

# Exercise 3 - Solution

## Q1.

No,  $AB^+ = \{A, B, C\}$ , a proper subset of  $\{A, B, C, D, E\}$

Yes,  $ABD^+ = \{A, B, C, D, E\}$

## Q2.

Let us use the following shorthand notation:

$C$  = CourseNo,  $SN$  = SecNo,  $OD$  = OfferingDept,  $CH$  = CreditHours,  $CL$  = CourseLevel,

$I$  = InstructorSSN,  $S$  = Semester,  $Y$  = Year,  $D$  = Days\_Hours,  $RM$  = RoomNo,

$NS$  = NoOfStudents

Hence,  $R = \{C, SN, OD, CH, CL, I, S, Y, D, RM, NS\}$ , and the following functional dependencies hold:

$\{C\} \rightarrow \{OD, CH, CL\}$

$\{C, SN, S, Y\} \rightarrow \{D, RM, NS, I\}$

$\{RM, D, S, Y\} \rightarrow \{I, C, SN\}$

First, we can calculate the closures for each left hand side of a functional dependency, since these sets of attributes are the candidates to be keys:

(1)  $\{C\}^+ = \{C, OD, CH, CL\}$

(2) Since  $\{C, SN, S, Y\} \rightarrow \{D, RM, NS, I\}$ , and  $\{C\}^+ = \{C, OD, CH, CL\}$ , we get:

$\{C, SN, S, Y\}^+ = \{C, SN, S, Y, D, RM, NS, I, OD, CH, CL\} = R$

(3) Since  $\{RM, D, S, Y\} \rightarrow \{I, C, SN\}$ , we know that  $\{RM, D, S, Y\}^+$  contains  $\{RM, D, S, Y, I, C, SN\}$ . But  $\{C\}^+$  contains  $\{OD, CH, CL\}$  so these are also contained in  $\{RM, D, S, Y\}^+$  since  $C$  is already there. Finally, since  $\{C, SN, S, Y\}$  are now all in  $\{RM, D, S, Y\}^+$  and  $\{C, SN, S, Y\}^+$  contains  $\{NS\}$  (from (2) above), we get:

$\{RM, D, S, Y\}^+ = \{RM, D, S, Y, I, C, SN, OD, CH, CL, NS\} = R$

Hence, both  $K_1 = \{C, SN, S, Y\}$  and  $K_2 = \{RM, D, S, Y\}$  are (candidate) keys of  $R$ .

## Q3.

(a) The key for this relation is Book\_title, Authurname. This relation is in 1NF and not in 2NF as no attributes are FFD on the key. It is also not in 3NF.

(b)

3NF decomposition:

Book0(Book\_title, Authurname)

Book1-1(Book\_title, Publisher, Book\_type)

Book1-2(Book\_type, Listprice)

Book2(Authorname, Author\_affil)

#### Q4.

(a)

- {M} IS NOT a candidate key since it does not functionally determine attributes Y or P.
- {M, Y} IS a candidate key since it functionally determines the remaining attributes P, MP, and C.
- {M, C} IS NOT a candidate key since it does not functionally determine attributes Y or P.

(b)

REFRIG is not in 2NF, due to the partial dependency {M, Y}  $\rightarrow$  MP (since {M}  $\rightarrow$  MP holds). Therefore REFRIG is neither in 3NF nor in BCNF.

Alternatively: BCNF can be directly tested by using all of the given dependencies and finding out if the left hand side of each is a superkey (or if the right hand side is a prime attribute). In the two fields in REFRIG: M  $\rightarrow$  MP and MP  $\rightarrow$  C. Since neither M nor MP is a superkey, we can conclude that REFRIG is is neither in 3NF nor in BCNF.

(c) Yes. Please follow the algorithm provided in the lecture notes.

#### Q5.

1) List the candidate keys for R.

EH/ABH/BDH/CDH

2) Determine the highest normal form of R with respect to F.

1NF. Non-prime attribute G is functionally determined by D.

3) Is the decomposition {ABCD, DEGH} (with the same FD set F) of R lossless-join?

No.

