

# GSOE9210 Engineering Decisions

## Problem Set 02

1. Choose either a problem of your own or one from the discussion forums and apply each of the following decision rules:
  - (a) *Maximin*
  - (b) *miniMax Regret*
  - (c) How would you go about determining your own personal optimism index ( $\alpha$ ) for the problem? What is your personal optimism index value? Post your optimism value on the forum.
2. For the technology start-up problem from week 1, do the following:
  - (a) Determine the action chosen by each of the rules *MaxiMax* and *Maximin*
  - (b) What would be the best response if the final prototype trial succeeds? Fails? What would be the *miniMax Regret* action?
  - (c) Based on your own decision, with which of the rule(s) above were your choices compatible (with which decision rule(s) did your choice agree)? Post your answer on the forum and/or post a reply comparing your answer with someone else's.
3. For the travelling problem in lectures, the values of outcomes based on walking distance (km) are given below:

	$b_L$	$b_P$
Tr	2	2
Bu	1	4

- (a) Describe each action as a lottery.
- (b) What is the *MaxiMax* (MM) action for this problem?
- (c) How would this be affected if the traveller had to visit the hospital's clinic, a further kilometre south of the hospital, afterwards?
- (d) Draw the decision table for the same problem using walking time instead, assuming a person walks at an average speed of 3km/h.
- (e) Which action is chosen under the *MaxiMax* rule when considering walking time?

Solution:

- (a) Below the lottery associated with an action 'A' is denoted  $\ell_A$ :

$$\ell_{Tr} = [b_L : E | b_P : E] = [E] \quad \ell_{Bu} = [b_L : D | b_P : C]$$

- (b) Take the bus (Bu), with least distance 1km. Note that, here, more preferred outcomes have lower values. Equivalently, preferences values could be associated with negative distances.
- (c) Adding the 1km walk to the clinic would just add 1 to every entry. This would not affect the outcomes' relative desirability, and hence would not affect the choice under *MaxiMax*.
- (d) Assuming no disruptions to the journey (e.g., traffic lights at road crossings, etc.), the decision table expressed in minutes is:

	$b_L$	$b_P$
Tr	40	40
Bu	20	80

- (e) Since walking time is a scaling factor, which preserves the relative values of the outcomes, this also has no effect on *MaxiMax*.
4. Repeat the exercise above for the *Maximin* (Mm) rule.

Solution:

- (a) Same as above.
  - (b) Take the train (Tr), with worst possible case a walk of 2km.
  - (c) Adding the 1km walk to the clinic would just add 1 to every entry, which, again, would not affect the outcomes relative desirability, and hence would not affect *Maximin*'s choice.
  - (d) As above.
  - (e) Since walking time is a scaling factor, which preserves the relative values of the outcomes, this has no effect on *Maximin* either.
5. Suppose Alice is the principal in the school fund-raising problem discussed in lectures:

	$d$	$w$	
S	120	85	$d$ day is dry
F	150	75	$w$ day is wet

- (a) Represent each action as a lottery.
- (b) Which action is preferred under *MaxiMax* and *Maximin*?
- (c) What optimism level (i.e., value of index  $\alpha$  under *Hurwicz*'s rule) would Alice have if she were 'indifferent' between (i.e., have equal preference for) the two options?
- (d) Derive a general expression for the value of the optimism index  $\alpha^*$  for which Alice would be indifferent between actions  $A_1$  and  $A_2$ , with best and worst outcomes  $M_1$  and  $m_1$ , and  $M_2$  and  $m_2$ , respectively.
- (e) Suppose there was a third option, an *indoor trivia night* (T), which generates profit \$100 regardless of the weather. How optimistic would Alice have to be to prefer the sports day over the trivia night?

Solution:

(a) The corresponding lotteries are shown below:

$$\ell_S = [d : \$120 | w : \$85] \quad \ell_F = [d : \$150 | w : \$75]$$

(b) Consider the decision table below:

	$d$	$w$	$m$	$M$
S	120	85	85	120
F	150	75	75	150

It follows that *Maximin* would prefer S and *MaxiMax* F.

(c) The *Hurwicz* values are given by:

$$V_H(S) = 120\alpha + 85(1 - \alpha)$$

$$V_H(F) = 150\alpha + 75(1 - \alpha)$$

The two are equivalent when:

$$V_H(S) = V_H(F)$$

$$120\alpha + 85(1 - \alpha) = 150\alpha + 75(1 - \alpha)$$

$$10(1 - \alpha) = 30\alpha$$

$$40\alpha = 10$$

$$\therefore \alpha = \frac{1}{4}$$

(d) In general, two actions  $A_1$  and  $A_2$  are equivalent when:

$$V_H(A_1) = V_H(A_2)$$

$$M_1\alpha + m_1(1 - \alpha) = M_2\alpha + m_2(1 - \alpha)$$

$$(M_1 - M_2 + m_2 - m_1)\alpha = m_2 - m_1$$

$$\alpha = \frac{m_2 - m_1}{(m_2 - m_1) + (M_1 - M_2)}$$

$$\text{rearranging} \quad \alpha = \frac{1}{1 - \frac{(M_1 - M_2)}{(m_1 - m_2)}}$$

(e) Action S is preferred when:

$$V_H(S) > V_H(F)$$

$$120\alpha + 85(1 - \alpha) > 100$$

$$35\alpha > 15$$

$$\therefore \alpha > \frac{3}{7}$$

6. How could you simplify Laplace's decision rule of insufficient reason? That is, can you give an equivalent, but simpler, criterion for choosing between actions?

**Solution:** Provided all states are exhaustive and mutually exclusive, Laplace's rule amounts to choosing the action which maximises the sum of the values of its outcomes.

