360.242 **Numerical Simulation and Scientific Computing I**, 2024W

Exercise #3: Task 1

Group 05

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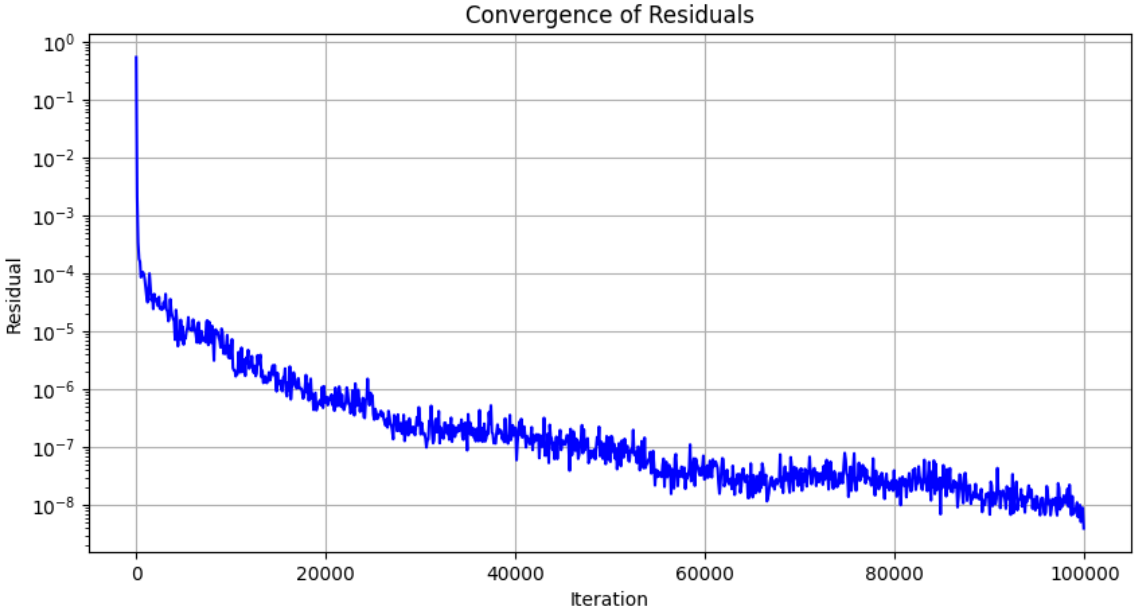
1 Conjugate Gradients Implementation

The results from the Conjugate Gradient (CG) method show how well it works for solving symmetric positive definite systems. The following two graphs track how the residual and error change over the iterations.

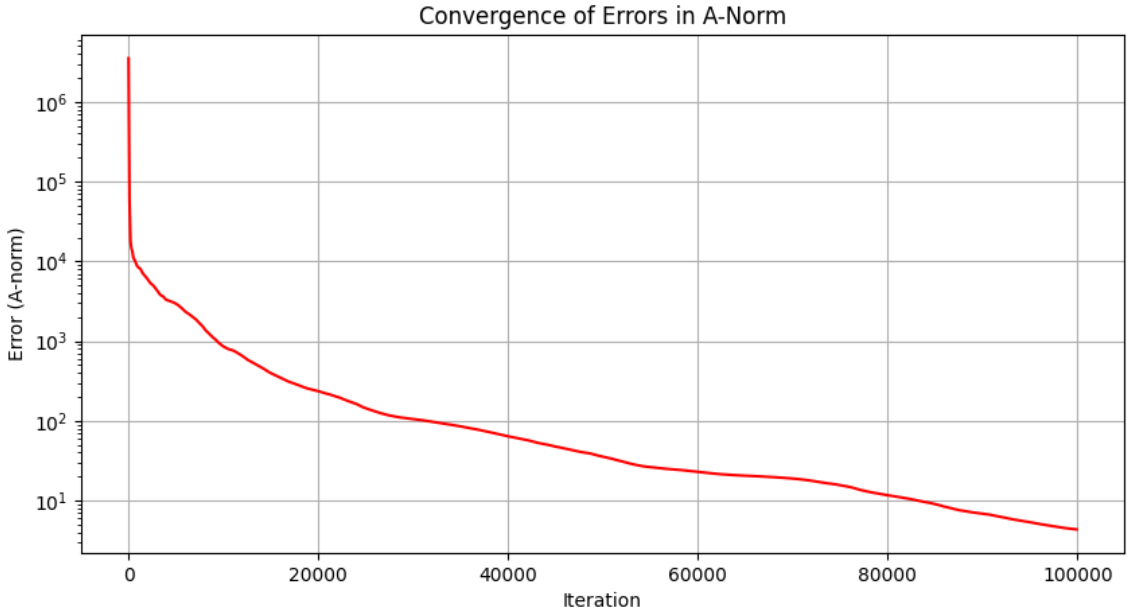


The first graph illustrates the convergence of the normalized residual:

$$\frac{||r_k||_2}{||r_0||_2}$$

over 100 000 iterations.
Initially, the residual decreases rapidly, indicating that the method effectively reduces the initial error. This is expected behaviour for the CG algorithm, which is designed to quickly improve the solution during the early iterations.

As the iterations progress, the rate of decrease slows, and the residual reaches very small values. In this phase, the graph exhibits minor oscillations. These oscillations are likely due to the limitations of numerical precision, as the residual approaches the limits of machine accuracy.



The second graph presents the convergence of the error in the A-norm, defined as:

$$||e_k||_A = \sqrt{e_k^T A e_k}$$

where e_k is the error at iteration k. The error decreases steadily throughout the 100 000 iterations. This consistent reduction highlights the efficiency of the CG method in improving the solution accuracy over time.

The results confirm that the CG method successfully solves the linear system. The rapid initial convergence, followed by slower progress and numerical noise, aligns with theoretical expectations for this algorithm.

Build and Run

1. **make clean**: cleans up generated files
2. **make all**: compiles programme defined in `main.cpp`
3. **make run**: runs the executable
4. **make plot**: runs the python file to generate the graphs