

LR Parsing

Traditional Parsing Algorithms

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Recap: Traditional Parsing Algorithms

lessons learned

How can we parse context-free languages effectively?

- predictive parsing

Which grammar classes are supported by these algorithms?

- LL(k) grammars, LL(k) languages

How can we generate compiler tools from that?

- implement automaton
- generate parse tables

What are other techniques for implementing top-down parsers?

- Parser Combinators
- PEGs
- ALL(*)

Overview

today's lecture

Overview

today's lecture

efficient parsing algorithms

- LR parsing
- LR parse table generation
- SLR & LALR parse tables
- Generalized LR parsing
- Scannerless Generalized LR parsing

I

LR Parsing

LR parsing

idea

problems with LL parsing

- predicting right rule
- left recursion

LR parsing

- see whole right-hand side of a rule
- look ahead
- shift or reduce

LR parsing

example

stack

\$

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$ — grammar

input

7	*	3	+	7	*	3	\$
---	---	---	---	---	---	---	----

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$

7	*	3	+	7	*	3	\$
---	---	---	---	---	---	---	----

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7
----	---

*	3	+	7	*	3	\$
---	---	---	---	---	---	----

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7
\$	E

*	3	+	7	*	3	\$
*	3	+	7	*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7	
\$	E	*

*	3	+	7	*	3	\$
	3	+	7	*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3

*	3	+	7	*	3	\$
		+	7	*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E

*	3	+	7	*	3	\$
		+	7	*	3	\$
		+	7	*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E		

*	3	+	7	*	3	\$
		+	7	*	3	\$
		+	7	*	3	\$
		+	7	*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E	+	

*	3	+	7	*	3	\$
		+	7	*	3	\$
		+	7	*	3	\$
			7	*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E	+	7

*	3	+	7	*	3	\$
		+	7	*	3	\$
		+	7	*	3	\$
				*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

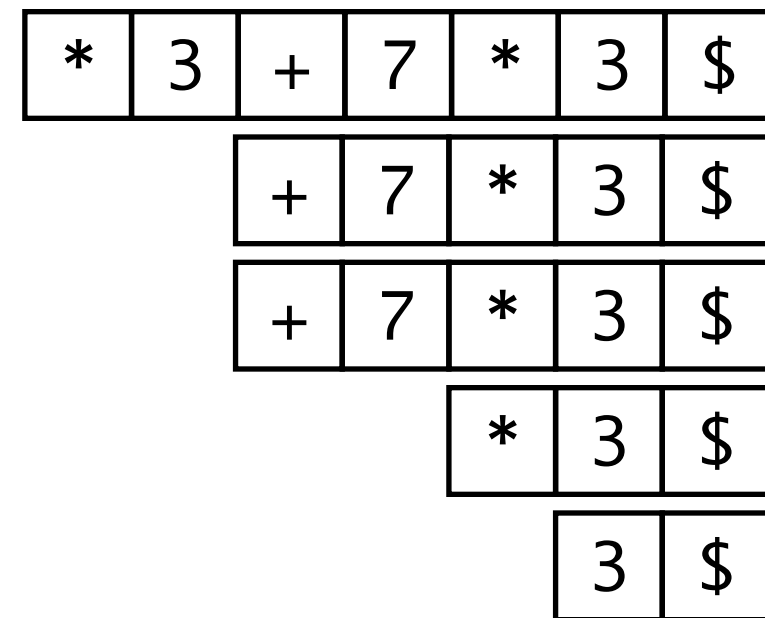
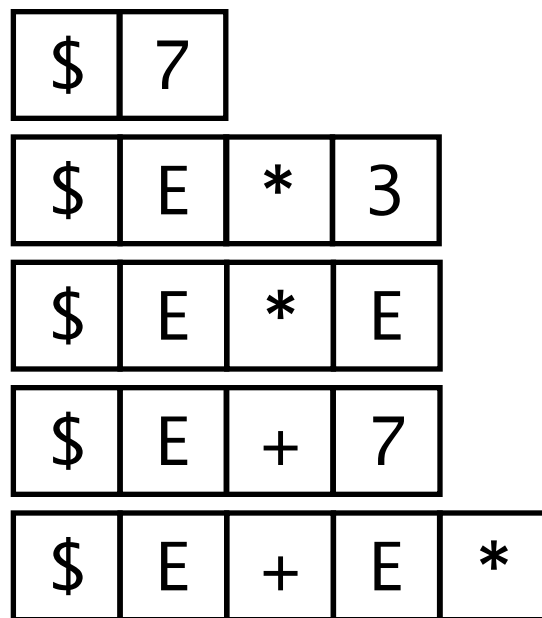
\$	7		
\$	E	*	3
\$	E	*	E
\$	E	+	7
\$	E	+	E

*	3	+	7	*	3	\$
		+	7	*	3	\$
		+	7	*	3	\$
				*	3	\$
				*	3	\$

LR parsing

example

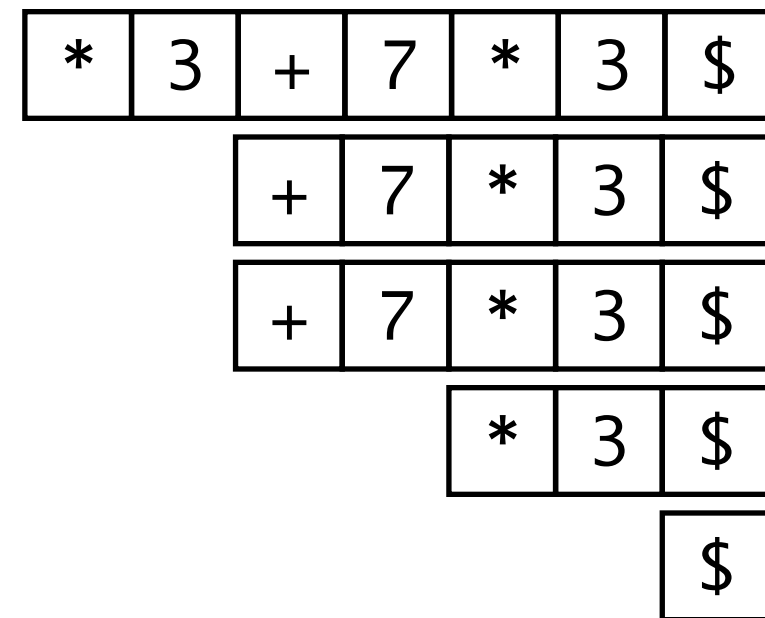
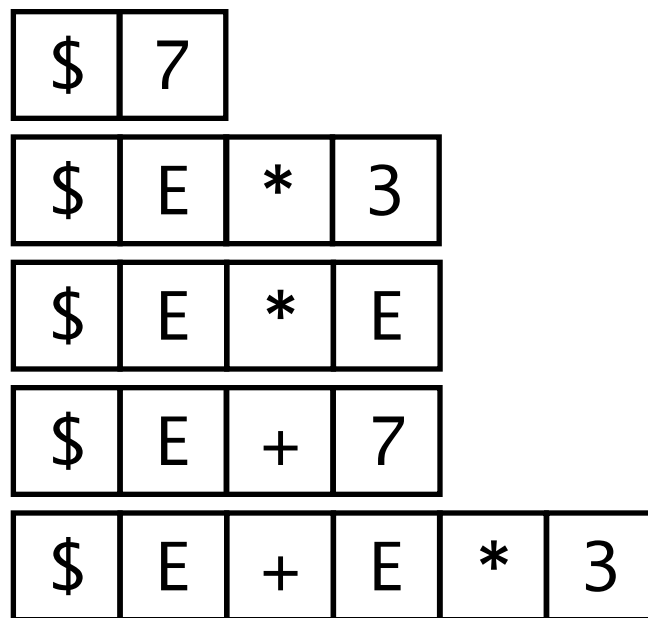
$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$



LR parsing

example

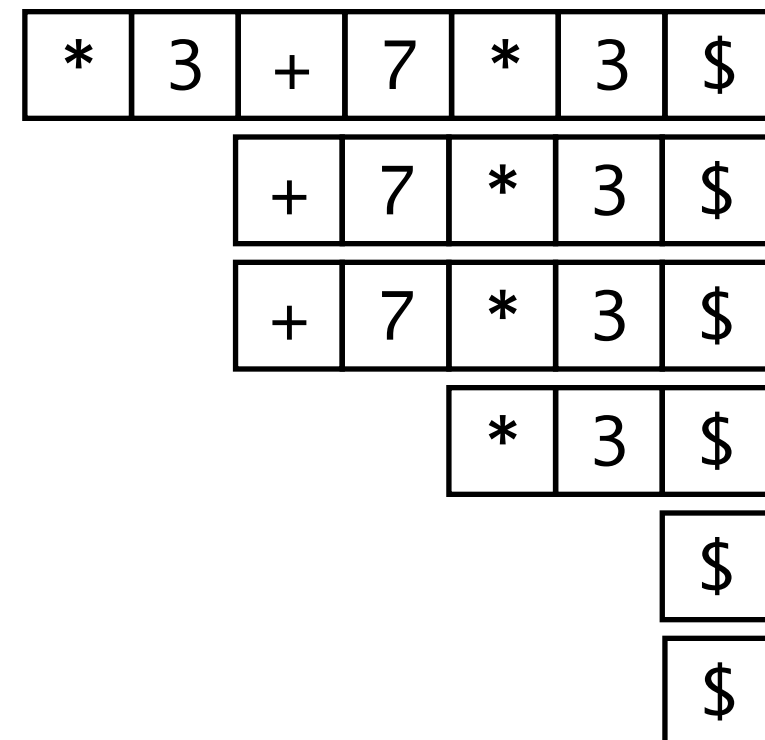
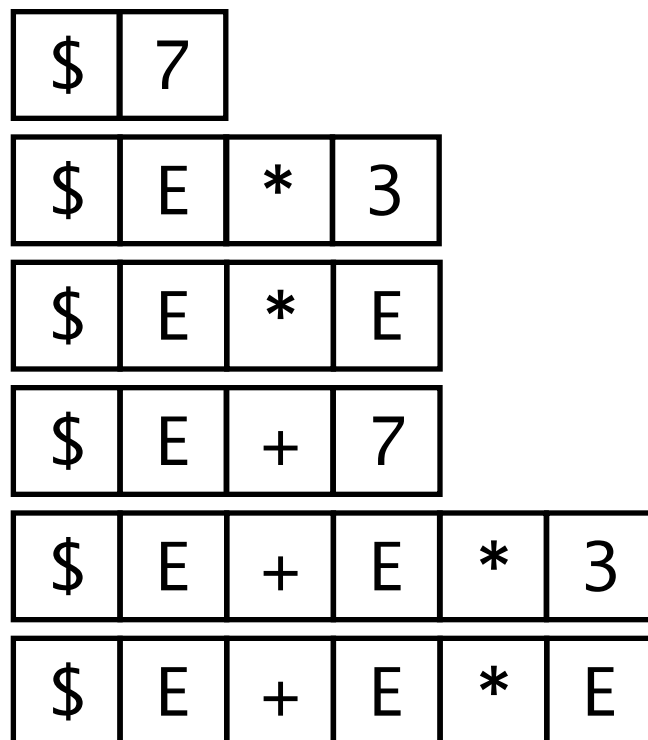
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 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$



LR parsing

example

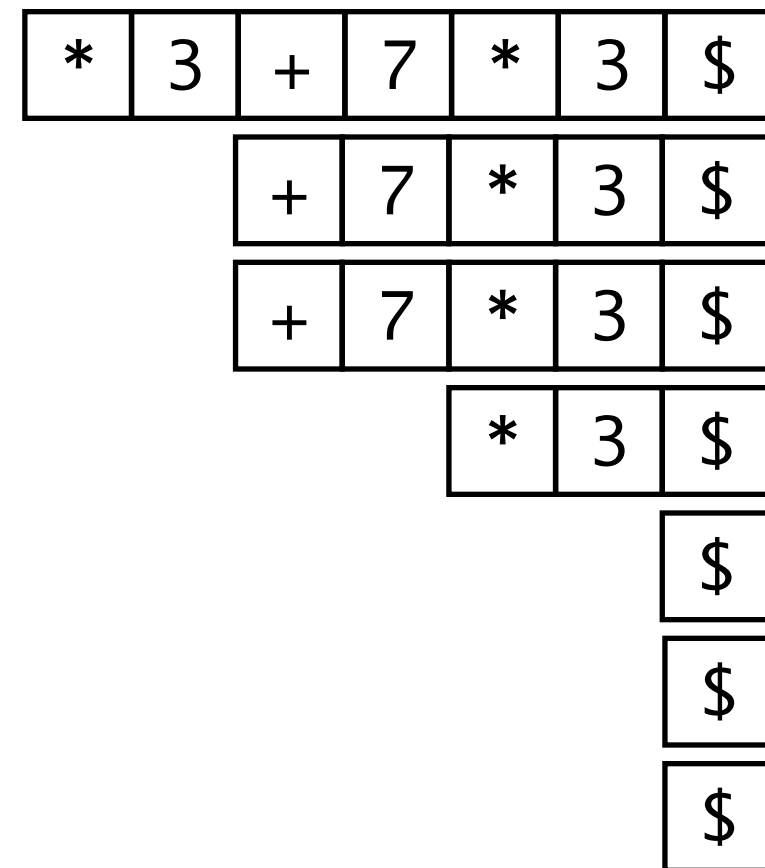
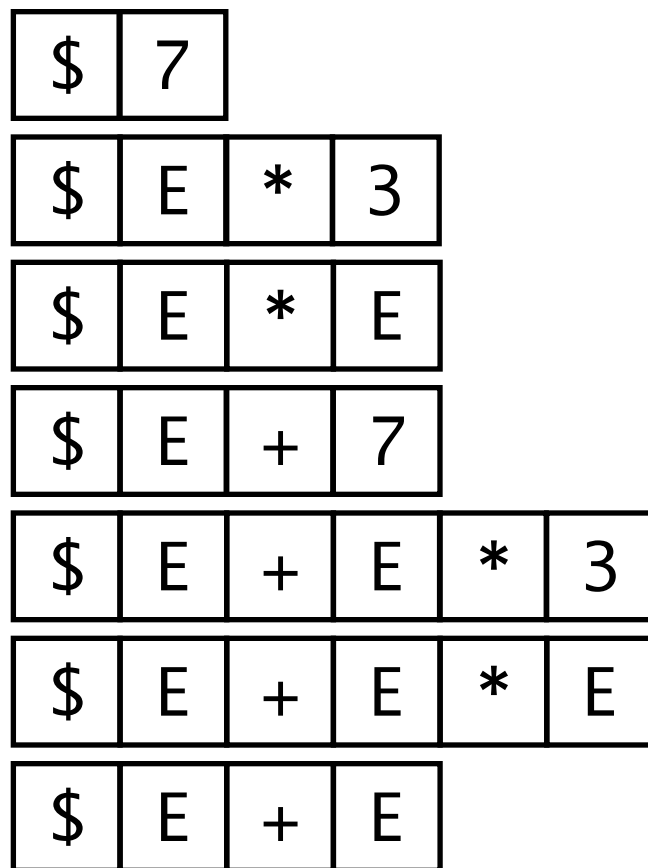
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LR parsing

