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Frequency-Domain Reduction of Potential Field Measurements to a Horizontal Plane*

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ABSTRACT

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An iterative fast-Fourier procedure is derived for referring to a horizontal plane potential field data measured over a topographic surface.

The convergence of the procedure is studied and it is found that results are satisfactory when the plane is close to the topography, either above, below, or intersecting the measurement surface.

Examples of calculations with artificial and real geophysical data are also presented.

INTRODUCTION

Potential field data in land surveys are measured and referred to an irregular topographic surface. In particular for the gravity case it is clear from the papers by Levallois (1962) and Naudy and Neumann (1965) that the point of application of the anomalies, when all corrections are made, is over the topographic surface and not at sea-level.

However, for further transformations (upward and downward continuation, reduction to the pole, frequency-domain inversion, etc.) it is necessary to know the values of the potential field over a plane generally close to the surface (e.g. sea-level plane).

Space-domain solutions of the problem have been developed, among others, by Dampney (1969), using the concept of equivalent source representation for a Bouguer gravity field. The necessary matrix inversion increases the calculation effort when working with a great number of points.

Bhattacharyya and Chan (1977) define an equivalent source representation at the observation surface, which is computed through an iterative resolution

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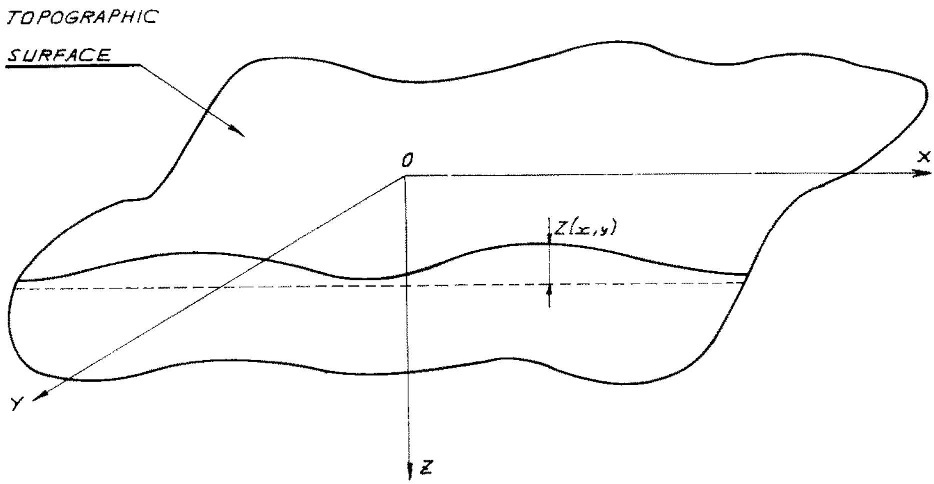

Fig. 1. Statement of the problem.

of a Fredholm integral equation. Gravity and magnetic cases are studied separately.

In the present work we derive a frequency-domain procedure, in which advantage is taken of ideas developed by Parker (1972) for direct gravity and magnetic calculations, Oldenburg (1974) for inverse gravity calculations and Parker and Huestis (1974) for inverse magnetic problems.

An iterative scheme, where a series of inverse Fourier transforms is computed at each stage, is the core of our method. Since the level of the horizontal plane can be chosen at will, convergence is better than in inversion problems and filtering is seldom needed. From the results obtained, field values can be rapidly referred to another plane parallel to the given plane by the usual methods of continuation.

No assumption is made about the nature of the potential field, which can be either gravimetric, magnetic, electric, or its components and derivatives, etc.

THEORY OF THE METHOD

Let $OXYZ$ be a system of rectangular coordinates (Fig. 1) with the Z-axis directed vertically downward.

Let $z(x,y)$ be the topographic surface at which a potential field $g^*(x,y)$ is measured and let us consider XY to be the horizontal plane on which we need to calculate the value of the field, $g(x,y)$. Let us suppose, besides, that no source material is interposed between XY plane and $z(x,y)$.

