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Q.4) A) $F(u,v) = \frac{1}{\sqrt{N} \times \sqrt{N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)\ e^{-j2\pi\left(\frac{ux}{N} + \frac{vy}{N}\right)}$

$$f(x,y) = \begin{cases} 0 & \text{if } x \neq 100 \\ 255 & \text{if } x = 100 \end{cases}$$

$$\Rightarrow F(u,v) = \frac{1}{\sqrt{201}\sqrt{201}} \sum_{x=0}^{200} \sum_{y=0}^{200} f(x,y)\ e^{-j2\pi\left(\frac{ux}{201} + \frac{vy}{201}\right)} \quad (\text{As } N = 201)$$

$$\Rightarrow F(u,v) = \frac{1}{201} \sum_{y=0}^{200} f(100,y)\ e^{-j2\pi\left(\frac{100u}{201} + \frac{vy}{201}\right)} \quad \left(\text{As } x \neq 100,\ f(x,y) = 0\right)$$

$\Rightarrow$ We know $f(100,y) = 255 \quad \forall y$

$$\Rightarrow F(u,v) = \frac{1}{201} \times 255 \times e^{-j\frac{200\pi u}{201}} \times \sum_{y=0}^{200} e^{-j\frac{2\pi vy}{201}}$$

Case I : $v = 0$

$$\Rightarrow F(u,v) = \frac{1}{201} \times 255 \times e^{-j\frac{200\pi u}{201}} \times \sum_{y=0}^{200} 1 \quad (\text{upper limit } 201)$$

$$\Rightarrow F(u,0) = 255\ e^{-j\frac{200\pi u}{201}}$$

Case II $v \neq 0$

$$\Rightarrow F(u,v) = \frac{255}{201} \times e^{-j\frac{200\pi u}{201}} \sum_{y=0}^{200} \left(e^{-j\frac{2\pi v}{201}}\right)^{y}$$

↳ sum of G.P

$$\Rightarrow F(u,v) = \frac{255}{201} \times e^{-j\frac{200\pi v}{201}} \left(\frac{1 - \left(e^{-j\frac{2\pi v}{201}}\right)^{201}}{1 - e^{-j\frac{2\pi v}{201}}}\right)$$

→ We know $e^{-j2\pi V} = \cos(2\pi V) + j(\sin 2\pi V) = 1 + 0j = 1$

⇒ Sum of ln P = 0

⇒ $F(U,V) = 0$ if $V \neq 0$

⇒ $F(U,V) = \begin{cases} 0 & \text{if } V \neq 0 \\ 255\, e^{-j2\pi\left(\frac{100V}{201}\right)} & \text{if } V = 0 \end{cases}$ $\quad (U,V \in [0, 200])$

⇒ We only have lines on points where V = 0 ⇒ We should get a horizontal line.