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Behavior of Concrete Under Biaxial Stresses

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Experimental studies into the biaxial strength of concrete are reviewed and technical difficulties encountered in the development of a suitable test setup are discusssed. A new testing apparatus is described which allows testing of concrete specimens under various biaxial stress states. Results of an investigation for which this equipment was used are reported. The test data indicate that the strength of concrete under biaxial compression, $\sigma_1 = \sigma_2$, is only 16 percent larger than under uniaxial compression. Tests in the region of combined compression and tension confirmed previously obtained data. The biaxial tensile strength of concrete is approximately equal to its uniaxial tensile strength.

Keywords: biaxial stresses; compressive strength; concretes; plain concrete; research; strains; stresses; stress-strain relationships; tensile strength; test equipment.

■ STUDIES OF THE BEHAVIOR of concrete under multiaxial stress states are essential to develop a universal failure criterion for concrete. Moreover, they are important for the design of various types of concrete structures: biaxial stresses act in the shear region of flexural members as well as in shells, plates, and various containment structures. It is, therefore, not surprising that numerous experimental investigations into the strength of concrete under biaxial stress states have been conducted during the past 60 years. Unfortunately, the test data reported by various investigators deviate from each other considerably. Furthermore, most studies have been limited to tests in the range of biaxial compression, and no data on the behavior of concrete under biaxial tension are available. One of the major problems in conducting tests on concrete subjected to biaxial stresses is the development of a well defined and uniform biaxial stress state in the specimen. It is believed that the discrepancies between test results from different sources often can be traced back to unintended differences in the stress states which have been developed in the test specimen.

In this paper previous investigations will be reviewed briefly. A new test apparatus will be described and tests on concrete specimens subjected to biaxial stresses will be presented which cover the entire range of stress combinations from biaxial compression to biaxial tension.

REVIEW OF PREVIOUS INVESTIGATIONS

Previous tests into the behavior of concrete under biaxial stresses can be subdivided into three groups depending on the type of specimen used.

Concrete cubes or plates were used for studies of the biaxial compressive strength of concrete by Föppl,[1] Wästlund,[2] Glomb,[3] Weigler and Becker,[4] Iyengar,[5] Vile,[6] and Robinson.[7] Föppl showed that a prismatic specimen subjected to uniaxial or biaxial compressive loads may be confined along its loaded surfaces due to friction between the bearing platens of the testing machine and the concrete. It is well known that such restraint may result in an increase of the apparent strength of the test piece. Föppl, therefore, tried to eliminate confinement by applying a lubricant to the loaded surfaces of the specimen. He showed, however, that such treatment may lead to the opposite effect: soft packings or lubricating agents between specimen and bearing platen cause lateral

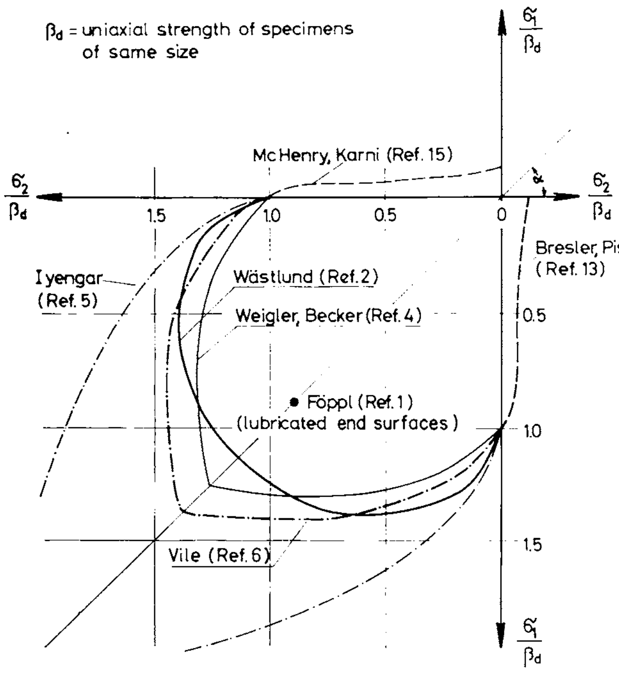

Fig. 1—Biaxial strength of concrete—review of previous investigations

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tensile stresses and a nonuniform stress distribution in the specimen resulting in a reduction of its apparent strength. Later investigators have continued to use test setups with conventional bearing platens and in some instances employed various surface treatments of the concrete or soft packings between the bearing platens and the specimen to eliminate restraint. The strength values obtained for the case of equal compression in both principal directions, $\sigma_1 = \sigma_2$, vary from 80 to 350 percent of the uniaxial compression strength of an identical test piece. Some of these test results are summarized in Fig. 1. In Reference 8 it is shown that friction between test specimen and bearing platens not only causes confinement of the concrete, but that part of the applied load may be sustained by the bearing platens which enclose the test specimen. If the load sustained by the bearing platens is not taken into account in determining the concrete stress, the strength of the test specimen will be overestimated.

Biaxial compressive stresses $\sigma_1 = \sigma_2$ can be generated by subjecting a cylindrical specimen to hydrostatic pressure in radial directions. This approach was used initially by Kármán[9] and Böker[10] in tests on marble. Richart, Brandtzaeg, and Brown[11] and later Fumagalli[12] applied this procedure to tests on concrete. To develop a truly biaxial stress state, restraint of the cylinders in the longitudinal direction must be avoided. At the same time penetration of the pressure fluid into cracks or pores on the surface of the concrete must be prevented, e.g., by placing the specimen