Let $G(\alpha, \beta)$ be the Fourier transform of $g(x,y)$:

$$\mathscr{F}[g] = G(\alpha,\beta) = \int\int_{x,y} g(x,y)\,\mathrm{e}^{-2\pi(\alpha x+\beta y)i}\mathrm{d}x\;\mathrm{d}y$$

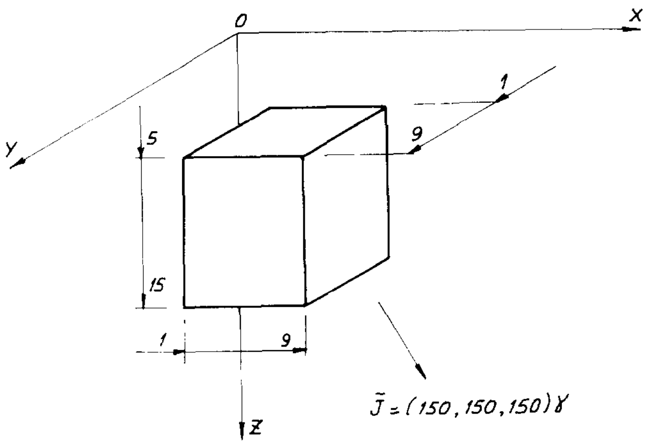

Fig. 2. Magnetized body used in the numerical example. Coordinates are in km.

Correspondingly, the inverse Fourier transform yields:

$$\mathscr{F}^{-1}[G] = g(x,y) = \iint_{\alpha,\beta} G(\alpha,\beta)\,e^{2\pi(\alpha x+\beta y)i}\,d\alpha\,d\beta \tag{1}$$

On the other hand, for a point having coordinates (x,y) the following expression (see e.g. Baranov, 1975, p. 48) gives the potential field at height $z(x,y)$:

$$g^*(x,y) = \iint_{\alpha,\beta} e^{\gamma z(x,y)}\,G(\alpha,\beta)\,e^{2\pi(\alpha x+\beta y)i}\,d\alpha\,d\beta \tag{2}$$

with:

$$\gamma = 2\pi\sqrt{\alpha^2+\beta^2}$$

Then, obviously:

$$g^*(x,y) - g(x,y) = \iint_{\alpha,\beta} [e^{\gamma z(x,y)} - 1]\,G(\alpha,\beta)\,e^{2\pi(\alpha x+\beta y)i}\,d\alpha\,d\beta$$

that is:

$$g(x,y) = g^*(x,y) - \iint_{\alpha,\beta} [e^{\gamma z(x,y)} - 1]\,G(\alpha,\beta)\,e^{2\pi(\alpha x+\beta y)i}\,d\alpha\,d\beta \tag{3}$$

The left-hand side of eq. 3 is the unknown field. The first term of the right-hand side is the measured field, and under the integral in the second term appears an expression containing the Fourier transform of the unknown field, $G(\alpha,\beta)$.

We can see, therefore, that eq. 3 defines a typical iterative procedure, summarized by:

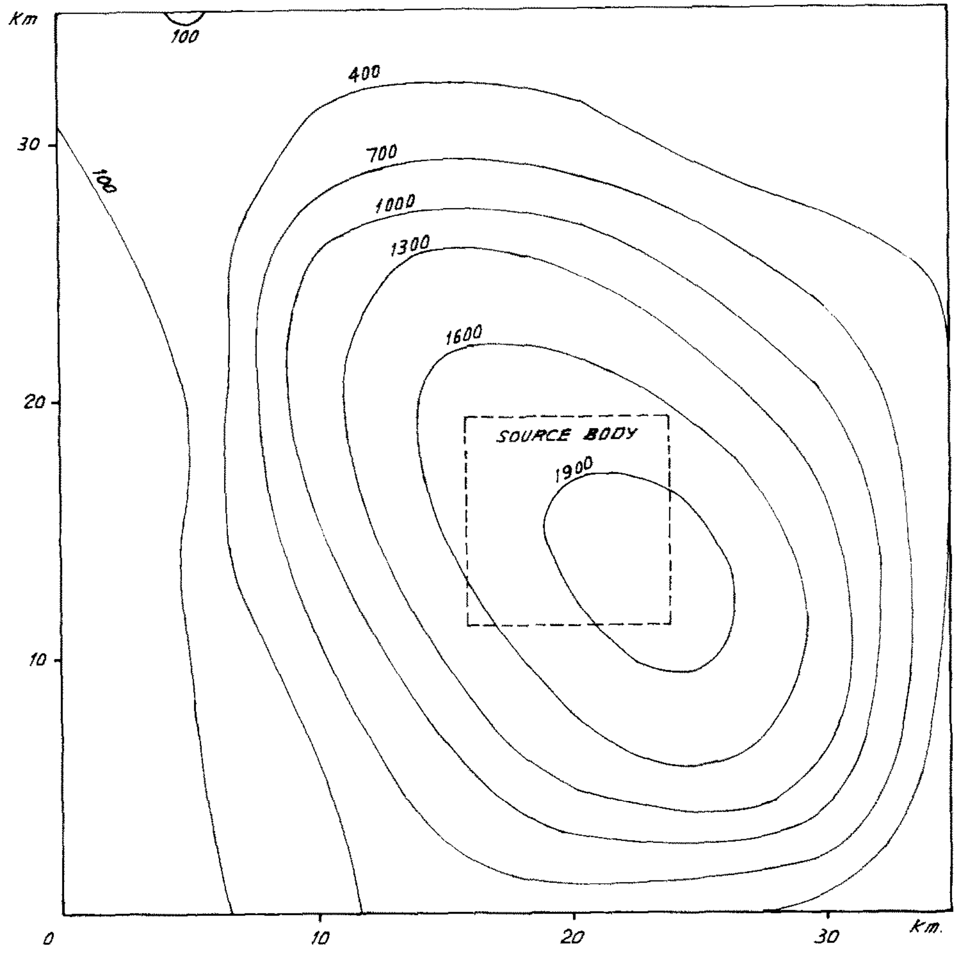

Fig. 3. Numerical example. Topographic relief. Heights are in metres.

$g = \phi(g)$

A first approximation is guessed (i.e., $g^*(x,y)$), its Fourier transform is put in place of $G(\alpha,\beta)$ and the right-hand side of eq. 3 is evaluated. The result is considered as a new approximation and the process can be repeated until some criterion of convergence is satisfied.

In the two-dimensional case, where the same profile is assumed in all the sections parallel to the XZ plane, an equivalent formulation of eq. 3 is:

$$g(x) = g^*(x) - \int_{-\infty}^{+\infty} [e^{2\pi|\alpha|z(x)} - 1] G(\alpha) e^{2\pi\alpha x i} d\alpha \tag{4}$$

