[\varphi N_t^{1+v} - \mu_w \left(1-\tau_t\right) w_t N_t \mathcal{U}_t\right] + \beta \log\left(1+\pi_{t+1}^w\right) \tag{39}$$

For convenience later on, let's introduce the following shorthand for wage adjustment costs

$$\psi_t^w = \frac{\mu_w}{1-\mu_w} \frac{1}{2\kappa_w} \left[\log\left(1+\pi_t^w\right)\right]^2 N_t \tag{40}$$

4.6 Monetary Policy

The monetary policy is given by Taylor's rule

$$i_t = r^* t + \phi_m \pi_t + \phi_y (Y_t - Y_{ss}) \tag{41}$$

where the coefficient ϕ_m and ϕ_y are exogenous and certain and r^* is the steady state interest rate, Y_{ss} is the steady state output and $Y_t - Y_{ss}$ represents the deviation of the output relative to the normal of that. It also meet the Fisher's equation

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t} \tag{42}$$

4.7 Government's Budget

The only market clearing conditions not implied implicitly by notation are that of goods and assets. Differentiating from Auclert et al. [2018], I introduce government transferring target to balance the budget of government.

$$\begin{aligned} Y_t &= \mathcal{C}_t + G_t + I_t + \mathcal{P}_t + \omega \mathcal{B}_t + \psi_t^p + \psi_t^w + \mathcal{TR}_t \\ \mathcal{A}_t + \mathcal{B}_t &= p_t + B^g \end{aligned} \tag{43}$$

5 Stationary Equilibrium

When the economy tends to be stable, the individual's behavior is consistent with the overall economic behavior: the production sector maximizes profits, the individual makes optimal planning for consumption and labor, and the commodity market clears. For heterogeneous agent model, I usually adopt the definition of density function to describe the dynamic transfer of economy through the transfer of density function. Therefore, I set D_t as a cross-sectional measure of the time t of the economy, which describes the distribution relationship between the number of economic individuals and asset holdings.

Figure 1 illustrates the DAG for my third example: a two-asset HANK model with household side similar to Kaplan and Violante [2018]. For households, the model features liquid and illiquid assets with convex adjustment costs of portfolio adjustment. On the supply side, it features wage as well as price rigidities, as well as capital with quadratic adjustment costs. Hence investment follows the standard q theory equations. [25]

Monetary policy follows a standard interest rate rule. The government levies a distortionary labor income tax to finance its debt and its expenditure on the final good. I assume a balanced budget. Some government bonds are held by households directly (along with firm equity) as illiquid assets, and the rest are transformed into liquid assets

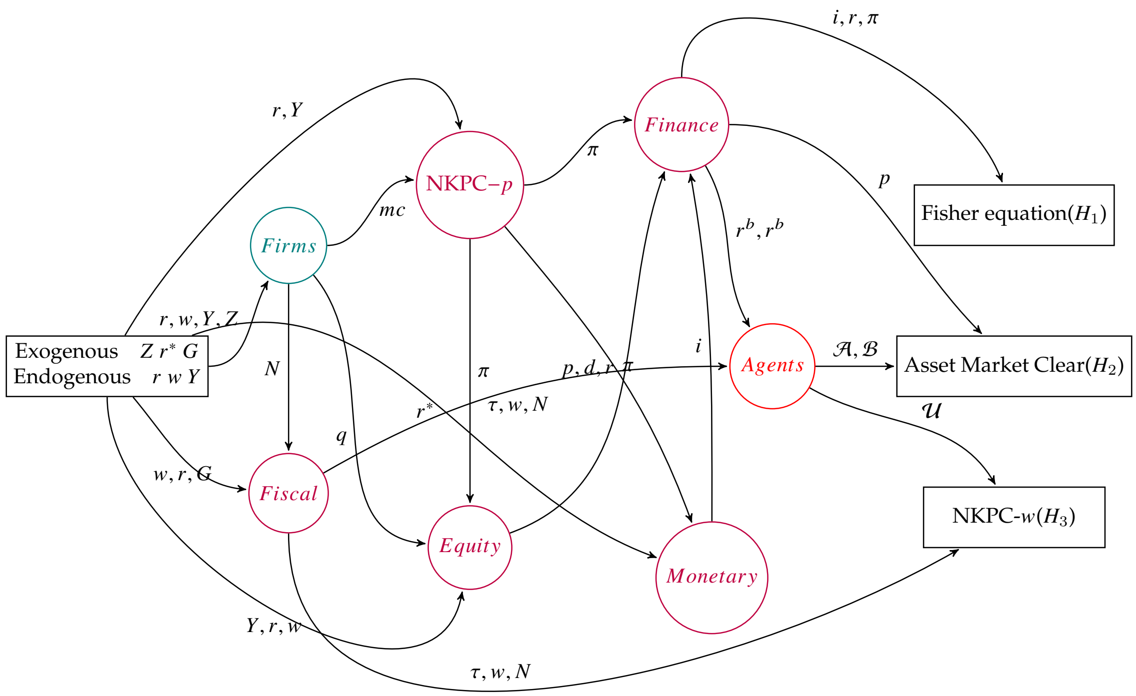

Figure 1: DAG representation of HANK economy

by a competitive financial intermediary. This liquidity transformation incurs a proportional cost, which determines the equilibrium spread between liquid and illiquid assets in all periods along perfect-foresight paths.

Competitive equilibrium includes individual decision variables $\{\tilde{c}_t^s, \tilde{k}_t^s, \tilde{l}_t^s, \tilde{d}_t^s\}$, firms' plans for production $\{K, N\}$, factor prices w_t, r_t.

1. Individual efficiency impact satisfies the transition rule of Markov chain $\Pi(e, e^{'})$.

2. In a stable state, agents meet budget constraints.

3. In this case, individual dynamic programming lifelong asset changes, consumption, and labor supply, this behavior is based on state variables e. Finally, when the economy reaches a steady state, the optimal strategy does not depend on time, but only on the current state. I can get the optimal policy function $a'(e, b, a)$, $c(e, b, a)$, $n(e, b, a)$.

4. The firm's optimal factor price sequence In the case of efficient market, the company's profit is zero.

5. According to the density function $D_t(e, b, a)$, I can find the variables of the whole economy, which is the sum of the individual economy.

$$\mathcal{A}_t\left(\left\{r_s^a, r_s^b, w_s, \tau_s, N_s\right\}\right) = \int a dD_t(e, b, a) \tag{44}$$

$$\mathcal{B}_t\left(\left\{r_s^a, r_s^b, w_s, \tau_s, N_s\right\}\right) = \int b_t(e, b, a) dD_t(e, b, a) \tag{45}$$

$$\mathcal{U}_t\left(\left\{r_s^a, r_s^b, w_s, \tau_s, N_s\right\}\right) = \int e \cdot u(c_t(e, b, a), n_t(e, b, a))^{-\sigma} dD_t(e, b, a) \tag{46}$$

$$\mathcal{C}_t\left(\left\{r_s^a, r_s^b, w_s, \tau_s, N_s\right\}\right) = \int c_t(e, b, a) dD_t(e, b, a) \tag{47}$$

$$\mathcal{TR}_t\left(\left\{r_s^a, r_s^b, w_s, \tau_s, N_s\right\}\right) = \int tr_t(e, b, a) dD_t(e, b, a) \tag{48}$$

$$\mathcal{P}_t\left(\left\{r_s^a, r_s^b, w_s, \tau_s, N_s\right\}\right) = \int \Phi\left(a_t(e, b, a), a\right) dD_t(e, b, a) \tag{49}$$

The last two are required only for checking the omitted goods market clearing condition

6. The government budget is balanced. Transfer payment tr_t is used to balance the budget.

$$\begin{aligned} Y_t &= \mathcal{C}_t + G_t + I_t + \mathcal{P}_t + \omega \mathcal{B}_t + \psi_t^p + \psi_t^w + \mathcal{TR}_t \\ \mathcal{A}_t + \mathcal{B}_t &= p_t + B^g \end{aligned} \tag{50}$$

6 SHADE Model and Sequence-Space Jacobian Algorithm

Auclert et al. [2019] promoted an algorithm to solve these problems. The issue is so-called Sequence space Heterogeneous-Agent Dynamic Equilibrium(SHADE). These models include most heterogeneous agent neoclassical and New Keynesian models.

