

# LL Parsing

## Traditional Parsing Algorithms

Eduardo Souza, Guido Wachsmuth, Eelco Visser

# Recap: Lexical Analysis

## lessons learned

What are the formalisms to describe regular languages?

- regular grammars
- regular expressions
- finite state automata

Why are these formalisms equivalent?

- constructive proofs

How can we generate compiler tools from that?

- implement DFAs
- generate transition tables

# Overview

## today's lecture

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efficient parsing algorithms

- predictive/recursive descent parsing
- LL parsing

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### efficient parsing algorithms

- predictive/recursive descent parsing
- LL parsing

### grammar classes

- LL(k) grammars
- LR(k) grammars

# Overview

## today's lecture

### efficient parsing algorithms

- predictive/recursive descent parsing
- LL parsing

### grammar classes

- LL(k) grammars
- LR(k) grammars

### more top-down parsing algorithms

- Parser Combinators
- Parsing Expression Grammars
- ALL(\*)

# I

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## Predictive (Recursive Descent) Parsing

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# Recap: A Theory of Language

formal languages





# Recap: A Theory of Language

## formal languages

vocabulary  $\Sigma$

finite, nonempty set of elements (words, letters)

alphabet



# Recap: A Theory of Language

## formal languages

vocabulary  $\Sigma$

finite, nonempty set of elements (words, letters)

alphabet

string over  $\Sigma$

finite sequence of elements chosen from  $\Sigma$

word, sentence, utterance



# Recap: A Theory of Language

## formal languages

vocabulary  $\Sigma$

finite, nonempty set of elements (words, letters)

alphabet

string over  $\Sigma$

finite sequence of elements chosen from  $\Sigma$

word, sentence, utterance

formal language  $\lambda$

set of strings over a vocabulary  $\Sigma$

$$\lambda \subseteq \Sigma^*$$



# Recap: A Theory of Language

## formal grammars



# Recap: A Theory of Language

## formal grammars

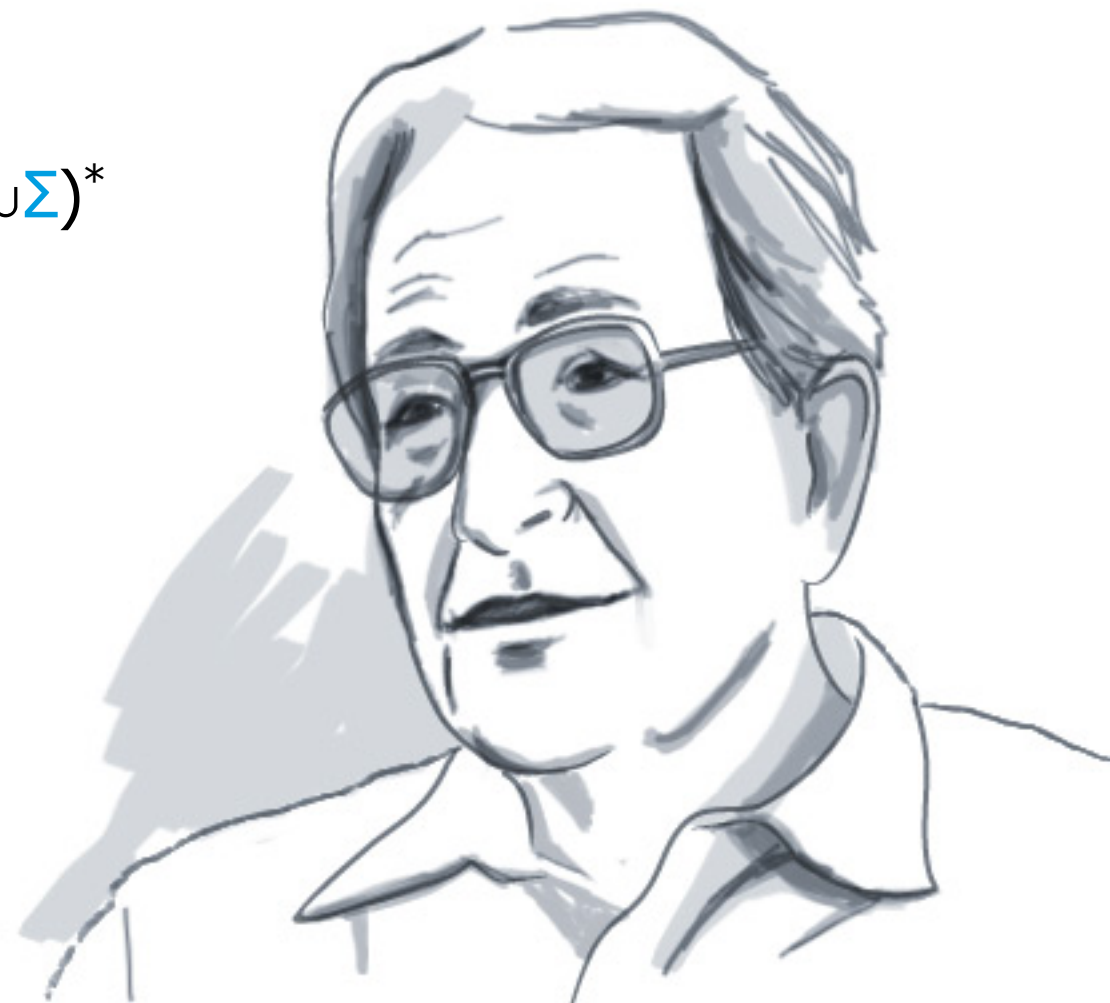
formal grammar  $G = (N, \Sigma, P, S)$

nonterminal symbols  $N$

terminal symbols  $\Sigma$

production rules  $P \subseteq (N \cup \Sigma)^* N (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

start symbol  $S \in N$



# Recap: A Theory of Language

## formal grammars

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start symbol  $S \in N$

nonterminal symbol





# Recap: A Theory of Language

## formal grammars

formal grammar  $G = (N, \Sigma, P, S)$

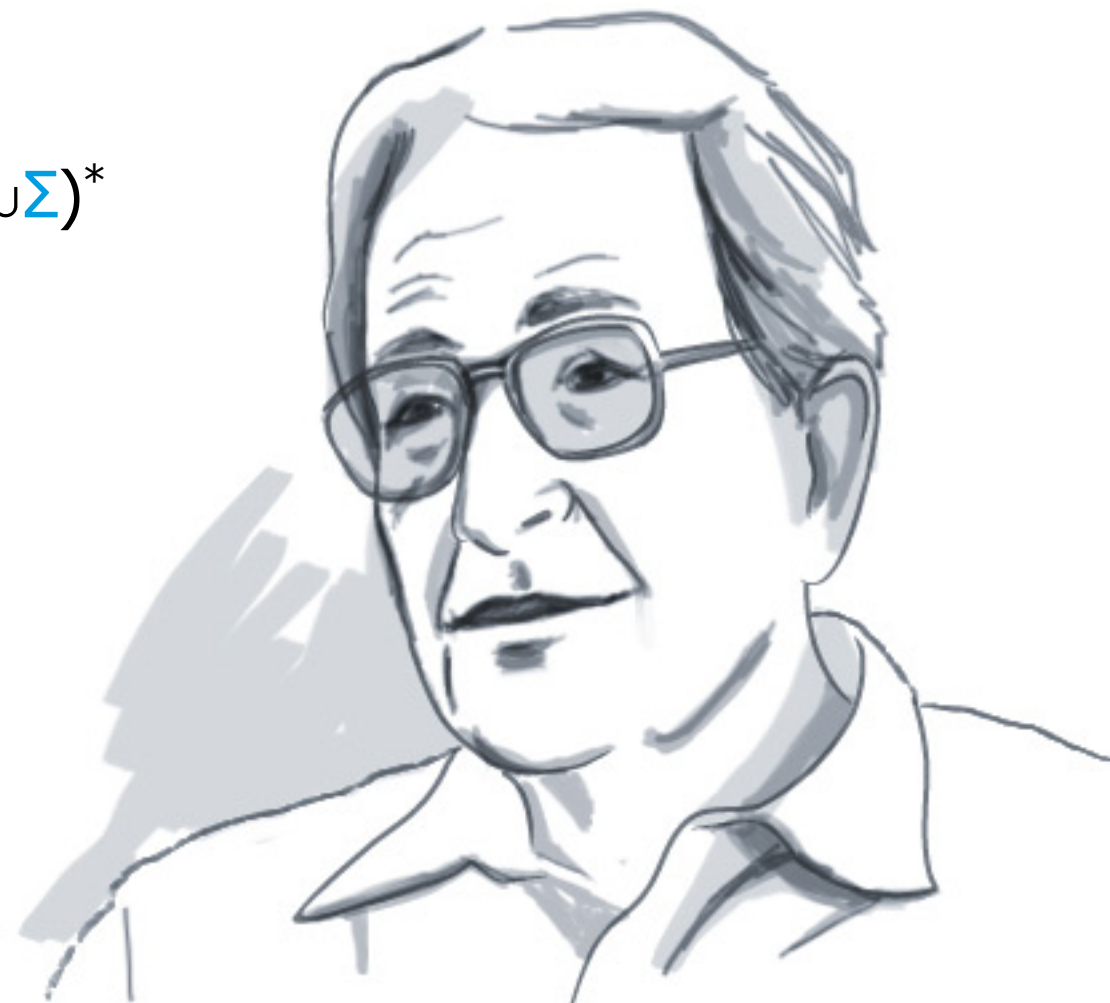
nonterminal symbols  $N$

terminal symbols  $\Sigma$

production rules  $P \subseteq \boxed{(N \cup \Sigma)^*} N \boxed{(N \cup \Sigma)^*} \times (N \cup \Sigma)^*$

