GSOE9210 Engineering Decisions

Problem Set 03

1. For the 'bus or train' example from lectures, suppose that normal walking speed is 3km/hr. There's a 15-minute wait for the bus down Liverpool Road and 10 minutes for the bus up Parramatta Road. The wait for the train is 20 minutes. The train takes 10 minutes to get to Ashfield station and the bus trip down Liverpool St is 15 minutes, while the bus up Parramatta Rd takes 10 minutes.

   Let the value function v be the total travel time from A to B in minutes.

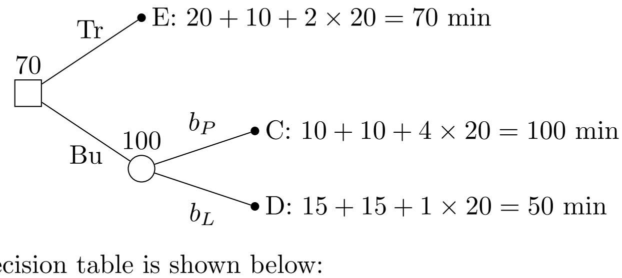
   Assume the walking distances are as in the lecture notes (below in kms):

ω	$d(\omega, \mathrm{B})$
C	4
D	1
E	2

   (a) Represent this problem in extensive form, showing travel times.
   (b) Draw the normal form (table) representation of this problem.
   (c) Using the two methods above, calculate the value of the problem under *Maximin* (*i.e.*, the value of the action chosen under *Maximin*)

Solution:

(a) At 3km/hr it takes 20 min $(= \frac{60\text{min/hr}}{3\text{km/hr}})$ to walk one km. The decision tree (extensive form) representation is shown below. Numbers above the nodes represent the propagated values under *Maximin*.

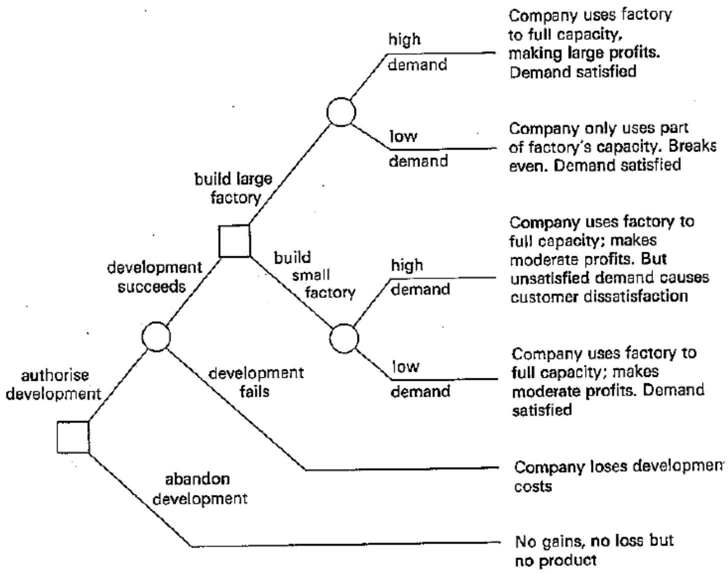


(b) The decision table is shown below:

	b_L	b_P
Tr	70	70
Bu	50	100

(c) The value under *Maximin* is 70.

2. The manufacturing problem discussed in lectures is reproduced below.

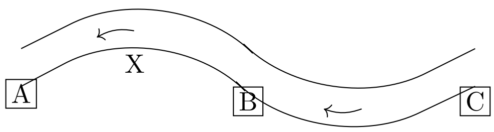


Using the value function given in lectures:

(a) Find the *Maximin* and *miniMax Regret* strategies for this problem.
(b) Evaluate this problem under *MaxiMax*, *Maximin*, *miniMax Regret* using both normal and extensive forms.

Solution: The value under *Maximin* is 0 for strategy Ab; under *miniMax Regret* strategy Au;s/L gives value 4.

3. In lectures we looked at Alice's river trading problem:



The fuel (in litres) required to travel to C from each starting point is:

	A	X	B	C
To C from:	4	3	2	0

(a) Without the aid of a ferry, what is the fuel usage to get from X to each of A, B, and C?
(b) If Alice starts at X, consider the ways in which Alice might get to C:
   A float down to A, then get to C from A
   B travel to B, then get to C from B
   C travel to C

Calculate the fuel required to get to C via the strategies above on a day in which: i. the ferry is operating; ii. the ferry isn't operating. Produce a 'loss table' with these values as 'losses'.

(c) Transform these values to produce a decision table for this problem in which more preferred outcomes receive higher values. Which transformation did you use?
[Hint: use a transformation that produces the 'fuel saved/remaining fuel in the tank' values from lectures.]

(d) Calculate the regret table for this problem.