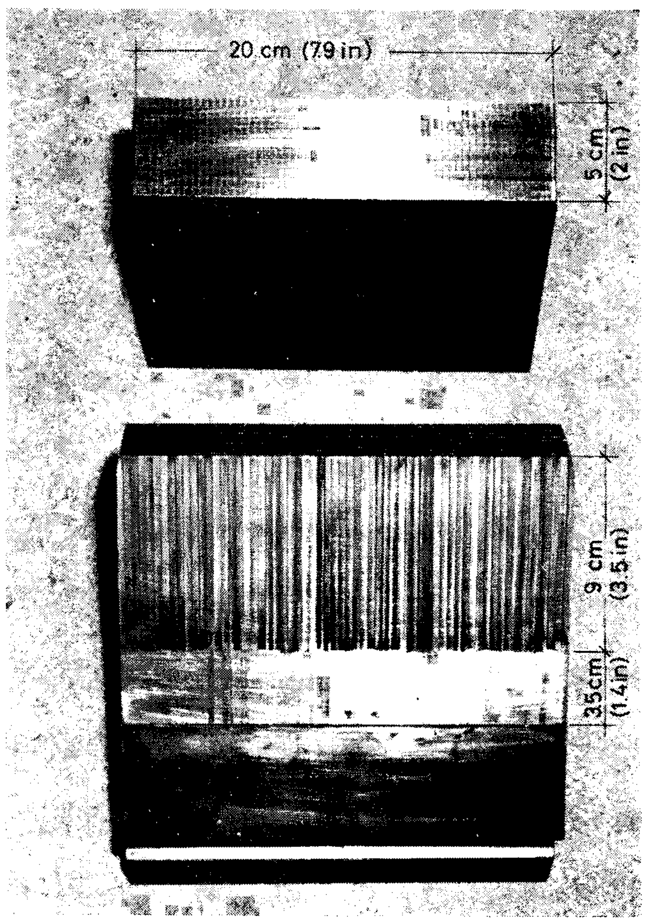

Fig. 2—Brush bearing platens; dimensions of filaments: 5 x 3 mm (0.195 x 0.118 in.); spacing of filaments: 0.2 mm (0.008 in.)

into a suitable membrane. Both of these requirements may not always have been satisfied in previous tests. This was realized by Richart, Brandtzaeg, and Brown who limited their conclusions to the statement that "the strength of the concrete in two dimensional compression was at least as great as the strength in simple compression."

Hollow cylinders subjected either to torsion and axial compression or to internal hydraulic pressure and axial compression were investigated by Bresler and Pister,[13] Goode and Helmy,[14] and by McHenry and Karni[15] to study the behavior of concrete under combined compressive and tensile stresses. Too large ratios of wall thickness to diameter of the specimen may lead, at least in the elastic range, to noticeable deviations from a uniform stress distribution across the thickness of the cylinder. However, the results from the various investigations are in comparatively good agreement and give a clear indication of the behavior of concrete subjected to a combination of tensile and compressive stresses. Bellamy[16] used hollow cylinders subjected to external pressure and axial compression. Values for the biaxial compressive strength up to 2.69 times the uniaxial strength were recorded.

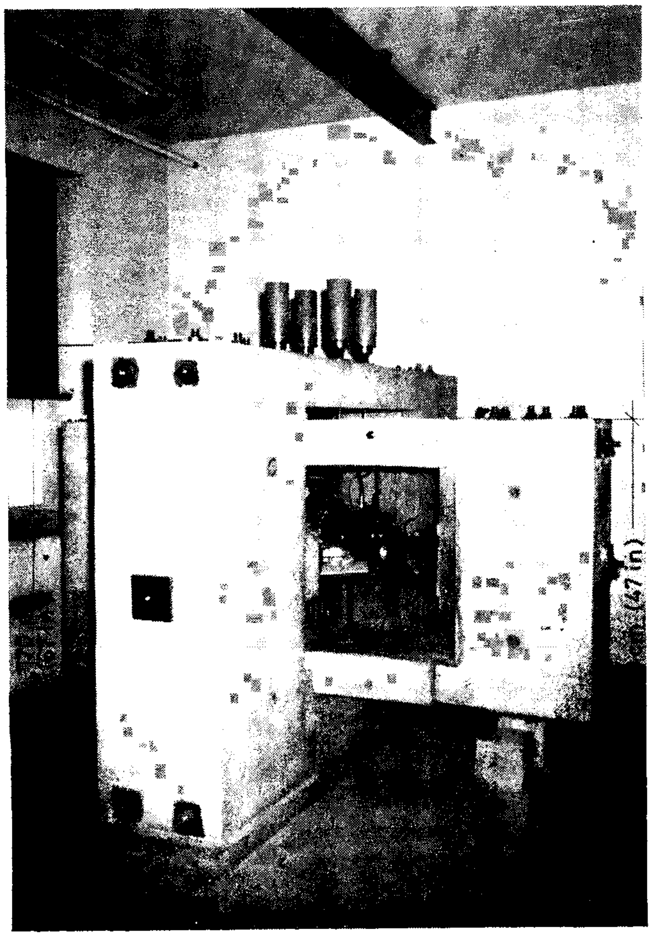

Figure 3—Loading frame for tests of concrete under biaxial stresses

A more detailed review of previous investigations has been presented e.g., in Reference 8. In this review it is concluded that square concrete plates subjected to in-plane loading appear to be suitable specimens to determine the biaxial strength of concrete over the entire range of biaxial stress combinations. It is proposed to load such specimens without restraint by replacing the solid bearing platens of conventional testing machine with "brush bearing platens." These platens consist of a series of closely spaced small steel bars (see Fig. 2) which are flexible enough to follow the concrete deformations without generating appreciable restraint of the test piece. Nevertheless their buckling stability is sufficient to transmit the required compressive forces into the concrete test piece. For tensile tests the filaments can be glued to the concrete. Various calibration tests showed the effectiveness of brush bearing platens in eliminating restraint. No adverse effects could be found such as local stress concentrations in the concrete near the tips of the small steel bars. Brush bearing platens were used in the investigation reported below. After completion of this work it was learned that a system similar to brush bearing platens had been used previously by Kjellman[17] in 1935 for tests on soil samples under triaxial compression.

EXPERIMENTAL INVESTIGATION

Scope

Concrete specimens 20 x 20 x 5 cm (7.9 x 7.9 x 2 in.) were subjected to biaxial stress combinations in the regions of biaxial compression, compression-tension and biaxial tension. Three types of concrete with an unconfined uniaxial compressive strength of 190, 315 and 590 kg/cm^2 (2700, 4450, and 8350 psi) were tested at 28 days. Within each region of stress combinations four different stress ratios σ_1/σ_2 were chosen, and six specimens were tested for each variable. A constant strain rate was maintained in loading the specimens. It was chosen such that the maximum load was reached after approximately 20 min. Loads and concrete strains in the three principal directions were recorded.

Concrete mixes and manufacture of specimens

The concrete contained gravel aggregate with a maximum size of 15 mm (0.6 in.). The water-cement ratio for the three types of concrete was 1.2, 0.9, and 0.43, respectively. Their cement content was 145, 190, and 445 kg/m^3, respectively. The specimens were cast horizontally in steel molds which had been precision machined so that no further preparation of the loaded surfaces was necessary. All specimens were compacted by hand. The specimens were moist cured for 7 days and then stored at a temperature of 20 C (68F) and a relative humidity of 65 percent. They were tested 28 days after casting.