example

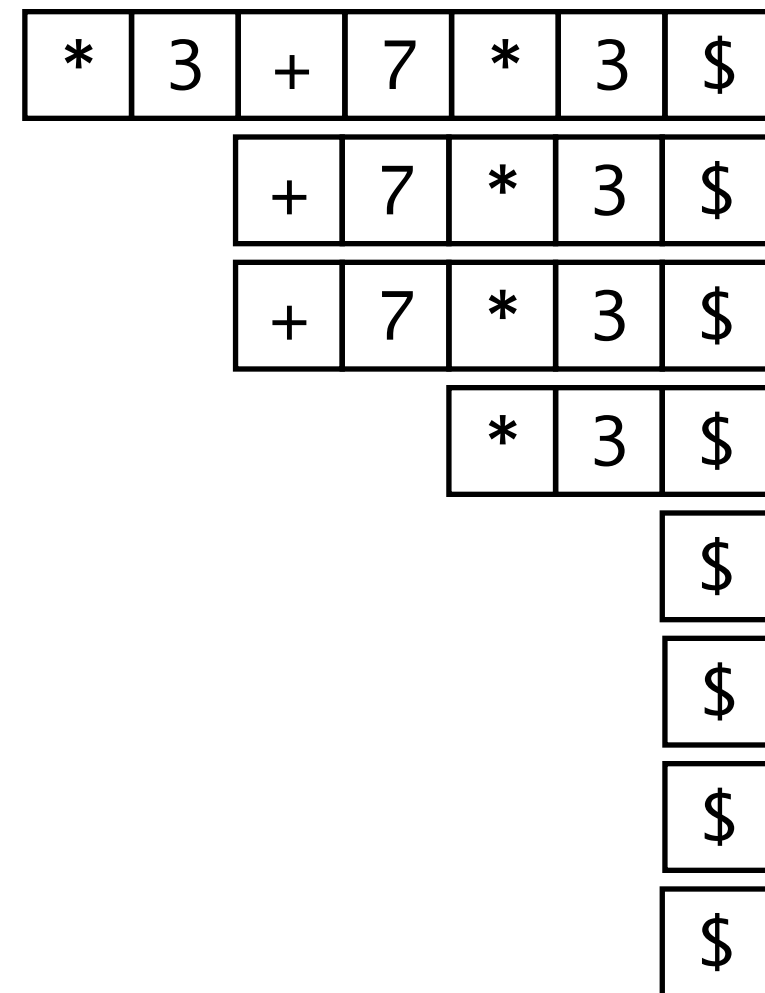
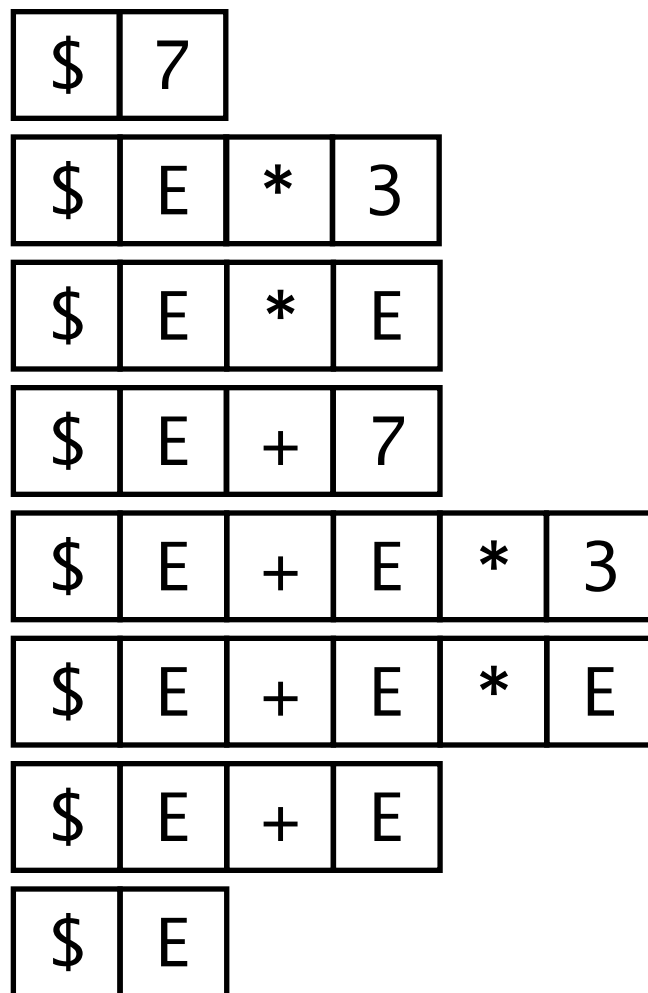
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 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$



LR parsing

example

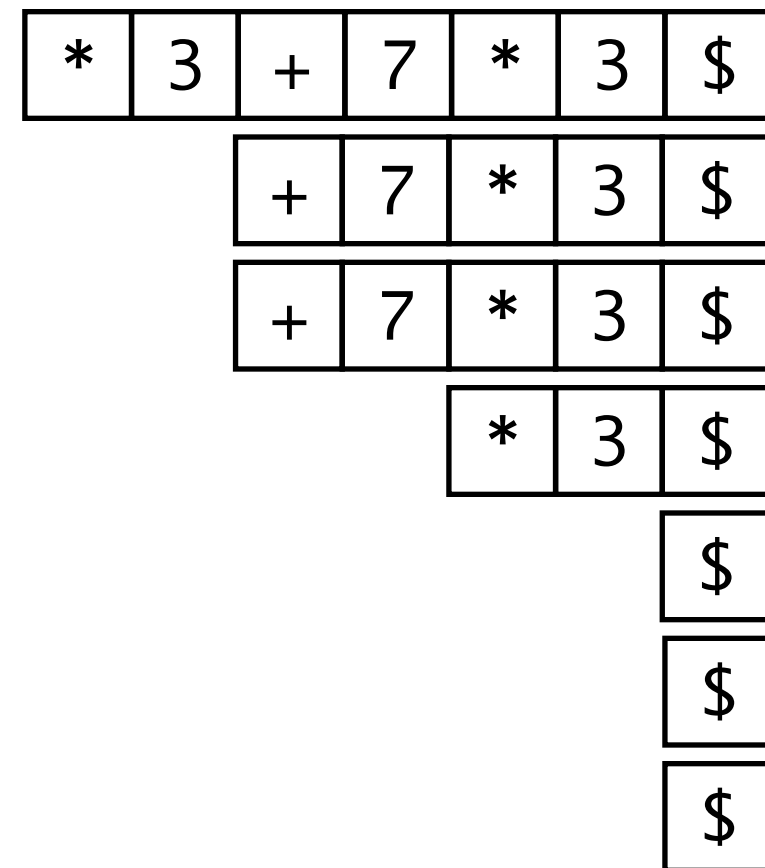
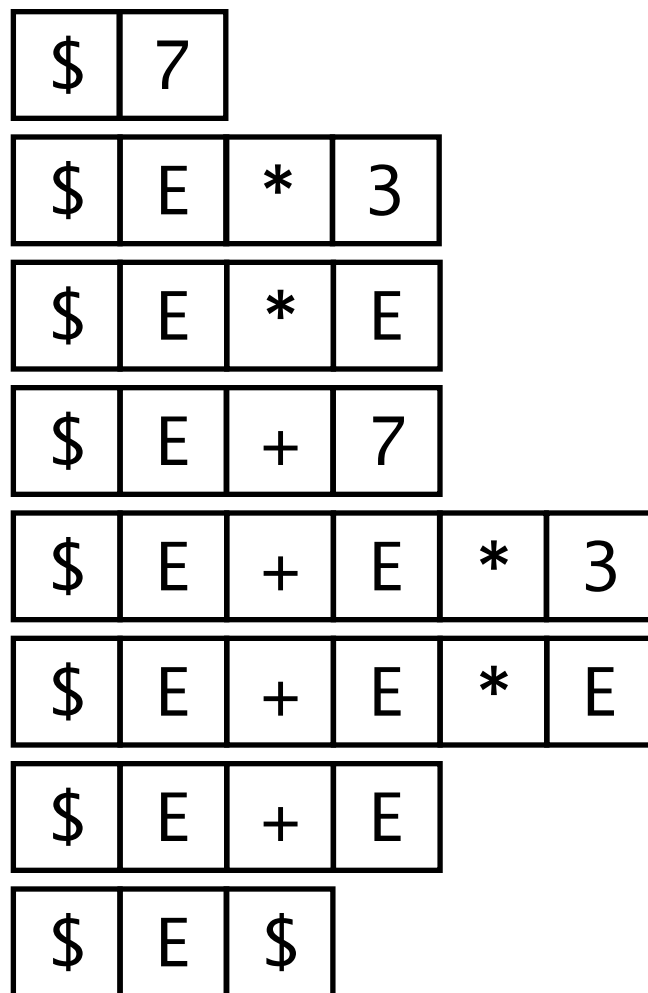
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LR parsing

example

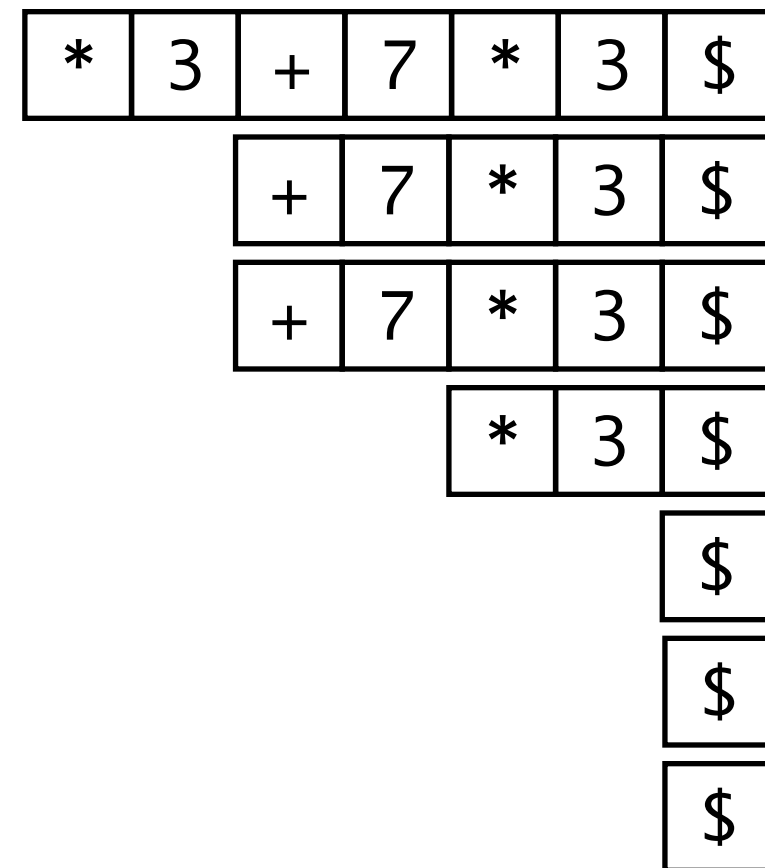
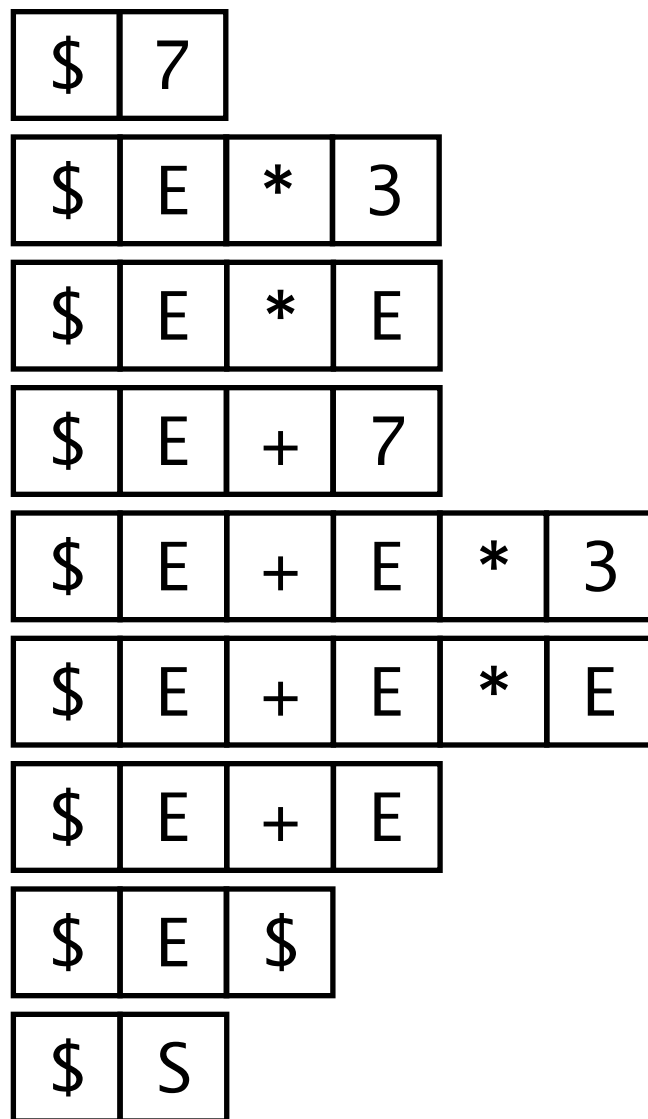
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 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$



