GSOE9210 Engineering Decisions

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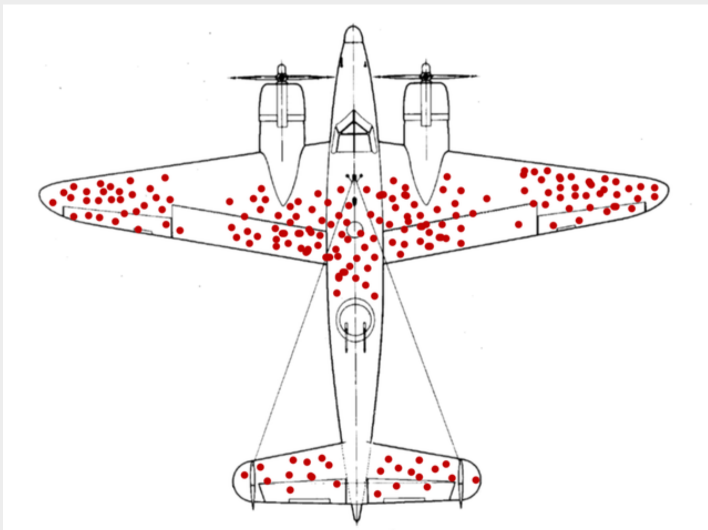
Introduction

Introduction

1. Introduction
   - Motivation
   - Decision problems: examples
   - Course overview
2. Decision problems: representation
   - Decision problem elements
   - Uncertainty
   - Decision trees
   - Decision tables
3. Decision problem classes
4. Decisions under certainty

High stakes decisions: War

Abraham Wald (1902–1950)



Plan: redistribute armour

Decision problems: professional

Example (Oil exploration)

You're the chief petroleum engineer of an oil company which owns a drilling option on an area of sea. Should you drill before the option expires?

Considerations:

- likelihood of finding oil, amount and quality, projected oil demand
- size and location of drilling
- cost of drilling and raising the oil, *etc.*

Decision problems

Example (Drug development)

You're the chief chemical engineer in a pharmaceuticals company which is considering whether to mass produce a new cancer treatment drug. Initial findings are inconclusive as to the drug's effectiveness. Should you go into development and/or production?

Considerations:

- likelihood of drug's effectiveness
- level of investment, timing, competition, *etc.*
- cost to the company of synthesis and trials; value of human life, *etc.*

Decision problems

Example (Manufacturing processes)

You're the head process engineer of Acme Inc., a company which manufactures car components. New regulations could mean increased demand for Acme's components in the near future. The managing director has requested a report on existing plant capacity and possible production options. What is your recommendation?

Considerations:

- likelihood and degree of increased demand
- options for increasing plant production
- cost-benefit analysis of capital investment, *etc*.

Decision problems: everyday

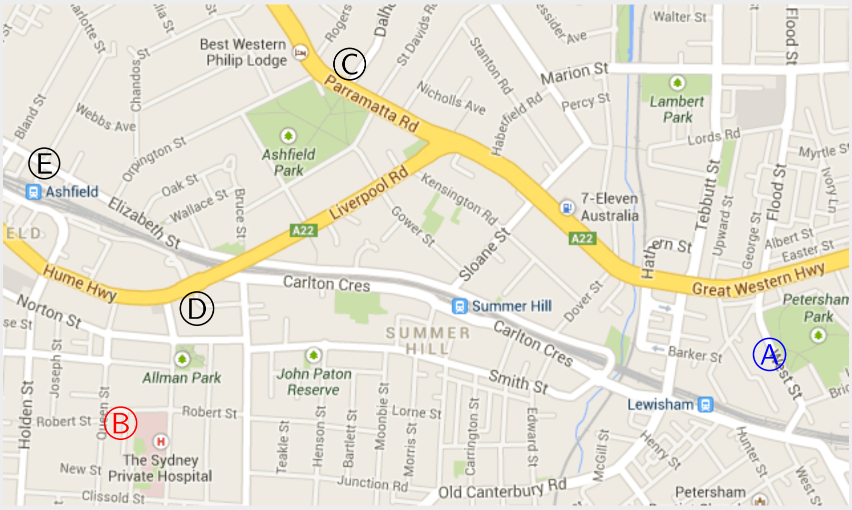
Example (To insure or not)

You own a necklace which you intend to sell at the end of the year. Should you insure it against theft?

