Open-Economy Macroprudential Policy: International Financial Arbitrage Channel

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Unfinished Draft still needs to Elaborate

1 Introduction

Financial crises are the main incentives to push economists to discuss how to mitigate the consequences of recession such as asset price crashed and aggregate demand impotency (see Borio [2003]). As a typical solution of flatting financial cycle, macroprudential policy has become new pillar of modern debates of optimal policy and financial cycle under the context of global low interest rate close to zero lower bound (ZLB) (see Caballero et al. [2021]). There are a series of articles to discuss the optimal policy under the environment of extremely low interest rate globally and different expectation. However, these discussions normally lack for the consideration international financial channels recently stressed by Itskhoki and Mukhin [2021a] and Itskhoki and Mukhin [2021b]. It is natural to cast doubt on the implication of traditional optimal policy without international financial market power like arbitrage or noise trade. In another aspect, our knowledge of optimal combination of policy toolkit is still limited because too many ingredients need to be accounted for such as agents' expectation, the efficacy of policy under open-economy context or ZLB and the difficulty to solve social planners problem with several policy tools at the same time. The target of this article is to sort out the relationship among various policy tools like monetary policy, macroprudential policy and exchange rate policy in a simplified environment. Under the rational expectation and unrational expectation, optimal policy combination also varies correspondingly.

In the first part of article, I follow the analysis of Farhi and Werning [2020], but incorporate the elements of financial arbitrage. This is a 3-period Minsky circle with boom and bust. In this context, there are two types of agents-borrowers and savers, which produce the excessive demand of financial market. Key difference from other research is I introduce a channel to link domestic and international market. Specifically, savers and borrowers have different preference on tradable and non-tradable goods following home-biased phenomenon, which creates a tunnel that bridges exchange rate and domestic consumption. As a abitrager, she constructs zero-portfolio to earn the interest margin, then makes decision from the risk-averse profit preference function. International financial arbitrage will wield its effect from this channel and the effect will be enlarged by the

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proportion of arbitragers. To manage the demand externality, two policy tools can be used: one is monetary policy to manage the supply to the international bond market, and another is FX intervention to control future profit of asset (UIP condition). To streamline and address the core viewpoints, I simplified other parts of the analysis: ZLB is binding during the bust period and all of uncertainty resolved in the 3rd period. Wage is rigid and only borrowers can enter into the labor market. Only borrowers take the risk of bond market which embodied on the output of Lucas's Tree.

The structure is designed as follows: In Section 2, I will discuss the optimal policy combination under rational and irrational expectations. In Section 3, I discuss the question in a fully dynamic environment with long-term expectation.

Related Literature My research is related to three aspects of research. The first aspect mainly discusses international finance channels, see Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021a], Itskhoki and Mukhin [2021b]. In these papers, international capital follow, financial arbitrager, and noise trader enhance exchange rate volatility. And then these effects are transfer from good price channel to consumers who have no involvement on international finance market. I will follow the same spirit and incorporate this channel into the analysis of optimal exchange rate policy. From optimal policy perspective, there are two mainstream methods: One is FX intervention under global low interest rate Amador et al. [2019], or how capital flow intertwines with FX intervention Cavallino [2019], and limited international financial market involvement Fanelli and Straub [2021]. Another is capital control such as Jeanne and Korinek [2010], Costinot et al. [2014], Schmitt-Grohé and Uribe [2016], Farhi and Werning [2017]. Egorov and Mukhin [2021] These policies mainly focus on how to use tax to control "hot money" to maintain the stability of economy. And these policies are also embodied in this article in the context of international financial market.

The second aspect is macroprudential policy especially how to prevent excessive demand and financial arbitrage. Jeanne and Korinek [2020], Caballero and Simsek [2020], Caballero and Simsek [2019], Lorenzoni [2008], Korinek and Simsek [2016], Farhi and Werning [2016] including recent work Bianchi and Coulibaly [2022]. This thread of research put more emphasis on the demanding externalities, i.e. the excessive demanding is attributed to over-pessimistic/optimistic expectations or limited access to financial market, which causes extra volatility on commodity price have to be burdened by savers.