I can view this procedure as a mapping from exogenous shocks and unknowns $\left\{\mathbf{X}^i\right\}_{i \in \mathcal{Z} \cup \mathcal{U}}$ to targets $\{\mathbf{X}^o\}_{o \in \mathcal{H}}$. I write this mapping in more condensed form as $\mathbf{H}(\mathbf{U}, \mathbf{Z})$, where $\mathbf{U}$ is defined as the stacked vector of unknown sequences $\left\{\mathbf{X}^i\right\}_{i \in \mathcal{U}}$, $\mathbf{Z}$ is defined as the stacked vector of exogenous sequences $\left\{\mathbf{X}^i\right\}_{i \in \mathcal{Z}}$, and $\mathbf{H}(\mathbf{U}, \mathbf{Z})$ itself is the implied stacked vector of targets $\left\{\mathbf{X}^i\right\}_{i \in \mathcal{H}}$ since the procedure satisfies $\mathbf{X}^o = h^o\left(\left\{\mathbf{X}^i\right\}_{i \in \mathcal{I}_b}\right)$ by construction, equilibrium is then equivalent to

$$\mathbf{H}(\mathbf{U}, \mathbf{Z}) = 0 \tag{51}$$

As Auclert et al. [2019], the DAG makes it easy to visualize some of the dependencies embedded in the model: for instance, the dividends from firms are distributed to households (according to a certain rule), so the output d of the firm block is an input to the

HA block. Similarly, the real interest rate r affects the taxes required for the government to achieve its balanced-budget target, so r is an input to the fiscal block, which has an output τ that is an input to the HA block. Monetary policy follows a standard interest rate rule. The government levies a distortionary labor income tax to finance its debt and its expenditure on the final good. I assume a balanced budget. Some government bonds are held by households directly(along with firm equity)as illiquid assets, and the rest are transformed into liquid assets by a competitive financial intermediary. This liquidity transformation incurs aproportional cost, which determines the equilibrium spread between liquid and illiquid assets in all periods along perfect-foresight paths.

7 Calibration

In this paper, in order to adapt to the development of China's economy, my calibration strategy is to select commonly used parameters in the literature.

7.1 Labor Wage

First, I calibrate the efficiency data. The first aspect is the random impact of income, which is given by $log(\mu_j) = \theta log(\mu_{j-1}) + v_j$. Based on the research of He et al. [2019], I take $\theta = 0.86$ and variance $\sigma_v^2 = 0.06$. Then I use the technology of Imrohoroglu et al. [1998], now I discretize the process into Markov chain

$$\Pi(e, e^{'}) = \begin{bmatrix} 0.9259 & 0.0741 & 0 \\ 0.235 & 0.953 & 0.0235 \\ 0 & 0.0741 & 0.9259 \end{bmatrix} \tag{52}$$

and the value $\mu = \{0.36, 1.0, 2.7\}$.

Also, I calibrate the age-specific labor efficiencies e_t^s. Based on He et al. [2019], they use the data in CHNS. They used CHNS to obtain data on the average working hours per worker per week, which were calculated on the basis of two questions. " C5: For how many days in a week, on the average, did you work ?" and "C6: For how many hours in a day, on the average, did you work?" I choose to use the method of Ludwig(2012): regress the data on the third-order polynomials, which are given in the following specifications.

$$\log e_s = \eta_0 + \eta_1 j + \eta_2 j^2 + \eta_3 j^3 + \epsilon_s \tag{53}$$

Here, e_j is age-specific productivity and ϵ_j is residual. And the coefficient vector represented by $\hat{\eta} = [\hat{\eta_1}, \hat{\eta_2}, \hat{\eta_3}]'$ determines the polynomial slope estimated according to the actual working time data.

Table 1: Calibration

Parameter		Value	Target
β	Discount factor	0.971	$r = 0.03$
σ	Inverse IES	2	
χ_0	Portfolio adj. cost pivot	0.25	
χ_1	Portfolio adj. cost scale	6.41	
χ_2	Portfolio adj. cost curvature	2	
$\underline{b}$	Borrowing constraint	0	
ρ_e	Autocorrelation of earnings	0.966	
σ_e	Cross-sectional std of log earnings	0.92	
φ	Disutility of labor	2.073	$N = 1$
ν	Inverse Frisch elasticity	1	
μ_w	Steady state wage markup	1.1	
κ_w	Slope of wage Phillips curve	0.1	
Z	TFP	0.468	$Y = 1$
α	Capital share	0.33	$K = 10Y$
μ_p	Steady-state markup	1.015	$\mathcal{A} + \mathcal{B} = 16Y$
δ	Depreciation	0.02	
κ_p	Slope of price Phillips curve	0.1	
ω	liquidity premium	0.005	
τ	Labor tax	0.356	
G	Government spending	0.2	
B^g	Bond supply	2.8	
ϕ	Taylor rule coefficient	1.5	
ϕ_y	Taylor rule coefficient on output	0	
n_e	Points in Markov chain for \$e\$	3	
n_b	Points on liquid asset grid	50	
n_a	Points on illiquid asset grid	70	