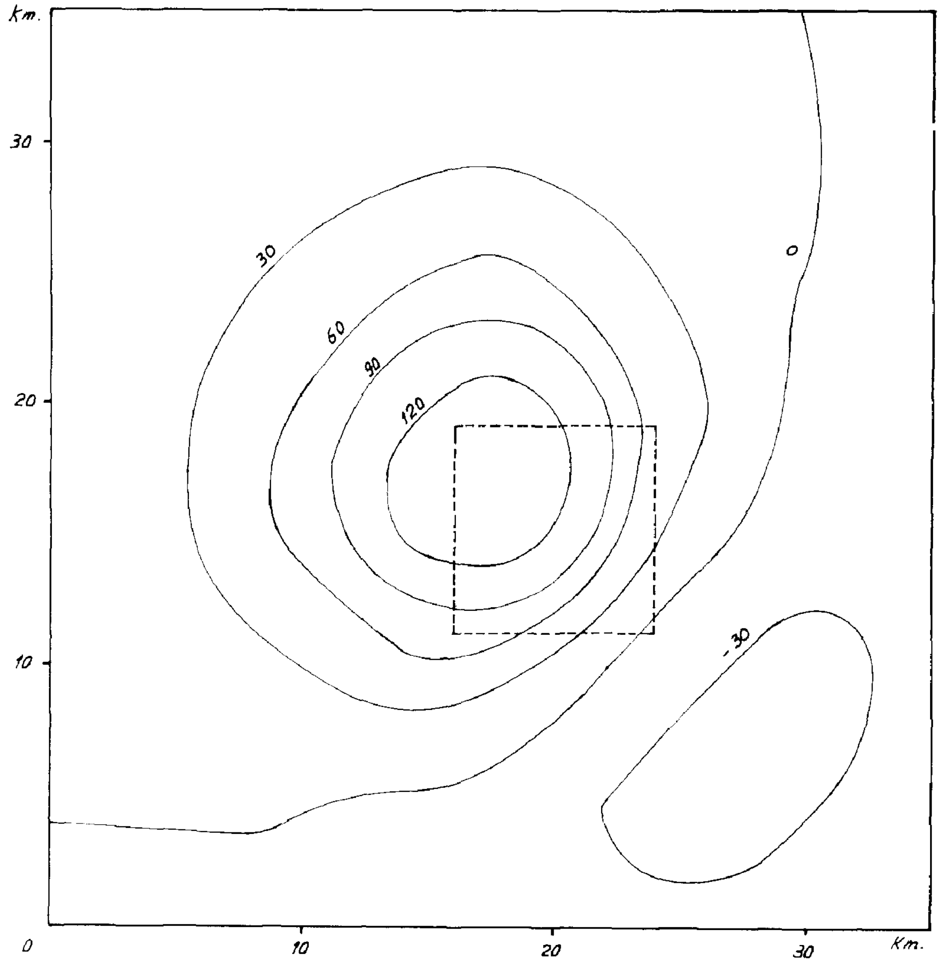

Fig. 4. Numerical example. Vertical magnetic field, in γ, computed at the topographic level.

where:

$$G(\alpha) = \int_{-\infty}^{+\infty} g(x)\,\mathrm{e}^{-2\pi\alpha x i}\,\mathrm{d}x \tag{5}$$

the Fourier transform of $g(x)$.

PRACTICAL COMPUTATION

The integral in eq. 3 is not a Fourier transform, and its direct evaluation is ill-advised.

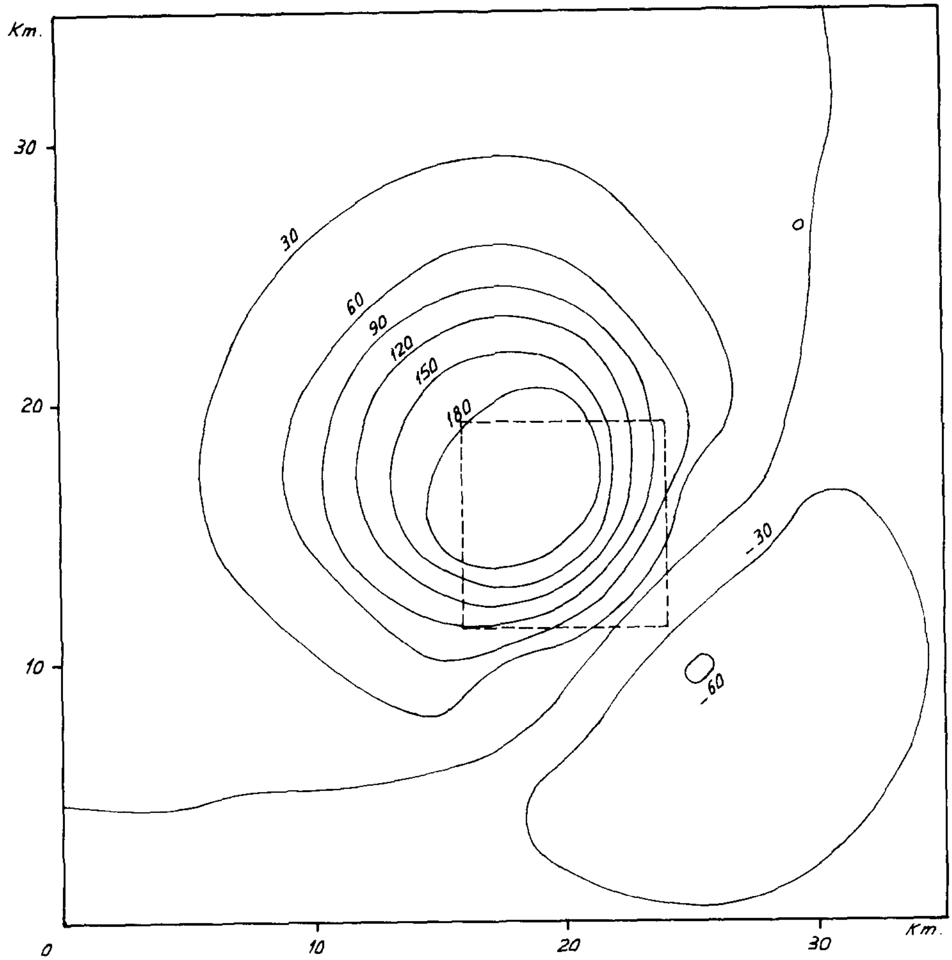

Fig. 5. Numerical example. Vertical magnetic field, in γ, computed at sea-level.