2 Optimal Policy Combination under Rational Expectation

I follow the analysis of Farhi and Werning [2020] but consider new ingredient of international financial market environment. There are 3 periods $t \in \{0, 1, 2\}$ and the "boom and bust" states $\omega \in \{H, L\}$ describe two different state of economics cycle. The difference is encoded in the output of Lucas tree $\{D^H, D^L\}$ due to the uncertainty of future.

Households are composed by two parts: savers and borrowers. Because borrowers can join financial market and skin in the game, the consumption of second period is under the expectation $\mathbb{E}\left[\log C_2^B\right]$. Savers gain labor income and the disutility function of labor is $h(l_1^S)$.

$$\begin{aligned}(1-\beta_0)\ln C_0^S + \beta_0(1-\beta_1)\left[\ln C_1^S - h\left(l_1^S\right)\right] + \beta_0\beta_1 \log C_2^S \\ (1-\beta_0)\ln C_0^B + \beta_0(1-\beta_1)\ln C_1^B + \beta_0\beta_1\mathbb{E}\left[\log C_2^B\right]\end{aligned} \tag{1}$$

Consumption derives from two kinds of goods: tradable and non-tradable goods $C_t = C_{Tt}^{\gamma} C_{Nt}^{1-\gamma}$ and the elasticity is γ. The proportion of tradable and non-tradable counterparts is decided by cost-minimizing problem

$$\frac{\gamma C_{Nt}}{(1-\gamma)C_{Tt}} = \frac{P_{Tt}}{P_{Nt}} \tag{2}$$

I assume the non-tradable good price is rigid $P_{Nt} = 1$, and then tradable good price follows the exchange rate $P_{Tt} = \mathcal{E}_t$. Define aggregate price P_t follows the relationship $P_t C_t = P_{Tt}C_{Tt} + P_{Nt}C_{Nt} = \frac{C_{Nt}}{1-\gamma}$, which is equivalent to

$$P_t = \frac{(1-\gamma)^{\gamma-1}}{\gamma^{\gamma}} P_{Nt}^{1-\gamma} P_{Tt}^{\gamma} \tag{3}$$

International abitragers construct zero-portfolio $\frac{b_t^A}{R_t} + \frac{\mathcal{E}_t b_t^*}{R_t^*} = 0$, and they set the optimal position by following risk-averse preference

$$V_t\left(\pi_{t+1}^{D^*}\right) = \mathbb{E}_t\left(\pi_{t+1}^{D^*}\right) - \frac{\omega}{2} var_t\left(\pi_{t+1}^{D^*}\right) \tag{4}$$

UIP condition in international finance market is given by the FOC of preference above:

$$\frac{\mathbb{E}_t\left\{R_t^* - \frac{\mathcal{E}_t R_t}{\mathcal{E}_{t+1}}\right\}}{\omega var_t\left(\pi_{t+1}^{D*}\right)} = \frac{\mathbb{E}_t\left\{R_t^* - \frac{\mathcal{E}_t R_t}{\mathcal{E}_{t+1}}\right\}}{\omega R_t^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)} = \frac{b_t^*}{R_t^*} \tag{5}$$

Therefore, the position of foreign currency bond is $b_t^* = R_t^* \frac{R_t^* - R_t \mathbb{E}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)}{\omega R_t^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)}$. Domestic bond market with these three types of trader should be clear

$$\phi^S b_t^S + \phi^B b_t^B + \phi^A b_t^A = 0 \tag{6}$$

Combine with UIP condition (5), the bond market equilibrium can be represented by

$$\phi^S b_0^S + \phi^B b_0^B + \phi^A \frac{\overline{\mathcal{E}_0} R_t \mathbb{E}_t\left(\frac{1}{\mathcal{E}_1}\right) - R_0^*}{\omega R_t \overline{\mathcal{E}_0} var_t\left(\frac{1}{\mathcal{E}_1}\right)} = 0$$

$$\phi^S b_1^S + \phi^B b_1^B + \phi^A \mathcal{E}_1 \frac{\frac{1}{\mathcal{E}_2^2} R_t \mathbb{E}_t\left(\mathcal{E}_1\right) - R_1^*}{\frac{1}{\mathcal{E}_2^2}\omega R_1 var_t\left(\mathcal{E}_1\right)} = 0$$