start symbol  $S \in N$

context



# Recap: A Theory of Language

## formal grammars

formal grammar  $G = (N, \Sigma, P, S)$

nonterminal symbols  $N$

terminal symbols  $\Sigma$

production rules  $P \subseteq (N \cup \Sigma)^* N (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

start symbol  $S \in N$

replacement





# Recap: A Theory of Language

## formal grammars

formal grammar  $G = (N, \Sigma, P, S)$

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terminal symbols  $\Sigma$

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start symbol  $S \in N$

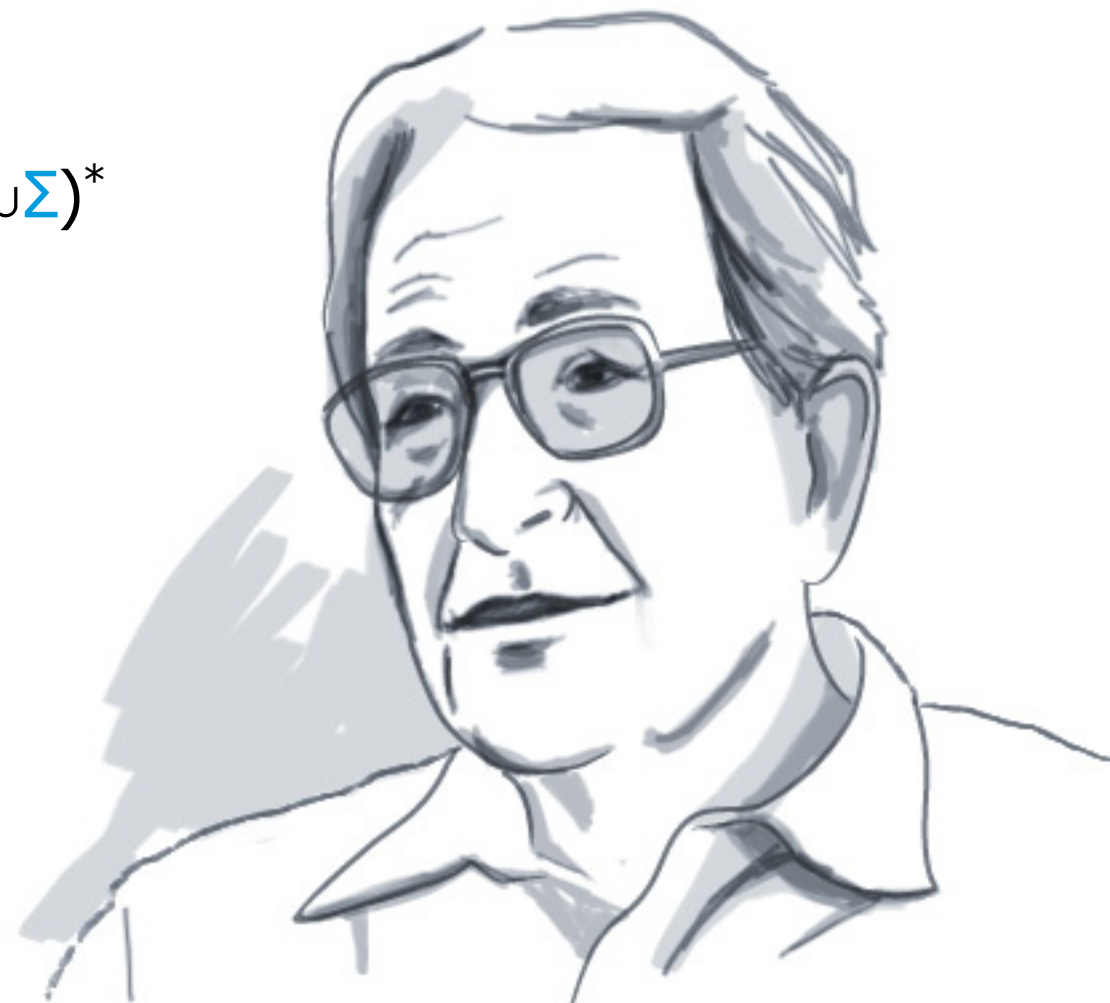
grammar classes

type-0, unrestricted

type-1, context-sensitive:  $(a A c, a b c)$

type-2, context-free:  $P \subseteq N \times (N \cup \Sigma)^*$

type-3, regular:  $(A, x)$  or  $(A, xB)$



# Recap: A Theory of Language

formal languages



# Recap: A Theory of Language

## formal languages

formal grammar  $G = (N, \Sigma, P, S)$



# Recap: A Theory of Language

## formal languages

formal grammar  $G = (N, \Sigma, P, S)$

derivation relation  $\Rightarrow_G \subseteq (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

$$w \Rightarrow_G w' \Leftrightarrow$$

$$\exists (p, q) \in P: \exists u, v \in (N \cup \Sigma)^*:$$

$$w = u p v \wedge w' = u q v$$



# Recap: A Theory of Language

## formal languages

formal grammar  $G = (N, \Sigma, P, S)$

derivation relation  $\Rightarrow_G \subseteq (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

$$W \Rightarrow_G W' \Leftrightarrow$$

$$\exists (p, q) \in P: \exists u, v \in (N \cup \Sigma)^*:$$

$$W = u p v \wedge W' = u q v$$

formal language  $L(G) \subseteq \Sigma^*$

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow_G^* w\}$$



# Recap: A Theory of Language

## formal languages

formal grammar  $G = (N, \Sigma, P, S)$

derivation relation  $\Rightarrow_G \subseteq (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

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formal language  $L(G) \subseteq \Sigma^*$

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow_G^* w\}$$

classes of formal languages





# Predictive parsing

## recursive descent

$\text{Exp} \rightarrow \text{"while"} \text{ Exp } \text{"do"} \text{ Exp}$

```
public void parseExp() {  
    consume(WHILE);  
    parseExp();  
    consume(DO);  
    parseExp();  
}
```



# Predictive parsing

## look ahead

Exp  $\rightarrow$  "while" Exp "do" Exp

Exp  $\rightarrow$  "if" Exp "then" Exp "else" Exp

```
public void parseExp() {  
    switch current() {  
        case WHILE: consume(WHILE); parseExp(); ...; break;  
        case IF    : consume(IF);   parseExp(); ...; break;  
        default   : error();  
    }  
}
```





# Predictive parsing

## parse table

rows

- nonterminal symbols  $N$
- symbol to parse

columns

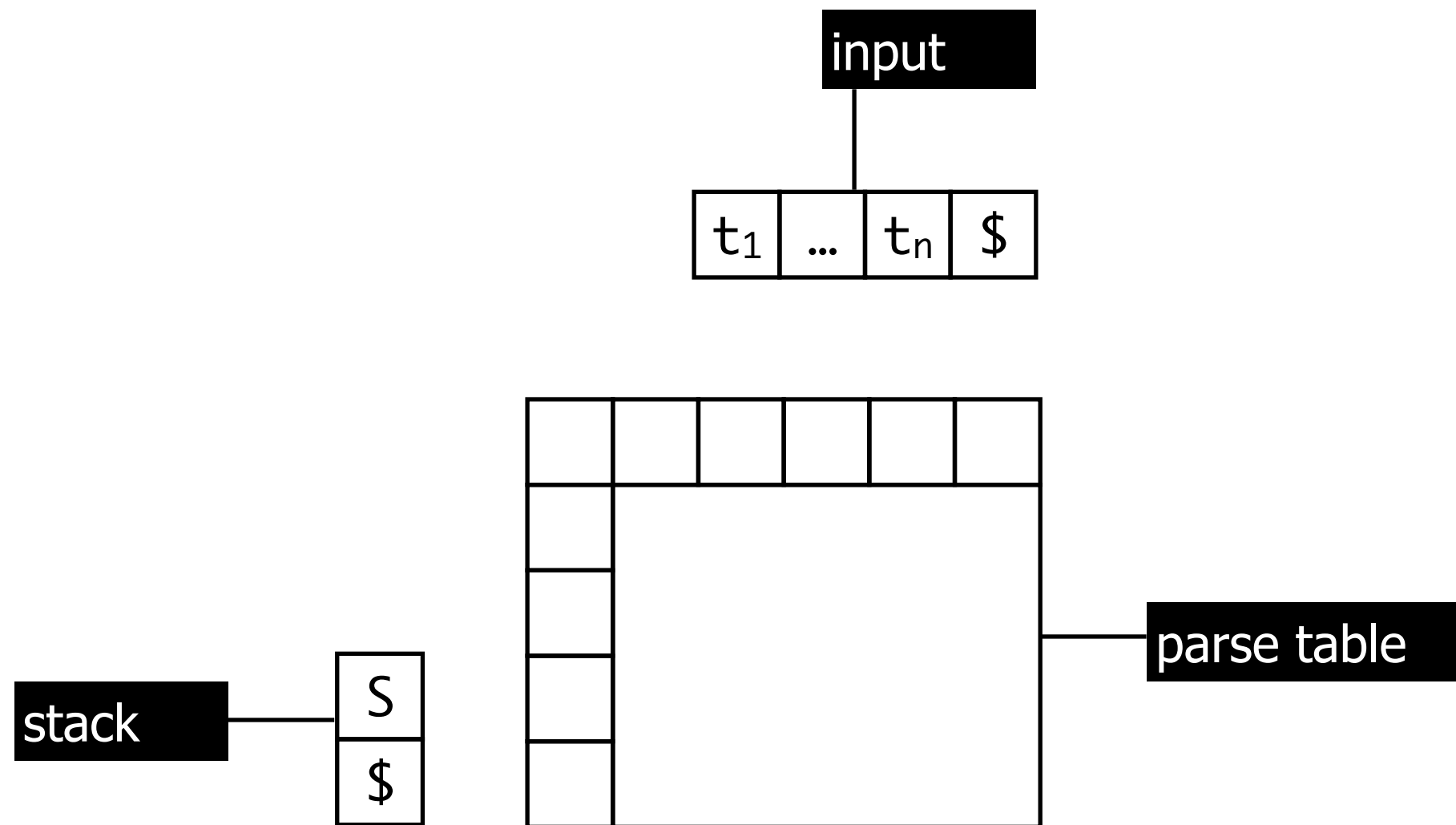
- terminal symbols  $\Sigma^k$
- look ahead  $k$

