Question 1.

(a).

The first order derivatives of f with respect to x is:

$$2a_1y^2x + a_4y + a_5$$

The second order derivatives of f with respect to x is:

$$2a_1y^2$$

The first order derivatives of f with respect to y is:

$$2a_1x^2y + a_4x$$

The second order derivatives of f with respect to y is:

$$2a_1x^2$$

(b).

The first order derivatives of f with respect to x is:

$$2a_1y^2x + 2a_2yx + a_3y^2 + a_4y + a_5$$

The second order derivatives of f with respect to x is:

$$2a_1y^2 + 2a_2y$$

The first order derivatives of f with respect to y is:

$$2a_1x^2y + a_2x^2 + 2a_3xy + a_4x + a_6$$

The second order derivatives of f with respect to y is:

$$2a_1x^2 + 2a_3x$$

(c).

$$\sigma`(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$
$$\sigma(x) = \frac{1}{(1+e^{-x})}$$
$$(1-\sigma(x)) = \frac{1+e^{-x}-1}{(1+e^{-x})} = \frac{e^{-x}}{(1+e^{-x})}$$
$$So\, \sigma`(x) = \sigma(x)(1-\sigma(x))$$

(d).

$$y_1^{\backslash} = 8x - 3$$
$$y_1^{\backslash\backslash} = 8$$
$$So\ it\ does\ not\ having\ local\ maximum\ points\ and\ the\ minimum\ points\ is \frac{3}{8}.$$

$$y_2^{\backslash} = 12x^3 - 6x^2$$
$$y_2^{\backslash\backslash} = 36x^2 - 12x$$
$$So\ it\ does\ not\ having\ local\ maximum\ points\ and\ the\ minimum\ points.$$

$$y_3^{\backslash} = 4 + \frac{1}{2\sqrt{1-x}}$$
$$y_3^{\backslash\backslash} = \frac{1}{4\sqrt{(1-x)^3}}$$
$$So\ it\ does\ not\ having\ local\ maximum\ points\ and\ the\ minimum\ points.$$

$$y_4^{\backslash} = 1 - \frac{1}{x^2}$$
$$y_3^{\backslash\backslash} = \frac{2}{x^3}$$
$$So\ it\ has\ local\ maximum\ points\ of\ -1\ and\ the\ minimum\ points\ of\ 1.$$

Question 2.

(a).

$$P(A) = \frac{20\% + 10\%}{1} = 0.3$$
$$P(B) = \frac{(1 - 20\%10\% - 40\%) + 10\%}{1} = 0.4$$
$$P(A \cup B) = \frac{10\%}{1} = 0.1$$
$$P(A) = \frac{40\%}{1} = 0.4$$

(b).

(i).

$$r = 1 - \frac{1}{6} - \frac{1}{12} - \frac{1}{12} - \frac{1}{6} - \frac{1}{6} = \frac{1}{3}$$

(ii).

$$P(X = 2, Y = 3) = \frac{1}{6}$$

(iii).

$$P(X=3) = 0 + r + 0 = 0 + \frac{1}{3} + 0 = \frac{1}{3}$$
$$P(X=3, Y=2) = r = \frac{1}{3}$$

(iv).

$$E(X) = 1 * \frac{1}{3} + 2 * \frac{1}{3} + 3 * \frac{1}{3} = 2$$
$$E(Y) = 1 * \frac{1}{3} + 2 * \frac{5}{12} + 3 * \frac{3}{12} = \frac{23}{12}$$
$$E(XY) = 1 * \frac{1}{6} + 2 * \frac{1}{12} + 3 * \frac{1}{12} + 2 * \frac{1}{6} + 4 * 0 + 6 * \frac{1}{6} + 3 * 0 + 6 * \frac{1}{3} + 9 * 0 = \frac{47}{12}$$

(v).

