Lecture 10: Data-Flow Analysis

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CS4200 Compiler Construction
TU Delft
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Earlier Lecture

Check that names are used correctly and that expressions are well-typed

Earlier Lecture

Why Type Checking? Some Discussion Points

Dynamically Typed vs Statically Typed

- Dynamic: type checking at run-time
- Static: type checking at compile-time (before run-time)

What does it mean to type check?

- Type safety: guarantee absence of run-time type errors

Why static type checking?

- Avoid overhead of run-time type checking
- Fail faster: find (type) errors at compile time
- Find all (type) errors: some errors may not be triggered by testing
- But: not all errors can be found statically (e.g. array bounds checking)

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Why types?

Why types?

- Statically prove the absence of certain (wrong) runtime behavior
  - “Well-typed programs cannot go wrong.” [Reynolds1985]
  - Also logical properties beyond runtime problems

What are types?

- Static classification of expressions by approximating the runtime values they may produce
- Richer types approximate runtime behavior better
- Richer types may encode correctness properties beyond runtime crashes

What is the difference between typing and testing?

- Typing is an over-approximation of runtime behavior (proof of absence)
- Testing is an under-approximation of runtime behavior (proof of presence)

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This Lecture

Check that variables are initialised, statements are reached, etc.

This Lecture

Eliminate common subexpressions, reduce loop strength, etc.

Reading Material

The following papers add background, conceptual exposition, and examples to the material from the slides. Some notation and technical details have been changed; check the documentation.

This paper introduces FlowSpec, the declarative data-flow analysis specification language in Spoofax. Although the design of the language described in this paper is still current, the syntax used is already dated, i.e. the current FlowSpec syntax in Spoofax is slightly different.

SLE 2017

https://doi.org/10.1145/3136014.3136029

FlowSpec: Declarative Dataflow Analysis Specification

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Abstract

We present FlowSpec, a declarative specification language for the domain of dataflow analysis. FlowSpec has declarative support for the specification of control flow graphs of programming languages, and dataflow analyses on these control flow graphs. We define the formal semantics of FlowSpec, which is rooted in Monotone Frameworks. We also discuss implementation techniques for the language, partly used in the prototype implementation built in the Spoofax Language Workbench. Finally, we evaluate the expressiveness and conciseness of the language with two case studies. These case studies are analyses for Green-Marl, an industrial, domain-specific language for graph processing. The first case study is a classical dataflow analysis, scaled to this full language. The second case study is a domain-specific analysis of Green-Marl.

CCS Concepts • **Software and its engineering → Domain specific languages**;

Keywords control flow graph, dataflow analysis

ACM Reference Format:
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1 Introduction

Dataflow analysis is a static analysis that answers questions on what *may* or *must* happen before or after a certain point in a program's execution. With dataflow analysis we can answer whether a value written to a variable *here* may be read *later*. Such dataflow analyses can be used to inform optimisations.

For example, consider Live Variables analysis, illustrated in Figure 1. This type of dataflow analysis can identify dead code, which can be removed as an optimisation. In the example this would be statement 1 since it writes x which is overwritten by statement 3 without being read in between. The Live Variables analysis provides a set of variables which are read before being written after each statement in $LV_\bullet$. The figure shows this in the $LV_\bullet$ set of statement 1, which does not contain x.

Dataflow may also be part of a language's static semantics. For example, in Java a final field in a class must be initialised by the end of construction of an object of that class. Since constructor code can have conditional control flow, a dataflow analysis is necessary to check that all possible execution paths through constructors actually assign a value to the final field [Gosling et al. 2005, sect. 16.9].

Dataflow analyses are often operationally encoded, whether in a general purpose language, an attribute grammar system or a logic programming language. This encoding is both an overhead for the engineer implementing it, as well as an overhead in decoding for anyone who wishes to understand the analysis.

In formal, mathematical descriptions of a dataflow analysis, the common patterns are often factored out. This shows commonalities between different analyses, allows the study of those commonalities and differences, as well as general

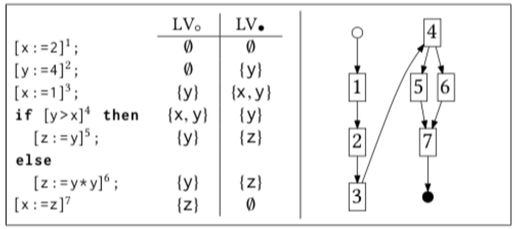

Figure 1. Classical dataflow analysis Live Variables (LV). On the left is an example program in the While language, with added brackets to number program fragments. On the right is the control flow graph (CFG) of the program. In the centre is the analysis result. The $LV_\circ$ and $LV_\bullet$ are before and after the CFG node's variables accesses respectively.

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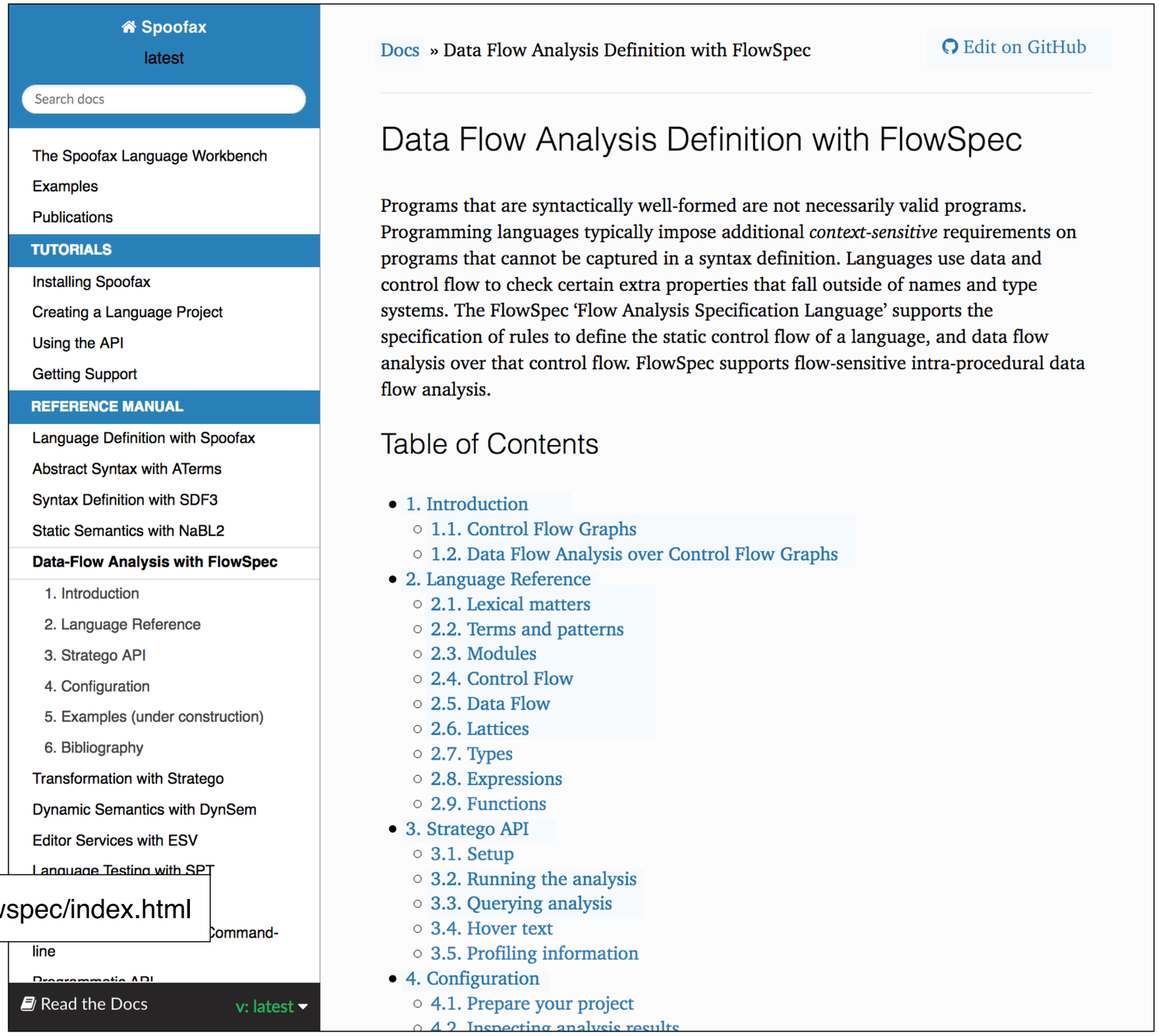
Documentation for FlowSpec at the metaborg.org website.

http://www.metaborg.org/en/latest/source/langdev/meta/lang/flowspec/index.html



Control-Flow

Control-Flow

What is Control-Flow?

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- “Order of evaluation”

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Discuss a series of example programs

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- What is the control flow?

Control-Flow

What is Control-Flow?

- “Order of evaluation”

Discuss a series of example programs

- What is the control flow?
- What constructs in the program determine that?

What is Control-Flow?

```
function id(x) { return x; }
id(4); id(true);
```

What is Control-Flow?

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function id(x) { return x; }
id(4); id(true);
```

Function calls

What is Control-Flow?

```
function id(x) { return x; }
id(4); id(true);
```

Function calls

- Calling a function passes control to that function
- A **return** passes control back to the caller

What is Control-Flow?

```
if (c) { a = 5 } else { a = "four" }
```

What is Control-Flow?

```
if (c) { a = 5 } else { a = "four" }
```

Branching

What is Control-Flow?

```
if (c) { a = 5 } else { a = "four" }
```

Branching

- Control is passed to one of the two branches
- This is dependent on the value of the condition

What is Control-Flow?

```
while (c) { a = 5 }
```

What is Control-Flow?

```
while (c) { a = 5 }
```

Looping

What is Control-Flow?

```
while (c) { a = 5 }
```

Looping

- Control is passed to the loop body depending on the condition
- After the body we start over

What is Control-Flow?

```
function1(a);
function2(b);
```

What is Control-Flow?

```
function1(a);
function2(b);
```

Sequence

What is Control-Flow?

```
function1(a);
function2(b);
```

Sequence

- No conditions or anything complicated
- But still order of execution

What is Control-Flow?

```
distance = distance + 1;
```

What is Control-Flow?

```
distance = distance + 1;
```

Reads and Writes

What is Control-Flow?

```
distance = distance + 1;
```

Reads and Writes

- The expression needs to be evaluated, before we can save its result

What is Control-Flow?

```
int i = 2;
int j = (i=3) * i;
```

What is Control-Flow?

```
int i = 2;
int j = (i=3) * i;
```

Expressions & side-effects

What is Control-Flow?

```
int i = 2;
int j = (i=3) * i;
```

Expressions & side-effects

- Order in sub-expressions is usually undefined
- Side-effects make sub-expression order relevant

Kinds of Control-Flow

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- Sequential statements

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- Conditional — if / switch / case

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- Conditional	if / switch / case
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- Async-await	threading

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- Coroutines / Generators	yield

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- Dispatch	function calls / method calls

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- Loop jumps	break / continue

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- Loop jumps	break / continue
- ... many more ...	

Why Control-Flow?

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Shorter code

- No need to repeat the same statement 10 times

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Parametric code

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- Let user decide repetition amount

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Reason about program execution

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Reason about program execution

- What happens when?

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Reason about program execution

- What happens when?
- In what order?

Control-Flow and Language Design

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Imperative programming

- Explicit control-flow constructs

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Declarative programming

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- More options for compilers to choose order

Control-Flow and Language Design

Imperative programming

- Explicit control-flow constructs

Declarative programming

- What, not how
- Less explicit control-flow

- More options for compilers to choose order
- Great if your compiler is often smarter than the programmer

Separation of Concerns in Data-Flow Analysis

Separation of Concerns in Data-Flow Analysis

Representation

- Represent control-flow of a program

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- Represent control-flow of a program
- Conduct and represent results of data-flow analysis

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Declarative Rules

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Language-Independent Tooling

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Language-Independent Tooling

- Data-Flow Analysis

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- Optimisation
- …

Control-Flow Graphs

What is a Control-Flow Graph?

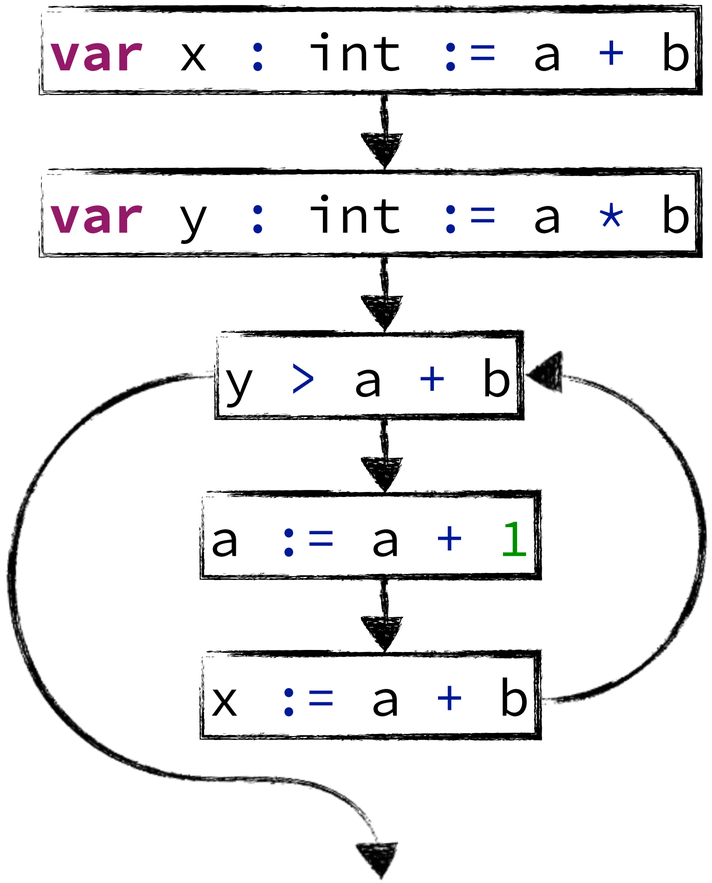
A **control flow graph** (**CFG**) in computer science is a representation, using graph notation, of all paths that might be traversed through a program during its execution.

https://en.wikipedia.org/wiki/Control_flow_graph

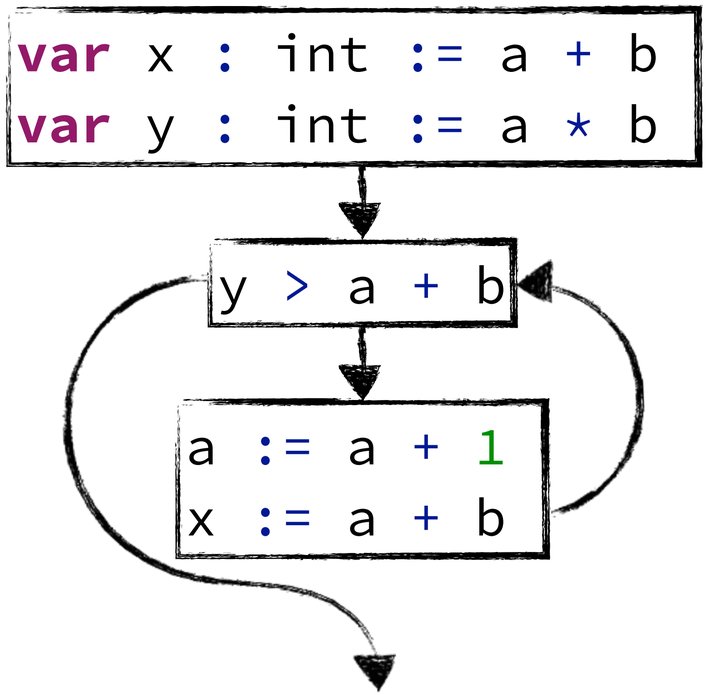
Control-Flow Graph Example

