CS 663 Home Work Assignment 2

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1 Question 3

Separable Filter:
A 2D filter **L** is said to be separable filter if it can be written as the outer product of two 1D vectors **u** and **v**:

$$\mathbf{L} = \mathbf{u} \text{ x } \mathbf{v}$$

where **u** is a column vector and **v** is a row vector.

1.1 Part a

Our Laplacian filter is

$$\mathbf{L} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

Let us assume that Laplacian filter 1 is a separable filter then there exits two vectors **u** and **v** such that **u** is a column vector and **v** is a row vector and $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$ and

$$\mathbf{L} = \mathbf{u} \text{ x } \mathbf{v} = \begin{bmatrix} u_1v_1 & u_1v_2 & u_1v_3 \\ u_2v_1 & u_2v_2 & u_2v_3 \\ u_3v_1 & u_3v_2 & u_3v_3 \end{bmatrix}$$

Now we are with

$$u_1v_1 = 1 \quad u_1v_2 = 1 \quad u_1v_3 = 1 \tag{2}$$

$$u_2v_1 = 1 \quad u_2v_2 = -8 \quad u_2v_3 = 1 \tag{3}$$

$$u_3v_1 = 1 \quad u_3v_2 = 1 \quad u_3v_3 = 1$$

Consider, 2 clearly from those three equations we can say that, $v_1/v_2 = 1$ and $v_2/v_3 = 1$ which means,

$$v1 = v2 = v3 \tag{4}$$

substituting the results of 4 in 3 we will get $u_2 = 1$ and $u_2 = -8$ simultaneously which is a contradiction. Hence our assumption is wrong.

Therefore Laplacian filter 1 is **not** a separable filter.

1.2 Part b

Our Laplacian filter is

$$\mathbf{L} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{5}$$

Since in the given question they asked can we implement **L** entirely using 1D convolution we can write **L** as sum of two filters which can be separable i.e, we **can implement L** entirely using 1D convolution.
Let

$$\mathbf{L1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{L2} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore,

$$\mathbf{L} = \mathbf{L1} + \mathbf{L2}$$

We can write **L1** as outer product of two 1D vectors **u1** and **v1** i.e,

$$\mathbf{L1} = \mathbf{u1} \text{ x } \mathbf{v1}$$

where $\mathbf{u1} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{v1} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

Similarly we can write **L2** as outer product of two 1D vectors **u2** and **v1** i.e,

$$\mathbf{L2} = \mathbf{u2} \text{ x } \mathbf{v2}$$

where $\mathbf{u2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{v2} = \begin{bmatrix} 1 & -3 & 1 \end{bmatrix}$

Therefore,

$$\mathbf{L} = \mathbf{u1} \text{ x } \mathbf{v1} + \mathbf{u2} \text{ x } \mathbf{v2}$$

Where **u1** and **u2** are column vectors and **v1** and **v2** are row vectors.
We can directly ignore convolution with **v1** and **u2** as they are identity filters i.e, convolution on a matrix using this filter results same matrix.
Therefore,

$$\mathbf{L} = \mathbf{u1} + \mathbf{v2}$$

For example if we are applying convolution with **L** on Image **I** then

$$\mathbf{I} * \mathbf{L} = \mathbf{I} * \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \mathbf{I} * \begin{bmatrix} 1 & -3 & 1 \end{bmatrix}$$

Where $*$ represents convolution operator

The Laplacian mask 5 **can be** implemented entirely using 1D convolutions.