If we make a Taylor expansion of the factor within the brackets and then integrate term by term, we obtain:

$$g(x,y) = g^{*}(x,y) - \sum_{n=1}^{\infty} \frac{z^n(x,y)}{n!} \iint_{\alpha,\beta} \gamma^n G(\alpha,\beta)\, e^{2\pi(\alpha x+\beta y)i}\, d\alpha\, d\beta$$

where the integral represents the inverse Fourier transform of $\gamma^n G(\alpha,\beta)$ and its physical meaning is the nth vertical derivative of the field $g(x,y)$. Hence:

$$g(x,y) = g^{*}(x,y) - \sum_{n=1}^{\infty} \frac{z^n(x,y)}{n!} \mathscr{F}^{-1}[\gamma^n G(\alpha,\beta)] \tag{6}$$

Since in the present case with the measured values over a rectangular grid,

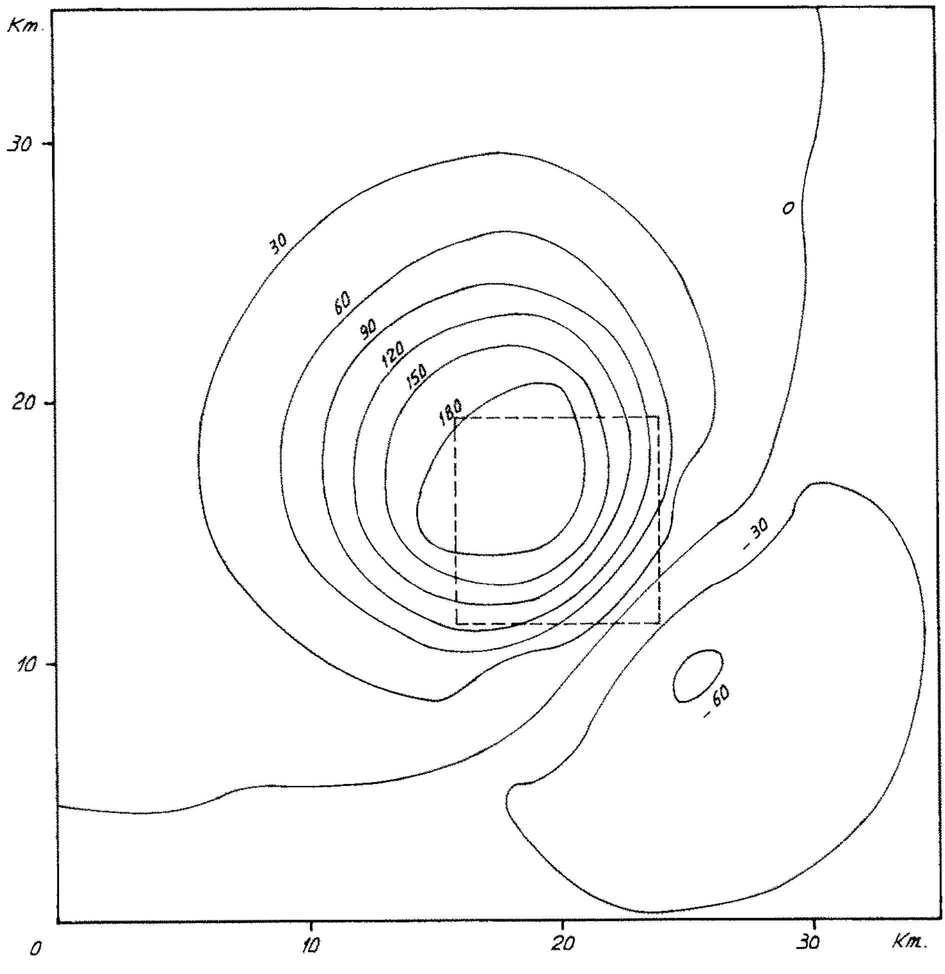

Fig. 6. Numerical example. The field of Fig. 4 has been reduced to sea-level through application of the proposed method.

direct and inverse Fourier transforms are easily computed via FFT (Fast Fourier Transform, see e.g. Cochran et al., 1967), the series in eq. 6, if rapidly convergent, provides a method for evaluating the integral in eq. 3.