```
let
  var x : int := a + b
  var y : int := a * b
 in
  while y > a + b do
    (
      a := a + 1;
      x := a + b
    )
end
```

Control-Flow Graph Example



Basic Blocks



Control Flow Graphs

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- Usually innermost statements and expressions

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- Splits: conditionally split the control flow

Control Flow Graphs

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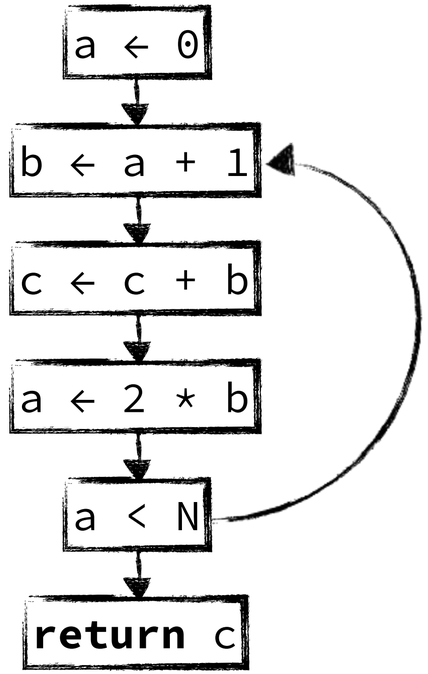
- Or blocks for consecutive statements (basic blocks)

Edges

- Back edges: show loops
- Splits: conditionally split the control flow
- Merges: combine previously split control flow

Equivalent to Unstructured Control-Flow

```
    a ← 0
L1: b ← a + 1
    c ← c + b
    a ← 2 * b
    if a < N goto L1
    return c
```



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Declarative Rules

- To define control-flow of a language
- To define data-flow of a language

Language-Independent Tooling

- Data-Flow Analysis
- Errors/Warnings
- Code completion
- Refactoring
- Optimisation
- …

Separation of Concerns in Data-Flow Analysis

Representation

- **Control Flow Graphs (CFGs)**
- Conduct and represent results of data-flow analysis

Declarative Rules

- To define control-flow of a language
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Data-Flow

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What is Data-Flow?

- Possible values (data) that flow through the program

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- Relations between those data (data dependence)

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What is Data-Flow?

- Possible values (data) that flow through the program
- Relations between those data (data dependence)

Discuss a series of example programs

- What is wrong or can be optimised?
- What is the flow we can use for this?

Check that code is reachable or observable

What is wrong here?

```
public int ComputeFac(int num) {
    return num;
    int num_aux;
    if (num < 1)
        num_aux = 1;
    else
        num_aux = num * this.ComputeFac(num-1);
    return num_aux;
}
```

What is wrong here?

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Dead code (control-flow)

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}
```

Dead code (control-flow)

- Most of the code is never reached because of the early return
- This is usually considered an error by compilers

What is “wrong” here?

```
x := 2;
y := 4;
x := 1;
// x and y used later
```

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Dead code (data-flow)

- The first value of x is never observed
- This is sometimes warned about by compilers

What is “wrong” here?

```
x := 2;
y := 4;
x := 1;
// x and y used later
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Dead code (data-flow)

Live variable analysis

- The first value of x is never observed
- This is sometimes warned about by compilers

Eliminate common subexpressions, reduce loop strength, etc.

What is suboptimal here?

```
let
  var x : int := a + b
  var y : int := a * b
 in
  if y > a + b then
    (
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Common subexpression elimination

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Common subexpression elimination

- a + b is already computed when you get to the condition
- There is no need to compute it again

What is suboptimal here?

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Common subexpression elimination

Available expression analysis

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      x := a + b
    )
end
```

What is suboptimal here?

```
for i := 1 to 100 do
  (
    x[i] := y[i];
    if w > 0 then
      y[i] := 0
  )
```

What is suboptimal here?

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for i := 1 to 100 do
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Loop unswitching

What is suboptimal here?

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Loop unswitching

- The if condition is not dependent on i, x or y
- Still it is checked in the loop, which is slowing the loop

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for i := 1 to 100 do
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Loop unswitching

Data-dependence analysis

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Separation of Concerns in Data-Flow Analysis

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- **Control Flow Graphs (CFGs)**
- Conduct and represent results of data-flow analysis

Declarative Rules

- To define control-flow of a language
- To define data-flow of a language

Language-Independent Tooling

- Data-Flow Analysis
- Errors/Warnings
- Code completion
- Refactoring
- Optimisation
- …

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- **Control Flow Graphs (CFGs)**
- **Data-flow information on CFG nodes**

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- Control Flow Graphs (CFGs)
- Data-flow information on CFG nodes

Declarative Rules

- A domain-specific meta-language for Spoofax: FlowSpec

Language-Independent Tooling

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Tiger in FlowSpec

Control-Flow Rules

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

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Map abstract syntax to control-flow (sub)graphs

- Match an AST pattern

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

- Match an AST pattern
- List all CFG edges of that AST

Control-Flow Rules

Map abstract syntax to control-flow (sub)graphs

- Match an AST pattern
- List all CFG edges of that AST
- Mark subtrees as CFG nodes

Control-Flow Rules

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- Or splice in their control-flow subgraph
- Use special “context” nodes to connect a subgraph to the outside graph

Control-Flow Graphs in FlowSpec

FlowSpec

Example program

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x := 1;

if y > x then

  z := y;

else

  z := y * y;

y := a * b;

while y > a + b do

 (a := a + 1;

  x := a + b)
```

Control-Flow Graphs in FlowSpec

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  start -> s,
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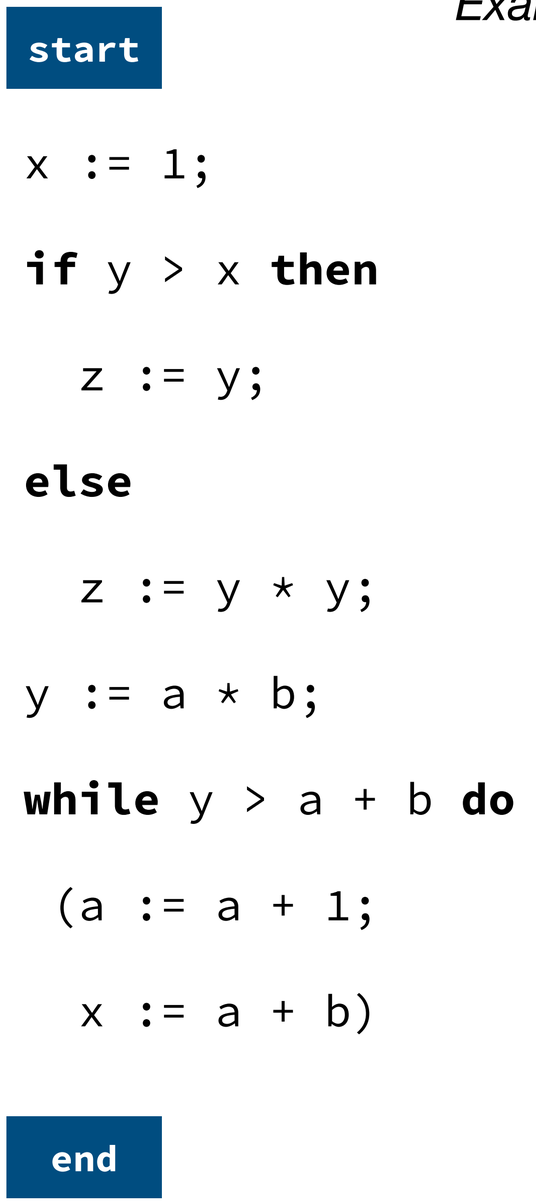

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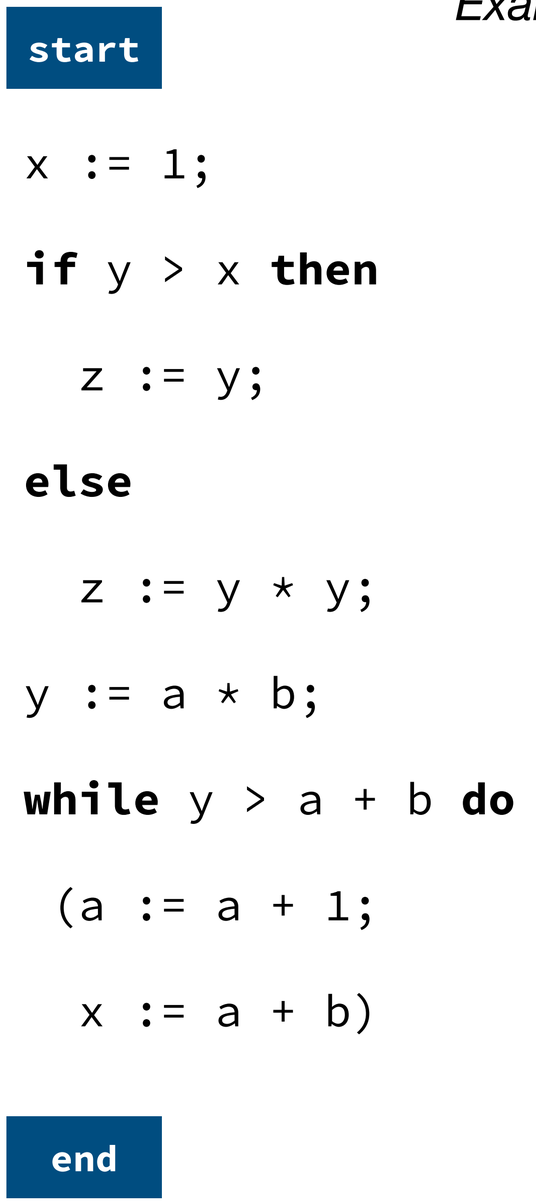

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root Mod(s) =
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a@Assign(_, _) =
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```

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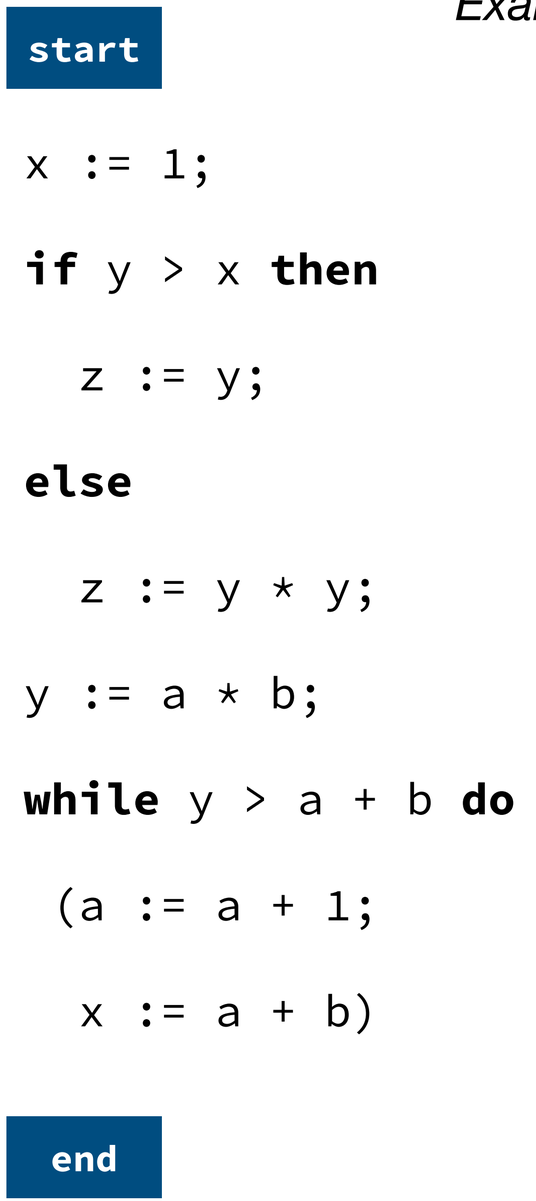

