CS-663 Assignment-1 Report

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1 Question 4

The given motion model is

$$x_2 = ax_1^2 + by_1^2 + cx_1y_1 + dx_1 + ey_1 + f$$

$$y_2 = Ax_1^2 + By_1^2 + Cx_1y_1 + Dx_1 + Ey_1 + F$$

We have 12 parameters *a, b, c, d, e, f, A, B, C, D, E, F*.

We need at-least $n/2$ i.e., $12/2 = 6$ control points (From slides)
Proof - To solve a set of equations having 12 variables we need at-least 12 equations (Otherwise we will get infinite solutions or no solution), for each point we get 2 equations (x, y).

If we have < 6 control points it is not sufficient to find all parameters.

Let us assume we have 6 control points $p_1, p_2, p_3, p_4, p_5, p_5$ and $t_1, t_2, t_3, t_4, t_5, t_6$ be their corresponding transformed points.

Let A be a 6×6 matrix and B be a 6×2 matrix:

$$A = \begin{pmatrix} p_{x_1}^2 & p_{y_1}^2 & p_{x_1} \cdot p_{y_1} & p_{x_1} & p_{y_1} & 1 \\ p_{x_2}^2 & p_{y_2}^2 & p_{x_2} \cdot p_{y_2} & p_{x_2} & p_{y_2} & 1 \\ p_{x_3}^2 & p_{y_3}^2 & p_{x_3} \cdot p_{y_3} & p_{x_3} & p_{y_3} & 1 \\ p_{x_4}^2 & p_{y_4}^2 & p_{x_4} \cdot p_{y_4} & p_{x_4} & p_{y_4} & 1 \\ p_{x_5}^2 & p_{y_5}^2 & p_{x_5} \cdot p_{y_5} & p_{x_5} & p_{y_5} & 1 \\ p_{x_6}^2 & p_{y_6}^2 & p_{x_6} \cdot p_{y_6} & p_{x_6} & p_{y_6} & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} a & A \\ b & B \\ c & C \\ d & D \\ e & E \\ f & F \end{pmatrix}$$

The product C is a 6×2 matrix, given by:

$$C = A \cdot B = \begin{pmatrix} t_{x_1} & t_{y_1} \\ t_{x_2} & t_{y_2} \\ t_{x_3} & t_{y_3} \\ t_{x_4} & t_{y_4} \\ t_{x_5} & t_{y_5} \\ t_{x_6} & t_{y_6} \end{pmatrix}$$

Assume t_{x_i} is x coordinate of point t_i, t_{y_i} is y coordinate of t_i, similarly for p_s also. it is of type $AB = C$ and find x where A is a square matrix.

$B = A^{-1}C$ (B has all values of our parameters)

Assume we have more than 6 control points (n points) then,

Let A be an $n \times 6$ matrix and B be a 6×2 matrix:

$$A = \begin{pmatrix} p_{x_1}^2 & p_{y_1}^2 & p_{x_1} \cdot p_{y_1} & p_{x_1} & p_{y_1} & 1 \\ p_{x_2}^2 & p_{y_2}^2 & p_{x_2} \cdot p_{y_2} & p_{x_2} & p_{y_2} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{x_n}^2 & p_{y_n}^2 & p_{x_n} \cdot p_{y_n} & p_{x_n} & p_{y_n} & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} a & A \\ b & B \\ c & C \\ d & D \\ e & E \\ f & F \end{pmatrix}$$

The product C is an $n \times 2$ matrix, given by:

$$C = A \cdot B = \begin{pmatrix} t_{x_1} & t_{y_1} \\ t_{x_2} & t_{y_2} \\ \vdots & \vdots \\ t_{x_n} & t_{y_n} \end{pmatrix}$$

$$Ax = b$$

We shall find **best fit parameter** which gives **minimum square error**.

For this we have **various optimisation techniques** which find parameter values.

Find B such that $AB = C$ (A is non Square)

$B = pinv(A).C$ (Pseudo inverse of A gives the best approximate for B).

$\implies B = (A^T A)^{-1} A^T.C$

This gives the values of our parameters