Question 2

Given,

first picture g_1 is taken by adjusting camera lens scene outside (f_1) is in focus. reflection off the window surface (f_2) will be blurred

$$g_1 = f_1 + h_2 * f_2 \quad | \; h_2 - \text{Blur kernel acting on } f_2 \quad - (1)$$

Second picture g_2 is taken by adjusting camera lens ~~Scene~~ reflection off window in focus. Camera lens Scene outside (f_1) will be defocused

$$g_2 = h_1 * f_1 + f_2 \quad | \; h_1 - \text{Blur kernel acting on } f_1 \quad - (2)$$

$g_1, g_2, h_1, h_2 \rightarrow$ known

Apply 2D fourier transform on (1), (2)

$$G_1(u,v) = F_1(u,v) + H_2(u,v) \cdot F_2(u,v) \quad - (3)$$

$$G_2(u,v) = H_1(u,v) \cdot F_1(u,v) + F_2(u,v) \quad - (4)$$

Solving (3) and (4) we get,

$$\boxed{F_2(u,v) = \frac{G_2(u,v) - G_1(u,v)\,H_1(u,v)}{1 - H_1(u,v)\,H_2(u,v)}} \quad - (5)$$

$$\boxed{F_1(u,v) = \frac{G_1(u,v) - G_2(u,v)\,H_2(u,v)}{1 - H_1(u,v) \cdot H_2(u,v)}} \quad - (6)$$

Now we can apply inverse fourier transform on (5) (6) to get f_2, f_1 respectively.

The only case above said fails is when denominator is 0.

That is $H_1(u,v) \cdot H_2(u,v) = 1$

If $H_1(u,v) \cdot H_2(u,v) = 1$ then $F_1(u,v)$, $F_2(u,v)$ will become undefined. which is a contridiction to the fact $F_1(u,v)$, $F_2(u,v)$ defined.

$\therefore$ Numerator also will become zero since $F_1(u,v)$, $F_2(u,v)$ are defined.

Even though defined we can't get f_1, f_2 from them.

<u>Cases where we can't find f_1, f_2:</u>

Since h_1, h_2 are blur kernels. Summation over domain is 1

Consider,

$$H_1(0,0) = \sum_x \sum_y h_1(x,y) e^{\frac{-j2\pi(0)(x)}{N}} \cdot e^{\frac{-j2\pi(0)y}{N}}$$

$$= \sum_x \sum_y h_1(x,y) \quad \text{(Summation over domain)}$$

$$= 1$$

$$H_2(0,0) = \sum_x \sum_y h_2(x,y) e^{\frac{-j2\pi(0)(x)}{N}} \cdot e^{\frac{-j2\pi(0)y}{N}}$$

$$= \sum_x \sum_y h_2(x,y) \quad \text{(Summation over domain)}$$

$$= 1$$

$\therefore$ we got $H_1(0,0) \cdot H_2(0,0) = 1$

when $u=0, v=0$ we can't find f_1, f_2

One more case is when numerator close to zero and denominator is also close to zero.

In this case very small noise in measuring g_1 or g_2 increases the noise in finding F_1, F_2 which implies finding f_1 and f_2 very difficult.