1.

(a).

$\because$ S and T are the set and for the set S and T have the function f: $S \to T$, and the function $must\ satisfied\ the\ (Fun)\ and\ (Tot)$

$\therefore$ it means for the $s \in S$, there is exactly one $t \in T$ such that $(s, t) \in f$

$\because$ for the relation $R_f \subseteq S \times S$ satisfied as the function and it defined as:

$(s, s') \in R_f$ if and only if $f(s) = f(s')$

$\therefore$ suppose $s \neq s'$, that: $f(s) = f(s') = s'$ can not prove that (s', s') in the relationship

suppose $s = s'$, that: $f(s) = f(s') = s' = s$ can prove that (s, s') in the relationship, and (s, s') can prove that $f(s) = s' = s = f(s')$

$\therefore$ $s = s'$

$\because$ $s = s'$ and $(s, s') \in R_f$

$\therefore$ $(s, s') = (s, s) \in R_f$

$\therefore$ the relation R_f satistied that for all $x \in S$: $(x, x) \in R_f$

$\therefore$ $the\ relation\ R_f\ is\ reflexive$

$\because$ $s = s'$ and $(s, s') \in R_f$

$\therefore$ $(s, s') = (s, s) = (s', s) \in R_f$

$\therefore$ the relation R_f satistied that for all $x, y \in S$: If $(x, y) \in R_f$, then $(y, x) \in R_f$

$\therefore$ the relation R_f is symmetric

suppose that there is a $s'' \in S$ and $(s', s'') \in R_f$ if and only if $f(s') = f(s'')$

$\therefore$ for the prove above can know that $s' = s''$

$\because$ $s = s'$ and $(s, s') \in R_f$, $s' = s''$ and $(s', s'') \in R_f$

$\therefore$ $s = s' = s''$ and that can know $(s, s'') \in R_f$

$\therefore$ the relation R_f satistied that for all $x, y, z \in S$: If (x, y) and $(y, z) \in R_f$, then $(x, z) \in R_f$

$\therefore$ the relation R_f is transitive

$\because$ the relation R_f is reflexive, symmetric, transitive

$\therefore$ the relation R_f is an equivalence relation.

(b).

$\because$ relation $R \subseteq S \times S$ is an equivalence relation

$\therefore$ R is reflexive that there is (s, s) in the relation

$\therefore$ $s = s'$ satisfied the relextive in R if it want to satistied the function $f_R: S \to T$ such that $(s, s') \in R_f$ if and only if $f(s) = f(s')$

$\therefore$ the function $f_R: S \to T$ satisfied the (Fun) and (Tot) that it can have only one $t \in T$

$\therefore$ there exists a set T and a function $f_R: S \to T$ when T contain all element in S and these element in S onlyonly satisfied reflexive in the relation.

2.

(a)(i).

$\because$ $a, b \in N$

$\therefore$ when $a = 0, b > 0$: $f(a) = 0, f(b) = 1, \max\{f(a), f(b)\} = 1, \quad$ and $f(a + b) = f(b) = 1$

when $a = 0, b = 0$: $f(a) = 0, f(b) = 0, \max\{f(a), f(b)\} = 0, \quad$ and $f(a + b) = f(0) = 0$

when $a > 0, b > 0$: $f(a) = 1, f(b) = 1, \max\{f(a), f(b)\} = 1,$

suppose $x = a + b > 0, \quad$ and $f(a + b) = f(x) = 1$

when $a > 0, b = 0$: $f(a) = 1, f(b) = 0, \max\{f(a), f(b)\} = 1, \quad$ and $f(a + b) = f(a) = 1$

$\therefore$ for all $a, b \in N$: $f(a + b) = \max\{f(a), f(b)\}$

(a)(ii).

$\because$ $a, b \in N$

$\therefore$ when $a = 0, b > 0$: $f(a) = 0, f(b) = 1, \mathrm{m}in\{f(a), f(b)\} = 0, \quad$ and $f(ab) = f(0) = 0$

when $a = 0, b = 0$: $f(a) = 0, f(b) = 0, \min\{f(a), f(b)\} = 0, \quad$ and $f(ab) = f(0) = 0$

when $a > 0, b > 0$: $f(a) = 1, f(b) = 1, \min\{f(a), f(b)\} = 1,$

suppose $x = a + b > 0, \quad$ and $f(ab) = f(x) = 1$

when $a > 0, b = 0$: $f(a) = 1, f(b) = 0, \min\{f(a), f(b)\} = 0, \quad$ and $f(ab) = f(0) = 0$

$\therefore$ for all $a, b \in N$: $f(ab) = \min\{f(a), f(b)\}$

(b)(i).