Loading equipment

Brush bearing platens—A photograph of brush bearing platens used in this investigation is shown in Fig.

2. The platens consist of individual steel filaments with a cross section of 3 x 5 mm (0.12 x 0.20 in.). The length of the filaments varies from 100 to 140 mm (3.9 to 5.5 in.), depending on the maximum concrete stress for which the particular brush bearing platen can be used without buckling of the filaments. The higher the strength of concrete to be tested the shorter the individual filaments. The use of shorter brush bearing platens for higher strength concrete does not significantly increase the restraint of the test piece since the concrete strains at a given stress decrease as the strength of concrete increases. The individual filaments are spaced approximately 0.2 mm apart (0.008 in.) and are soldered together over a length of 35 mm (1.4 in.) so that a solid block is formed. The lateral flexibility of the filaments is such that for biaxial compression or biaxial tension the average principal stresses in the specimen do not deviate by more than 0.5 percent from the values calculated under the assumption of no restraint. For tests in the range of compression-tension, this error may be up to 3 percent. The flatness of the surface of the brush bearing platens was maintained within 2×10^{-3} mm (7.9×10^{-5} in.). For the tensile tests the brush bearing platens were glued to the concrete specimens using epoxy resins. Penetration of the glue between the brush filaments was avoided by sealing these spaces with a rubber cement. This treatment had no measurable effect on the flexibility of the filaments.

To verify effectiveness and reliability of the brush bearing platens, concrete prisms with various height to side length ratios including cubes as well as concrete plates 20 x 20 x 5 cm (7.9 x 7.9 x 2 in.) were loaded in uniaxial compression with and without brush bearing platens. If brush bearing platens were used, the strength of the specimens was independent of their shape and equal to the strength of prismatic specimens with a height to side length ratio of 4.0.[18] This appears to provide sufficient proof that end restraint of concrete specimens can be eliminated by brush bearing platens.

Loading frame—The testing machine used in this investigation is shown in Fig. 3. Individual frames were designed for the two principal stress directions. One frame is stationary while the other frame can move freely. The latter frame is suspended from the stationary frame by means of long, hinged steel rods and four vertical springs so that it can follow small movements of the specimen in any direction without generating appreciable secondary stresses in the specimen. This also facilitates alignment of the test setup prior to loading. Both frames consist of precast, prestressed concrete elements with a compressive strength of 600 kg/cm^2 (8500 psi).

For the test, the specimen is placed in the center of the two crossing loading frames. It rests on an adjustable platform which is lowered after a small preload has been applied to the specimen. Double-acting hydraulic loading jacks fitted into the loading frame can generate maximum loads of 75,000 kg (165,000 lb) in compression and 40,000 kg (88,000 lb) in tension. Solid bearing platens with spherical seats to which the brush bearing platens can be mounted are attached to the loading frames.

Control of the ratio σ_1/σ_2—The ratio of the applied stresses σ_1/σ_2 can be maintained constant throughout a test by a load distributing frame as shown in Fig. 4. A hydraulic jack (1) which is connected to a pump applies a load to a beam which is supported by two additional hydraulic jacks (2) and (3). Pressure lines connect the jacks (2) and (3) with the hydraulic jacks in the main testing machine. The position of the hydraulic jack (1) is adjustable along the beam and controls the ratio of applied stresses σ_1/σ_2.

TEST RESULTS

Failure modes

The crack patterns observed in the specimens after failure were similar to those obtained in previous investigations. In the tests under uniaxial compression numerous microcracks parallel to the direction of the applied load were formed. Complete collapse of the specimen was accompanied by the formation of one major crack which has an angle of approximately 30 deg with respect to the direction of the externally applied load (Fig. 5). Specimens subjected to biaxial compression showed similar microcracks parallel to the free surfaces of the specimens. At failure an additional major crack developed which had an angle of 18-27 deg to the free surfaces of the specimen. Specimens subjected to combined tension and compression behaved similarly to the specimens loaded in biaxial compression as long as the applied tensile stress was less than 1/15th of the compressive stress. For larger tensile stresses single cleavages perpendicular to the principal