LR parsing

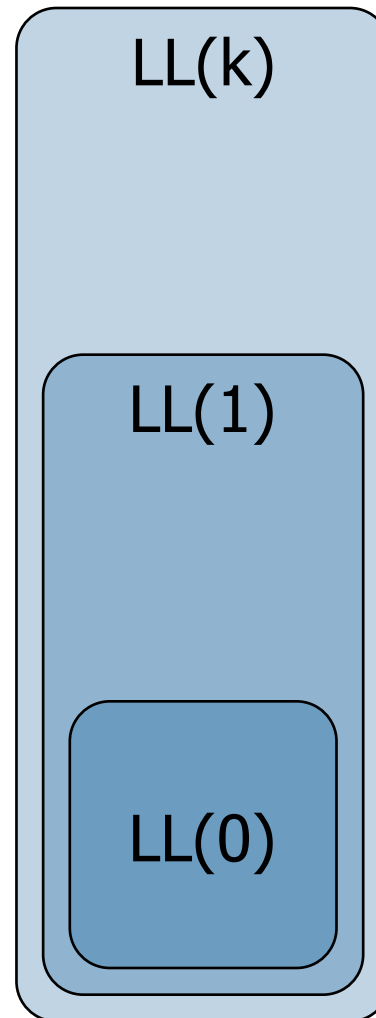
example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$



