Online and Offline Industrial Structure Dynamics in COVID-19

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Abstract

In order to flatten the curve of COVID-19 and reduce potential deaths, most countries have adopted lockdown policy. However, during lockdown period we found the market value of website establishments still go up. In this paper, our primary goal is to explain how this happen during lockdown. Based on workhorse SIR model, a heuristic single sector model is introduced as a benchmark model to study the impact of lockdown on the whole economy. In second part, we build a planning problem with industrial structure–online and offline sectors. We show how lockdown policy will crowd out the offline sector and how that effect embodies on macro variables.

1 Introduction

The COVID-19 pandemic has brought massive and unexpected life twists, but it did not grind everything to a halt. While it is true the coronavirus has caused catastrophic health, social and economic effects, we can still find that the market value of internet company increases and squeeze out traditional industry. Recent researchers launched a series of the redevelopment of SIR (Susceptible-Infectious-Recovered) model that was promoted by Kermack et al. [1927]. The SIR framework and its various extensions model the spread are researched in many inquiries recently (e.g. Berger et al. [2020], Atkeson [2020]). The simplest version of the model that includes three differential equations can provide a good approximation for the pandemic's dynamic. Besides the ability to describe the pandemic's dynamic nature, it can also provide a basis for predicting the economy and planning for the economy. Several recent papers have started conducting optimal policy analysis within this framework (e.g. Alvarez et al. [2020], Jones et al. [2020], Farboodi et al. [2020], Rowthorn and Toxvaerd [2012], Eichenbaum et al. [2020].) But they only use the amount of people represent the value of the welfare. Differentiating from them, we build the model based on the classic theory of utility.

According to a report released by iResearch (2017), China's O2O market size amounted to 978 billion RMB at the end of 2017, with an increase of 46.8% from 2016. Driven by the huge

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profit potential, more and more merchants in China are rushing in the O2O business. In this process, increasingly number of offline merchants are crowed out especially during the period of COVID-19. Lockdown policy in China intensify crowding out effect: people are not allowed to leave home freely and then the consumption of offline part will decrease in that case. However, it is totally another story for online part since we still need commodity to sustain normal lives. The truth is the market value of website establishments create new historical record during 2020. Considering the negative shock for the economy the consumption should be affected but the real scenario cannot support this educated guess. According to a spectrum of anecdotal evidence the online market seems still to go up. An intuition of our work is to build a model to illustrate this abnormal scenario.

In our model, we introduce two sectors: online and offline sectors. From economics perspective, this first effect we will consider is incoming effect, which means that if the economy suffers from a negative shock, the online and offline sectors will decrease at the same time. Another one is substitution effect, which means that consumer will choose to buy another product if that price is relatively low. We describe that price as "potential price" during certain disadvantage factors e.g. infected population or lockdown policy. These two cases will cause the product in online sector will increase.

At the same time another self-evident issue is launched: how the industrial structure will interact with lockdown policy? In Guerrieri et al. [2020], they provide one theory illustration that how the negative shock will affect the economy. They prove, firstly, in one-sector economy, a pandemic's shock is negative and the drop in supply dominates not demanding side. They also prove a less obvious result: it also holds in incomplete market because of different marginal propensities to consume (MPC). In these models, there is a mechanism from income loss to demand decline. Although it makes the total demand decline larger than the total demand decline in typical agency cases, the decline is still smaller than the output decline caused by a supply shock. Intuitively, the largest MPC of people who lose their income may be large, but only one means that their consumption decline is always a suppressed version of income loss. Then we turn to economies with multiple sectors. When the impact is concentrated in certain sectors, as in the case of a shutdown in response to an epidemic, there is more room for total spending to contract. Some goods are no longer available, which makes overall consumption less attractive. One explanation is that the suspension of production increases the shadow price of goods in the affected sectors, making current total consumption more expensive, thus inhibiting consumption. On the other hand, due to the unavailability of goods in some sectors, expenditures can be transferred to other sectors through alternative channels.

The industrial structure is also closely related to the problem of inequality in the economy. Because industries have different production characteristics, factor demand, and economic dependence, representative consumers in different industries will bring different profits and output. In other words, the industrial structure is the root cause of inequality in the economy. Different industrial structure necessarily means different output value and wages. Under the condition of the incomplete market, people cannot freely choose the industry they are engaged in and we will show how that will give rise to the gap between these two sectors.