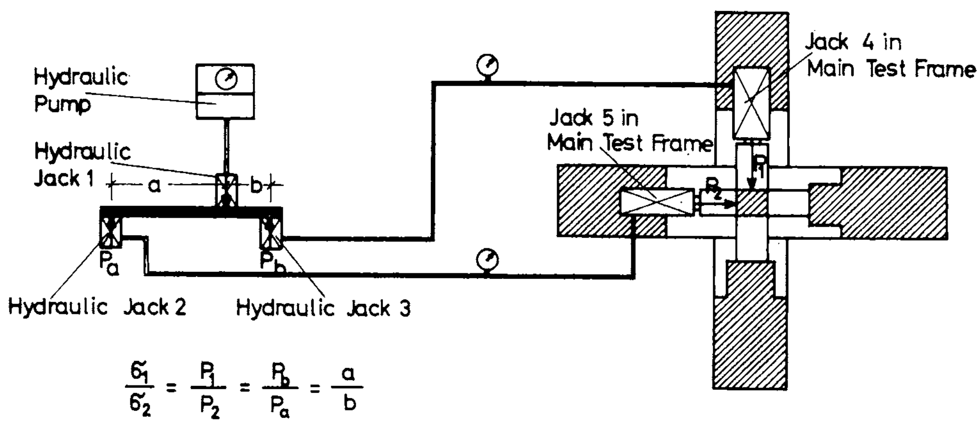

Fig. 4—Hydraulic system to maintain constant ratio σ_1/σ_2

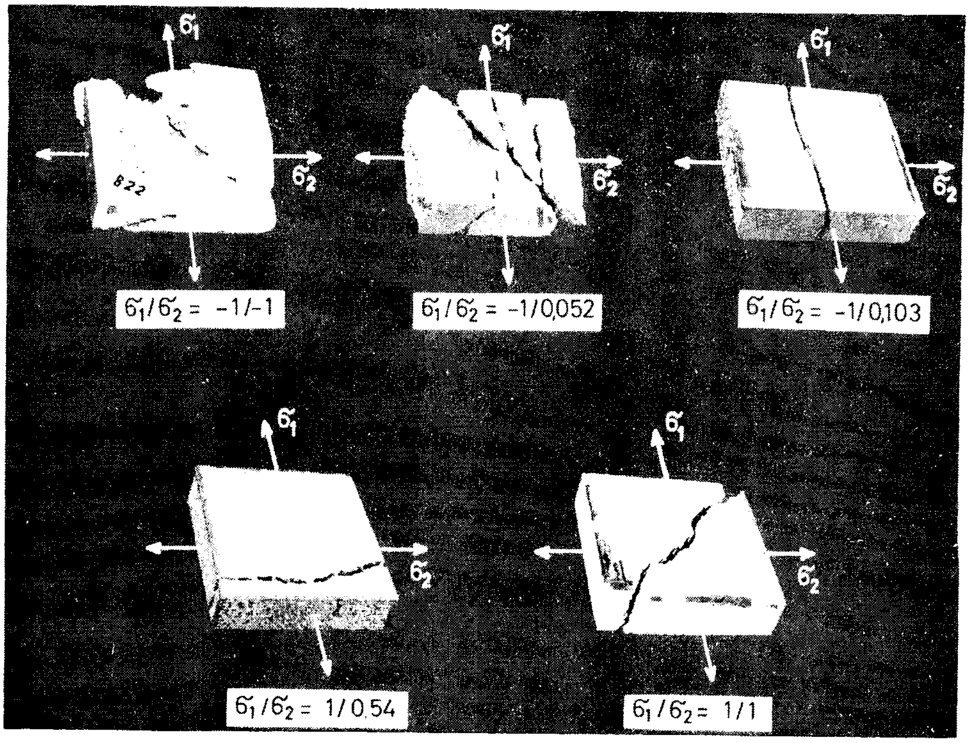

Fig. 5—Failure modes of specimens subjected to biaxial stresses

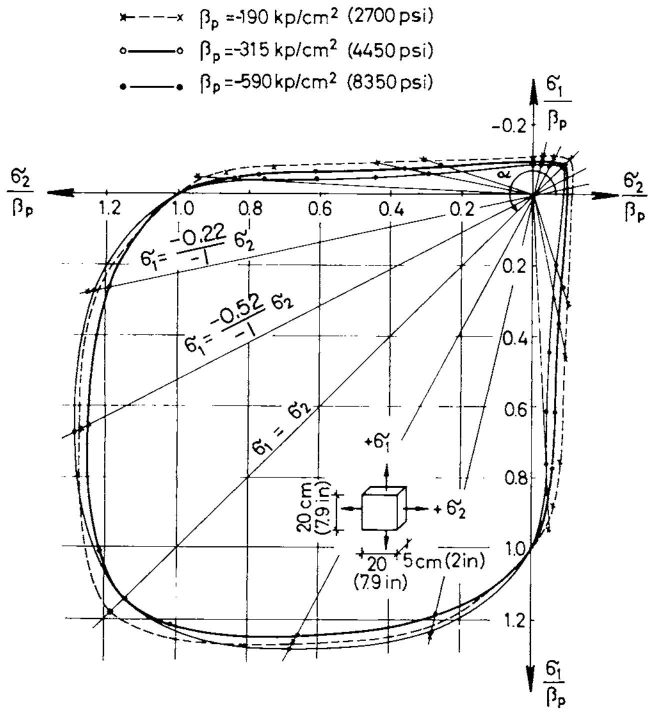

Fig. 6—Biaxial strength of concrete; results of experimental investigation

tensile stress were observed. Similar behavior was found for specimens loaded in biaxial tension. For equal tension in both principal directions no preferred direction of cleavage fracture could be observed except that the crack was always perpendicular to the free plane of the specimen.

Strength data

All strength data are reported as fractions of the unconfined uniaxial compressive strength, β_P which was obtained from the same specimens as used for the biaxial tests. As stated previously, β_P is identical with the uniaxial compressive strength of prisms 5 x 5 x 20 cm (2 x 2 x 7.9 in.) and, therefore is also referred to as "prism strength." In the following, all numerical stress, strength and strain values are recorded as negative values when compression, and as positive values when tension.

In Fig. 6 the relationship between the principal stresses at failure, σ_1/β_P and σ_2/β_P, is given for the three types of concrete investigated. Fig. 7 shows these relationships for the range of compression-tension and biaxial tension on a larger scale. According to Fig. 6, the strength of concrete under biaxial compression is larger than under uniaxial compression. The relative strength increase is almost identical for the three types of concrete which were investigated. The large variation in water-cement ratio and cement content had no significant effect on the biaxial strength. In the range of compression-tension and biaxial tension, however, the relative strength decreases as the uniaxial strength increases. Also the ratio of uniaxial tensile strength to the prism strength of the concrete, β_P is variable and amounts to —0.11, —0.09, and —0.08 for β_P = — 190, —315, and —590 kg/cm² (—2700, —4450, and —8350 psi), respectively. The strength of concrete under biaxial ten-

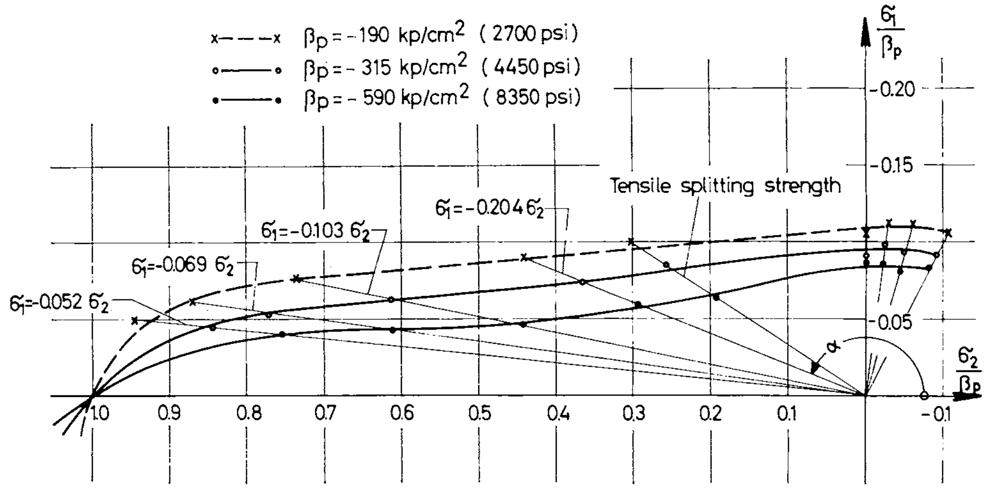

Fig. 7—Strength of concrete under combined tension and compression, and under biaxial tension; results from experimental investigation

sion is almost independent of the stress ratio σ_1/σ_2 and equal to the uniaxial tensile strength.