According to the prove in problem 1, if the $R_f \subseteq N \times N$, the relation $(m, n) \in R_f$

is an equivalence class, and it contain (m, m) in the relation that $m \in N$

$\therefore$ the relation contain equivalence class $[m] = \{m\}$ for $m \in N$ and m in the relation

that all subset only have 1 element

$\because$ $E \subseteq Pow(N)$ be the set of equivalence class of R_f

$\therefore$ $E = \{\{m_1\}, \{m_2\}, \{m_3\}, \ldots, \{m_n\}\}$ for $m_1, m_2, m_3, \ldots, m_n$ are the element in the relation R_f

$\because$ $[n] \in E$ denote the equivalence class of n

$\therefore$ $[n] = n$

$\because$ the relation $\boxplus, \boxdot \in E^2 \times E$

$\because$ $x \in X$ and $y \in Y$, and x, y both only have one element

$\therefore$ $[x + y] = \{x\} + \{y\} = \{x + y\} \in Z$ and $Z \in E$

$\therefore$ z only exist one element $\{x + y\} \in Z$ such that $((x, y), z) \in R_f$

$\therefore \boxplus$ is a function

(b)(ii).

According to the prove in problem 1, if the $R_f \subseteq N \times N$, the relation $(m, n) \in R_f$

is an equivalence class, and it contain (m, m) in the relation that $m \in N$

$\therefore$ the relation contain equivalence class $[m] = \{m\}$ for $m \in N$ and m in the relation

that all subset only have 1 element

$\because$ $E \subseteq Pow(N)$ be the set of equivalence class of R_f

$\therefore$ $E = \{\{m_1\}, \{m_2\}, \{m_3\}, \ldots, \{m_n\}\}$ for $m_1, m_2, m_3, \ldots, m_n$ are the element in the relation R_f

$\because$ $[n] \in E$ denote the equivalence class of n

$\therefore$ $[n] = n$

$\because$ the relation $\boxplus, \boxdot \in E^2 \times E$
$\because$ $x \in X$ and $y \in Y$, and x, y both only have one element
$\therefore$ $[xy] = \{x\} \times \{y\} = \{xy\} \in Z$ and $Z \in E$
$\therefore$ z only exist one element $\{xy\} \in Z$ such that $((x, y), z) \in R_f$
$\therefore$ $\boxdot$ is a function

(c).
$\because$ $A, B, C \in E$
$\because$ the problem 2(b) have prove that $E = \{\{m_1\}, \{m_2\}, \{m_3\}, \dots, \{m_n\}\}$ for $m_1, m_2, m_3, \dots, m_n$ are the element in the relation $R_{f,}$ the relation contain equivalence class $[m] = \{m\}$ for $m \in N$ and m in the relation
$\because$ the binary operations on E that $\boxplus, \boxdot: E \times E \to E$
$\therefore$ for the question:

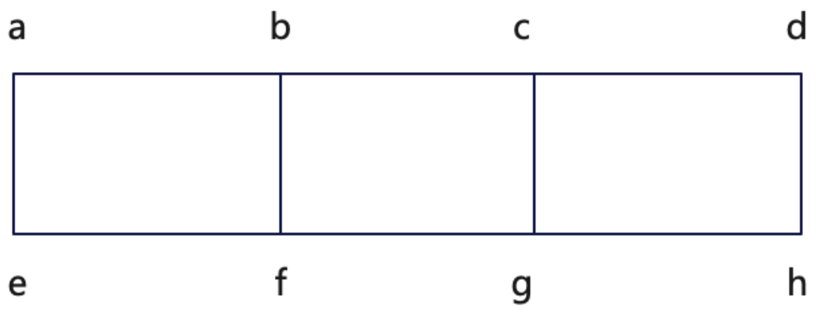
(i).
for every element $m = \{m_i: i = 1,2,3, \dots, n\}$ in the A has：
$A \boxdot [1] = [m_i] \boxdot [1] = [m_i \times 1] = [m_i] = A$ for all element m_i in the A