Technology non-tradable output Y_{Nt} At date 0 and 1.

$$\phi^S C_{Nt}^S + \phi^B C_{Nt}^B \le Y_t = \phi^S l_t^S$$

$$\phi^S C_{N2,\omega}^S + \phi^B C_{N2,\omega}^B \le Y_{2,\omega} = D_{2,\omega}$$

For these two types of agents, borrowers and savers, they face different types of constraint. As savers, they receive the income from labor l_0^S and government transfer t_t^S. and invest in riskless bond b_t^S;. The tradable goods' consumption is financed by international tradable goods endowment process Y_{Tt} and the tradable goods price counted in dollars P_{Tt}. Domestic goods' price is denoted by P_t composed by tradable and non-tradable price P_{Nt} and P_{Tt} I mentioned above. In the whole sections, I assume price of non-tradable goods is rigid i.e. $P_{Nt} = 1$ constantly, which streamlines our normative analysis of exchange rate $\mathcal{E}_t$.

$$P_0 C_0^S + \frac{b_1^S}{R_0} \leq P_{T0} Y_{T0} + b_0^S + l_0^S + t_0^S, \quad l_0^S = \frac{Y_0}{\phi^S}$$
$$P_1 C_1^S + \frac{b_2^S}{R_1} \leq P_{T1} Y_{T1} + b_1^S + l_1^S, \quad l_1^S = \frac{Y_1}{\phi^S}$$
$$P_2 C_{2,\omega}^S \leq b_2^S$$

Similarly, the constraints of borrowers are:

$$P_0 C_0^B + \frac{b_1^B}{R_0}(1-\tau_0) + \left(x_1^B - \frac{1}{\phi^B}\right)\mathcal{P}_0 \leq P_{T0} Y_{T0} + t_0^B + b_0^B$$
$$P_1 C_1^B + \frac{b_2^B}{R_1} + \left(x_2^B - x_1^B\right)\mathcal{P}_1 \leq P_{T1} Y_{T1} + b_1^B$$
$$P_2 C_{2,\omega}^B \leq b_2^B + x_2^B D_{2,\omega}$$

x_t^B is risky Lucas tree holding at date t, and $\mathcal{P}_t$ is the price of the Lucas tree at date t and it is different from P_t (it is commodity price). Government wields policy of capital tax $\{\tau_0\}$ and transfer $\left\{t_0^S, t_0^B\right\}$ to savers and borrowers. For the convenience of analysis, I assume borrowers and savers follow the same tradable goods endowment process.

Arbitrager. Each borrower and arbitrager is endowed with $\frac{1}{\phi^B}$ and $\frac{1}{\phi^A}$ of the Lucas tree at time 0 . Government budget constraint

$$\phi^S t_0^S + \phi^B t_0^B + \frac{b_1^B}{R_0}\tau_0 + \frac{b_1^A}{R_0}\tau_0^A \leq 0$$

Assume $P_0 = 1$ and $P_2 = 1$, i.e. $\mathcal{E}_0 = \overline{\mathcal{E}_0}, \mathcal{E}_2 = \overline{\mathcal{E}_2}$ exchange rate is a constant number in the first period and last period. It can be viewed as the central bank will deliver the commitment to peg down the exchange rate in the future, which is a method to manage the expectation of agents efficiently.

Optimal consumption problem of savers deliver following Euler's equation (see Appendix for proof).

$$C_1^S = \frac{1-\beta_1}{\beta_1}\frac{1}{P_1}C_2^S \tag{7}$$

equation (7) predicts that if period 1's exchange rate increases (foreign goods' price increase more than domestic), consumption of 1 period will be dampened.