Therefore, in our model, we present an optimal control model with heterogeneous production sectors for the COVID-19 epidemic. We adopt the model structure of Alvarez et al. [2020], a variation in the SIR epidemiology model reviewed and proposed by Atkeson [2020] to analyze the optimal lockdown policy. Morton and Wickwire [1974] promoted the origin edition of the planning problem where they construct a control scheme for the immunization of susceptibles in the Kermack-McKendrick epidemic model for a closed population. And they use the approach of dynamic programming and Pontryagin's Maximum Principle and allows one, for specific values of the cost and removal rates, to apply necessary and sufficient conditions for optimality and show that a one-switch candidate is an optimal control. In the remaining cases, they show that an optimal control, if it exists, has at most one switch. And then Hansen and Day [2011] extend the existing work on the time-optimal control of the basic SIR epidemic model with a mass action contact rate. They provide analytic solutions for the model that minimizes the outbreak size (or infectious burden) under the assumption that there are limited resources and optimal control policies for an isolation only model, a vaccination only model and a combined isolation–vaccination model (or mixed model). Several conclusions are given by them: under certain circumstances, the optimal isolation only policy is not unique; further- more the optimal mixed policy is not merely a combination of the optimal isolation only policy and the optimal vaccination only policy. Some recent papers talk about the planning problem centrally, see Acemoglu et al. [2020]; Barro et al. [2020]; Eichenbaum et al. [2020]; Berger et al. [2020]; Hall et al. [2020]; Chari et al. [2020]; Jones et al. [2020] and Richard Baldwin [2020]. The typical approach in the epidemiology literature is to study the dynamics of the pandemic, for infected, deaths, recovered, as functions of some exogenous chosen diffusion parameters, which are in turn related to various policies, such as the partial lockdown of schools, universities, businesses, and other measures of diffusion mitigation, and where the diffusion parameters are stratified by age and individual covariates.

We differ from these studies in two ways: first, we place the planning problem within the general equilibrium framework of the problems of utility and social planners. This economy is characterized by variables such as consumers, wages, prices, and debt. Risk aversion exists in the individual decision-making process. We construct an expected utility function that shows risk aversion in the choice between labor supply and consumption. For the firm part, we set up a linear technical production function. So we can get wages in a balanced economy. As far as the government is concerned, the fiscal deficit is the main source of the government's fiscal deficit. On this basis we can discuss fiscal policy. Second, we consider a model with industrial structure. Therefore, we can consider the problem of inequality in the economy due to sector heterogeneity i.e. the wage gap of these two sectors.

2 Baseline Planning Model with Government

This section firstly employs the SIR model to model the population dynamics. We then define the expected utility for susceptible agents and fixed utility for those infected under the assumption that the probability of infection is proportional to lockdown policy. We employ an objective function of the social planner to compromise between total expected utility and death toll.

2.1 SIR Model

As in Atkeson [2020] and Alvarez et al. [2020], at any point in time t, the whole population $N(t)$ is divided into those susceptible $S(t)$, those infected $I(t)$ and those recovered $R(t)$, i.e.,

$$N(t) = S(t) + I(t) + R(t), \quad \forall t > 0. \tag{1}$$

The recovered category $R(t)$ here includes individuals who have been infected, survived the disease, and are assumed to be immune to COVID-19 within a certain period of time. We normalize the initial population to $N(0) = 1$ where only those alive population are considered. The social planner can control a fraction $L(t)$ of the population, where $L \leq 1$ allows us to meet a more actual situation to keep some significant departments and industries working on such as power plant, food supply vendors and groceries. The lockdown efficiency θ measures the proportion that population cannot contact others freely. If $\theta = 1$, the lockdown policy completely wields its effects on population. But the actual scenario we never harbor the ability to control all people to move in different cities to curb the transmission of virus; so we take parameter $\theta < 1$.

Given the above setup, the law of dynamic of susceptible agents, infected agents, and total population are

$$\dot{S} = -\beta S(t)(1-\theta L(t))I(t)(1-\theta L(t)), \tag{2}$$

$$\dot{I} = \beta S(t)(1-\theta L(t))I(t)(1-\theta L(t)) - \gamma I(t), \tag{3}$$

$$-\dot{N} = D(t) = \phi(I(t))I(t), \tag{4}$$

$$\phi(I(t)) = [\varphi + \kappa I]\gamma. \tag{5}$$

The parameter β represents the number that the susceptible agents contact the infected agents per unit time. We set the probability that people get infected from infected agents is 1 after contacting. For infected people, they can recover at the rate of γ. The death of disease $D(t)$ is defined as product of rate of death per unit time $\phi(I(t))$ and number of infected agents. The "case fatality rate" (CFR) $\phi(I(t)) \in (0, 1)$ in Alvarez et al. [2020] is the rate of fatality of infected people. It appears that the CFR manifest the direct proportion with $I(t)$, which reflects the jam effects in health care system.

2.2 Agents' Utility, One Sector Production and Equilibrium

We define the expected utility for susceptible agents and fixed utility for infected agents. The utility of each susceptible agent consists of two parts: maintaining the presented scenario, namely not be infected; and the possibility of being tracked into infected group. The utility formula is modeled as follows

$$E[u_s(c_t^s, n_t^s)] = p(L)u_s(c_t^s, n_t^s) + [1 - p(L)]u_i(c_t^i, n_t^i) \tag{6}$$

$$E[u_i(c_t^i, n_t^i)] = u_i(c_t^i, n_t^i) \tag{7}$$

where c_t^s and n_t^s are the consumption and labor supply of susceptible agents; c_t^i and n_t^i are for infected people. The probability function $p(L)$ depends on direct proportion of lockdown policy $L(t)$ and the effectiveness of lockdown policy α with the form

$$p(L) = \alpha L(t). \tag{8}$$

The parameter α takes values between 0 and 1. If $\alpha = 1$, the policy is fully effective of lockdown policy, however, some contacts may still happen even under a full economic lockdown, we set $\alpha < 1$.