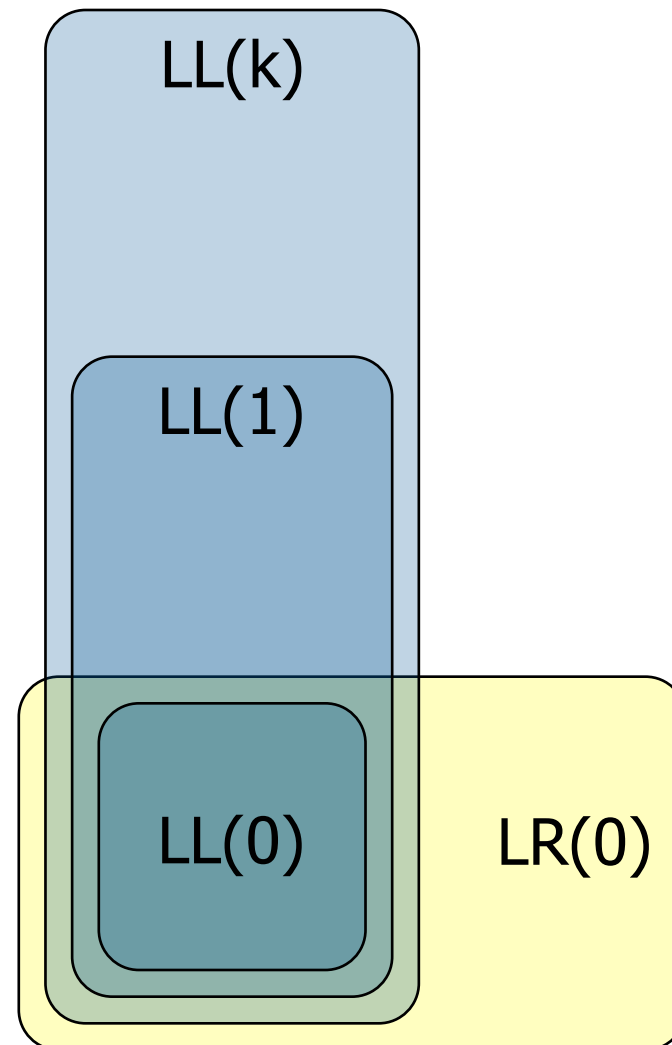
Grammar classes

context-free grammars



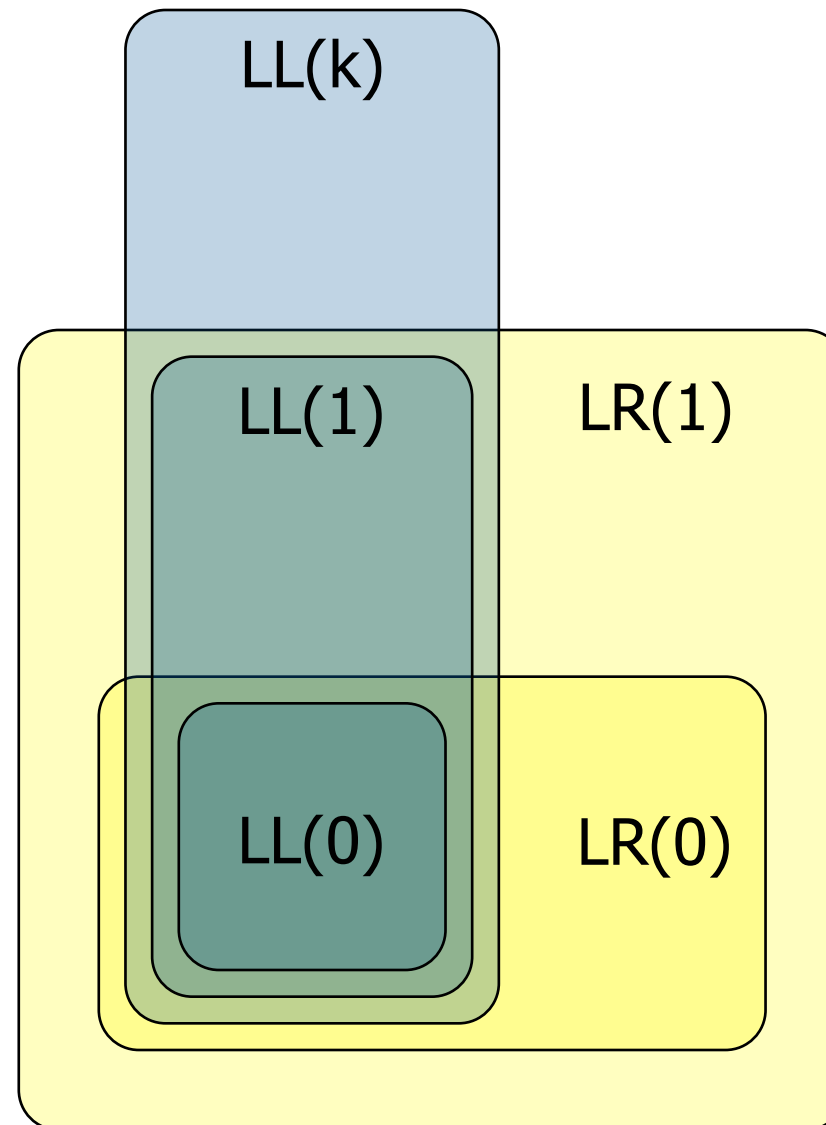
Grammar classes

context-free grammars



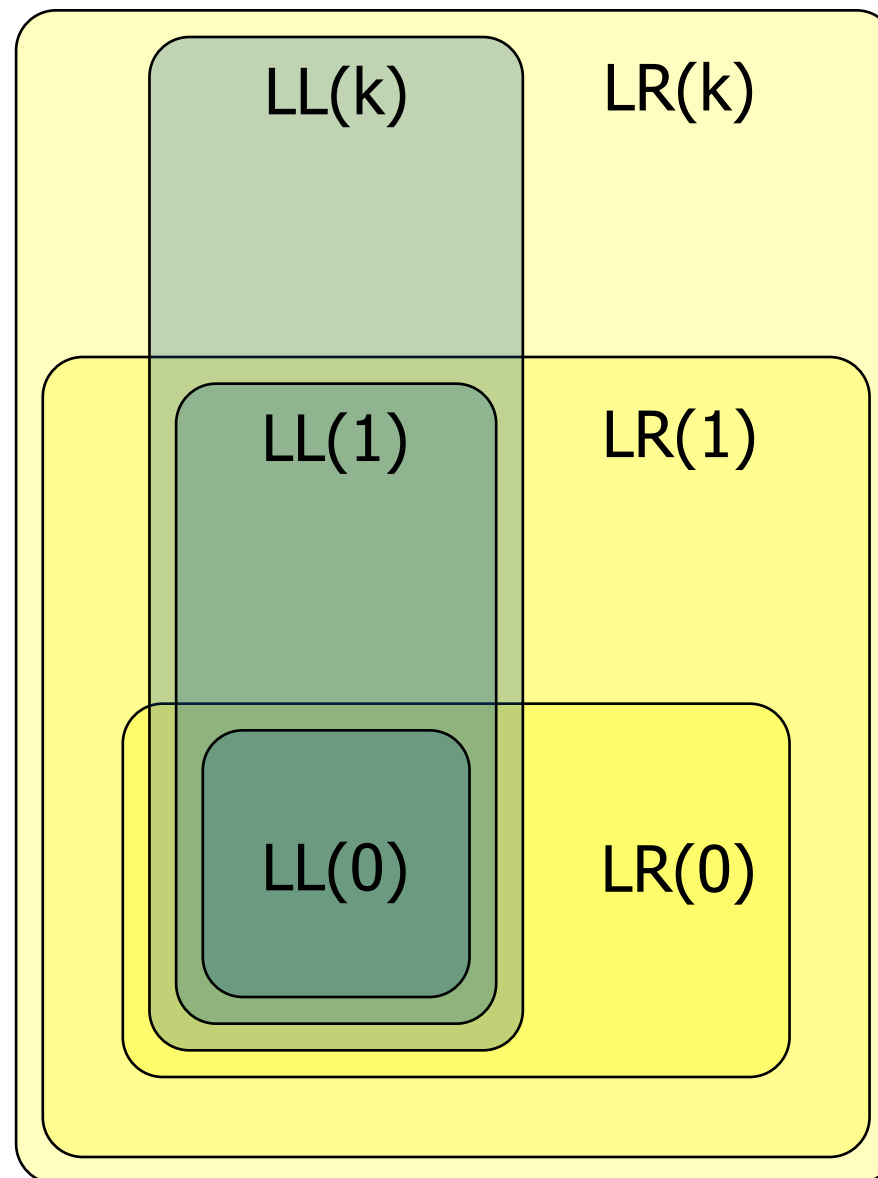
Grammar classes

context-free grammars



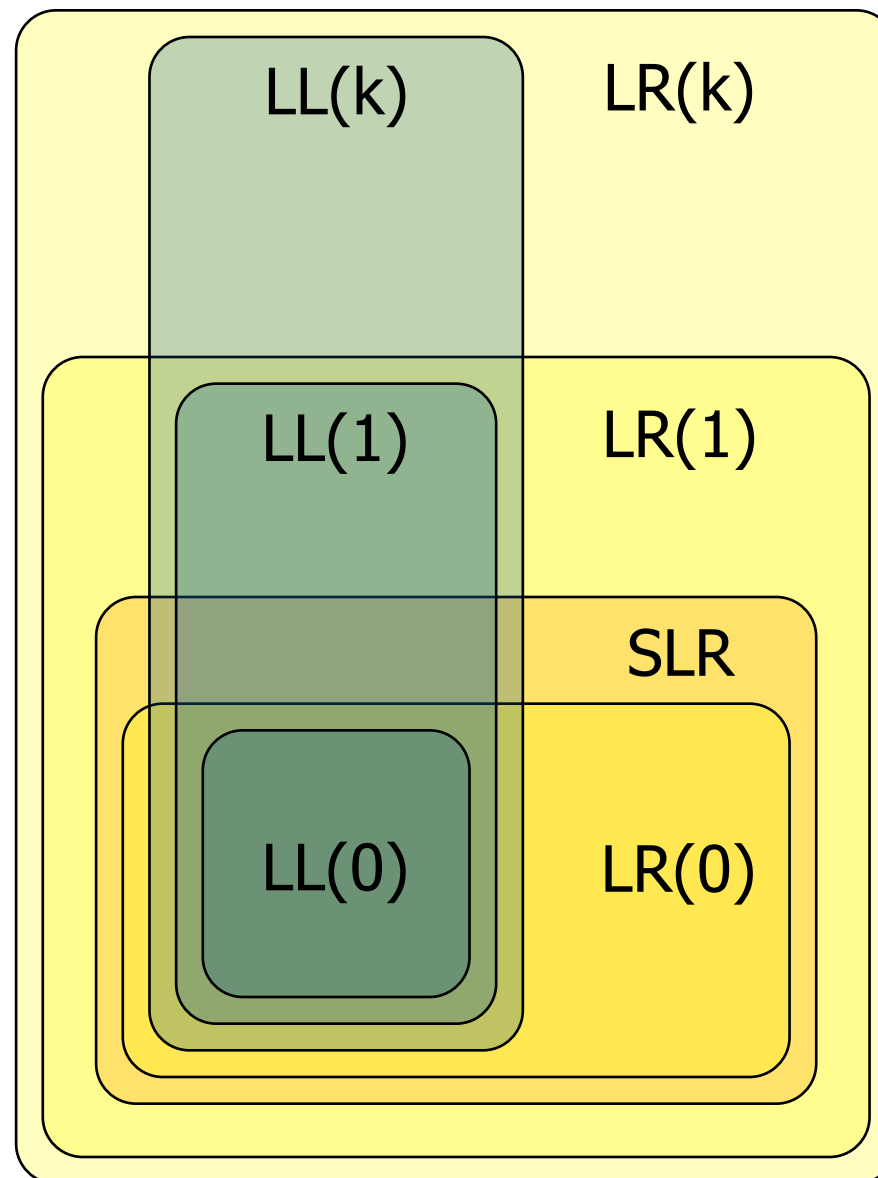
Grammar classes

context-free grammars



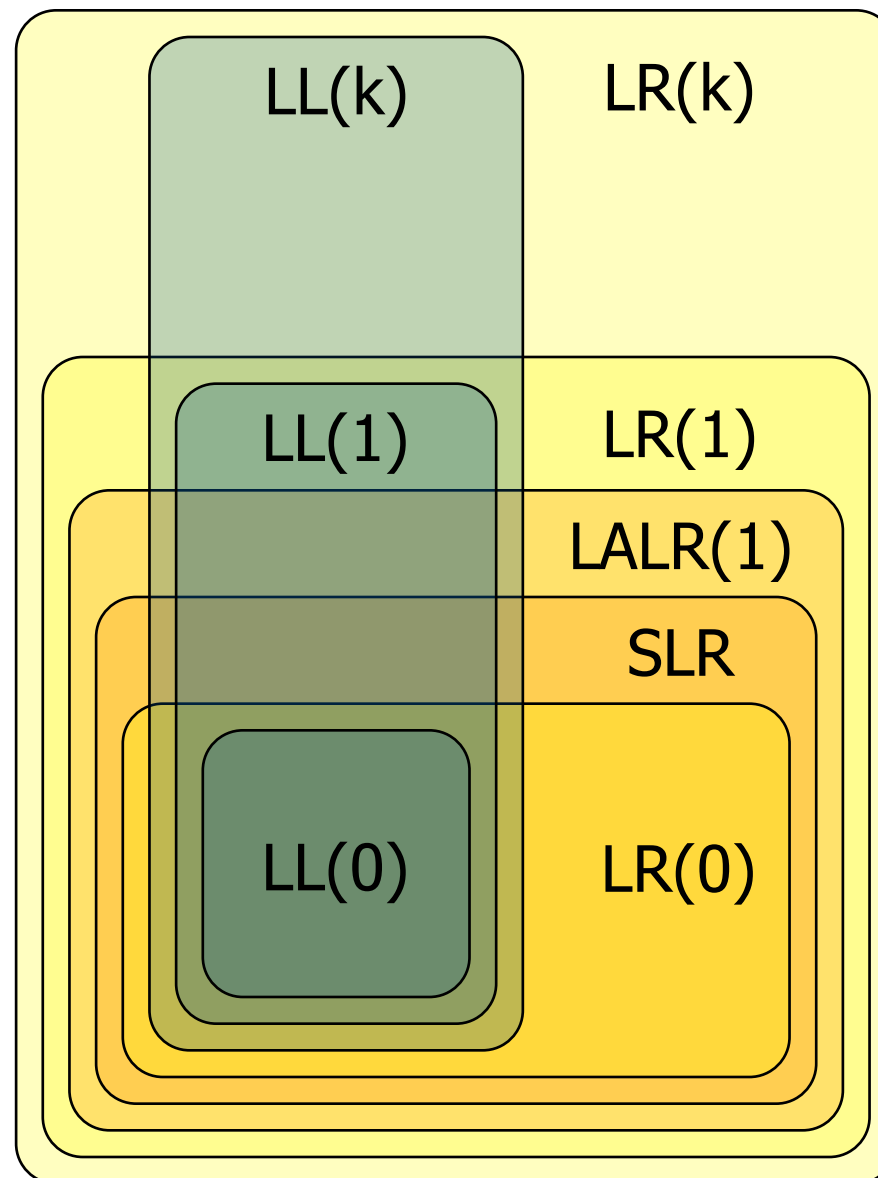
Grammar classes

context-free grammars



Grammar classes

context-free grammars



II

LR Parse Tables

LR parsing

parse table

rows

- states of a DFA

columns

- topmost stack symbol
- Σ, N

entries

- reduce, rule number
- shift, goto state
- goto state
- accept state

	T_1	...	N_1	...
1	s 3			
2			g 5	
3	r 1			
4	r 2	a		
5				
6			g 1	
7	s 1			
8				
...				

LR(0) parse tables

items, closure & goto

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

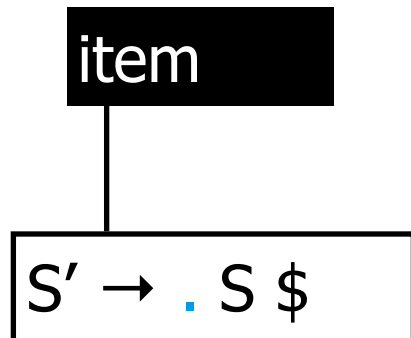
items, closure & goto

$S' \rightarrow \cdot S \$$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

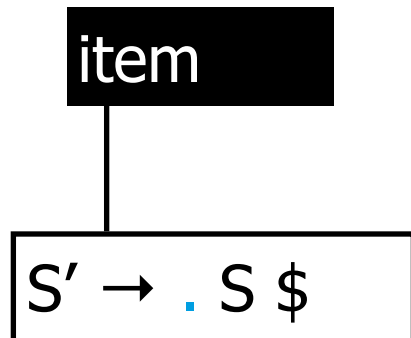
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

items, closure & goto



closure

- for every item $A \rightarrow a \cdot X \beta$
- for every rule $X \rightarrow \gamma$
- add item $X \rightarrow \cdot \gamma$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

items, closure & goto

$S' \rightarrow \cdot S \$$
$S \rightarrow \cdot x$
$S \rightarrow \cdot (L)$

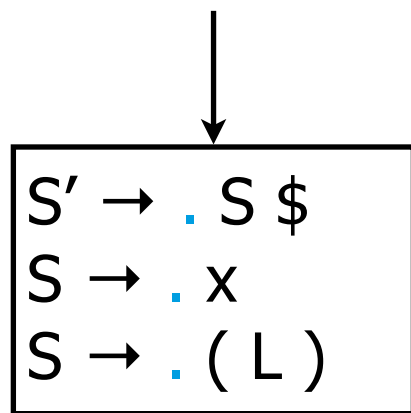
closure

- for every item $A \rightarrow a \cdot X \beta$
- for every rule $X \rightarrow \gamma$
- add item $X \rightarrow \cdot \gamma$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

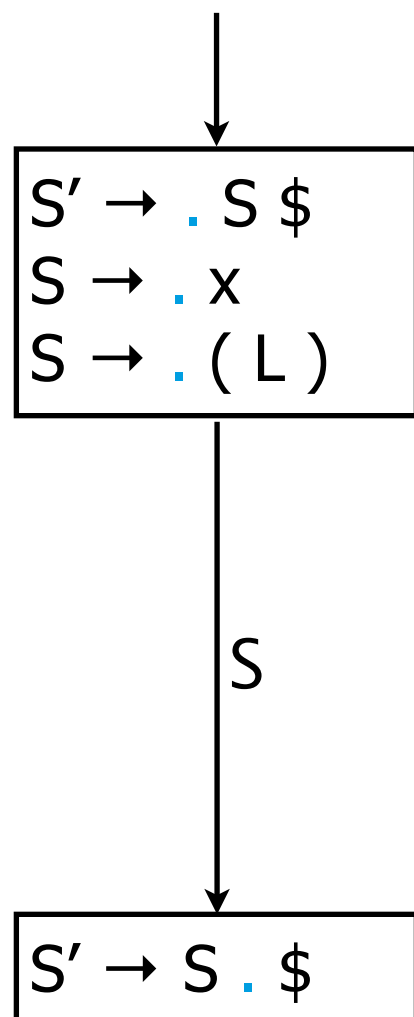
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

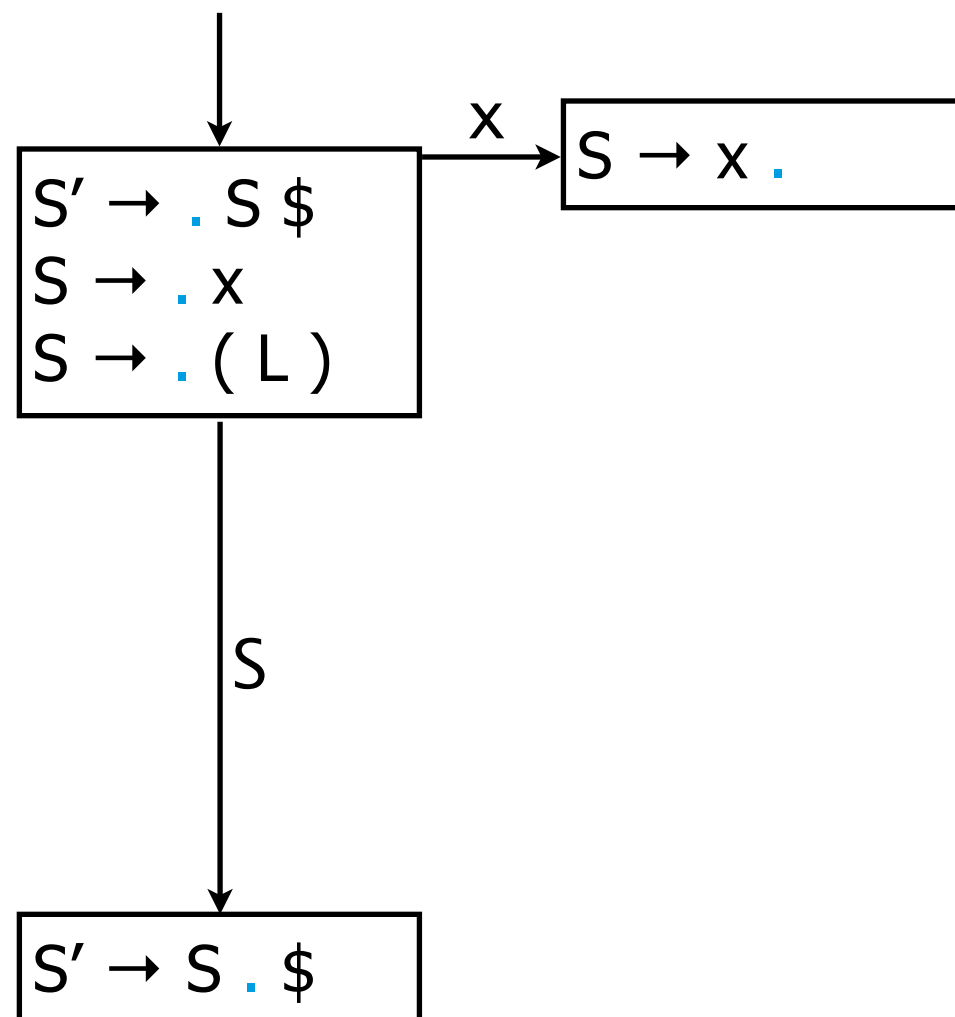
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