In Fig. 8 the average values σ_1/β_P and σ_2/β_P as obtained from the three types of concrete are shown. To demonstrate the restraining effect of solid bearing platens, additional tests on similar specimens loaded by using solid platens were conducted. The results of these tests are also shown in Fig. 8. For the unrestrained specimens the highest relative strength was obtained for a stress ratio $\sigma_1/\sigma_2 = -1/-0.5$ where $\sigma_1/\beta_P = 1.27$. For equal compression in both principal directions a strength of $1.16\beta_P$ was observed. For $\sigma_2 = 0$, $\sigma_1 = \beta_P$. From the tests with solid bearing platens significantly higher strength values were obtained. A maximum value of $\sigma_1/\beta_P = 1.48$ was observed for a stress ratio of $\sigma_1/\sigma_2 = -1/-0.5$. For $\sigma_1 = \sigma_2$ the apparent strength of the specimens was $1.45\beta_P$. It should be kept in mind, however, that this increase in strength is apparent but not real and only due to the restraint of the specimen.

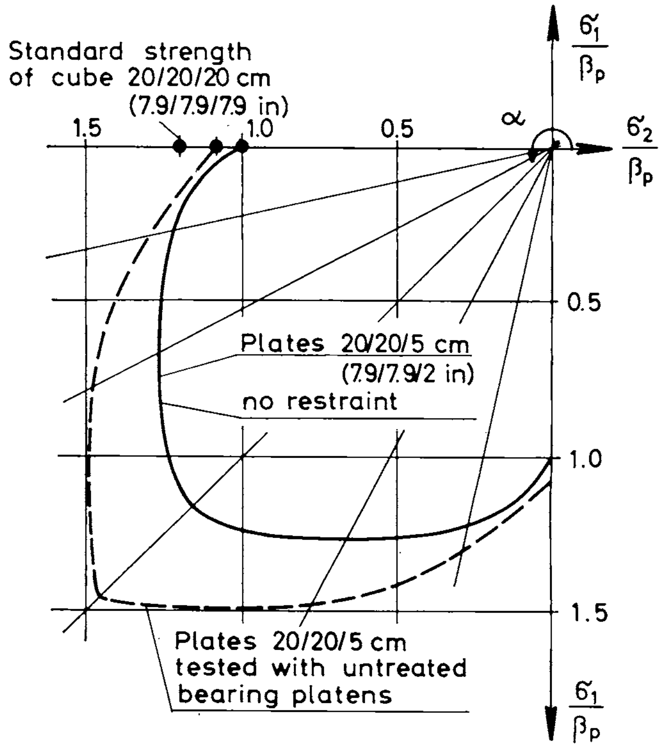

Fig. 8—Strength of concrete under biaxial compression; comparison of data from restrained and unrestrained specimens

Concrete strains

Strains in the three principal directions were recorded for all tests. The following discussion, however, will be limited to strains measured on specimens with an average prism strength $\beta_P = -315$ kg/cm^2 (4450 psi).

Stress-strain relationships for specimens subjected to biaxial compression are shown in Fig. 9. The corresponding curves for the region of combined compression and tension, and for biaxial tension are presented in Fig. 10 and 11. In Fig. 12 the principal strains at failure for various stress combinations are summarized. In this diagram the three principal strains at failure are given as function of the stress ratio σ_1/σ_2 which is expressed in terms of the angle α where

$$\tan\alpha = \sigma_1/\sigma_2 \text{ or } \alpha = \tan^{-1}(\sigma_1/\sigma_2)$$

The angle α corresponds to the slope of the straight lines through the origin of a $\sigma_1 = f(\sigma_2)$ diagram as shown in Fig. 6 and 7. The four regions of biaxial stress combinations are limited by the following values of α:

Biaxial tension .... $0 < \alpha < \frac{\pi}{2}$

Tension-compression .. $\frac{\pi}{2} < \alpha < \pi$

Biaxial compression ... $\pi < \alpha < \frac{3}{2}\pi$

Compression-tension .. $\frac{3}{2}\pi < \alpha < 2\pi$

For comparison, the principal stresses at failure σ_1 and σ_2 are also given as functions of the stress ratio σ_1/σ_2 in Fig. 13. In the region of biaxial compression in Fig. 9 the strains in the direction of the larger principal stress increase in magnitude as the stress at failure increases in magnitude They range from $\varepsilon_1 = -2.2$ mm/m (0.0022 in. per in.) for uniaxial compression to -3.0 mm/m for $\sigma_1/\sigma_2 = -1/-0.52$. For $\sigma_1/\sigma_2 = -1/-1$ a value of -2.6 mm/m was observed. For combined compression and tension the failure strains vary as expected: the failure strains in the direction of the compressive stress decrease in magnitude in Fig. 10 as the simultaneously acting tensile stress increases. The failure strains for the range of