Assume the consumption is inelastic during the period of pandemic, i.e. agents cannot make the optimal decision for c_t and n_t. Infected agents accept unemployment insurance b. Apparently, social planned is incentive to avoid the worse situation that become infected people for they will suffer from welfare loss. We constrain the consumption and labor supply as follows

$$c_t^s = (1 - L(t))w(t), \quad n_t^s = 1 - L(t) \tag{9}$$

and

$$c_t^i = b, \quad n_t^s = 0. \tag{10}$$

The remedy against the ongoing COVID-19 recession that is currently being debated is fiscal stimulus. To consider the effects of fiscal stimulus in our model, we introduce a stylized government sector. Government debt is

$$B_t = I(t)b. \tag{11}$$

As Guerrieri et al. [2020], competitive firms produce the final good only from labor using the linear technology, so the wage can be same as price of product,

$$Y_t = N_t, \tag{12}$$

$$p_t = w_t. \tag{13}$$

Total demand comes from two parts: the consumption of susceptible agents $S(t)c_t^s$ and of infected agents $I(t)b$,

$$C_t = S(t)c_t^s + I(t)b. \tag{14}$$

Another side total supply, namely total labor supply N_t comes from only the susceptible people,

$$N_t = S(t)(1 - L(t)). \tag{15}$$

Define market equilibrium is

$$N_t = C_t. \tag{16}$$

Under this scenario, the wage of economy is

$$w_t = \frac{S(1 - L) - Ib}{S}. \tag{17}$$

2.3 Planning Problem

The planning problem is modified version of Alvarez et al. [2020] and Acemoglu et al. [2020]. We employ an objective function of social planner. Assume that agents live forever, unless they die from the infection. The planner discounts all values at the rate $r > 0$ and with probability ν per unit of time both a vaccine or a cure appear, then all infected agents could be cured or get the immunity. Thus, the planning problem can be consist in maximizing the following present value

$$\max_{L} \int_0^\infty e^{-(r+v)t} \Big\{ \underbrace{S(t)u(c_t^s, n_t^s) + I(t)b}_{\text{Total Utility}} - \underbrace{\chi_d D(t)}_{\text{Loss Value}} \Big\} dt \tag{18}$$

We get HJB equation:

$$\begin{aligned}(r+v)V(S,I) = \min_{L\in[0,\bar{L}]} \Big\{ & S(t)u(c_t^s, n_t^s) + I(t)b - \chi_d \phi(I(t))I(t) + \\ & - \partial_S V(S,I)[-\beta S(t)I(t)(1-\theta L(t))^2] \\ & + \partial_I V(S,I)[\beta S(t)I(t)(1-\theta L(t))^2 - \gamma I(t)] \Big\}\end{aligned} \tag{19}$$

The domain of $V(S,I)$ is $S + I \leq 1$. Following Alvarez et al. [2020], we will use the value function iteration method to solve this problem. Note the boundary of value function has an analytical formula. $V(S,0) = \frac{(1+\alpha bS)^2}{4\alpha(r+\nu)}$ and $V(0,I) = \frac{a^2b^2}{8\alpha\gamma+4\alpha(r+v)}I^2 + \frac{4\alpha\chi_d\phi+2ab}{4\alpha\gamma+4\alpha(r+v)}I + \frac{1}{4\alpha(r+v)}$.

3 Modified Model for Industrial Structure

In this part, we want to explain the fact that why the wage of online sector i.e. some establishments based on electronic business will increase in the period of pandemic.

3.1 Technology, Price and Equilibrium

The price of products of online sector is proportional to wage w_{onlne} and inversely for technology

$$P_{online} = \left(\frac{w_{online}}{z_{online}(j)} \right) \tag{20}$$

z_{online} is the technology to produce j. Efficiency is a random variable to meet following distribution:

$$F_i(z) = e^{-T_i z^{-\theta}} \tag{21}$$

This is a workhorse distribution that is always used in literature about technology and innovation. It is related to idiosyncratic technology of online sector $z_{online}(j)$ and country-specified parameter T_i. The probability theory of extremes provides a form for $F_i(z)$ that yields a simple expression for π_{ni} and for the resulting distribution of prices. We assume that country i 's efficiency distribution is Fréchet (also called the Type II extreme value where $T_i > 0$ and $\theta > 1$. We treat the distributions as independent across countries. The (country-specific) parameter T_i governs the location of the distribution. A bigger T_i implies that a high efficiency draw for any good j is more likely. The parameter θ (which we treat as common to all countries) reflects the amount of variation within the distribution. A bigger θ implies less variability.