STUDY OF CONVERGENCE

Two problems arise from the convergence of the procedure. The first is the convergence of the series in eq. 6.

A reasonable assumption is to consider that the field has a bounded spectrum, since its measurement has been made over a discrete set of points.

Then:

$$\left|\mathscr{F}^{-1}[\gamma^n G(\alpha,\beta)]\right| \leqslant \iint_D \gamma^n \left|G(\alpha,\beta)\right| \mathrm{d}\alpha \mathrm{d}\beta \leqslant A\gamma^n_{\max} G_{\max}$$

A being the area of the bounded domain D on the α,β plane.

Also, the topography $z(x,y)$ must be considered bounded, and therefore:

$$\left|\frac{z^n(x,y)}{n!}\mathscr{F}^{-1}[\gamma^n G(\alpha,\beta)]\right| \leqslant \frac{z^n_{\max}}{n!} A\gamma^n_{\max} G_{\max} \tag{7}$$

We see that for all (x,y) each term of the series is bounded by the corresponding term of a numerical convergent series, so that uniform and absolute convergence in eq. 6 is ensured.

From eq. 7 we infer that the convergence is improved when $z_{\max}$ is small (the plane lies near the topography) and $\gamma_{\max}$ is also small (no very high-frequency components are present in the spectrum).

The presence of high-frequency components makes its appearance in proportion as the grid spacing approaches $z_{\max}$. According to studies by Ku et al. (1971) for evenly spaced downward continuation, if $z_{\max}$ is greater than half a grid spacing, a (frequency-domain) filter should be necessary.

In practical computation, the sum of the series is calculated until the absolute value of the last term, at each point of the grid, is less than a given tolerance.

The second problem is the convergence of the iterative procedure itself.

To study it, we return to eq. 3, or its two-dimensional version, eq. 4, since conclusions are the same for both cases.

In eq. 4, if a bounded spectrum is assumed, the integration goes from $-\alpha_{\max}$ to $\alpha_{\max}$. Also if a finite extent of the field is assumed, as it occurs in practical computations via FFT, the integral in eq. 5, which defines the Fourier transform, is taken between $-L$ and L.

Thus, in eq. 4, replacing $G(\alpha)$ by its value from eq. 5 we obtain:

$$\begin{aligned} g(x) &= g^*(x) - \int_{-\alpha_{max}}^{\alpha_{max}} [\mathrm{e}^{2\pi|\alpha|z(x)} - 1]\left[\int_{-L}^{L} g(\xi)\mathrm{e}^{-2\pi\alpha\xi i}\mathrm{d}\xi\right]\mathrm{e}^{2\pi\alpha x i}\mathrm{d}\alpha \\ &= g^*(x) - \int_{-L}^{L}\left\{\int_{-\alpha_{max}}^{\alpha_{max}} [\mathrm{e}^{2\pi|\alpha|z(x)} - 1]\mathrm{e}^{2\pi\alpha(x-\xi)i}\mathrm{d}\alpha\right\} g(\xi)\mathrm{d}\xi \end{aligned} \tag{8}$$

Eq. 8 is a Fredholm integral equation of the second kind, having the kernel:

$$K(x,\xi) = \int_{-\alpha_{max}}^{\alpha_{max}} [\mathrm{e}^{2\pi|\alpha|z(x)} - 1]\mathrm{e}^{2\pi\alpha(x-\xi)i}\mathrm{d}\alpha \tag{9}$$

From the theory of integral equations (see e.g. Rey Pastor et al., 1959, p. 554) a sufficient condition of convergence of the iterative resolution is verified when the kernel is bounded by $1/2L$.

