Numerical Simulation and Scientific Computing

Homework 2 - Task 2

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Exercise 2: Performance Benchmarks, Finite Difference Discretization

Task 2.1 Estimating/ Modeling

For benchmarking this task the following CPU was used: 12th Gen Intel(R) Core(TM) i7-1255U and following spec were extracted from the Intel Webpage[1]. The memory bandwidth is 83.2 GB/s and the peak performance therefore 272 GFLOP/s [2] The calculations are based on lectures slides from Lecture 2.

Machine Balance

The theoretical single-threaded machine balance for this CPU is:
Max. Memory size = 64 max. memory bandwidth = 83.2GB/s

$$B_m = \frac{\text{memory bandwidth [GBytes/s]}}{\text{peak performance [GFLOPs/s]}} = \frac{83,2\,\text{GB/s}}{272\,\text{GFLOP/s}} \approx 0.306\,\frac{\text{Bytes}}{\text{FLOPs}}$$

Code Balance

For a matrix-matrix multiplication (MMM) of two $N \times N$ matrices the code balance is:

$$B_c = \frac{\text{data traffic [Bytes]}}{\text{floating point operations [FLOPs]}} = \frac{3 \cdot 8 \cdot N^2\,\text{[Bytes]}}{2 \cdot N^3\,\text{[FLOPs]}} = \frac{12\,\text{[Bytes]}}{N\,\text{[FLOPs]}}$$

Matrix Size for Peak Performance

A MMM would utilize peak performance under idealized conditions when the machine balance and code balance are equal. Thus, the ideal N is given by:

$$B_c = B_m \quad \Longrightarrow \quad \frac{12}{N} = 0.306 \quad \Longrightarrow \quad N = \frac{12}{0.306} = 39.24.. \approx 39$$

Theoretical Runtime for MMM

Assuming peak performance, the MMM runtime is given by:

$$t = \frac{\text{operations [FLOPs]}}{\text{peak performance [FLOPs/s]}} = \frac{2 \cdot N^3}{272 \cdot 10^9}$$

Thus, the theoretical runtimes are:

[1] Intel® Core™ i7-1255U Prozessor (12 MB Cache, bis zu 4,70 GHz) – Produktspezifikationen (no date) Intel. Available at: https://www.intel.de/content/www/de/de/products/sku/226259/intel-core-i71255u-processor-12m-cache-up-to-4-70-ghz/specifications.html (Accessed: 2 December 2024).

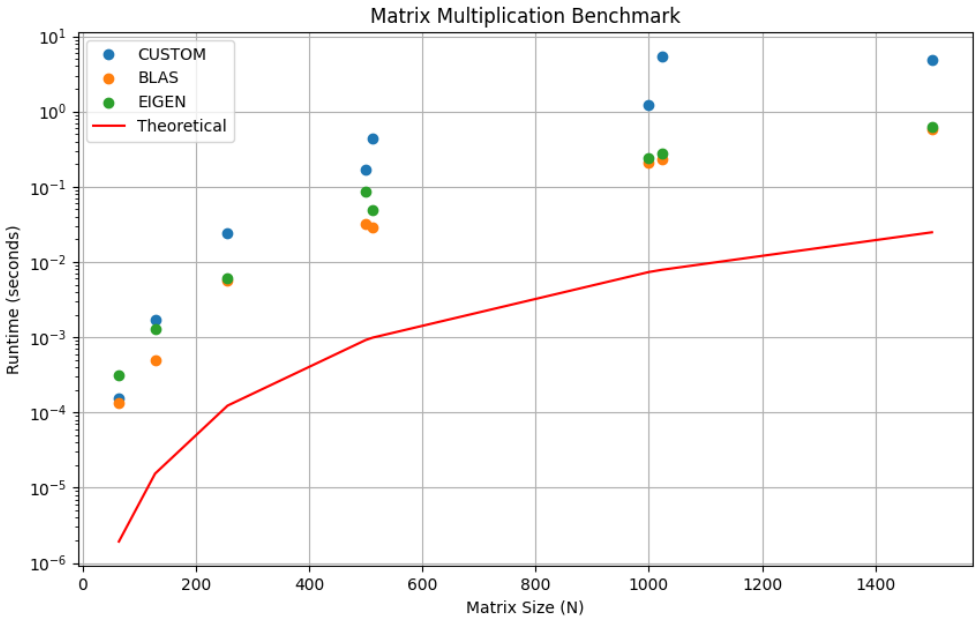
[2] Export Compliance Metrics for Intelreg; Microprocessors (no date) Intel. Available at: https://www.intel.com/content/www/us/en/support/articles/000005755/processors.html (Accessed: 2 December 2024).

N	$t\,(\mathrm{s})$
1000	$\frac{1}{136} \approx 0.0074$
2000	$\frac{1}{17} \approx 0.0589$
5000	$\frac{125}{136} \approx 0.9191$

Task 2.2 Benchmarking

For this task the CPU 12th Gen Intel(R) Core(TM) i7-1255U was used for benchmarking and in the following graph you can see the theoretical peak performance based on the function below as a red line and the measured time points from the different implementations.

$$t = \frac{2 \cdot N^3}{272 \cdot 10^9}$$



Notes

Build instructions for the provided Makefile.

1. **make clean**: cleans up generated files
2. **make run**: runs a specific test for the specified N value for each method
3. **make runAll**: runs the program for the provided Ns for each method implemented