Lecture 11: Monotone Frameworks

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CS4200 Compiler Construction
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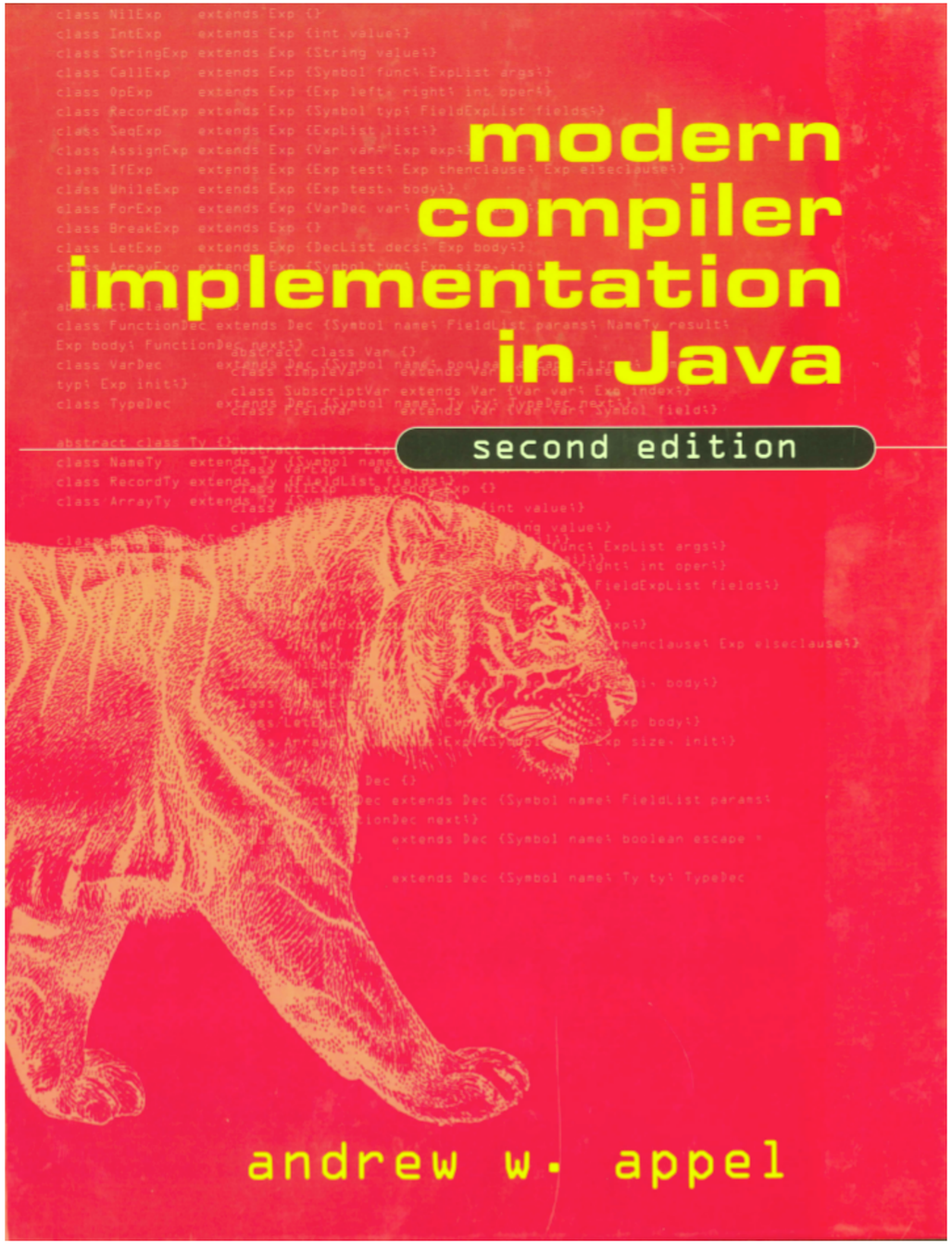
Background

