C. Knoll, X. Jia, R. Heedt

Trajectory Planning for Closed Kinematic Chains Applied to Cooperative Motions in Health Care

Kassel (virtual), 2021-03-17

Introduction

- Important situation in health care: standing up from sitting position

Source: https://www.robotik-produktion.de/allgemein/roboter-in-der-pflege/

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$\rightarrow$ Different stress levels for care giver and receiver

- Optimal motion? $\rightarrow$ mathematical model necessary

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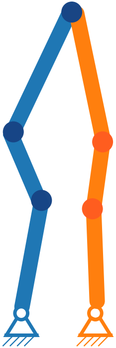
$\rightarrow$ Motion planning for **closed kinematic chains** necessary

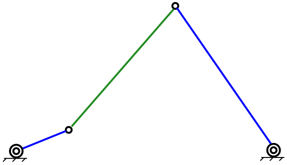


Outline

- ☑ Motivation
- → **Modelling of Closed Kinematic Chains**
- ☐ Motion Planning for Kinematic Chains
- ☐ Simplification via Quasi-Stationary Trajectory Planing
- ☐ Conclusion and Outlook

Simple Case: Four Bar Linkage

- 4 revolute joints
- 4 links (including basis)
- Closed kinematic chain
- One degree of freedom



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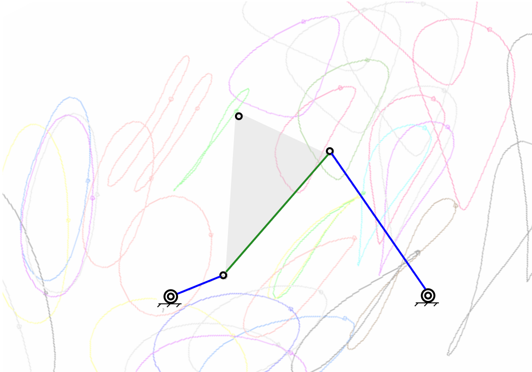

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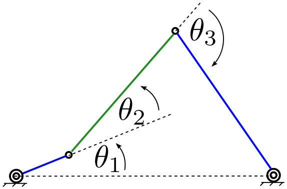
Simple Case: Four Bar Linkage

- 4 revolute joints
- 4 links (including basis)
- Closed kinematic chain
- One degree of freedom (e. g. θ_1)
- Quite “complex” motion of the 2nd link
- $\exists$ functional dependencies “$\theta_2(\theta_1)$” and “$\theta_3(\theta_1)$” in closed form



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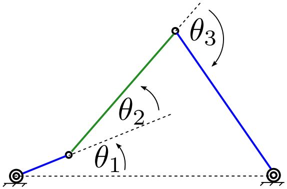
Four Bar Linkage – Modeling Options

Option 1: Use functional dependencies $\theta_2(\theta_1)$, $\theta_3(\theta_1)$

- One independent variable: θ_1

$\Rightarrow$ Lagrange Eq. of 2nd kind applicable $\rightarrow$ ODE System ($n = 2$)



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- Three independent variable: $\theta_1, \ldots, \theta_3$

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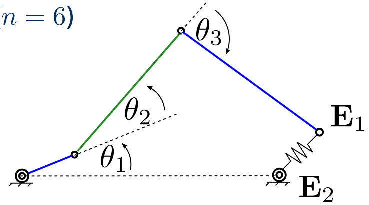

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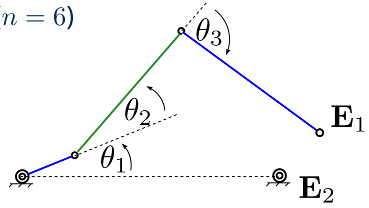
- Three independent variables: $\theta_1, \ldots, \theta_3$

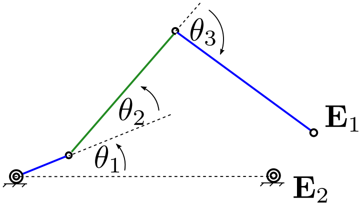
$\Rightarrow$ Lagrange Eq. of 1st kind necessary

$\Rightarrow$ DAE System, $n_\mathbf{x} = 6$, $n_\lambda = 2$, Index: 3



Lagrange Equations of 1st Kind

- Algebraic condition: $\mathbf{E}_1(\boldsymbol{\theta}) - \mathbf{E}_2(\boldsymbol{\theta}) =: \mathbf{a}(\boldsymbol{\theta}) \overset{!}{=} \mathbf{0}$