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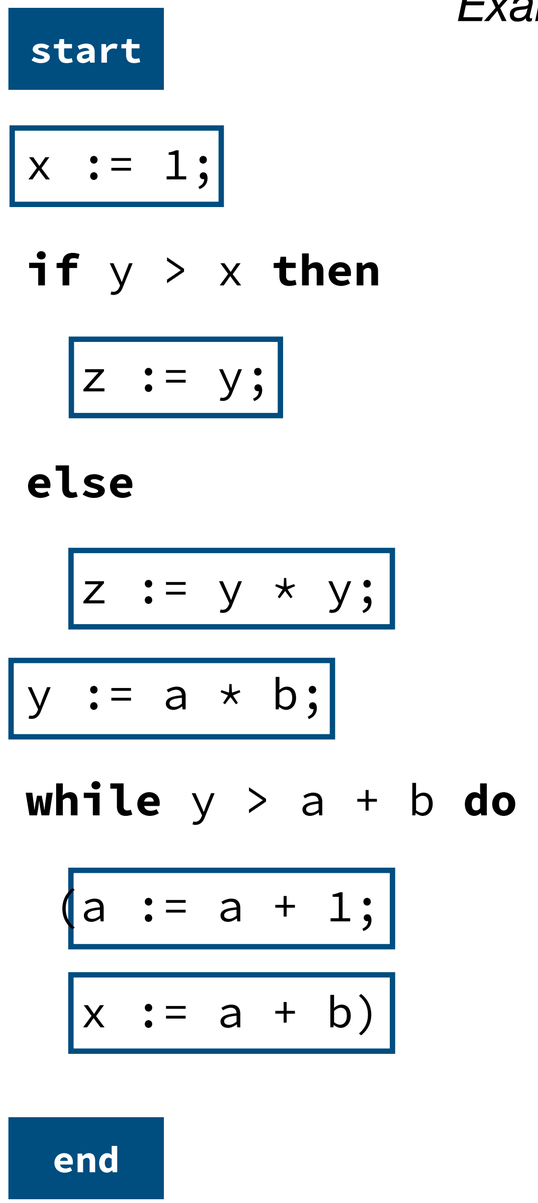

Control-Flow Graphs in FlowSpec

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```

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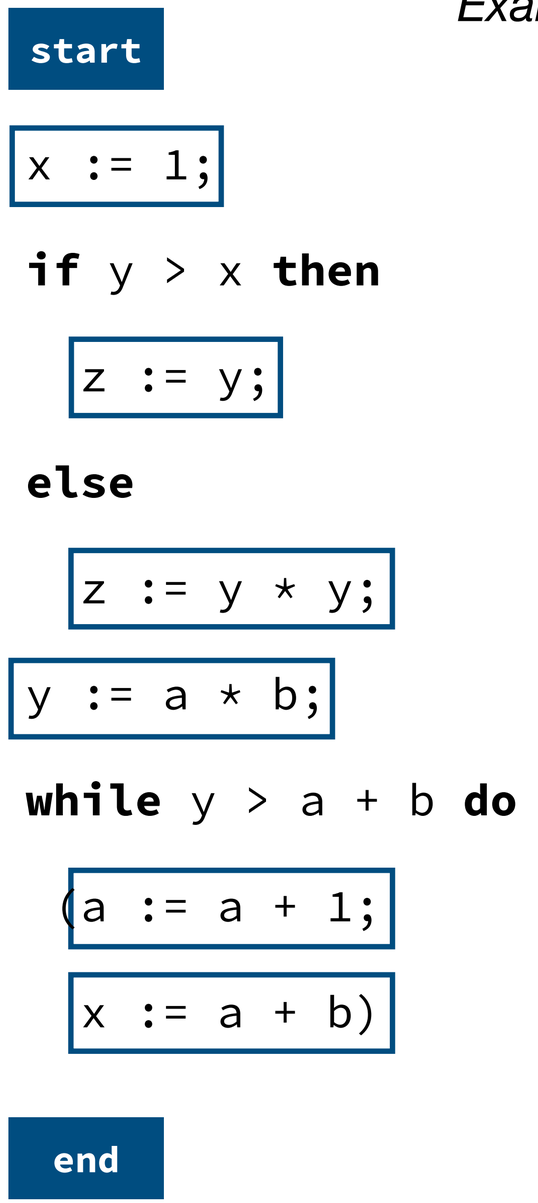

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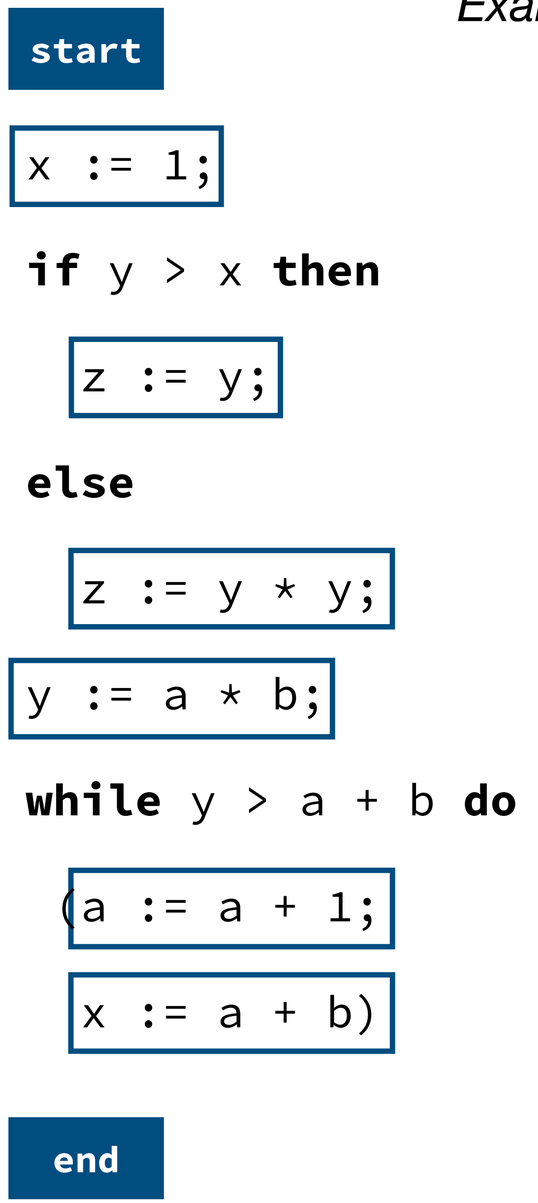

Control-Flow Graphs in FlowSpec

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Seq(s1, s2) =
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```

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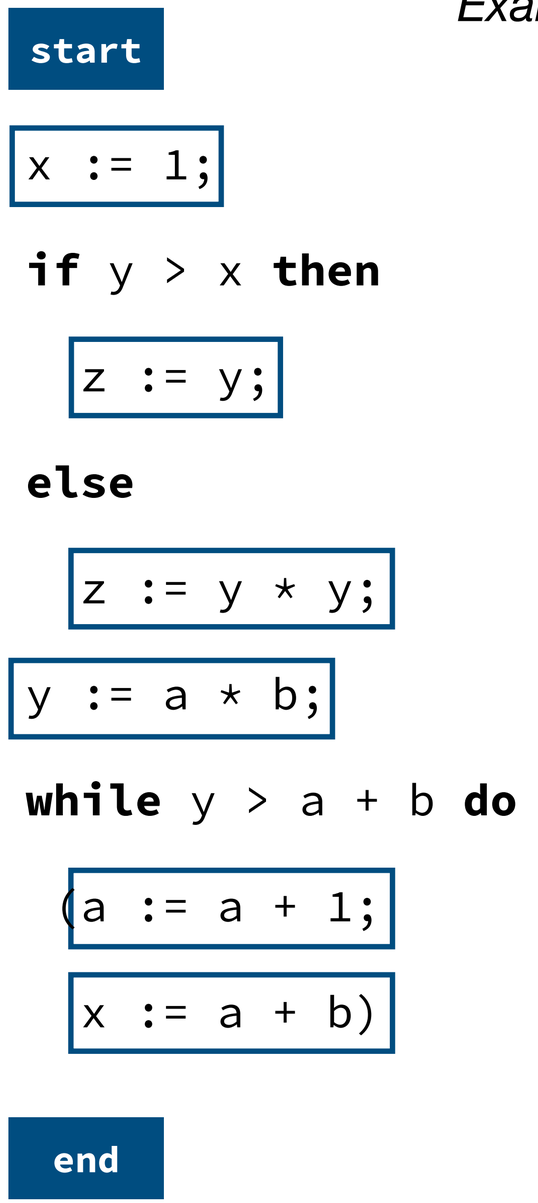

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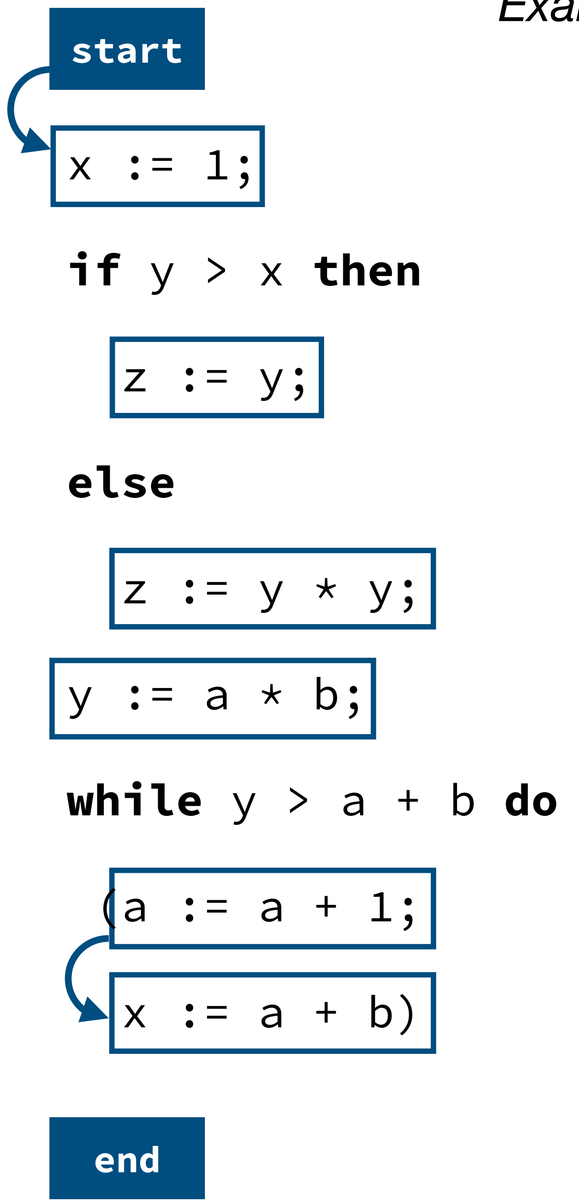

Control-Flow Graphs in FlowSpec

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```

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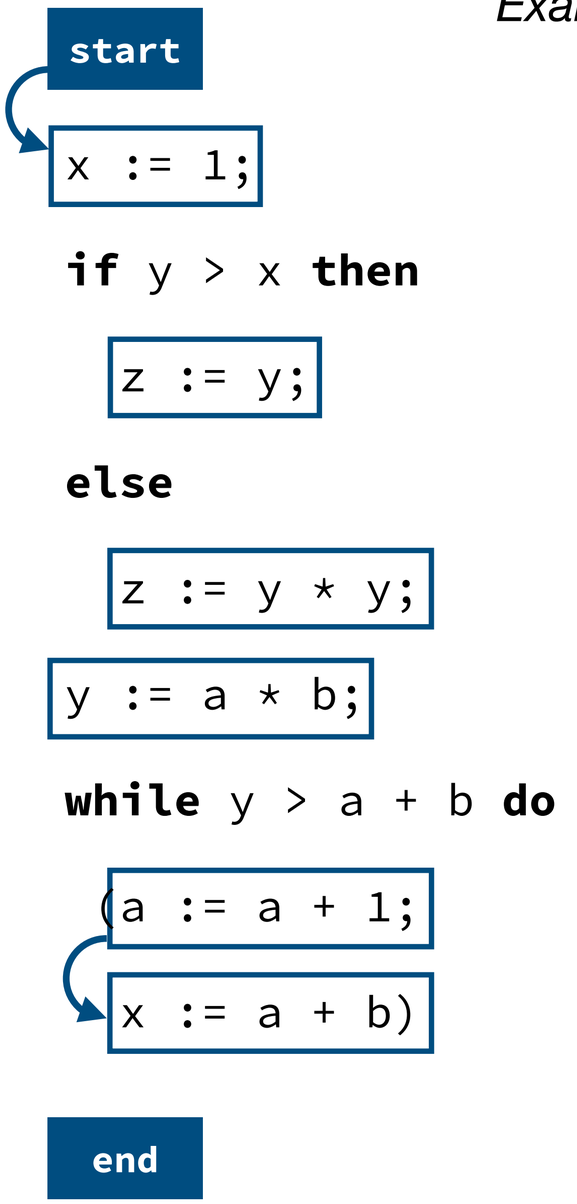

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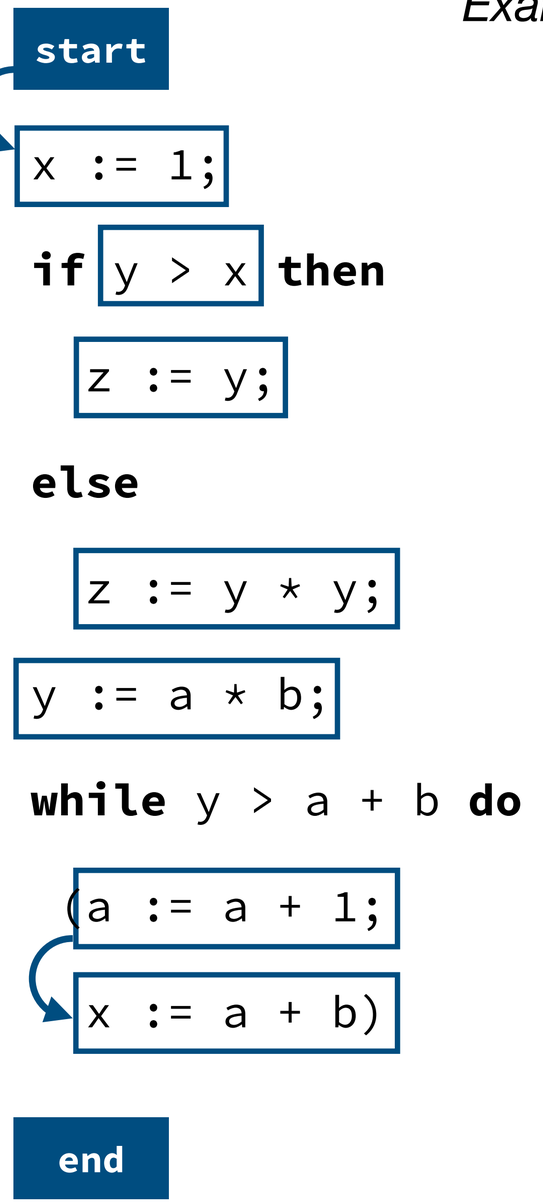