(ii):
for every element $a = \{a_i: i = 1,2,3, \dots, n\}$ in the A and every element $b = \{b_i: i = 1,2,3, \dots, n\}$ in the B has：
$A \boxplus B = [a_i] \boxplus [b_i] = [a_i + b_i] = [b_i + a_i] = [b_i] \boxplus [a_i] = B \boxplus A$
for all element a_i in the A and all element b_i in the B

(iii).
for every element $a = \{a_i: i = 1,2,3, \dots, n\}$ in the A and every element $b = \{b_i: i = 1,2,3, \dots, n\}$ in the B and every element $c = \{c_i: i = 1,2,3, \dots, n\}$ in the C has：
$A \boxdot (B \boxplus C) = [a_i] \boxdot ([b_i] \boxplus [c_i]) = [a_i] \boxdot ([b_i + c_i]) = [a_i] \boxdot (b_i + c_i)$
$= ([a_i] \boxdot [b_i]) + ([a_i] \boxdot [c_i]) = (A \boxdot B) \boxplus (A \boxdot C)$
for all element a_i in the A and all element b_i in the B and all element c_i in the C

3.

(a)(i).



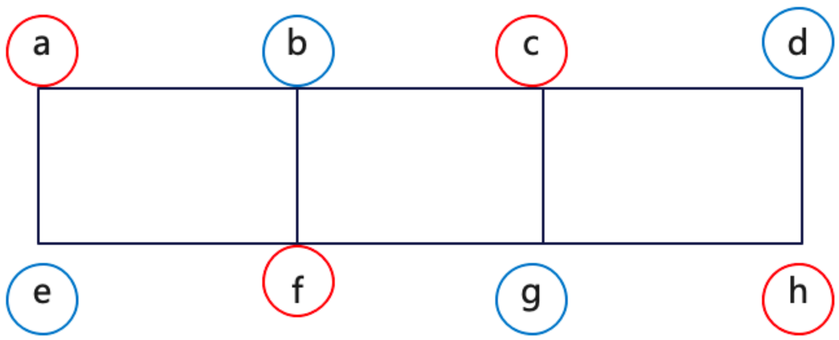
The vertices can represent each house, and the edges represent the networks interfere.

(a)(ii).

The problem we need to solve is to find the chromatic number of the graph.

(b).

the minimum number of wifi channels required for the neighbourhood is 2. I use red and blue circle to represent the two different wifi channels.

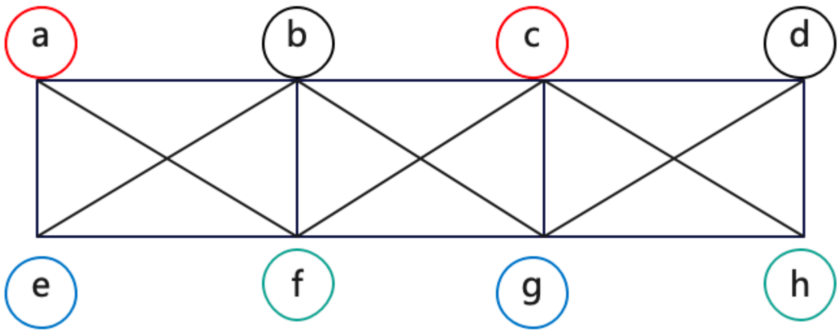


(c).

The vertices can represent each house, and the edges represent the networks interfere.

The problem we need to solve is to find the chromatic number of the graph.

the minimum number of wifi channels required for the neighbourhood is 4. *I use red , black, green and* blue circle to represent the two different wifi channels.



4.

(a)

Petersen graph does not contain a subdivision of K5.

$\because$ *according to the to the definitionof the Complete graph* k_n: k_5 *is a graph has* 5 *vertices* which are all pairwise connected.

For the problem 4, *the Petersen Graph has the maximum degree of* 3 *for all vertices. But in order* to get a subdivision of k_5. We need to have minimum 5 certices that all with at least degree of 4.

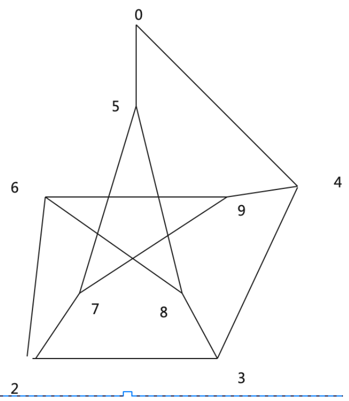
$\therefore$ There is no psooibility for Petersen graph to contain a subdivision of k_5.