Decomposition	A	B	C	D	E	G	H
$R_1(A, B, C, D)$	a	a	a	<b>a</b>	b	b	b
$R_2(D, E, G, H)$	b	b	b	<b>a</b>	a	a	a

Decomposition	A	B	C	D	E	G	H
$R_1(A, B, C, D)$	a	a	a	a	b	<b>a</b>	b
$R_2(D, E, G, H)$	<b>a</b>	b	b	a	a	a	a

4) Find a minimal cover  $F_m$  for F.

$F_m = \{AB \rightarrow C, D \rightarrow A, D \rightarrow G, E \rightarrow B, AB \rightarrow D, E \rightarrow A, CD \rightarrow E\}$

5) Decompose into a set of 3NF relations if it is not in 3NF. Make sure your decomposition is dependency-preserving and lossless-join.

For  $F_m = \{AB \rightarrow C, D \rightarrow A, D \rightarrow G, E \rightarrow B, AB \rightarrow D, E \rightarrow A, CD \rightarrow E\}$  :

From  $AB \rightarrow C, AB \rightarrow D$ , derive  $R_1\{A, B, C, D\}$

From  $D \rightarrow A, D \rightarrow G$ , derive  $R_2\{A, D, G\}$

From  $E \rightarrow B, E \rightarrow A$ , derive  $R_3\{A, B, E\}$

From  $CD \rightarrow E$ , derive  $R_4\{C, D, E\}$

None of the relation schemas contains a key of  $R$ , add one relation schema  $R_5\{E, H\}$

6) Decompose it into a collection of BCNF relations if it is not in BCNF. Make sure your decomposition is lossless-join.

For  $= \{AB \rightarrow CD, E \rightarrow D, ABC \rightarrow DE, E \rightarrow AB, D \rightarrow AG, ACD \rightarrow BE\}$  :

Consider  $AB \rightarrow CD$ ,  $AB$  is not a superkey, split  $R$  into  $R_1\{A, B, C, D\}$  and  $R_2\{A, B, E, G, H\}$

Consider  $D \rightarrow A$  in  $R_1\{A, B, C, D\}$ ,  $D$  is not a superkey, split  $R_1$  into  $R_{11}\{A, D\}$  and  $R_{12}\{B, C, D\}$

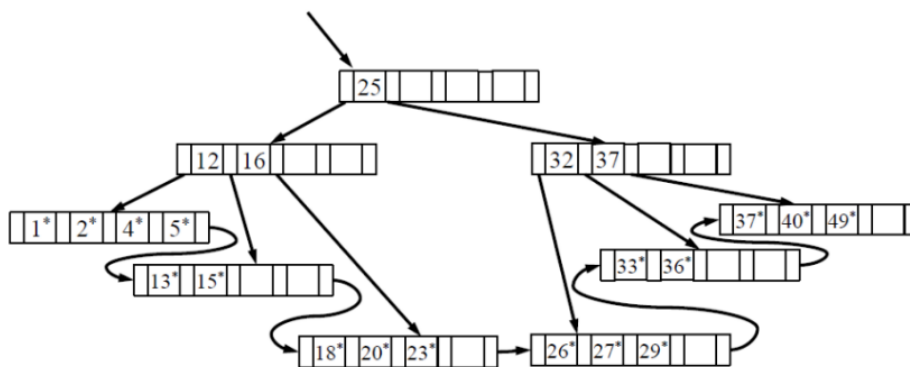
Consider  $E \rightarrow AB$ ,  $E$  is not a superkey, split  $R_2$  into  $R_2\{A, B, E\}$  and  $R_3\{E, G, H\}$

Consider  $E \rightarrow G$ ,  $E$  is not a superkey, split  $R_3$  into  $R_{31}\{E, G\}$  and  $R_{32}\{E, H\}$

One of the possible lossless-join decompositions to BCNF is:  $R_{11}, R_{12}, R_2, R_{31}, R_{32}$

## Q6.

1)



2) According to the definition of B+-tree, the biggest B+-tree for the same set of index entries will be the tree where every node is just “half full”. Note that the answer is not unique.