IMAGE RESTORATION

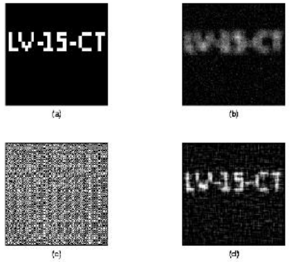


Image restoration

SUMMARY:

- Summary of frequency domain filtering
- Linear degradation model
- Linear restoration model
- Linear shift-invariant systems
- Inverse filter
- Parametric Wiener filter

Image restoration

Process of removing or reducing image degradations which occur during image formation.

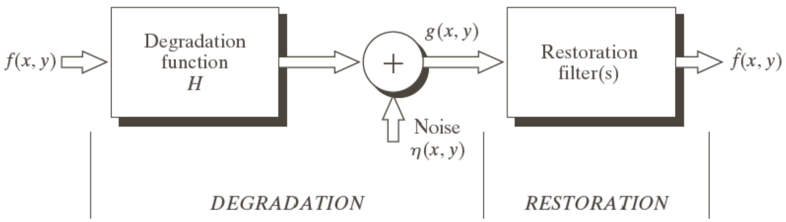


Image degradation

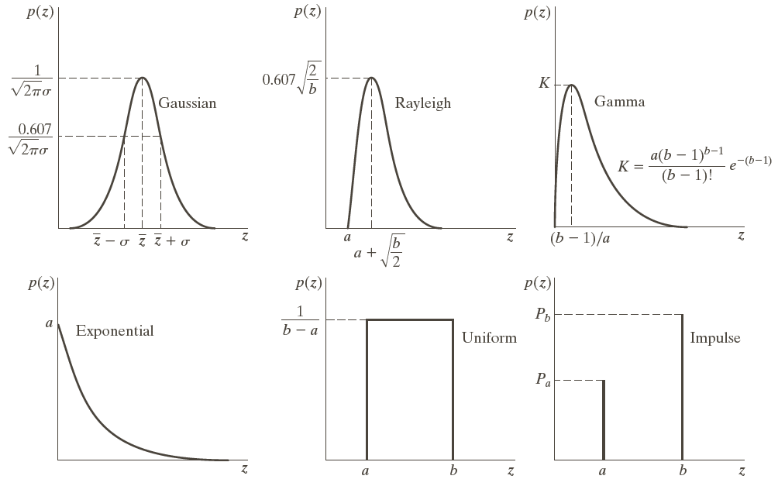
Degrading factors:

- blurring produced by the optical system or object motion during acquisition
- noise from electronic and optical devices.

Restoration

- Noise only: spatial filtering, frequency filtering
- Linear shift-invariant (LSI) degradations

Noise models



Gaussian; Rayleigh; Gamma; Exponential; Uniform; Impulse (shot noise).

Noise only filtering

Degraded image $g(x, y)$:

$$\boxed{g(x, y) = f(x, y) + \eta(x, y)}$$

- Mean filter: $\widehat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$
- Median filter: $\widehat{f}(x, y) = \text{median}_{(s,t) \in S_{xy}} g(s, t)$
- Bandreject frequency domain filters

Linear shift-invariant systems

A linear shift-invariant (LSI) system $\mathcal{O}$ maps an input signal $f(t)$ to an output signal $g(t)$, denoted as $f(t) \xrightarrow{\mathcal{O}} g(t)$, satisfying:

- Linearity: if $f_1(t) \xrightarrow{\mathcal{O}} g_1(t)$ and $f_2(t) \xrightarrow{\mathcal{O}} g_2(t)$, then, for arbitrary constants a, b,

$$a\, f_1(t) + b\, f_2(t) \xrightarrow{\mathcal{O}} a\, g_1(t) + b\, g_2(t)$$

- Shift invariance: if $f(t) \xrightarrow{\mathcal{O}} g(t)$, then for each time shift T,

$$f(t - T) \xrightarrow{\mathcal{O}} g(t - T)$$

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Convolution

- Output of a LSI system is given by the convolution of the input with a kernel $h(t)$:

$$g(t) = (h * f)(t) = \int_{-\infty}^{\infty} h(t-s)\, f(s)\, ds$$

- Filter kernel $h(t)$ is called the impulse response function or in 2-D point spread function.
- The Fourier transform $H(\mu)$ is called the transfer function.

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Linear shift-invariant degradations

- Degraded image $g(x,y)$ is convolution of ideal image $f(x,y)$ by kernel $h(x,y)$ plus noise $\eta(x,y)$:

$$\boxed{g(x,y) = (h * f)(x,y) + \eta(x,y).}$$

In frequency domain:

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- Finite images of size $N \times N$:

$$\boxed{\mathbf{g} = \mathbf{H}\,\mathbf{f} + \mathbf{n}}$$

where $\mathbf{g}, \mathbf{f}, \mathbf{n}$ are column vectors of length $M \times N$, and $\mathbf{H}$ is an $MN \times MN$ convolution matrix.