And we normalize the offline wage $w_{offline}$ to 1 since we only need to focus on the wage of online sector. In order to embody the effect of epidemic we think the offline product price is also affected by potential price factor $d_{offline}(I, L)$. Specifically, the price of products of offline sector is

$$P_{offline} = \left(\frac{1}{z_{offline}(j)}\right) d_{offline}(I, L) = \left(\frac{1}{z_{offline}(j)}\right) e^{\alpha I + \mu L} \tag{22}$$

We assume perfect competition, which means all the products in certain sector are set to the same price. They shopping around two sectors for the best deal, namely consumer will be free to choose the cheapest product and then we have

$$P^* = \min \left\{P_{online}, P_{offline}\right\} \tag{23}$$

The lowest price will be less than p unless all source's price is greater than that. These assumptions imply we can get following price distribution of online and offline sectors:

$$G_{online}(p) = \Pr\left[P_{online} \leq p\right] = \left(\frac{w_{online}}{z_{online}(j)}\right) \leq p \tag{24}$$

$$G_{offline}(p) = \Pr\left[P_{offline} \leq p\right] = \left(\frac{1}{z_{offline}(j)}\right) e^{\alpha I + \mu L} \leq p \tag{25}$$

So the technology distribution of these two sectors when the market is clear should be

$$z_{online}(j) \geq \frac{w}{p} \tag{26}$$

$$z_{offline}(j) \geq \frac{e^{\alpha I + \mu L}}{p} \tag{27}$$

$$G_{online} = 1 - \exp\left\{-p^{\theta}\left[T_{online}\,(w)^{-\theta}\right]\right\} \tag{28}$$

$$G_{offline} = 1 - \exp\left\{-p^{\theta}\left[T_{offline}\left(e^{\alpha I + \mu L}\right)^{-\theta}\right]\right\} \tag{29}$$

We can get the market share

$$G(p) = 1 - \left(1 - G_{online}(p)\right)\left(1 - G_{offline}(p)\right) \tag{30}$$

$$\lambda_{online} = \int_0^{\infty} \left[1 - G_{offline}(p)\right] dG_{online}(p) = \frac{T_{online}\,(w_{\text{on}})^{-\theta}}{T_{online}\,(w_{\text{on}})^{-\theta} + T_{offline}\,(e^{\alpha I + \mu L})^{-\theta}} \tag{31}$$

Assume all the products are homogeneous. The online and offline sectors should be clear. Here we use $w = w_{online}$ for concise.

$$\underbrace{\phi w S}_{\text{Wage}} = \underbrace{[\phi S w + (1-\phi)S]\lambda_{online}}_{\text{Consumption}} \tag{32}$$

where ϕ is proportion that individuals work in online sector. The market is incomplete, which means that people work in each sector cannot move into another freely. Only healthy people can work during pandemics and receive wage w. And all these money will be consumed on each time point. The ratio is λ_{online} as we mentioned before. The LHS is the total wage of online sector and RHS is total consumption of agents. For offline sector we have

$$[\phi S w + (1-\phi)S]\,(1-\lambda_{online}) = (1-\phi)S \tag{33}$$

It is a self-consistent market clean condition. From equation 32 the wage is

$$w = \frac{1-\phi}{\phi}\frac{\lambda}{1-\lambda} \tag{34}$$

Combined with market share of online and offline sector

$$\frac{1}{w} = \frac{\phi}{1-\phi}\left(\frac{1}{\lambda}-1\right) = \frac{\phi}{1-\phi}\left(\frac{T_{offline}\left(e^{\alpha I+\mu L}\right)^{-\theta}}{T_{online}\left(w\right)^{-\theta}}\right) \tag{35}$$

If $T_{offline} = T_{online}$,

$$w = \left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\theta+1}} \exp\left\{\frac{\alpha\theta I + \mu\theta L}{1+\theta}\right\} \tag{36}$$

$$\lambda_{online} = \frac{\left(\left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\theta+1}} \exp\left\{\frac{\alpha\theta I+\mu\theta L}{1+\theta}\right\}\right)^{-\theta}}{\left(\left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\theta+1}} \exp\left\{\frac{\alpha\theta I+\mu\theta L}{1+\theta}\right\}\right)^{-\theta} + \left(e^{\alpha I+\mu L}\right)^{-\theta}} \tag{37}$$

Analytical Natures???

3.2 Planning Problem

The total value can be attributed to three parts: (1) ρ is the ratio of production value of online and offline sectors; (2) $(1-\rho)$ is the ratio of value of other sectors accept fixed wage $\bar{w}$; and (3) the loss value

$$\max_{L} \int_0^{\infty} e^{-(r+v)t} \left\{ \begin{array}{c} \underbrace{\rho\left[\phi S(t) w + (1-\phi)S(t)\right]}_{\text{Value of Online and Offline Sectors}} \\ + \underbrace{\bar{w}(1-\rho)(1-L(t))[\tau(S(t)+I(t))+1-\tau]}_{\text{Value of Other Sector}} \\ - \underbrace{vsl \times I\varphi(I)}_{\text{Loss}} \end{array} \right\} dt \tag{38}$$

4 Numerical Method and Parameter Setting

We use value function iteration method to solve the HJB equations. In order to use this method, we have to get the difference form of the HJB equations. The process can be seen in the Appendix A.

