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The iteration method for downward continuation of a potential field from a horizontal plane

Shi-zhe Xu[1], Jinyu Yang[1,2*], Changfu Yang[1], Pengfei Xiao[1], Shengchang Chen[1] and Zhihong Guo[3]

[1]*Zhejiang University, Department of Earth Sciences, Hangzhou 310027, PR China,* [2]*Qingdao Institute of Marine Geology, Fuzhou Rd, Qingdao 266071, PR China, and* [3]*China Aero Geophysical Survey & Remote Sensing Center for Land and Resources, 31 Xueyuan Rd, Beijing 100083, PR China*

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ABSTRACT

This paper introduces an iteration method for the downward continuation of potential-field data from a horizontal plane, and compares it with the conventional frequency-domain method (Fourier transform) using 2D and 3D model tests. The paper evaluates the two methods in terms of the results, i.e. downward-continuation distance and stability. The iteration method proves to be more stable and able to downward continue the potential for a greater distance than the Fourier transform method.

INTRODUCTION

The most frequently used method for the continuation of potential fields from plane to plane is the Fourier transform (FT) method. The features of the FT method are: (1) a high computation speed; (2) the upward continuation calculation is stable and the result is good; (3) downward continuation is unstable and the result is poor: the maximum continuation distance is only a few times the point interval of the data set. Downward continuation of the potential-field data to a deeper level is of interest to many geophysicists. Pawlowski (1995) proposed a preferential continuation method, derived from a Wiener filter and Green's equivalent-layer principles, for potential-field anomaly enhancement. Fedi and Florio (2002) presented a stable downward-continuation method based on the computation of stable vertical derivatives obtained by using the integrated second vertical derivative method and a Taylor series expansion of the field. Cooper (2004) gave three new methods based on derivative and compensation approaches for the reduction of FT-induced noise in the downward-continuation process. Even though the above-mentioned methods improve the stability of the result and the signal-to-noise ratio to a certain extent, the downward-continuation distance is still short; it is about 2 grid intervals using Pawlowski's (1995) method, 5 grid intervals using Fedi and Florio's (2002) method, and 10 sample intervals for 2D potential and 3.25 grid intervals for 3D potential using Cooper's (2004) method. The iterative method for downward continuation was first proposed by Strakhov and Devitsyn (1965). They used an integral formula to calculate the potential fields in the process of upward continuation. However, the potential-field community has not yet adopted their method. Guspi (1987) proposed an iterative fast Fourier procedure to continue the potential fields, measured over a topographic surface, to a horizontal plane which is close to the topography. Taking advantage of the stability of upward continuation using the FT technique, we propose an iterative approach to plane-to-plane potential-field downward continuation. Using this method, we can, in a stable manner, downward continue the potential-field data to a much deeper level: for the 2D case, for 100 sample intervals; for the 3D case, for 20 grid intervals. Although the computation time for the iteration method is slightly bit longer than that for the fast Fourier transform (FFT) method, it is acceptable. For example, it takes 5 minutes to perform the downward-continuation calculation of a data set consisting of 1024 by 1024 points on a Pentium III 900 MHz PC.

*E-mail: allyyangjinyu@yahoo.com

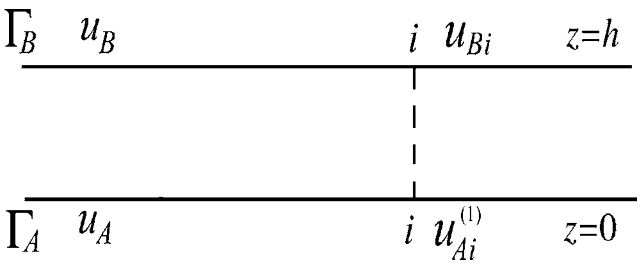

Figure 1 The observation plane Γ_B and the continuation plane Γ_A.

Here, both the iteration method and the FT method are applied to the downward continuation of the same model potentials, and the results are then compared.