Considerations:

- value of necklace
- cost of insurance
- likelihood of theft

Decision problems

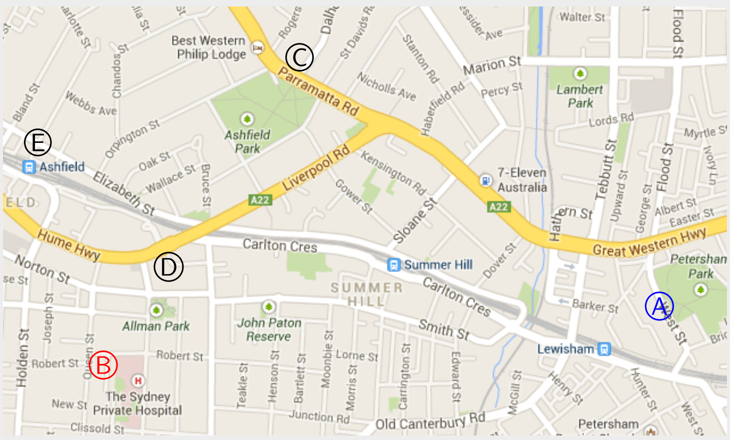


Example (Getting from A to B)

You have to get from Petersham Park (A) to the Hospital (B) by either train or bus. The train goes to Ashfield Station (E). You don't know the bus route: either via Parramatta Rd (C) or Liverpool Rd (D).

Introduction | Decision problems: examples

Decision problems: discussion



Suppose you:

- are an ER surgeon
- are a tourist
- *have an injured foot ...*

Quantitative problems

Example (Inventory)

Your football club provides uniforms for each of its members. The initial order needs to be placed before the final number of members is known. The initial (early) order costs $\$500$ plus $\$20$ per uniform. Late orders incur an additional $\$300$ fee plus the usual $\$20$ per uniform. Uniforms are sold for $\$40$ each.

How many uniforms should you order initially?

Group decisions

Example (Song contest)

Seven judges vote for four songs: A, B, C, D.

	J1	J2	J3	J4	J5	J6	J7	Tot.
A	4	1	2	4	1	2	4	18
B	3	4	1	3	4	1	3	19
C	2	3	4	2	3	4	2	20
D	1	2	3	1	2	3	1	13

What if song D is disqualified?

	J1	J2	J3	J4	J5	J6	J7	Tot.
A	3	1	2	3	1	2	3	15
B	2	3	1	2	3	1	2	14
C	1	2	3	1	2	3	1	13

Group decisions

Example

Three people (P1, P2, P3) vote for three candidates A, B, C in a poll. The preferences are:

	P1	P2	P3
1st	B	C	A
2nd	C	A	B
3rd	A	B	C

What should be the group preference?

- Most preferred, second preference, . . .
- Majority: two voters prefer B to C, two C to A, . . .

Who decides?

"If . . . decision-theoretic structures do not in the future occupy a large part of the education of engineers, then the engineering profession will find that its traditional role of managing scientific and economic resources for the benefit of man has been forfeited to another profession."

—Ron Howard (1966)
Professor of Management Science and Engineering
Stanford University

Course overview

Course aims:

To prepare engineering graduates for decision-making roles.

