

China's Policy Instruments : Tax Reduction, Retirement Prolonging and Welfare Changes

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Abstract

China is facing a series of significant debt problems. We have studied the changes in debt and benefits under different policy instruments under the framework of large-scale OLG. Under the three retirement ages, as the retirement age increases, the maximum increase in benefits is 17.98%, and the debt is 75.69%. Under the five tax rates, the optimal tax rate is 28%, the maximum increase in benefits is 22.65%, and the maximum debt ratio is 75%.

1 Introduction

The prospect of an aging population in many advanced economies is worrying about the sustainability of economic growth. This was not a problem in China in the past, but it is not today. These concerns are increasing, and the rising old-age dependency ratio is turning into a growing tax burden, while rich pensions and medical benefits crowd out public investment spending on infrastructure or education, negatively affecting capital accumulation and productivity growth. Acemoglu and Johnson (2007) estimated the impact of life expectancy at birth on economic growth. They also found no evidence of positive effects. Pensions and retirement policies are important for a country's welfare improvement and economic development. How to adjust various policy tools is our main research object. Most developed countries have begun raising the retirement age or tightening conditions for early retirement, reducing pressure on social security taxes (OECD, 2007; OECD, 2009). Some scholars believe that postponed retirement can reduce the dependency ratio (the ratio of non-working-age population to working-age population) and reduce the pressure on pension payments to a certain extent. Solve the problem fundamentally. According to statistics from the World Social Security Research Center of

the Chinese Academy of Social Sciences, the basic pension insurance fund for employees in urban enterprises will have a deficit in 2028 and run out by 2035 in terms of the contribution rate at a baseline of 16%. Many media joked that this meant that "the offspring of the 1980s would have no money to feed the elderly." At the same time, China is entering an aging society and the working-age population is decreasing. According to the National Bureau of Statistics, China's population aged 60 and over in 2018 was 249 million, accounting for 17.9% of the total population. According to the 2015 prediction of the National Office of Aging, by 2022, the number of elderly people over 60 will increase to 268 million, and this proportion will rise to 18.5%. By 2036, the proportion of the elderly population will further increase to 423 million, and the proportion will increase to 29.1%. By 2053, the proportion of the elderly population will reach a peak of 487 million, accounting for 34.8% of the national population. The World Social Security Research Center of the Chinese Academy of Social Sciences pointed out that the pressure to support the basic endowment insurance of urban employees in the system will continue to increase in the future. In short, retirees will be supported by nearly two contributors in 2019, but only one person will support one retiree around 2050. China's existing retirement system dates back to the 1950s, when the average life expectancy of the Chinese was only 45 years old, and almost all labor was high-intensity manual labor, mechanization and modernization. Therefore, we hope to improve the economic situation by changing the backward retirement policy, so whether changing the retirement age can improve economic welfare is the issue that our model will explore.

Population aging is caused by three factors: declining fertility, prolonging life expectancy and a large number of early retirement behaviors. For the aging problem, many papers have given the results of research. The first aspect is related to the savings rate. Many scholars have studied the savings behavior from different aspects: İmrohoroglu (2018) has studied the relationship between long-term care risks and savings rate. They believe that China's high savings rate is related to the lack of effective long-term care insurance mechanism. As for the relationship between pension and private savings, we can conclude from the study of Attanasio and Brugiavini (2003), Attanasio and Rohwedder (2003) that there is a substitutive relationship between them. Through the pension system, the savings rate can be effectively reduced and the consumption of the younger generation can be encouraged. On the other hand, it is related to the supply of labor force. Hu (1979) first constructed a classical labor market supply model mainly focused on the impact of the pension system on the supply of labor force. Their model has significant economic characteristics. The topic of He et al. (2015) is that how the rapid change of population structure and pension reform promoted the growth of urban residents' savings rate and labor supply in China since 1997.

Debt (Figure 2) is also a major problem facing China in the near future. For the total

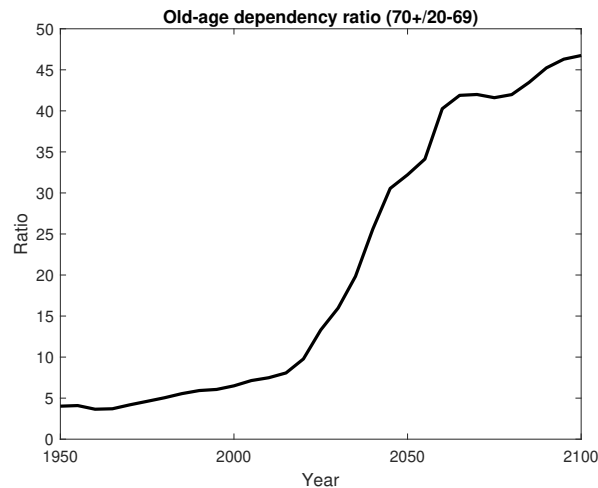


Figure 1: Old-age dependency ratio (ratio of population aged 70+ per 100 population 20-69)

debt of nearly 170 trillion yuan, according to data, in 2014, China's sovereign assets were calculated based on a narrow caliber, with a net value of 28.5 trillion yuan. In 2015, the central government and local government debt at various levels were 26.66 trillion yuan, accounting for the proportion of GDP in the same period At 39.4%, the government debt level and debt ratio were 56.8% and 89.2%, respectively, but still lower than the EU's 60% early warning line and internationally accepted warning values. It is expected to exceed 60% in 2020. However, some scholars have pointed out that China is a high-saving country (Figure 3), and its external debt only accounts for 3% of its total debt,

and its debt is affected little by international factors. In fact, due to the transformation of pensions, we must necessarily digest the conversion costs, and the debt problem may not be so optimistic.

Pension reform is imperative to tackle with the problem brought by population aging. One aspect of the reform is to delay the retirement age. Obviously, this will significantly reduce the pension gap. There are few quantitative articles in this area. We will study this aspect. Faced with the problem of economic transformation and the improvement of industrial productivity, China has significantly reduced the burden of corporate social security contributions, and lowered the proportion of urban workers' basic pension insurance contributions to 16%. China's pensions are mainly based on three pillars: basic pension, enterprise annuity and voluntary personal savings pension scheme. The first pillar here is absolute in both coverage and volume. But China's pension system is also facing great problems: first, the pressure of aging; second, the coverage rate is still insufficient, especially the second and third pillars; third, the return rate of funds is

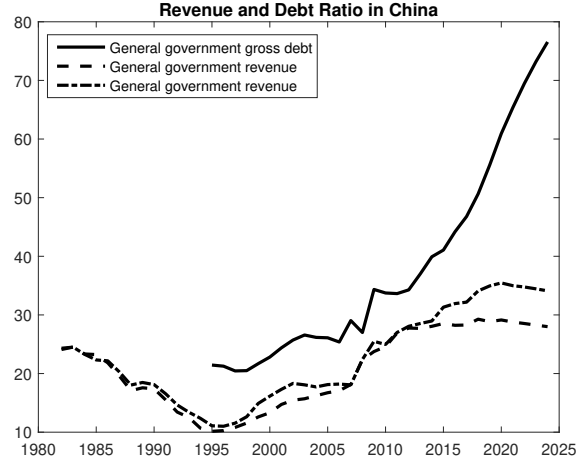


Figure 2: Revenue and Debt Ratio in China(1980-2025)

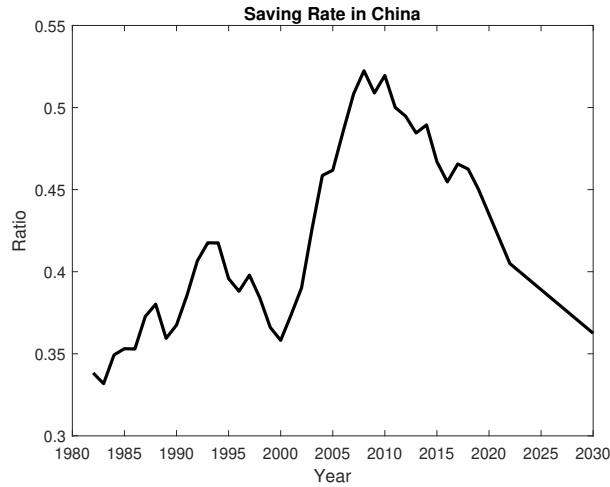


Figure 3: Saving Rate in China(1980-2030)

insufficient. There have been many articles on the reform of the pension system. Fehr, Kallweit, and Kindermann (2013) first proposed a Hybrid Pension system, which consists of PAYG and wage-dependent pensions. He et al.(2015) used this system to study China's labor supply in 2015, but they did not explore the optimal pension policy.