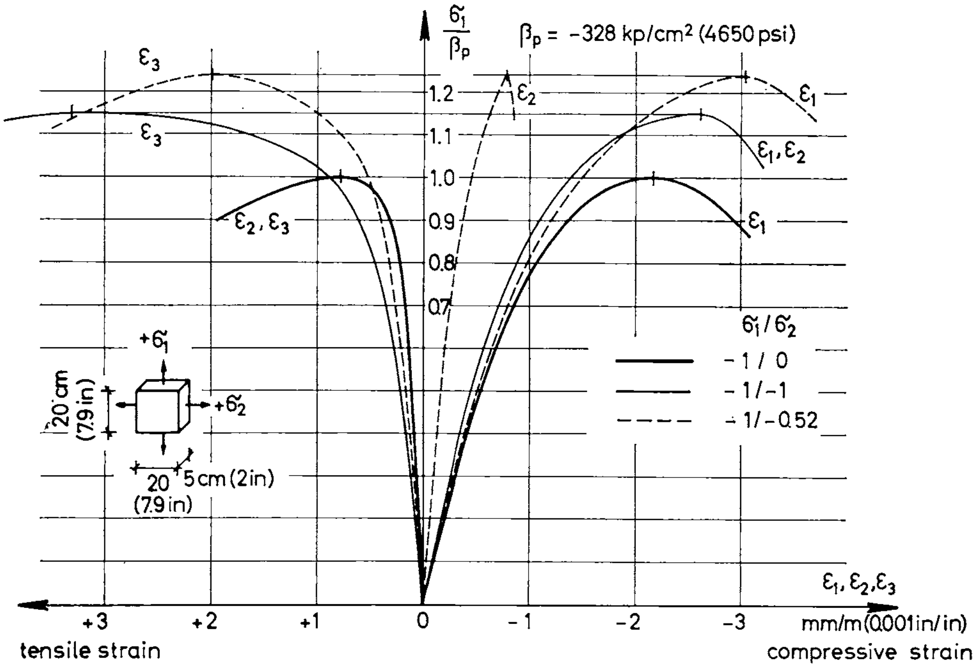

Fig. 9—Stress-strain relationships of concrete under biaxial compression

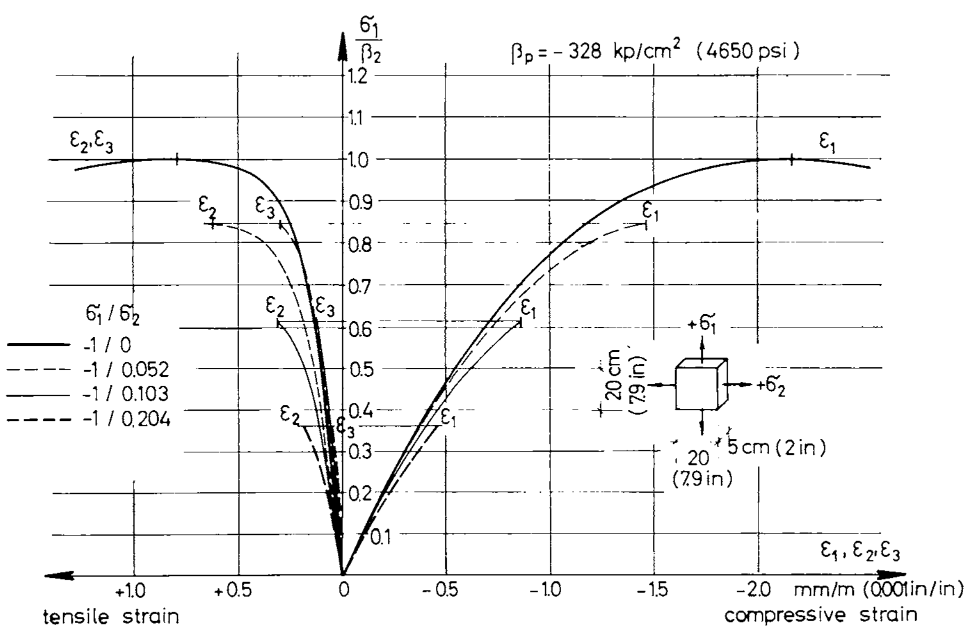

Fig. 10—Stress-strain relationships of concrete under combined tension and compression

biaxial tension deviate little from values calculated according to Hooke's law.

For biaxial compression the volumetric strains $\Delta V/V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ for various stress ratios σ_1/σ_2 are shown in Fig. 14. Up to stresses $\sigma_1 = 0.35\,\beta_P$ volumetric strain and applied stress are approximately proportional. If the stress increases beyond this value the rate of volume reduction increases until at 80 to 90 percent of the ultimate a point of inflection is reached. The minimum volume was observed at approximately 95 percent of the failure stress. Further straining of the specimen resulted in a volume increase and eventually in positive values for $\Delta V/V$. Similar relationships were found by other investigators.[6,11] It is generally agreed that the inflection point coincides with the stress at which major microcracking of the concrete is initiated.

In Fig. 14 the strain ε for uniaxial compression expressed as a fraction of ε_{1u} at failure is also given as a function of the volumetric strain $\Delta V/V$. For a constant Poisson's ratio this relationship is linear. Apparently, Poisson's ratio is constant beyond the elastic limit and increases only at stresses beyond the point of inflection of the volumetric strain relationship.

