Graph Theory

Problem 1
True or false?

(a) The complete bipartite graph $K_{5,5}$ has no cycle of length five.

(b) If T is a tree with at least four edges, then $\chi(T) = 3$.

(c) Let C_n denote a cycle on n vertices. For all $n \geq 5$ it holds $\chi(C_n) \neq \chi(C_{n-1})$.

(d) It is possible to remove two edges from K_6 so that the resulting graph has a clique number of 4.

Problem 2
What is the minimum number of edges that need to be removed from K_5 so that the resulting graph has a chromatic number of

(a) 3?

(b) 2?

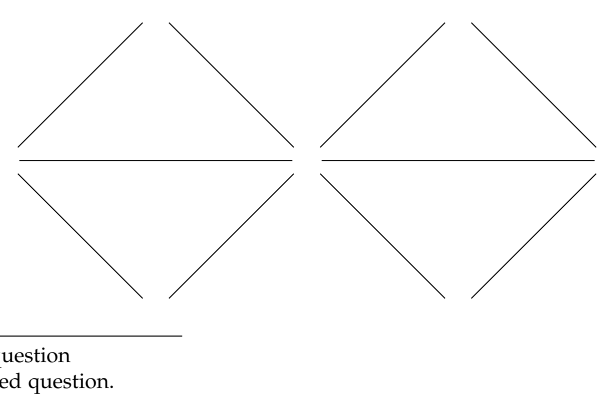
(c) 1?

Problem 3
Consider the complete 3-partite graphs $K_{4,1,1}$, $K_{3,2,1}$, $K_{2,2,2}$.

(a) What is the chromatic number of each of these graph?

(b) Which of these graphs are planar?

Problem 4
Consider the following graph, G:



† indicates a previous exam question
* indicates a difficult/advanced question.

(a) What is the chromatic number of G?

(b) What is the clique number of G?

(c) Does G have a Hamiltonian path and/or a Hamiltonian cycle?

(d) Does G have an Eulerian path and/or an Eulerian cycle?

Problem 5

Draw a single graph with 6 vertices and 10 edges that satisfies each of the following:

(a) is planar,

(b) contains a Hamiltonian circuit, and

(c) does not contain an Eulerian path.

Problem 6† (20T2)

(a) Prove that a graph with 6 vertices and at least 13 edges must have at least two vertices of degree 5.

(b) Prove that a graph with 6 vertices and at least 13 edges has clique number at least 4.

(c) Prove that a graph with 6 vertices and at least 13 edges is non-planar.

(d) Draw a planar graph with 6 vertices and 12 edges that has clique number 3. Justify each property.