In terms of optimal pension policy, İmrohoroglu(1999) first explored the distortion effect of optimal pension policy and welfare by using large-scale OLG model. Nishiyama and Smetters (2007) studied the welfare effects of 50% tax policy. Kitao (2014) studied welfare changes under various policies. We are different from them in that we need to explore the optimal solution under the dual policy tools, and further come to the tools of how to control the optimal policy.

In the aspect of solution method, except for a few examples, the general equilibrium

model of dynamic inhomogeneous media has no analytical solution, and the analytical results are not allowed to be obtained. Algorithms for solving heterogeneous agent models with endogenous distribution have recently been introduced into economic literature. Famous studies in this field include Aiyagari (1994), Huggett (1996), and Ríos-Rull (1999). We use shooting method to solve the model, and the way to solve it will be given in the article.

The article follows the following logic. In the second part, we will build a large-scale OLG model, which depicts the heterogeneity of income and age structure, and introduces a composite pension system. In the third part, we define the equilibrium state of the economy, which is the key to solving the model. The fourth part puts forward the idea of solving the model. The fifth part carries on the model calibration, mainly adopts the parameter which most articles use. Finally, the static and dynamic results of the economy are analyzed in the following ways: firstly, individuals in the economy analyze the economy from asset decision-making, consumption, labor supply and savings rate; secondly, the overall economy, we analyze the optimal policy tools and dynamic transfer.

2 Empirical Evidence

In order to predict the effect of macro variables, we establish following specification:

$$\begin{aligned}\Delta \text{ Pension}_t = & \beta_0 + \beta_1 r_t + \beta_2 \text{Debt}_t + \beta_3 Z_t + \beta_4 \Delta C_t \times r_t + \beta_5 \Delta Y_t \times r_t + \beta_6 \Delta C_t \times \text{Debt}_t \\ & + \beta_7 \Delta Y_t \times \text{Debt}_t + \beta_8 \Delta Y_t \times Z_t + \beta_9 \Delta C_t \times Z_t + \epsilon_t\end{aligned}$$

r_t is the interest of the end of year. Debt_t is the debt of government, which represents the investment and government purchasing. Z_t is the investment of technology, $\Delta C_t, \Delta Y_t$ are the change proportion of them. The reason why we choose them because that can represent the response of shock.

Since the national plan has great impact on Chinese economy and path of development. We also want to focus on the effect of nation plan and leadership alternate. By controlling time fixed effect, we choose to set leadership dummy variable D_i .

$$\begin{aligned}\Delta \text{ Pension}_{t,i} = & \beta_0 + \beta_1 r_{t,i} + \beta_2 \text{Debt}_{t,i} + \beta_3 Z_{t,i} + \beta_4 \Delta C_{t,i} \times r_t + \beta_5 \Delta Y_{t,i} \times r_t + \beta_6 \Delta C_{t,i} \times \text{Debt}_t \\ & + \beta_7 \Delta Y_{t,i} \times \text{Debt}_t + \beta_8 \Delta Y_{t,i} \times Z_t + \beta_9 \Delta C_{t,i} \times Z_t + \beta_{10} D_i + \epsilon_{t,i}\end{aligned}$$

	(1)	(2)	(3)	(4)
Δ Pension				
$Debt_t$	-0.00589** (-2.364)	-0.00630** (-2.252)	-0.00553* (-1.889)	-0.00469 (-1.520)
r_t	0.0920** (2.478)	0.0921** (2.410)	0.119** (2.340)	0.108* (1.978)
Z_t	1.23e-05*** (3.585)	1.24e-05*** (3.547)	1.33e-05*** (3.581)	1.06e-05** (2.751)
$\Delta Y \times G$		-8.828 (-1.304)	-9.718 (-1.541)	-17.31* (-1.974)
$\Delta C \times G_t$		8.288 (1.312)	8.780 (1.541)	15.99* (2.051)
$\Delta Y \times r_t$			-2.217 (-0.578)	-0.169 (-0.0434)
$\Delta C \times r_t$			0.648 (0.147)	-1.408 (-0.311)
$\Delta Y \times Z_t$				4.933 (1.526)
$\Delta C \times Z_t$				-6.885* (-1.760)
Constant	-0.0493 (-0.386)	-0.0357 (-0.268)	-0.143 (-0.751)	-0.0729 (-0.329)
Time FE	NO	NO	NO	NO
R-squared	0.508	0.536	0.607	0.651
Ajusted R2	0.476	0.476	0.476	0.476

Robust t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
Δ Pension				
$Debt_t$	-0.00642** (-2.106)	-0.00704** (-2.339)	-0.00619 (-1.730)	-0.00569 (-1.441)
r_t	0.117*** (3.206)	0.120*** (3.358)	0.138*** (3.014)	0.130** (2.645)
Z_t	4.98e-06 (1.345)	4.50e-06 (1.236)	6.19e-06 (1.577)	5.41e-06 (1.291)
$\Delta Y \times G$		-12.01* (-1.738)	-11.55 (-1.525)	-16.51 (-1.554)
$\Delta C \times G_t$		11.43 (1.575)	10.73 (1.342)	15.41 (1.423)
$\Delta Y \times r_t$			0.0693 (0.0106)	0.936 (0.136)
$\Delta C \times r_t$			-1.203 (-0.168)	-2.084 (-0.275)
$\Delta Y \times Z_t$				3.483 (0.698)
$\Delta C \times Z_t$				-4.340 (-0.805)
Constant	-0.0448 (-0.278)	-0.0270 (-0.171)	-0.119 (-0.570)	-0.0819 (-0.368)
R-squared	0.570	0.629	0.660	0.674
Time FE	YES	YES	YES	YES
Adjusted R2	0.450	0.450	0.450	0.450

t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Before the fixed effect is controlled, we can see that interest, technology investment and debt have significant impact on them. Government investments and debt manifest negative correlation, and interest rate and technology investment have positive effect on it. After controlling the time fixed effect, the technology investments turn into insignificant variable. Now we prove pension has directive correlation with macro economy. We will model the economy to predict the introduction of fully funded system in next part.

3 The Model Economy

In this section, we develop a large scale overlapping generations (OLG)-model developed by Auerbach and Kotlikoff(1987) with finite life individuals, firms and government. The utility of the individual derives from consumption and rest. The company maximizes profits and determines the optimal interest rate and wage rate. The government relies on pension taxes and payroll taxes to finance pensions, and Transfer payments balance government financing. Individuals in the economy face uncertainty about income and time. And we assume the market is effective, the fertility is exogenous.

3.1 Demographic Structure

Suppose an individual is born and starts working at the age of $j = 1$. The time to retire is $j = R$ and the maximum life expectancy is $j = J$. The individual faces the conditional survival probability $\phi_j \in (0, 1)$, which indicates the probability of survival from $j - 1$ years old to j years old, then the probability of survival to j years old is $\prod_{j=1}^n \phi_j$. Let the total number of individuals in the economy be N_t , and the fertility rate n be a certain value.

Then based on the above information, we can deduce the age structure of the economy. The proportion of people with age j to the total population is

$$\mu_j = \frac{\phi_j}{1 + n} \mu_{j-1} \quad (1)$$

μ_j denotes the proportion of age- j individuals. The sum of the proportions of each age is $\sum_{j=1}^J \mu_j = 1$.

3.2 Preferences

We use a recursive utility function, which is constructed by Kreps and Porteus (1978), Epstein and Zin (1989). This utility function is different from the classical utility function. The ordinary CCRA utility function only considers the characteristics of the individual risk aversion, and this function considers the timeliness of utility, that is, the trade-off between the current utility and the future expected present value. From a practical point of view, individuals in the economy will also make current decisions based on future utility. According to the hypothesis, the utility of a single phase comes from consumption and rest. Assuming that the total number of times for each period of the individual is 1, the rest time is $1 - l$, l denotes the working time. The functional form of the single-phase

utility is

$$u(c, l) = c^\gamma (1 - l)^{1-\gamma} \quad (2)$$

The parameter γ represents the elasticity of substitution between consumption and rest. Then the intertemporal utility planning problem can be expressed in the following form

$$V_t(k_t^s, s) = \max_{k_{t+1}^{s+1}, c_t^s, l_t^s} \left\{ u(c_t^s, l_t^s)^\rho + \beta \phi_s E_t[V_{t+1}^\psi(k_{t+1}^s, s+1)]^{\frac{\rho}{\psi}} \right\}^{\frac{1}{\rho}} \quad (3)$$

The variable k_t^s represents the assets of the s -year-old individuals at the term t , t represents the calendar time, and s represents the age. η indicates a change in work efficiency, indicating a switch between high productivity and low productivity.