PRINCIPLE OF THE METHOD

Figure 1 shows two horizontal surfaces, Γ_A and Γ_B, at heights of $z = 0$ and $z = h$, respectively. There is no potential source between Γ_A and Γ_B. The potential on the surface Γ_A ($z = 0$) is $u(x, y, 0)$. Let $U(k_x, k_y, 0)$ be the Fourier transform of $u(x, y, 0)$, i.e.

$$U(k_x, k_y, 0) = F[u(x, y, 0)], \tag{1}$$

where k_x and $k_y$are the wavenumbers in the x- and y-directions, respectively. It is well known that the potential $u(x, y, h)$ on the surface Γ_B ($z = h$) can be obtained from the equation,

$$u(x, y, h) = F^{-1}[(e^{-\sqrt{k_x^2+k_y^2}h}U(k_x, k_y, 0)], \tag{2}$$

where F^{-1} is the inverse Fourier transform.

Equation (2) is the continuation formula of the FT method, where h is the continuation distance. When $h > 0$, the process is called upward continuation and it is very stable. When $h < 0$, the process is called downward continuation and it is unstable.

The potential u_B on Γ_B is already known. We calculate the potential u_A on Γ_A using the iteration method to downward continue the potential from Γ_B to Γ_A, as follows:

1 Vertically project the point i on Γ_B on to point i on Γ_A, then copy the corresponding potential u_{Bi} to $u_{Ai}^{(1)}$ as the initial value of point i on Γ_A. For point i, let $u_{Ai}^{(1)} = u_{Bi}$; for all the points, let $u_A^{(1)} = u_B$.

2 Since the region between Γ_A and Γ_B is a source-free space, the potential satisfies Laplace's equation. We can use equation (2) to calculate the potential $u_B^{(1)}$ on Γ_B using the initial potential $u_A^{(1)}$ on Γ_A.

3 Using the difference between u_B and $u_B^{(1)}$ to correct $u_A^{(1)}$, we obtain a new $u_A^{(2)}$ using the equation,

$$u_A^{(2)} = u_A^{(1)} + s(u_B - u_B^{(1)}), \tag{3}$$

where s is the iterative step. Experimentally, we let $0<s\leq 1$. The smaller s is, the more stable the continuation result will be, and the longer the computational time. If s is too large, e.g. $s = 10$, the iteration may be divergent.

4 Repeating steps 2 and 3, we obtain the iterative equation,

$$u_A^{(n+1)} = u_A^{(n)} + s(u_B - u_B^{(n)}). \tag{4}$$

After n iterations, we obtain $|u_B - u_B^{(n)}| < \varepsilon$, where ε is a very small number. We then have $|u_A^{n+1} - u^n{}_A| < s\varepsilon$ and now $u_A^{(n+1)}$can be taken as the approximate value of u_A, i.e.

$$u_A \approx u_A^{(n+1)}. \tag{5}$$

In general, a stable and small ε can be reached after 20–50 iterations. Here, we use a Fourier transform for the upward continuation calculation in step 2 during this downward-continuation process of the iteration method.

Since the vertical component Z of the magnetic anomaly, the aeromagnetic anomaly T and the gravity anomaly g satisfy Laplace's equation in the source-free space, they can be considered as the potential u to which this iteration method can be applied for downward continuation.

COMPARISON OF THE DOWNWARD-CONTINUATION RESULTS OBTAINED BY THE ITERATION METHOD AND BY THE FT METHOD

2D model test

Figure 2 shows the 2D model test of downward continuation using the iteration method. The depth of the source is about 1000 m below the surface. Figure 2(a) shows the theoretical anomaly on the line at a height of 2000 m; the point spacing of the data is 10 m. We downward continue the anomaly from 2000 m to the lines at heights of 1000 m and 0 m (the continuation distances are 100 and 200 point intervals, respectively). The resulting data, without any smoothing process, are shown in Fig. 2(b, c). The dots represent the continued values, which are very close to the theoretical values (solid lines). The mean error near the maximum field is about 2%. This model test shows that even though the downward-continuation distance is 100–200 sample intervals, the results of continuation for the theoretical model are very good. However, the downward-continuation distance using the FT method is much smaller.

