GSOE9210 Engineering Decisions

Problem Set 05

1. Consider the river problem described in lectures:

	p	$1-p$	
	f	$\overline{f}$	V_B
A	4	0	$4p$
B	3	1	$2p+1$

   (a) For $p = \frac{3}{4}$, what is the slope of the *Bayes* indifference line through A?
   (b) Draw the *Bayes* indifference curves for $p = \frac{1}{4}$ and $\frac{3}{4}$ through A and B.
   (c) Draw the *Bayes* indifference curve for which an agent would be indifferent between A and B, respectively. What is the slope of the line?
   (d) For which probability (*i.e.*, value of p) would an agent be indifferent between A and B under the *Bayes* decision rule?
   (e) What is the *Bayes* value associated with the indifference curve through A and B?
   (f) For which values of p would an agent prefer A to B?

2. Repeat the above exercises for regret. How do the strategies chosen by the *Bayes* decision rule when using the original values (v) compare to the strategies chosen when regrets are used instead?

3. Consider the generic two-strategy problem below:

	p	$1-p$
	s_1	s_2
A	a_1	a_2
B	b_1	b_2

   Assume neither strategy dominates the other.

   (a) Prove that an agent will be indifferent between A and B under *Bayes* when:
$$p = \frac{\Delta y}{\Delta x + \Delta y}$$
   where
$$\Delta y = |a_2 - b_2|$$
$$\Delta x = |a_1 - b_1|$$

(b) Prove that:

$$p = \frac{m}{m-1}$$

where $m = -\frac{\Delta y}{\Delta x}$ is the slope of the line joining A and B in the Cartesian plane.

4. Consider the decision table below, with $P(s_1) = p$:

	p	$1-p$
	s_1	s_2
A	5	3
B	4	1
C	2	5

   (a) For which value of p would the agent be indifferent between A and C?
   (b) Plot the *Bayes* values for the strategies as p varies from 0 to 1.
   (c) For which values of p are A, B, and C preferred, respectively, under the *Bayes* decision rule?

5. Alice sells drinks at a local market once every month. She can order stock to sell several drink types: a) hot chocolate; b) iced tea; c) lemonade; d) orange juice.

   From past experience she knows that when she sells only one type of drink, on warm days her sales total for each type are: \$10 on hot chocolate, \$40 on iced tea, \$30 on lemonade, and \$40 on orange juice. On cool days, however, her sales totals are: \$30 on hot chocolate, \$0 on iced tea, \$20 on lemonade, and \$10 on orange juice.

   She has to order her stock weeks in advance, long before she can predict the temperature on the day of the market.

   (a) Produce a decision table for this problem.
   (b) What proportion of drinks should she stock to maximise her guaranteed (*i.e.*, minimum) sales total regardless of the temperature?
   (c) Find the *Bayes* strategies for $p = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$.
   (d) What is the least favourable probability distribution on warm and cool (not warm) days?
   (e) Repeat the above analysis for the *miniMax Regret* rule.
   (f) Define the admissibility frontier for this problem.

6. Show that a strategy is admissible iff it is a *Bayes* strategy for some probability distribution.

7. Show that a *Maximin* strategy is always a *Bayes* strategy for some probability distribution.

8. Prove that for any two actions A and B, if A weakly dominates B, and all state probabilities are non-zero, then the *Bayes* decision rule will *strictly* prefer A over B.