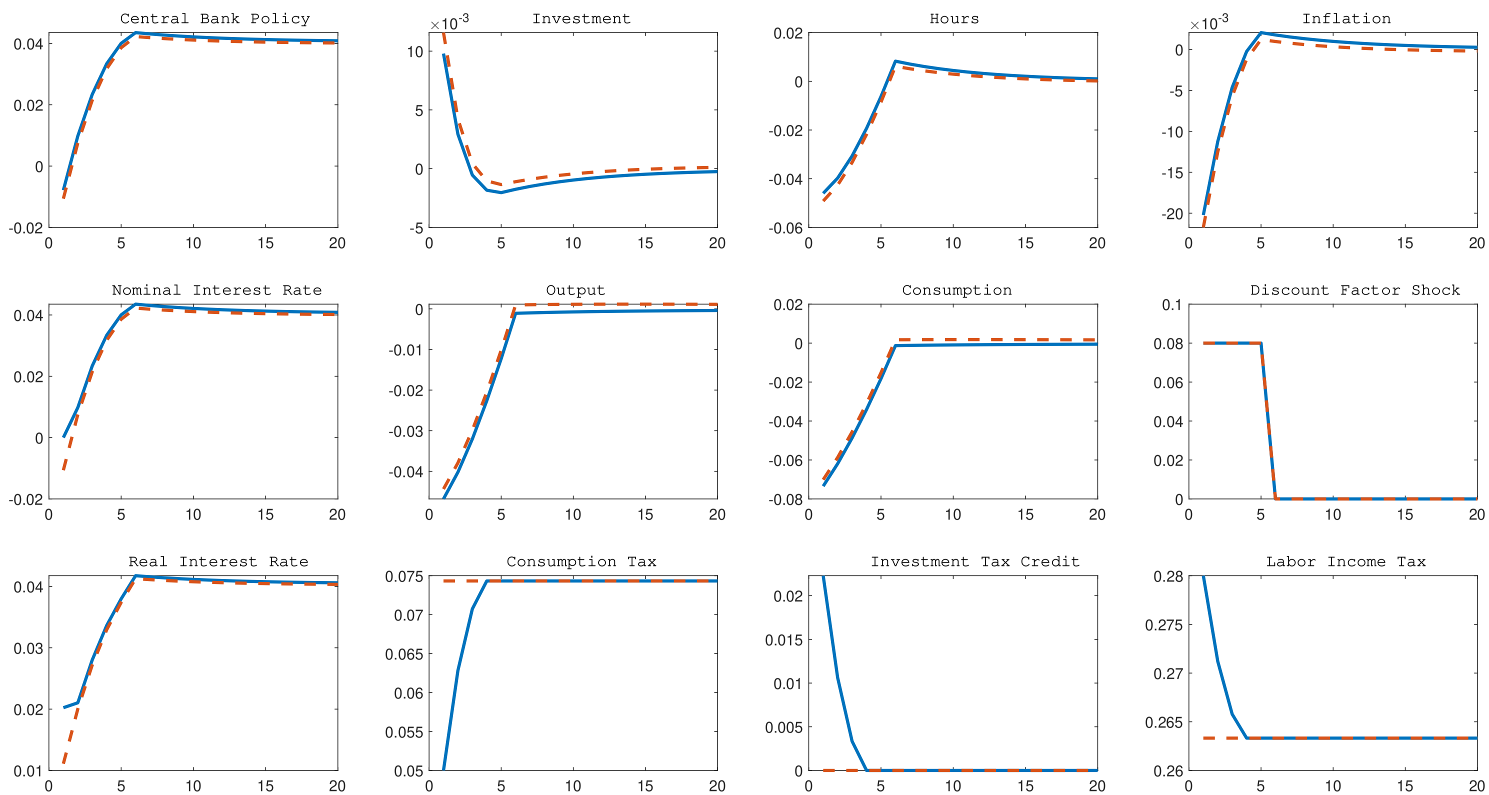

Figure 2: Equivalence between UFP and MP

References

Adrien Auclert and Matthew Rognlie. Inequality and aggregate demand. Working Paper 24280, National Bureau of Economic Research, February 2018.

Adrien Auclert, Matthew Rognlie, and Ludwig Straub. The intertemporal keynesian cross. Working Paper 25020, National Bureau of Economic Research, 2018.

Adrien Auclert, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. Working Paper 26123, National Bureau of Economic Research, 2019.

Anmol Bhandari, Serdar Birinci, Ellen McGrattan, and Kurt See. What do survey data tell us about us businesses? Working Paper 26063, National Bureau of Economic Research, July 2019. URL http://www.nber.org/papers/w26063.

Florin Bilbiie. Monetary Policy and Heterogeneity: An Analytical Framework. 2019 Meeting Papers 178, Society for Economic Dynamics, 2019. URL https://ideas.repec.org/p/red/sed019/178.html.

Timo Boppart, Per Krusell, and Kurt Mitman. Exploiting mit shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control*, 89:68 – 92, 2018. ISSN 0165-1889. Fed St. Louis-JEDC-SCG-SNB-UniBern Conference, titled: "Fiscal and Monetary Policies".

Christopher D. Carroll. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320, 2006.

Isabel Correia, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles. Unconventional fiscal policy at the zero bound. *American Economic Review*, 103(4):1172–1211, June 2013. doi: 10.1257/aer.103.4.1172. URL https://www.aeaweb.org/articles?id=10.1257/aer.103.4.1172.

Sergio de Ferra, Kurt Mitman, and Federica Romei. Household heterogeneity and the transmission of foreign shocks. *Journal of International Economics*, 124:103303, 2020. ISSN 0022-1996. doi: https://doi.org/10.1016/j.jinteco.2020.103303. URL https://www.sciencedirect.com/science/article/pii/S0022199620300222. NBER International Seminar on Macroeconomics 2019.

Emmanuel Farhi and Iván Werning. A theory of macroprudential policies in the presence of nominal rigidities. *Econometrica*, 84(5):1645–1704, 2016. doi: https://doi.org/10.3982/ECTA11883. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA11883.

Martin Feldstein. The role for discretionary fiscal policy in a low interest rate environment. Working Paper 9203, National Bureau of Economic Research, September 2002. URL http://www.nber.org/papers/w9203.

Gourinchas and Pierre-Olivier. Flattening the pandemic and recession curves. Working Paper 26918, National Bureau of Economic Research, 2020.

Veronica Guerrieri, Guido Lorenzoni, Ludwig Straub, and Iván Werning. Macroeconomic implications of covid-19: Can negative supply shocks cause demand shortages? Working Paper 26918, National Bureau of Economic Research, April 2020.

Marcus Hagedorn, Jinfeng Luo, Iourii Manovskii, and Kurt Mitman. Forward guidance. *Journal of Monetary Economics*, 102(C):1–23, 2019. doi: 10.1016/j.jmoneco.2019.01. URL https://ideas.repec.org/a/eee/moneco/v102y2019icp1-23.html.

Robert E. Hall. The high sensitivity of economic activity to financial frictions*. *The Economic Journal*, 121(552):351–378, 2011. doi: https://doi.org/10.1111/j.1468-0297.2011.02421.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0297.2011.02421.x.

Hui He, Lei Ning, and Dongming Zhu. The impact of rapid aging and pension reform on savings and the labor supply. *IMF Working Papers*, 19:1, 01 2019.

Jonathan Heathcote, Kjetil Storesletten, and Giovanni L. Violante. Optimal Tax Progressivity: An Analytical Framework*. *The Quarterly Journal of Economics*, 132(4):1693–1754, 06 2017. ISSN 0033-5533.

Ayse Imrohoroglu, Selahattin Imrohoroglu, and Douglas H Joines. The Effect of Tax-Favored Retirement Accounts on Capital Accumulation. *American Economic Review*, 88 (4):749–768, September 1998.

Greg Kaplan and Giovanni L. Violante. Microeconomic heterogeneity and macroeconomic shocks. *Journal of Economic Perspectives*, 32(3):167–94, 2018.

Greg Kaplan, Benjamin Moll, and Giovanni L. Violante. Monetary policy according to hank. *American Economic Review*, 108(3):697–743, 2018.

Finn E. Kydland and Edward C. Prescott. Time to build and aggregate fluctuations. *Econometrica*, 50(6):1345–1370, 1982. ISSN 00129682, 14680262.

Alisdair McKay, Emi Nakamura, and Jón Steinsson. The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58, October 2016. doi: 10.1257/aer.20150063. URL https://www.aeaweb.org/articles?id=10.1257/aer.20150063.

Julio J. Rotemberg. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4):517–531, 1982.

Ayşe İmrohoroğlu and Kai Zhao. Household saving, financial constraints, and the current account in china. *International Economic Review*, 61(1):71–103, 2020.