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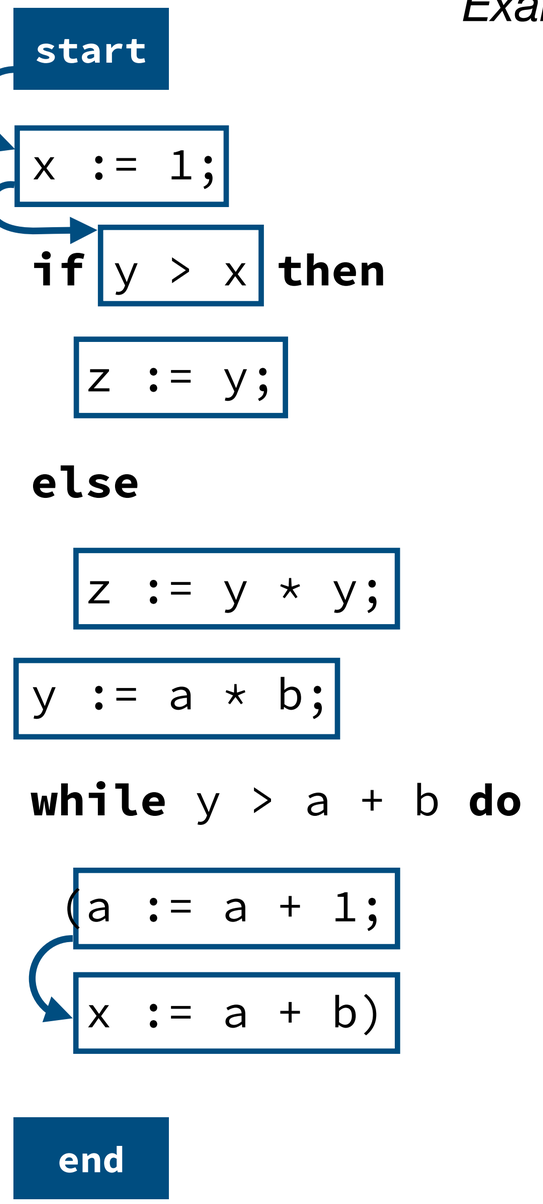

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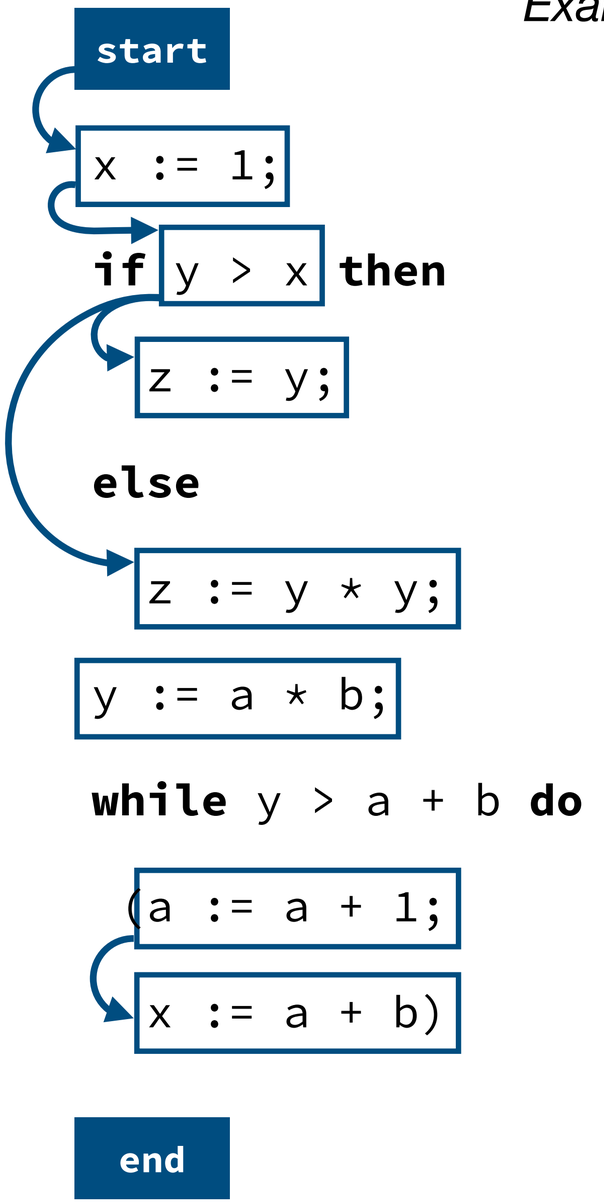

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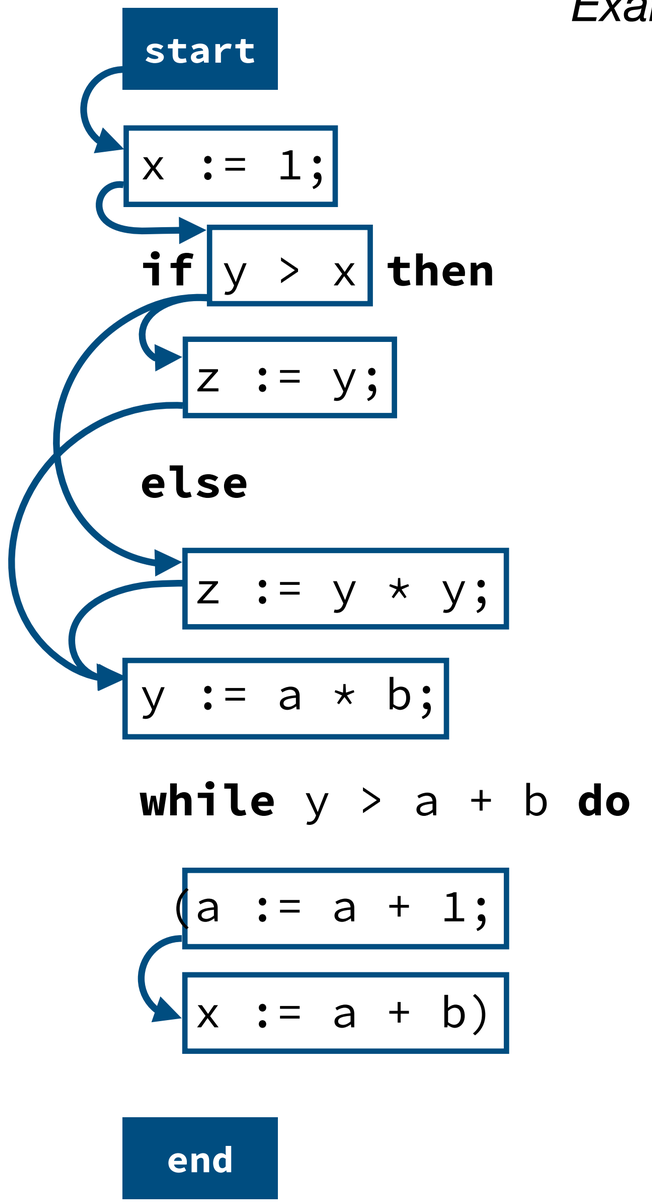

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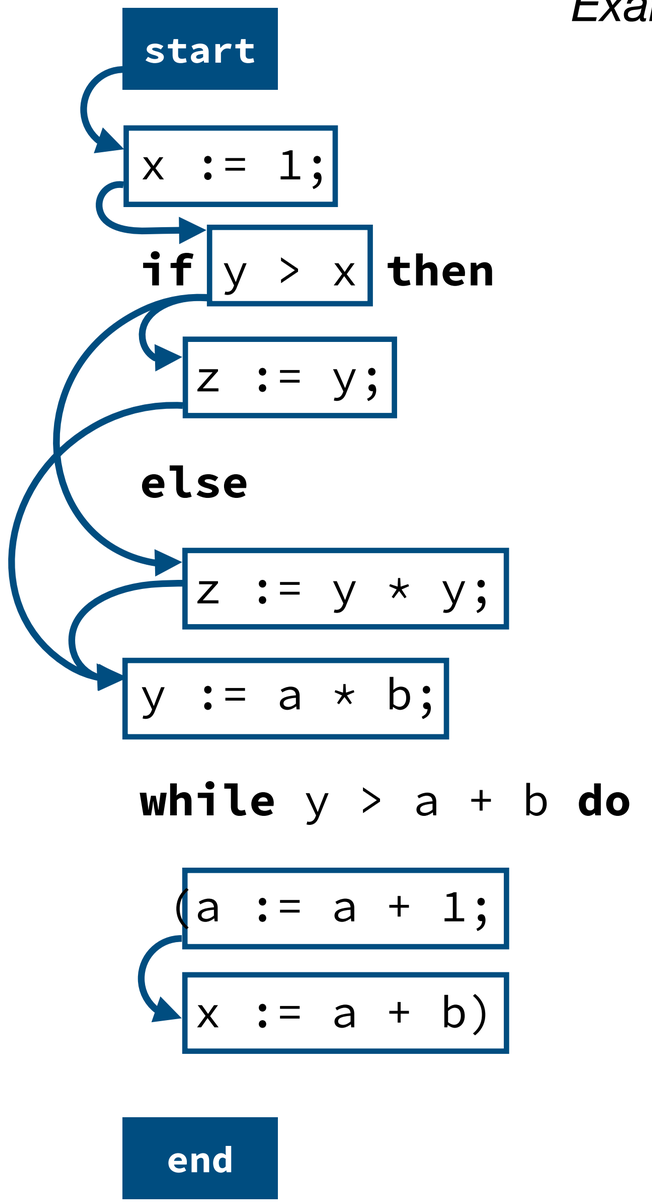

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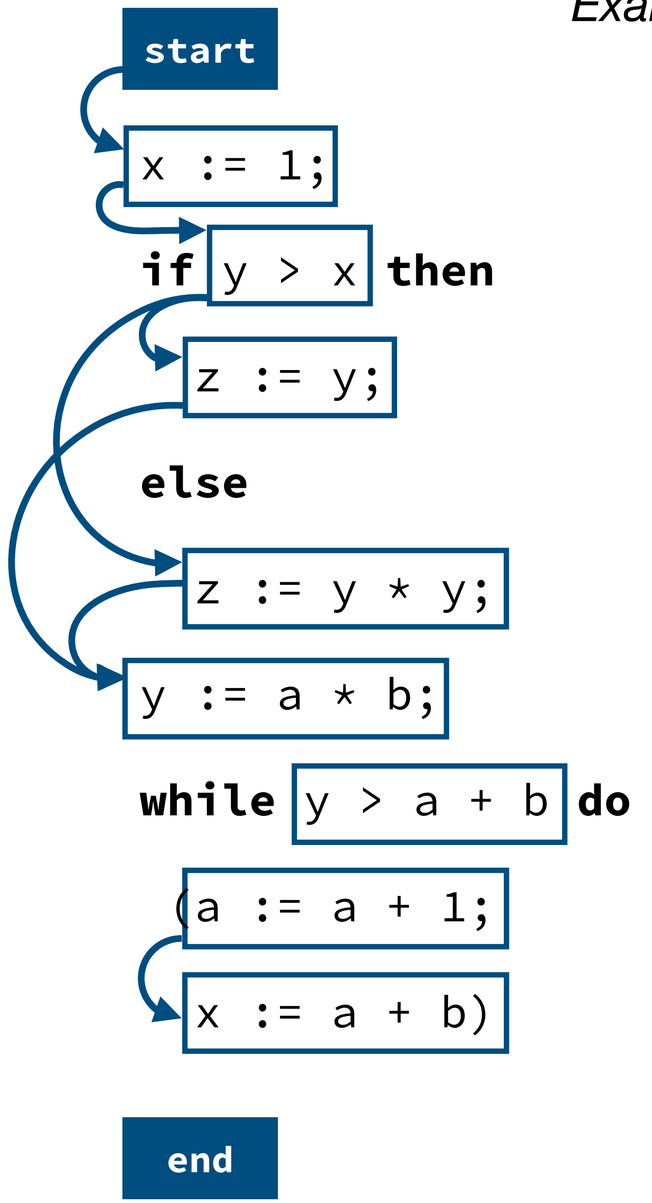