7. Alice has a choice of buying an investment property in either of two suburbs: A and B. In five years, house prices are likely to go up by \$2K in B, and by \$1K in A. However, there is an existing proposal to build a shopping centre in A in the next year. If the shopping centre is approved ( $a$ ), house prices in A will increase in value over the next five years by \$6K.

For the problem described above:

- Which is the *Maximin* action?
- Which is the best action if approval from the shopping centre is granted? If approval is not granted?
- Which is the *miniMax Regret* action?
- Which of the two decision rules above would be most relevant for a property investor?

**Solution:** The decision table looks as follows, where the entries represent the value in five years:

	$a$	$\bar{a}$	$m$
A	6	1	1
B	2	2	2

- The *Maximin* action is B.
- In state  $a$ , the best action is A. In state  $\bar{a}$ , the best action is B.
- The regret matrix/table is given by:

	$a$	$\bar{a}$	$M$
A	0	1	1
B	4	0	4

It follows that the *miniMax Regret* action is A.

- Speculative investors tend to be risk takers, looking for high returns even where there's risk. Such a person would probably choose according to *miniMax Regret* rather than *Maximin*.

8. Consider the following decision table:

	$s_1$	$s_2$	$s_3$	$s_4$	$V$
A <sub>1</sub>	2	2	0	1	
A <sub>2</sub>	1	1	1	1	
A <sub>3</sub>	0	4	0	0	
A <sub>4</sub>	1	3	0	0	

- Evaluate each action under the following decision rules, and determine which action will be chosen under each rule: i. *MaxiMax* (MM) ii. *Maximin* (Mm) iii. *Hurwicz's* rule for values of  $\alpha = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ .
- Which decision rules above *agree* on this problem; *i.e.*, choose the same actions?

- (c) Two decision rules are said to be *equivalent* if they choose the same action for every possible decision problem. Which of the rules above are equivalent?

Solution:

- (a) For the case  $\alpha = \frac{1}{4}$ :

	$s_1$	$s_2$	$s_3$	$s_4$	$M$	$m$	$H$
$A_1$	2	2	0	1	2	0	$\frac{1}{2}$
$A_2$	1	1	1	1	1	1	1
$A_3$	0	4	0	0	4	0	1
$A_4$	1	3	0	0	3	0	$\frac{3}{4}$

- i.  $A_3$
  - ii.  $A_2$
  - iii.  $A_2$  and  $A_3$
- (b) *MaxiMax* and *Maximin* don't agree on this problem. *Hurwicz's* rule will agree with the other rules for some values of  $\alpha$ : e.g., with *MaxiMax* for  $\alpha = 1$ ; with *Maximin* for  $\alpha = 0$ . For other values, such as  $\alpha = \frac{1}{4}$ , *Hurwicz's* rule agrees with neither.
- (c) None. The problem above is a counterexample showing that none of the rules are equivalent, as all the rules produce different choices for at least one (e.g., this) decision problem.

9. For the problem above, which is the *miniMax Regret* action?

Solution: The regret matrix/table is given by:

	$s_1$	$s_2$	$s_3$	$s_4$	$M$
$A_1$	0	2	1	0	2
$A_2$	1	3	0	0	3
$A_3$	2	0	1	1	2
$A_4$	1	1	1	1	1

Action  $A_4$  has the least maximum regret (1).

10. For the raffle problem discussed in lectures:

- (a) Draw the decision tree and table
- (b) Should you draw a ticket in the raffle?
- (c) What if you knew there were three blue tickets? Four? None?
- (d) How many blue tickets would there have to be to make it worth entering?
- (e) If there were  $n$  blue tickets ( $0 \leq n \leq 4$ ), how would the value of the prize which makes it worthwhile entering depend on  $n$ ?

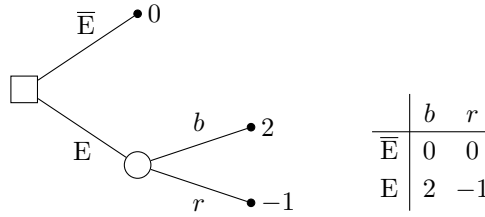
Solution: The raffle corresponds to the lottery:

$$\ell = [b : \$2 | r : -\$1]$$

where:

- $b$  blue ticket drawn  
 $r$  red ticket drawn

- (a) A concise decision tree (there may be others) and table are shown below:



where  $E$  stands for 'Enter the raffle'.

- (b) To enter is to risk losing your entry fee ( $-1$ ) for the chance to win 2. I probably would enter, due to the small amounts involved.  
(c) If I knew there were three blue tickets (*i.e.*, chances of winning are 3 to 1 in favour) I would definitely enter. I would also enter if there were four blue tickets. This would represent a certain win. If there were no blue tickets that would mean certain loss; I wouldn't enter.  
(d) For me, probably two or more. Note that, based on expected values:

$$\begin{aligned} E(E) &= P(b)(2) + P(r)(-1) \\ &= \frac{1}{2}(2) + \frac{1}{2}(-1) = \frac{1}{2} \\ E(\bar{E}) &= 0. \end{aligned}$$

- (e) In general, let  $\$w$  be value of winning and  $\$l$  that of losing:

$$\begin{aligned} E(E) &= P(b)w + P(r)l \\ &= \frac{n}{4}w + \frac{4-n}{4}(-1) \quad (\text{setting } l = -1) \\ &= \frac{1}{4}(wn - 4 + n) \\ &= \frac{1}{4}((w+1)n - 4) \\ E(\bar{E}) &= 0. \end{aligned}$$

We would require (for  $n > 0$ ):

$$\begin{aligned} E(E) &> E(\bar{E}) \\ (w+1)n - 4 &> 0 \\ w &> \frac{4}{n} - 1 \end{aligned}$$

For  $n = 0$  there would be no finite value that would make entering worthwhile.