entries

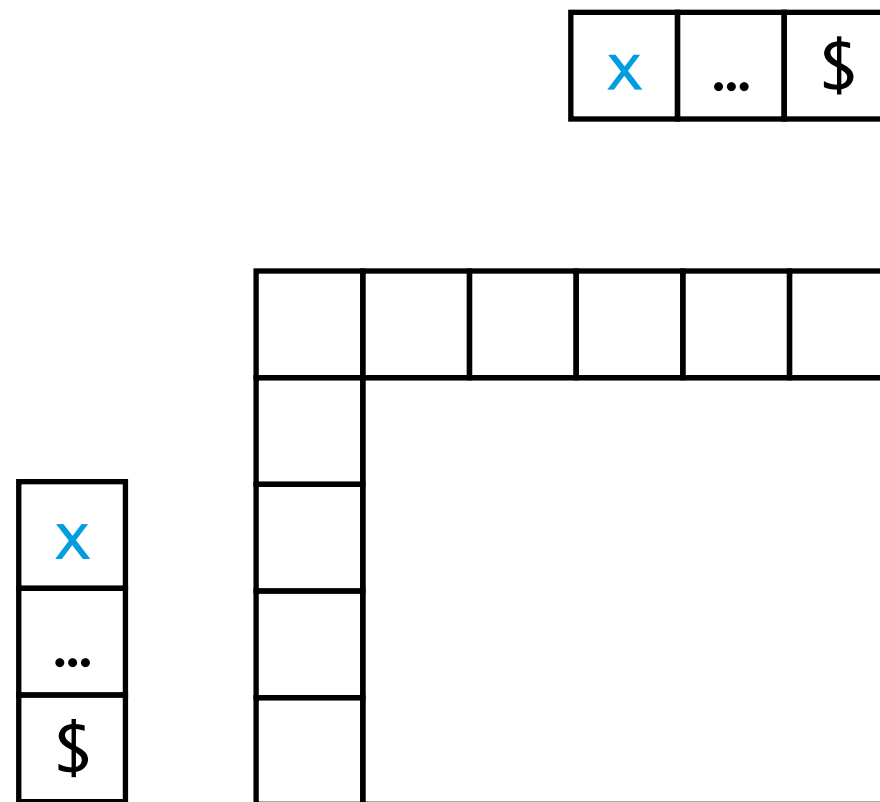
- production rules  $P$
- possible conflicts

	$T_1$	$T_2$	$T_3$	...
$N_1$	$N_1 \rightarrow \dots$		$N_1 \rightarrow \dots$	
$N_2$		$N_2 \rightarrow \dots$		
$N_3$		$N_3 \rightarrow \dots$	$N_3 \rightarrow \dots$	
$N_4$	$N_4 \rightarrow \dots$			
$N_5$		$N_5 \rightarrow \dots$		
$N_6$	$N_6 \rightarrow \dots$	$N_6 \rightarrow \dots$		
$N_7$			$N_7 \rightarrow \dots$	
$N_8$	$N_8 \rightarrow \dots$	$N_8 \rightarrow \dots$	$N_8 \rightarrow \dots$	
...				