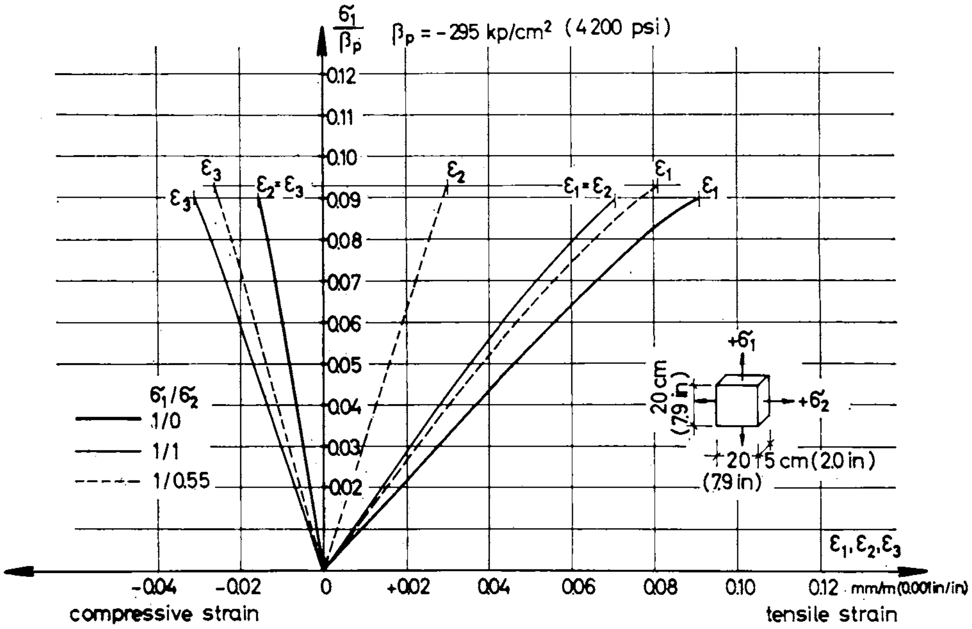

Fig. 11—Stress-strain relationships of concrete under biaxial tension

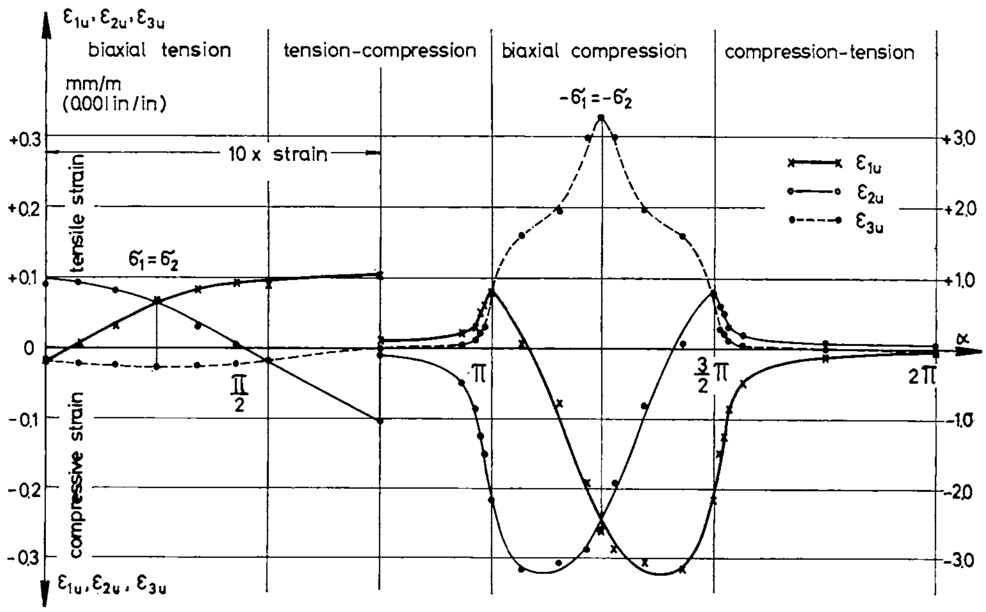

Fig. 12—Failure strains of concrete under biaxial stress states $\varepsilon = f(\sigma_1/\sigma_2) = f(\alpha)$ where $\tan\alpha = \sigma_1/\sigma_2$

In Fig. 15 the relative stresses at the elastic limit, at the point of inflection, at minimum or maximum volume and at failure are shown for the entire range of biaxial stress combinations.

Finally the relationships between the stress ratio and the modulus of elasticity and Poisson's ratio for stresses below the elastic limit were studied. The three equations for stress and strain according to Hooke's law were used and solved by successive approximation. For the tests on specimens with $\beta_P = 315$ kg/cm^2 the modulus of elasticity is 325,000 kg/cm^2 (4,600,000 psi). It is independent of the applied stress ratio. The coefficient of variation amounts to 3.3 percent. Within the region of biaxial compression a constant value for Poisson's ratio of $\mu = 0.20$ was calculated. The corresponding value for the region of biaxial tension was 0.18. For combined compression and tension Poisson's ratio ranged from 0.18 to 0.20.

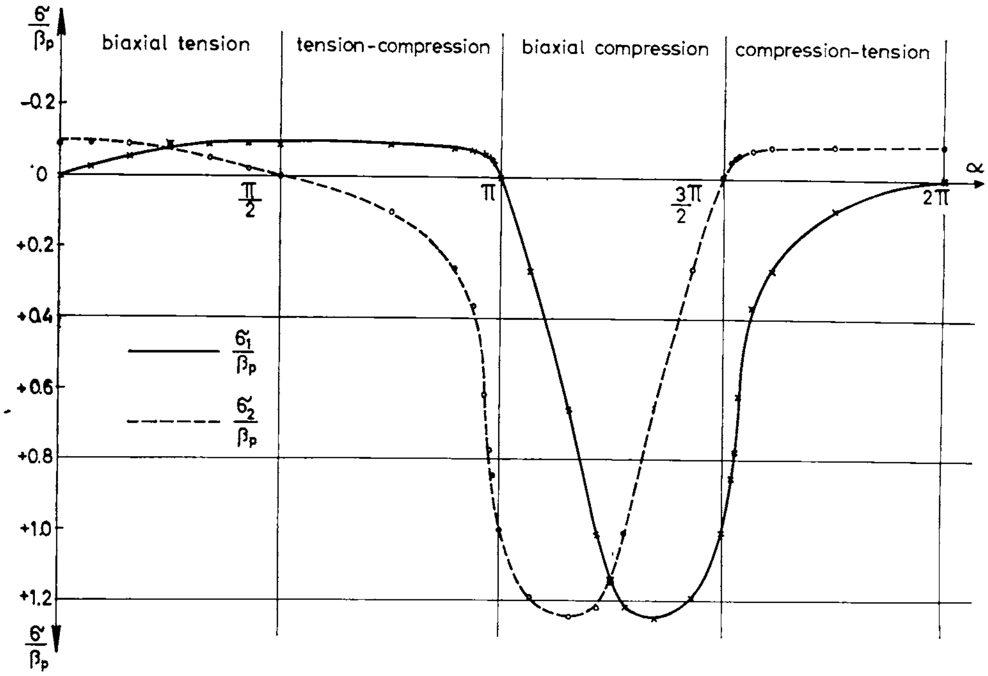

Fig. 13—Strength of concrete under biaxial stress states $\varepsilon = f(\sigma_1/\sigma_2) = f(\alpha)$ where $\tan\alpha = \sigma_1/\sigma_2$