Chapter 10: Liveness Analysis

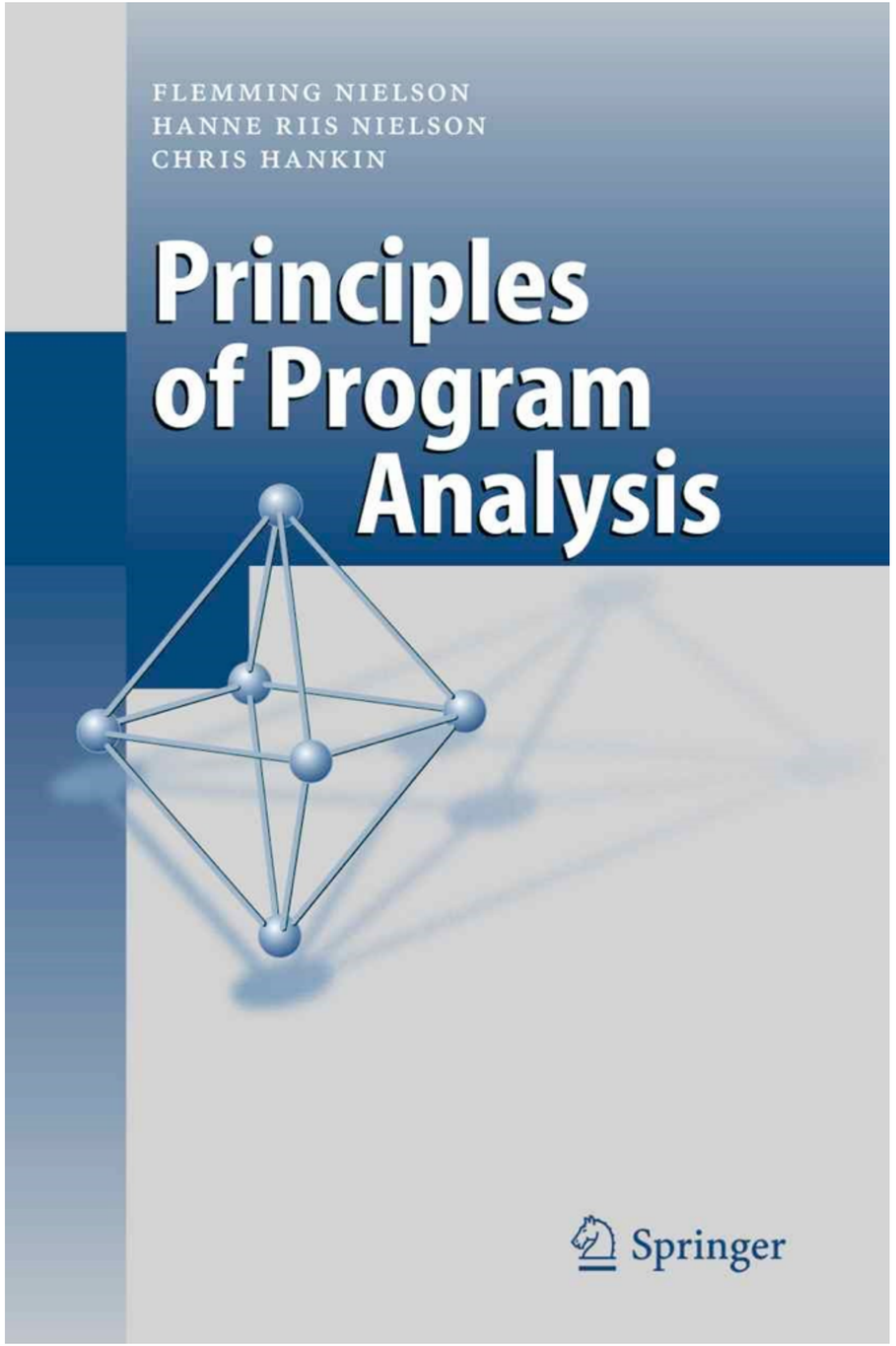
Chapter 17: Dataflow Analysis



The applied/“cookbook” version

Chapter 1: Introduction

Chapter 2: Data Flow Analysis



The theoretical/“general” version

Data-Flow Analysis

Previous Lecture

Control-Flow

- Order of execution
- Reasoning about what is reachable

Data-Flow

- Flow of data through a program
- Reasoning about data, and dependencies between data

FlowSpec

- Control-Flow rules to construct the graph
- Annotate with information from analysis by Data-Flow rules

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed
- What extra invariants do some data adhere to
- Data dependence between data/variables where the data lives

Available Expressions

```
let
  var x : int := a + b
  var y : int := a * b
 in
  while y > a + b then
     (
       a := a + 1;
       x := a + b
     )
end
```

- `a + b` is *already computed* when you get to the condition
- There is no need to compute it again

Live variables

```
x := 2;
y := 4;
x := 1;
if y > x then
  z := y
else
  z := y * y;
x := z
```

- The first value of x is never observed, *because it isn't read after the assignment*

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed
- What extra invariants do some data adhere to
- Data dependence between data/variables where the data lives

Flow-sensitive types

```
void hello(String? name) {
    if (is String name) {
        // name is of type String here
        print("Hello, ``name``!");
    }
    else {
        print("Hello, world!");
    }
}
```

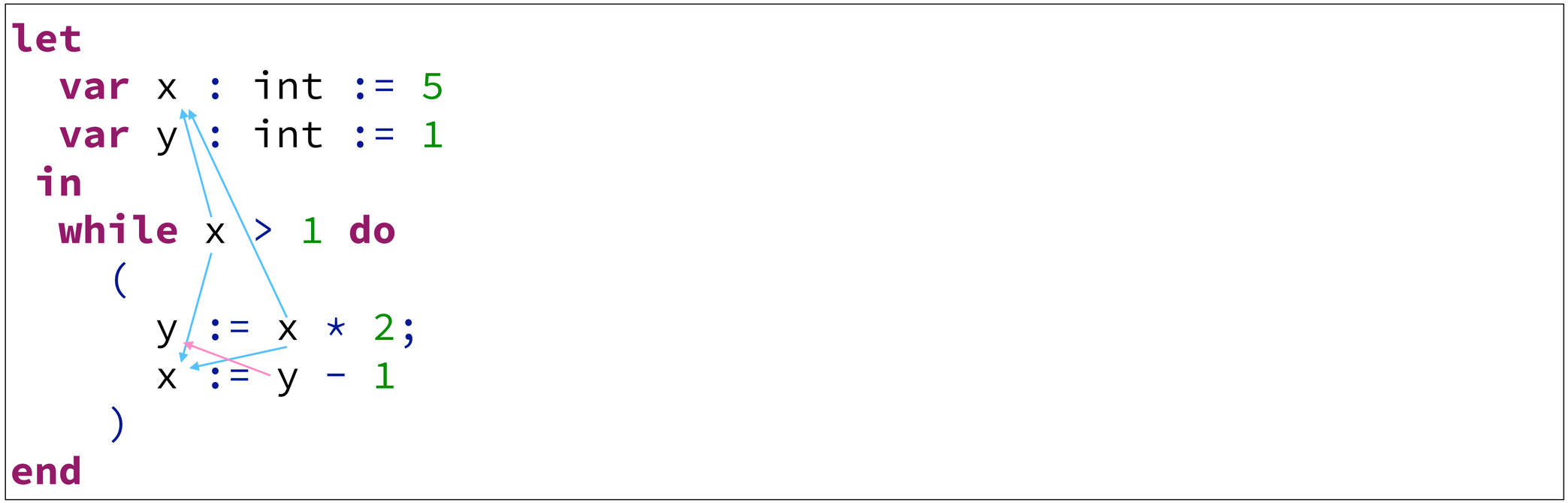
- Ceylon (https://ceylon-lang.org/)
- Union and intersection types
- `String?` ≡ `String | Null`
- **`is`** like Java's **`instanceof`**
- General name: path-sensitive data-flow analysis