As Alvarez et al. [2020], We use a value of statistical life of 20 times annual per capita GDP, which is in line with utilitarian values of life.

Table 1: Parameters

Parameter	Value	Definition
β	0.2	Daily increase of active cases if unchecked
γ	0.055	Daily rate of infected recovery (includes those that die)
φ	0.01γ	IFR: fatality per active case (per day)
κ	0.05γ	Implies a 3 percent fatality rate with 40 percent infected
r	0.05	Annual interest rate 5 percent
ν	0.667	Prob rate vaccine + cure (exp. duration 1.5 years)
$\bar{L}$	0.9	1 - GPD share health, retail, government, utilities, and food mfg
θ	0.5	Effectiveness of lockdown
χ	0	Value of Statistical Life $20 \times w$

5 Numerical Results

We show the main quantitative results of the planning problem described in the previous section. Throughout our primary focus is on the controlled and uncontrolled scenario with comparing different risk aversion parameters α, value of life χ to explore the optimal lockdown policies and changes in output, wages, and government deficits on this basis. We first analyze the one sector situation. And then move to the comparisons under different industrial structures to capture the optimal lockdown policy on the unnecessary sector and their corresponding outcomes.

5.1 Baseline Model: Risk Aversion and Life Value

Risk aversion is an important feature of individual decision makers. When representative consumers take a hedging attitude towards unknown risks, it will directly affect individual decision making. In our model, where active unemployment is a consequence of this problem, policymakers have an incentive to choose initiatives when they consider the consequences of illness.

Presented in Figure 1, the optimal lockdown policy shows the monotonous decline after a short period of high lockdown rate. As the degree of risk aversion increases, the lockdown policy becomes stricter. Tighter lockdown policies have led to a decline in output, which ultimately fell by 2.5%. Increased risk aversion also leads to an obvious whole decline in wages. In both risk aversion scenarios, the wages appear to fall first and then rise. The fiscal deficit curves first increases and then decreases, and stricter lockdown policy leads to a whole increase in the fiscal deficits.

For the yield of the economy, the shock of the pandemic is totally negative for the economy. With the proportion $I(t)$ increases the lost of the yield is also enhanced, from 2% to 2.5%.

In our baseline model, the wage is same as price index($w_t = p_t$), so we need only focus on the variation of wage of working population. We can get the conclusion from the numerical solution that the severity of pandemic is negative correlated with the price index(wage), which means the increasing of $I(t)$ means the minimum of price decreases. In our results, it decreases from 0.981 to 0.977. And then, it will increase and converge to a fixed value. The reason for this can be explained by the fiscal policy. At the first stage, there are still many people who work on positions and accept the wage w_t. Pandemic shock decreases the wage at first. And then, fiscal policy exerts its ability to meet the requirement and demanding. Along with people who are infected and given unemployment insurance, the demanding increases again.

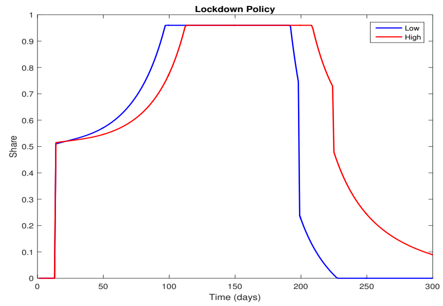
Fiscal deficit comes from the unemployment policy b. According the setting of the model, it should be concert with the trend of infected people. Deficit increases to 2.25% with the proportion $I(t)$ enhances. And then, the same as wage and yield, decrease to a fixed value.

We set $b = 0.05$ and $b = 0.15$ as two comparison. We can get the conclusion that with benefit decreases the yield of economy will also decrease, but fiscal deficit decreases also, as we expected. And the minimum price also increase, which means that a more rigid lockdown policy will be used.

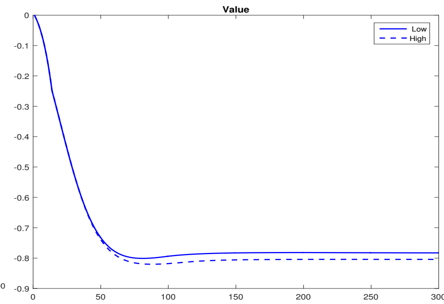
The value of life is an obvious concept, and as the value of life goes up, we tend to stick to a conservative lockdown policy. Presented in Figure 2, both of the optimal lockdown curves under the two scenarios show the monotonous decline and cross at date 100. As the value of life increases, the lockdown policy becomes stricter, resulting in an overall decline in output. The output curve with $\chi_d = 1.5$ first falls and then rises. The wage curve with $\chi_d = 1.5$ shows an obvious whole decline comparing to the curve with $\chi_d = 0.6$. Both of the wage curves appear to fall first and then rise. The fiscal deficit curves first increases and then decreases, and stricter lockdown policy leads to a whole increase in the fiscal deficits.