Topic structure:

- Single-agent decisions
  - certainty
  - complete uncertainty
  - partial uncertainty: likelihoods
- Multi-agent decisions: games

Learning outcomes:

- Classify decision problems according to their characteristics
- Model/represent decision problems according to their class
- Analyse and solve decision models
- Evaluate solutions for decision models

Single-agent decisions: overview

1. Decision problems
2. Decision problem representations: trees and tables
3. Decisions under uncertainty (ignorance and risk)
4. Quantifying likelihood: probability
5. Preference and utility
6. Information and its value

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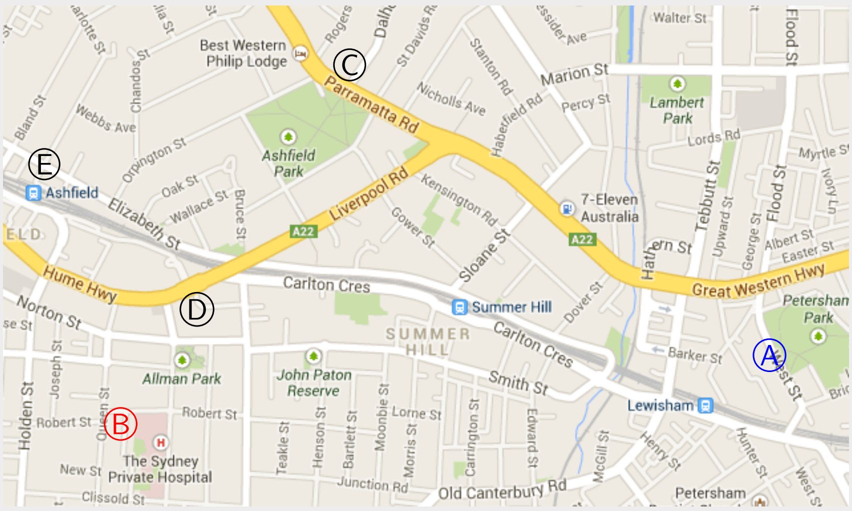
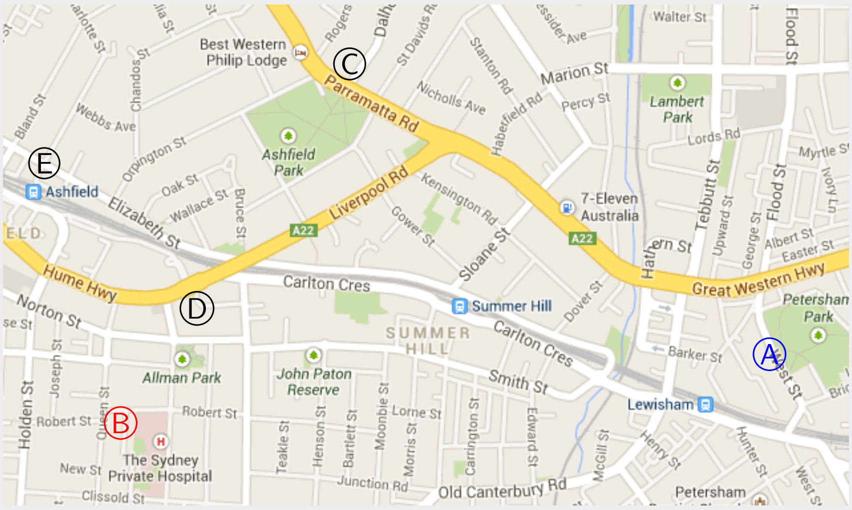
Decision problems



Example (Getting from A to B)

You have to get from Petersham Park (A) to the Hospital (B) by either train or bus. The train goes to Ashfield Station (E). You don't know the bus route: either via Parramatta Rd (C) or Liverpool Rd (D).

Decision problems: abstraction/elements



What do *decision problems* have in common?