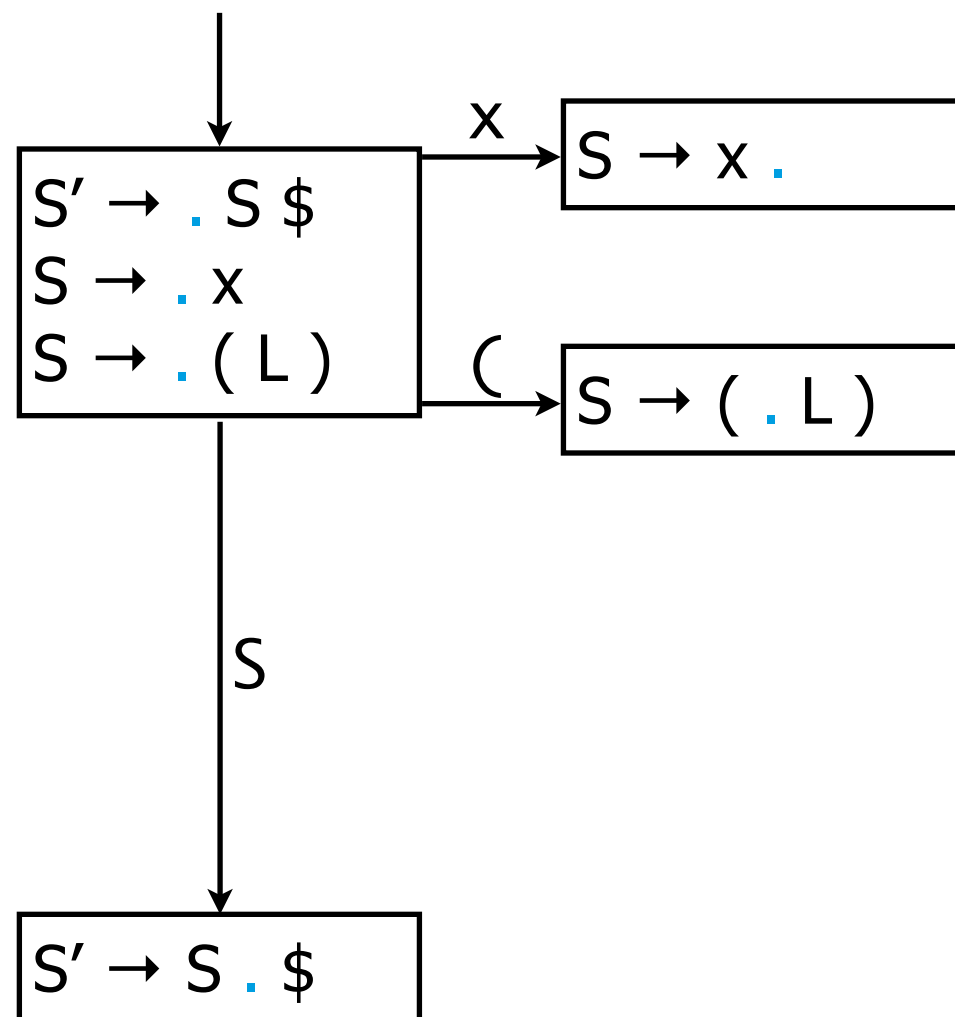
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

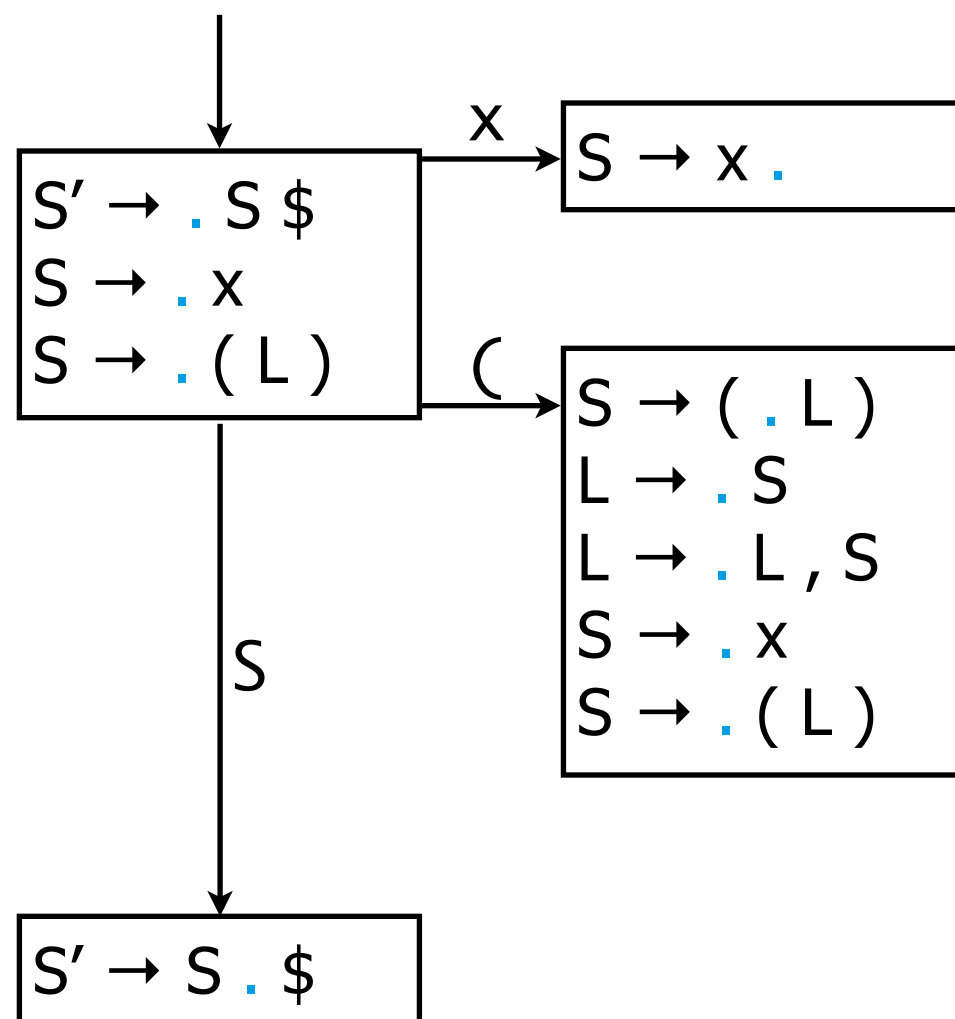
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$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

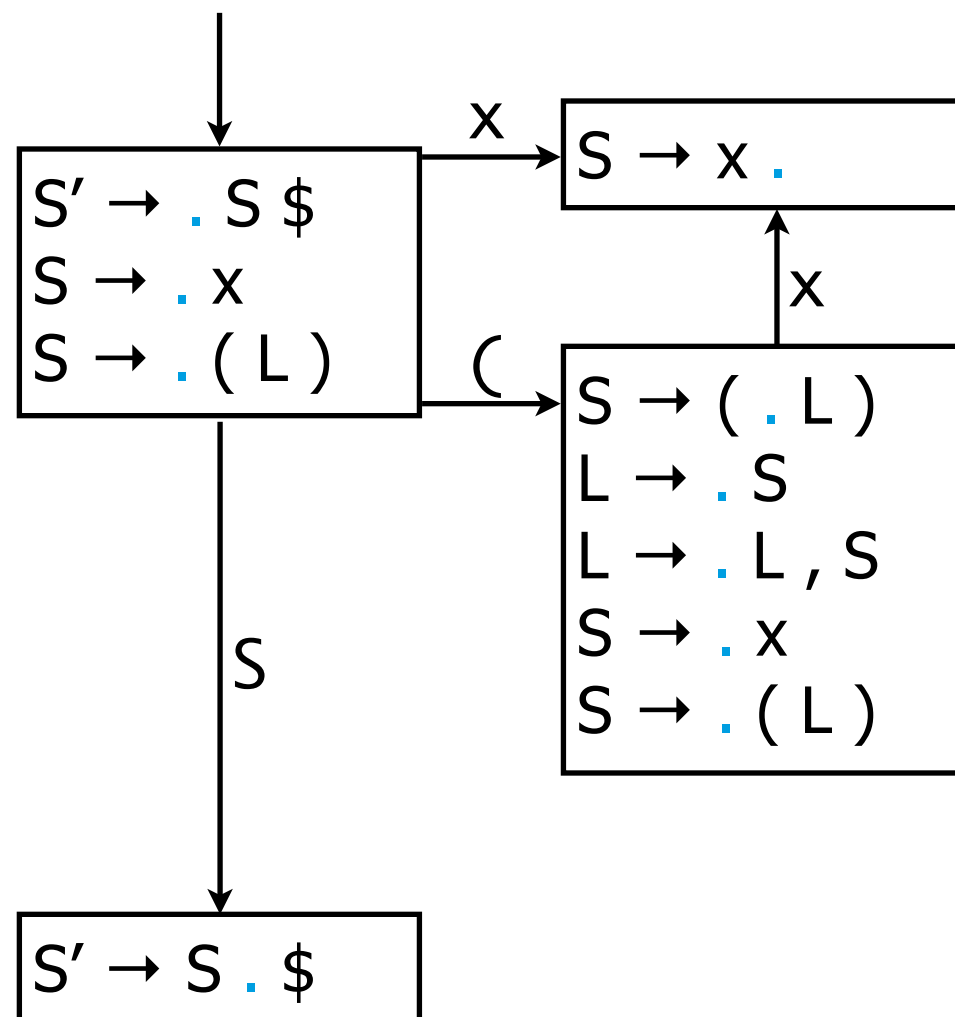
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

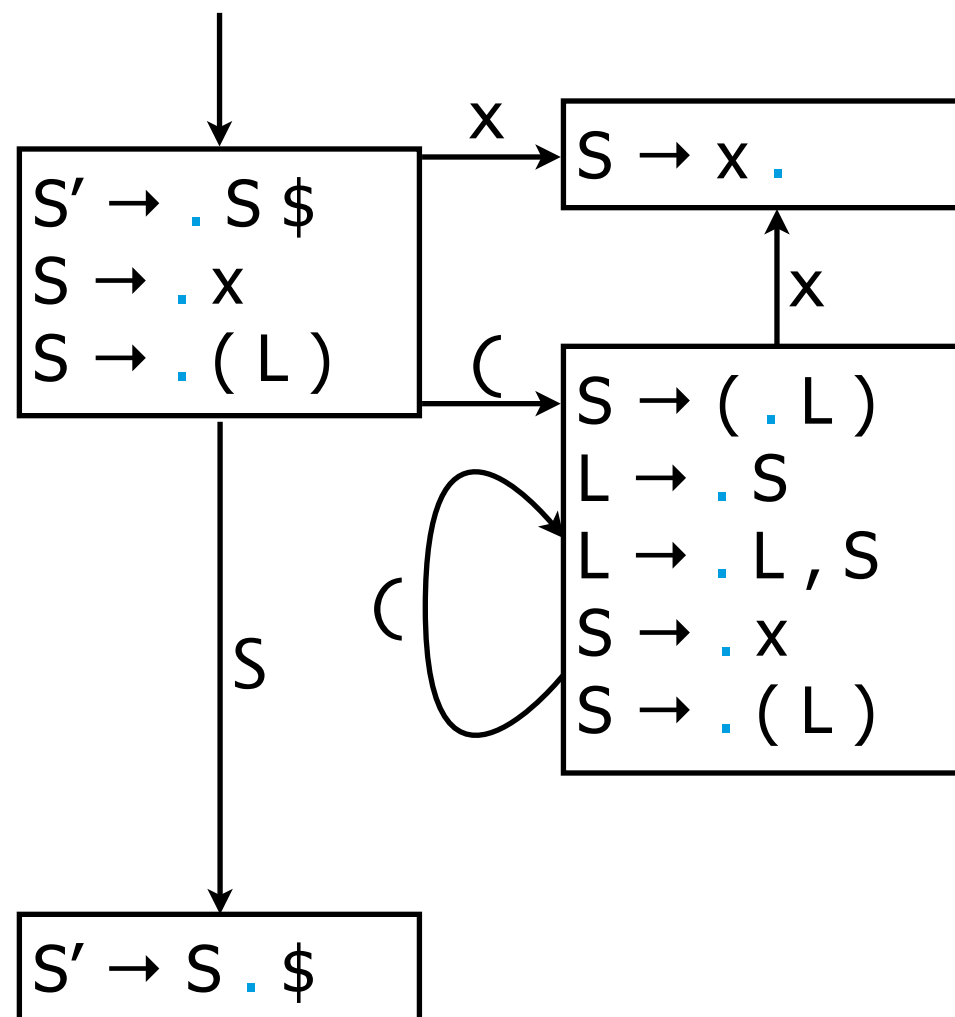
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

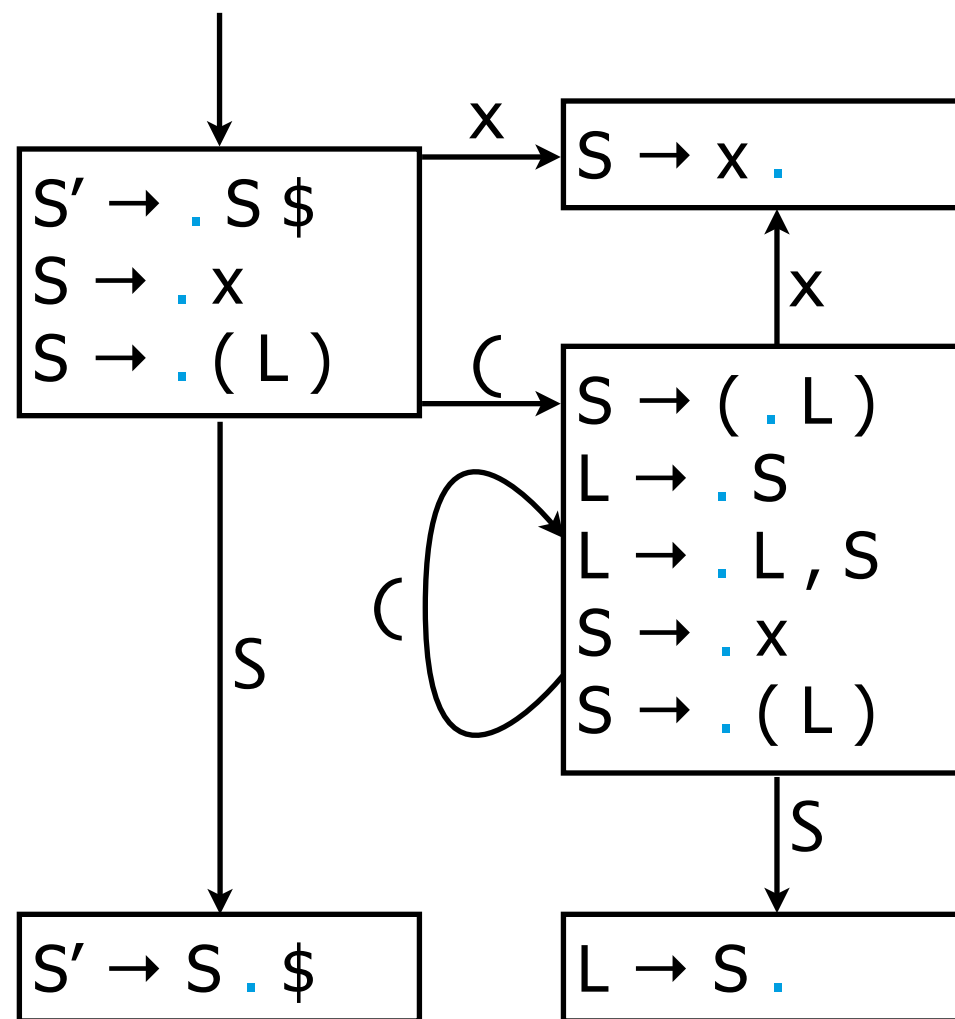
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