Lemma 1. *Under first-best allocation, the price of Lucas's Tree is proportion to the period 1's real output $\frac{Y_1}{P_1}$. Specifically,*

$$\mathcal{P}_1 = \frac{\beta_1(1-\gamma)}{1-\beta_1}\frac{Y_1}{P_1} \tag{8}$$

Proof. Total non-tradable goods consumption equal to the domestic output $\phi^S C^S_{N1} + \phi^B C^B_{N1} = Y_1$, $\phi^S C^S_{N2} + \phi^B C^B_{N2} = Y_2$. Aggregate consumption C_t is proportional to non-tradable consumption $C_t = \frac{C_{Nt}}{(1-\gamma)P_t}$. According to Euler's equation (7) $C^S_1 = \frac{1-\beta_1}{\beta_1}\frac{1}{\mathcal{E}_1}\frac{C_{N2}}{(1-\gamma)}$ and $C_{N1} = \frac{1-\beta_1}{\beta_1}C_{N2}$. Therefore, the ratio of output in period 1 and 2 is

$$\frac{Y_2}{Y_1} = \frac{\phi^S C^S_{N2} + \phi^B C^B_{N2}}{\phi^S C^S_{N1} + \phi^B C^B_{N1}} = \frac{\phi^S C^S_{N2} + \phi^B C^B_{N2}}{\phi^S \frac{1-\beta_1}{\beta_1}\frac{C^S_{N2}}{(1-\gamma)} + \phi^B \frac{1-\beta_1}{\beta_1}\frac{C^B_{N2}}{(1-\gamma)}} = \frac{\beta_1(1-\gamma)}{1-\beta_1}$$

Borrowers Euler's equation delivers that

$$\frac{1-\beta_1}{\beta_1}\frac{1}{C^B_1 E\left[\frac{1}{C^B_2}\right]} = \frac{D_{2,\omega}P_1}{\mathcal{P}_1} = \frac{Y_2 P_1}{\mathcal{P}_1} = \frac{\beta_1(1-\gamma)}{1-\beta_1}\frac{Y_1}{\mathcal{P}_1}$$

$$\mathcal{P}_1 = \frac{\beta_1(1-\gamma)}{1-\beta_1}\frac{Y_1}{P_1}$$

■

Lemma 2. *Under first-best allocation, Walras's Law guarantees the optimal consumption of borrowers C^B_1 to be*

$$C^B_1 = \Theta\mathcal{P}_1 - \frac{(1-\beta_1)}{P_1\phi^B}\left(\phi^S\mathcal{E}_1 Y_{T1} - \phi^B b^B_t - \phi^A b^A_t\right) \tag{9}$$

where $\Theta = \frac{(\gamma+\beta_1-\gamma\beta_1)}{\phi^B}\frac{(1-\beta_1)}{\beta_1(1-\gamma)^2}$

Proof. Given Euler equation (7) and $P_2 C^S_{2,\omega} = b^S_2$

$$C^S_1 = \frac{1-\beta_1}{P_1}\left(\mathcal{E}_1 Y_{T1} + b^S_1 + \frac{Y_1}{\phi^S}\right)$$

$$\begin{aligned}
\phi^B C^B_1(1-\gamma)P_1 &= Y_1 - (1-\beta_1)(1-\gamma)\left(\phi^S\mathcal{E}_1 Y_{T1} + \phi^S b^S_1 + Y_1\right)\\
&= Y_1 - (1-\beta_1)(1-\gamma)\left(\phi^S\mathcal{E}_1 Y_{T1} - \phi^B b^B_t - \phi^A b^A_t + Y_1\right)\\
&= (\gamma+\beta_1-\gamma\beta_1)\frac{(1-\beta_1)P_1}{\beta_1(1-\gamma)}\mathcal{P}_1 - (1-\beta_1)(1-\gamma)\left(\phi^S\mathcal{E}_1 Y_{T1} - \phi^B b^B_t - \phi^A b^A_t\right)\\
C^B_1 &= \frac{(\gamma+\beta_1-\gamma\beta_1)}{\phi^B}\frac{(1-\beta_1)}{\beta_1(1-\gamma)^2}\mathcal{P}_1 - \frac{(1-\beta_1)}{P_1\phi^B}\left(\phi^S\mathcal{E}_1 Y_{T1} - \phi^B b^B_t - \phi^A b^A_t\right)
\end{aligned}$$

■

The marginal propensity to consume $\frac{1-\beta_1}{P_1}$. Following from assumption that interest between dates 1 and 2 is $R_1 = 1$, we can get consumption at date 2.