- *actions* (alternatives, strategies) ($\mathcal{A}$): Tr, Bu
- *possible states* (events, cases, scenarios) ($\mathcal{S}$): *e.g.*, Liverpool Rd bus (b_L) or Parramatta Rd bus (b_P)
- *outcomes* (consequences) (Ω): arrive at C, D, or E

Decision elements

outcomes: possible future states of the world we are interested in (mutually exclusive)

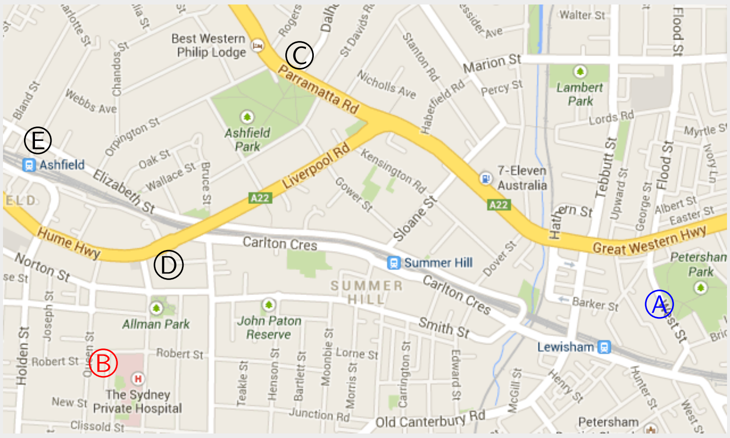
Possible outcomes may depend on other 'conditions' of two types:

actions: occurrences/circumstances under decision-maker's direct control (*e.g.*, decision-maker boards bus/train)

states: occurrences/circumstances outside decision-maker's control (subject to chance; *e.g.*, bus goes up Parramatta Rd)

Each action and state combine to *uniquely determine* an outcome

Decision problems: elements



- $\Omega = \{$C, D, E$\}$
  $\mathcal{A} = \{$Tr, Bu$\}$
  $\mathcal{S} = \{b_L, b_P\}$
- each action associated with a set of possible outcomes:
  e.g., Tr: {E}, Bu: {C, D}

Decision problems

Example (To insure or not)

You own a necklace which you intend to sell at the end of the year. Should you insure it against theft?

- *actions* ($\mathcal{A}$): Insure, don't insure
- *states* ($\mathcal{S}$): necklace stolen, necklace not stolen
- *outcomes* (Ω): uninsured necklace sold (not stolen), insured necklace sold, necklace stolen and not insured, necklace stolen but insured

Quantitative problems

Example (Inventory)

Your football club provides uniforms for each of its members. The initial order needs to be placed before the final number of members is known. The initial (early) order costs $\$500$ plus $\$20$ per uniform. Late orders incur an additional $\$300$ fee plus the usual $\$20$ per uniform. Uniforms are sold for $\$40$ each.

- *actions* ($\mathcal{A}$): Order quantity (q): $O_0, O_1, O_2, \ldots, O_q, \ldots$
- *states* ($\mathcal{S}$): Membership (m): $r_0, r_1, r_2, \ldots, r_m, \ldots$
- *outcomes* (Ω): Result of 2-place function of q and m, $f(q,m)$ ...

Oil exploration; uncertainty

Example (Oil exploration)

You're the chief petroleum engineer of an oil company which owns a drilling option on an area of sea. Should you drill before the option expires?

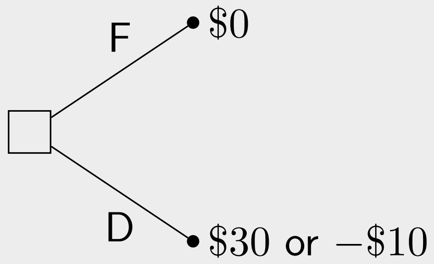
- *actions* ($\mathcal{A}$): Drill (D), Forfeit rights (F) (don't drill ($\overline{\text{D}}$))
- *states* ($\mathcal{S}$): Oil present (o), no oil ($\overline{o}$)
- *outcomes* (Ω): Profit ($\$30$), loss ($-\$10$), status quo ($\$0$)

Uncertainty

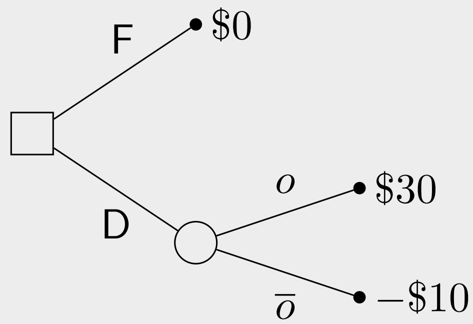
There is *uncertainty* due to incomplete information about which of multiple possible states is *actual*.