3.3 Earnings

The family faces certain budget constraints. Every household's income comes from wages, capital returns and government transfers. At the beginning of each family, the asset holding k_t^0 is 0, and the consumption is fully financed by the government's transfer payment. d_t^s is the number of bonds held, bonds and capital get the same rate of return. This constraint can be divided into two periods: the first part is before retirement, here we set the salary of the individual whose age is j when the calendar time is t , then the salary of the unit time has the following form

$$w_t^s = A_t w_t \eta e_t^s \quad (4)$$

Here w_t denotes the payroll rate, and A_t denotes the technology, wages increase as social productivity increases. e_t^s represents the work efficiency of an individual with age s at t . When $0 < s \leq R$, part of the return on capital should be handed over to the government at the τ_k rate as a capital tax. On the other hand, wages are taxed in two parts. The first part is wage tax and the other part is pension tax. The ratios are τ_w and τ_p , respectively. The transfer of government is a lump-sum form. Here we assume that the elderly do not have a legacy for future generations, and that all accidental bequests will be completely confiscated by the government. When $R < s \leq J$, households will accept additional pensions. All consumption is financed by existing capital.

As a result, the budget constraint can be represented by:

$$(1 + \tau_t^c)c_t^s + k_{t+1}^{s+1} + d_{t+1}^{s+1} = \begin{cases} (1 - \tau_w - \tau_t^{pen})w_t^s l_t^s + [1 + (1 - \tau_k)r_t](k_t^s + d_t^s) + tr_t & 0 < s \leq R \\ [1 + (1 - \tau_k)r_t](k_t^s + d_t^s) + tr_t + pen_t & R < s \leq J \end{cases} \quad (5)$$

In addition, each term can borrow capital $k_t^s \geq 0$, which is a constraint for households.

3.4 Production

The production function is set in the form of neoclassical function, which is the same as the general model. Production function is Cobb-Douglas function, and output Y_t comes from capital K_t and effective human capital $A_t L_t$. There is depreciation in capital, and the depreciation rate is a constant δ . So:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \quad (6)$$

The technological progress is exogenous and its rate of growth is g_A :

$$A_t = (1 + g_A)A_{t-1} \quad (7)$$

The firms want to maximize profits. So we can get the first-order conditions:

$$r_t = \alpha K_t^{\alpha-1} (A_t L_t)^{1-\alpha} - \delta \quad (8)$$

$$w_t = (1 - \alpha) K_t^\alpha (A_t L_t)^{-\alpha} \quad (9)$$

So we can deduce the average growth rate of real wage will be g_A .

3.5 Government's Budget

The government's fiscal balance can be seen from both income and expenditure. Income consists of tax and accidental bequests, and tax is composed of wage tax and capital tax. From the perspective of expenditure, it consists of government purchase and transfer payments, which are used to balance the budget.

$$G_t + Tr_t + (1 + r_t)D_t = T_t + Beq_t + D_{t+1} \quad (10)$$

Government's tax revenue is given by:

$$T_t = \tau_w w_t A_t L_t + \tau_K r_t K_t + \tau_c C_t \quad (11)$$

Government purchase is an exogenous variable, which is related to the growth rate of technology g_A and the growth rate of population n_t .

$$G_t = G_{t-1}(1 + g_A)(1 + n_t) \quad (12)$$

3.6 Social Security

Considering the actual situation in China, the social pension system here is different from the traditional pay-as-you-go(PAYG) pension system. This system was first proposed by He, H., Ning, L., Zhu, D.(2015). They studied the reform of China's pension system.

Pension is not only related to pension tax paid by contemporary people, but also to the individual's salary before retirement. There is a certain proportion relationship between the two parts of pension. Assuming that the proportion paid by contemporaries is m , then the proportion of pension related to the wage of individual working period is $(1 - m)$ naturally.

$$pen_t^s = \theta_{pen}[m\bar{E}_{s,t} + (1 - m)Q_j] \quad (13)$$

θ_{pen} denotes a certain ratio of replacement, $\bar{E}_{s,t}$ denotes the average pensions financed by the economy individuals, Q_j denotes the pension related to the individual's ability of producing.

Based on the above assumptions, we can deduce that:

$$\bar{E}_{s,t} = \frac{\sum_{t=1}^{R-1} \sum_{\eta} e_t^s w_t \eta_t (1 - l_t)}{\sum_{t=1}^{R-1} \mu_t} \quad (14)$$

$$Q_j = \frac{\sum_{t=1}^{R-1} \sum_{\eta} e_t^s w_t \eta_t (1 - l_t)}{R - 1} \quad (15)$$

It can be seen from the expression that $\bar{E}_{s,t}$ is related to the number of individuals in the economy. The more contemporary individuals are, the more pensions they receive from their wages. Q_j denotes the average wage earned by an individual's effective work throughout the working period in an economy. That is to say, the more money an individual earns when he works young, the more pensions he receives. This system reflects the fairness of the individual.

This system is different from the article published by İmrohoroglu et al. (1998) to test the IRA pension system in the United States. There are two policy variables θ_{pen} and m in this system. Obviously, the bigger θ_{pen} is, the heavier the burden of government is, and the bigger m is, the greater the burden of contemporary working individuals will be.

4 Stationary Equilibria

When the economy tends to be stable, the individual's behavior is consistent with the overall economic behavior: the production sector maximizes profits, the individual makes optimal planning for consumption and labor, and the commodity market clears. Similar to İmrohoroglu et al. (1995), let's define the competitive equilibrium of the economy. For the overall economic variables, due to the natural exogenous growth trend of population and technology, we remove the trend of each variable. Then we get the following definition:

$$\begin{aligned}\tilde{Y} &= \frac{Y}{A_t N_t}, \tilde{C} = \frac{C}{A_t N_t}, \tilde{L} = \frac{L}{N_t}, \tilde{Pen} = \frac{Pen}{A_t N_t} \\ \tilde{G} &= \frac{G}{A_t N_t}, \tilde{T} = \frac{T}{A_t N_t}, \tilde{Beq} = \frac{Beq}{A_t N_t}, \tilde{D} = \frac{D_t}{A_t N_t}\end{aligned}\quad (16)$$

and stationary individual variables:

$$\tilde{k}_t = \frac{k_t}{A_t}, \tilde{c}_t = \frac{c_t}{A_t}, \tilde{pen}_t = \frac{pen_t}{A_t}, \tilde{tr}_t = \frac{tr_t}{A_t}, \tilde{d}_t^s = \frac{d_t}{A_t}\quad (17)$$

For heterogeneous agent model, we usually adopt the definition of density function to describe the dynamic transfer of economy through the transfer of density function. Therefore, we set f_t as a cross-sectional measure of the time t of the economy, which describes the distribution relationship between the number of economic individuals and asset holdings.

Based on above assumptions, stationary equilibrium for the government policy $\{\tau_k, \tau_w, \tau_{pen}, \theta_{pen}, m, \tilde{r}, \tilde{G}\}$ according to the price system allocation and the sequence of aggregate productivity indicators A_t that satisfy the following conditions:

Definition 4.1 *Competitive equilibrium includes individual decision variables $\{\tilde{c}_t^s, \tilde{k}_t^s, \tilde{l}_t^s, \tilde{d}_t^s\}$, firms' plans for production $\{K, N\}$, factor prices w_t, r_t . Government's security policy $\{m, \theta_{pen}\}$ and lump-sum transfer beq_t . Then the time-invariant distributions of individuals $f_t(k_t)$ for each age $j = 1, 2, \dots, J$ such that*

1. *Population structure is stable: population growth rate $n = \frac{N_{t+1}}{N_t} - 1$ is constant, conditional survival probability ϕ_t^s is independent of time as a constant, that is $\phi_t^s = \phi_t$.*

2. The rate of technological progress is a constant g_A .
3. In a stable state, budget constraints can be rewritten as follows:

$$(1 + \tau_t^c)\tilde{c}_t^s + (1 + g_A)\tilde{k}_{t+1}^{s+1} + (1 + g_A)\tilde{d}_{t+1}^{s+1} = \begin{cases} (1 - \tau_w - \tau_t^{pen})\tilde{w}_t^s l_t^s + [1 + (1 - \tau_k)r_t](\tilde{k}_t^s + \tilde{d}_t^s) + \tilde{t}r_t & 0 < s \\ [1 + (1 - \tau_k)r_t](\tilde{k}_t^s + \tilde{d}_t^s) + \tilde{t}r_t + p\tilde{e}n_t & R < s \end{cases} \quad (18)$$

In addition, $l_t^s \in [0, 1]$, and $\tilde{k}_t^s \geq 0$.