A Optimal Problem and FOC

A.1 Household

The optimization problem for household is

$$\mathcal{L} = \max_{c_t,b_t,a_t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{n_t^{1+v}}{1+v} + \beta E_t V_{t+1}\left(e_{t+1}, b_t, a_t\right) \right\}$$
$$+\lambda_t \left[-c_t - a_t - \frac{b_t}{(1+r_t)^R} + (1-\tau_t)\, w_t n_t e_t + \left(1+r_t^a\right) a_{t-1} + \frac{(1+r_t^b) b_{t-1}}{(1+r_t)^R} + tr_t - \Phi\left(a_t, a_{t-1}\right) \right] \tag{54}$$

And the FOC for control variable c_t, n_t is:

$$\frac{\partial \mathcal{L}}{\partial c_t} = c_t^{-\sigma} - \lambda_t = 0 \tag{55}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = \varphi n_t^{v} - \lambda_t \left(1-\tau_t\right) w_t e_t = 0 \tag{56}$$

Combine these two specifications

$$\varphi n_t^{v} = c_t^{-\sigma}\left(1-\tau_t\right) w_t e_t \tag{57}$$

$$n_t = \left(\frac{c_t^{-\sigma}\left(1-\tau_t\right) w_t e_t}{\varphi} \right)^{\frac{1}{v}} \tag{58}$$

I can find the key structure for these problems: I can always transform the muti-variables' dynamic programming problem into univariate of that. In my problem, the state variables are $\{a_t, b_t, e_t\}$ and control variables $\{c_t, n_t\}$. Using Lagrange method I can convert it into one control variable $\{c_t\}$. Therefore, I need to solve the dynamic programming problem by giving the discrete space of state variable and then I can get the policy function on this discrete space.

For dynamic programming problems, finite-term dynamic programming and infinite-term dynamic programming are different. In contrast, infinite dynamic programming can be regarded as a homogeneous problem, because there is no difference between different time and infinite time. However, for a finite terms problem, this is not the same, different time will start to make the problem is not homogeneous. So I can only start planning from the last phase.

When calculating multivariate optimization problems, I can express all decision variables in terms of state variables. If I use budget constraints instead, I determine the interval and search for the optimal value in a certain range.

The algorithm is a variant of the endogenous grid point method of Carroll [2006] that I developed for this two-asset problem. The key trick is, whenever the household is partially constrained, to include Lagrange multipliers in the backward iteration. I also exploit the fact that, endogenously, the constraint on the illiquid asset will never be binding unless the constraint on the liquid asset is also binding(otherwise, a simple variation will improve utility)—and if both are binding, then the policy is trivial. Overall, I start from a guess for the (discretized) partials of the value function and iterate backward until convergence. Throughout, I will use (z', b', a') to refer to tomorrow's grid and (z, b, a) today's grid.

1. **Initial guess** Guess $V_a(z', b', a')$ and $V_b(z', b', a')$

2. **Common** $z' \to z$. By definition

$$W_b(z, b', a') = \beta \Pi V_b(z', b', a')$$
$$W_a(z, b', a') = \beta \Pi V_a(z', b', a')$$

3. **Unconstrained** $a' \to a$. Assuming that no constraints bind, $\lambda_t = \mu_t = 0$, and the FOCs become

$$u'(c) = W_b\left(z, b', a'\right)$$
$$u'(c)\left[1 + \Phi_1\left(a', a\right)\right] = W_a\left(z, b', a'\right)$$

Combine these to get

$$0 = F\left(z, b', a, a'\right) \equiv \frac{W_a(z, b', a')}{W_b(z, b', a')} - 1 - \Phi_1\left(a', a\right)$$

which characterizes $a'(z, b', a)$. Use this to map $W_b(z, b', a')$ into $W_b(z, b', a)$ by interpolation, then compute consumption as

$$c\left(z, b', a\right) = W_b\left(z, b', a\right)^{-\frac{1}{\sigma}}$$

4. **Unconstrained** $\underline{b'} \to b$. Now using $a'(z, b', a)$ and $c(z, b', a)$ from the previous step, use the budget constraint to obtain

$$b\left(z, b', a\right) = \frac{c(z, b', a) + a'(z, b', a) + \frac{b'}{(1+r_t)^{R-1}} - (1 + r^a)\, a + \Psi\left(a'(z, b', a), a\right) - (1 - \tau_t)\, w_t n_t e_t + \cdots}{(1 + r^b)^R}$$

I invert this function via interpolation to get $b'(z, b, a)$. The same interpolation weights can be used to do $a'(z, b', a) \to a'(z, b, a)$

5. **Liquidity constrained** $a' \to a$. This branch is analogous to the unconstrained case. Assuming that the liquidity constraint is binding, $\lambda_t > 0$, and (89) and (90) become

$$u'(c) = \lambda + W_b\left(z, 0, a'\right)$$
$$u'(c)\left[1 + \Phi_1\left(a', a\right)\right] = W_a\left(z, 0, a'\right)$$

To help with scaling, let us define $\kappa \equiv \lambda / W_b(z, 0, a')$ and rewrite the first equation as

$$u'(c) = (1+\kappa) W_b(z, 0, a')$$

Divide and rearrange to get

$$0 = F(z, \kappa, a, a') \equiv \frac{1}{1+\kappa} \frac{W_a(z, 0, a')}{W_b(z, 0, a')} - 1 - \Phi_1(a', a)$$

I solve this for $a'(z, \kappa, a)$, and compute consumption as

$$c(z, \kappa, a) = [(1+\kappa) W_b(z, \kappa, a)]^{-\frac{1}{\sigma}}$$

6. **Liquidity constrained** $\underline{x} \to \underline{b}$. Now using $a'(z, x, a)$ and $c(z, \kappa, a)$ from the previous step, use the budget constraint to obtain

$$b(z, \kappa, a) = \frac{c(z, \kappa, a) + a'(z, \kappa, a) + \underline{b} - (1+r^a) a + \Phi(a'(z, \kappa, a), a) - z}{1 + r^b}$$

I invert this function via interpolation to get $\kappa(z, b, a)$. The same interpolation weights can be used to map $a'(z, \kappa, a)$ into $a'(z, b, a)$. I already know that $b'(z, b, a) = \underline{b}$

7. **Update guesses** The final $b'(z, b, a)$ is the element-wise maximum of its unconstrained and liquidity-constrained counterparts. Replace the unconstrained $a'(z, b, a)$ with constrained one at the exact same points. Compute consumption from the budget constraint as

$$c(z, b, a) = z + (1+r^a) a + \left(1 + r^b\right) b - \Phi(a'(z, b, a), a) - a'(z, b, a) - b'(z, b, a)$$

Finally use the envelope conditions (91) and (92) to update the guesses

$$\begin{aligned} V_b(z, b, a) &= \left(1 + r^b\right) c(z, b, a)^{-\sigma} \\ V_a(z, b, a) &= [1 + r^a - \Phi_2(a'(z, b, a), a)] \, c(z, b, a)^{-\sigma} \end{aligned}$$

Go back to step 2, repeat until convergence.