Oil exploration analysis

- Decide between two options:

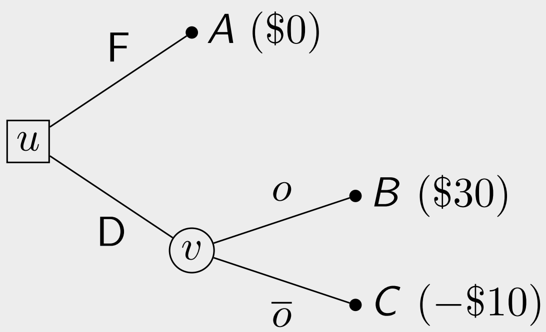


- Combined into a decision tree:



Choosing between uncertain situations is one of the fundamental problems of complex decision-making.

Decision trees



In a decision tree:

- leaf nodes represent outcomes
- branches represent either actions or states/events
- internal nodes can be *decision nodes* (boxes) or *chance nodes* (circles)

Exercises

- What type of node is u? v? B?
- What does the branch labelled D represent?
- What does the branch labelled $\overline{o}$ represent?

Problem representation

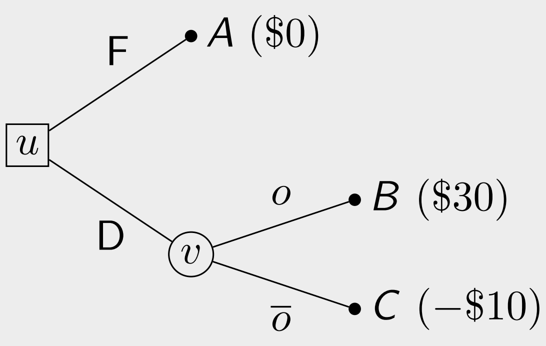
Exercises

Draw decision trees for the problems below:

- Alice's insurance problem
- Alice's football club inventory problem

How would you modify the representations above if Alice had two insurance policies to choose from?

Problem representation: decision tables



- Observation: each action–state pair uniquely determines an outcome
- Model as a 2-ary (dyadic) function: $\omega : \mathcal{A} \times \mathcal{S} \to \Omega$

Represented as a table:

		$\mathcal{S}$	
	ω	o	$\overline{o}$
$\mathcal{A}$	F	A	A
	D	B	C

Decision tables:

- row = action
  column = state
- Interpretation: B $= \omega(\text{D}, o)$ means "B is the outcome of action D in state o";

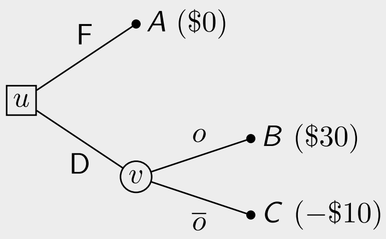
Decision tables

$\mathcal{S}$

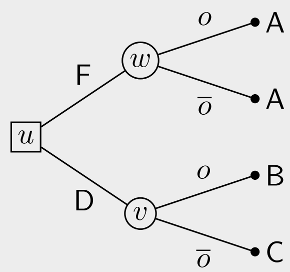
		s_1	s_2	$\dots$	s_k	$\dots$	s_n
	A_1	ω_{11}	ω_{12}	$\dots$	ω_{1k}	$\dots$	ω_{1n}
	A_2	ω_{21}	ω_{22}	$\dots$			
$\mathcal{A}$	$\vdots$						
	A_j	ω_{j1}	ω_{j2}	$\dots$	ω_{jk}	$\dots$	
	$\vdots$						
	A_m	ω_{m1}	ω_{m2}	$\dots$	ω_{mk}	$\dots$	ω_{mn}