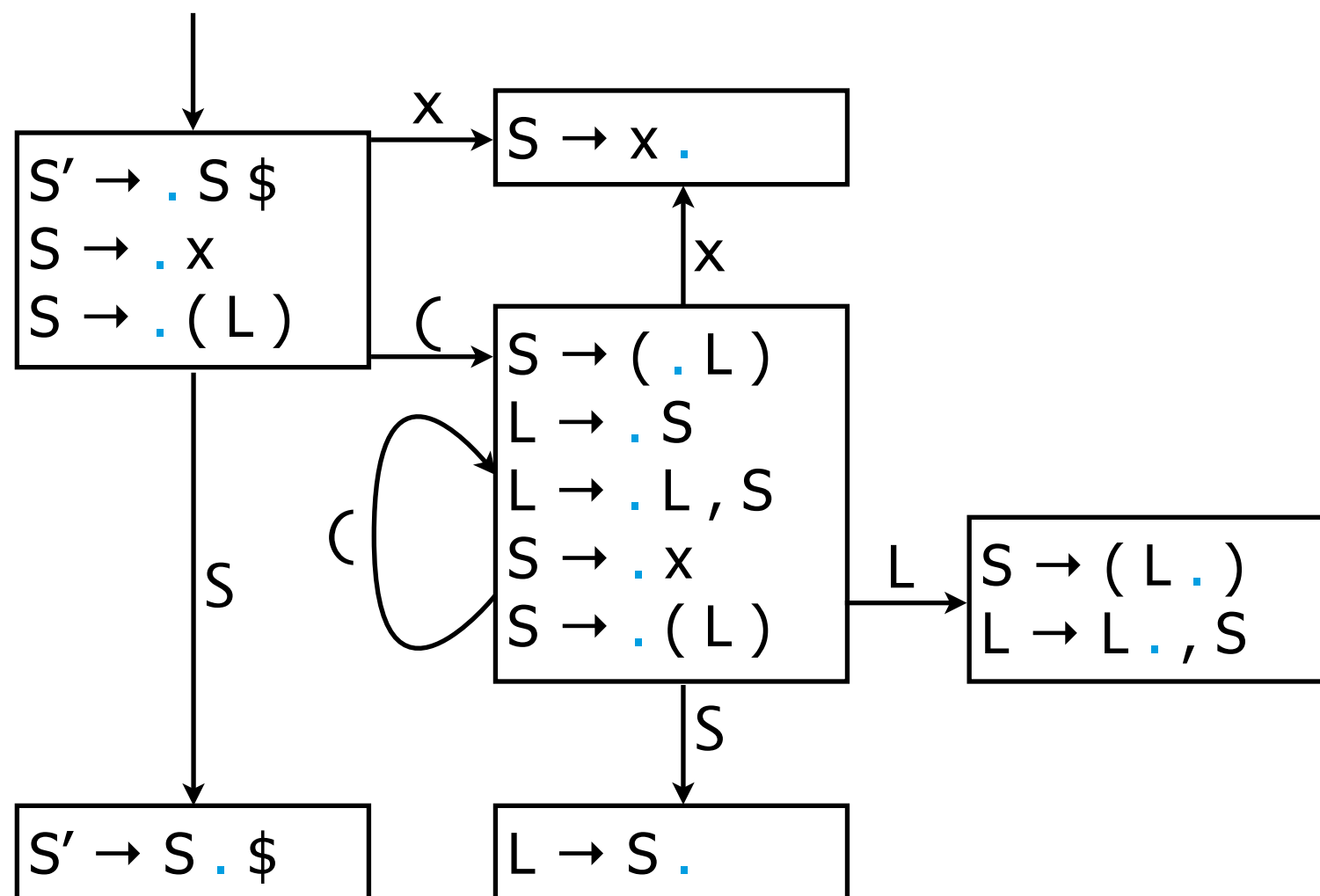
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
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LR(0) parse tables

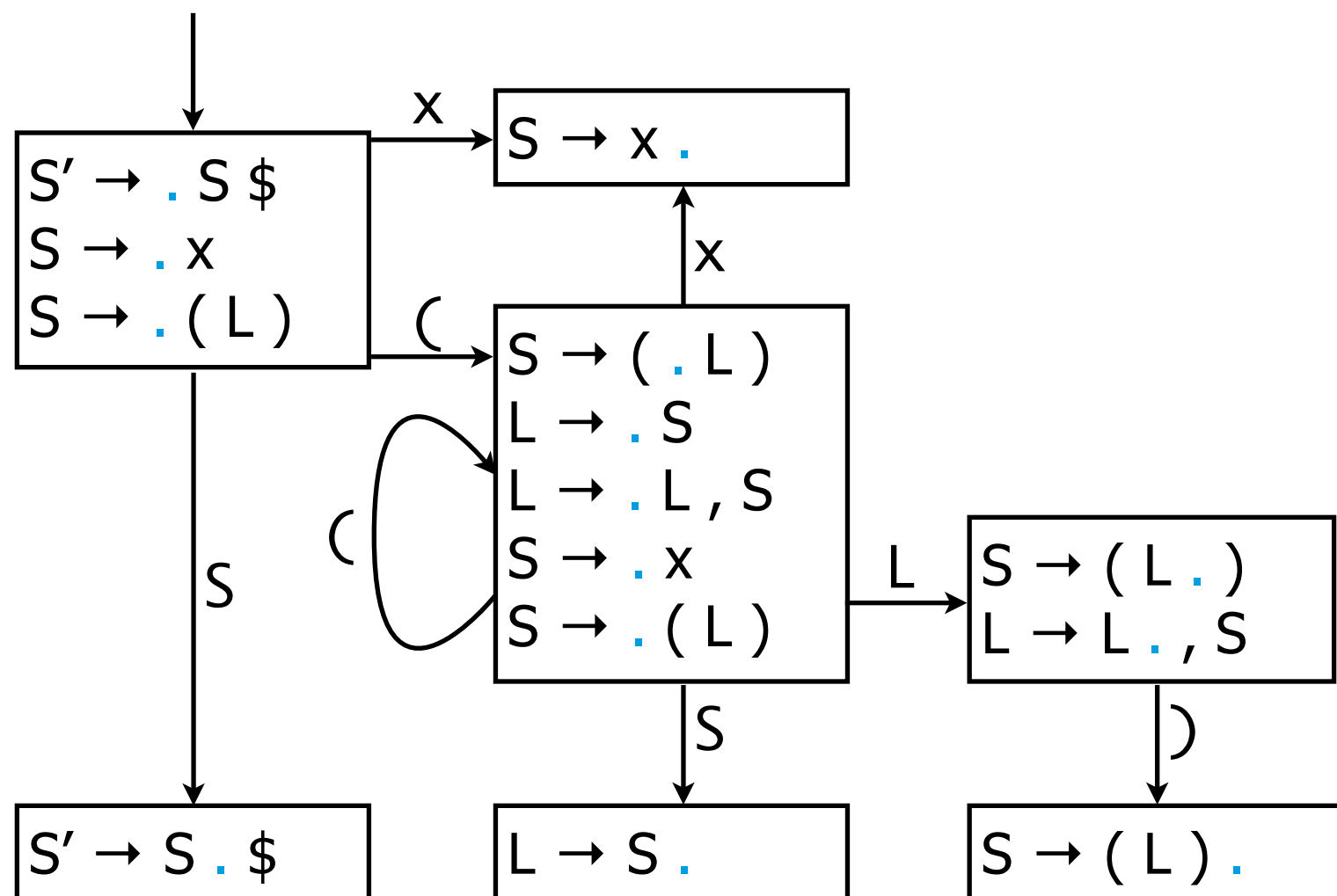
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

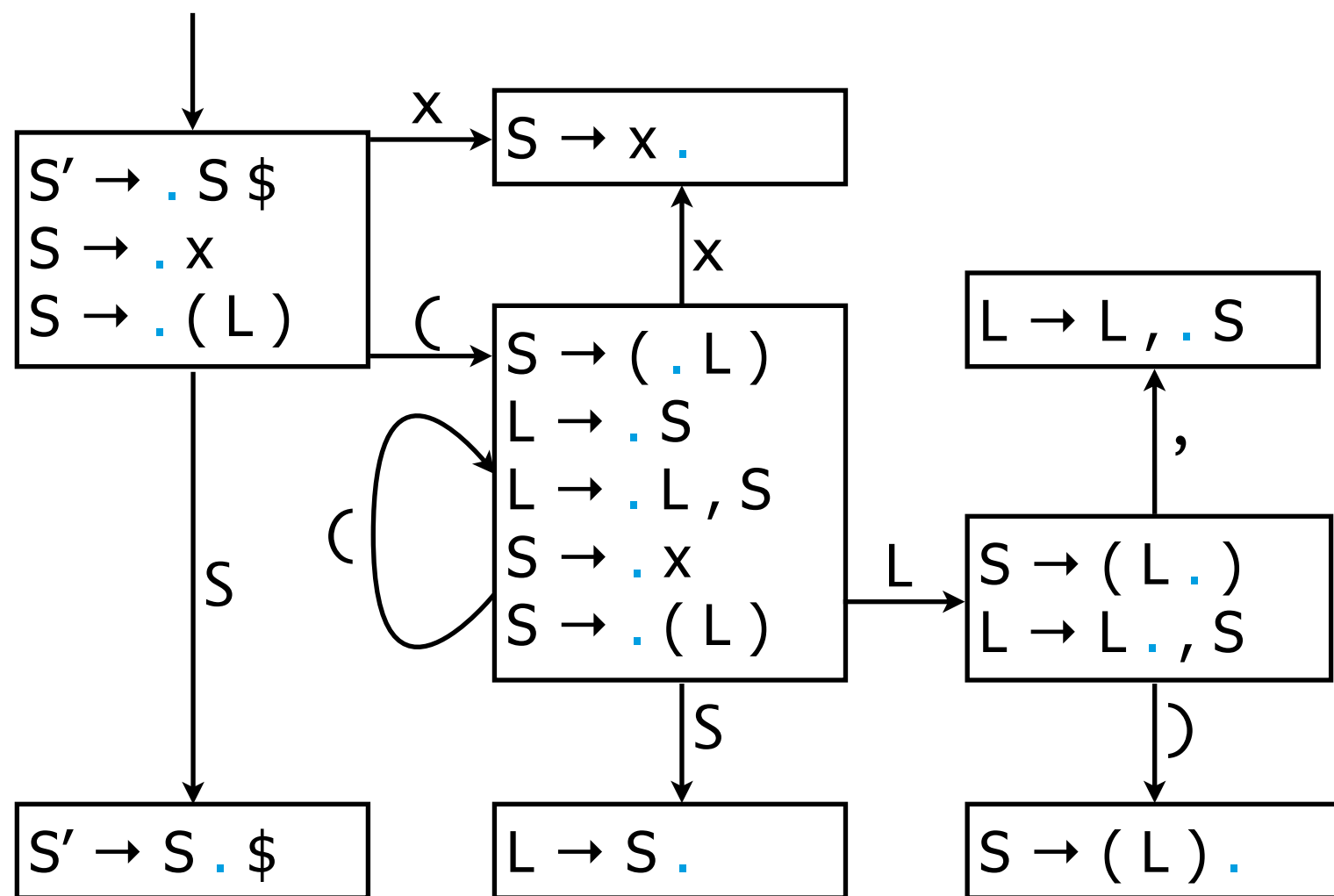
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