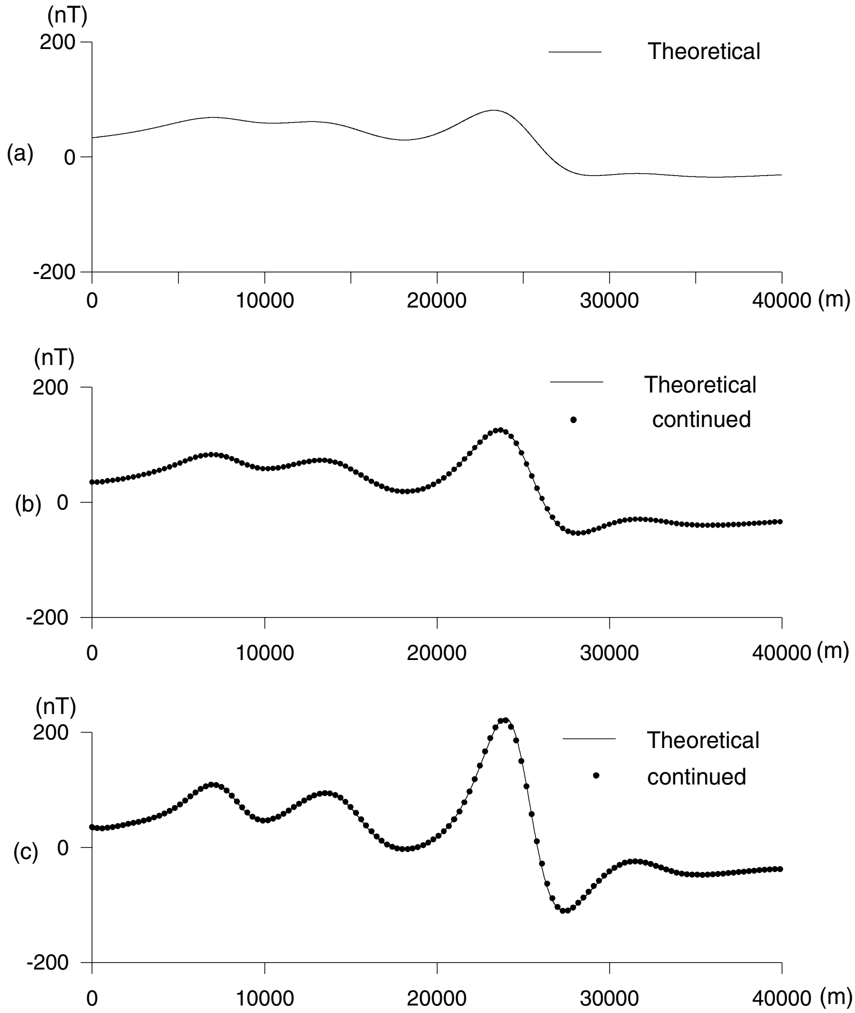

Figure 2 2D model test of downward continuation by the iteration method: (a) at height 2000 m; (b) at height 1000 m, continuation from 2000 m to 1000 m; (c) at height 0 m, continuation from 2000 m to 0 m. Total number of points: 4000. The distance between data points is 10 m. Not all the points are shown.

Figure 3 shows the results of the 2D model test of downward continuation using the FT method. We downward continue the theoretical magnetic anomaly (shown in Fig. 3a) from 2000 m to 1975 m (the continuation distance is 2.5 point intervals). The result, shown in Fig. 3(b), is not good. We increase the continuation distance to 30 m (3 point intervals) and plot the resulting data in Fig. 3(c) without smoothing. The result is divergent. In general, the downward-continuation distance is less than 3 sample intervals using the FT method, when no smoothing process is applied.

3D model test

Figure 4 shows the 3D model test of downward continuation using the iteration method. The shallowest depth of the 5 magnetic sources is about 1000 m below the surface. Figure 4(a) shows the theoretical magnetic anomaly on the horizontal plane at a height of 2000 m. The gridding interval is 100 m. We downward continue the theoretical potential value from 2000 m to 0 m (the continuation distance is 20 grid intervals). The result is shown in Fig. 4(c), without data smoothing. Figure 4(b) is the contour map of the theoretical magnetic anomaly on the surface at a height of 0 m, which compares very well with the continued data. We can see that the result is still good when the downward-continuation distance reaches 20 grid intervals using the iteration method. The mean error near the maximum field is about 6%.