4. In this case, individual dynamic programming lifelong asset changes, consumption, and labor supply, this behavior is based on state variables η . Finally, when the economy reaches a steady state, the optimal strategy does not depend on time, but only on the current state. We can get the optimal policy function $\tilde{k}'(\tilde{k}, s, \eta)$, $\tilde{c}(\tilde{k}, s, \eta)$, $l(\tilde{k}, s, \eta)$.

5. The firm's optimal factor price sequence In the case of efficient market, the company's profit is zero.

6. According to the density function, we can find the variables of the whole economy, which is the sum of the individual economy.

$$\tilde{K}_{t+1} = \frac{1}{N_{t+1}} \sum_{\tilde{k}_t} \sum_s \sum_{\eta} \tilde{k}'_t(\tilde{k}, s, \eta) f_t(\tilde{k}, s, \eta) \quad (19)$$

$$\tilde{D}_{t+1} = \frac{1}{N_{t+1}} \sum_{\tilde{d}_t} \sum_s \sum_{\eta} \tilde{d}'_t(\tilde{k}, s, \eta) f_t(\tilde{k}, s, \eta) \quad (20)$$

$$\tilde{L}_{t+1} = \frac{1}{N_t} \sum_{\tilde{k}_t} \sum_s \sum_{\eta} \eta e_t^s \tilde{l}_t(\tilde{k}, s, \eta) f_t(\tilde{k}, s, \eta) \quad (21)$$

$$\tilde{C}_{t+1} = \frac{1}{N_{t+1}} \sum_{\tilde{k}_t} \sum_s \sum_{\eta} \tilde{c}_t(\tilde{k}, s, \eta) f_t(\tilde{k}, s, \eta) \quad (22)$$

$$\tilde{B}eq_{t+1} = \frac{1}{N_{t+1}} \sum_{\tilde{k}_t} \sum_s \sum_{\eta} (1 - \phi_t^s)[1 + (1 - \tau_k)r_{t+1}] \tilde{k}'_t(\tilde{k}, s, \eta) f_t(\tilde{k}, s, \eta) \quad (23)$$

$$\tilde{T}_t = \tau_w w_t \tilde{L}_t + \tau_k r_t \tilde{K}_t \quad (24)$$

7. The government budget is balanced. Transfer payment $\tilde{t}r$ is used to balance the budget.

$$\tilde{T}_t + \tilde{B}eq_t + (1 + g_A)(1 + n)\tilde{D}_t = \tilde{G}_t + \tilde{T}r_t + (1 + r_t)\tilde{D}_t \quad (25)$$

8. Social security system is balanced:

$$\tilde{P}en = \tau_t^{pen} w_t \tilde{L}_t \quad (26)$$

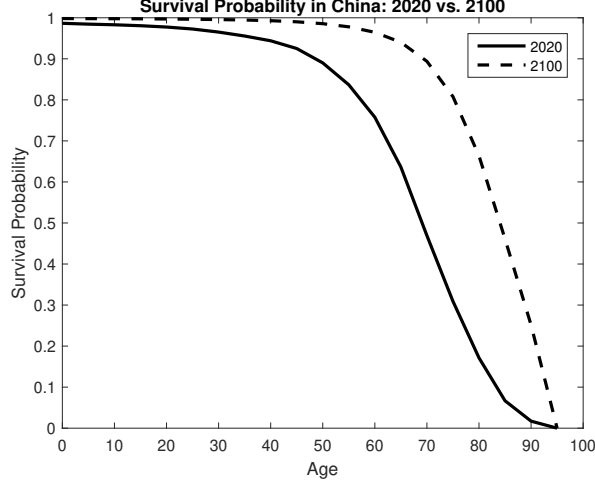


Figure 4: Conditional Survival Probability of 2020 vs. 2100

9. Goods market clears, capital depreciates at the rate of δ :

$$\begin{aligned}\tilde{Y}_t &= \tilde{K}_t^\alpha \tilde{L}_t^{1-\alpha} \\ &= (1 + g_A)(1 + n)\tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t + \tilde{C}_t + \tilde{G}_t\end{aligned}\tag{27}$$

5 Calibration

In this paper, in order to adapt to the development of China's economy, our calibration strategy is to select commonly used parameters in the literature.

5.1 Demographics

We first assume that the period of the model is one year. In this case, we calibrate the model. According to İmrohoroglu(1998), individuals are born at the age of 21, when $s = 1$. Individuals in the economy retire at $s = 41$ and live up to $s = 65$ years¹, with a calendar of 85 years. If individual is older than 85 years old, the survival probability is zero.

Another aspect is conditional survival probability $\phi_{s=1}^J$, the data can be derived from China Census 2018.

We set the fertility rate from article İmrohoroglu(2018). Since urban population accounts for about 40% of China's population, from 1980 to 2011, each couple had 1.3 chil-

¹China generally stipulates that higher-level occupational retirement can be extended.

dren ($0.4 \times 1 + 0.6 \times 1.5 = 1.3$), we adopted the one-child policy to set the fertility rate in the economy at $n = 0.65$.

Based on the above data, we can calculate the population structure of the model. By comparing it with the actual data, we can see that the population structure of the model is roughly consistent with the model.

5.2 Labor Wage

First, we calibrate the efficiency data. The first aspect is the random impact of income, which is given by $\log(\mu_j) = \theta \log(\mu_{j-1}) + v_j$. Based on the research of He, Ning, Zhu(2015), we take $\theta = 0.86$ and variance $\sigma_v^2 = 0.06$. Then we use the technology of Tauchen(1986) and İmrohoroglu(2018), now we discretize the process into Markov chain and the value $\mu = \{0.36, 1.0, 2.7\}$.

On the other hand, we calibrate the age-specific labor efficiencies e_t^s . Based on He, Ning, and Zhu(2015), they use the data in CHNS. They used CHNS to obtain data on the average working hours per worker per week, which were calculated on the basis of two questions. "C5: For how many days in a week, on the average, did you work?" and "C6: For how many hours in a day, on the average, did you work?" We choose to use the method of Ludwig(2012): regress the data on the third-order polynomials, which are given in the following specifications.

$$\log e_s = \eta_0 + \eta_1 j + \eta_2 j^2 + \eta_3 j^3 + \epsilon_s \quad (28)$$

Here, e_j is age-specific productivity and ϵ_j is residual. And the coefficient vector represented by $\hat{\eta} = [\hat{\eta}_1, \hat{\eta}_2, \hat{\eta}_3]'$ determines the polynomial slope estimated according to the actual working time data.

5.3 Technology and Preferences

Based on Song, Storesletten and Zilibotti(2011), depreciation ratio $\delta = 10\%$ and capital share $\alpha = 0.5$. The growth rate of technology is set according to the way of İmrohoroglu(2018), they set the TFP factor is $\gamma_t = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\alpha}} = 1.062$, so the $g_A = 0.031$.

As for parameters of preferences, we set them from Krueger and Kubler(2006). The utility function parameter $\sigma = 2$, then $\psi = 1 - \sigma = -1$, $\rho = -1$ and subjective discount factor $\beta = 0.92$.

5.4 Pension System

According to the research of He, Ning, Zhu(2015), they set the replacement ratio $\theta_{pen} = 0.5$ $m = 1$ and average hours ratio $\bar{l} = 0.379$.