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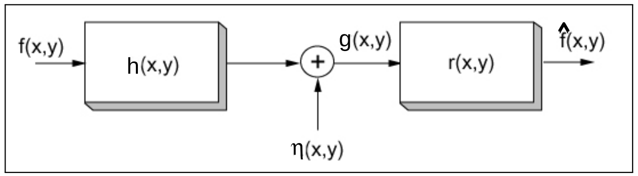
Linear restoration model

Deconvolution: compute an estimate $\widehat{f}(x,y)$ of the ideal image by using a reconstruction kernel $r(x,y)$:

$$\boxed{\widehat{f}(x,y) = (r * g)(x,y)}$$



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Constrained least squares restoration

- Solution Estimated solution $\widehat{\mathbf{f}}$ should satisfy the model equation:

$$\left\|\mathbf{g} - \mathbf{H}\widehat{\mathbf{f}}\right\|^2 = \|\mathbf{n}\|^2$$

- Constraints: introducing a matrix $\mathbf{Q}$ such that the term

$$\left\|\mathbf{Q}\,\widehat{\mathbf{f}}\right\|^2$$

is minimized.

Minimization problem and solution

- Minimize

$$\mathcal{E}(\widehat{\mathbf{f}}) = K \left\| \mathbf{Q}\,\widehat{\mathbf{f}} \right\|^2 + \left(\left\| \mathbf{g} - \mathbf{H}\widehat{\mathbf{f}} \right\|^2 - \|\mathbf{n}\|^2 \right)$$

  where K is a constant, called the regularization parameter, which determines the relative weight of both terms

- Least squares solution:

$$\boxed{\widehat{\mathbf{f}} = \left(\mathbf{H}^t\mathbf{H} + K\,\mathbf{Q}^t\mathbf{Q}\right)^{-1}\mathbf{H}^t\mathbf{g}}$$

Inverse filter

- Put $K = 0$ in formula for least squares solution:

$$\begin{aligned}\widehat{\mathbf{f}} &= (\mathbf{H}^t\mathbf{H})^{-1}\mathbf{H}^t\mathbf{g} = \mathbf{H}^{-1}\mathbf{g}\\ &= \mathbf{H}^{-1}(\mathbf{H}\,\mathbf{f} + \mathbf{n}) = \mathbf{f} + \mathbf{H}^{-1}\mathbf{n}\end{aligned}$$

  Expressed in the frequency domain:

$$\boxed{\widehat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}}$$

  This is the 'inverse' filter.

- This filter is useless in practice: small values in $H(u,v)$ amplify the noise.

Wiener filter

Assume $\mathbf{f}$ and $\mathbf{n}$ to be random vectors with zero mean and covariance matrices $\mathbf{R}_f = \mathbb{E}(\mathbf{f}\,\mathbf{f}^t)$ for the signal and $\mathbf{R}_n = \mathbb{E}(\mathbf{n}\,\mathbf{n}^t)$ for the noise. Let $\mathbf{Q}$ be the noise-to-signal ratio $\mathbf{Q} = \left(\frac{\mathbf{R}_n}{\mathbf{R}_f}\right)^{\frac{1}{2}}$ and let $K = 1$.

$$\boxed{\widehat{\mathbf{f}} = \left(\mathbf{H}^t\mathbf{H} + \mathbf{R}_f^{-1}\mathbf{R}_n\right)^{-1}\mathbf{H}^t\mathbf{g}}$$

Parametric Wiener filter

Minimum norm least squares solution:

$$\widehat{\mathbf{f}} = \left(\mathbf{H}^t\mathbf{H} + K\,\mathbf{I}\right)^{-1}\mathbf{H}^t\mathbf{g}$$

or, expressed in the frequency domain:

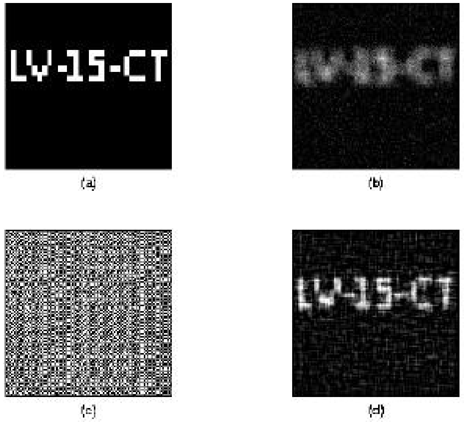
$$\boxed{\widehat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + K}\right]G(u,v)}$$

This is the Parametric Wiener filter or Pseudo-inverse filter.

K is the regularization parameter.

Case $K = 0$: inverse filter.

Example



(a): Original. (b): Degraded image. (c): Inverse filtering. (d): Pseudo-inverse filtering ($K = 0.01$).