A.2 Firms

The firm's optimization problem is

$$\begin{aligned} J_t(k_{t-1}) = \max_{p_t, k_t, n_t} \Big\{ & \tfrac{p_t}{P_t} F(k_{t-1}, n_t) - \tfrac{W_t}{P_t} n_t - \xi \left(1 - \delta + \tfrac{\iota_t}{k_{t-1}}\right) k_{t-1} - \xi(p_t, p_{t-1}) Y_t \\ & + \tfrac{1}{1+r_t} J_{t+1}(k_t) \Big\} \end{aligned} \tag{59}$$

subject to

$$\frac{p_t}{P_t} Y_t = \left(\frac{F\left(K_{t-1}, L_t\right)}{Y_t} \right)^{\frac{1}{\mu}-1} Y_t \tag{60}$$

$$k_t = (1-\delta) k_{t-1} + \iota_t \tag{61}$$

Lagrange equation for this problem:

$$\begin{aligned}\mathcal{L} = \max_{p_t, k_t, n_t} \Big\{ & \tfrac{p_t}{P_t} F\left(k_{t-1}, n_t\right) - \tfrac{W_t}{P_t} n_t - \phi \left(1 - \delta + \tfrac{\iota_t}{k_{t-1}}\right) k_{t-1} - \xi\left(p_t, p_{t-1}\right) Y_t + \tfrac{1}{1+r_t} J_{t+1}\left(k_t\right) \\ & -\lambda_t \left[\tfrac{p_t}{P_t} Y_t - \left(\tfrac{F\left(K_{t-1}, L_t\right)}{Y_t} \right)^{\frac{1}{\mu}-1} Y_t \right] - Q_t \left[k_t - (1-\delta) k_{t-1} - \iota_t\right] \Big\}\end{aligned} \tag{62}$$

FOC is

$$\frac{\partial \mathcal{L}}{\partial p_t} = \frac{1}{P_t} F\left(k_{t-1}, n_t\right) - \frac{\partial \xi\left(p_t, p_{t-1}\right)}{\partial p_t} Y_t - \lambda_t \frac{Y_t}{P_t} + \frac{1}{1+r_t} \frac{\partial J_{t+1}\left(k_t\right)}{\partial p_t} = 0 \tag{63}$$

$$\frac{\partial \mathcal{L}}{\partial k_t} = \frac{1}{1+r_t} \frac{\partial J_{t+1}\left(k_t\right)}{\partial k_t} - Q_t = 0 \tag{64}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = \frac{p_t}{P_t} \frac{\partial F\left(k_{t-1}, n_t\right)}{\partial n_t} - \frac{W_t}{P_t} = 0 \tag{65}$$

$$\frac{\partial \xi\left(p_t, p_{t-1}\right)}{\partial p_t} = \left[\frac{1}{2\kappa^p} \left(\frac{p_t - p_{t-1}}{p_{t-1}} \right)^2 \right]' = \frac{1}{p_{t-1}} \frac{1}{\kappa^p} \frac{p_t - p_{t-1}}{p_{t-1}} \tag{66}$$

$$\frac{\partial J_{t+1}\left(k_t\right)}{\partial p_t} = -\frac{\partial \xi\left(p_{t+1}, p_t\right)}{\partial p_t} = \left[\frac{1}{2\kappa^p} \left(\frac{p_{t+1} - p_t}{p_t} \right)^2 \right]' = -\frac{1}{\kappa^p} \frac{p_{t+1}}{p_t^2} \left(\frac{p_{t+1} - p_t}{p_t} \right) \tag{67}$$

$$\frac{\partial \mathcal{L}}{\partial p_t} = \frac{1}{P_t} F\left(k_{t-1}, n_t\right) - \frac{1}{p_{t-1}} \frac{1}{\kappa^p} \frac{p_t - p_{t-1}}{p_{t-1}} Y_t - \lambda_t \frac{Y_t}{P_t} - \frac{1}{1+r_t} \frac{1}{\kappa^p} \left(\frac{p_{t+1} - p_t}{p_t} \right) \left(-\frac{p_{t+1}}{p_t^2} \right) Y_{t+1} = 0 \tag{68}$$

$$\left(1 - \lambda_t\right) \frac{1}{P_t} Y_t = \frac{1}{\kappa^p} Y_t \left(\frac{p_t - p_{t-1}}{p_{t-1}} \right) \frac{1}{p_{t-1}} + \frac{1}{1+r_t} \frac{1}{\kappa^p} \left(\frac{p_{t+1} - p_t}{p_t} \right) \left(-\frac{p_{t+1}}{p_t^2} \right) Y_{t+1} \tag{69}$$

$$\frac{p_{t+1} - p_t}{p_t} = \pi_t^p \tag{70}$$

This setup generates a nonlinear Phillips curve for price inflation

$$\pi_t (1 + \pi_t) = \kappa^p mc_t + \frac{1}{1+r_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1 + \pi_t) \tag{71}$$

B HANK Estimation

It will be time-consuming if I use traditional algorithm to estimate the exogenous shock of HANK model, but it will be efficient in Sequence-Jacobian framework. The reason is the time complexity is $O(N)$ but it is $O(N^2)$ in directive Bayesian estimation. As Auclert et al. [2019], I set the impulse responses computed so far correspond to the $MA(\infty)$ representation of the model with aggregate shocks. I have get the blocks(individual policy function, aggregate variable and price), mapping stochastic processes $\{\tilde{\mathbf{X}}_t\}$ into one another, rather than sequences $\{\mathbf{X}_t\}$. my results can be summarized as follows.

Assume that exogenous shocks in the model are given by a set of $MA(\infty)$ processes

$$d\tilde{Z}_t^z = \sum_{s=0}^{\infty} m_s^z \epsilon_{t-s}^z$$

where there are as many processes as there are exogenous shocks $z \in \mathcal{Z}$. Here, $\{\epsilon_t^z\}$ are mutually iid standard normally distributed innovations, and $\{m_s^z\}$ are the MA coefficients of the shock to z I denote by $\mathbf{m}^z$ the column vector that results from stacking the m_s^z 's. For any output $o \in \mathcal{O}$ of the stochastic SHADE model, consider the $MA(\infty)$ representation of the equilibrium sequence $\{d\tilde{X}_t^o\}$

$$d\tilde{X}_t^o = \sum_{s=0}^{\infty} \sum_{z \in \mathcal{Z}} m_s^{o,z} \epsilon_{t-s}^z$$

Given shock $z \in \mathcal{Z}$ and assume that all shock innovations are zero, except for $\epsilon_t^z = 1$ in some period t. In that case, the expected path of $d\tilde{Z}_t^z$ going forward is given by

$$\mathbb{E}_t \left[d\tilde{Z}_{t+s}^z \right] = m_s^z$$

Hence, this expected path corresponds to a shock $\mathbf{m}^z = d\mathbf{Z}^z$ to z in the perfect-foresight model. In other words, the impulse response of output o is given by $\mathbf{G}^{0,z} d\mathbf{Z}^z = \mathbf{G}^{0,z}\mathbf{m}^z$.

Therefore the MA coefficients $\mathbf{m}^{0,z} \equiv (m_s^{o,z})$ of output o in response to shock z can be obtained by solving the associated perfect foresight SHADE model, that is, for any $o \in \mathcal{O}, z \in \mathcal{Z}$ I have:

$$\mathbf{m}^{0,z} = \mathbf{G}^{0,z}\mathbf{m}^z$$

where $G^{0,z}$ is the general equilibrium Jacobian.

I first compute all second moments from the impulse responses, and then compute the likelihood function from these second moments.