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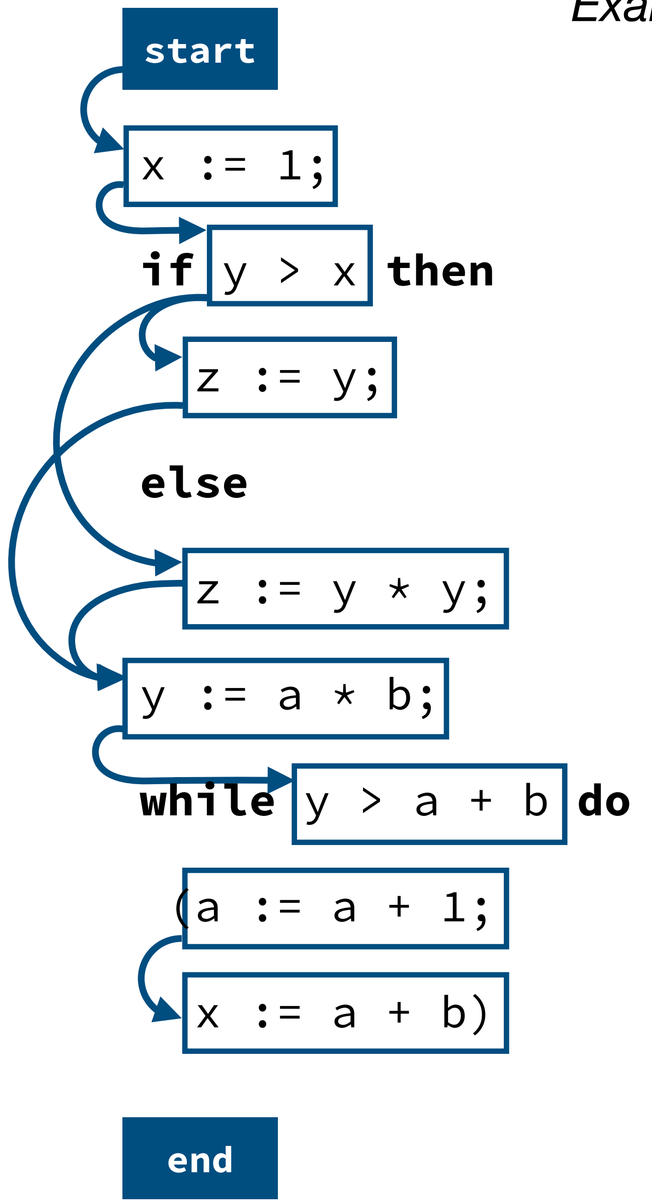

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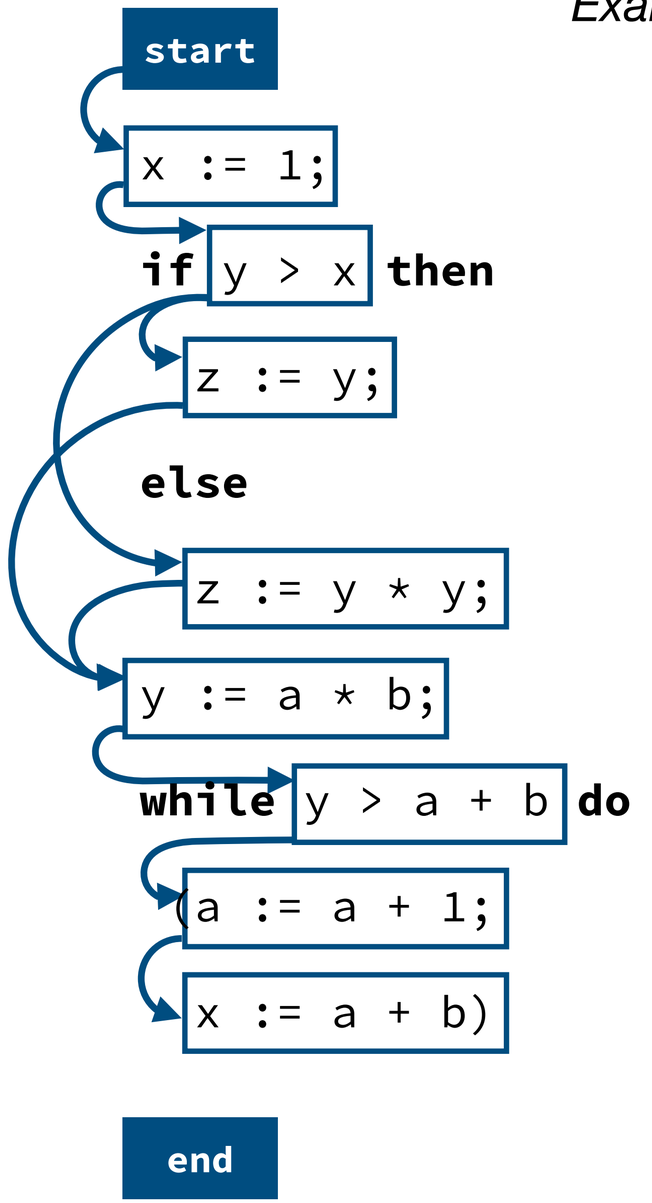

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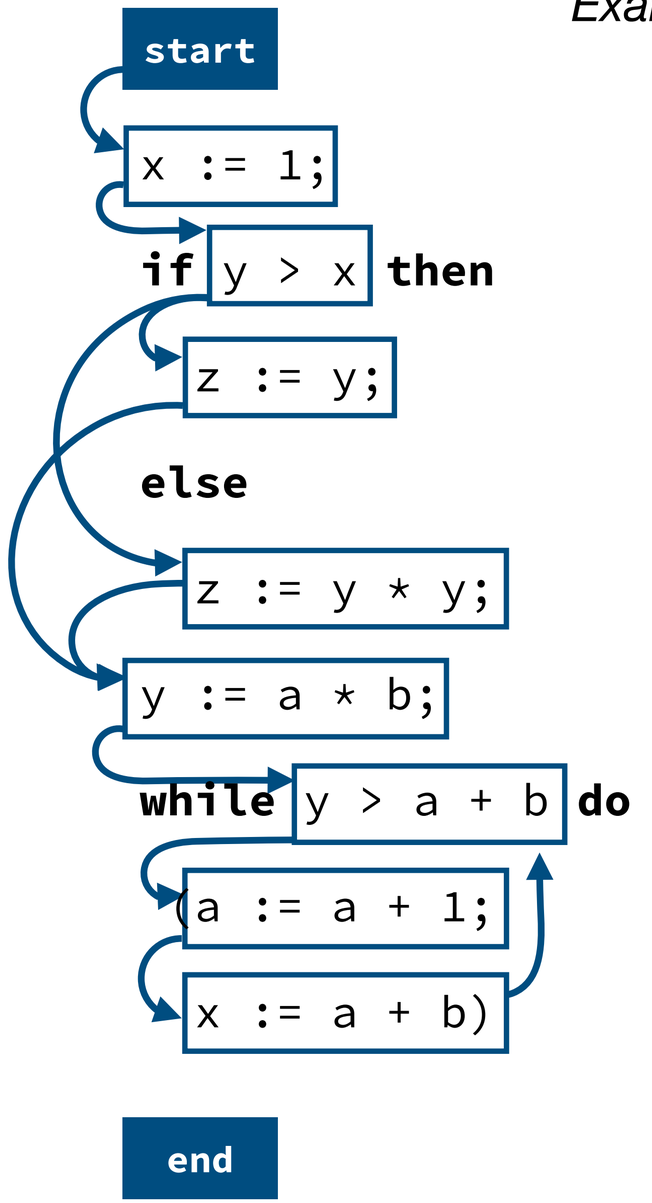

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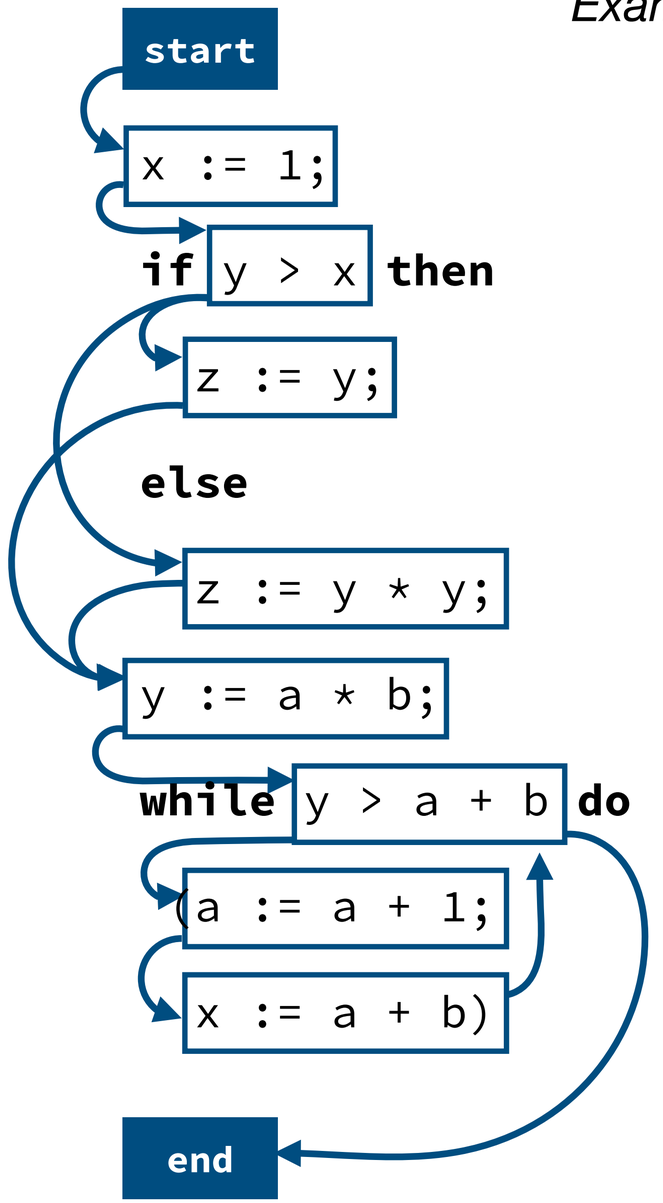

Data-Flow Rules

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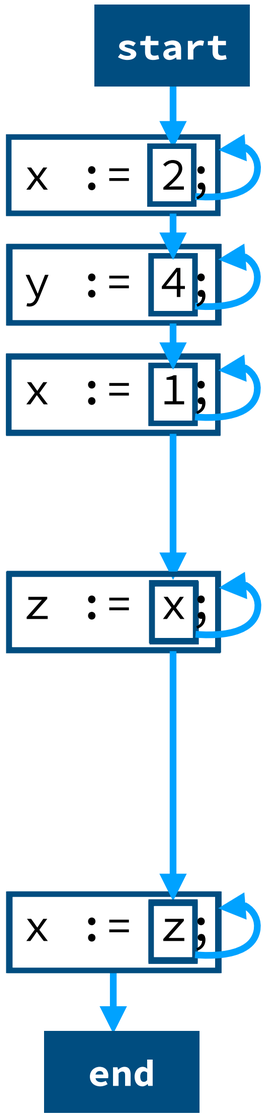

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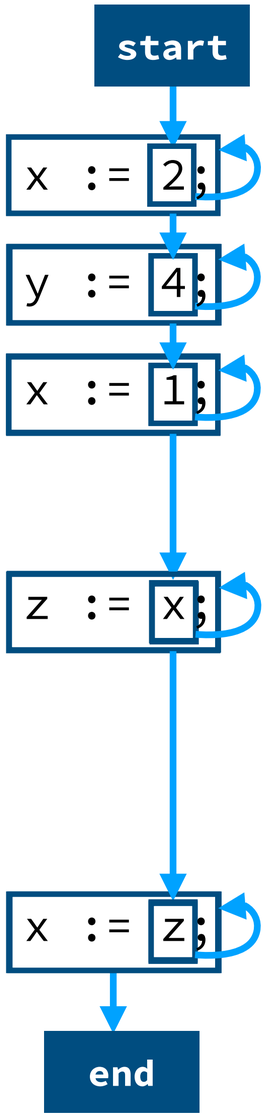

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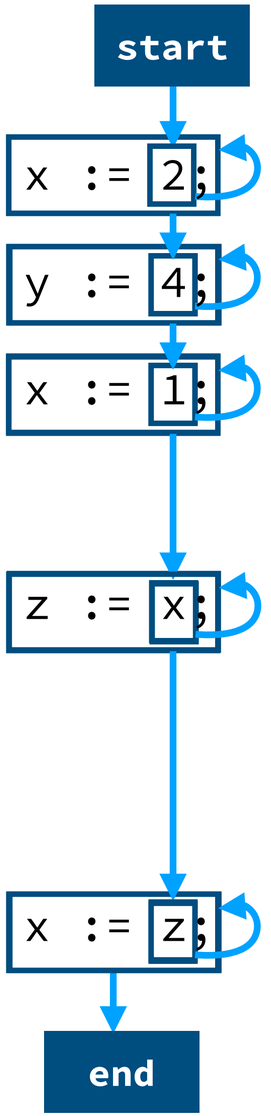

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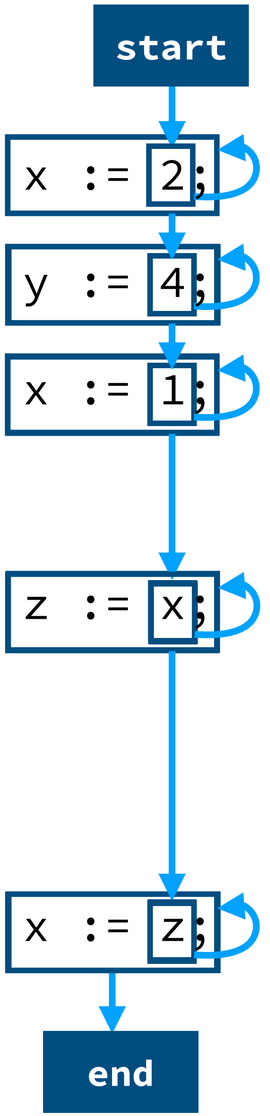

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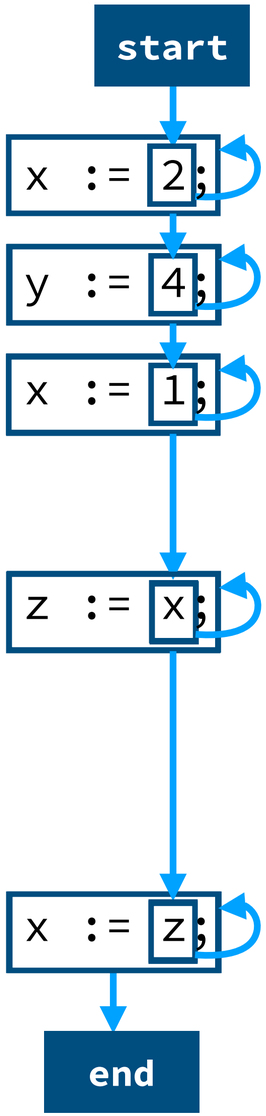