(b)

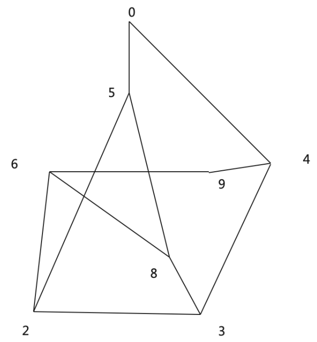
Petersen graph contains a subdivision of $K_{3,3}$.

Start at Petersen Graph:

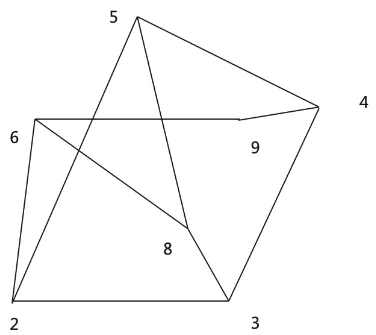
Delete an edge(0,1), then replace a vertex '1' of degree 2 with an edge connecting is neighbours (2,1) and (1,6), at last get the new edge (2,6) like to graph below:



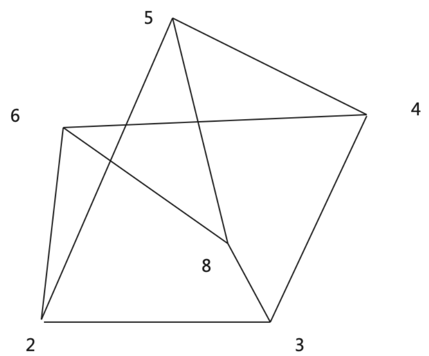
Delete an edge(7,9), then replace a vertex '7' of degree 2 with an edge connecting is neighbours (2,7) and (7,5), at last get the new edge (2,5) like to graph below:



then replace a vertex '0' of degree 2 with an edge connecting is neighbours (5,0) and (0,4), at last get the new edge (5,4) like to graph below:



then replace a vertex '9' of degree 2 with an edge connecting is neighbours (6,9) and (9,4), at last get the new edge (6,4) like to graph below:



Now, we can get the subdivision of $k_{3,3}$ from Petersen Graph with two set, set1 contains the vertices of (3,5,6) and set2 with the vertices of (2,4,8)

5.

(a).

when $j = i$: $R^j = R^i$

when $j = i + 1$: $R^j = R^{i+1} = R^i \cup (R; R^i)$

when $j = i$: $R^j = R^i$

$\therefore\ R^i \subseteq R^j$

when $j > i$:

like $j = i + 1$: $R^j = R^i \cup (R; R^i)$

like $j = i + 2$: $R^j = (R^i \cup (R; R^i)) \cup (R; R^i \cup (R; R^i)))$

...

like $j = i + n$ when $n \in N$: $R^j = R^i \cup \ldots \cup (R; R^i)$

$\therefore\ R^i \subseteq R^j$ when $i \leq j$

$\therefore\ when\ j \geq i$: $R^i \subseteq R^j \subseteq R^{j+1} \subseteq R^{j+2} \subseteq \cdots \subseteq R^{j+n}$ $with\ the\ n \in N$

$\therefore$ exist $P_i(j)$ be the proposition that $R^i \subseteq R^j$ that $P_i(j)$ holds for all $j \geq i$

(b).

when $n = 0$:

$R^0; R^m = I; R^m = R^m$

$P(0) = R^0; R^m = R^{0+m} = R^m$

$\therefore\ P(n)$ holds for $n = 0$

when $n > 0$:

$P(n)$ supposed that $R^n; R^m = R^{n+m}$

$P(n + 1) = R^{n+1}; R^m$

$= ((R^n \cup (R; R^n)); R^m$

According the the proof in the assignment1:

$P(n + 1) = ((R^n \cup (R; R^n)); R^m$

$= (R^n; R^m) \cup ((R; R^n); R^m)$

$= (R^n; R^m) \cup (R; (R^n; R^m))$

$= R^{n+m} \cup (R; R^{n+m})$

$= R^{n+m+1}$

$\because\ P(n)$ can prove to $P(n + 1)$

$\therefore\ P(n)$ holds for all $n > 0$

$\therefore\ P(n)$ holds for all $n \in N$

(c).

when $j = 1$: $R^j = R^i$

when $j > i$:

suppose $j = i + n$ with $n \in N$

if the suppose $R^i = R^{i+n}$ exist, it can devide that $R^{i+1} = R^{i+n+1}$ to provve $R^i = R^j$ for all $j > i$

and exist $i \in N$ such that $R^i = R^{i+1}$

$R^{i+n+1} = R^{i+n} \cup (R; R^{i+n})$

$\because$ we suppose $R^i = R^{i+n}$

$\therefore R^{i+n+1} = R^{i+n} \cup (R; R^{i+n})$

$= R^{i} \cup (R; R^{i})$

$= R^{i+1}$

$\therefore$ the suppose exist.