6 Results

6.1 Backfitting

For backfitting, some articles use agent's decision data, and some use macro data for backtesting. We decided to use the debt in the macro data for backtesting because debt is the main issue we studied. In fact, because China's microdata are highly heterogeneous, especially in China's urban and rural areas, whether it's consumption changes or labor supply, it may not be easy to fit using microdata. We use the data of China's total debt

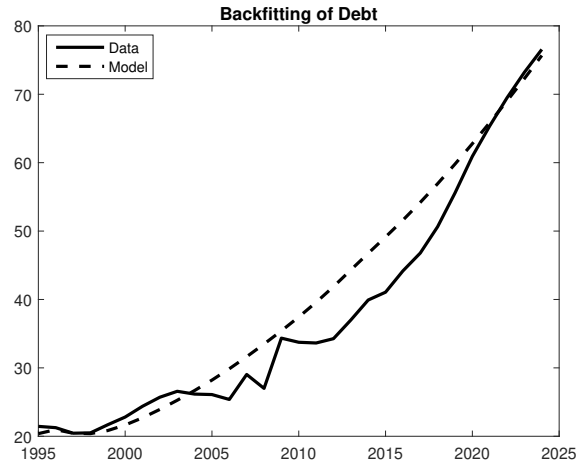


Figure 5: Backfitting of Debt Ratio

to GDP. By calibrating parameters, we have fitted(Figure 5) the changes in debt growth fundamentally.

6.2 Agent Profiles

Figure demonstrates that the asset first increases and then decreases with age. This hump-shaped distribution method is consistent with different retirement ages, whether it is $R = 55$, $R = 60$, or $R = 65$. This is consistent with the laws of most similar studies. It

can be seen that the turning point occurs around retirement. After retirement, individuals begin to consume previously saved assets, gradually reducing their assets. In the absence of a legacy, asset changes eventually go to zero.

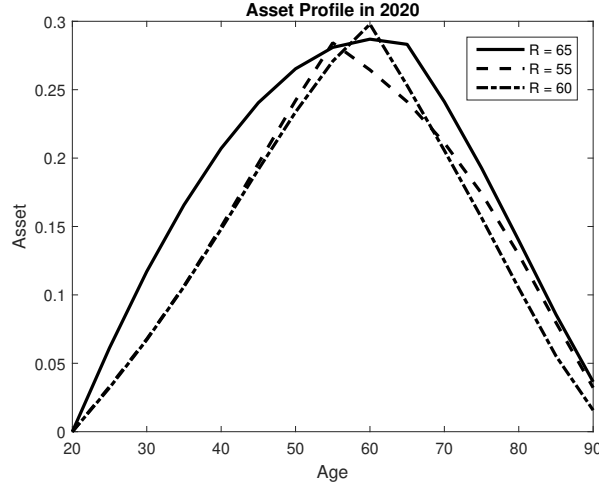


Figure 6: Asset Profile for $R = 55$, $R = 60$, and $R = 65$ (2020)

According to economic principles, consumption is an important component of gross national product. The increase in consumption will promote economic development and reduce the pressure on the government from the pension gap. We explored the changes in consumption pathways under different retirement ages and different tax rates. The differences between different ages are obvious. When $R = 65$, the path is the basic hump shape, which is consistent with the traditional theory. However, the situation is different between $R = 55$ and $R = 60$. The trends of these two cases are generally the same, but they decrease first and then increase. A possible explanation for this situation is that under different retirement ages, the path of age-efficiency profile is different, which leads to different labor supply. We can confirm this situation in the next section.

On the other hand, the total change in consumption is not monotonous. When $R = 60$, consumption is obviously larger than the other two cases at every moment. It can be inferred that as the retirement age of individuals increases, consumption tends to increase first and then decrease.

Labor supply will directly determine unemployment and employment conditions, which is affected by many factors. We will then explore how labor supply will change in the context of aging. In recent years, the aging of the population has caused widespread concern in society. One of the important reasons is that the aging of the population will affect the economic growth of a country through factors such as labor supply, savings and capital investment. The aging of the population has a negative impact on China's economic growth, and to some extent inhibits the positive impact of human capital on

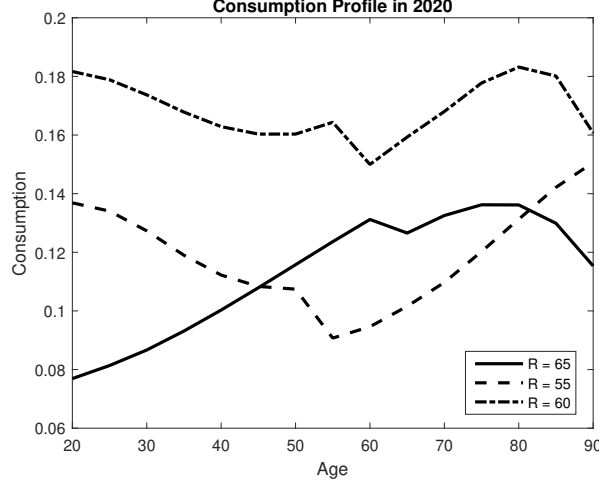


Figure 7: Consumption Profile for $R = 55$, $R = 60$, and $R = 65$ (2020)

economic growth. In the context of population aging, human capital investment has a significant impact on economic growth. Therefore, increasing human capital investment and improving labor knowledge reserve and professional quality are the inevitable choices to cope with the aging of the population and promote economic growth. Intuitively, aging will increase the burden on the younger generation and should increase the labor supply. The first policy tool to solve this problem is to delay the retirement age. Different retirement ages lead to different labor supply.

The figure shows the changes in labor supply at three different retirement ages, reflecting the huge differences in labor supply strategies at different retirement ages. When $R = 55$ and $R = 60$, the labor supply trend is the same. In contrast, the labor supply is more at each moment when $R = 55$, because under the same life length, if the working hours are relatively reduced, the labor supply will increase. However, the labor supply in the case of $R = 65$ shows the opposite trend, and the labor supply has been declining. This situation may be caused by different age-profiles.

But does the lifetime labor supply increase or decrease? We can judge that as the retirement age increases, the total labor supply may decrease first and then increase. This can be obtained from the aggregate amount of the economy.

Many scholars have done some research on the savings rate. İmrohoroğlu (2018) uses a model economy full of altruistic factors to compare it with China's economy and to examine the role of various factors in the change of savings rate. It is concluded that the interaction between long-term capital risk and demography plays an important role in improving the savings rate. The savings rate will increase from 20% in the 1980s to about 25% in 2010. The possibility that the government or family members are insufficient in old age insurance can generate a substantial increase in the savings rate in China. He, H.,

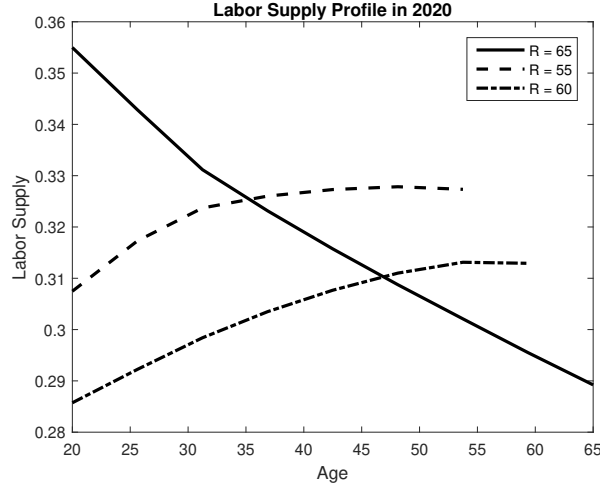


Figure 8: Labor Supply Profile for $R = 55$, $R = 60$, and $R = 65$ (2020)

Ning, L., Zhu, D.(2015) points out that for working age, the impact of rapid aging and pension reform on savings rate and labor supply increases with age, and pension reform plays a dominant role because individuals are getting closer to retirement age. This is mainly due to the rapid aging of the post-retirement savings rate. Dynamic efficiency is the core issue in analyzing the impact of fiscal policy, that is, pension policy reform. Diamond (1965) pointed out that a competitive economy can achieve a stable state, in which there is obviously too much capital. If the population growth rate exceeds the stable state of marginal international capital products, or if the sustained investment of the economy exceeds profits, then the economy is considered to be dynamic inefficient. Accumulative improvements can be achieved in a dynamic, inefficient economy that allows the current generation. Part of the capital stock, and then keep the consumption of all future generations unchanged. If the equilibrium of social security is not dynamic and effective, there is no need for incomplete market and improved risk allocation to provide a normative reason for the introduction of PAYG social security. Andrew Abel et al. (1989) provided a sufficient condition for the dynamic efficiency of the two-stage OLG model, which can be experientially tested. In implementing this test, they strongly supported the hypothesis that the U.S. economy (and the economies of other industrialized countries) was dynamic and effective. Next, we explore the dynamic effectiveness of the model economy under the conditions of pension reform. In the framework of this paper, what we want to study next is the impact of pension reform on the savings rate.