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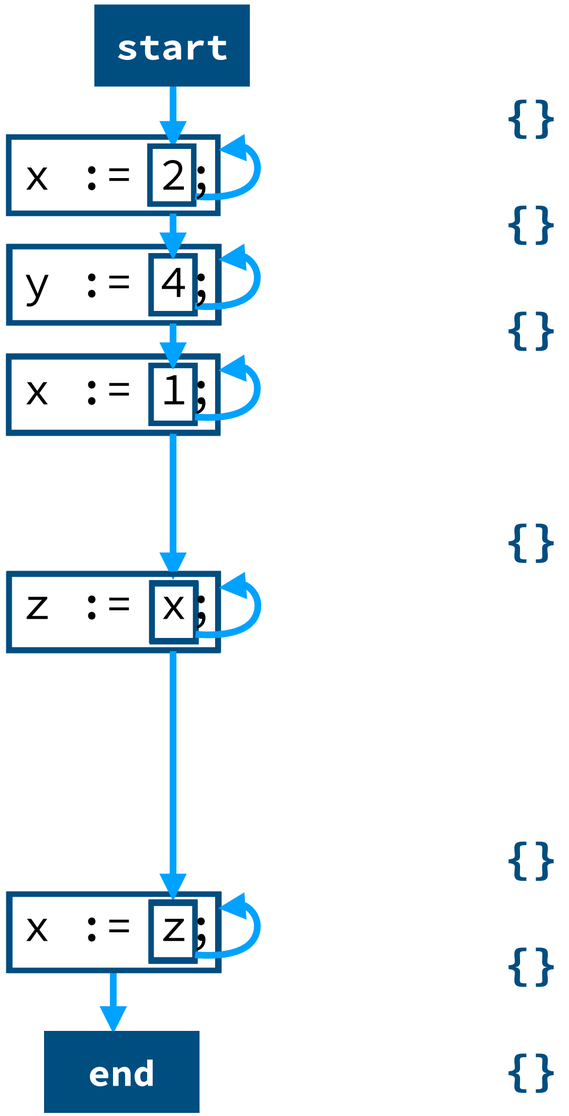

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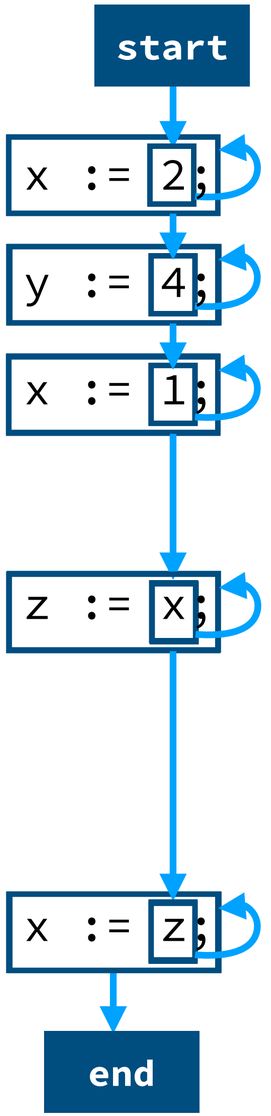

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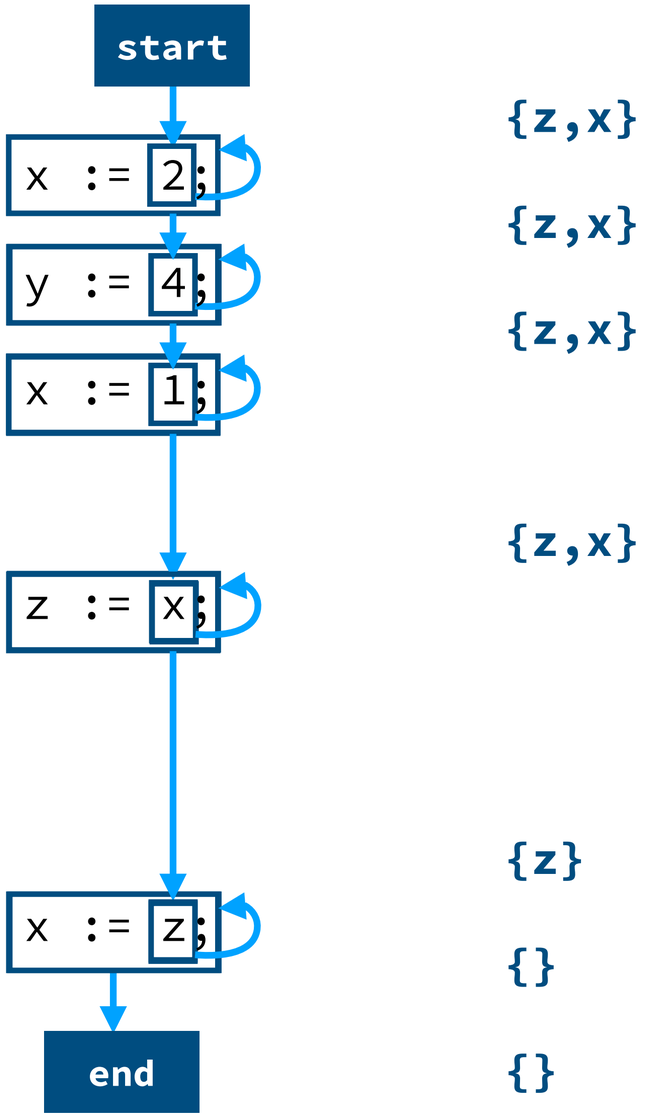

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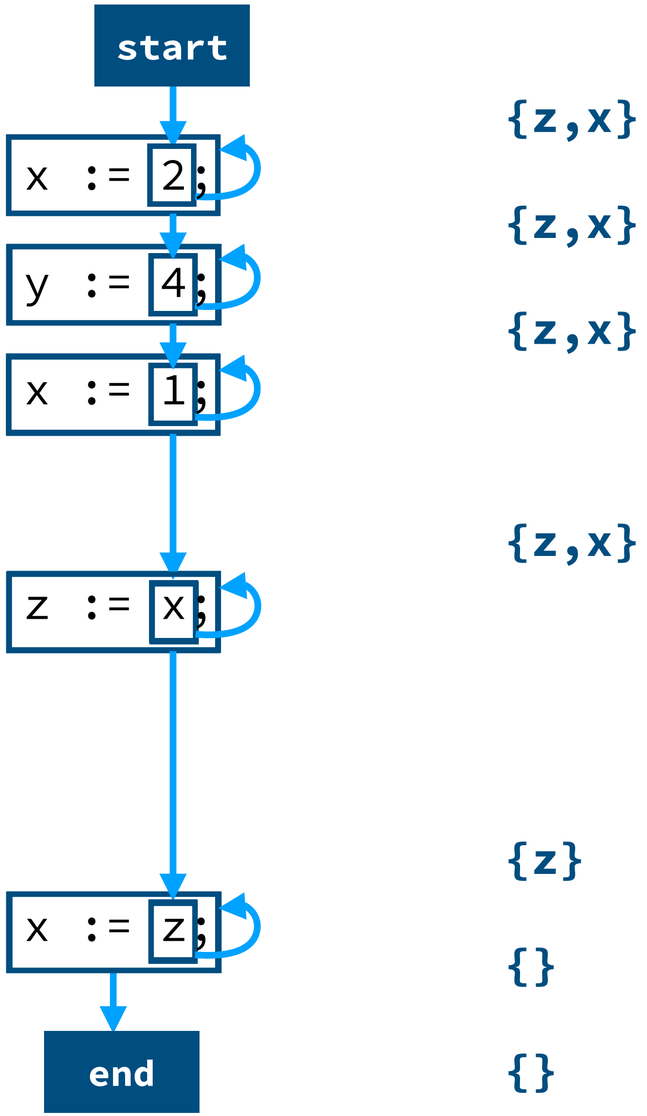

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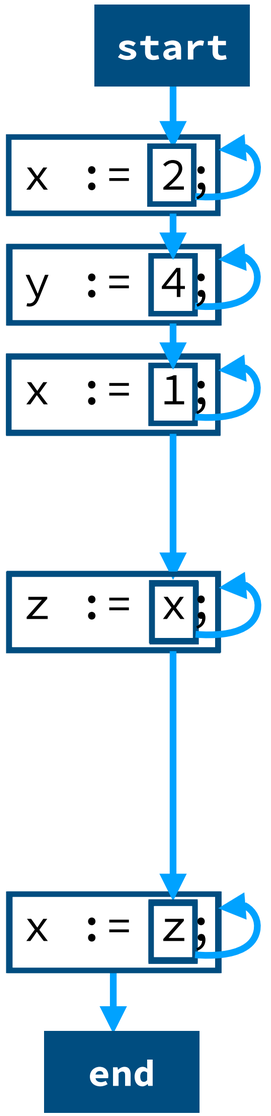

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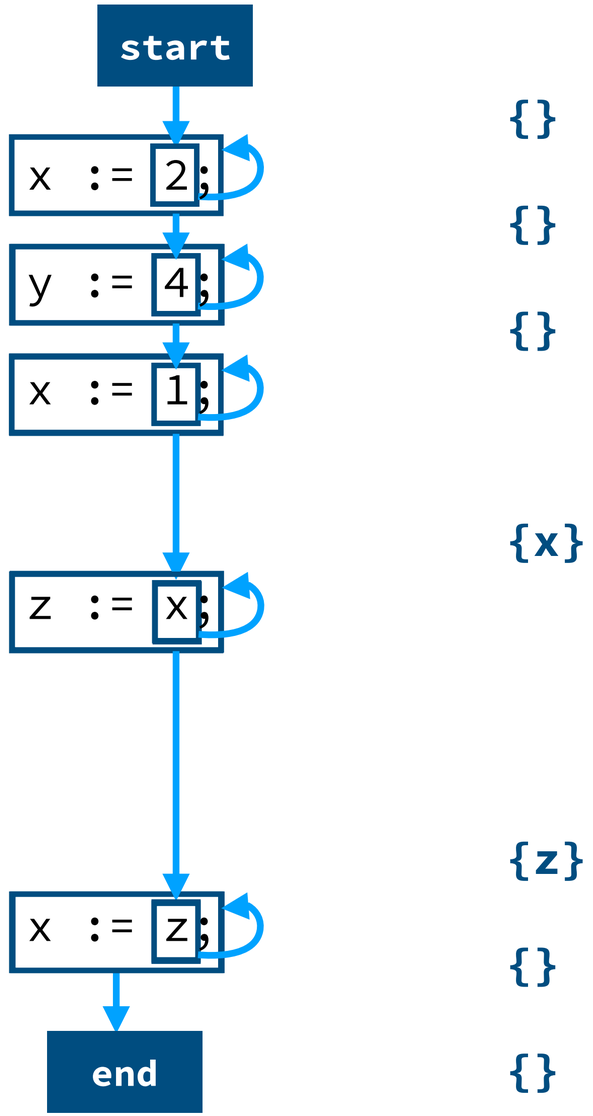

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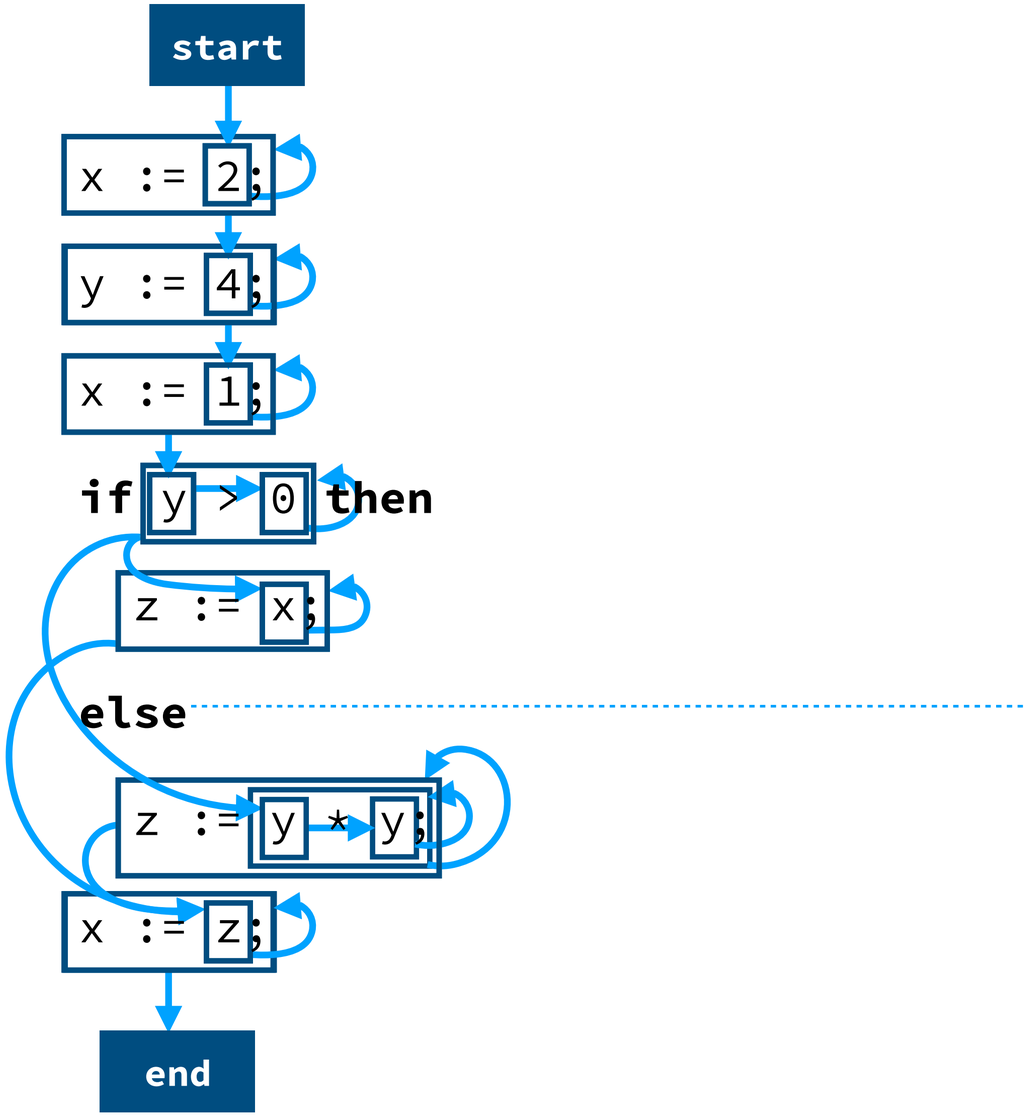