What is Data-Flow Analysis?

Static approximation of runtime behaviour

- What has or will be computed
- What extra invariants do some data adhere to
- Data dependence between data/variables where the data lives

Reaching definitions



- The inverse relation of live variables
- RD gives us the possible origins of the current value of a variable

Reaching definitions

```
1  let
2    var x : int := 5          ──────────────────────  x↦2
3    var y : int := 1          ──────────────────────  x↦2;y↦3
4   in
5    while x > 1 do            ──────────────────────  x↦2,8;y↦3,7
6      (
7        y := x * 2;           ──────────────────────  x↦2,8;y↦7
8        x := y - 1            ──────────────────────  x↦8;y↦7
9      )
10 end
```

- Analysis result is a map (shown here after each statement)
- Propagate information along the control-flow graph

Traditional Kill/Gen Sets

Traditional set based analysis

Traditional set based analysis has

- Sets as the type of information that's calculated
- A transfer function of:
  - previousSet ∖ kill(currentNode) ∪ gen(currentNode)
- Usually the initial set is empty

Depends on the direction of the analysis

Available Expressions

"An **expression** is **available** if it *must* have already been computed, and not later modified, on all paths to the program point"

```
kill(Assign(var, e1)) :=
  { e2 ∈ AllAE | var ∈ FV(e2) }

gen(Assign(var, e1)) :=
  { e2 ∈ SE(e1) | var ∉ FV(e2) }
```

`AllAE`

- All Available Expressions in the program

`FV`

- Free Variables of the argument

`SE`

- Subexpressions of the argument

Available Expressions

"An **expression** is **available** if it *must* have already been computed, and not later modified, on all paths to the program point"

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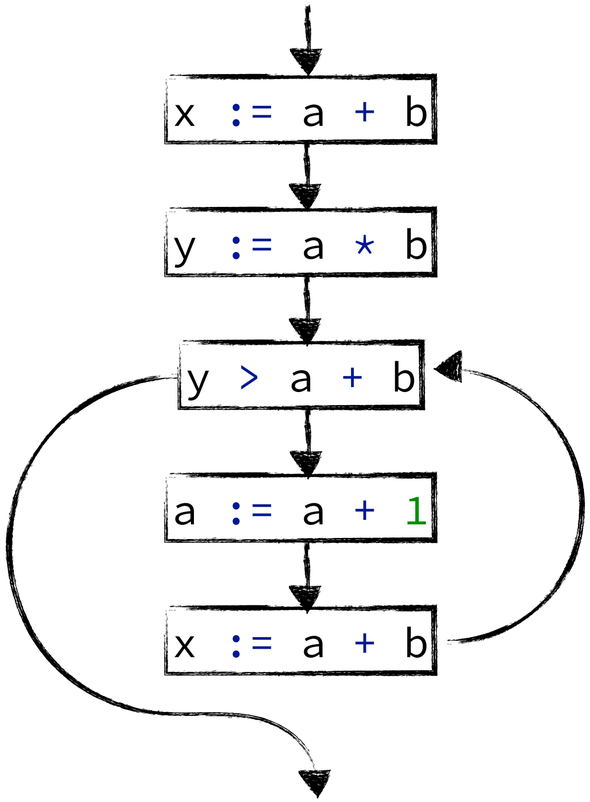

{}

{a + b}

{a + b, a * b}

{a + b, a * b}

{}

{a + b}

Available Expressions

"An **expression** is **available** if it *must* have already been computed, and not later modified, on all paths to the program point"

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kill(Assign(var, e1)) :=
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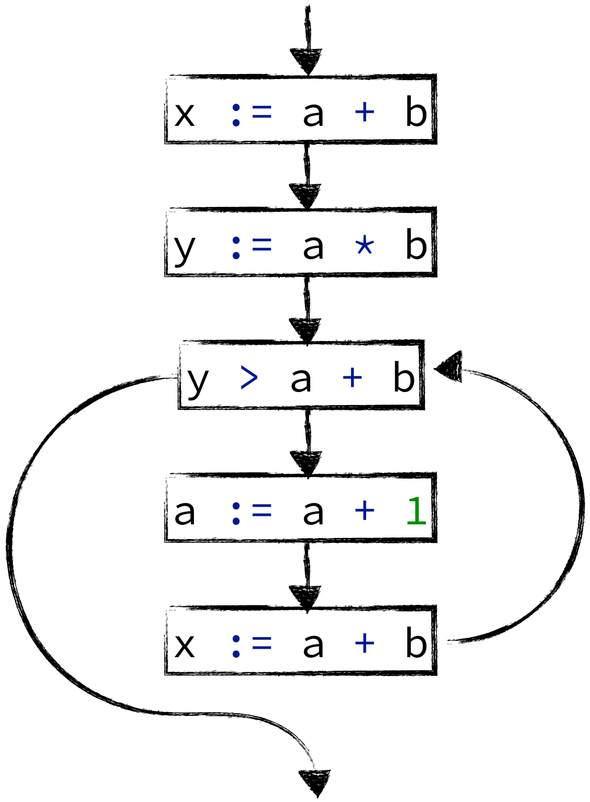