The figure shows the changes in the savings rate at different retirement ages, and we see that the overall change trend for different ages is consistent. In comparison, the savings rate of $R = 65$ is larger, and $R = 55$ is larger than $R = 60$. But high savings rates do not benefit the economy. Since savings have not been translated into actual benefits

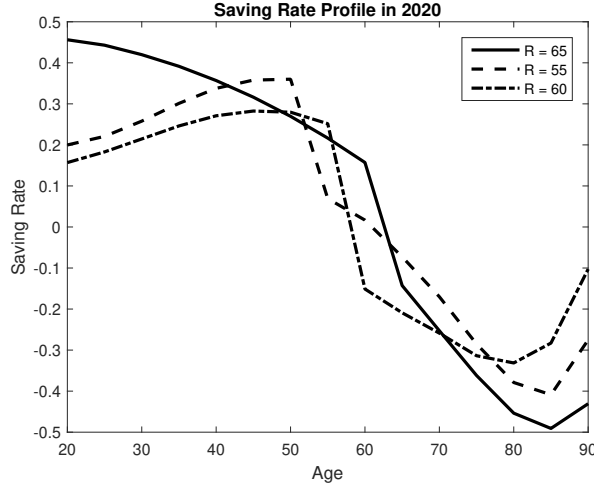


Figure 9: Saving Rate Profile for $R = 55$, $R = 60$, and $R = 65$ (2020)

in the economy, it is not possible to improve the Pareto efficiency of the economy.

6.3 Dynamic Transition

6.3.1 Policy Tool: Retirement Age

So far, our analysis has focused on comparatively static analysis. Next, we will study the economic dynamics and the changing characteristics of the model economy. Specifically, we assume that the survival probability and population growth rate of the model population are the same as those predicted by the original data. Since 2100, the population variable has remained unchanged, equal to the population of 2100. The setting is in line with the fact that the current birth rate is declining. Therefore, we further believe that by 2200, the transfer of the new stability system has been completed. In addition, the level of government expenditure and transfer remained unchanged in 2015.

There are two main fiscal policies used in our simulations. The first is to rely on long-term debt financing, and the second is to finance short-term debt and long-term tax financing.

Note that what we simulate here is when we continue to use debt, that is, government expenditure is always financed by government debt. From the perspective of capital stock, as the retirement age increases, the capital stock decreases. From the perspective of the growth trend, the growth of capital stock is accelerating. Capital stocks fell after debt was stopped in favor of wage tax financing.

Consumption changes tend to increase first and then decrease, and the consumption of $R = 55$ and $R = 60$ has not changed much. It is worth noting that an increase in

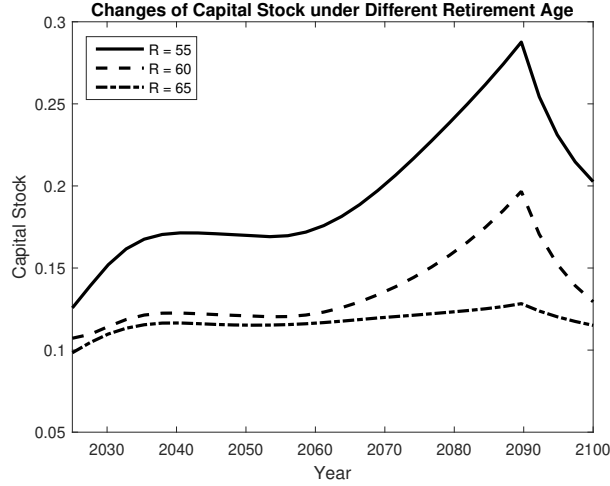


Figure 10: Changes of Capital Stock under Different Retirement Age(2025-2100, Long Term)

retirement age will cause a significant decline in consumption, mainly due to a reduction in capital per capita.

From the perspective of labor supply, it proves that we see in the agent profile that $R = 65$ is the largest labor supply, followed by $R = 55$. The overall trend is gradually decreasing.

The issue of debt is the main concern of this article. Different fiscal policies adopted by the government will lead to different debt accumulation. Under the policy of different retirement ages and long-term use of debt, the ultimate change in debt varies significantly. Among them, the changes in the total debt of $R = 55$ and $R = 60$ tend to be the same, and the debt decreases significantly when $R = 65$. This is because the total supply in the economy increases, but the debt Does the decline represent a drop in debt pressure? It can be seen from the debt ratio that this does not hold. At the initial moment, the debt ratio is the largest at $R = 55$. The initial debt ratios of $R = 55$, $R = 60$ and $R = 65$ are 79.91%, 77.24% and 75.69% respectively. However, the debt ratio of the former two has fallen rapidly, but with the extension of the debt period, the debt ratio has picked up. The debt period stopped and the debt ratio dropped again. In contrast, the debt at $R = 65$ has always declined but declined slowly.

Moreover, the overall debt level does not change monotonically, there is a so-called "optimal" retirement age, because at $R = 60$, the total debt is the lowest. This shows that blindly extending the retirement age cannot reduce debt levels.

6.3.2 Policy Tool: Tax Rate

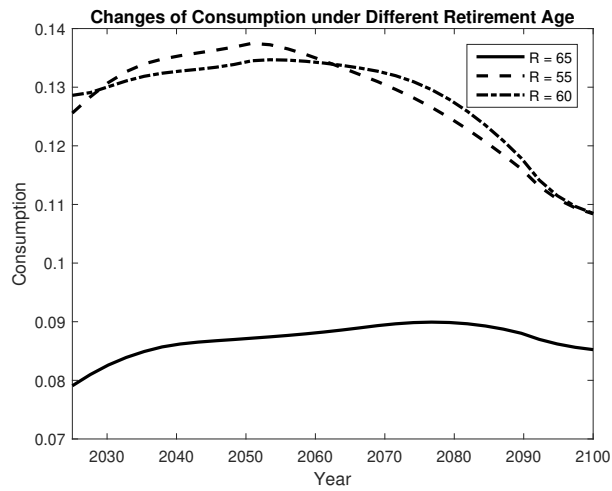


Figure 11: Changes of Consumption under Different Retirement Age(2025-2100, Long Term)

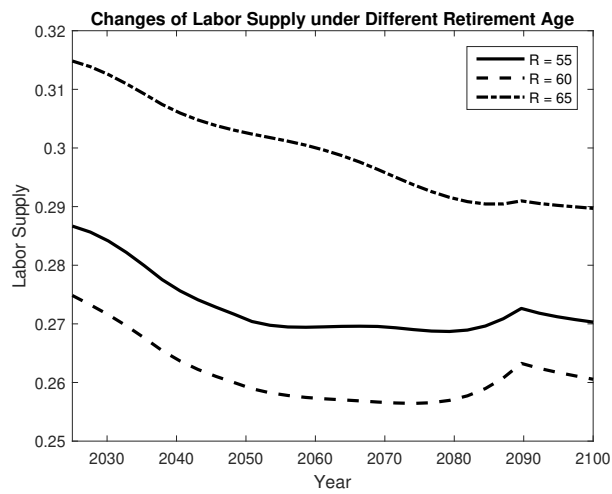


Figure 12: Changes of Labor Supply under Different Retirement Age(2025-2100, Long Term)

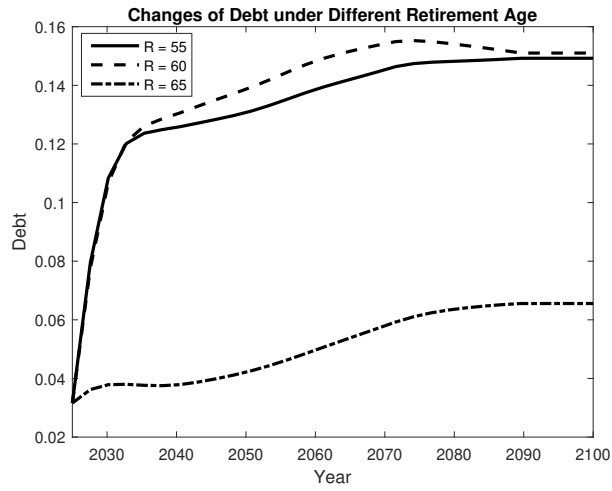


Figure 13: Changes of Debt under Different Retirement Age(2025-2100, Long Term)