Lemma 3. *Under Walras's Law, period 1 fixed-point equation is*

$$Y_1 = \frac{1-\beta_1}{\beta_1}\frac{1}{\mathbb{E}\left[\frac{1}{D_{2,\omega}-\beta_1(1-\gamma)\left(\mathcal{E}_1 Y_{T1}\phi^S + \phi^S b^S_1 + Y_1\right)}\right]} + (1-\beta_1)(1-\gamma)\left(\mathcal{E}_1 Y_{T1}\phi^S + \phi^S b^S_1 + Y_1\right) \tag{10}$$

Proof. Euler equation of borrowers is: $\frac{1}{P_1}\frac{1-\beta_1}{\beta_1}\frac{1}{\mathbb{E}\left[\frac{1}{C_2^B}\right]} = C_1^B$

$$\phi^B \frac{1-\beta_1}{\beta_1}\frac{1}{\mathbb{E}\left[\frac{1}{C_2^B}\right]}(1-\gamma) + \phi^S \frac{1-\beta_1}{P_1}\left(\mathcal{E}_1 Y_{T1} + b_1^S + \frac{Y_1}{\phi^S}\right) P_1(1-\gamma) = Y_1$$

Under the optimal allocation of savers in the first period,

$$C_1^S = \frac{1-\beta_1}{P_1}\left(\mathcal{E}_1 Y_{T1} + b_1^S + \frac{Y_1}{\phi^S}\right)$$

the second period is

$$C_2^S = \beta_1\left(\mathcal{E}_1 Y_{T1} + b_1^S + \frac{Y_1}{\phi^S}\right)$$

the borrower's consumption is:

$$(1-\gamma)C_2^B = \frac{D_{2,\omega}}{\phi^B} - \frac{\phi^S}{\phi^B}\beta_1\left(\mathcal{E}_1 Y_{T1} + b_1^S + \frac{Y_1}{\phi^S}\right)(1-\gamma)$$

Combing the equation, then yield a fixed point equation

$$\frac{1-\beta_1}{\beta_1}\frac{1}{\mathbb{E}\left[\frac{1}{D_{2,\omega}-\beta_1(1-\gamma)\left(\mathcal{E}_1 Y_{T1}\phi^S + \phi^S b_1^S + Y_1\right)}\right]} + (1-\beta_1)(1-\gamma)\left(\mathcal{E}_1 Y_{T1}\phi^S + \phi^S b_1^S + Y_1\right) = Y_1$$

■

Proposition 1. *(**Expenditure switching channel**) Under first-best optimal allocation, if period 2 exchange rate is pegged to ensure $P_2 = 1$, the optimal output varies with saving/debt of period 1:*

$$\frac{dY_1}{db_1^S} = \frac{\phi^S(1-\beta_1)(1-\gamma)\left(1 - \frac{\mathbb{E}\left[\left(\frac{1}{C_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{C_2^B}\right]\right)^2}\right)}{1-(1-\beta_1)(1-\gamma) + (1-\beta_1)(1-\gamma)\frac{\mathbb{E}\left[\left(\frac{1}{C_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{C_2^B}\right]\right)^2}} \tag{11}$$

and $\frac{dY_1}{db_1^S} < 0$ *constantly.*

Proof. Period 1's fixed point equation is

$$\phi^B \frac{1-\beta_1}{\beta_1}(1-\gamma)\frac{1}{\mathbb{E}\left[\frac{1}{C_2^B}\right]} + \phi^S \frac{1-\beta_1}{P_1}\left(\mathcal{E}_1 Y_{T1} + b_1^S + \frac{Y_1}{\phi^S}\right) P_1(1-\gamma) = Y_1$$

the derivatives of the left and right hand side is:

$$-(1-\beta_1)(1-\gamma)\frac{\mathbb{E}\left[\left(\frac{1}{C_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{C_2^B}\right]\right)^2}\left[\phi^S + \frac{dY_1}{db_1^S}\right] + \phi^S(1-\beta_1)(1-\gamma) + (1-\beta_1)(1-\gamma)\frac{dY_1}{db_1^S} = \frac{dY_1}{db_1^S}$$