$$E(X^2) = 1 * \frac{1}{3} + 4 * \frac{1}{3} + 9 * \frac{1}{3} = \frac{14}{3}$$
$$E(Y^2) = 1 * \frac{1}{3} + 4 * \frac{5}{12} + 9 * \frac{3}{12} = \frac{17}{4}$$

(vi).

$$Cov(X, Y) = E(XY) - E(X) * E(Y) = \frac{47}{12} - 2 * \frac{9}{4} = -\frac{7}{12}$$

(vii).

$$Var(X) = E(X^2) - E(X)^2 = \frac{14}{3} - 2 * 2 = \frac{2}{3}$$
$$Var(Y) = E(Y^2) - E(Y)^2 = \frac{17}{4} - \frac{23}{12} * \frac{23}{12} = \frac{83}{144}$$

(viii).

$$Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{VarX} * \sqrt{Var(Y)}} = -\frac{7\sqrt{3}}{\sqrt{166}}$$

Question3.

(a).

The dimensions of A is 3 * 5, the dimention of B is 6 * 1, the dimension of A^T is 5 * 3.

(b).

(i). AB and BA is not exists.

(ii)

$$AC =$$
$$[21, 14, 14$$
$$20, 10, 10$$
$$56, 28, 28]$$
$$CA =$$
$$[31, 39, 40$$
$$10, 12, 12$$
$$18, 18, 16]$$

(iii).

$$AD =$$
$$[20, 32$$
$$17, 19$$
$$43, 45]$$

DA is not exist.

(iv).

DC is not exist.

$$CD =$$
$$[43, 41$$
$$11, 13$$
$$18, 22]$$
$$D^T C =$$
$$[38, 18, 18$$
$$32, 18, 18]$$

(v).

$$Bu =$$
$$[14$$
$$4]$$

uB is not exist.

(vi).

Au is not exist.

(vii).

$$Av =$$
$$[18$$
$$13$$
$$31]$$

vA is not exist.

(viii).

$$Av + Cv =$$
$$[47$$
$$22$$
$$45]$$

(c).

(i).

$$||u||_1 = |1| + |3| = 4$$
$$||u||_2 = \sqrt{1^2 + 3^2} = \sqrt{10}$$
$$||u||_2^2 = 1^2 + 3^2 = 10$$
$$||u||_\infty = 3$$

(ii).

$$||u||_1 = |2| + |4| + |1| = 7$$
$$||u||_2 = \sqrt{2^2 + 4^2 + 1^2} = \sqrt{21}$$
$$||u||_2^2 = 2^2 + 4^2 + 1^2 = 21$$
$$||u||_\infty = 4$$

(iii).

$$v + w =$$
$$[3$$
$$2$$
$$3]$$
$$||v+w||_1 = |3| + |2| + |3| = 8$$
$$||v+w||_2 = \sqrt{3^2 + 2^2 + 3^2} = \sqrt{22}$$
$$||v+w||_2^2 = 3^2 + 2^2 + 3^2 = 22$$
$$||v+w||_\infty = 3$$

(iv).

$$Av =$$
$$[18$$
$$13$$
$$31]$$
$$A(v-w) =$$
$$[15$$
$$13$$
$$27]$$
$$||Av||_2 = \sqrt{18^2 + 13^2 + 31^2} = \sqrt{1454}$$
$$||A(v-w||_\infty = 27$$

(d).

$$< u, v >= 3$$
$$< u, w >= 0$$
$$< v, w >= -\frac{1}{2}$$

(e).

A dot product of 0 proves that the two vectors are orthogonal, a positive dot product indicates that the angle between the two vectors is acute angle, and a negative dot product indicates that the angle between the two vectors is obtuse angle.

(f).

the inverse of A is:

$$[-\frac{1}{11}, \frac{3}{11}$$
$$\frac{4}{11}, -\frac{1}{11}]$$

(g).

A is a Singular matrix, so it has not the inverse matrix.

(h).

$$(X^T X)^T = X^T (X^T)^T = X^T X$$
$$So\ that\ X^T X\ is\ always\ symmetric.$$