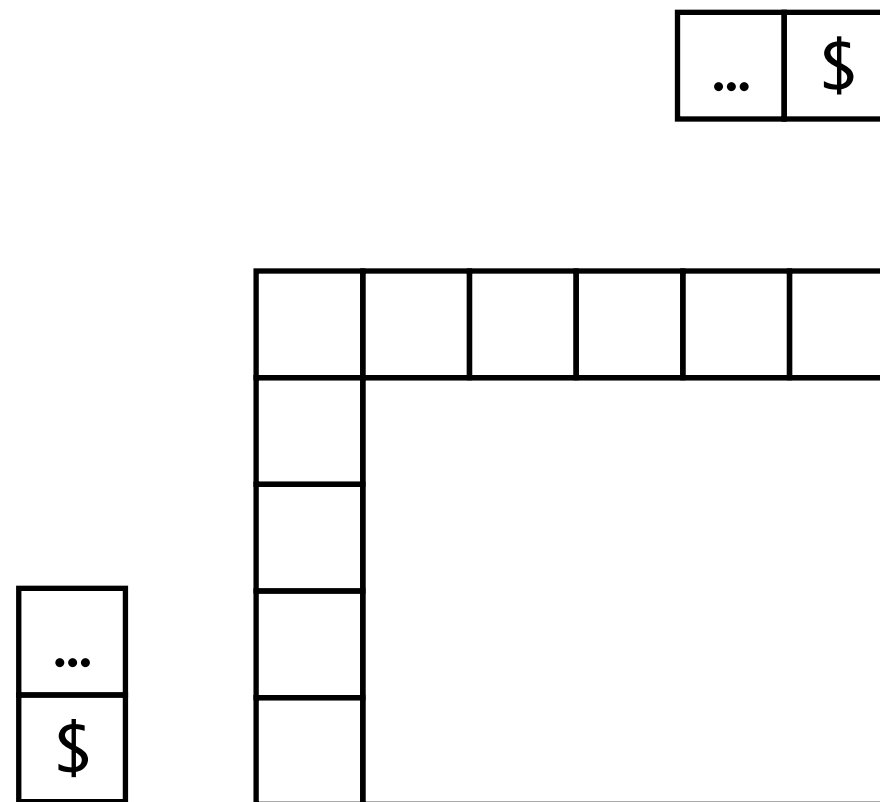
# Predictive parsing automaton



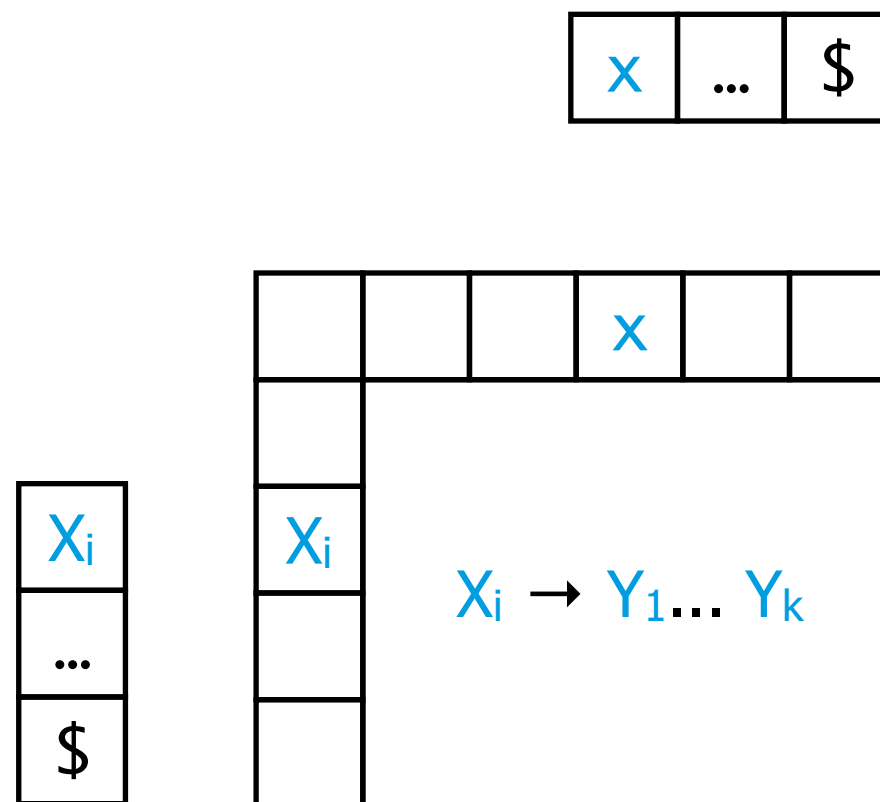
# Predictive parsing automaton



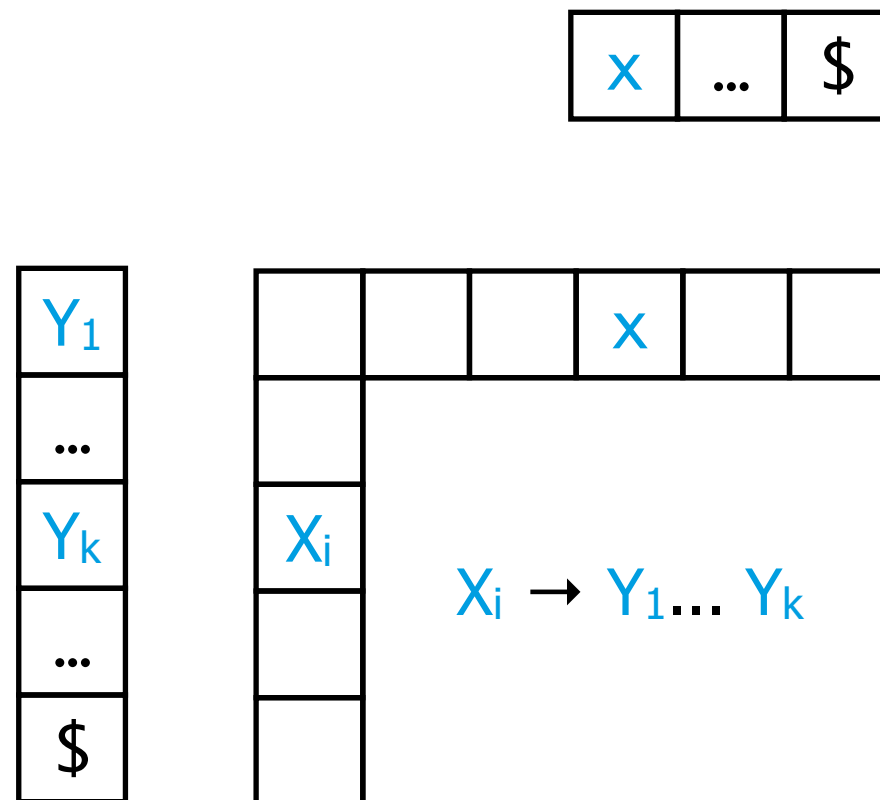
# Predictive parsing automaton



# Predictive parsing automaton



# Predictive parsing automaton



# II

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## LL Parse Tables

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# Predictive parsing

## filling the table

entry  $(X, w) \in P$  at row  $X$  and column  $T$

$$T \in \text{FIRST}(w)$$

$$\text{nullable}(w) \wedge T \in \text{FOLLOW}(X)$$



# Predictive parsing

## filling the table

entry  $(X, w) \in P$  at row  $X$  and column  $T$

$T \in \boxed{\text{FIRST}(w)}$  — letters that  $w$  can start with

$\text{nullable}(w) \wedge T \in \text{FOLLOW}(X)$

# Predictive parsing

## filling the table

entry  $(X, w) \in P$  at row  $X$  and column  $T$

$$T \in \text{FIRST}(w)$$

$$\boxed{\text{nullable}(w)} \wedge T \in \text{FOLLOW}(X)$$

$$W \Rightarrow_{G^*} \varepsilon$$

# Predictive parsing

## filling the table

entry  $(X, w) \in P$  at row  $X$  and column  $T$

$$T \in \text{FIRST}(w)$$

$$\text{nullable}(w) \wedge T \in \text{FOLLOW}(X) \text{ — letters that can follow } X$$

# Predictive parsing

## nullable

nullable( $X$ )

$$(X, \varepsilon) \in P \Rightarrow \text{nullable}(X)$$

$$(X_0, X_1 \dots X_k) \in P \wedge \text{nullable}(X_1) \wedge \dots \wedge \text{nullable}(X_k) \Rightarrow \text{nullable}(X_0)$$

nullable( $w$ )

$$\text{nullable}(\varepsilon)$$

$$\text{nullable}(X_1 \dots X_k) = \text{nullable}(X_1) \wedge \dots \wedge \text{nullable}(X_k)$$

# Predictive parsing

## first sets

### FIRST(X)

$$X \in \Sigma : \text{FIRST}(X) = \{X\}$$

$$(X_0, X_1 \dots X_i \dots X_k) \in P \wedge \text{nullable}(X_1 \dots X_i) \Rightarrow \text{FIRST}(X_0) \supseteq \text{FIRST}(X_{i+1})$$

### FIRST(w)

$$\text{FIRST}(\varepsilon) = \{\}$$

$$\neg \text{nullable}(X) \Rightarrow \text{FIRST}(Xw) = \text{FIRST}(X)$$

$$\text{nullable}(X) \Rightarrow \text{FIRST}(Xw) = \text{FIRST}(X) \cup \text{FIRST}(w)$$

# Predictive parsing

## follow sets

**FOLLOW**(X)

$(X_0, X_1 \dots X_i \dots X_k) \in P \wedge \text{nullable}(X_{i+1} \dots X_k) \Rightarrow \text{FOLLOW}(X_i) \supseteq \text{FOLLOW}(X_0)$

$(X_0, X_1 \dots X_i \dots X_k) \in P \Rightarrow \text{FOLLOW}(X_i) \supseteq \text{FIRST}(X_{i+1} \dots X_k)$

# Example

p0: Start  $\rightarrow$  Exp EOF

p1: Exp  $\rightarrow$  Term Exp'

p2: Exp'  $\rightarrow$  "+" Term Exp'

p3: Exp'  $\rightarrow$

p4: Term  $\rightarrow$  Fact Term'

p5: Term'  $\rightarrow$  "\*" Fact Term'

p6: Term'  $\rightarrow$

p7: Fact  $\rightarrow$  Num

p8: Fact  $\rightarrow$  "(" Exp ")"

	nullable	FIRST	FOLLOW
Start			
Exp			
Exp'			
Term			
Term'			
Fact			

# Example

## nullable

- p0: Start  $\rightarrow$  Exp EOF
- p1: Exp  $\rightarrow$  Term Exp'
- p2: Exp'  $\rightarrow$  "+" Term Exp'
- p3: Exp'  $\rightarrow$
- p4: Term  $\rightarrow$  Fact Term'
- p5: Term'  $\rightarrow$  "\*" Fact Term'
- p6: Term'  $\rightarrow$
- p7: Fact  $\rightarrow$  Num
- p8: Fact  $\rightarrow$  "(" Exp ")"

$$(X, \varepsilon) \in P \Rightarrow \text{nullable}(X)$$

$$(X_0, X_1 \dots X_k) \in P \wedge$$

$$\text{nullable}(X_1) \wedge \dots \wedge \text{nullable}(X_k) \Rightarrow \text{nullable}(X_0)$$

	nullable	FIRST	FOLLOW
Start			
Exp			
Exp'			
Term			
Term'			
Fact			



# Example

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- p0: Start  $\rightarrow$  Exp EOF
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$$(X, \varepsilon) \in P \Rightarrow \text{nullable}(X)$$

$$(X_0, X_1 \dots X_k) \in P \wedge$$

$$\text{nullable}(X_1) \wedge \dots \wedge \text{nullable}(X_k) \Rightarrow \text{nullable}(X_0)$$

	nullable	FIRST	FOLLOW
Start	no		
Exp	no		
Exp'	yes		
Term	no		
Term'	yes		
Fact	no		

# Example

## FIRST sets

- p0: Start  $\rightarrow$  Exp EOF
- p1: Exp  $\rightarrow$  Term Exp'
- p2: Exp'  $\rightarrow$  "+" Term Exp'
- p3: Exp'  $\rightarrow$
- p4: Term  $\rightarrow$  Fact Term'
- p5: Term'  $\rightarrow$  "\*" Fact Term'
- p6: Term'  $\rightarrow$
- p7: Fact  $\rightarrow$  Num
- p8: Fact  $\rightarrow$  "(" Exp ")"

$$(X_0, X_1 \dots X_i \dots X_k) \in P \wedge$$

$$\text{nullable}(X_1 \dots X_i) \Rightarrow \text{FIRST}(X_0) \supseteq \text{FIRST}(X_{i+1})$$

	nullable	FIRST	FOLLOW
Start	no		
Exp	no		
Exp'	yes		
Term	no		
Term'	yes		
Fact	no		

# Example

## FIRST sets

- p0: Start  $\rightarrow$  Exp EOF
- p1: Exp  $\rightarrow$  Term Exp'
- p2: Exp'  $\rightarrow$  "+" Term Exp'
- p3: Exp'  $\rightarrow$
- p4: Term  $\rightarrow$  Fact Term'
- p5: Term'  $\rightarrow$  "\*" Fact Term'
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- p7: Fact  $\rightarrow$  Num
- p8: Fact  $\rightarrow$  "(" Exp ")"

$$(X_0, X_1 \dots X_i \dots X_k) \in P \wedge$$

$$\text{nullable}(X_1 \dots X_i) \Rightarrow \text{FIRST}(X_0) \supseteq \text{FIRST}(X_{i+1})$$

	nullable	FIRST	FOLLOW
Start	no	Num (	
Exp	no	Num (	
Exp'	yes	+	
Term	no	Num (	
Term'	yes	*	
Fact	no	Num (	

# Example

## FOLLOW sets

$$(X_0, X_1 \dots X_i \dots X_k) \in P \wedge$$

$$\text{nullable}(X_{i+1} \dots X_k) \Rightarrow \text{FOLLOW}(X_i) \supseteq \text{FOLLOW}(X_0)$$

$$(X_0, X_1 \dots X_i \dots X_k) \in P \Rightarrow \text{FOLLOW}(X_i) \supseteq \text{FIRST}(X_{i+1} \dots X_k)$$

p0: Start  $\rightarrow$  Exp EOF

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p7: Fact  $\rightarrow$  Num

p8: Fact  $\rightarrow$  "(" Exp ")"

	nullable	FIRST	FOLLOW
Start	no	Num (	
Exp	no	Num (	
Exp'	yes	+	
Term	no	Num (	
Term'	yes	*	
Fact	no	Num (	

# Example

## FOLLOW sets

$$(X_0, X_1 \dots X_i \dots X_k) \in P \wedge$$

$$\text{nullable}(X_{i+1} \dots X_k) \Rightarrow \text{FOLLOW}(X_i) \supseteq \text{FOLLOW}(X_0)$$

$$(X_0, X_1 \dots X_i \dots X_k) \in P \Rightarrow \text{FOLLOW}(X_i) \supseteq \text{FIRST}(X_{i+1} \dots X_k)$$

