1) A)

Clean Image intensity:- $I(x,y)$

let Noisy Image intensity be $I'(x,y)$

let the pdf of Intensity I be $P_I(i)$

we can say $I'(x,y) = I(x,y) + N(x,y)$

$P_N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$ (Let it be the pdf of $N(x,y)$)

w.k.t when two indelendent random variable A and B summation gives

i.e $S = A + B$

$$P_S(x) = \int_{-\infty}^{\infty} P_A(t)\, P_B(x-t)\, dt$$

Proof:-

$S = A + B$

consider $P_S(x) = P(S \le x)$

$= P(A + B \le x)$

$\Rightarrow P_S(x) = \int_{-\infty}^{\infty} P_B(x-t) P_A(t)\, dt$

$P_S(x) = \frac{d}{dx} P_S(x)$

$P_S(x) = \frac{d}{dx} \int_{-\infty}^{\infty} P_B(x-t)\, P_A(t) dt$

$P_S(x) = \int_{-\infty}^{\infty} P_B(x-t)\, P_A(t)\, dt$

Let us use this relation to calculate $P_{I'}(x)$

as $I' = I + N$

$P_{I'}(x) = \int_{-\infty}^{\infty} P_I(t).\ P_N(x-t)\, dt$

$$\boxed{= \int_{-\infty}^{\infty} P_I(t) \cdot e^{\frac{(-(x-t)^2)}{2\sigma^2}} \times \frac{1}{\sqrt{2\pi}\sigma}\, dt.}$$

→ The resultant Pdf resembles "convolution operation" in Image Processing.

If the Noise is uniform ~~for~~ from $-r$ to r, $r > 0$

w.r.t
$$P_N(x) = \begin{cases} \frac{1}{2r} & -r \le x \le r \\ 0 & x > r \text{ (or) } x < -r. \end{cases}$$

$$I'(x,y) = I(x,y) + N(x,y)$$

$$P_{I'}(x) = \int_{-\infty}^{\infty} P_I(t) \cdot P_N(x-t)\, dt.$$

$$= \int_{-\infty}^{t-r} P_I(t) \cdot 0\, dt + \int_{t-r}^{t+r} P_I(t) \times \frac{1}{2r}\, dt + \int_{t+r}^{\infty} P_I(t)\, 0\, dt$$

$$\boxed{P_{I'}(x) = \frac{1}{2r} \int_{t-r}^{t+r} P_I(t)\, dt}$$