(e) Determine *Maximin*, *miniMax Regret* strategies and values for this problem.

(f) Which strategies, if any, are dominated?

Solution:

(a) A: 0; B: $3 - 2 = 1$; C: 3

(b) The 'loss' table is:

	f	$\overline{f}$
A	0	4
B	1	3
C	3	3

(c) Under the transformation $v(\omega) = 4 - l(\omega)$, where $l : \Omega \to \mathbb{R}$ is the corresponding value in the loss table, the 'fuel saved/left' table is:

	f	$\overline{f}$
A	4	0
B	3	1
C	1	1

(d) The regret table is:

	f	$\overline{f}$
A	0	1
B	1	0
C	3	0

(e) *Maximin*: B and C with value 1; *miniMax Regret*: A and B with value 1.

(f) Strategy B is at least as good as C if the ferry is not operating, and better if it is; *i.e.*, B weakly dominates C.

4. Consider the decision table below:

	s_1	s_2	s_3
A	3	4	2
B	4	4	3
C	5	6	3

(a) Which strategies in the decision table shown are dominated?
(b) If regret is measured, which strategies are dominated?

Solution:

(a) A is strictly dominated by C and weakly dominated by B. B is weakly dominated by C.
(b) The regret table is:

	s_1	s_2	s_3
A	2	2	1
B	1	2	0
C	0	0	0

Because lower values are better when considering regret, A is strictly dominated by C. In fact, in this problem the dominance relationships when using original values are preserved under regrets. Would this be the case in general?

5. Consider the decision table shown below:

	s_1	s_2
A	0	4
B	3	1
C	2	3
D	1	2

A strategy is termed *admissible* if it is not strictly dominated by any other strategy. A strategy which is not admissible is called *inadmissible*.

(a) Which strategies are admissible?
(b) Under what circumstances should a decision-maker choose an inadmissible action/strategy?

Solution:

(a) The only strictly dominated strategy is D, therefore the admissible strategies are A, B, and C.
(b) Any strategy which is inadmissible is dominated, which means there is an alternative strategy which will be a better response in every state. Inadmissible strategies usually can be discarded.

6. We saw in lectures the following table listing the rules which always eliminate dominated strategies (for strict and weak dominance respectively).

Rule	Strict	Weak
MaxiMax	$\surd$	$\times$
Maximin	$\surd$	$\times$
Hurwicz's	$\surd$	$\times$
miniMax Regret	$\surd$	$\times$
Laplace's	$\surd$	$\surd$

Verify the properties above, giving counterexamples where relevant.

Solution: For *MaxiMax*, if A strictly dominates B then A's outcomes are strictly preferred in every state. In particular $V_{MM}(\text{A}) = \max \text{A} > \max \text{B} = V_{MM}(\text{B})$. So B would be eliminated under *MaxiMax*.

However, consider the following:

	s_1	s_2
A	1	1
B	1	0

Strategy A weakly dominates B, but $V_{MM}(\text{A}) = \max \text{A} = 1 = \max \text{B} = V_{MM}(\text{B})$. In this case the elimination principle will not eliminate B. This counter-example proves that weakly dominated strategies are not always eliminated under *MaxiMax*.

For *miniMax Regret* consider the following decision table (the corresponding regret table is shown on the right):

	s_1	s_2	s_3
A	1	1	0
B	0	1	0
C	0	0	1

	s_1	s_2	s_3
A	0	0	1
B	1	0	1
C	1	1	0

All three actions have the same maximum regret, so they are all equivalent as far as *miniMax Regret* is concerned; *i.e.*, none of the actions is eliminated by *miniMax Regret*. However, action B is dominated by A.

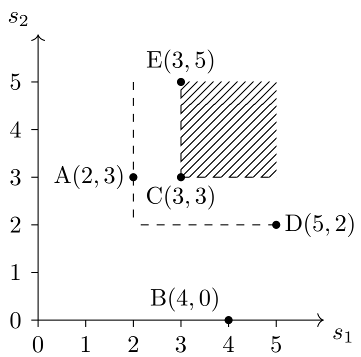
The proof for the other rules is similar, and left as an exercise.

7. For this problem you'll be using the decision table below:

	s_1	s_2
A	2	3
B	4	0
C	3	3
D	5	2
E	3	5

   (a) Graph the strategies.
   (b) Which strategies are dominated: i. weakly; ii. strictly.
   (c) Draw the dominance region for C (*i.e.*, the region of all strategies which dominate C).
   (d) List the indifference classes for this problem.

Solution:



A is dominated by C (weakly) and E (strictly);

B is dominated by D (strictly);

C is dominated by E (weakly);

D is not dominated;

E is not dominated.