In the model test, we express the root-mean-square error *rms* in the continuation as

$$rms = \sqrt{\frac{\sum_{i=1}^{m}\left[(u_{\mathrm{A,cont}})_i - (u_{\mathrm{A,theo}})_i\right]^2}{m}}, \qquad (6)$$

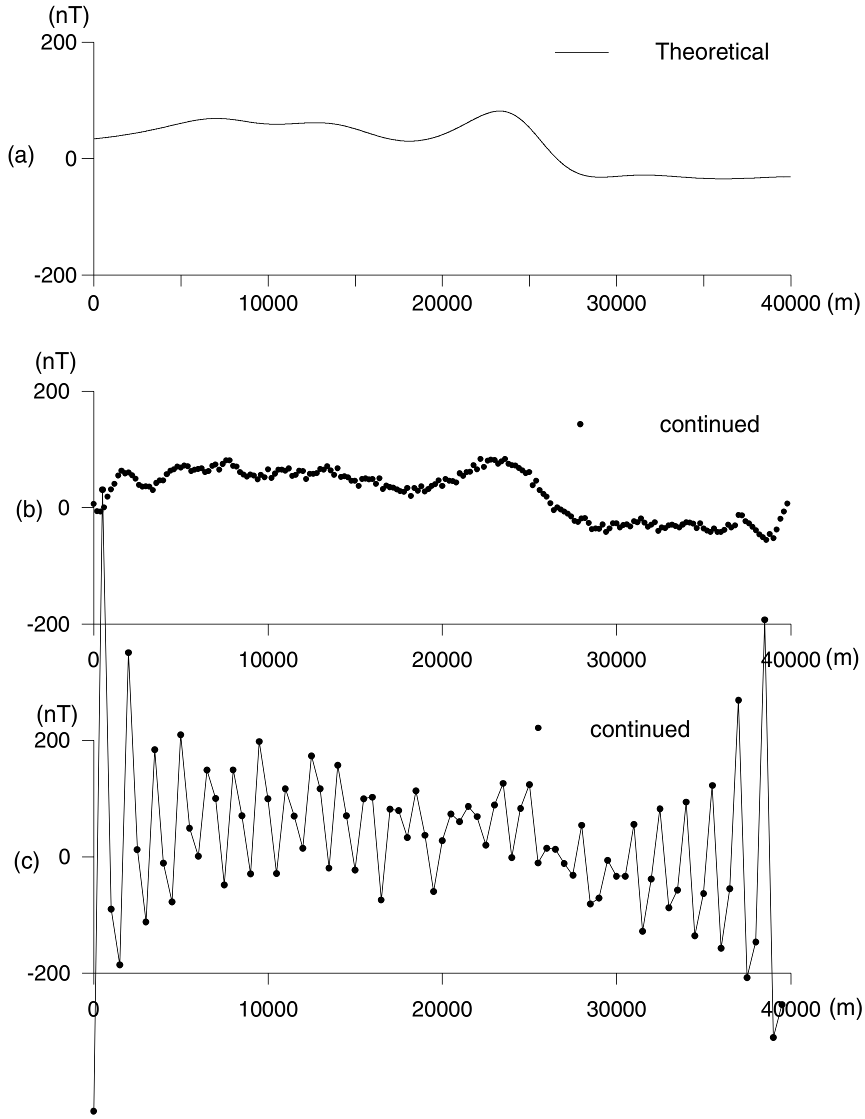

Figure 3 2D model test of downward continuation by the FT method: (a) at height 2000 m; (b) at height 1975 m, continuation from 2000 m to 1975 m; (c) at height 1970 m, continuation from 2000 m to 1970 m. Total number of points: 4000. The distance between data points is 10 m. Not all the points are shown.

where ($u_{\mathrm{A,cont}}$) and ($u_{\mathrm{A,theo}}$) are continued and theoretical values on the horizontal plane Γ_A, respectively, and m is the number of calculation points. Figure 5 shows the relationship between *rms* and the number of iterations n. From Fig. 5, we can see that as the number of iterations increases, *rms* decreases and then reaches a steady value, which indicates that the iterations are stable.