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```

```
x := 2;

y := 4;

x := 1;

if y > 0 then

  z := x;

else

  z := y * y;

x := z;
```

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
 live:     Set(name)

property rules
 live(Ref(n) -> next) =
   live(next) \/ {n}

 live(Assign(n, _) -> next) =
   { m | m <- live(next), n != m }

 live(_.end) =
   {}
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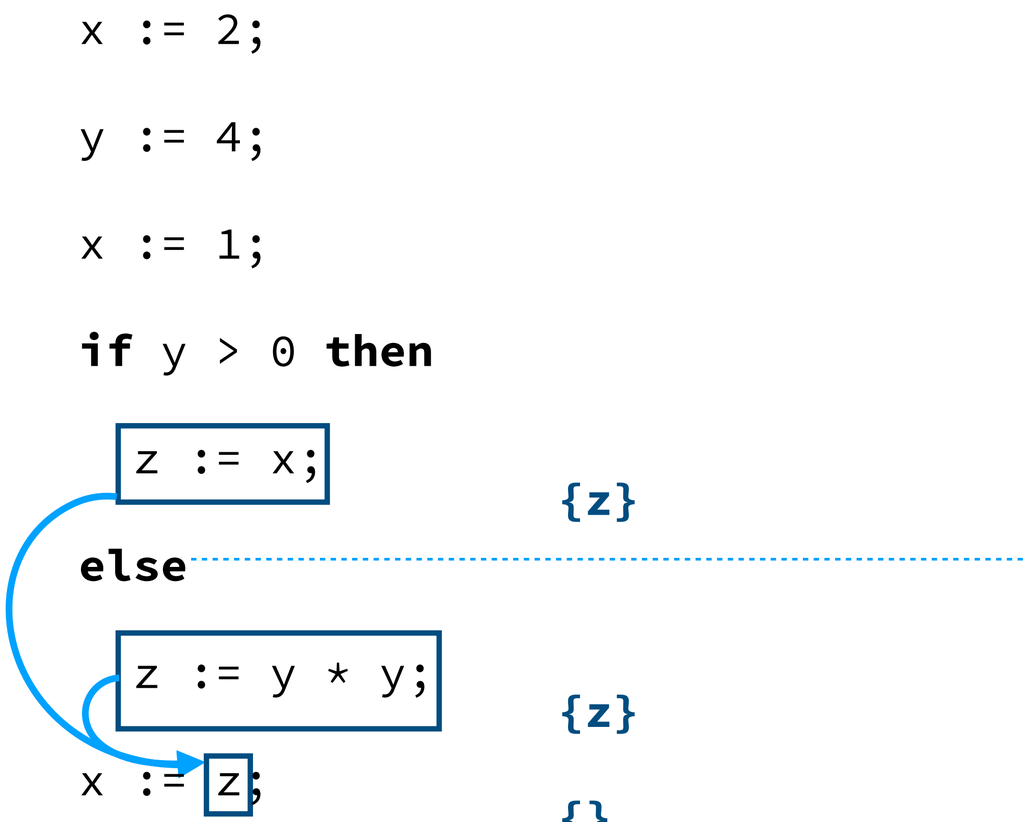

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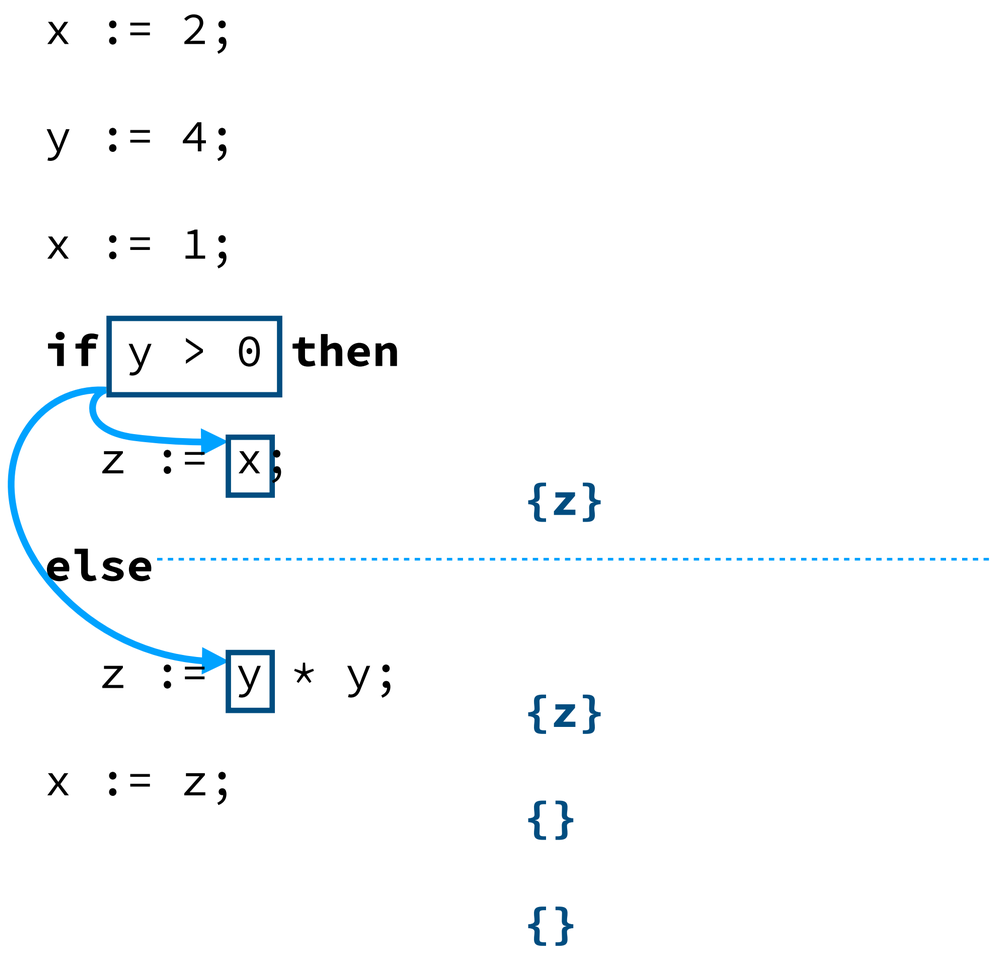

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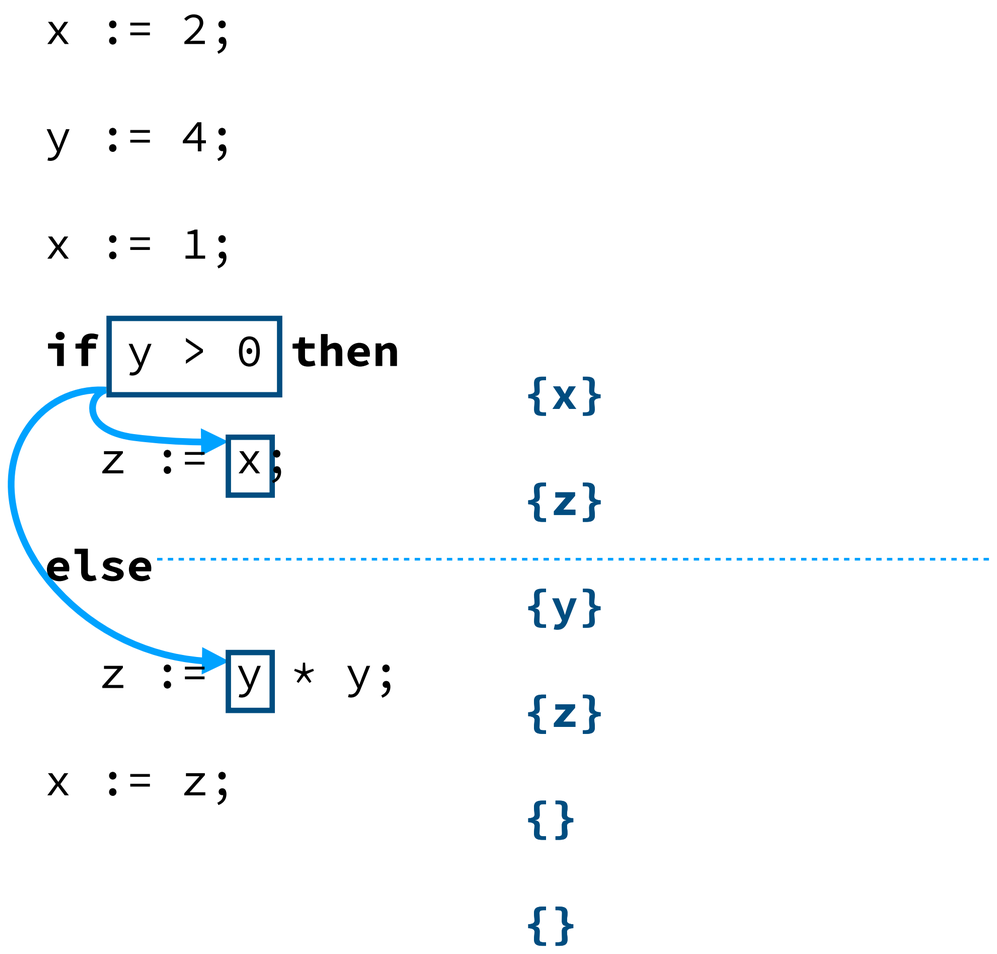

Live Variables in FlowSpec

A variable is *live* if the current value of the variable *may* be read further along in the program

```
properties
 live: MaySet(name)

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 live(Ref(n) -> next) =
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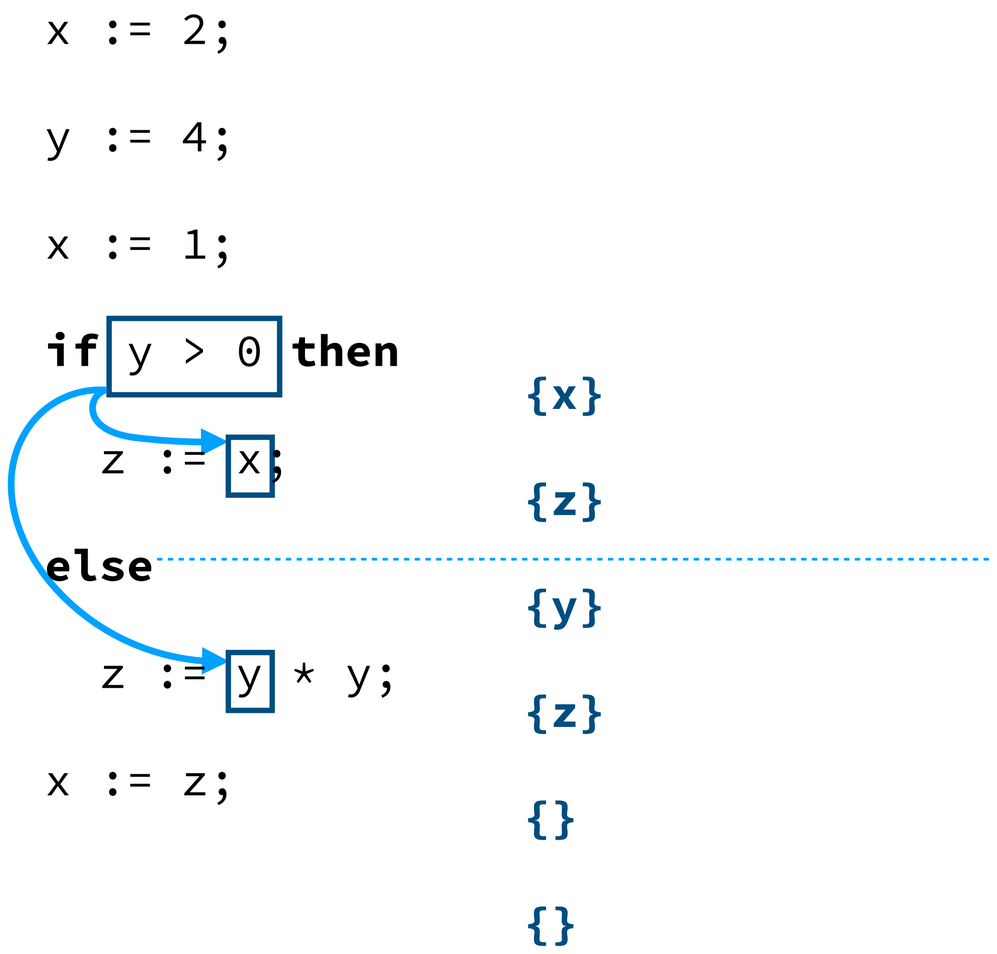

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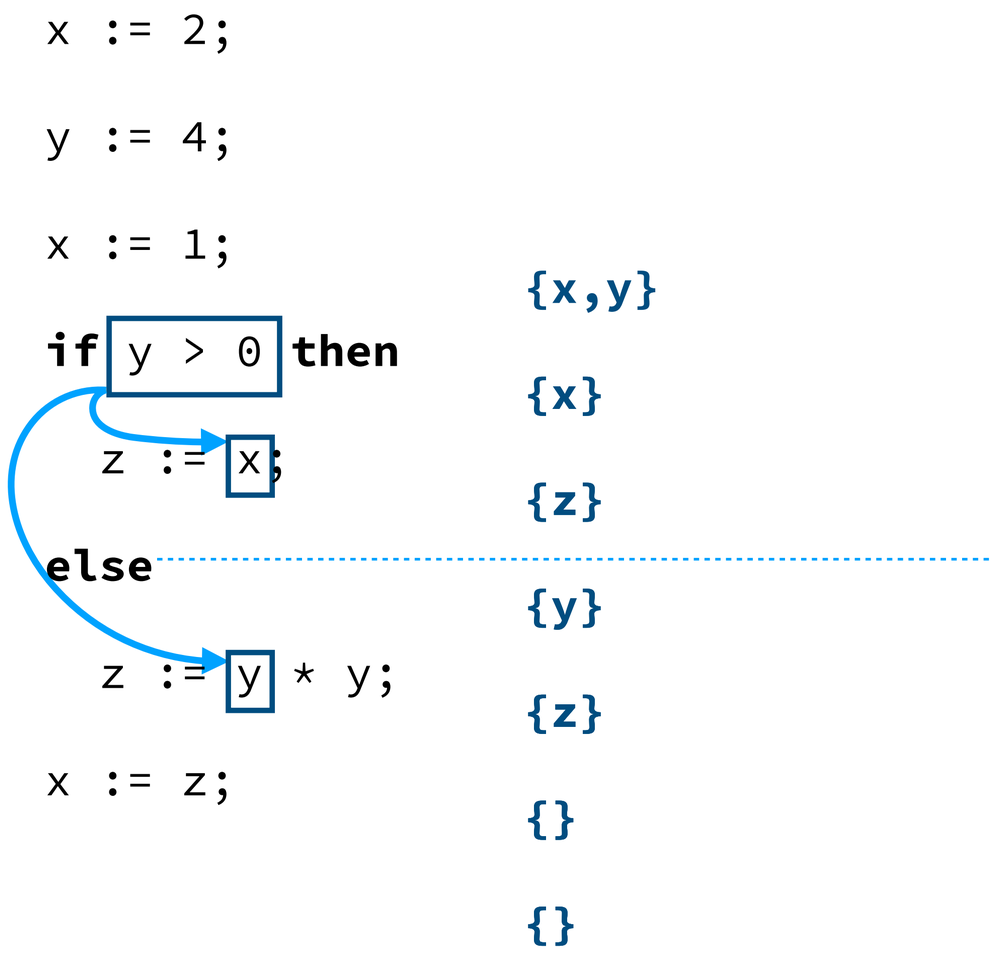

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