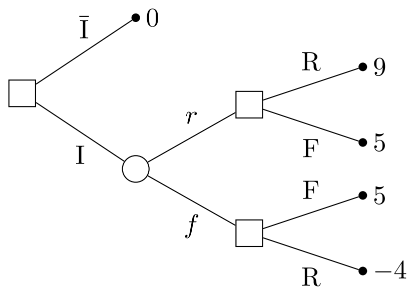
The *Maximin* indifference classes are: $\{B\}$ (0), $\{A, D\}$ (2), and $\{C, E\}$ (3). Their respective *Maximin* values are shown in parentheses.

8. Alice is considering whether to invest \$1000 over an investment period, and if so on which option to invest. She is looking at an investment market which will either rise (r) by 9% or fall (f) by 4% over the investment period. She must submit her instruction to invest (I) or not to her stock broker before the market movement is known. After learning of the market position, she can choose to invest in option F, which gives a fixed return of 5%, or risky option R which follows the market's movement.

   (a) Represent this situation as a decision tree (*i.e.*, in extensive form) and convert this to a decision table (normal form).
   (b) Graph the strategies showing the *Maximin* indifference curves.
   (c) Which policy should Alice choose under *Maximin*?
   (d) Repeat the above for *miniMax Regret*

   Solution:

   (a) There are many possible decision trees that could be used to represent this problem. One is shown below. Because Alice has to decide whether or not to invest in the first place, the initial options are: I: invest; $\bar{\text{I}}$: don't invest.

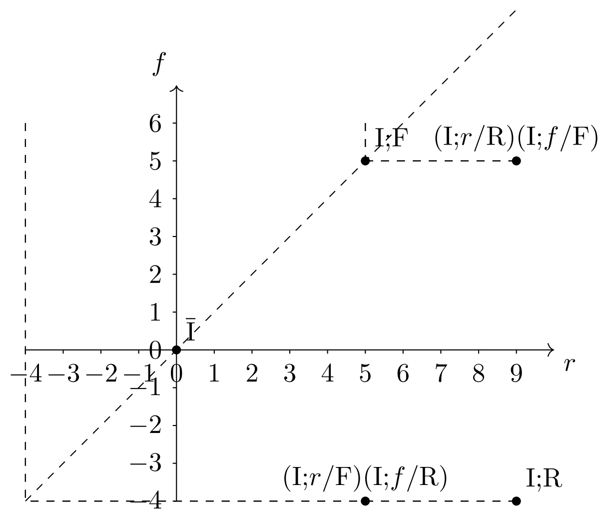


   The normal form is shown below:

	r	f
~~Ī~~	~~0~~	~~0~~
I;F	5	5
(I;r/R)(I;f/F)	9	5
~~(I;r/F)(I;f/R)~~	~~5~~	~~−4~~
I;R	9	−4

Notice that Ī and (I;r/F)(I;f/R) are strictly dominated and hence have been struck out. Strategy (I;r/R)(I;f/F) weakly dominates all others. In some sense it is the best strategy overall: by choosing it one will not do worse in any state, and will do better in some.

(b) Graphically:

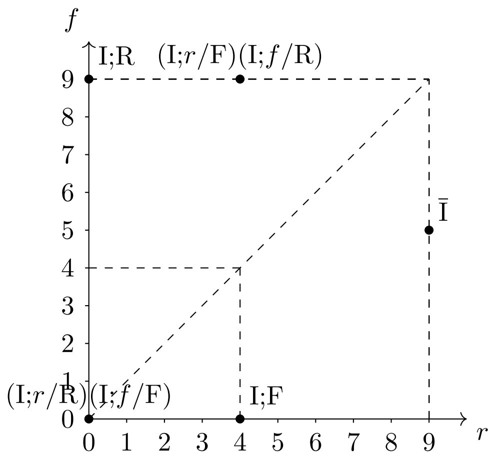


(c) It is clear from the graph that I;F and (I;r/R)(I;f/F) are both *Maximin* strategies.

(d) In terms of regret:

	r	f
~~Ī~~	~~9~~	~~5~~
I;F	4	0
(I;r/R)(I;f/F)	0	0
~~(I;r/F)(I;f/R)~~	~~4~~	~~9~~
I;R	0	9

Note that for regret we prefer smaller values (minimisation). So a strategy (policy) is dominated if its values are greater than those of another strategy, as indicated above.
The regret graph is shown below:



Notice that strategy I;F is a *Maximin* strategy, whereas the unique *miniMax Regret* strategy is (I;r/R)(I;f/F), which is a no-regret strategy in all states—*i.e.*, it weakly dominates all other strategies. In most cases this would be regarded as the preferred strategy, as it is at least as good, if not better, than all the other strategies in all states.

Would a person who wishes to guarantee the best possible return on her investment apply *Maximin* or *miniMax Regret*?