Figure 6 shows the 3D model test of downward continuation using the FT method. We downward continue the theoretical magnetic anomaly (shown in Fig. 4a) from 2000 m to 1900 m (the continuation distance is one grid interval) without data smoothing. The result is good and is shown in Fig. 6(a). We increase the continuation distance to 200 m (two grid intervals) and plot the continued data in Fig. 6(b). We can see that the result is very poor. When we increase the continuation distance to 300 m (three grid intervals), the calculation is divergent. It is apparent that the downward-continuation distance that the FT method can reach is less than three grid intervals if no smoothing process is applied.

DOWNWARD CONTINUATION WITH REAL DATA

There is no noise in the model data so we could not prove the excellence of our iteration method by using the model test alone. We know that real data always contains noise, thus we now give an example of downward continuation using real data. Before we apply the iteration method, we use an FFT

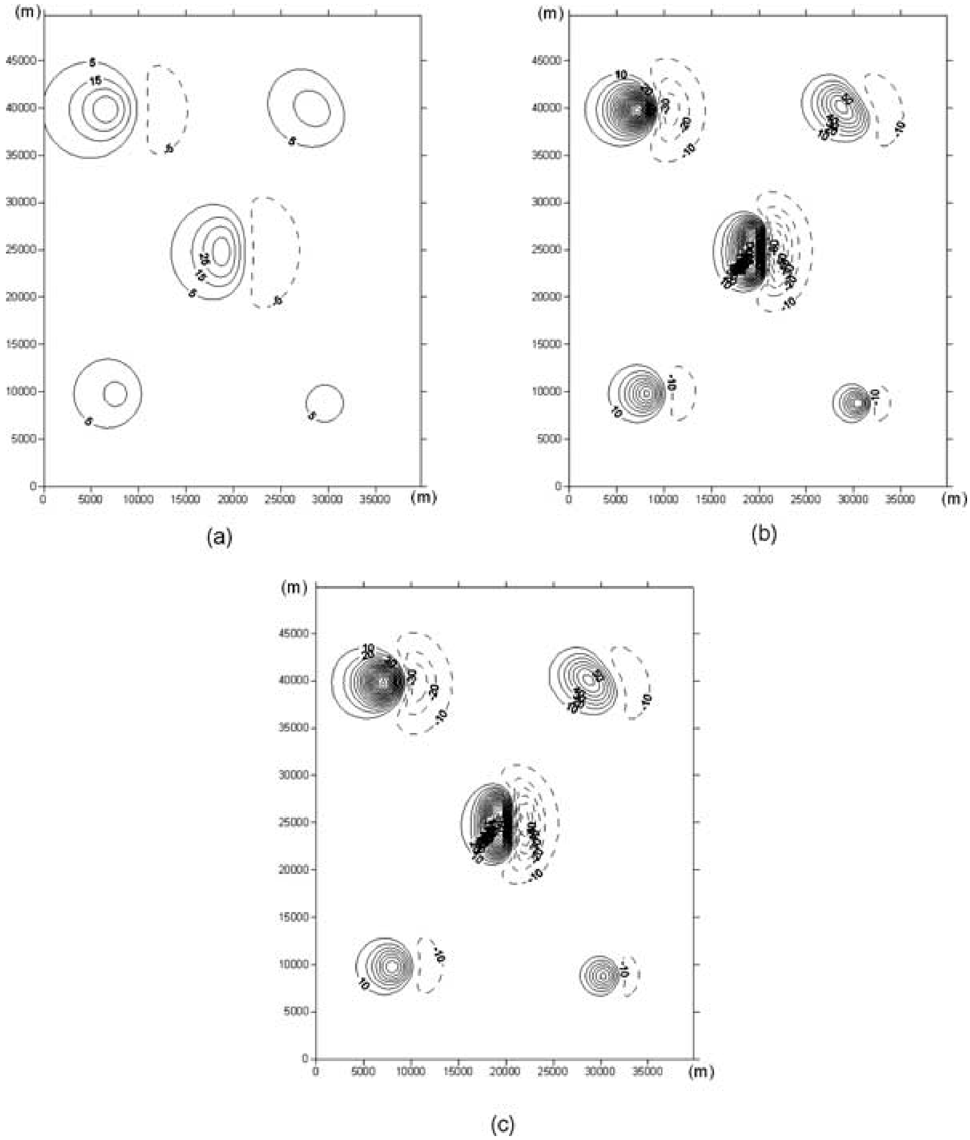

Figure 4 3D model test of downward continuation by the iteration method: (a) theoretical anomaly at height 2000 m; (b) theoretical anomaly at height 0 m; (c) continued anomaly from 2000 m to 0 m.

smoothing method (Bevington 1969) to smooth the real data in order to suppress the noise.