```
x := a + b

y := a * b

while y > a + b do (

    a := a + 1;

    x := a + b

)
```

Available Expressions in FlowSpec

An expression is *available* if it *must* have been evaluated previously and its variables not reassigned

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 available: MustSet(term)
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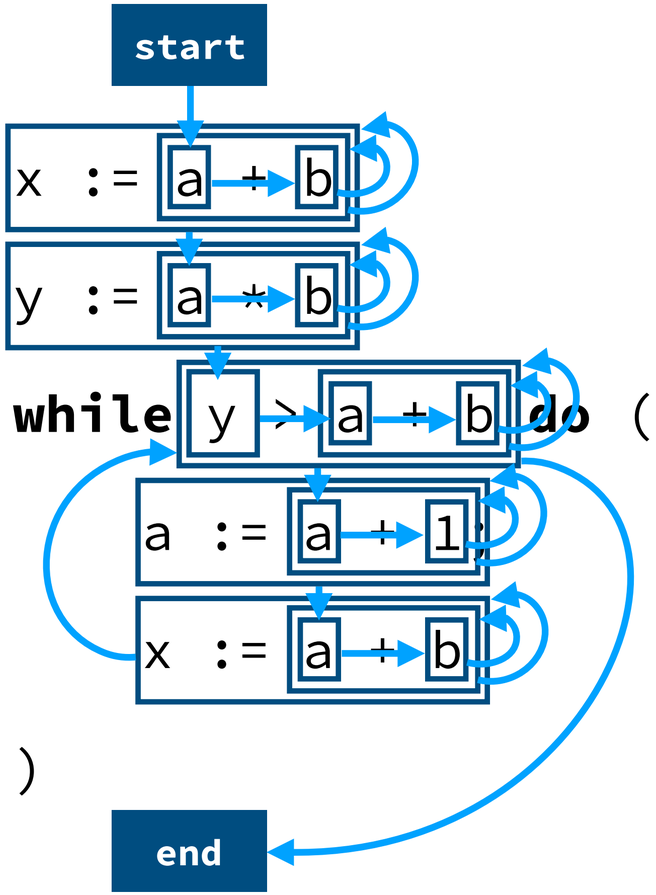

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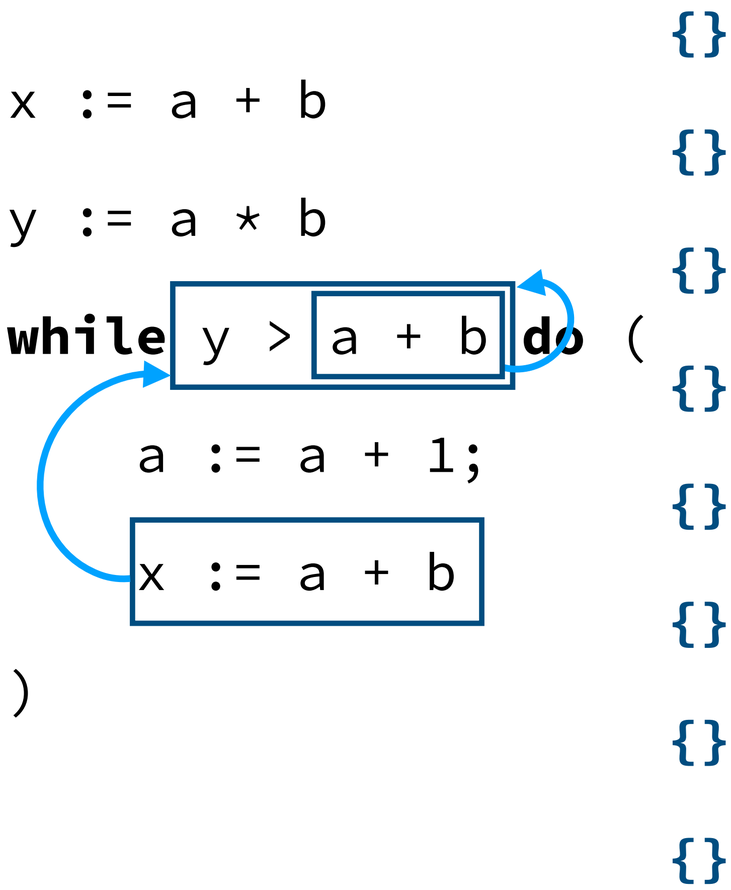

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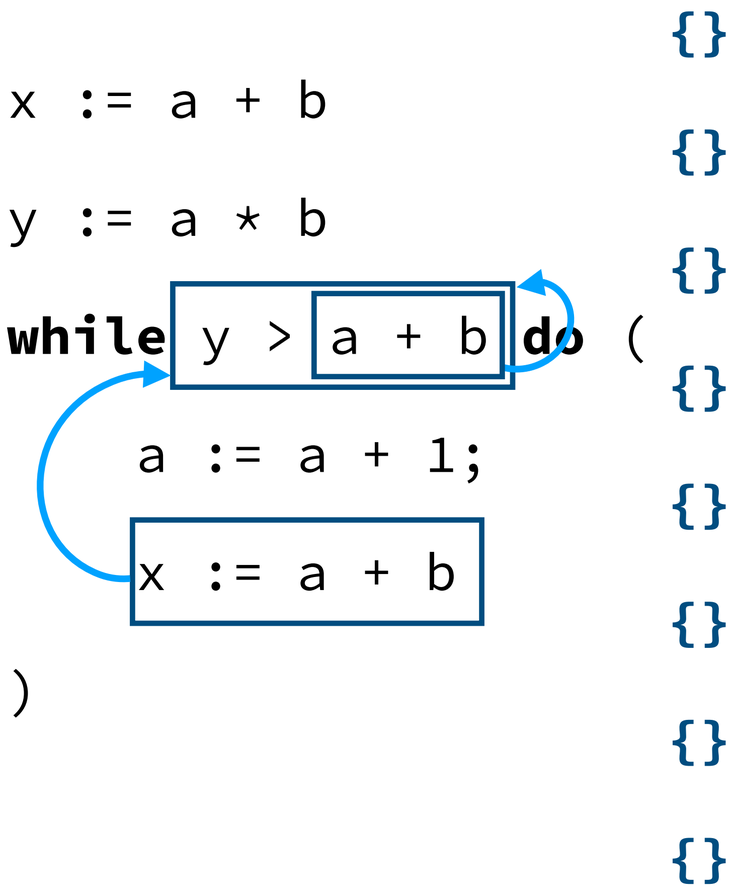

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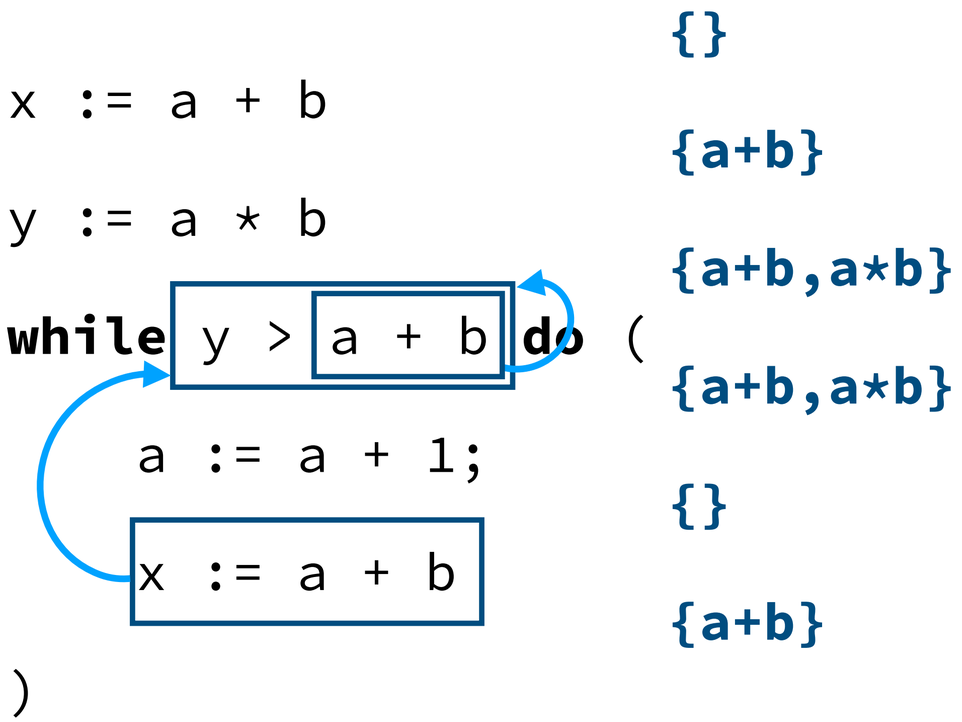

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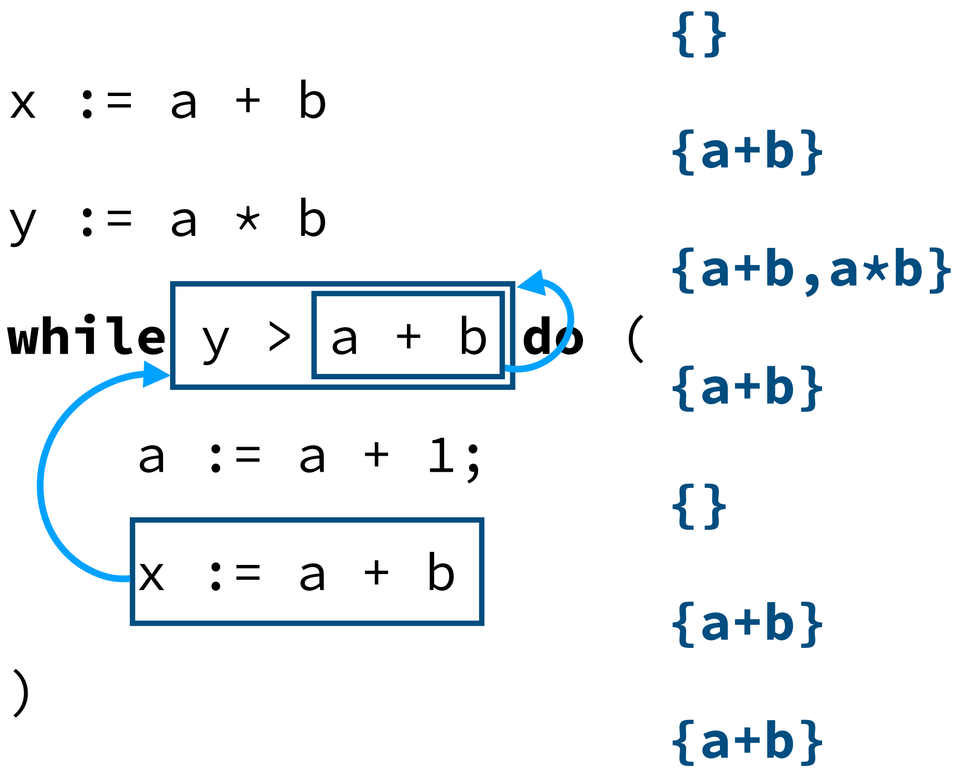

Conclusion

Summary

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Control-Flow

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- Order of execution

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- Reasoning about what is reachable

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Data-Flow

- Flow of data through a program
- Reasoning about data, and dependencies between data

FlowSpec

- Control-Flow rules to construct the graph
- Annotate with information from analysis by Data-Flow rules

Next Week

Monotone Frameworks

- The semantics behind FlowSpec
- A general framework for (intraprocedural, flow-sensitive) data-flow analysis