This proves proposition 1. ■

Aggregate Demand

3 Optimal Policy Combination under Unrational Expectation

4 Fully Dynamic Environment

Household The representative household's preference is composed by utility from consumption and disutility of labor from labor supply. The price of each commodity is 1 for non-tradable commodity. C_{Nt}, p_{Ht} is the dollar price in the US of U.S. tradables. And p_{Ft} is dollar price in the US of foreign tradables. Technology

$$\mathrm{E}_0 \sum_t \beta^t z_t \left[u\left(c_t\right) - v\left(h_t\right) \right]$$

E_0 denotes the expection operator at the time t, and β is the discount factor and z_t represents a discount factor.

$$u\left(c_t\right) = \frac{C^{1-\sigma}}{1-\sigma}, \quad v\left(h_t\right) = \frac{h_t^{1+\phi}}{1+\phi}$$

Where σ is the inverse elasticity of intertemporal substitution and ϕ is the inverse of the Frisch elasticity. The consumption good C_t is composed by tradable and nontradable parts $C_t = C_{Tt}^{\gamma} C_{Nt}^{1-\gamma}$.$P_t$ is aggregate price level of tradable and nontradable goods. In each period, household endows with exogenous tradable goods Y_{Tt}.

$$P_t C_t + \frac{1}{1+\tau_t} \frac{B_t}{R_t} = B_{t-1} + W_t L_t + P_{Tt} Y_{Tt} + \Pi_t + T_t$$

Law of one price $P_{Tt} = \mathcal{E}_t P_{Tt}^*$. Assume a stable price level in the foreign country $P_{Tt}^* = 1$, and therefore home-currency tradable price tracks the nominal exchange rate $P_{Tt} = \mathcal{E}_t$. The non-tradable goods are produced by perfectly competitive firms

$$\Pi_t = P_{Nt} L_t^{\alpha} - W_t L_t$$

We assume prices are perfectly rigid, $P_{Nt} = 1$ and the labor demand in equilibrium (This assumption is robust to one-period advance price setting and monopolistic competition) is given by $L_t^{\alpha} = C_N$.

Total consumption is

$$P_t C_t = P_T C_T + P_N C_N$$

$$\frac{P_N}{P_T} = \frac{(1-\gamma) C_T}{\gamma C_N}$$

Exchange rate equilibrium derives from total consumption

$$\mathcal{E}_t = \frac{\gamma}{1-\gamma} \frac{C_{Nt}}{C_{Tt}}$$

Financial Market Arbitrager in the market holds 0 position on asset portfolio:

$$\frac{D_t}{R_t} + \frac{\mathcal{E}_t D_t^*}{R_t^*} = 0$$

(D_t, D_t^*) is arbitrageurs' holding with one foreign currency return, $D_t^* = R_t^*$ and it delivers $R_t = -\frac{R_t^* D_t}{\varepsilon_t D_t^*}$ define $R_{t+1}^* = R_t^* - \frac{\varepsilon_t R_t}{\varepsilon_{t+1}}$ $\pi_{t+1}^{D}{}^* = D_t^* + \frac{D_t}{\mathcal{E}_{t+1}} = D_t^* - \frac{R_t}{\mathcal{E}_{t+1}} \frac{D_t^* \mathcal{E}_t}{R_t^*} = \frac{D_t^*}{R_t^*}\left(R_t^* - \frac{\mathcal{E}_t R_t}{\mathcal{E}_{t+1}}\right) = \frac{\widehat{R_{t+1}^* D_t^*}}{R_t^*}$ $\Theta_t = \beta \mathbb{E}_t\left[\frac{z_t}{z_{t+1}} \frac{U_{T,t}}{U_{T,t+1}}\right] V_t\left(\pi_{t+1}^{D_*}\right) = \mathbb{E}_t\left\{\Theta_{t+1}\pi_{t+1}^{D_*}\right\} - \frac{\omega}{2}\operatorname{var}_t\left(\pi_{t+1}^{D_*}\right) V_t\left(\pi_{t+1}^{D}{}^*\right) = \mathbb{E}_t\left\{\Theta_{t+1}\frac{\widetilde{R_{t+1}^*} D_t^*}{R_t^*}\right\} - \frac{\omega}{2}\operatorname{var}_t\left(\frac{\widetilde{R_{t+1}^*} D_t^*}{R_t^*}\right) \mathbb{E}_t\left\{\Theta_{t+1}\frac{\widetilde{R_{t+1}^*}}{R_t^*}\right\} = \omega D_t^* \operatorname{var}_t\left(\frac{\widetilde{R_{t+1}^*}}{R_t^*}\right) \frac{\mathbb{E}_t\left\{\Theta_{t+1}\widetilde{R_{t+1}^*}\right\}}{\omega_{var}\widetilde{R_{t+1}^*}} = \frac{D_t^*}{R_t^*}$. Financial market cleaning condition

$$B_t + D_t = 0$$

As ID (2022), we define net foreign asset (NFA)

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} = \frac{B_t}{R_t}$$

Government balance Government taxes on financial income $\pi_{t+1}^{D_*}$

$$T_t = \tau_t \mathcal{E}_t \pi_{t+1}^{D_*} - \frac{\tau_t}{1+\tau_t}\frac{B_t}{R_t}$$

Lemma 1. $B_t^* = D_t^*$ Net foreign asset equals to foreign-currency bond. Proof Domestic market clear $B_t + D_t = 0$. At the same time, arbitrager hold 0 position on asset of foreign and domestic currency bond $\frac{D_t}{R_t} + \frac{\varepsilon_t D_t^*}{R_t^*} = 0$. According to the definition of NFA, we have $\frac{R_t \varepsilon_t B_t^*}{R_t^*} - \frac{R_t \varepsilon_t D_t^*}{R_t^t} = 0$.

Lemma 2 Home country budget constraint expressed in terms of foreign currency is Proof Foreign currency bond is flexible supply at an exogenous interest rate R^*.

$$P_t C_t + \frac{1}{1+\tau_t}\frac{B_t}{R_t} = B_{t-1} + W_t L_t + P_{Tt} Y_{Tt} + \Pi_t + T_t$$
$$\frac{1}{1+\tau_t}\frac{B_t}{R_t} = B_{t-1} + P_{Tt} Y_{Tt} - P_{Tt} C_{Tt} + T_t$$

Define $NX_t = P_{Tt} Y_{Tt} - P_{Tt} C_{Tt} = \uparrow_t (Y_{Tt} - C_{Tt})$

$$\frac{1}{1+\tau_t}\frac{B_t}{R_t} = B_{t-1} + NX_t + T_t$$

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$$\frac{F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} = F_{t-1} + \mathcal{E}_t F_{t-1}^* + \tau \mathcal{E}_t \pi_t^* - T_t, \quad \pi_t^* = \tilde{R}_t^* \cdot \frac{N_{t-1}^* + D_{t-1}^*}{R_{t-1}^*},$$

combine government budget constraint

$$\frac{B_t}{R_t} = B_{t-1} + NX_t + \tau_t \mathcal{E}_t \pi_{t+1}^{D_*}$$

Market cleaning $B_t + D_t = 0, B_t^* = D_t^*, \frac{D_t}{R_t} + \frac{\varepsilon_t D_t^*}{R_t^*} = 0$ and NFA $\frac{\varepsilon_t B_t^*}{R_t^*} = \frac{B_t}{R_t}$

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} = \frac{R_{t-1}\mathcal{E}_{t-1} D_{t-1}^*}{R_{t-1}^*} + NX_t + \tau_t \mathcal{E}_t \pi_{t+1}^{D_*}$$
$$\frac{B_t^*}{R_t^*} = B_{t-1}^* + (Y_{Tt} - C_{Tt}) + \tau_t \frac{\widetilde{R_{t+1}^*} D_t^*}{R_t^*}$$

