6)A)

Given F is a continous Fourier operator.

Need to prove:- $F(F(F(F(f(t))))) = f(t)$

Let us try to Prove:-

$$F(F(f(t))) = f(-t)$$

If $F(F(f(t))) = f(-t)$

$$\Rightarrow F(F(F(F(f(t)))))$$

$$= F(F(f(-t))) \quad (\because F(F(f(t))) = f(-t))$$

$$= f(-(-t))$$

$$= f(t)$$

So If we can prove $F(F(f(t))) = f(-t)$, Required condition is proved.

Claim:- $F(F(f(t))) = f(-t)$.

Proof:-

As F is a continous Fourier operator

$$\Rightarrow F(f(t)) = g(u) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ut} dt$$

$$F(F(f(t))) = F(g(u)) = \int_{-\infty}^{\infty} g(u) e^{-j2\pi ut} du \quad -①$$

w.k.t $g(u) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ut} dt$

it can be written as $\int_{-\infty}^{\infty} f(s) e^{-j2\pi us} ds$, as it is independent of variable.

substituting it in eqn ① gives

$$F(g(u)) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(s)\, e^{-i2\pi u s}\, ds\right) e^{-i2\pi u t}\, dt$$

$$= \int_{-\infty}^{\infty} f(s)\left(\int_{-\infty}^{\infty} e^{-i2\pi u (s+t)}\, dt\right) ds.$$

w.r.t $\int_{-\infty}^{\infty} e^{-i2\pi u(s+t)}\, dt = \delta(s+t)$

$$= \int_{-\infty}^{\infty} f(s)\cdot \delta(s+t)\, dt = f(-t).$$ (Property of δ function)

$\therefore \boxed{F(F(t)) = f(-t)}$

Practical uses:-

1) checking if a function is even function or not
if $F(F(f(t))) = f(t)$, we can say $f(t)$ is even function

2) Reversing the signal If we are receiving a signal applying Fourier tranform twice gives the reversed signal.

3)