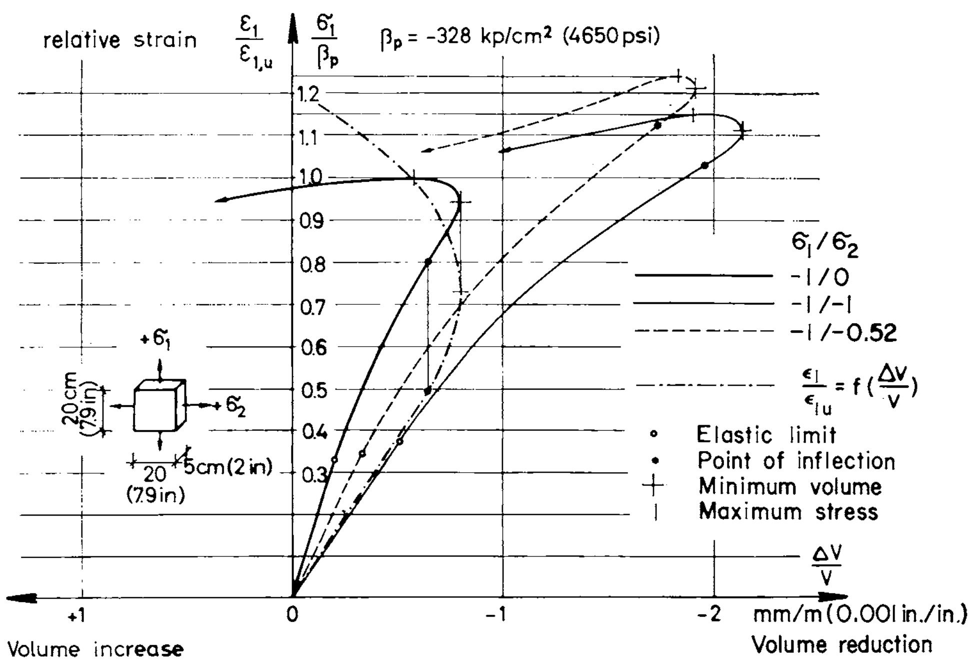

Fig. 14—Volumetric strain of concrete under biaxial compression

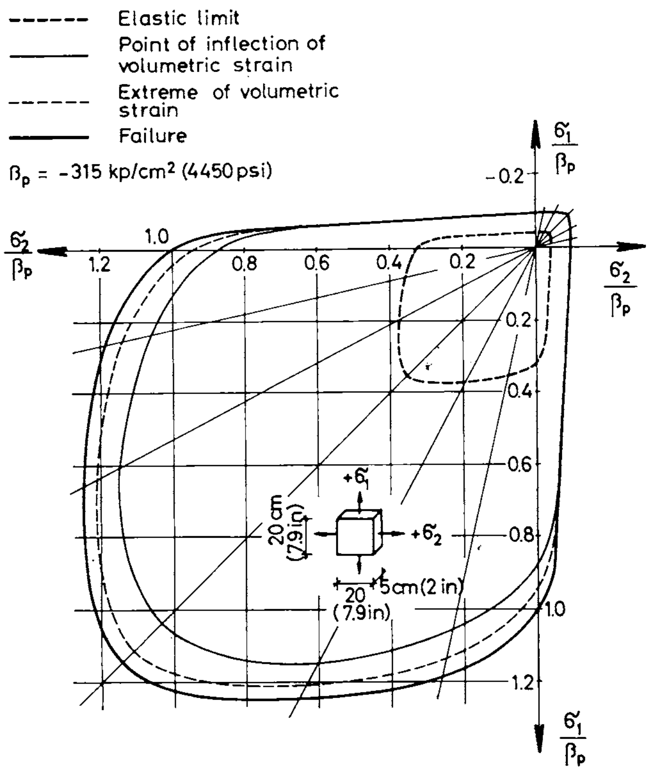

Fig. 15—Stresses at the elastic limit, minimum volume and failure of concrete subjected to biaxial stress states

SUMMARY AND CONCLUSIONS

A review of previous experimental investigations into the strength of concrete under biaxial stress states reveals considerable deviation among test data from different sources. For the case of equal compression in two principal directions strength values ranging from 80 to 350 percent of the uniaxial strength of concrete have been reported. It is likely that these differences can be attributed to difficulties in developing a well defined biaxial stress state in the test specimens. No data in the region of biaxial tension have been reported in the literature. In the present investigation use was made of a recently developed test setup, which allows testing of square concrete plates under any combination of in plane biaxial compressive and tensile stresses. Restraint of the test piece is avoided by brush-like load bearing platens. The test data reported herein show that the strength of concrete subjected to biaxial compression may be up to 27 percent higher than the uniaxial strength of concrete. For equal compressive stresses in two principal directions the strength increase is approximately 16 percent. These values are considerably smaller than many of the test data reported previously. The tests in the region of combined compression and tension substantiate the results obtained by other investigators which show that the compressive stress at failure decreases as the simultaneously acting tensile stress is increased. The strength of concrete under biaxial tension is approximately equal to its uniaxial tensile strength. Furthermore, it was shown that for low stresses the modulus of elasticity for and Poisson's ratio low stresses is independent of the applied stress ratio.

This investigation is being continued. At present, tests on concrete under sustained biaxial stresses are being conducted. In addition studies of the failure mechanism and of a universal failure criterion for concrete are continued. The test setup is also suitable to investigate the behavior of concrete under triaxial stresses. Where all principal stresses can be varied independently of each other.

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Sinopsis—Résumé—Zusammenfassung

Comportamiento del Concreto Sujeto a Esfuerzos Biaxiales

Se revisan los estudios experimentales sobre la resistencia biaxial del concreto así como las dificultades técnicas encontradas para el desarrollo de un sistema apropiado de ensaye. Se describe un nuevo aparato de ensaye el cual permite ensayar muestras de concreto bajo diferentes condiciones de esfuerzo biaxial. Se reportan los resultados de una investigacién en la cual se usó este equipo. Los datos de ensaye indican yue la resistencia del concreto bajo compresión biaxial, $\sigma_1 = \sigma_2$, es sólo 16 por ciento mayor que bajo compresión uniaxial. Los ensayes en la región de compresión y tensión combinadas confirmaron los datos previamente obtenidos. La resistencia biaxial a tensión del concreto es aproximadamente igual a su resistencia uniaxial a tensión.

Comportement de Béton sous Contrainte Bi-Axiale

Des études expérimentales sur la résistance bi-axiale de béton sont passées en revue et des difficultés techniques rencontrées dans la réalisation d'essais convenables sont discutées. Une nouvelle méthode d'essai est décrite qui, permet d'essayer les spécimens de béton sous plusieurs étapes de contrainte bi-axiale. Les résultats d'une étude pour laquelle cet équipement a été utilisé est décrite. Les données de l'essai indique la résistance du béton sous une compression biaxiale $\sigma_1 = \sigma_2$, est seulement I6 pourcent plus importante que sous une compression uniaxiale. Des essais dans la région de compression et tension combinées ont confirmé les données obtenues précédemment. La résistance à la tension bi-axiale du béton est approximativement égale à sa résistance à la tension uniaxial.

Das Verhalten von Beton bei zweiachsiger Beanspruchung

Frühere Versuche über die Festigkeit von Beton bei zweiachsiger Beanspruchung werden zusammengefasst, und die Probleme, die beim Aufbau einer geeigneten Versuchseinrichtung zu beachten sind, werden aufgezeigt. Eine neue Prüfeinrichtung wird beschrieben, die es erlaubt Betonproben zwängungsfrei mit verschiedenen zweiachsigen Spannungskombinationen zu belasten. Versuche, die mit Hilfe dieser Prüfeinrichtung durchgeführt wurden, zeigen, dass die Umschlingungsfestigkeit des Betons nur um 16 Prozent grösser als die einachsige Druckfestigkeit des Betons ist. Die Ergebnisse von Versuchen im Druck-Zug-Bereich stimmen mit jenen anderer Autoren überein. Die zweiachsige Zugfestigkeit ist gleich der einachsigen Zugfestigkeit.