### Eindimensionale Betrachtung

Annahme: Gaußverteilung zu $t = 0$

$$n(r, t = 0) \propto \exp\left(\frac{-x^2}{2\sigma_x^2}\right) \tag{1}$$

Mit der eindimensionalen Geschwindigkeitsverteilung (Maxwell-Boltzmann)

$$g(v, T) = \left[\frac{m}{2\pi kT}\right]^{\frac{1}{2}} \exp\left(\frac{-mv^2}{2kT}\right) \tag{2}$$

gilt dann zum Zeitpunkt $t > 0$:

$$\begin{align}
n(r, t > 0) &\propto \int \mathrm{d}v \exp\left(\frac{-(x - vt)^2}{2\sigma_x^2}\right) \cdot g(v, T) \tag{3}\\
&\propto \int \mathrm{d}v \exp\left(\frac{-(x - vt)^2}{2\sigma_x^2}\right) \cdot \exp\left(\frac{-mv^2}{2kT}\right) \tag{4}\\
&= \int \mathrm{d}v \exp\left(\frac{-x^2}{2\sigma_x^2} + \frac{2xvt}{2\sigma_x^2} - \frac{v^2t^2}{2\sigma_x^2} - \frac{mv^2}{2kT}\right) \tag{5}\\
&= \int \mathrm{d}v \exp\Bigg(\frac{-x^2}{2\sigma_x^2} + v \underbrace{\left(\frac{xt}{\sigma_x^2}\right)}_{b} - v^2 \underbrace{\left(\frac{t^2}{2\sigma_x^2} + \frac{m}{2kT}\right)}_{a^2}\Bigg) \tag{6}
\end{align}$$

Bronstein, Wolfram o.ä. liefert:

$$\begin{align}
n(r, t > 0) &\propto \exp\left(\frac{-x^2}{2\sigma_x^2}\right) \cdot \exp\left(\frac{b^2}{4a^2}\right) \tag{7}\\
&= \exp\left(\frac{-x^2}{2\sigma_x^2}\right) \cdot \exp\left(\frac{\left(\frac{xt}{\sigma_x^2}\right)^2}{4\left(\frac{t^2}{2\sigma_x^2} + \frac{m}{2kT}\right)}\right) \tag{8}\\
&= \exp\left(\frac{-x^2}{2\sigma_x^2} + \frac{x^2t^2}{4\sigma_x^4\left(\frac{t^2}{2\sigma_x^2} + \frac{m}{2kT}\right)}\right) \tag{9}\\
&= \exp\left(\frac{-x^2}{2\sigma_x^2} + \frac{x^2t^2}{2\sigma_x^4\left(\frac{t^2}{\sigma_x^2} + \frac{m}{kT}\right)}\right) \tag{10}\\
&= \exp\left(\frac{-x^2 \cdot \sigma_x^2\left(\frac{t^2}{\sigma_x^2} + \frac{m}{kT}\right) + x^2t^2}{2\sigma_x^4\left(\frac{t^2}{\sigma_x^2} + \frac{m}{kT}\right)}\right) \tag{11}\\
&= \exp\left(\frac{-x^2t^2 - x^2\sigma_x^2\frac{m}{kT} + x^2t^2}{2\sigma_x^4\left(\frac{t^2}{\sigma_x^2} + \frac{m}{kT}\right)}\right) \tag{12}\\
&= \exp\left(\frac{-x^2\sigma_x^2\frac{m}{kT}}{2\sigma_x^4\left(\frac{t^2}{\sigma_x^2} + \frac{m}{kT}\right)}\right) \tag{13}\\
&= \exp\left(\frac{-x^2\frac{m}{kT}}{2\sigma_x^2\left(\frac{t^2}{\sigma_x^2} + \frac{m}{kT}\right)}\right) \tag{14}\\
&= \exp\left(\frac{-x^2\frac{m}{kT}}{2t^2 + 2\sigma_x^2\frac{m}{kT}}\right) \tag{15}\\
&= \exp\left(\frac{-x^2}{2\left(\frac{kT}{m}t^2 + \sigma_x^2\right)}\right) \tag{16}
\end{align}$$

Die Atomdichte entspricht also wieder einer gaußschen Verteilung, allerdings mit breiteren Varianz

$$\sigma_x^2(t) = \frac{kT}{m}t^2 + \sigma_x^2 \tag{17}$$