Question 4

$f(x,y)$

We have an image $n \times n$ such that only $K \ll n^2$ elements in it are non-zero, where K is known. and locations of the non-zero elements are also known.

a) We have a set of only m Discrete Fourier transforms

DFT:

$$f(u,v) = \frac{1}{\sqrt{w_1 w_2}} \sum_{x=0}^{w_1-1} \sum_{y=0}^{w_2-1} e^{\frac{-j2\pi ux}{w_1}} \times e^{\frac{-j2\pi vy}{w_2}} \times f(x,y)$$

Now we know $f(u,v)$ values for m (u,v) pairs.

$f(u,v)$ Now we can write the above equation as,

$$f(u,v) = \frac{1}{n} \sum_{(x_i,y_i) \in V} a_{x_i y_i} \cdot e^{\frac{-j2\pi ux_i}{w_1}} \cdot e^{\frac{-j2\pi vy_i}{w_2}}$$

$a_{x_i y_i}$ is a variable which is a real.

Now we have K variables and m (u,v) pairs.

∴ Note that for a particular pair (u,v) we will have two equations in the K variables.

∴ Since $f(u,v)$ is a complex and RHS is also complex we can compare real and imaginary parts.

∴ For a pair (u,v) we have 2 equations in K variables

∴ for m (u,v) pairs we have 2m equations in K variables.

∴ We are left with 2m equations with K variables linear equation.

We can solve only when $2m \geq K$

$$\begin{bmatrix} F(u_1,v_1) \\ F(u_2,v_2) \\ \vdots \\ F(u_m,v_m) \end{bmatrix} = M_{m\times k} \begin{bmatrix} a_{1x1} \\ \vdots \\ a_{nx1} \end{bmatrix}_{k\times 1}$$

$$M_{pq} = e^{-j2\pi \frac{u_p x_q}{w_1}} \times e^{-j2\pi \frac{v_p y_q}{w_2}}$$

→ linear equation can be solved by using any method.

→ Out of the 2m equations take atleast k linearly independent equations to solve for k variables.

b) Minimum possible value m is $\lceil k/2 \rceil$ if $m < k/2$ then we will have more variables than avaible equations. Hence we can't find soln if $m < k/2$. Therefore minimum value is $\lceil k/2 \rceil$. Where $\lceil \cdot \rceil$ = Ceiling function

c) No, The above described method doesn't work if Coordinates are Unknown.

→ If they are unknown then all the elements in matrix M will be unknown also. We will get a non-linear equation and no. of variables increases

→ The above method assumes that (x_i, y_i) are known

→ Therefore the above method doesn't works when coordinates are unknown.