Figure 7(a) shows a magnetic anomaly on a horizontal plane at a height of 200 m. The grid interval is 50 m. We use the FT method to upward continue the anomaly to a horizontal plane at a height of 700 m; the result is shown in Fig. 7(b). We then downward continue the anomaly by the iteration method from 700 m to 200 m, without data smoothing; the result is shown in Fig. 7(c). The *rms* is 2.76 nT between Fig. 7(c) and Fig. 7 (a). The continuation depth is 10 times the grid interval. We downward continue the magnetic anomaly (Fig. 7b) from 700 m to 200 m (the continuation distance is ten grid

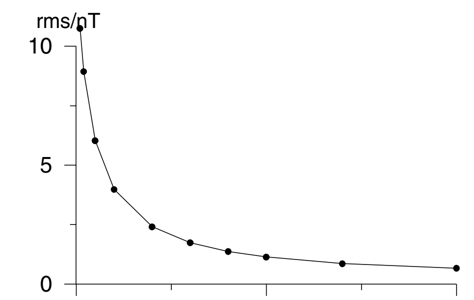

Figure 5 The relationship between number of iterations and *rms* (for Fig. 4).

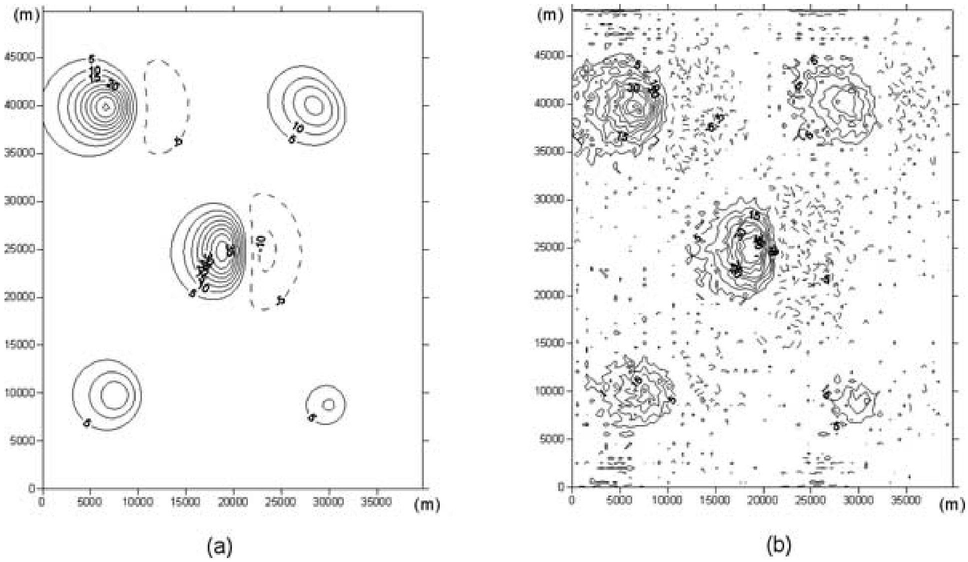

Figure 6 3D model test of downward continuation by the FT method: (a) continued anomaly from 2000 m (Fig. 4a) to 1900 m; (b) continued anomaly from 2000 m (Fig. 4a) to 1800 m.