- A decision table represents the 2-ary function $\omega : \mathcal{A} \times \mathcal{S} \to \Omega$, where $\mathcal{A} = \{A_1, \dots, A_m\}$ and $\mathcal{S} = \{s_1, \dots, s_n\}$; entry in j-th row and k-th column is $\omega_{jk} = \omega(A_j, s_k)$
- Formally, a decision problem is a 4-tuple $(\mathcal{A}, \Omega, \mathcal{S}, \omega)$

Trees and tables



	o	$\overline{o}$
F	\$0	\$0
D	\$30	−\$10

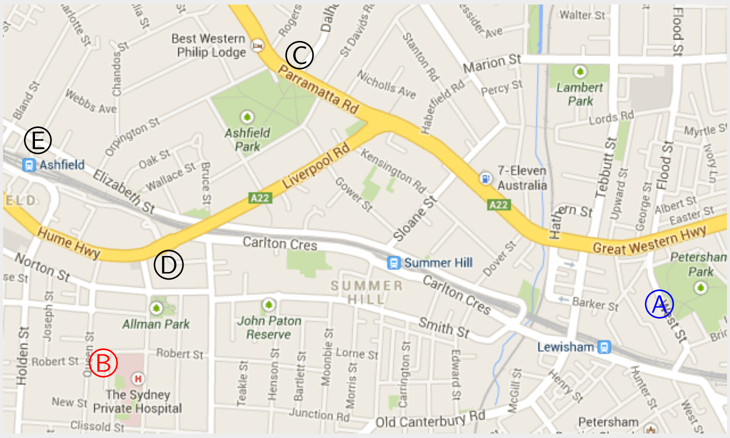


- Multiple trees may correspond to the same table
- Going from tables (*normal form*) to trees (*extensive form*) is straight forward, but the converse can be tricky
- Which representation is better: trees or tables?
- Which representation facilitates decision analysis most?

Outcomes

Comparing outcomes: value/payoff functions

- Preferences over outcomes can be easily expressed if the outcomes can be quantified numerically

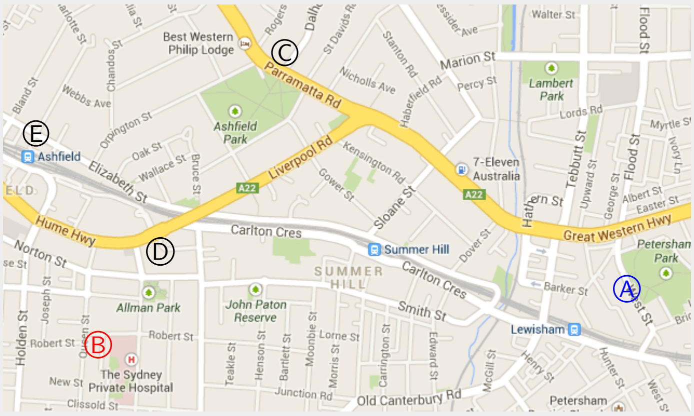


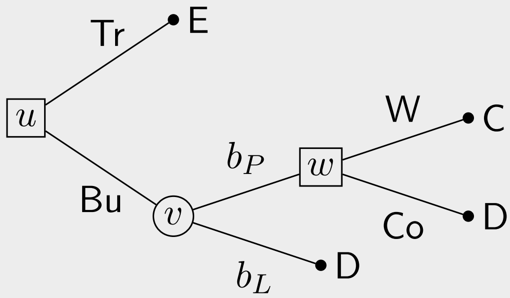
Distance to B:

ω	$d(\omega, \text{B})$
B	0km
C	4km
D	1km
E	2km

- Prefer E to C because $d(\text{E}, \text{B}) < d(\text{C}, \text{B})$

Outcomes and values

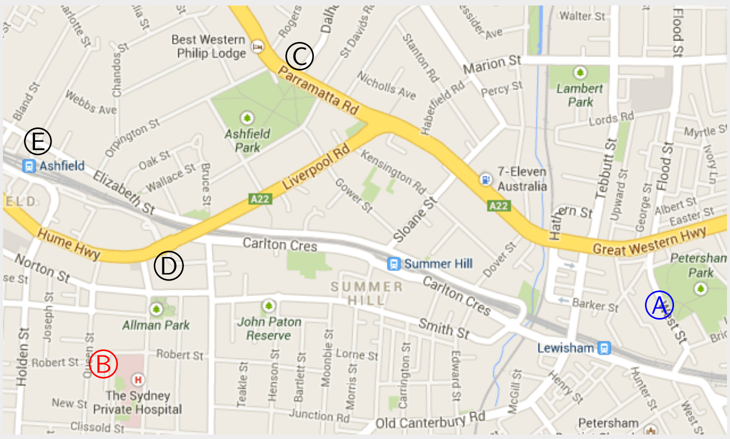




Question

Suppose that the route up Parramatta Rd loops around to D providing the new option to either walk from C or continue to D. Do the two D leaf nodes correspond to the same outcome if we evaluate them according to: (a) distance; (b) total travel time?

Outcomes and values



Values of outcomes based on distance (km):

	b_L	b_P
Tr	2	2
Bu	1	4

- Walking distance? Straight line?
- Consider values based on travel times (mins):

	b_L	b_P
Tr	30	30
Bu	10	40

Decision and preference

- Outcomes are objective but agents have *subjective* (individual) *preferences* over them:

  If we play poker and I win the outcome is 'good' for me and 'bad' for you

- Often assign each outcome a numerical *value*; *e.g.*, money, distance, *etc*. *i.e.*, each agent has its own *value function*: $v : \Omega \to \mathbb{R}$
- Value functions/actions essentially are *random variables*
- A (subjective) decision problem is now: $(\mathcal{A}, \mathcal{S}, \Omega, \omega, v)$

Convention

Value assignments usually assign higher values to more preferred (more desirable) outcomes.

Decision problems: representation | Decision tables

Epistemic state

An agent's decisions are subjective; they depend on:

- their preferences (*e.g.*, values on outcomes)
- their *epistemic state* (*i.e.*, information about the state of the world when the decision is made); *e.g.*, bus route map; past experience; *etc.*

Definition (Epistemic state)

An agent's *epistemic state* is the knowledge (information) or belief it has about the actual state of the world.

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Decision problems and rules

Fundamental problem of decision theory

For any given decision problem, to come up with a *rational* choice from among the possible actions.

Definition (Decision rule)

A *decision rule* is a way of choosing, for each decision problem, an action or set of actions.

Questions:

- What constitutes a *rational* decision rule?
- How does an agent's epistemic state affect a decision rule?

Outcomes

Decision problem classes

Decision problems can be classified based on an agent's epistemic state:

- Decisions under *certainty*: the agent knows the unique actual state
- Decisions under *uncertainty*:
  - Decisions under *ignorance* (full uncertainty): the agent believes multiple states/outcomes are possible; likelihoods unknown
  - Decisions under *risk*: the agent believes multiple states/outcomes are possible; likelihood information available

Outcomes

Decisions under certainty

Example (Project budgeting)

You are a lead software engineer in a major software company. Your R & D team has proposed three possible projects, A, B, and C, each with a different life-time. The net profits over the life of the projects are listed in the adjacent table.

	profit ($M)
A	20
B	13
C	17

- Which project would you choose?

Complex outcomes

Project life-time cash-flows:

- A: three years, big initial set-up costs
- B: one year immediate return
- C: three years, small initial set-up costs

cashflow ($M)

	Year 1	Year 2	Year 3
A	-10	5	25
B	13	0	0
C	-5	10	12

New perspective:

- Outcomes described by vectors: *e.g.*, for A: $(-10, 5, 25)$.
- What is more important: maximising total return, preserving cash, *etc.*?
- Which project would you choose?

