GSOE9210 Engineering Decisions

Problem Set 04

1. At a school fête, a suspicious-looking man is offering bets on the toss of a coin of questionable fairness. If the contestant correctly chooses the face on the coin, the man will pay \$2 for each dollar bet, and will return the original dollar bet; *i.e.*, if you bet \$1 and win you will gain \$2, leaving you with \$3; if you lose you will lose the dollar you bet, leaving you with \$0.

   (a) Suppose Alice has a \$10 note in her pocket; represent the gamble as a decision table. Include the option of leaving (L) (*i.e.*, refusing to gamble).

   (b) If Alice were pessimistic (*i.e.*, she used *Maximin* as her decision rule), would Alice bet on heads or tails, or not bet?

   (c) Suppose Alice has a similarly pessimistic friend Bob, and both could bet together on the same toss, should that affect Alice's decision?

   (d) Suppose Alice was friendless but had—instead of one \$10 note—10 \$1 coins in her pocket. How might this affect her decision?

2. (a) Find the value $\frac{3}{4}$ of the way from 4 to 2 on the number line.

   (b) Find a general expression for the value μ of the way from a to b on the number line.

   Consider the decision problem below:

	s_1	s_2
A	2	3
B	4	0
C	3	3
D	5	2
E	3	5

   (c) Plot the actions as points on the Cartesian plane. Find the coordinates of the point μ of the way from B to A.

   (d) Find the point, $P(x, y)$, on the segment AB that intersects with the diagonal $y = x$.

   (e) Find the *Maximin* mixed action for the decision problem above.

3. Consider a scenario with four states, two pure actions, and their mixtures (M), where $\mu_A = \mu$:

	s_1	s_2	s_3	s_4
A	4	2	1	3
B	0	1	5	2

(a) Draw the mixture plot for this problem.

(b) Do all states need to be considered when determining the *Maximin* mixed action?

(c) Find the *Maximin* mixed action, and the *Maximin* value—*i.e.*, the value of the *Maximin* mixed action—for this problem.

4. Alice sells drinks at a local market once every month. She can order stock to sell several drink types: a) hot chocolate; b) iced tea; c) lemonade; d) orange juice.

   From past experience, she knows that when she sells only one type of drink, on warm days her sales total for each type are: $10 on hot chocolate, $40 on iced tea, $30 on lemonade, and $40 on orange juice. On cool days, however, her sales totals are: $30 on hot chocolate, $0 on iced tea, $20 on lemonade, and $10 on orange juice.

   She has to order her stock weeks in advance, long before she can predict the temperature on the day of the market.

   (a) Produce a decision table for this problem.

   (b) If for the same total amount of stock she can stock different drinks, what proportion of drinks should she stock to maximise her guaranteed daily sales total regardless of the temperature?

   (c) Draw the admissibility frontier for this problem. Are any actions inadmissible?

5. Alice is considering whether to invest $1000, and if so on which type of investment and for how long (full term or half). She is looking at a risky stock (R) which will either rise (r) in value by 6% or flatten (f) to 0%. If she invests in the risky stock she must do so for the full term, before she knows how the stock's price will change. Another option is to invest in a fixed rate option (F), which gives a constant return of 2%. For the fixed-term investment she has a further option to invest initially for a long term (L) (*i.e.*, for the full term), or a short term (S) (half term). At the end of the short term, she will know the movement of the risky stock, and will be able to re-invest for the remaining period by doing the following: (a) if the risky stock has risen, she will switch to it earning an average profit of 4%; (b) if the risky stock has fallen she'll reinvest in F but will receive a lower rate, which would reduce her gains to 1% per annum.

   (a) Represent this situation as a decision tree (*i.e.*, in extensive form) and as a decision table.

   (b) How many information sets are there in this problem?

   (c) Assuming diversified stock portfolios (*i.e.*, mixtures of investments) are allowed, which mixed strategies are admissible?

   (d) Which is the *Maximin* mixed strategy?

   (e) Which is the *miniMax Regret* mixed strategy?

6. Consider the following decision table, in which mixed strategies are allowed.

	s_1	s_2
A	4	0
B	1	4
C	2	1

(a) Which, if any, strategies are dominated?

(b) Prove that a possible strategy S, with payoffs 2 and 3 in states s_1 and s_2 respectively, would not be dominated.