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p6: Term'  $\rightarrow$

p7: Fact  $\rightarrow$  Num

p8: Fact  $\rightarrow$  "(" Exp ")"

	nullable	FIRST	FOLLOW
Start	no	Num (	
Exp	no	Num (	) EOF
Exp'	yes	+	) EOF
Term	no	Num (	+ ) EOF
Term'	yes	*	+ ) EOF
Fact	no	Num (	* + ) EOF

# Example

## LL parse table

entry  $(X, w) \in P$  at row  $X$  and column  $T$

$T \in \text{FIRST}(w)$

$\text{nullable}(w) \wedge T \in \text{FOLLOW}(X)$

p0: Start  $\rightarrow$  Exp EOF

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p4: Term  $\rightarrow$  Fact Term'

p5: Term'  $\rightarrow$  "\*" Fact Term'

p6: Term'  $\rightarrow$

p7: Fact  $\rightarrow$  Num

p8: Fact  $\rightarrow$  "(" Exp ")"

	+	*	Num	(	)	EOF
Start						
Exp						
Exp'						
Term						
Term'						
Fact						

# Example

## LL parse table

p0: Start  $\rightarrow$  Exp EOF  
 p1: Exp  $\rightarrow$  Term Exp'  
 p2: Exp'  $\rightarrow$  "+" Term Exp'  
 p3: Exp'  $\rightarrow$   
 p4: Term  $\rightarrow$  Fact Term'  
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entry  $(X, w) \in P$  at row  $X$  and column  $T$

$T \in \text{FIRST}(w)$

$\text{nullable}(w) \wedge T \in \text{FOLLOW}(X)$

	+	*	Num	(	)	EOF
Start			p0	p0		
Exp			p1	p1		
Exp'	p2				p3	p3
Term			p4	p4		
Term'	p6	p5			p6	p6
Fact			p7	p8		

# Example

parsing  $(4+3)*6$  EOF

p0: Start  $\rightarrow$  Exp EOF

p1: Exp  $\rightarrow$  Term Exp'

p2: Exp'  $\rightarrow$  "+" Term Exp'

p3: Exp'  $\rightarrow$

p4: Term  $\rightarrow$  Fact Term'

p5: Term'  $\rightarrow$  "\*" Fact Term'

p6: Term'  $\rightarrow$

p7: Fact  $\rightarrow$  Num

p8: Fact  $\rightarrow$  "(" Exp ")"

	+	*	Num	(	)	EOF
Start			p0	p0		
Exp			p1	p1		
Exp'	p2				p3	p3
Term			p4	p4		
Term'	p6	p5			p6	p6
Fact			p7	p8		



# Grammar classes

context-free grammars

# Grammar classes

context-free grammars

LL(0)

# Grammar classes

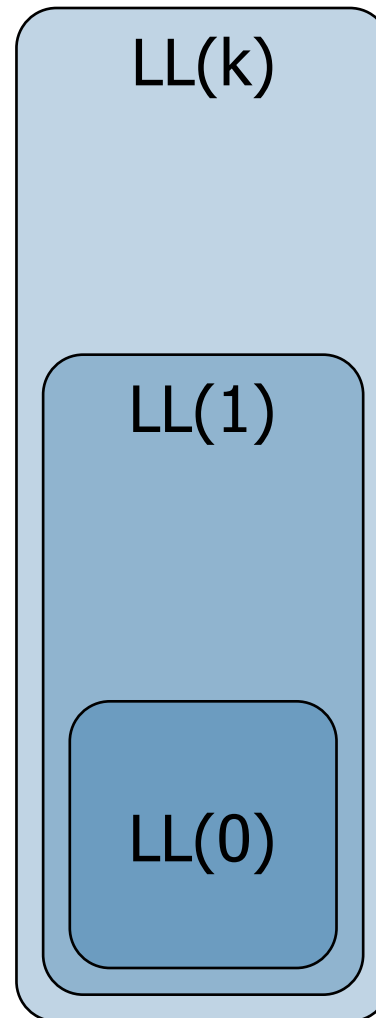
context-free grammars

LL(1)

LL(0)

# Grammar classes

context-free grammars



# Predictive parsing

## encoding precedence

$\text{Exp} \rightarrow \text{Num}$

$\text{Exp} \rightarrow "(" \text{Exp} ") "$

$\text{Exp} \rightarrow \text{Exp} "*" \text{Exp}$

$\text{Exp} \rightarrow \text{Exp} "+" \text{Exp}$

$\text{Fact} \rightarrow \text{Num}$

$\text{Fact} \rightarrow "(" \text{Exp} ") "$

$\text{Term} \rightarrow \text{Term} "*" \text{Fact}$

$\text{Term} \rightarrow \text{Fact}$

$\text{Exp} \rightarrow \text{Exp} "+" \text{Term}$

$\text{Exp} \rightarrow \text{Term}$



# Predictive parsing

## eliminating left recursion

$\text{Term} \rightarrow \text{Term} "*" \text{Fact}$

$\text{Term} \rightarrow \text{Fact}$

$\text{Exp} \rightarrow \text{Exp} "+" \text{Term}$

$\text{Exp} \rightarrow \text{Term}$

$\text{Term}' \rightarrow "*" \text{Fact Term}'$

$\text{Term}' \rightarrow$

$\text{Term} \rightarrow \text{Fact Term}'$

$\text{Exp}' \rightarrow "+" \text{Term Exp}'$

$\text{Exp}' \rightarrow$

$\text{Exp} \rightarrow \text{Term Exp}'$





# Predictive parsing

## left factoring

Exp  $\rightarrow$  "if" Exp "then" Exp "else" Exp

Exp  $\rightarrow$  "if" Exp "then" Exp

Exp  $\rightarrow$  "if" Exp "then" Exp Else

Else  $\rightarrow$  "else" Exp

Else  $\rightarrow$



# III

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## Parser Combinators

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# Parser Combinators

- Parsers are modeled as functions, and larger parsers are built from smaller ones using higher-order functions.
- Can be implemented in any lazy functional language with higher-order style type system (e.g. Haskell, Scala).
- Parser combinators enable a recursive descent parsing strategy that facilitates modular piecewise construction and testing.

# Parser Combinators

type Parser = String -> Tree

# Parser Combinators

What about intermediate parsers?

```
type Parser = String -> Tree
```

# Parser Combinators

Return the remainder of the input!

~~type Parser = String -> Tree~~

type Parser = String -> (Tree, String)

# Parser Combinators

What about parsers that fail?

~~type Parser = String -> Tree~~

type Parser = String -> (Tree, String)

# Parser Combinators

Returns either a singleton or  
an empty list!

~~type Parser = String -> Tree~~

~~type Parser = String -> (Tree, String)~~

type Parser = String -> [(Tree, String)]

# Parser Combinators

~~type Parser = String -> Tree~~

~~type Parser = String -> (Tree, String)~~

~~type Parser = String -> [(Tree, String)]~~

type Parser a = String -> [(a, String)]

A parser  $p$  returns either a singleton list  $[(a, \text{String})]$  indicating success, or an empty list  $[]$  indicating failure. Returning a list of results, also allow to express ambiguities.

# Primitive Parsers

success

```
success :: a -> Parser a
success v = \inp -> [(v, inp)]
```

or

```
success :: a String -> [(a, String)]
success v inp -> [(v, inp)]
```

fail

```
fail :: Parser a
fail = \inp -> []
```

match

```
match :: Parser Char
match = \inp -> case inp of
    []      -> []
    (x:xs) -> [(x,xs)]
```



# Parser Combinators

alternation

```
alt :: Parser a -> Parser a -> Parser a
p 'alt' q = \inp -> (p inp ++ q inp)
```

sequencing

```
seq :: Parser a -> Parser b -> Parser (a, b)
p 'seq' q = \inp -> [((v,w), inp'') | (v, inp') <- p inp
                                     , (w, inp'') <- q inp']
```

# Parser Combinators

- Parser combinators can use semantic actions to manipulate intermediate results, being able to produce values rather than trees.
- More powerful combinators can be build to allow parsing context-sensitive languages (by passing the context together with parsing results).
- Parser Combinators in Scala:  
A. Moors, F. Piessens, Martin Odersky. Parser combinators in Scala. Technical Report Department of Computer Science, K.U. Leuven, February 2008.

# IV

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## Parsing Expression Grammars

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# Parsing Expression Grammars (PEGs)

Rules are of the form:

$$A \leftarrow \mathbf{e}$$

where  $A$  is a non-terminal and  $\mathbf{e}$  is a parsing expression.

# PEG operators

Operator	Type	Precedence	Description
' '	primary	5	Literal string
" "	primary	5	Literal string
[ ]	primary	5	Character class
.	primary	5	Any character
( <i>e</i> )	primary	5	Grouping
<i>e</i> ?	unary suffix	4	Optional
<i>e</i> *	unary suffix	4	Zero-or-more
<i>e</i> +	unary suffix	4	One-or-more
& <i>e</i>	unary prefix	3	And-predicate
! <i>e</i>	unary prefix	3	Not-predicate
<i>e</i> <sub>1</sub> <i>e</i> <sub>2</sub>	binary	2	Sequence
<i>e</i> <sub>1</sub> / <i>e</i> <sub>2</sub>	binary	1	Prioritized Choice

# Syntactic Predicates

**&e** and **!e** : preserves the knowledge whether e fails or not.

Comment  $\leftarrow$  `'//'` (!EndOfLine `.`)\* EndOfLine

# Parsing Expression Grammars

- Prioritized choice '/'.