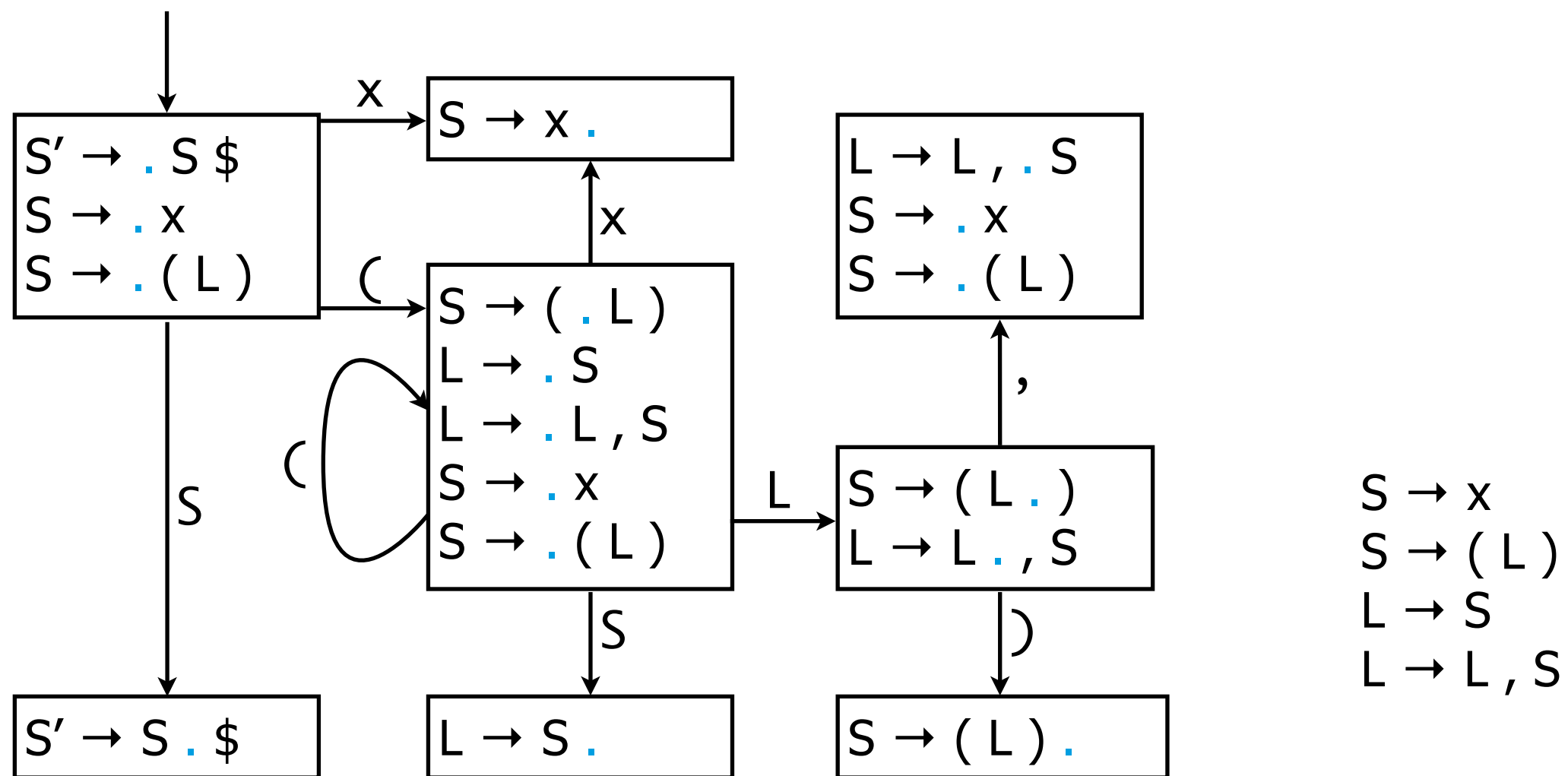
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

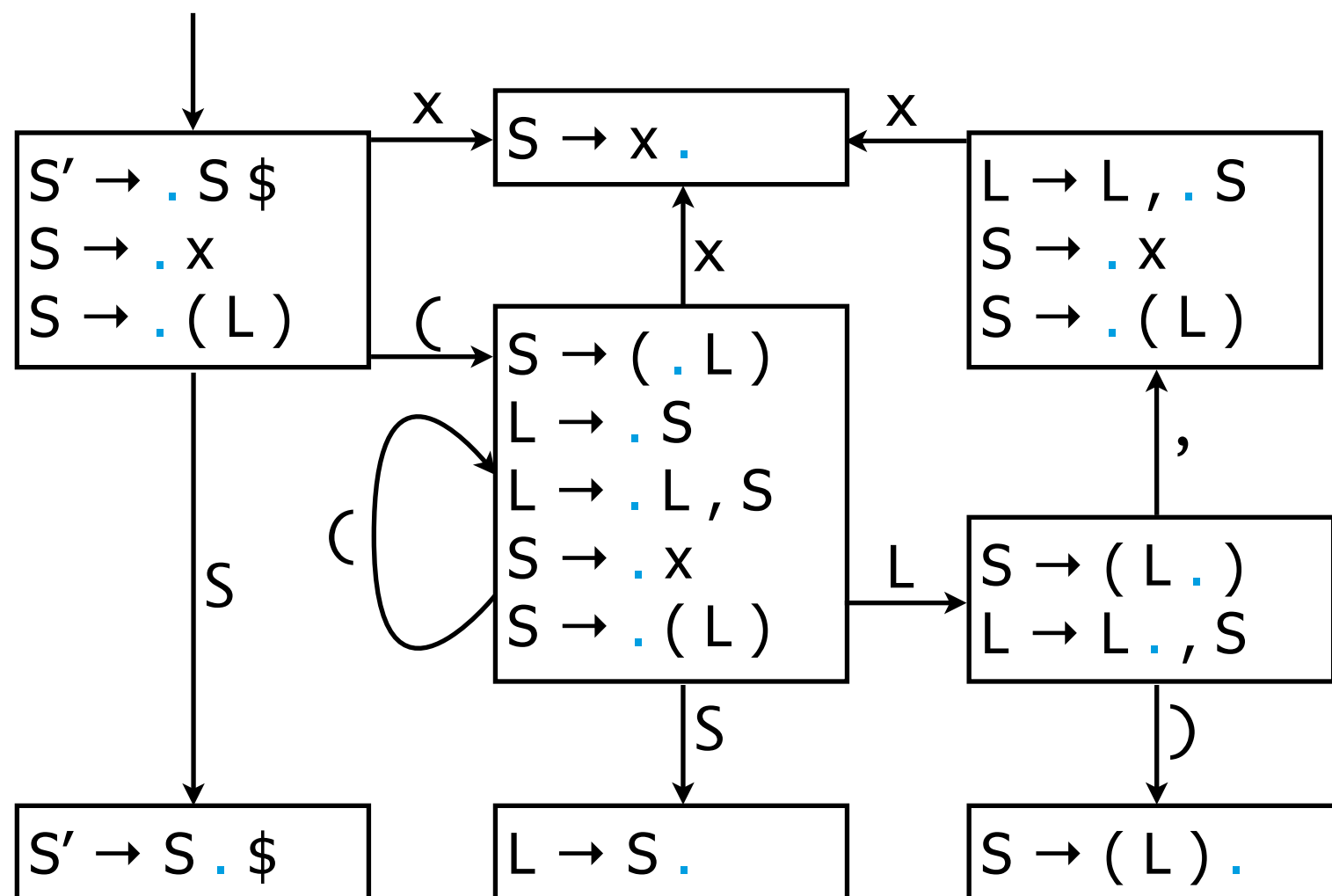
LR(0) parse tables

items, closure & goto



LR(0) parse tables

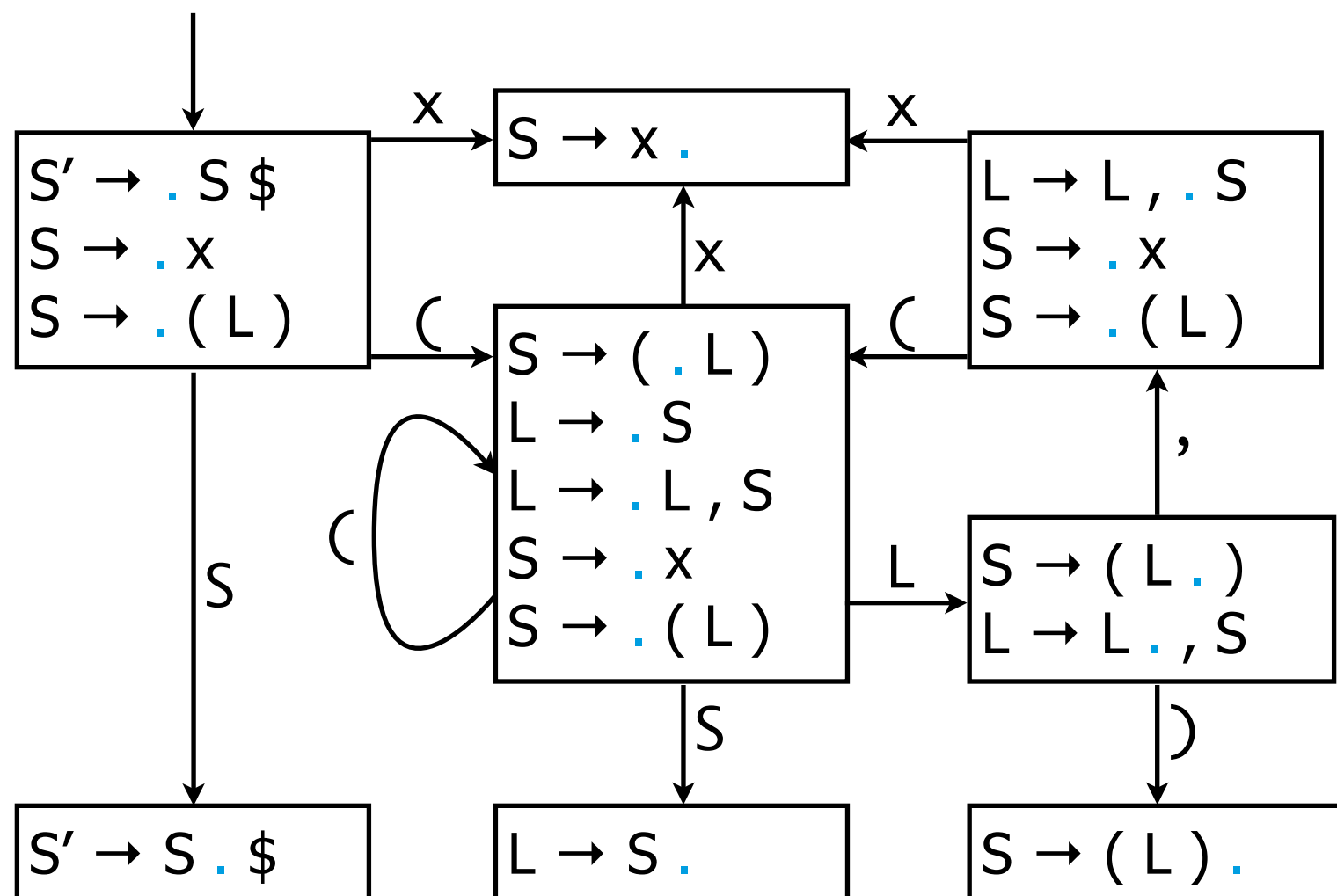
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

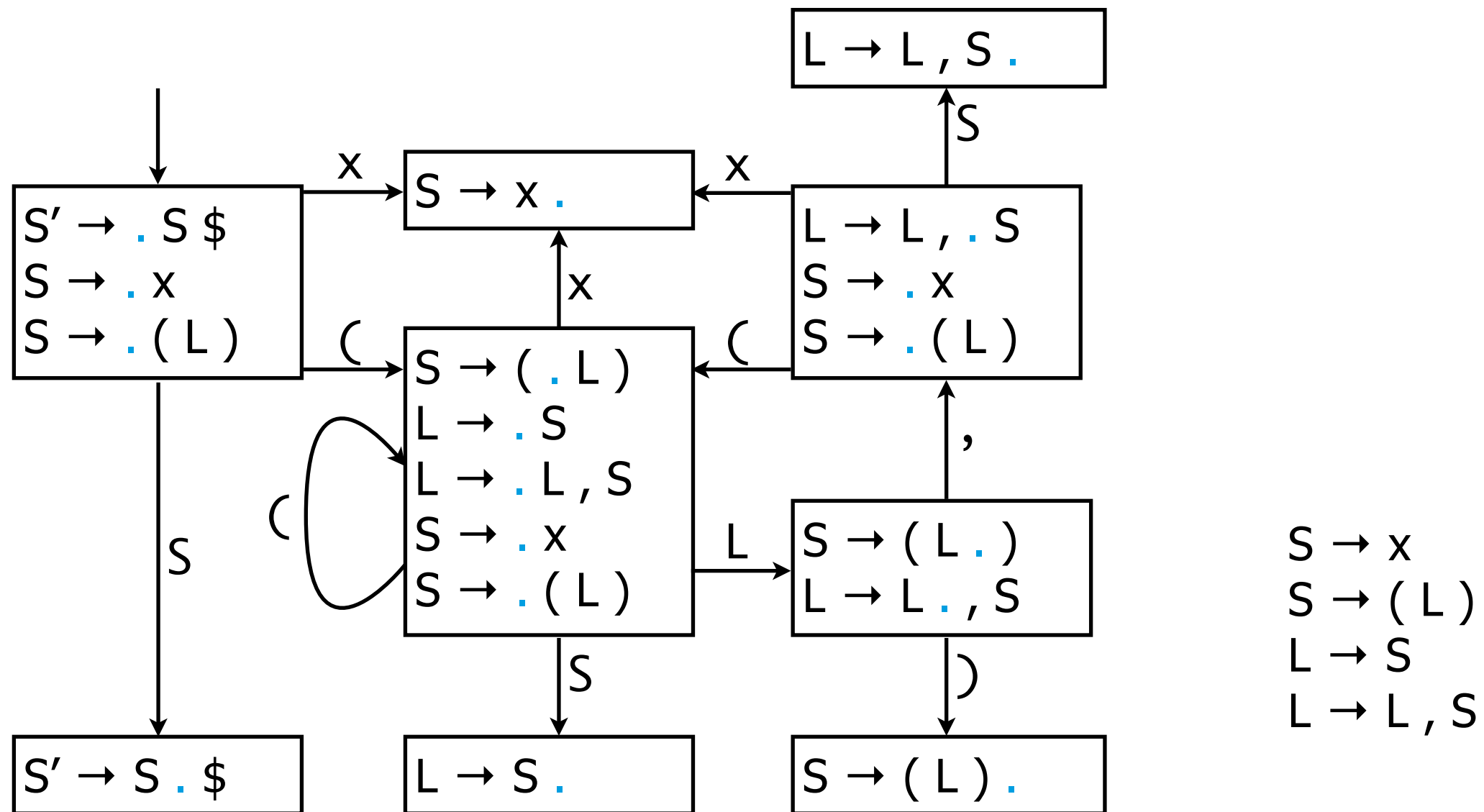
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

