

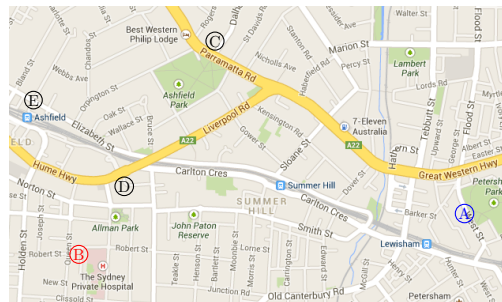
# GSOE9210 Engineering Decisions

## Problem Set 06

1. Alice has \$1000 and has been offered a gamble represented by the lottery:

$$\ell = [\frac{1}{3} : \$4000 | \frac{2}{3} : \$0]$$

- a) What is the expected monetary value (EMV) of the gamble?
  - b) If it costs \$1000 to gamble, would it be a fair bet?
  - c) If Alice prefers to not bet, for this bet she is: a) risk averse; b) risk seeking; or c) risk neutral?
  - d) Describe the shape of Alice's utility curve.
2. Consider the example in lectures in which Alice was indifferent between a certain \$10 and the lottery  $[\frac{1}{2} : \$50 | \frac{1}{2} : \$0]$ . Based on Alice's utility function given in lectures, estimate Alice's certainty equivalent for the lottery in which she shares her profits with Bob. Use this to determine her risk premium for the combined gamble.
  3. An agent with a non-decreasing utility function who is: a) risk averse; b) risk seeking; or c) risk neutral, has a risk premium in what range?
  4. Consider the travelling problem from lectures:



	$b_L$	$b_P$
Tr	20	20
Bu	10	40

where payoffs represent travel times in minutes.

A surgeon needs to travel from A to B to treat a patient suffering from an aggressive bacterial infection. If the infection isn't treated it will start to cause damage after 15 minutes. Death, due to organ failure, will result after 40 minutes.

- a) Suppose the surgeon would consider taking the train (travel time: 20 minutes) over the bus if there is only a 10% chance of the bus going down Liverpool Road (i.e., travel time: 10 minutes). Draw the decision table in terms of utility values.

- b) What might the surgeon's utility for (i.e., as a function of) time look like?
- c) For what range of probabilities of the bus going down Liverpool Road would the surgeon prefer to take the bus?
- d) Suppose the surgeon has the option of driving. Driving is subject to traffic and would take 15 minutes in light traffic and twice as long in heavy traffic. Suppose that the surgeon believes there's a 40% chance of heavy traffic. Using the utility graph of part b), or otherwise, estimate the surgeon's risk premium (in terms of travel time) for driving.
5. Consider a decision problem with the following decision table for options A, B, and C, where the outcomes are in dollar amounts and probabilities are shown above the respective states:

	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
	$s_1$	$s_2$	$s_3$
A	20	30	40
B	10	20	40
C	20	30	20

- (a) If the agent were risk neutral, which option should she choose?
- (b) Suppose now that agent's preferences were such that she would be indifferent between \$20 and the lottery  $[\frac{1}{2} : \$10 | \frac{1}{2} : \$40]$ , and between \$30 and the lottery  $[\frac{1}{5} : \$10 | \frac{4}{5} : \$40]$ . Which of A, B, or C would the agent choose?
- (c) For part b), is the agent risk neutral, averse, or prone/seeking?
- (d) Estimate the agent's certainty equivalent and risk premium for A.
6. A *two-way winner-takes-all* gamble between two participants A and B is a gamble in which, if the two participants bet amounts \$a and \$b respectively, the winner takes everything ( $$(a + b)$$ ). Let  $p$  be the probability that A wins. Prove that a two-way winner-takes-all bet is fair to both participants iff:

$$\frac{a}{b} = \frac{p}{1-p}$$

7. Which of the following real-valued functions are: a) non-decreasing; b) strictly increasing; c) one-to-one; d) onto.

$$f(x) = x, f(x) = x + 1, f(x) = x^2, f(x) = |x|$$

8. Show that for a non-decreasing function it is not true in general that if  $f(x) \geq f(y)$ , then  $x \geq y$ .
9. Show that for any real-valued function  $f : \mathbb{R} \rightarrow \mathbb{R}$ , the following are equivalent:
- (a)  $f$  is strictly increasing
- (b) for any  $x, y \in \mathbb{R}$ ,  $x > y$  iff  $f(x) > f(y)$
- (c) for any  $x, y \in \mathbb{R}$ ,  $x \geq y$  iff  $f(x) \geq f(y)$