$S \leftarrow \mathbf{a\ b\ /\ a}$

$S \leftarrow \mathbf{a\ /\ a\ b}$

# Parsing Expression Grammars

- Prioritized choice '/'.

$$S \leftarrow \mathbf{a\,b} / \mathbf{a} \quad \neq \quad S \leftarrow \mathbf{a} / \mathbf{a\,b}$$

- Backtracks when parsing an alternative fails.
- Uses memoization of intermediate results to parse in linear time, called packrat parsing.
- Does not allow left-recursion.



# V

---

ANTLR / ALL(\*)

---

# LL(\*)

- Parses LL-regular grammars: for any given non-terminal, parsers can use the entire remaining of the input to differentiate the alternative productions.
- Statically builds a DFA from the grammar productions to recognize lookahead.
- When in conflict, chooses the first alternative and backtracks when DFA lookahead recognition fails.

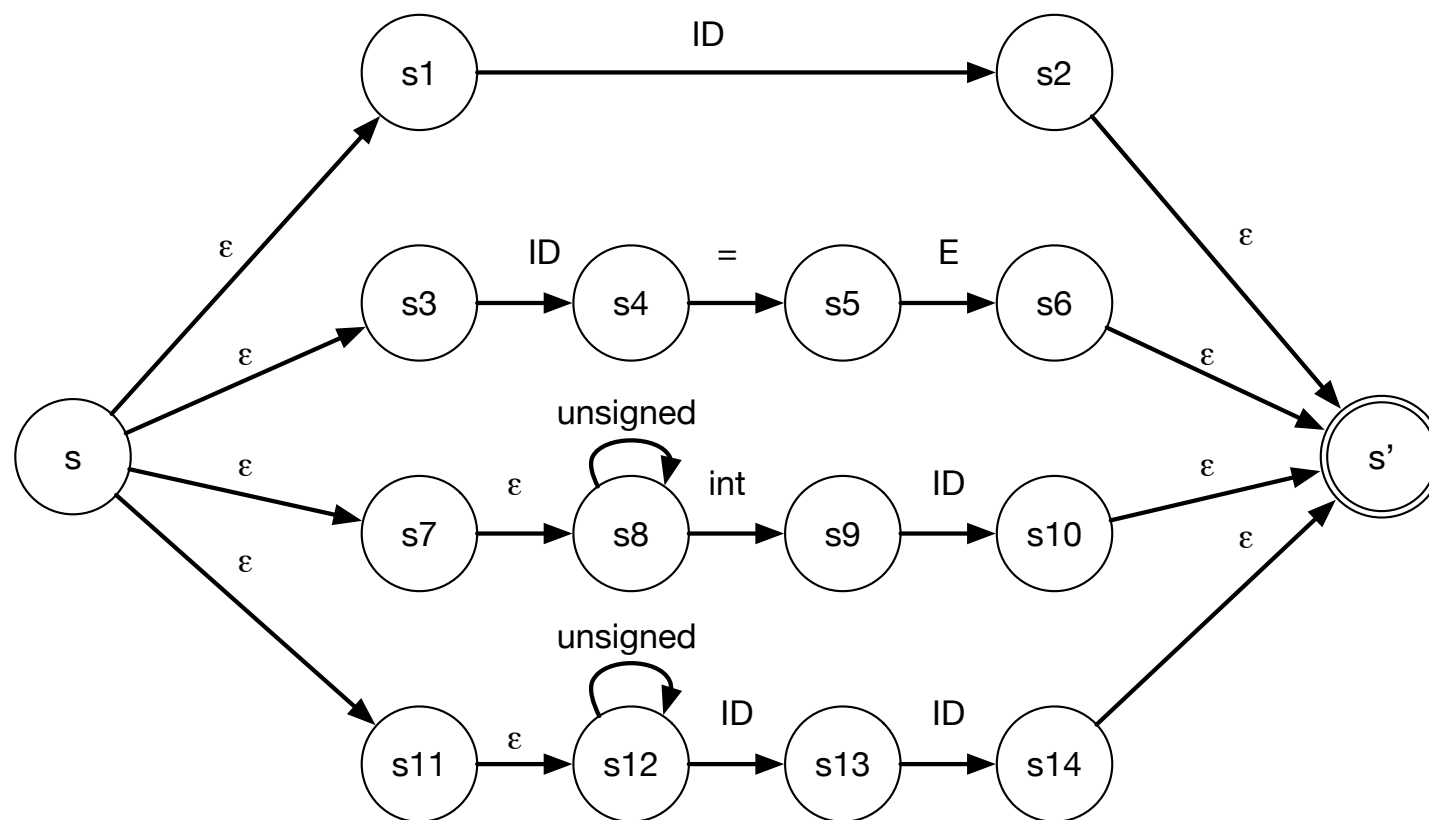
# LL(\*) Grammar Analysis

S : ID  
| ID = E  
| **unsigned\*** **int** ID  
| **unsigned\*** ID ID

# LL(\*) Grammar Analysis

terminals: ID, =, **int**, **unsigned**

- S : ID (1)  
| ID = E (2)  
| **unsigned\* int** ID (3)  
| **unsigned\* ID ID** (4)



augmented recursive  
transition network (ATN)

# LL(\*) Grammar Analysis

terminals: ID, =, **int**, **unsigned**

S : ID (1)  
| ID = E (2)  
| **unsigned**\* **int** ID (3)  
| **unsigned**\* ID ID (4)

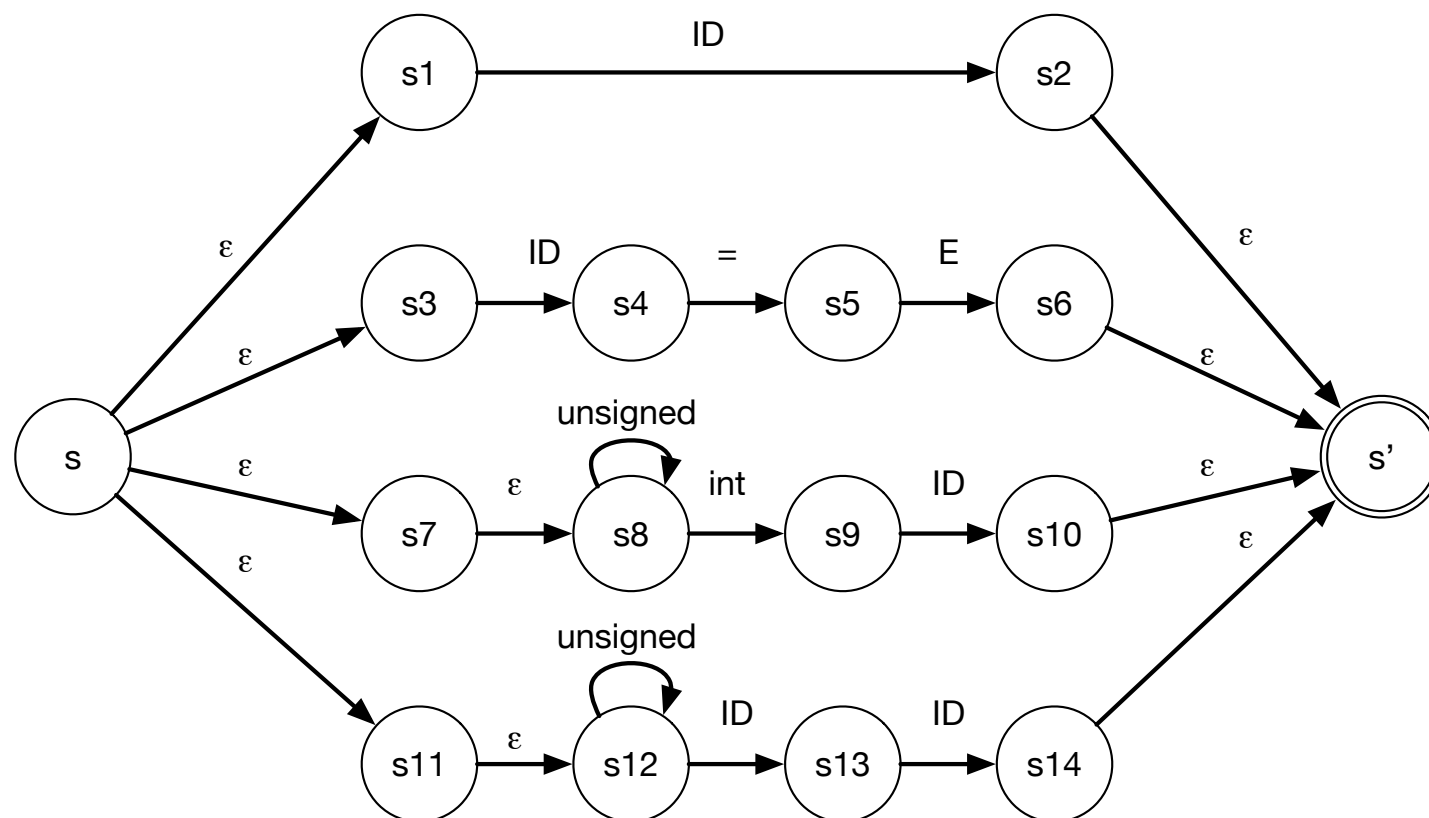
converts ATN to DFA

(i) Initial state = closure(s);

(ii) Create a final state for each production;

(iii) While DFA state contains ATN states from multiple paths, create transitions with terminals (or EOF if DFA contains s');