The first step consists of computing the model's autocovariance function. Let $\widehat{\mathcal{O}} \subset \mathcal{O}$ be the set of outputs whose second moments I would like to characterize, and denote by $d\tilde{\mathbf{X}}_t = \left(d\tilde{X}_t^o\right)_{o \in \widehat{\mathcal{O}}}$ the vector-valued stochastic process of all outputs in $\widehat{\mathcal{O}}$. Similarly, let $\mathbf{m}_t^{\widehat{\mathcal{O}},\mathcal{Z}} = (m_t^{o,z})_{o \in \widehat{\mathcal{O}}, z \in \mathcal{Z}}$ be the stacked $|\widehat{\mathcal{O}}| \times n_z$ matrix of MA coefficients of $d\tilde{\mathbf{X}}_t$. Then, the autocovariances of $d\tilde{\mathbf{X}}_t$ are given by

$$\text{Cov}\left(d\tilde{\mathbf{X}}_t, d\tilde{\mathbf{X}}_{t'}\right) = \sum_{s=0}^{T-(t'-t)} \left[\mathbf{m}_s^{\widehat{\mathcal{O}},\mathcal{Z}}\right] \left[\mathbf{m}_{s+t'-t}^{\widehat{\mathcal{O}}\mathcal{Z}}\right]'$$

The second step to estimation is to evaluate the likelihood function. Let

$$d\tilde{\mathbf{X}}_t^{obs} = Bd\tilde{\mathbf{X}}_t + \mathbf{u}_t$$

denote the vector of n_{obs} observables whose likelihood I would like to determine. Here $\{\mathbf{u}_t\}$ is iid normal with mean 0 covariance matrix $\Sigma_{u,}$ and B is a $n_{obs} \times |\widehat{\mathcal{O}}|$ matrix. since $d\tilde{\mathbf{X}}_t^{obs}$ is a linear combination of the ϵ_t^z and $\mathbf{u}_t$ terms, it has a multivariate normal distribution. Moreover, its second moments are a simple linear transformation of those of $d\tilde{\mathbf{X}}_t$

$$\text{Cov}\left(d\tilde{\mathbf{X}}_t^{obs}, d\tilde{\mathbf{X}}_{t'}^{obs}\right) = 1_{t=t'} \cdot \Sigma_{\mathbf{u}} + B\,\text{Cov}\left(d\tilde{\mathbf{X}}_t, d\tilde{\mathbf{X}}_{t'}\right) B'$$

I stack these covariances into a large symmetric $n_{\text{obs}}T_{\text{obs}} \times n_{\text{obs}}T_{\text{obs}}$ matrix $\mathbf{V}$, where T_{obs} is the number of time periods in my data. The log-likelihood function is then the conventional log multivariate density. Dropping the constant term, it can be expressed as a function of the observed data $d\tilde{\mathbf{X}}^{obs} = \left(d\tilde{\mathbf{X}}_t^{obs}\right)$ (stacked as $n_{obs}T_{obs}$ -dimensional vector) as

$$\mathcal{L} = -\frac{1}{2}\log\det\mathbf{V} - \frac{1}{2}\left[d\tilde{\mathbf{X}}^{obs}\right]'\mathbf{V}^{-1}\left[d\tilde{\mathbf{X}}^{obs}\right]$$

The efficiency of the maximum likelihood function has been proved by Auclert et al. [2019].

C Shocks and Response

C.1 Monetary Policy Shock

As I all know, if the central bank deviates from its system's Taylor rule, that is, to raise the nominal interest rate, then due to price stickiness, it also changes the real interest rate, and so can influence the performance of the real economy (Figure 3). Strict monetary policy conditions forced companies to postpone investment, and households began to accumulate more savings or reduce their credit stock. Due to the reduction in total demand, companies lower prices to avoid (reduce) loss of profits. Due to price stickiness and price index, the nominal adjustment is gradual, and inflation requires more than four quarters. The central bank observed that total demand was shrinking and lowered the nominal interest rate back to its steady state level, and the entire economy stabilized at its initial steady state. Aging makes a significant difference in the output gap response (GDP deviates from its flexible price equilibrium level). This is also reflected in the fact that young people and old people have different reactions. In particular, in the old society, retired agents had more savings, and workers also assumed more debt over longer lives. In addition, in young societies, currency restrictions have created incentives to postpone consumption and increase savings. However, in the old society, the impact of raising interest rates was even more asymmetric: the elderly could interpret the shock as extra income, so they increased consumption, while young people faced higher credit costs and reduced consumption. Since aging has also changed the relative size of similar groups of people, in the grey society, the total consumption and output gap have decreased by a small margin. Therefore, I find that aging will reallocate asset positions between generations, which reduces the effectiveness of monetary policy-defined as the rate of inflation falls significantly under the same currency deflation, and the total demand for interest rate changes less elastic. As Figure 3 points out that a negative interest shock will galvanize the short-term investment of firms and lead to the increasing of production. And the response is increasing with the size of the shocking.

Consumption of the economy(Figure 4) is in tune with the response of production but the degree is weak than production. But the short-term welfare(Figure 5) present value showcase the trend of decreasing, it prove that it costs the future value of welfare. The key variable I want to focus on in this paper is the change of asset $\mathcal{B}_t$ as Figure 6.

C.2 Fiscal Policy Shock

The government increases public expenditure by one percent of steady-state GDP (Figure 7). As a result, firms increase production to satisfy the extra demand. A higher level of production requires more labor, so firms increase wages to attract more workers. To offset the increase in production costs and the loss in profits, price-setting firms increase their prices. The central bank launches a tightening cycle, and holds the interest rate elevated until the demand-side inflationary pressure disappears, and inflation goes back to its original steady-state level. Due to the increase in wages, young households consume more. Old households consume less because of the increasing interest rate and credit demand (from the young households). Later on, though, the higher interest rates push young consumption down and old consumption up. Aging also changes the relative size of cohorts, and decreases the available labor force in the economy. In an old society the labor supply is more inelastic (the Frisch-elasticity is lower). Hence, firms are forced to increase wages more than in a young society to attract the required labor force. This additional increase in wages amplifies the increase in marginal costs and inflation, too. As a consequence, in an old society the central bank needs to raise the policy rate by a larger amount than in a young society. A stronger monetary policy reaction in a more gray society forces the young to give up more consumption.

Assets(Figure 10) are facilitated by the decreasing of government purchasing. The demand of liquid asset declines and the interest rate of that go along with that trend, which means the requirement of Assets will exacerbate due to that.

C.3 Technology Shock

In the article Guerrieri et al. [2020] introduced a concept that might be accurately described as ”supply creating its own excess demand”. In other words, a negative supply shock leads to a shortage of demand, resulting in more output and employment contraction than supply shock itself. I call supply shocks with these characteristics Keynesian supply shocks. Temporary negative supply shocks (such as those caused by a pandemic) will reduce production and employment. The decline in supply shocks may be frightening, but it is an effective response to a certain extent, because output and employment are bound to decline. Some people question whether any demand stimulus measures can justify a response to a supply shock in essence. They believe that the economic response should be based solely on social protection. Others argue that negative shocks may cost more output than efficiency. Gourinchas and Pierre-Olivier [2020], for example, has proposed a height measurement aimed at ”flattening the curve”.

They present a theory of Keynesian supply shocks: supply shocks that trigger changes in aggregate demand larger than the shocks themselves. I argue that the economic shocks associated to the COVID-19 epidemic—shutdowns, layoffs, and firm exits—may have this feature. In one-sector economies supply shocks are never Keynesian. I show that this is a general result that extend to economies with incomplete markets and liquidity constrained consumers. In economies with multiple sectors Keynesian supply shocks are possible, under some conditions. A 50% shock that hits all sectors is not the same as a 100% shock that hits half the economy. Incomplete markets make the conditions for Keynesian supply shocks more likely to be met. Firm exit and job destruction can amplify the initial effect, aggravating the recession. I discuss the effects of various policies. Standard fiscal stimulus can be less effective than usual because the fact that some sectors are shut down mutes the Keynesian multiplier feedback. Monetary policy, as long as it is unimpeded by the zero lower bound, can have magnified effects, by preventing firm exits. Turning to optimal policy, closing down contact-intensive sectors and providing full insurance payments to affected workers can achieve the first-best allocation, despite the lower per-dollar potency of fiscal policy.