Composite outcomes: Net Present Value (NPV)

- The *Net Present Value* (NPV): value of the project in *present* terms
- Future worth less than in the present
- Model by a *discount rate* (γ); assume discount rate of 20%

$$\begin{aligned} \mathit{NPV}(\mathsf{A}) &= -10 + \frac{5}{1.2} + \frac{25}{1.2^2} && \approx 11.5 \\ \mathit{NPV}(\mathsf{B}) & && \approx 13.0 \\ \mathit{NPV}(\mathsf{C}) &= -5 + \frac{10}{1.2} + \frac{12}{1.2^2} && \approx 11.7 \end{aligned}$$

- More generally:

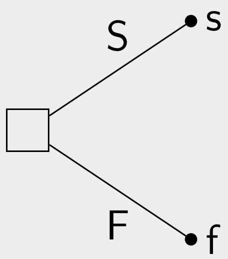
$$v(x_1, x_2, x_3) = x_1 + \frac{x_2}{1+\gamma} + \frac{x_3}{(1+\gamma)^2}$$

Decisions under certainty

Example (School fund-raising)

A school committee is looking to hold a fund-raiser. It has a choice between holding a fête or a sports day.

	s_0
S	s
F	f



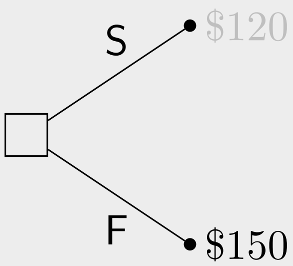
- In this example:
  $\mathcal{A} = \{\mathsf{S}, \mathsf{F}\}$
  $\Omega = \{\mathsf{s}, \mathsf{f}\}$
  $\mathcal{S} = \{s_0\}$
- Which action preferred: F or S?
- Which outcome preferred: f or s?

Decisions under certainty: value functions

Example (School fund-raising)

A school committee is looking to hold a fund-raiser. It has a choice between holding a fête or a sports day. It expects to make \$150 for a fête but only \$120 for a sports day.

	s_0	V
S	\$120	\$120
F	\$150	\$150



- Value function over *outcomes*: $v : \Omega \to \mathbb{R}$
- In this example:

$$v(\mathsf{s}) = 120$$
$$v(\mathsf{f}) = 150$$

- Value function over *actions*; *i.e.*, $V : \mathcal{A} \to \mathbb{R}$; here $V(\mathsf{A}) = v(\omega)$, where $\omega = \omega(\mathsf{A}, s_0)$

Rational decisions

- A *normative* theory of decision-making: *i.e.*, decisions ideal (*rational*) decision-makers *ought* to make
- Which principles govern rational decision-making?

Rationality Principle 1 (Elimination)

Faced with two possible alternatives, rational agents should never choose the less preferred one.

- *i.e.*, rational agents should discard less preferred actions
- Rational decisions:

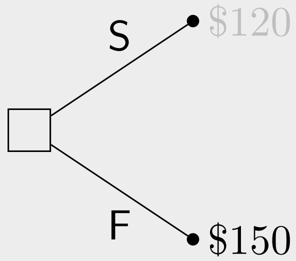
It is irrational to choose a less preferred alternative.

Rational decisions under certainty

Rationality Principle 1 (Elimination)

Given a value function $V : \mathcal{A} \to \mathbb{R}$ over actions, rational agents should prefer action A to B iff $V(\text{A}) > V(\text{B})$.

	s_0	V
S	\$120	120
F	\$150	150



- Since F is preferred to S ($V(\text{F}) > V(\text{S})$), S is eliminated (by elimination), hence the rational choice is the remaining option: F

Corollary

A rational agent should not choose any action which is not preference maximal.

Summary

- Decision problems
- Elements in a (any) decision: actions, states, outcomes
- Representing decision problems: trees and tables
- Uncertainty and information (epistemic state)
- Preference (value) over outcomes
- Decision classes
- Decisions under certainty