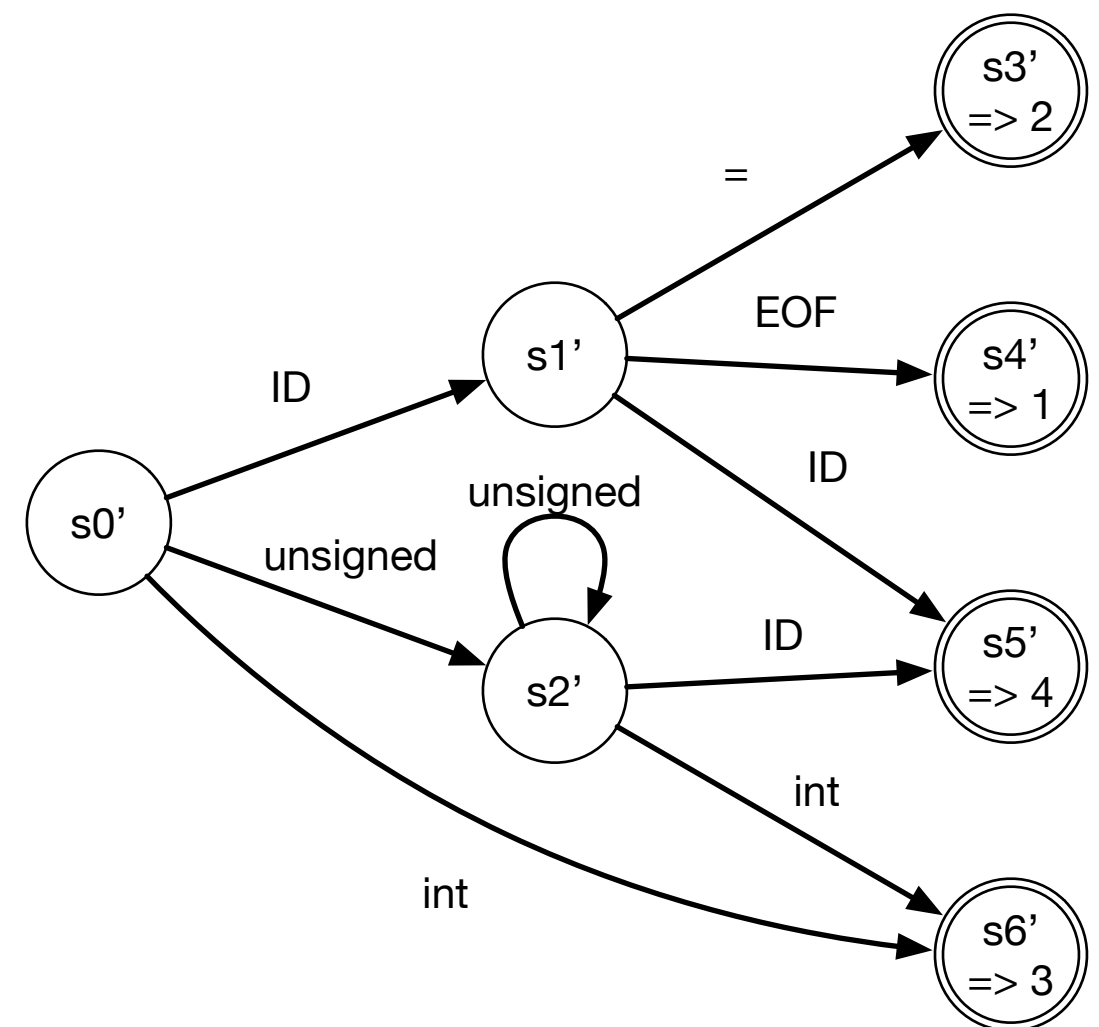
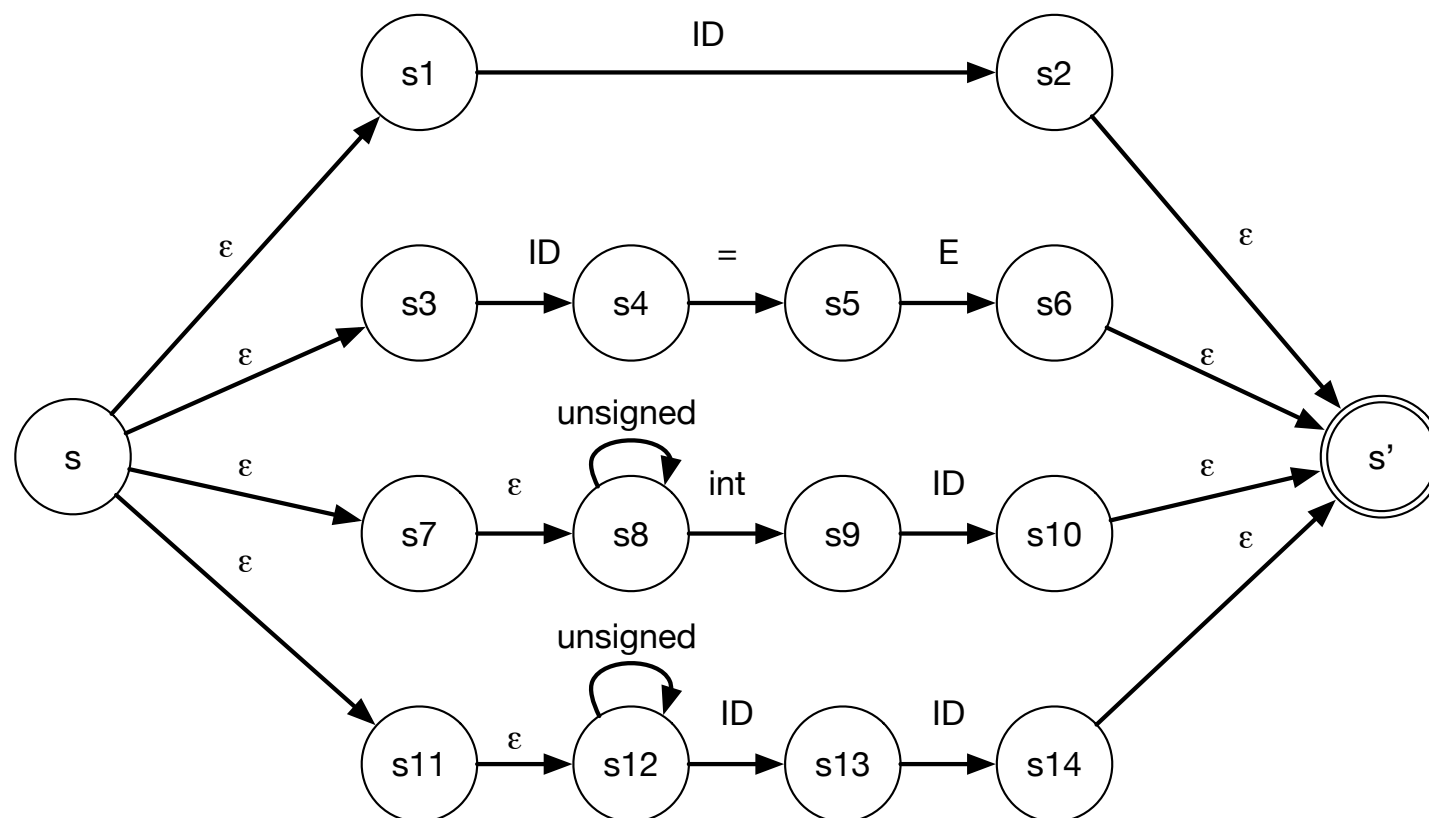
(iv) If DFA state contain ATN states from a single path, add transition to corresponding final DFA state;



# LL(\*) Grammar Analysis

terminals: ID, =, **int**, **unsigned**

- S : ID (1)  
 | ID = E (2)  
 | **unsigned**\* **int** ID (3)  
 | **unsigned**\* ID ID (4)



# Adaptive LL(\*)

```
S : E = E ;  
    | E ;  
  
E : E * E  
    | E + E  
    | E ( E )  
    | ID
```

```
void S() {  
    switch (adaptivePredict("S", callStack)){  
        case 1 :  
            E(); match('='); E();  
            match(';'); break;  
        case 2:  
            E(); match(';'); break;  
    }  
}
```

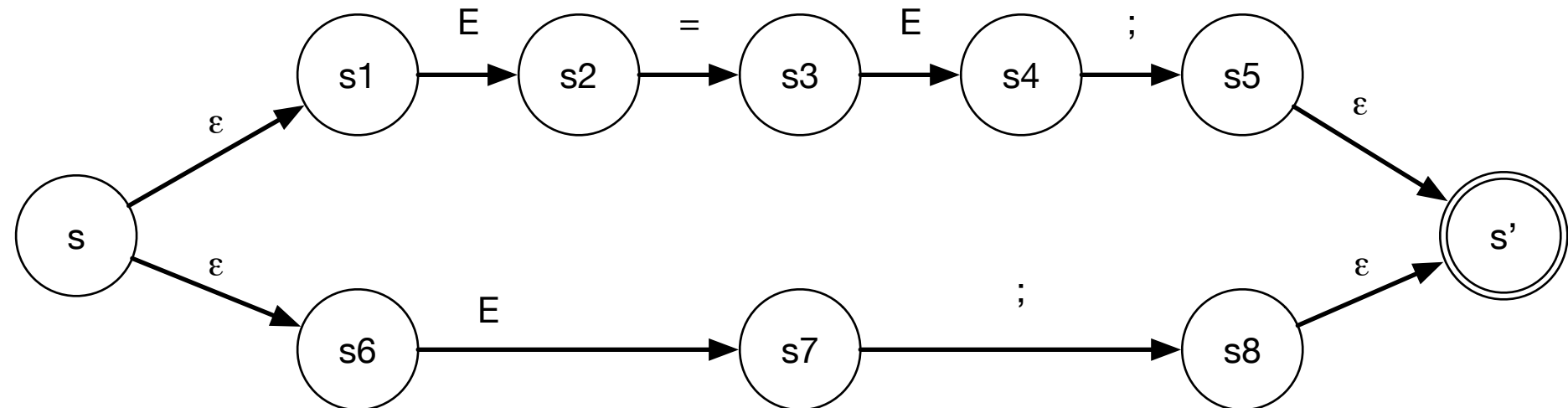
# Adaptive LL(\*)

Dynamically builds the lookahead DFA.