5.2 Online and Offline structure and Inequality

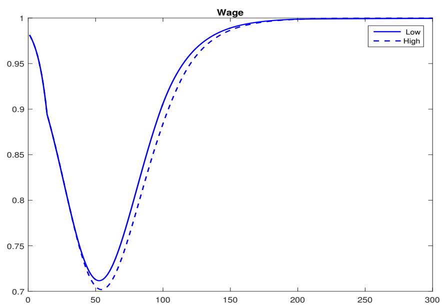
In the modified industrial structure model, we explore how the market share of these two parts will change during the pandemic in the context of lockdown policy. When the lockdown policy is implemented, the market share of offline part will go up and offline sector will be crowed out. After lockdown period, these two market share return to initial state.



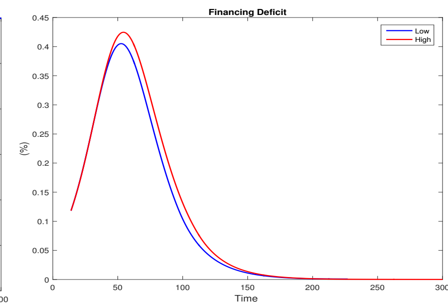
(a) Lockdown



(b) Output

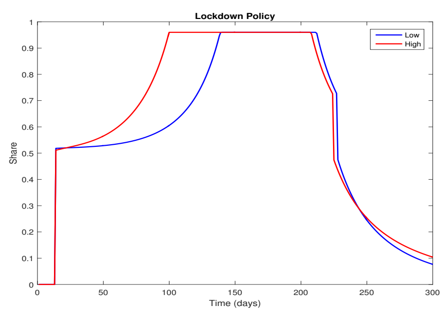


(c) Wage

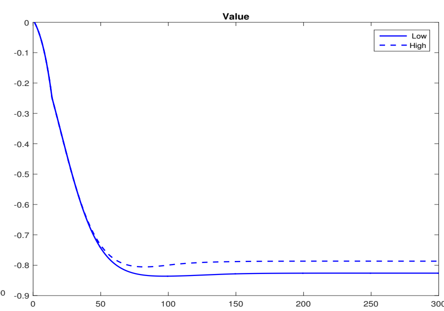


(d) Deficit

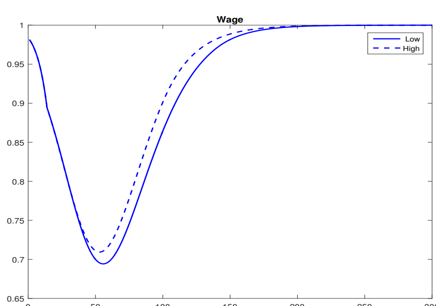
Figure 1: Risk aversion, $\alpha = 0.01$, $\alpha = 0.02$