$z(x)$ can assume positive and negative values; we call z_0 the value that maximizes

$$\left| e^{2\pi|\alpha|z(x)} - 1 \right|$$

then:

$$\left| e^{2\pi|\alpha|z(x)} - 1 \right| \leqslant \left| e^{2\pi\alpha_{\max} z_0} - 1 \right| = \mu \tag{10}$$

and:

$$|K(x,\xi)| \leqslant 2\alpha_{\max}\mu$$

Then, the condition:

$$\mu < \frac{1}{4\alpha_{\max} L} \tag{11}$$

although excessive, ensures convergence.

μ gives a measure of the rapidity of convergence. Small values of μ allow more rapid convergence.

For a given μ, eq. 11 establishes that either $\alpha_{\max}$ (maximum frequency component) or L (profile length) must be limited.

From eq. 10 we see that μ is small when z_0 is small – again we meet the proximity between the plane and the topography – and when $\alpha_{\max}$ is not very large.

NUMERICAL EXAMPLE

We consider a prismatic body whose shape is described in Fig. 2, having a constant magnetic moment per unit volume given by the vector $\boldsymbol{J}$. Also we consider the XY plane as the sea-level plane, and we define a topography over a square region of this plane, as shown in Fig. 3.

The vertical component of the magnetic field due to the body is computed over the topographic surface (Fig. 4), and at sea-level (Fig. 5), using a grid of 8×8 points, 5 km apart.

Then the procedure here developed is applied to the surface data for obtaining sea-level values. Fig. 6 shows the results obtained, which agree almost exactly with the computed values in Fig. 5.

This is a case of shallow source combined with high topographic relief and the peak value of the field at sea-level exceeds by more than 50% the peak value at terrain level. Final results were obtained after 10 iterations, computing

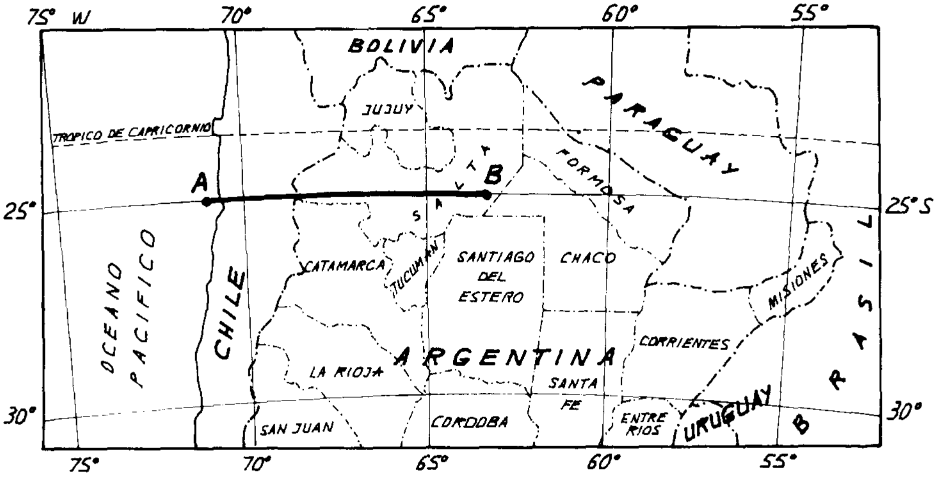

Fig. 7. Geophysical example. AB section of the 25° S parallel gravity profile studied by Introcaso and Pacino (1986) is considered.

each of the 8 terms of the series. The error in the most significant anomalies of the reduced field, compared with the computed values at sea-level was less than 1%. At the 4th iteration this error was already less than 4%. No filtering was employed in the calculations.

GEOPHYSICAL EXAMPLE

We take a section of a Bouguer profile along the 25° S parallel, studied by Introcaso and Pacino, 1986 (Fig. 7).

The section considered starts at sea-level and intersects the Andean Chain at heights near 4000 m; then it descends sharply towards the Chaco Plains. (Fig. 8).

Gravity anomalies at topographic level (corrected for the effect of the subducted Nazca Plate and an intermediate thermal diapir, as explained in the paper mentioned) are plotted in Fig. 8, and the corresponding values reduced to the sea-level through the application of the proposed method are shown in the same figure.

Although the differences are small (maximum values less than 10 mGal), they cannot be neglected in order to define correctly regional and residual anomalies.