$S : E = E ;$   
 $| E ;$

$E : E * E$   
 $| E + E$   
 $| E ( E )$   
 $| ID$

terminals:  $=, ;, *, +, (, ), ID$



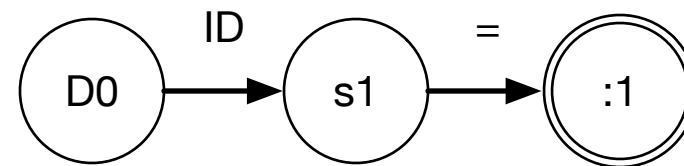


# Adaptive LL(\*)

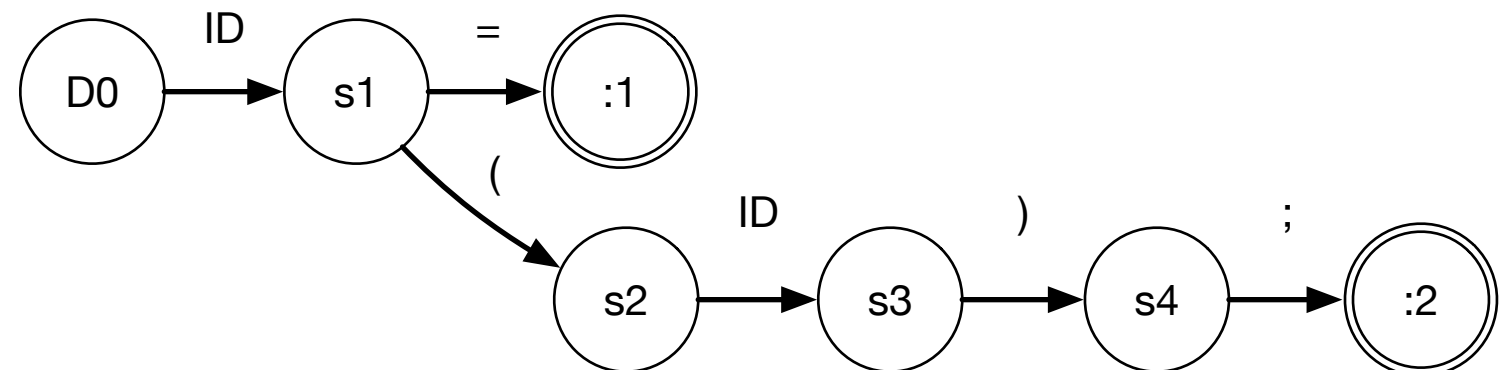
Dynamically builds the lookahead DFA.

After parsing:

`x = y;`



`f(x);`



# Adaptive LL(\*)

parse: **yba**

S : **x** B  
| **y** C

B : A **a**

C : A **b a**

A : **b**  
|  $\epsilon$

# Adaptive LL(\*)

input: **yba**

$S : \mathbf{x} B$   
      | **y** C

stack: S

$B : A \mathbf{a}$

$C : A \mathbf{b} a$

$A : \mathbf{b}$   
      |  $\epsilon$

# Adaptive LL(\*)

input: **yba**

$S : \mathbf{x} B$   
      | **y** C

stack: **yC**

$B : A \mathbf{a}$

$C : A \mathbf{b} a$

$A : \mathbf{b}$   
      |  $\epsilon$

# Adaptive LL(\*)

input: **ba**

$S : \mathbf{x} B$   
      |  $\mathbf{y} C$

stack: C

$B : A \mathbf{a}$

$C : A \mathbf{b} \mathbf{a}$

$A : \mathbf{b}$   
      |  $\epsilon$

# Adaptive LL(\*)

input: **ba**

S : **x** B  
| **y** C

stack: A **ba**

B : A **a**

C : A **b a**

A : **b**  
|  $\epsilon$

# Adaptive LL(\*)

input: **ba**

$S : \mathbf{x} B$   
       $| \mathbf{y} C$

stack: A **ba**

$B : A \mathbf{a}$

$C : A \mathbf{b} \mathbf{a}$

$b \in \text{First}(A)$   
 $b \in \text{Follow}(A)$

$A : \mathbf{b} \quad A_1$   
       $| \epsilon \quad A_2$

$A_1$  or  $A_2$ ?

# Adaptive LL(\*)

- Tries to parse using **strong LL** (without looking at the call stack), when there are multiple alternatives, splits the parser to try all possibilities.
- Parsing continues with a graph, handling multiple stacks in parallel to explore all alternatives.
- Eventually, if the grammar is unambiguous, only one stack "survives". If multiple stacks survive, an ambiguity has been found and the parser picks the production with lower value.



# ANTLR 4

- Accepts as input any context-free grammar that does not contain indirect or hidden left-recursion.
- Generates ALL(\*) parses in Java or C#.
- Grammars contain lexical and syntactic rules and generate a lexer and a parser.
- ANTLR4 grammars can be scannerless and composable by using individual characters as input symbols.

# VI

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## Summary

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# Summary

## lessons learned

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How can we parse context-free languages effectively?

- predictive parsing algorithms

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Which grammar classes are supported by these algorithms?

- LL(k) grammars, LL(k) languages

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How can we generate compiler tools from that?

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- generate parse tables

# Summary

## lessons learned

How can we parse context-free languages effectively?

- predictive parsing algorithms

Which grammar classes are supported by these algorithms?

- LL(k) grammars, LL(k) languages

How can we generate compiler tools from that?

- implement automaton
- generate parse tables

What are other techniques for implementing top-down parsers?

- Parser Combinators
- PEGs
- ALL(\*)

# Literature

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## formal languages

Noam Chomsky: Three models for the description of language. 1956

J. E. Hopcroft, R. Motwani, J. D. Ullman: Introduction to Automata Theory, Languages, and Computation. 2006

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J. E. Hopcroft, R. Motwani, J. D. Ullman: Introduction to Automata Theory, Languages, and Computation. 2006

## syntactical analysis

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

Alfred V. Aho, Ravi Sethi, Jeffrey D. Ullman, Monica S. Lam: Compilers: Principles, Techniques, and Tools, 2nd edition. 2006

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ALL(\*)

Terence John Parr, Sam Harwell, Kathleen Fisher. Adaptive LL(\*) parsing: the power of dynamic analysis. In OOPSLA 2014.

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## Parsing Expression Grammars

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## Parser Combinators

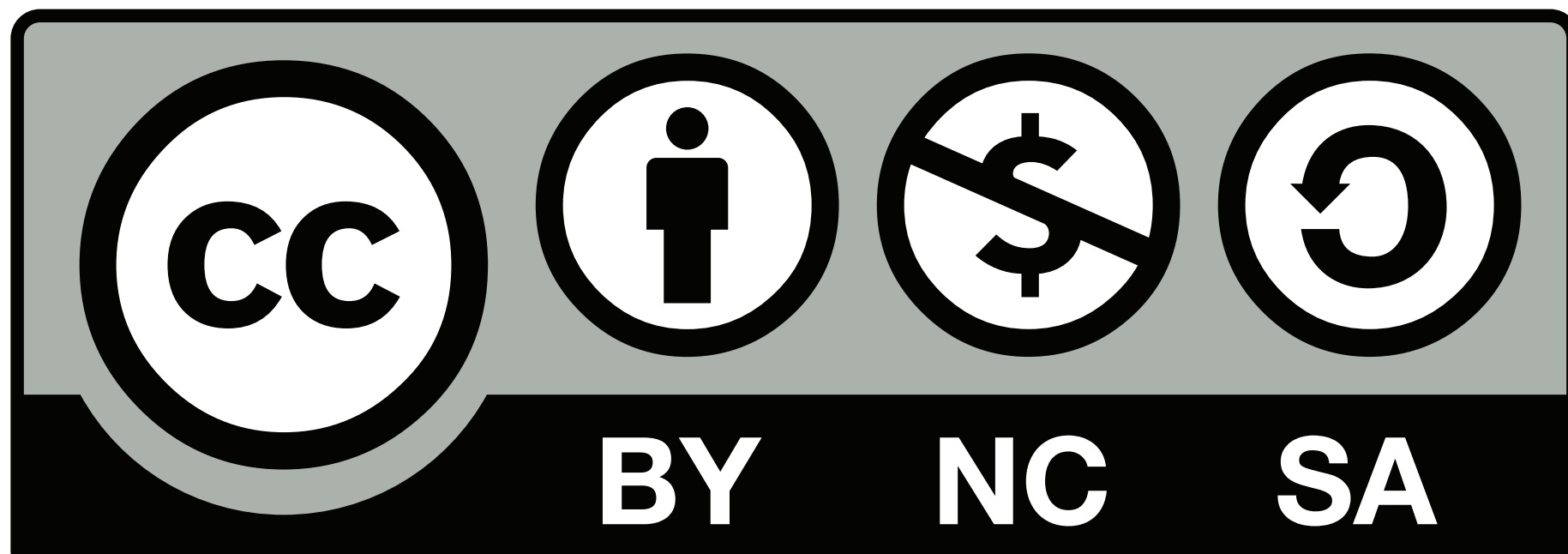
Graham Hutton. Higher-Order Functions for Parsing. Journal of Functional Programming, 1992.

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