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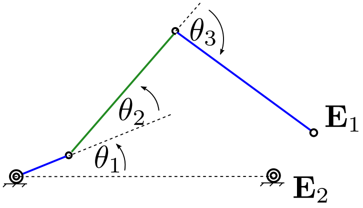

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- Equations of motion:

$$\underbrace{\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{K}(\boldsymbol{\theta})}_{=:\mathbf{L}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}})} = \boldsymbol{\tau} - \mathbf{A}^T(\boldsymbol{\theta})\boldsymbol{\lambda} = \mathbf{H}(\boldsymbol{\theta}) \underbrace{\begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\lambda} \end{pmatrix}}_{\boldsymbol{\mu}},$$

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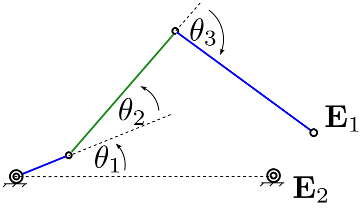

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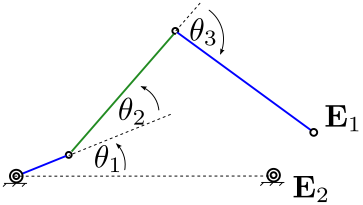

- State space form of ODE-part: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\mu})$, with $\mathbf{x} = \begin{pmatrix}\boldsymbol{\theta} \\ \dot{\boldsymbol{\theta}}\end{pmatrix}$
- Index 3 $\Rightarrow$ Finding initial values and simulation nontrivial

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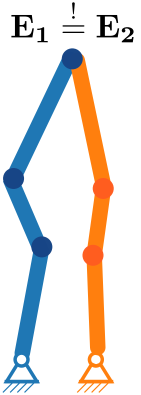
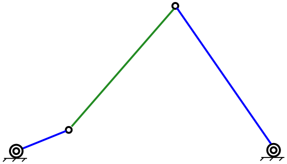


Side note: Degree of Freedom

General formula:

$$F = (N_{\text{links}} - 1) \cdot N_{\text{m. p.}} - \sum_{i=1}^{N_{\text{joints}}} (N_{\text{m. p.}} - f_i) \quad \text{with } N_{\text{m. p.}}\text{: motion possibilities}$$



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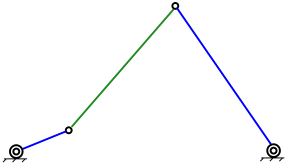
Planar linkage with only revolute joints:

$$F = (N_{\text{links}} - 1) \cdot 3 - \sum_{i=1}^{N_{\text{joints}}} (3 - 1) = (N_{\text{links}} - 1) \cdot 3 - N_{\text{joints}} \cdot 2$$



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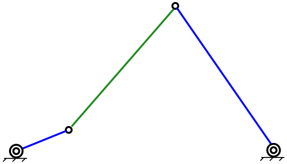

Single-loop closed chain: $N_{\text{links}} = N_{\text{joints}}$

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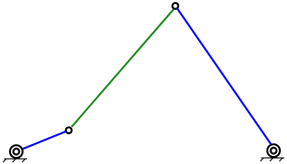
$$F = 4 - 3 = 1$$



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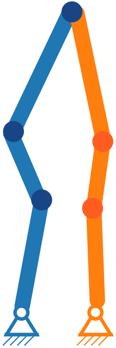

$$F = 7 - 3 = 4$$

Outline

☑ Motivation

☑ Modelling of Closed Kinematic Chains

→ **Motion Planning for Kinematic Chains**

☐ Simplification via Quasi-Stationary Trajectory Planing

☐ Conclusion and Outlook

Motion Planning via Optimization (1)

- Original DAE model: $$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \boldsymbol{\mu}) \quad \text{with } \mathbf{x} = \begin{pmatrix}\boldsymbol{\theta}\\ \dot{\boldsymbol{\theta}}\end{pmatrix} \text{ and } \boldsymbol{\mu} = \begin{pmatrix}\boldsymbol{\tau}\\ \boldsymbol{\lambda}\end{pmatrix}\\ \bar{\mathbf{a}}(\mathbf{x}) &= 0\end{aligned}$$