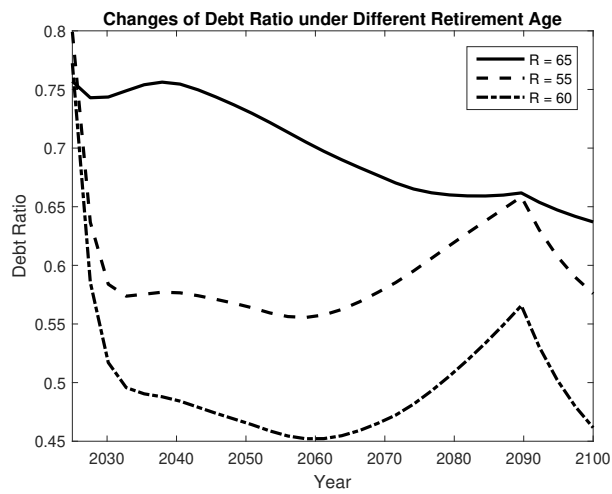


Figure 14: Changes of Debt Ratio under Different Retirement Age(2025-2100, Long Term)

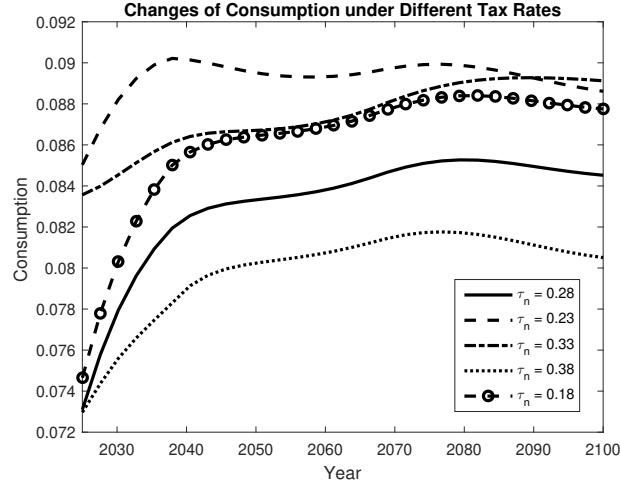


Figure 15: Changes of Consumption under Different Tax Rates(2025-2100, Short Term)

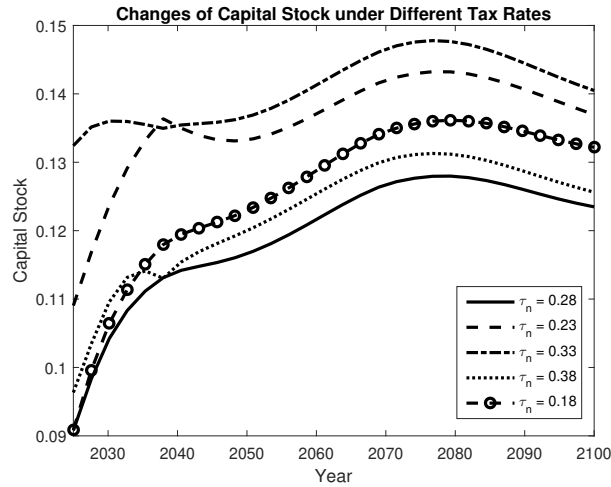


Figure 16: Changes of Capital Stock under Different Tax Rates(2025-2100, Short Term)

Another policy scenario is short-term debt financing and long-term tax financing. We have to compare the results of similar policies and different types of policies separately. For consumption, it is expected that the overall consumption will be lower than that under the first type of policy. For different tax rates, the total consumption is highest when the tax is 23%, and lowest when it is 38%. It can be seen that the impact of taxation on consumption is not monotonous, and it also tends to decrease first and then increase.

Similarly, changes in capital stocks and consumption trends are consistent. The capital stock is significantly lower than in the case of long-term debt. Furthermore, the capital stock also follows the trend of rising first and then falling.

The labor supply is different. Overall, the labor supply in the two policy situations is not much different. Moreover, the amount of labor supply and the tax rate show a linear

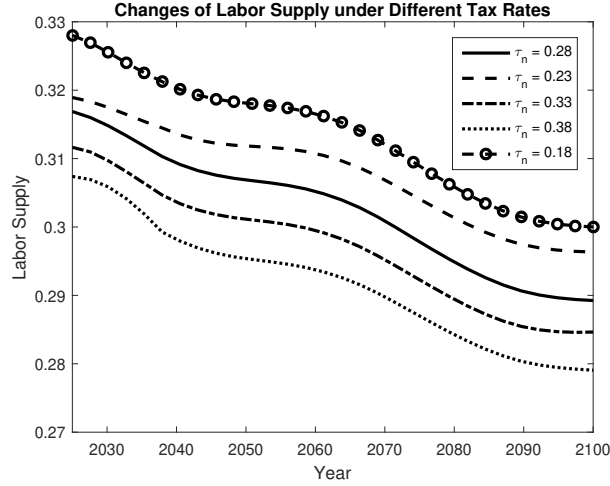


Figure 17: Changes of Labor Supply under Different Tax Rates(2025-2100, Short Term)

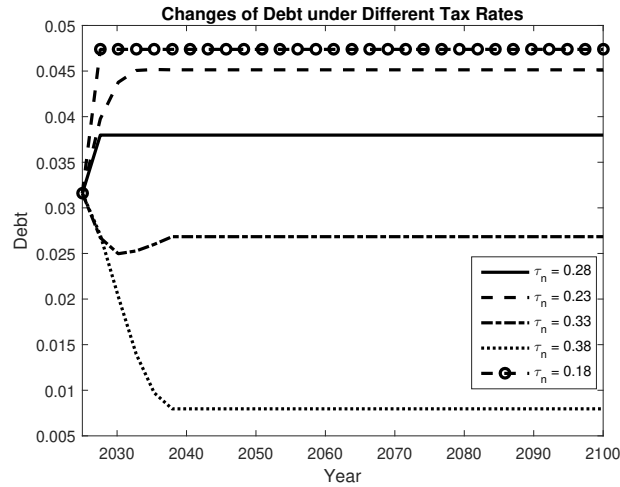


Figure 18: Changes of Debt under Different Tax Rates(2025-2100, Short Term)

relationship. The lower the tax rate, the greater the amount of labor supply. With the extension of time, the labor supply has decreased year by year.

At the same retirement age, taxes are also monotonous. Rising taxes mean falling debt, and total debt no longer changes after debt financing is stopped. We remain concerned about the proportion of debt. It can be seen that the debt ratio shows a completely opposite relationship. At the same retirement age, the debt ratio at the 38% tax rate is the highest, which is about 94%. At 18%, it is 73.5%. This shows that raising the tax rate does not necessarily reduce the debt ratio. On the contrary, it is possible to increase the debt (of course, we have not simulated the case where the tax rate is less than 18%).

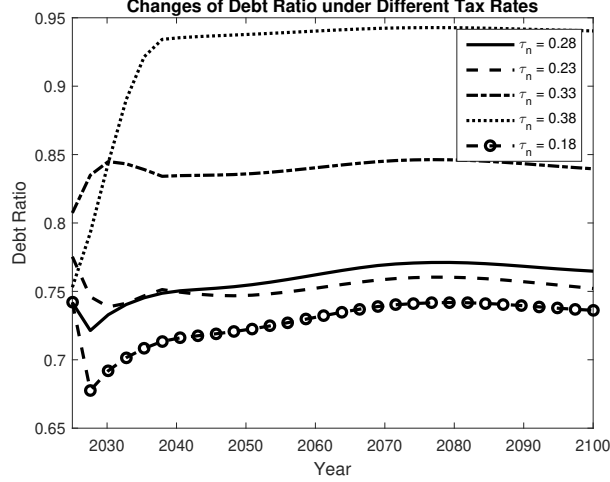


Figure 19: Changes of Debt Ratio under Different Tax Rates(2025-2100, Short Term)

6.4 Optimal Policy

Debt is one of our concerns, but an increase in government burden does not necessarily mean an increase in residents' welfare. Pay-as-you-go pension systems have many effects on benefits. First, social security provides partial insurance through intergenerational and intergenerational income redistribution. Social insurance prevents personal income uncertainty to a certain extent because it is redistributed from high-income workers to low-income workers and bears the impact of some income changes. Second, because most households in the economy are constrained by credit during periods of low income, they are unable to use debt, which worsens the welfare effect. Social insurance benefits reduce benefits and labor supply. Third, the additional savings mean that the total consumption of different dynamic economic efficiencies (in this model) will also be reduced. Whether the consumption effect can make up for the huge welfare distortions caused by the decrease in labor supply and savings can usually be determined quantitatively through models. We will explore the effects of the use of social security policy tool in both policy strategies (ie, short-term debt and long-term debt).