{}

{a + b}

{a + b, a * b}

{a + b, ~~a * b~~}

{}

{a + b}

Live Variables

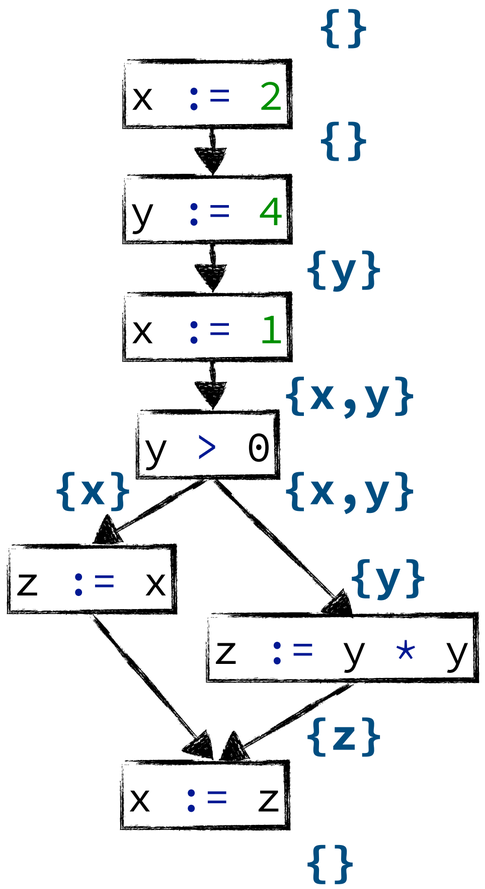
“A **variable** is **live** if there exists a path from there to a use of the variable, with no re-definition of the variable on that path. ”

```
kill(Assign(var, e1)) :=
  { var }

gen(Assign(var, e1)) :=
  { FV(e1) }

gen(b@BinOp(_, _, _)) :=
  { FV(b) }

gen(u@UnOp(_, _)) :=
  { FV(u) }
```



Traditional set based analysis

Sets as analysis information

Kill and gen sets per control node type

- previousSet ⟍ **kill**(currentNode) ∪ **gen**(currentNode)

Can propagate either forward or backward

Can merge information with either union or intersection

- Respectively called **may** and **must** analyses

Beyond Sets

Constant propagation and folding

```
let
  var a : int := 0
  var b : int := a + 1
 in
    c := c + b;
    a := 2 * b
end
```


```
let
  var a : int := 0
  var b : int := 0 + 1
 in
    c := c + b;
    a := 2 * b
end
```


```
let
  var a : int := 0
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end
```

Constant propagation and folding

```
let
  var a : int := 0
  var b : int := a + 1
 in
    c := c + b;
    a := 2 * b
end
```

$a \mapsto 0$

$a \mapsto 0,\ b \mapsto 1$

$a \mapsto 0,\ b \mapsto 1,\ c \mapsto ?$

$a \mapsto 2,\ b \mapsto 1,\ c \mapsto ?$

Constant propagation and folding

The type of the analysis information

- Variables bound to either a particular *constant* or a *marker for non-constants*

The transfer functions per control node

- Basically an interpreter implementation for constants
- Needs to propagate markers when found

Monotone Frameworks

Termination

Data-Flow Analysis needs fixpoint computation

- Because of loops

To terminate, there needs to be a fixpoint

- **And** we need to actually get to that fixpoint
- How can we check this?

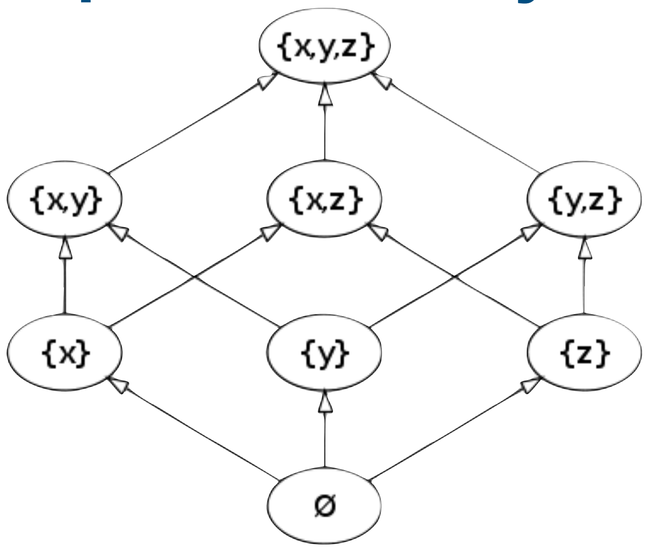
Lattice Theory

A set X is totally ordered under ≤ if for a, b, c ∈ X

- $a \leq b \wedge b \leq a \Rightarrow a = b$ (antisymmetry)
- $a \leq b \wedge b \leq c \Rightarrow a \leq c$ (transitivity)
- $a \leq b \vee b \leq a$ (totality)

A partial ordering drop the totality constraint

- e.g. subset inclusion:



Lattice Theory

A Lattice is a partially ordered set where

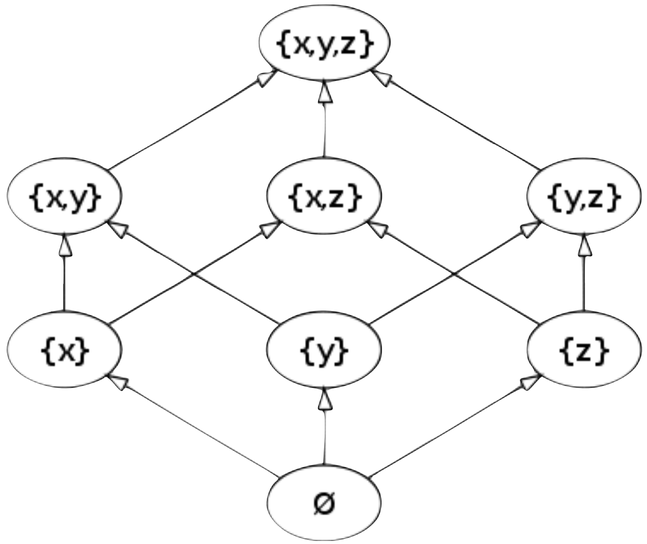
- every two elements have a unique least upper bound (or supremum or join)
- every two elements have a unique greatest lower bound (or infimum or meet)

Least upper bound (LUB)

- $a \sqsubseteq b \Leftrightarrow a \sqcup b = b$
- $a \sqcup b = c \Rightarrow a \sqsubseteq c \wedge b \sqsubseteq c$

Greatest lower bound (GLB)

- $a \sqsubseteq b \Leftrightarrow a \sqcap b = a$
- $a \sqcap b = c \Rightarrow c \sqsubseteq a \wedge c \sqsubseteq b$



A bounded lattice has a top and bottom

- These are $\top$ and $\bot$ respectively

Lattices for data-flow analysis

Consider $\top$ as the coarsest approximation

- It's a safe approximation, because it says we're not sure of anything

Then we can combine data-flow information with $\sqcup$

- It is the most information preserving combination of information

Lattices for data-flow analysis

Transfer functions should be monotone increasing

- i.e. for transfer function f, $a \sqsubseteq b \Rightarrow f(a) \sqsubseteq f(b)$
- This includes the identity function

Monotone transfer functions give a termination guarantee

- In a loop we reach a fixpoint if the functions start returning the same thing
- Worst case scenario: the loop reaches $\top$
- *This **only** works if the lattice is of **finite height***

General interval analysis has an infinite lattice

- $\top = [-\infty,\infty]$
- If a loop adds a finite number to a variable, you never get to ∞

Recap

An analysis consists of

- The type of the analysis information
- The *transfer functions* that express the ‘effect’ of a control node
- The initial analysis information

Recap

An analysis consists of

- The type of the analysis information, **and the lattice instance for that type**
- The *transfer functions* that express the ‘effect’ of a control node
  - **These should be monotone with respect to the lattice**
- The initial analysis information

Executing Monotone Frameworks

Executing Monotone Frameworks

Great formal model for reasoning

- Fairly simple
- Makes intuitive sense
- Has nice mathematical properties

But how to execute?

Framework overview

Control-flow graph

- graph
- start node
- reverse beforehand if backward analysis

Lattice instance for data-flow information:

- Lattice L
- Least Upper Bound $\sqcup$
- Bottom value $\bot \in L$

Initial data-flow information for start node

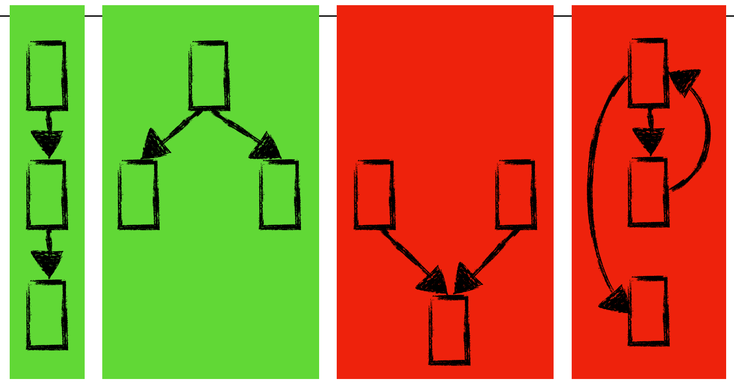
Transfer function $f : (L \to L)$ per control-flow graph node

- Denotes the data-flow effect of the CFG node

Naive approach

```
start_node.value = initial_value
walk(start_node)

function walk(node) =
  for next in node.successors:
    next.value = node.transfer(node.value)
    walk(node)
```

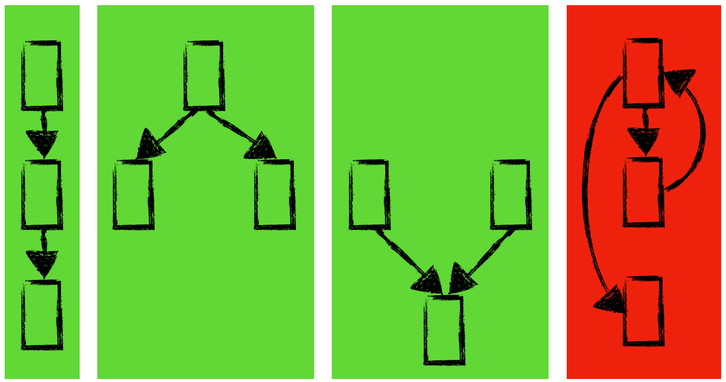


- Fine for straight-line programs
- Distributes information along splits in control-flow
- Overrides values from one path with those of another
- Recursive: great for stack overflows on loops