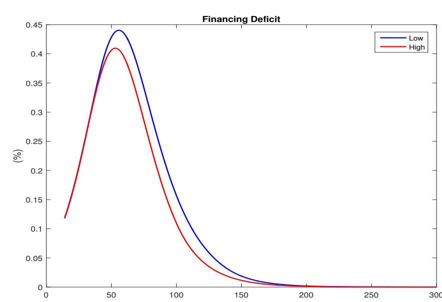
(a) Lockdown



(b) Output



(c) Wage



(d) Deficit

Figure 2: Value of Life, $\chi_d = 1.5$,$\chi_d = 0.6$

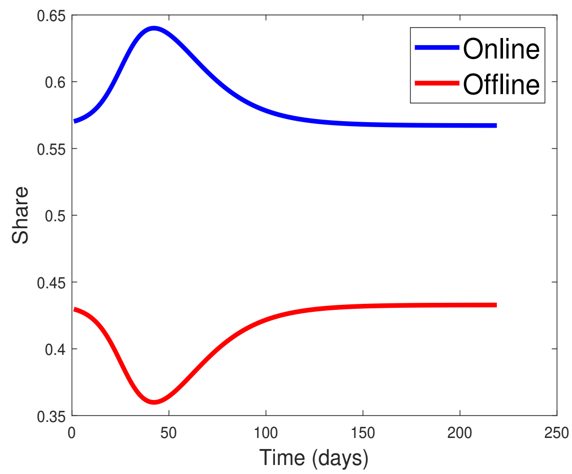


Figure 3: Online and Offline Structure

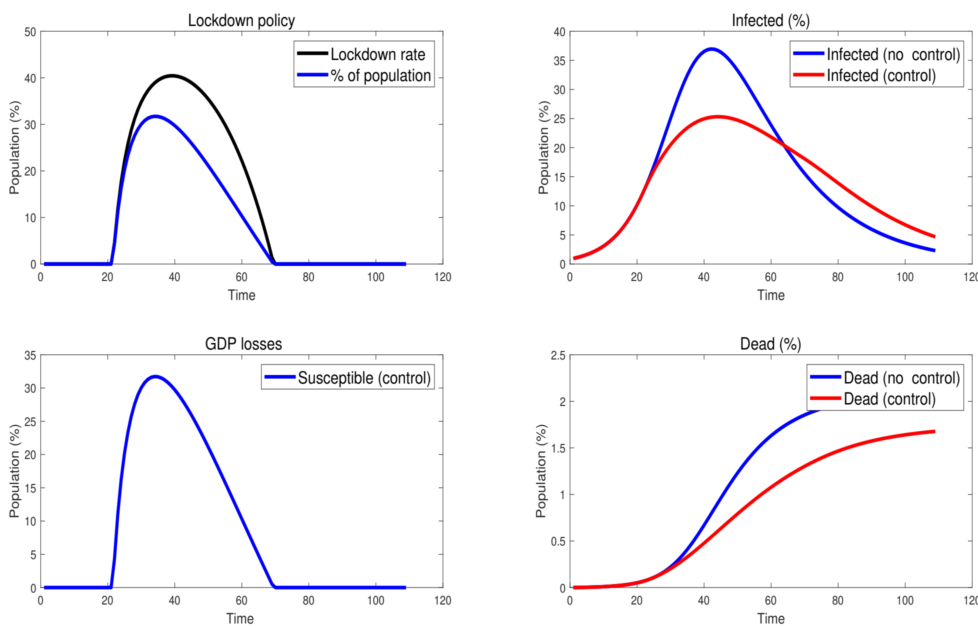


Figure 4: Lockdown Effect

6 Conclusion

According to our analysis, we can get the relationship between lockdown and susceptible, infected population, and and lockdown in a framework of general equilibrium. Another insight we provide for this arena is the substantial nexus between industrial structure and the critical index we mentioned above. Different industry organizations can give rise to distinct outcomes for the whole economy. This model dictates the crowding effect of online relative to offline part. When the industrial structure difference becomes larger, the degree of inequality will increase significantly because of the increased wage gap.

However, we still do not answer whether the debt will impact welfare in the long term. Since we are mainly considering short-term decisions, one possible strategy is to integrate long-term and short-term goals to measure this issue. Under the condition of an infinite time limit, the decision is time-homogeneous; In other words, starting at any time, our decision strategy is independent of time and related to other state variables. This may ignore the impact of variable debt accumulated over time. It is possible to approach this problem with a combination of short and long terms. It is not intuitive whether the short-term benefit increase is better or the future benefit loss from debt is more significant.

Besides, there are still some other uncharted fields we want to explore under this framework. The first nature is CRRA utility function, a typical utility function argued in many ex-works. It can reveal risk-aversion identity for inter-temporary decision making. Another problem is CES productive function, which can integrate various intermediate producers into one part flexibly. Considering the heterogeneous-agent nature about age, initial assets, in a word, these natures can describe the economy more realistically since these initial conditions affect the decision making under the pandemic scenario.

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A Appendix for Numerical Method

A.1 One-sector Model

As Alvarez et al. [2020], we use value function iteration algorithm to solve this problem.

The utility of agents is

$$E[u_s(c_t^s, n_t^s)] = (1 - \alpha L)Lw + \alpha Lb \tag{39}$$

because we can calculate the equilibrium wage, the utility can be written as:

$$u_t^s = (1 - \alpha L)\frac{S(1-L) - Ib}{S} + \alpha Lb \tag{40}$$

Solve this social planner's problem:

$$\begin{aligned}
&\max_{L(t)} \int_0^\infty e^{-(r+v)t}\left\{S(t)\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I(t)b - \chi_d D(t)\right\}dt \\
=&\max_{L(t)} \int_0^\infty e^{-(r+v)t}\left\{S(t)\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I(t)b + \chi_d \phi(I(t))I(t)\right\}dt
\end{aligned} \tag{41}$$

The HJB equation of this problem is:

$$\begin{aligned}
(r+v)V(S,I) = \min_{L\in[0,\bar{L}]} \Bigg\{ & S(t)\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I(t)b + \chi_d \phi(I(t))I(t) + \\
& - \partial_S V(S,I)[-\beta S(t)I(t)(1-\theta L(t))^2] \\
& + \partial_I V(S,I)[\beta S(t)I(t)(1-\theta L(t))^2 - \gamma I(t)] \Bigg\}
\end{aligned} \tag{42}$$

To calculate the partial difference $\partial_S V(S,I)$ and $\partial_I V(S,I)$, we choose to $V_S^-(i,j)$ and $V_I^+(i,j)$

$$V_S^-(i,j) = \frac{V(S_i, I_j) - V(S_{i-1}, I_j)}{S_i - S_{i-1}} \tag{43}$$

$$V_I^+(i,j) = \frac{V(S_i, I_{j+1}) - V(S_i, I_j)}{I_{j+1} - I_j} \tag{44}$$

$$\begin{aligned}
(r+\nu)V(S_i, I_j) = \min_{L\in[0,\bar{L}]} & \left\{ S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I_j b + \chi_d \phi(I_j)I_j \right. \\
& \left. + \left[\beta S_i I_j (1-\theta L)^2\right]\left[V_I^+(i,j) - V_S^-(i,j)\right] - \gamma I_j V_I^-(i,j) \right\}
\end{aligned} \tag{45}$$