If $P_N = \frac{W}{A}$ (competitive-flexible price), then $\Pi = 0$, and $Y = C = W = A$,(Good market equilibrium, Household FOC,

$$W_0 = E_0 \sum_{t=0} \beta^t \left[\log C_T - (1-\gamma)L\right]$$

$$\frac{B^*}{R^*} - B^*_{-1} = (Y_T - C_T) - (1-\tau)\left[R^*_{-1} - R_{-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\frac{B^*_{t-1} - F^*_{t-1}}{R^*_{t-1}}$$

$$\beta R^*_t \mathbb{E}_t \frac{C_T}{C'_T} = 1 + \omega\sigma_t^2 \frac{B^* - N^* - F^*}{R^*}$$

$$\beta R_t \mathbb{E}_t \left[\frac{C_T}{C'_T}\frac{\mathcal{E}}{\mathcal{E}'}\right] = 1$$

$$\mathcal{E} = \frac{\gamma}{1-\gamma}\frac{W}{C_T}$$

$$\sigma_t^2 = R^2 \operatorname{var}\left(\frac{\mathcal{E}}{\mathcal{E}'}\right)$$

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Appendix

A Proof of Section 2

Optimization problem

$$\max_{\left\{C_0,C_1,C_2,b_1^S,b_2^S\right\}} (1-\beta_0)\ln C_0^S + \beta_0(1-\beta_1)\left[\ln C_1^S - h\left(l_1^S\right)\right] + \beta_0\beta_1 \log C_2^S - \lambda_1\left(P_0 C_0^S + \frac{b_1^S}{R_0} - \mathcal{E}_0 Y_{T0} - b_0^S - l_0^S - t_0^S\right)$$
$$-\lambda_2\left(P_1 C_1^S + \frac{b_2^S}{R_1} - \mathcal{E}_1 Y_{T1} - b_1^S - l_1^S\right) - \lambda_3\left(P_2 C_{2,\omega}^S - b_2^S\right)$$
$$\frac{\beta_0(1-\beta_1)}{C_1^S} - \lambda_2 P_1 = 0; \beta_0\beta_1\mathbb{E}\left[\frac{1}{C_2^S}\right] - \lambda_3\mathbb{E}\left[P_2\right] = 0; \frac{\lambda_2}{R_1} = \lambda_3$$

The consumption of foreign bond investor is:

$$\max_{\left\{C_0,C_1,C_2,b_1^S,b_2^S\right\}} (1-\beta_0)\ln C_0^A + \beta_0(1-\beta_1)\ln C_1^A + \beta_0\beta_1\mathbb{E}\left[\log C_2^A\right]$$
$$-\lambda_1\left(P_0 C_0^B + \frac{b_1^B}{R_0}(1-\tau_0) + \left(x_1^B - \frac{1}{\phi^B}\right)\mathcal{P}_0 - P_{T0} Y_{T0} - t_0^B - b_0^B\right)$$
$$-\lambda_2\left(P_1 C_1^B + \frac{b_2^B}{R_1} + \left(x_2^B - x_1^B\right)\mathcal{P}_1 - P_{T1} Y_{T1} - b_1^B\right) - \lambda_3\left(P_2 C_{2,\omega}^B - b_2^B - x_2^B D_{2,\omega}\right)$$
$$\frac{\beta_0(1-\beta_1)}{C_1^A} - \lambda_2 P_1 = 0; \frac{\beta_0\beta_1}{C_2^A} - \lambda_3 P_2 = 0; -\frac{\lambda_2}{R_1} + \lambda_3 = 0; -\lambda_2\mathcal{P}_1 + \lambda_3 D_{2,\omega} = 0$$
$$\frac{1-\beta_1}{\beta_1}\frac{1}{C_1^B\mathbb{E}\left[\frac{1}{C_2^B}\right]} = \frac{P_1}{P_2} = \frac{D_{2,\omega}}{\mathcal{P}_1}$$