CONCLUSIONS

A frequency-domain method has been presented for reduction to a constant level of any potential field measured over an irregular terrain surface.

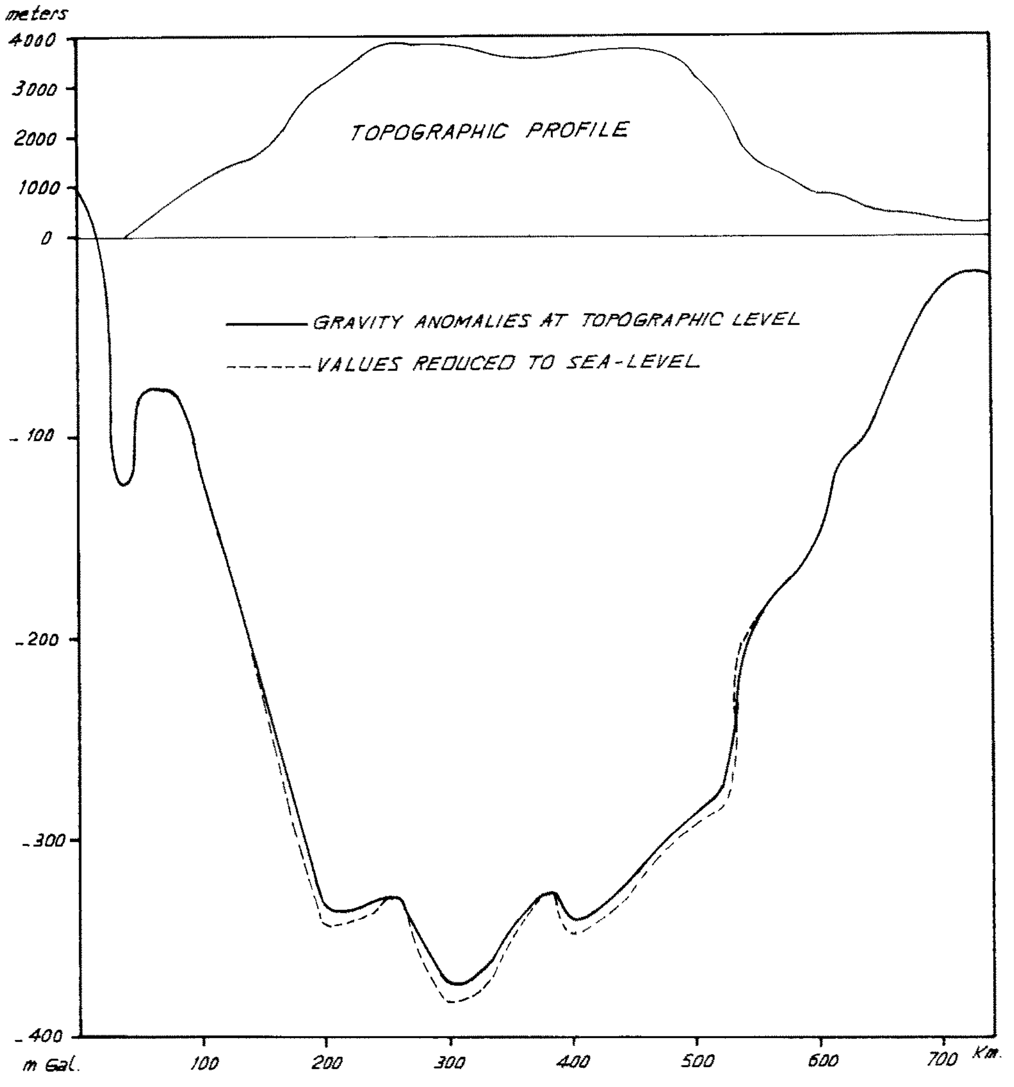

Fig. 8. Geophysical example. Topographic profile. Gravity anomalies considered and their reduction to sea-level.

No matrix inversion is needed and since Fourier transforms involved in the iterations can be computed very quickly, this method can work efficiently with a large number of measuring points.

Additional advantage of the frequency-domain approach arises from the fact that the horizontal plane can be chosen close to the topography without the necessity of having the data points close together as in convolution methods. So, convergence is fast and no filtering is necessary.

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