Using lattices

```
for node in nodes:
  node.value = bottom
start_node.value = initial_value
walk(start_node)

function walk(node) =
  for next in node.successors:
    next.value =
      next.value ⊔ node.transfer(node.value)
    walk(node)
```

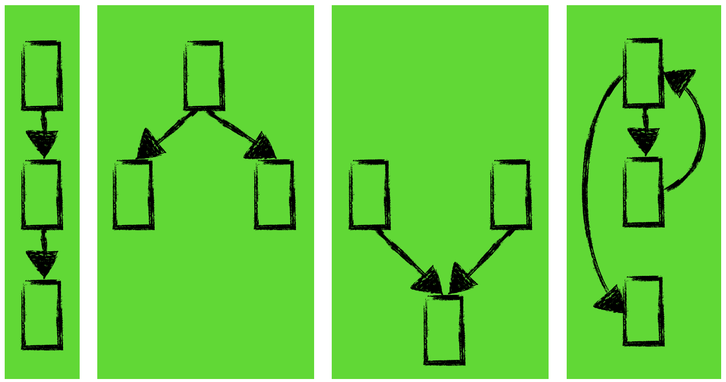


- Fine for straight-line programs
- Distributes information along splits in control-flow
- Combines values from one path with those of another
- Recursive: great for stack overflows on loops

Worklist

```
for node in nodes:
  node.value = bottom
start_node.value = initial_value
worklist = nodes

while !worklist.empty():
  node = worklist.pop()
  for next in node.successors:
    oldValue = next.value
    newValue = node.transfer(node.value)
    if !(newValue ⊑ oldValue):
      next.value = oldValue ⊔ newValue
      worklist = worklist ++ [ next ]
```



If `initial_value == bottom` and a transfer function is identity: traversal will stop there, so don't just start from the `start_node`

- Fine for straight-line programs
- Distributes information along splits in control-flow
- Combines values from one path with those of another
- Worklist: works for loops too

FlowSpec Design

Framework overview

Control-flow graph

- graph — **Control-flow rules**
- start node — **Root rule(s)**
- reverse beforehand if backward analysis — **In edge direction of data-flow rules**

Lattice instance for data-flow information:

- Lattice L — **In property definition**
- Least Upper Bound $\sqcup$ — **In lattice definition**
- Bottom value $\bot \in L$ — **In lattice definition**

Initial data-flow information for start node — **In special data-flow rule**

Transfer function $f : (L \to L)$ per control-flow graph node

- Denotes the data-flow effect of the CFG node — **In data-flow rule**

Variants of Data-Flow Analysis

Many interacting features

- Intra-procedural or inter-procedural
  - Inter-procedural with dynamic dispatch means dynamic control flow analysis depending on the data-flow analysis
- Flow-insensitive, flow-sensitive or even path-sensitive
- Different kind of context-sensitivity for dynamic dispatch
  - Different contexts for the same program point are separately tracked
  - Call-sensitivity: limited “stacktrace”, call-path is tracked
  - Object-sensitivity: objects are tracked by the allocation point in the program

Worklist optimizations in FlowSpec

Worklist algorithm optimizations

Filter irrelevant CFG nodes

- With transfer function tf(x) = x

Order nodes

- Topological order would make sense
- But there are cycles in our graphs
- Every cycle should be computed to a fixpoint
  - Really we need each strongly connected component (SCC)
- Tarjan's SCCs algorithm gives SCCs in reverse topological order!
- Within each SCC the order should also not be random:
  - We use the reverse post-order of the spanning tree

Tarjan's SCC algorithm

SCC identification

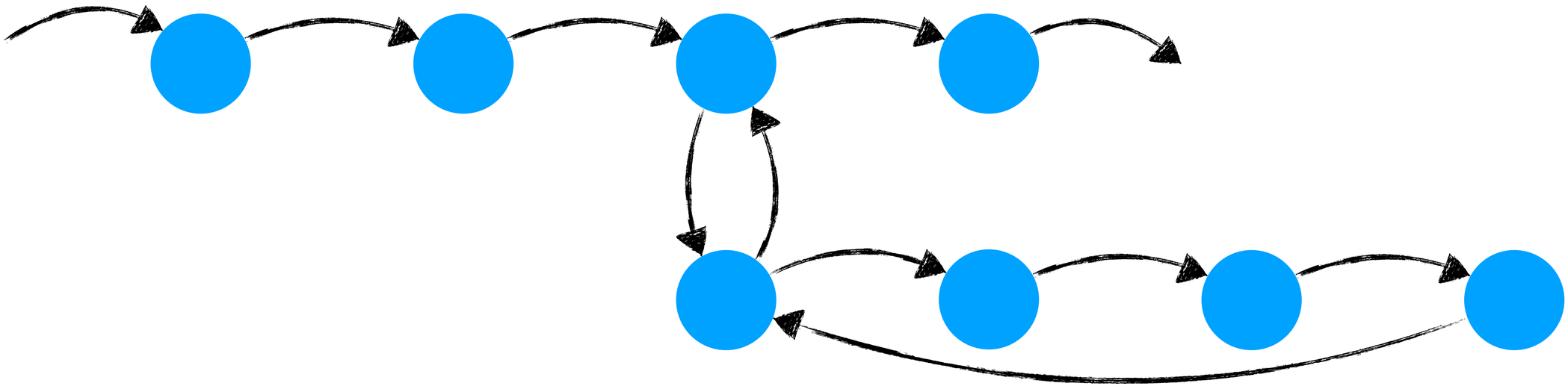
- Label nodes with a increasing integers during a depth-first searches
  - Multiple searches to make sure you reach all nodes in the graph
- The depth-first spanning forest (spanning trees from the searches) holds SCCs as subtrees