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  $\bar{\mathbf{a}}(\mathbf{x}) = 0$
- Euler discretization for N steps $k = 0, \ldots, N-1$:
  $\mathbf{x}_{k+1} = \Delta t \cdot \mathbf{f}(\mathbf{x}_k, \boldsymbol{\mu}_k)$ with $\mathbf{x}_k := \mathbf{x}(k\Delta t), \quad \boldsymbol{\mu}_k := \boldsymbol{\mu}(k\Delta t)$

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- Cost function: $J(\mathbf{Z}) = \sum_{k=0}^{N-1} \mathbf{x}_k^T \mathbf{Q} \mathbf{x}_k + \boldsymbol{\mu}_k^T \mathbf{R} \boldsymbol{\mu}_k \rightarrow \min$

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- Constraints:
  - Discretized equations of $\mathbf{c}_{1,k}(\mathbf{Z}) := \Delta t \cdot \mathbf{f}(\mathbf{x}_k, \boldsymbol{\mu}_k) - \mathbf{x}_{k+1} \overset{!}{=} 0$
  - Chain closing condition: $\mathbf{c}_{2,k}(\mathbf{Z}) := \bar{\mathbf{a}}(\mathbf{x}_k) \overset{!}{=} 0$

Motion Planning via Optimization (2)

How big ist the problem (e. g. for $N = 100$)?

$$\dim \mathbf{c}_{1,k} = \dim \mathbf{x} = 2 \cdot \dim \boldsymbol{\theta} = 12,$$
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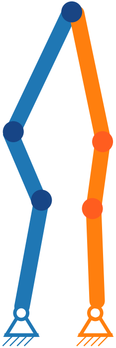

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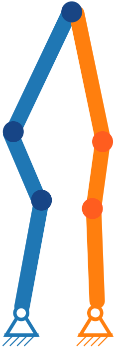
Number of constraints: $N_\mathsf{c} = N \cdot \dim \mathbf{c}_{1,k} + N \cdot \dim \mathbf{c}_{2,k} = 1400$



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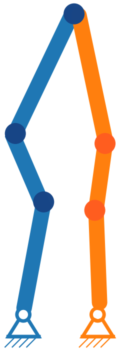
Number of variables: $\dim \mathbf{Z} = (N+1) \cdot \dim \mathbf{x} + N \cdot \underbrace{\dim \boldsymbol{\mu}}_{6+2} = 2012$



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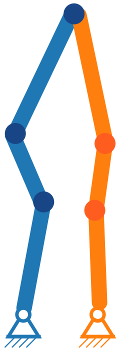

$\rightarrow$ “Medium sized” nonlinear optimal control problem

Calculation time: $t_\mathsf{calc} \approx 1\,\mathrm{min} \ldots 1\,\mathrm{h}$

(Setup: CasADi, IPOPT, PC@3.2GHz)

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→ **Simplification via Quasi-Stationary Trajectory Planing**

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Reconsidering the Problem

Equations of motion :

$$\begin{aligned} \mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{K}(\boldsymbol{\theta}) &= \boldsymbol{\tau} - \mathbf{A}^T(\boldsymbol{\theta})\boldsymbol{\lambda} = \mathbf{H}(\boldsymbol{\theta}) \underbrace{\begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\lambda} \end{pmatrix}}_{\boldsymbol{\mu}}, \\ \mathbf{a}(\boldsymbol{\theta}) &= \mathbf{0} \end{aligned} \qquad (*)$$

Reconsidering the Problem

Equations of motion (assuming slow motion):

$$\underbrace{\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})}_{\approx \mathbf{0}} + \mathbf{K}(\boldsymbol{\theta}) = \boldsymbol{\tau} - \mathbf{A}^T(\boldsymbol{\theta})\boldsymbol{\lambda} = \mathbf{H}(\boldsymbol{\theta}) \underbrace{\begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\lambda} \end{pmatrix}}_{\boldsymbol{\mu}},$$

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New (smaller) problem:

Find equilibrium configuration $\boldsymbol{\theta}$ and load $\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\lambda} \end{pmatrix}$ such that $(*)$ is satisfied.

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Find equilibrium configuration $\boldsymbol{\theta}$ and load $\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\lambda} \end{pmatrix}$ such that $(*)$ is satisfied.