$\therefore \therefore R^{j} = \therefore R^{i}$ for all $j \geq i$ when exists $i \in N$ such that $R^{i} = R^{i+1}$

(d).

(e).

(f).

6.

(a).

the tree T_{left} can be the form of $(T_{left_{left}}, T_{left_{right}})$

(B). $\text{count}(\tau) = 0$

(R). $\text{count\_of\_node}(T_{left}, T_{right}) = 1 + \text{count\_of\_node}(T_{left}) + \text{count\_of\_node}(T_{right})$

(b).

the tree T_1 can be the form of $(T_{left_{left}}, T_{left_{right}})$

(B). $\text{count\_of\_leaf\_for node}(\tau) = 0$

$\text{count\_of\_leaf\_for node}(\tau, \tau) = 1$

(P). $\text{count\_of\_leaf\_ for node}(T_{left}, T_{right}) = \text{count\_of\_leaf\_ for node}(T_{left}) +$

$\text{count\_of\_leaf\_ for node}(T_{right})$

(c).

the tree T_{left} can be the form of $(T_{left_{left}}, T_{left_{right}})$

(B). $\text{count\_of\_fully internal node}(\tau) = -1$

$\text{count\_of\_fully internal node}(\tau, \tau) = 0$

(R). $\text{count\_of\_fully internal node}(T_{left}, T_{right})$

$= 1 + \text{count\_of\_fully internal node}(T_{left}) + \text{count\_of\_fully internal node}(T_{right})$

(d).

based on (b) and (c):

the tree T_1 can be the form of $(T_{left_{left}}, T_{left_{right}})$

(B). $\text{count\_of\_leaf\_for node}(\tau) = 0$

$\text{count\_of\_leaf\_for node}(\tau, \tau) = 1$

$\text{count\_of\_fully internal node}(\tau) = -1$

$\text{count\_of\_fully internal node}(\tau, \tau) = 0$

(P). $\text{count\_of\_leaf\_ for node}(T_{left}, T_{right}) = \text{count\_of\_leaf\_ for node}(T_{left}) +$

$\text{count\_of\_leaf\_ for node}(T_{right})$

$\text{count\_of\_fully internal node}(T_{left}, T_{right})$

$= 1 +$ count_of_fully internal node(T_{left}) + count_of_fully internal node(T_{right})

according to (B): count_of_leaf_for node $= 1 +$ count_of_fully internal node

$\because P(T)$ *be the proposition that leaves*$(T) = internal(T) + 1$
$\therefore$ count_of_leaf_ for node(T_{left}) $= 1 +$ count_of_fully internal node(T_{left})
count_of_leaf_ for node(T_{right}) $= 1 +$ count_of_fully internal node(T_{right})
$\therefore$ count_of_leaf_ for node(T_{left}, T_{right}) = count_of_leaf_ for node(T_{left}) +
count_of_leaf_ for node(T_{right})
$= 1 +$ count_of_fully internal node(T_{left}) +
$1 +$ count_of_fully internal node(T_{right})
$= 1 + (1 +$ count_of_fully internal node(T_{left})
+count_of_fully internal node(T_{right}))
$= 1 +$ count_of_leaf_ for node(T_{left}, T_{right})

$\therefore$ count_of_leaf_for node $= 1 +$ count_of_fully internal node can also be proved in (P)
$\therefore$ P(T) holds for all binary trees T.

7.

Let lexicographic ordering $\leq$ lex $\subseteq \Sigma_1^* \times \Sigma_2^*$
$for\ all(x1, x2) \in \Sigma_1^* \times \Sigma_1^*$ and x1 $\in \Sigma_1$
(B): lexicographic order $= \lambda = x_1 \leq$ lex $\subseteq \Sigma_1^* \times \Sigma_2^*$
(R): lexicographic order $= x_1 \leq$ lex $\subseteq \Sigma_1^* \times \Sigma_2^*$