6.4.1 Policy tools: Retirement Age

First, define the total social welfare function following İmrohoroğlu et al. (1995).

$$W = \sum_{j=1}^J \sum_k \sum_{\eta} \beta^{j-1} \lambda_j(k_j, \eta_j) \left(\prod_{i=1}^j \psi_i \right) u(c_j(k_j, \eta_j), l_j(k_j, \eta_j)) \quad (29)$$

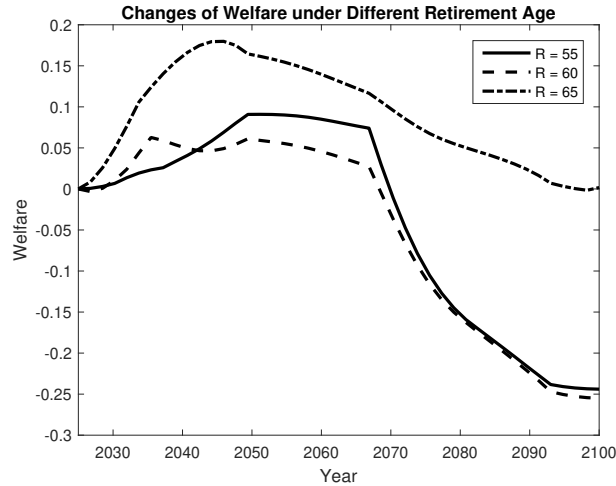


Figure 20: Changes of Welfare under Different Tax Rates(2025-2100, Long Term)

We can see that the welfare effect of taxation is delayed, and after about 5-10 years, the proportion of welfare increase reaches the maximum. In the case of $R = 55$, $R = 60$, and $R = 65$, they increased by 9.09 %, 6.28%, and 17.98%, respectively. Even if the ratio of debt is relatively high in the case of $R = 65$, the increase in benefits is more obvious, and it is Pareto's improvement throughout the simulation. Obviously, the other two cases are not that they only improve the welfare of some generations, and the welfare reduction for future generations is relatively large. In the case of $R = 55$, $R = 65$, the benefits in 2100 are respectively Reduced by 24.39% and 25.50%.

6.4.2 Policy tools: Tax Rate

What effects does taxation have on the economy as a whole? First, taxes will reduce the overall welfare of the economy. Because young people in the economy consume more personally, they account for most of the welfare in the economy. Tax policies will lead to a decline in the welfare of the younger generation in the economy. However, the increase in tax rate can reduce the government's debt to a certain extent, so in the long run, the consequence of low tax rate is a reduction in economic growth rate, which is not good for the economy. Therefore, a reasonable tax rate should increase long-term economic welfare, and the welfare of the older generation will be improved.

Specifically, the simulation results confirm the existence of the optimal tax rate, but this change is complicated and does not show a certain monotony. Among these five types of tax rates, 28% is the best, regardless of the maximum or minimum change, while the difference of 18% is not significant. The 33% strategy is the worst from the results.

Table 1: Changes of Welfare under Different Tax Rates

Tax Rates	Maximum change	Minimum change
$\tau_n = 0.18$	22.60%	-0.87%
$\tau_n = 0.23$	10.62%	-6.89%
$\tau_n = 0.28$	22.65%	0.43%
$\tau_n = 0.33$	5.46%	-8.46%
$\tau_n = 0.38$	14.71%	-2.57%

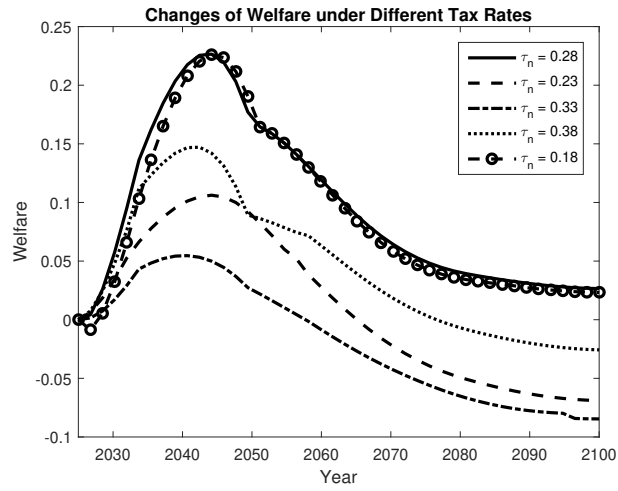


Figure 21: Changes of Welfare under Different Tax Rates(2025-2100, Short Term)

7 Conclusion

We develop a dynamic general equilibrium model. Debt and welfare effects are two very influential aspects of policy regulation. By simulating a large-scale OLG model under two policy scenarios, we have obtained a series of results, which show that different retirement systems, that is, different ages, and different tax rate policies will lead to welfare changes. Specifically, it mainly explores the changes in the use of short-term debt and long-term tax or long-term debt and short-term tax, debt and welfare levels under two policy strategies. From the results, there are relatively optimal tax rates and retirement ages, but due to the amount of calculations, we cannot simulate all situations. Under different retirement ages, as the retirement age increases, the maximum increase in benefits is 17.98%, and the increase in debt is 75.69%. Under the five tax rates, the optimal tax rate is 28%, the maximum increase in benefits is 22.65%, and the maximum debt ratio is 75%. Nevertheless, we are unable to determine specific optimal tax rates and retirement ages because the changes are complex and non-monotonic. But we can give a certain scope, which can provide a guide for policy formulation. Of course, there are many policy setting strategies, such as using tax and debt to finance at intervals, we can all simulate.

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A Appendix for Computation Method

For the individual in the economy, the recursive utility function should adopt the dynamic programming method to solve the optimal strategy. Individuals plan their lifelong strategies at an early stage. Because of the uncertainty, shooting method can not be used. This method can only be used in the deterministic summation. Therefore, we adopt K. Krusell and Smith (1998) method to iterate the density function.

For dynamic programming problems, finite-term dynamic programming and infinite-term dynamic programming are different. In contrast, infinite dynamic programming can be regarded as a homogeneous problem, because there is no difference between different time and infinite time. However, for a finite terms problem, this is not the same, different time will start to make the problem is not homogeneous. So we can only start planning from the last phase.

As far as the last issue is concerned, the value function is zeros, so the value function is:

$$V_t(k_t^J, J, \eta) = \max_{k_{t+1}^{J+1}, c_t^J, 0} u(c_t^J, 0) \quad (30)$$

According to our assumptions, $k^{J+1} = 0$. Next we can covert the dynamic programming problem into

When calculating multivariate optimization problems, we can express all decision variables in terms of state variables. If we use budget constraints instead, we determine the interval and search for the optimal value in a certain range. Here we use the so-called golden section search method, the basic idea of this method is to gradually narrow the interval. In the next step, select two points in the interval and select a lower value. The right side of the above equation becomes the new limit point of the interval. The golden section search method optimizes the selection of these new points, which can ensure that the number of searches is minimum. This process keeps iterating until the interval length is small enough, and then we stop.

For a working agent, the problem of maximization is more complex because he also chooses his optimal labor supply. There are various numerical methods to calculate this two-dimensional optimization problem. Here we choose to transform the problem into two nested one-dimensional optimization problems, and apply the golden section search in the outer cycle to calculate the capital in the next period and directly from the first-order conditions, and work with the help of Gauss-Newton algorithm. Optimize the inner cycle of selection. Once the individual optimization problem is solved, we can aggregate individual savings and labor supply to derive the total amount and update our

initial estimates of \tilde{K} , \tilde{L} and factor prices \tilde{w}_t, r . Next, the value of τ_{pen} and tr_t will be calculated through the budget balance between the government and the pension system. We update the old values by weighted average and iterate until convergence. The complete algorithm is described in the following procedure.

Algorithm A.1

Step 1: Guess the initial steady-state values, including \tilde{K} , \tilde{L} , and accident bequest Beq_t .

Step 2: The optimal strategy function is calculated through the dynamic programming process, and the value function iteration method is used in this process.

Step 3: Calculate the sequence of government strategies $w_t, r_t, \tilde{r}, \tau_{pen}$, and pen_t .

Step 4: Calculate the overall economic variables for each period $\tilde{K}, \tilde{L}, Beq_t$.

Step 5: According to market clearing, budget balance and other conditions, iterate the economic total variables until step 3 convergence.