We assume that $S_i - S_{i-1} = I_{j+1} - I_j = \Delta$.

$$V_I^+(i,j) - V_S^-(i,j) = \frac{1}{\Delta}\left[V(S_i, I_{j+1}) - V(S_i, I_j) - V(S_i, I_j) + V(S_{i-1}, I_j)\right] \tag{46}$$

$$\begin{aligned}
[1+(r+\nu)dt]V\left(S_i,I_j\right) = \min_{L\in[0,\bar{L}]} \Bigg\{ & S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt \\
&+\left[\beta S_iI_j(1-\theta L)^2\right]dt\left[V_I^+(i,j)-V_S^-(i,j)\right]-\gamma I_jdtV_I^-(i,j)+V(S_i,I_j)\Bigg\}
\end{aligned} \tag{47}$$

$$\begin{aligned}
[1+(r+\nu)dt]V\left(S_i,I_j\right) = \min_{L\in[0,\bar{L}]} \Bigg\{ & S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt \\
&+\left[\beta S_iI_j(1-\theta L)^2\right]\frac{dt}{\Delta}\left[V\left(S_i,I_{j+1}\right)-2V\left(S_i,I_j\right)+V\left(S_{i-1},I_j\right)\right] \\
&-\gamma I_j\frac{dt}{\Delta}\left[V\left(S_i,I_j\right)-V\left(S_i,I_{j-1}\right)\right]+V\left(S_i,I_j\right)\Bigg\}
\end{aligned} \tag{48}$$

$$\begin{aligned}
[1+(r+\nu)dt]V\left(S_i,I_j\right) = \min_{L\in[0,\bar{L}]} \Bigg\{ & S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt \\
&+[1-(r+\nu)dt]\left\{\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}\left[V\left(S_i,I_{j+1}\right)-2V\left(S_i,I_j\right)+V\left(S_{i-1},I_j\right)\right]\right. \\
&\left.-\frac{\gamma I_j}{1-(r+\nu)dt}\frac{dt}{\Delta}\left[V\left(S_i,I_j\right)-V\left(S_i,I_{j-1}\right)\right]+V\left(S_i,I_j\right)\right\}
\end{aligned} \tag{49}$$

$$\begin{aligned}
V\left(S_i,I_j\right) = \min_{L\in[0,\bar{L}]} \Bigg\{ & S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt \\
&+[1-(r+\nu)dt]\left[1-2\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}-\frac{\gamma I_j}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_i,I_i\right) \\
&+[1-(r+\nu)dt]\left[\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_i,I_{j+1}\right) \\
&+[1-(r+\nu)dt]\left[\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_{i-1},I_j\right) \\
&+[1-(r+\nu)dt]\left[\frac{\gamma I_j}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_i,I_{j-1}\right)\Bigg\}
\end{aligned} \tag{50}$$

With respect the situation that we set the interval of different of direction on the discrete space.

$$
\begin{aligned}
V\left(S_i, I_j\right) = \min_{L\in[0,\bar{L}]} & \left\{ S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha L b\right]dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1-(r+\nu)dt]\left\{1 - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}\right\} V\left(S_i, I_j\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j+1}\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} V\left(S_{i-1}, I_j\right) \\
& \left. + [1-(r+\nu)dt]\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j-1}\right) \right\}
\end{aligned} \tag{51}
$$

On the edge of the space:

$$
\begin{aligned}
V\left(S_i, I_j\right) = \min_{L\in[0,\bar{L}]} & \left\{ S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha L b\right]dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1-(r+\nu)dt]\left\{1 - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}\right\} V\left(S_i, I_j\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} V\left(S_{i-1}, I_{j+k}\right) \\
& \left. + [1-(r+\nu)dt]\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j-1}\right) \right\}
\end{aligned} \tag{52}
$$

Before value function iteration, we have to determine the initial value of discrete space:

$$
V(0, I) = \frac{a^2 b^2}{8\alpha\gamma + 4\alpha(r+v)} I^2 + \frac{4\alpha\chi_d\phi + 2ab}{4\alpha\gamma + 4\alpha(r+v)} I + \frac{1}{4\alpha(r+v)} \tag{53}
$$

$$
V(S, 0) = \frac{\left(\frac{(1+\alpha b S)^2}{4\alpha}\right)}{r+v}; \tag{54}
$$

And then we use FOC to find optimal policy $L(t)$

$$
L = \frac{(1+\alpha b I + ab S) - 2\theta[\beta S I]\left[V_I^+ - V_S^-\right]}{2\alpha - 2\theta^2[\beta S I]\left[V_I^+ - V_S^-\right]} \tag{55}
$$