Idea: patch neighboring equilibria together $\rightarrow$ definition of geometric path
$\rightarrow$ choose time scaling $t \mapsto \boldsymbol{\theta}(t)$ (additional design freedom)

Equilibrium Finding

Original cost function

$$J(\mathbf{Z}) = \sum_{k=0}^{N-1} \mathbf{x}_k^T \mathbf{Q} \mathbf{x}_k + \boldsymbol{\mu}_k^T \mathbf{R} \boldsymbol{\mu}_k$$

constraints

$\mathbf{c}_{1,k}, \mathbf{c}_{2,k}$ for $k = 1, \ldots, N$

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for k fixed

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with $\bar{\mathbf{Z}}_k := (\mathbf{x}_k, \boldsymbol{\mu}_k)$, $\dim \bar{\mathbf{Z}}_k = 14$

and parameters $\bar{\mathbf{P}}_k := (y_{\mathrm{E},k}, \bar{\mathbf{Z}}_{k-1}, \bar{\mathbf{Z}}_{k-2})$

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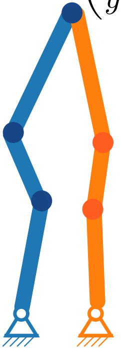
for k fixed

$\mathbf{E} = \begin{pmatrix} x_{\mathrm{E}} \\ y_{\mathrm{E}} \end{pmatrix}$



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$\rightarrow$ For each k increase y_{E} (“standing up”) via $\bar{\mathbf{c}}_{3,k}$ and penalize changes in $\bar{\mathbf{Z}}$

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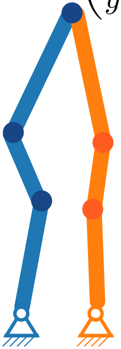
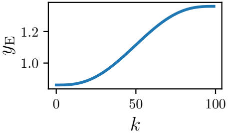


Trajectory Generation

For each $k = 0, \dots, N$:

1. Choose y_E from



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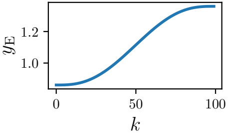

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③ Calculate $\bar{\mathbf{P}}_{k+1}$ and $\bar{\mathbf{c}}_{i,k+1}$ according to results

④ $k := k + 1 \to$ ①

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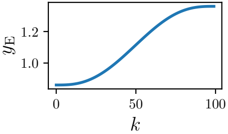
1. Choose y_E from 

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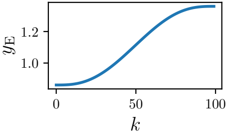

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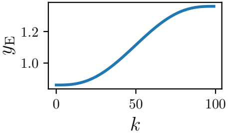

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7. Calculate actual *dynamic* torques

$$\boldsymbol{\mu}^\star(t) = \arg\min_{\boldsymbol{\mu}} \boldsymbol{\mu}^T \mathbf{R} \boldsymbol{\mu}$$

$$\text{s.t.}$$

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{K}(\boldsymbol{\theta}) = \mathbf{H}(\boldsymbol{\theta})\boldsymbol{\mu}$$

for each desired time step t

Results

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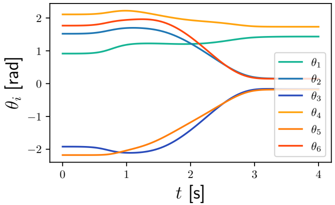
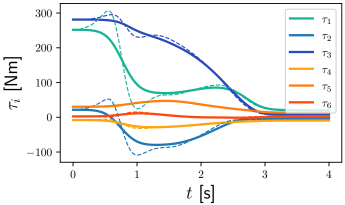
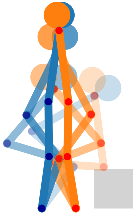
Numerical Results:







See also: `https://github.com/TUD-RST/MPCKC`

Outline

- ☑ Motivation
- ☑ Modelling of Closed Kinematic Chains
- ☑ Motion Planning for Kinematic Chains
- ☑ Simplification via Quasi-Stationary Trajectory Planing
- → **Conclusion and Outlook**

Conclusion and Outlook

Summary:

- Modelling of closed kinematic chains via Lagrange Eq. of 1st kind
- Formulation as optimal control problem
- Simplification via sequential solution of quasi-stationary auxiliary problems
  $\rightarrow \approx 1000\times$ speedup
- Python based software frame work: `https://github.com/TUD-RST/MPCKC`

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Outlook:

- Parameter studies (lengths, masses, $\mathbf{R}_{ii}$)
- More realistic humans (e. g. with arms)
- Consideration of tilting (ZMP)