- Nodes that can reach the same lowest label are an SCC together

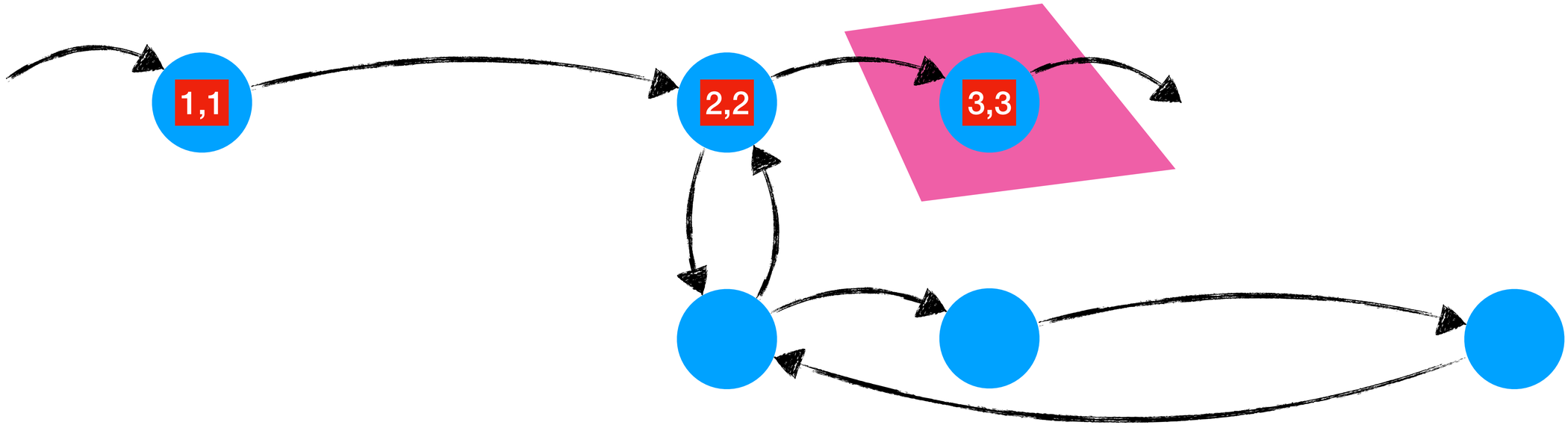
The version in FlowSpec is slightly adapted

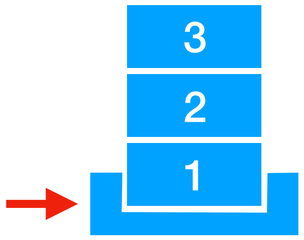
- To return the topological order instead of the reverse topo order
- To have reverse postorder inside SCCs

CFG filtering



Tarjan’s SCC algorithm





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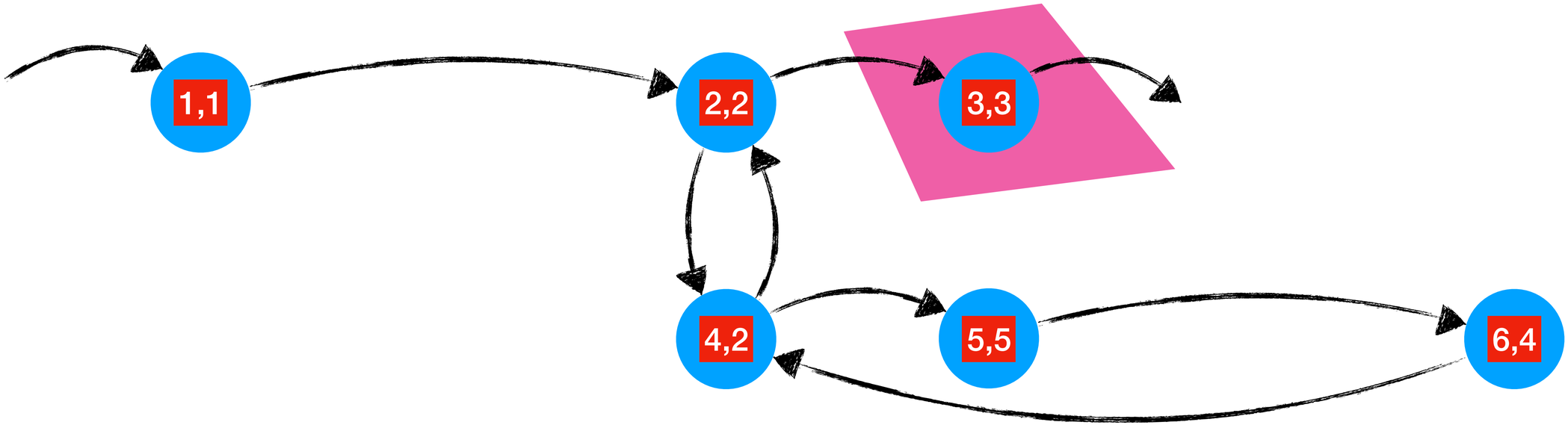

