Q) A)

Given 1D convolution mask

$(w_0, w_1, \cdots w_6)$

consider 1D matrix f as $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

If we apply convolution mask $(w_0, w_1, \cdots w_6)$ on f

we need to reverse (i.e rotate by $180°$) convolution mask and padd with $f-1$ zero to 1D matrix

we will get

$$\underbrace{[w_0 a_1 \quad w_1 a_1 + w_2 a_2 \cdots\cdots\cdots \quad w_6 a_n]}_{n+f-1 \text{ values } (f = 7 \to \text{filtersize})}$$

we can represent it a product of these matries

$$A = \underbrace{\begin{bmatrix} w_0 & 0 & 0 & \cdots & 0 \\ w_1 & w_1 & 0 & & \vdots \\ w_2 & w_2 & w_1 & & \\ \vdots & w_3 & w_2 & & 0 \\ w_6 & \vdots & \vdots & & w_1 \\ 0 & w_6 & \vdots & & w_5 \\ \vdots & 0 & & & w_6 \\ 0 & \vdots & & & \end{bmatrix}}_{(n+f-1) \times n} \text{(let)}, \quad \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}}_{n \times 1}$$

We know that we need to cut $\frac{f-1}{2}$ values from start and end of result, So we will get

$$[w_3 a_1 + w_4 a_2 + w_5 a_3 + w_6 a_4 + \cdots\cdots\cdots]$$

we can get this by removing 3 rows from top and three rows from bottom from the above matrix (A)

1.e

we will get our final convolution as multiplication of

$$\begin{bmatrix} w_3 & w_2 & w_1 & w_0 & 0 & 0 & \cdots & 0 \\ w_4 & w_3 & w_2 & w_1 & w_0 & \cdots & & \vdots \\ w_5 & w_4 & w_3 & & w_2 & \cdots & & 0 \\ w_6 & w_5 & w_4 & \cdots & & & & w_3 \\ 0 & w_6 & \cdots & & & & & w_1 \\ 0 & 0 & \cdots & & & & & w_2 \\ \vdots & \vdots & & & & & & w_3 \\ 0 & 0 & \cdots & & & & & \end{bmatrix}_{n \times n} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}_{n \times 1}$$

• Properties of this matrix are all the diagonals contains same values and only there are 7 non-zero diagonals (diagonals containing non-zero value).

→ A potential application of such a matrix-based construction will be in efficient implementation of convolutional neural networks (CNN) and structured way of matrix representation helps in optimizing and regularizing filters during tasks like deconvolution and Image reconstruction.