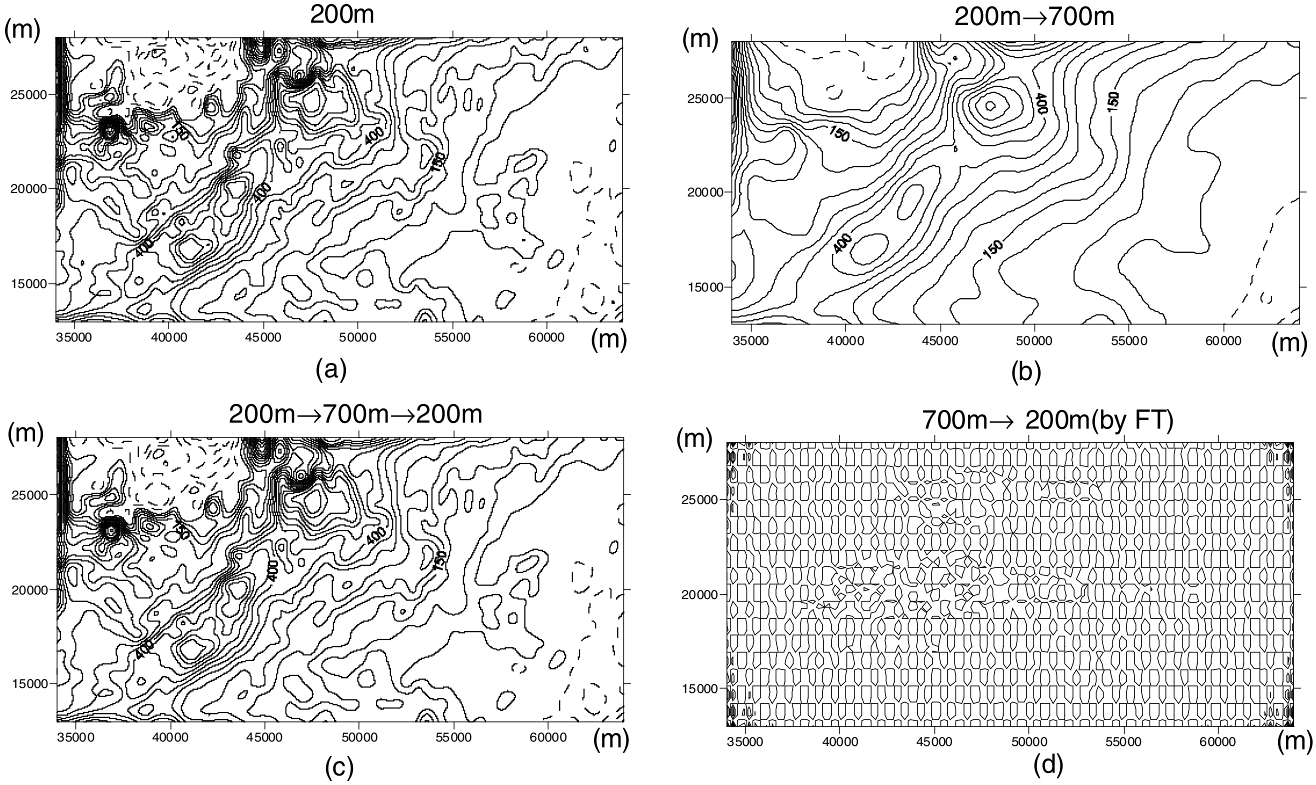

Figure 7 The result of real-data downward continuation (the grid interval is 50 m): (a) the magnetic anomaly on the 200 m horizontal plane; (b) the magnetic anomaly upward continued from 200 m to 700 m; (c) the magnetic anomaly downward continued from 700 m to 200 m by the iteration method; (d) the magnetic anomaly downward continued from 700 m to 200 m by the FT method.

intervals) using the FT method without data smoothing, as shown in Fig. 7(d). The result is very divergent. It is apparent that the downward-continuation distance obtained using the FT method is very limited.

COMPUTATION SPEED OF THE ITERATION METHOD

It is well known that FFT algorithms have a high computation speed. We apply an FFT algorithm to perform the FT upward-continuation calculation during the procedure of downward continuation by the iteration method. Thus the computation speed of the iteration method is also fast. For example, it takes 5 minutes to perform a downward-continuation calculation (50 iterations) for a data set consisting of 1024 by1024 points on a Pentium III 900 MHz PC, which is acceptable.

CONCLUSION

Although the FT method possesses a very high computation speed, it is not stable for downward continuation if the continuation distance is greater than several sample intervals. The iteration method can increase the downward-continuation distance to 20 sample intervals. The computation speed of the iteration method is also fast. Thus the iteration method is a possible new method for achieving the downward continuation of horizontal-plane potential fields.

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