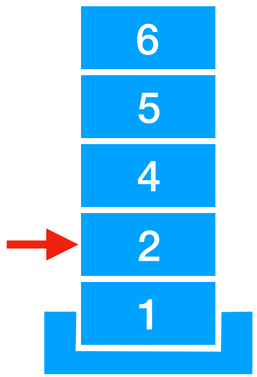

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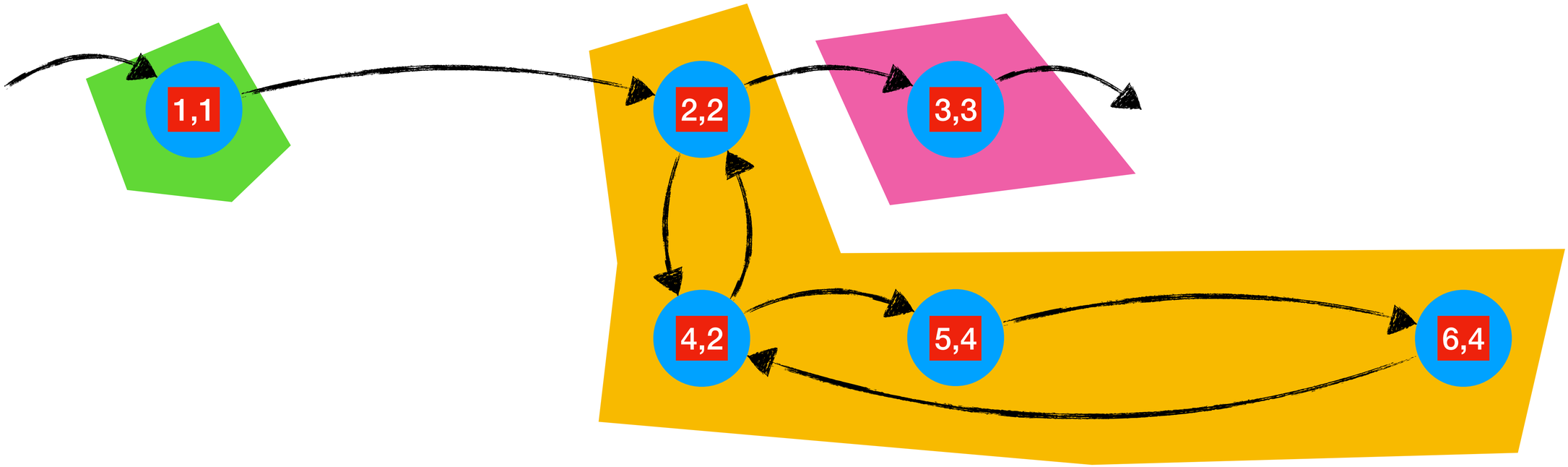
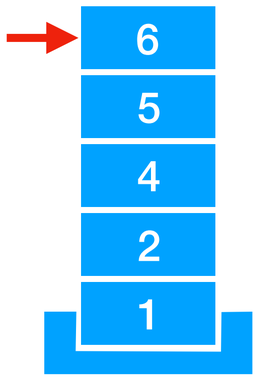
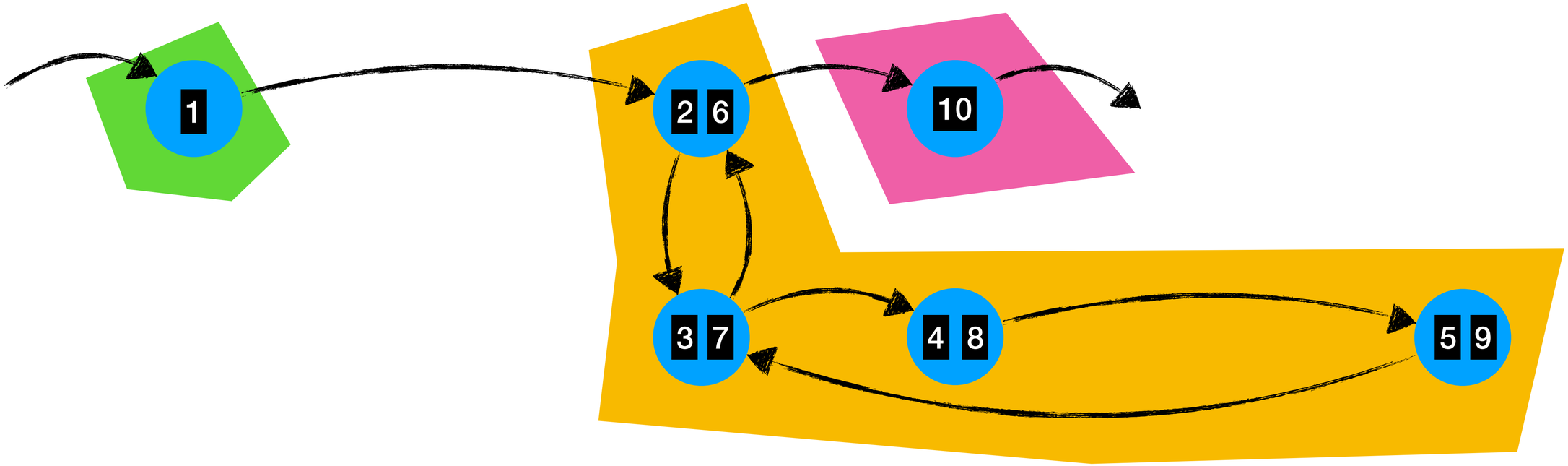




Reverse postorder in SCC



Conclusion

Summary

Data-flow analysis and its uses

- Sets are not enough
- Lattices are the generalization

Monotone Frameworks

- Finite height lattices + monotone transfer functions = termination
- Execute by worklist algorithm

FlowSpec design

- FlowSpec only does intra-procedural, flow-sensitive analysis
- Worklist algorithm with optimizations:
  - SCCs, reverse post-order within SCC, CFG filtering