D MIT Shocks

What if I consider perfect foresight path of the economy? Many researches have focused on MIT shocks(Boppart et al. [2018],Kydland and Prescott [1982]). I first point out that before the researchers conducted groundbreaking work, the researchers used analytical methods to characterize the perfect expected equilibrium caused by the ”MIT shock.” The MIT shock created by Thomas Sargent refers to an unpredictable shock without a

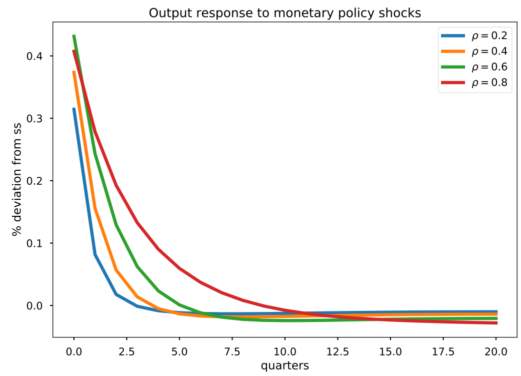

Figure 3: Output response to monetary policy shocks

shock to the steady state balance of the economy. In other words, in this economy, no shocks are expected, but now there will be shocks. Then, assuming that no more shocks will occur, the analysis focuses on understanding the equilibrium transition that occurs along the perfect prediction path. Therefore, the described process seems to be difficult to coincide with rational expectations.

The Kydland-Prescott paper developed a non-linear model without market frictions and, hence, could focus on solving a planning problem. This problem was approached using "linearization" around the steady state; Kydland and Prescott first solved for the steady state, then provided a quadratic (second-order Taylor expansion) approximation of the objective function expressed in terms of the vector of choice variables- around that steady state, along with a linear set of (or linearized) constraints. Thus, they obtained a linear-quadratic problem, which was well known to have an exact solution in the form of a quadratic value function along with linear policy rules.

A key element in the approach pioneered by Kydland and Prescott was the use of a recursive system, which fits value-function methods well. The key objects sought were thus functions of a state vector of exogenous and endogenous variables. To be concrete, one would for example in the very simplest case of a stochastic growth model with optimal saving and no adjustment costs find output, y, to be a function G of the capital stock, k, and TFP $z : y = G(k, z)$. Similarly, the law of motion for the endogenous state would be written using a function H as $k' = H(k, z)$.

Use the parameters I estimate above, I can get the response of MIT shock. The sensitivity of shock responses to fiscal, monetary, technology shocks are idiosyncratic (Figure 18, Figure 19, Figure 20). Monetary policy is the most effective way in these 3 tools, but it decades fast meanwhile. Assets weakly response to the shock of fiscal and technology shock, but the affect is relatively long-term. And the impacting of technology shock is instant and simultaneous.

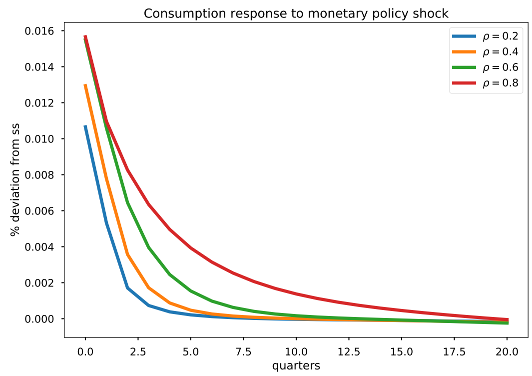

Figure 4: Consumption response to monetary policy shock

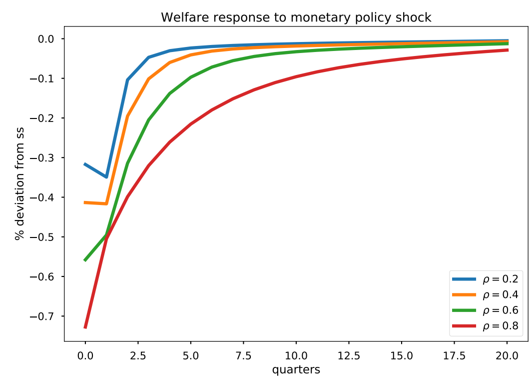

Figure 5: Welfare response to monetary policy shock

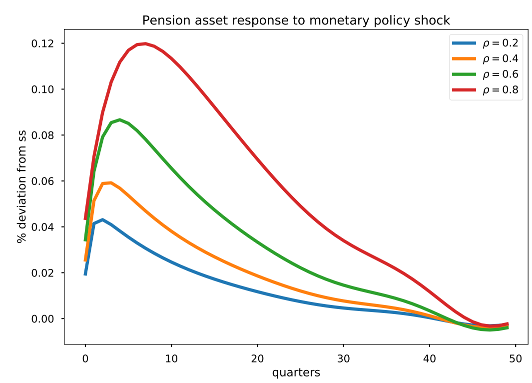

Figure 6: Asset response to monetary policy shock

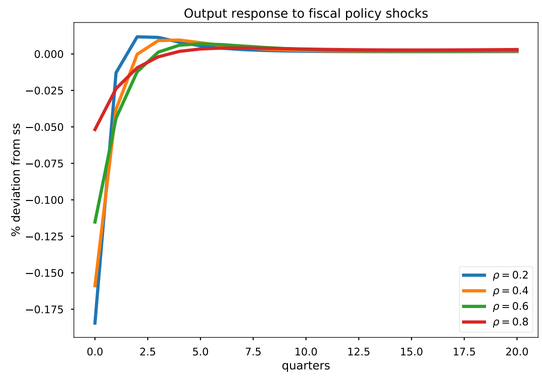

Figure 7: Changes of Debt under Different Tax Rates(2025-2100, Short Term)

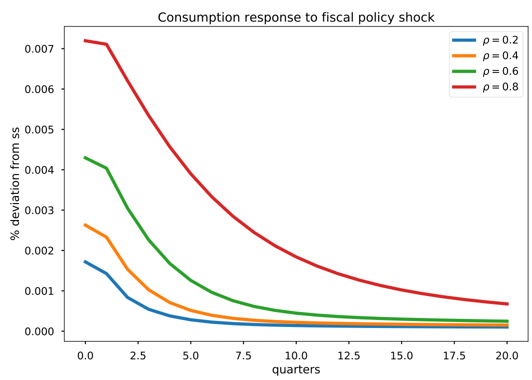

Figure 8: Changes of Debt Ratio under Different Tax Rates(2025-2100, Short Term)

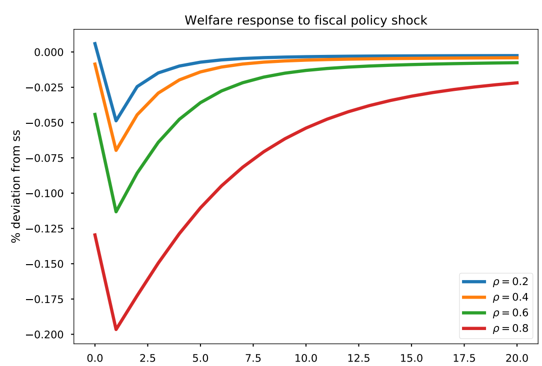

Figure 9: Changes of Debt Ratio under Different Tax Rates(2025-2100, Short Term)

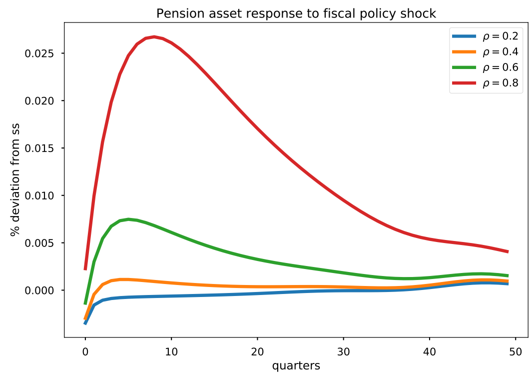

Figure 10: Asset response to fiscal policy shock

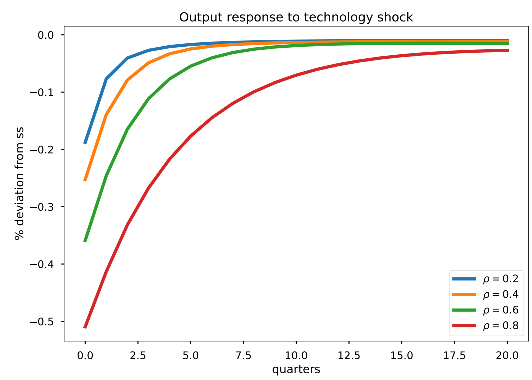

Figure 11: Consumption response to technology shock

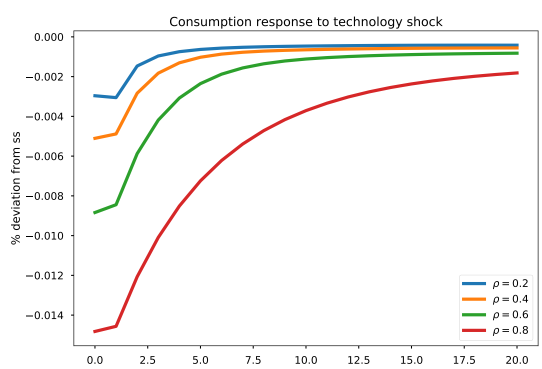

Figure 12: Output response to technology shock

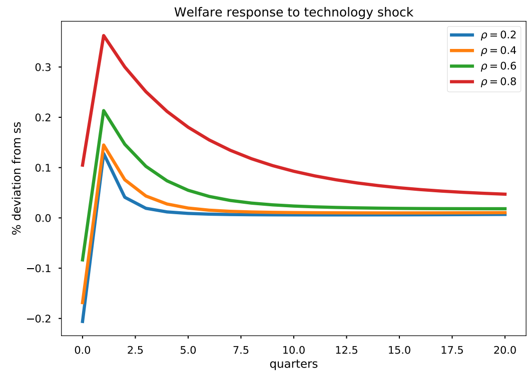

Figure 13: Welfare response to technology shock

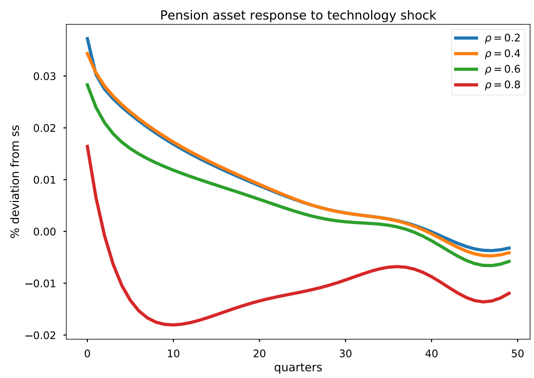

Figure 14: Asset response to technology shock

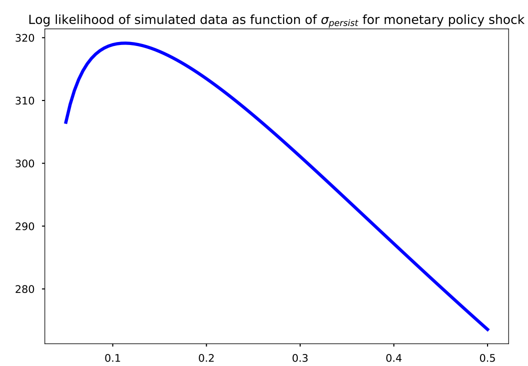

Figure 15: Log likelihood of simulated data as function of $\sigma_{persist}$ for monetary policy shock

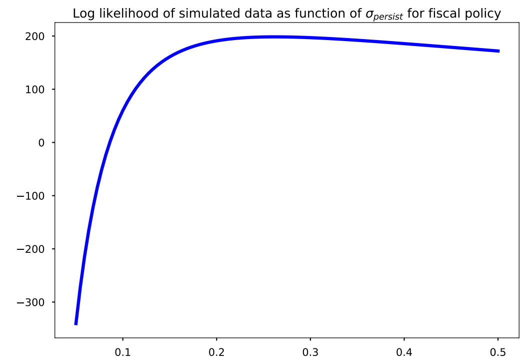

Figure 16: Log likelihood of simulated data as function of $\sigma_{persist}$ for fiscal policy

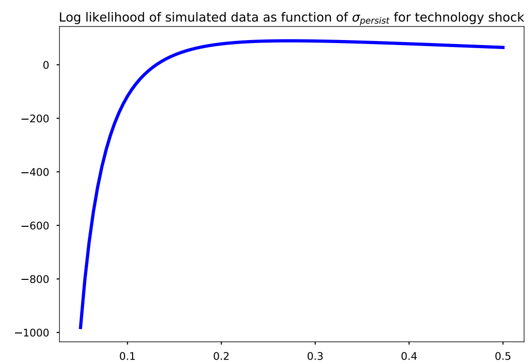

Figure 17: Log likelihood of simulated data as function of $\sigma_{persist}$ for technology shock

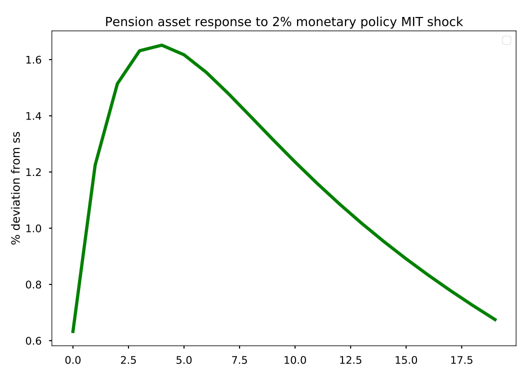

Figure 18: Asset response to 2% monetary policy MIT shock

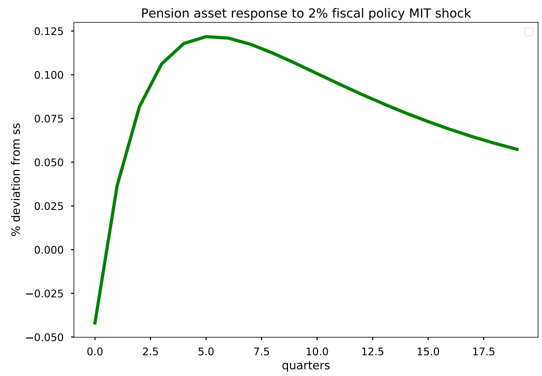

Figure 19: Asset response to 2% fiscal policy MIT shock

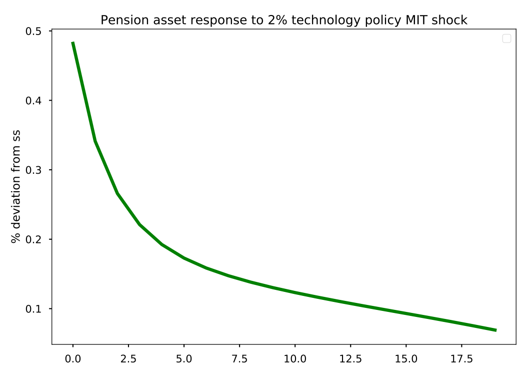

Figure 20: Asset response to 2% technology policy MIT shock