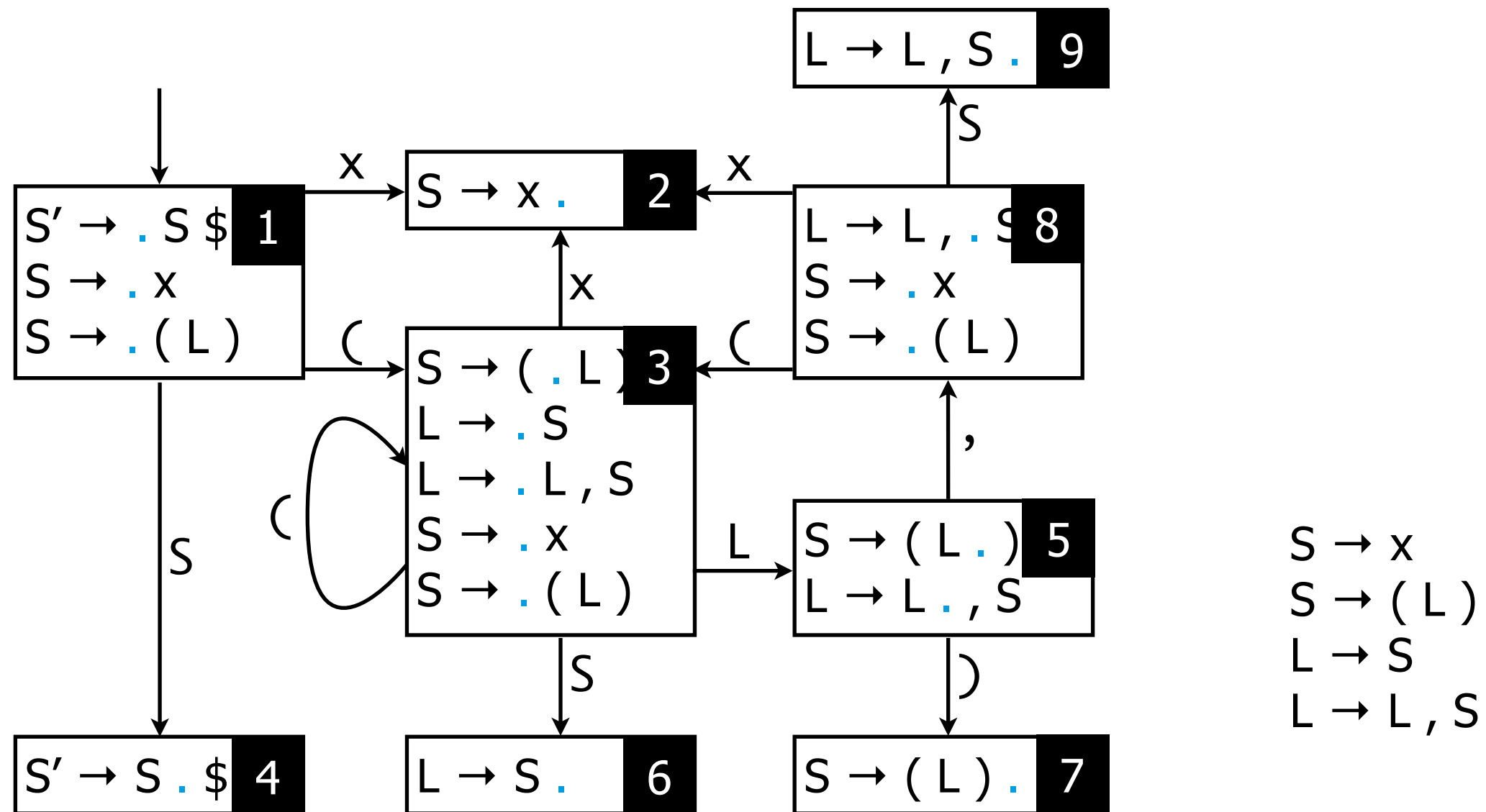
LR(0) parse tables

items, closure & goto



LR(0) parse tables

items, closure & goto



LR(0) parse tables

result

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

(x	,	x)	\$
---	---	---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

x	,	x)	\$
---	---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

2
3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

6
3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

5
3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

x)	\$
---	---	----

8
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

)	\$
---	----

2
8
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

)	\$
---	----

8
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

)	\$
---	----

9
8
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

)	\$
---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

)	\$
---	----

5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

\$

7
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

\$

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

result

\$

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

4
1

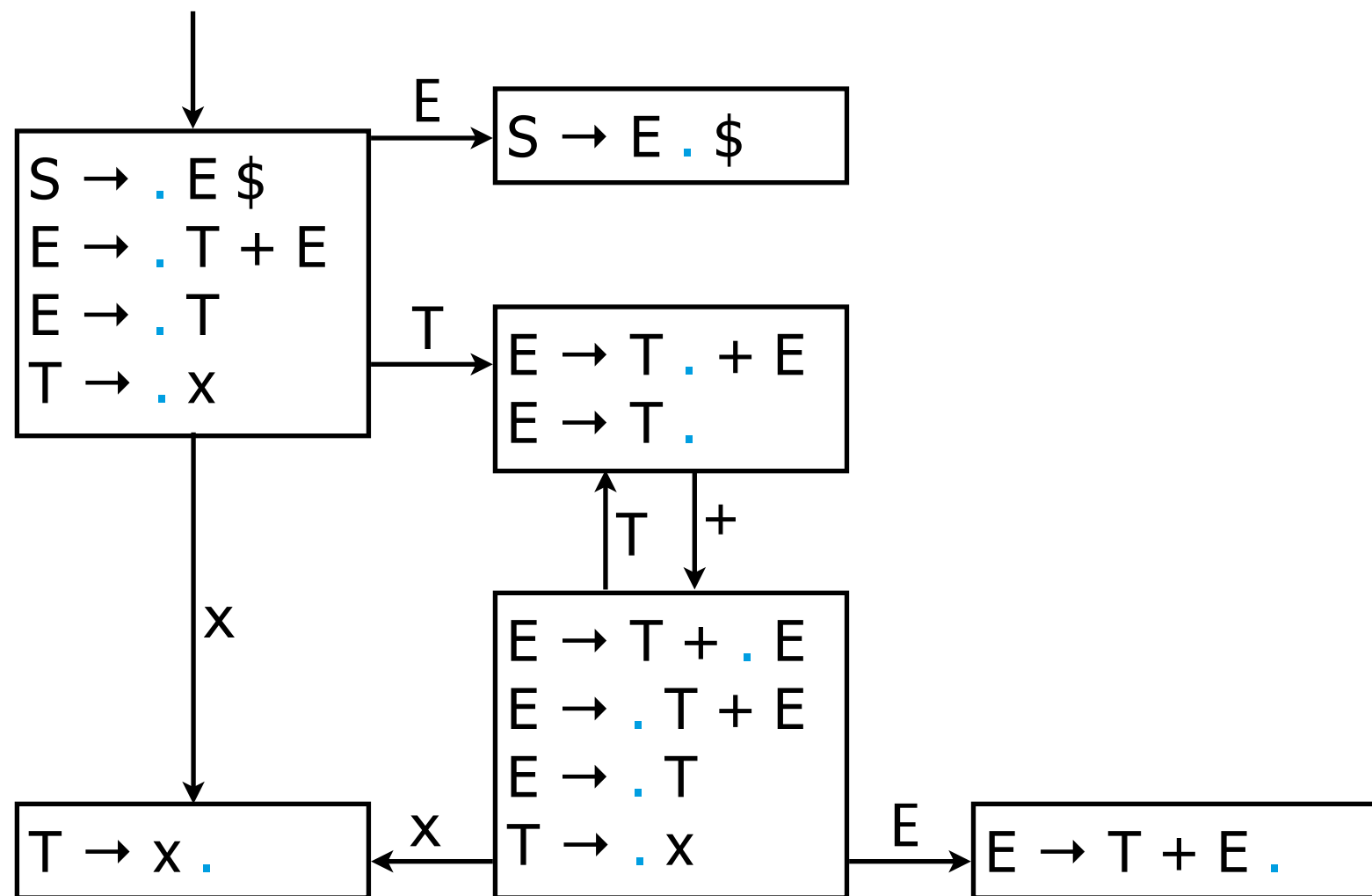
$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

III

Conflict Resolution

SLR parse tables

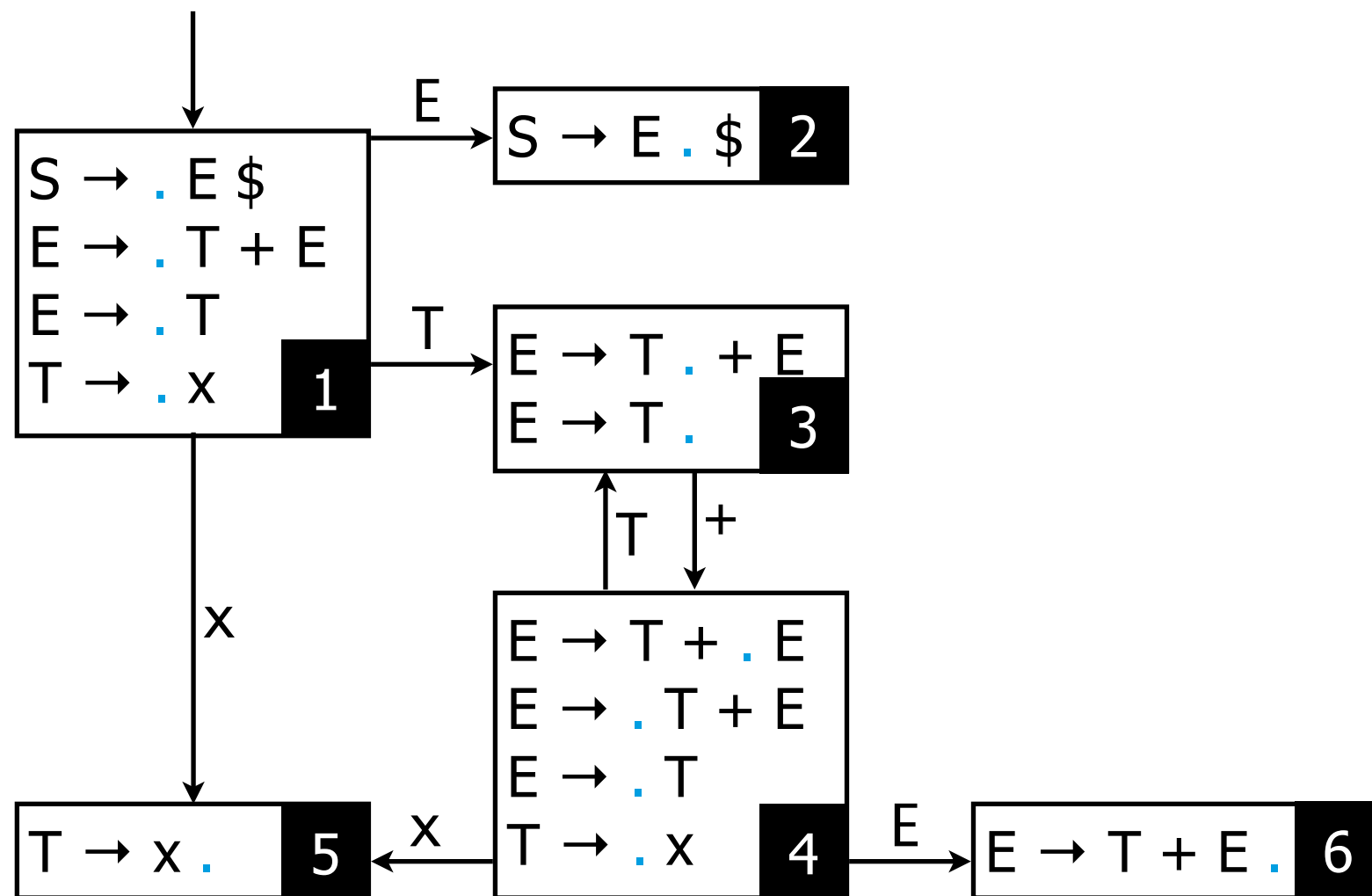
shift-reduce conflicts



$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

SLR parse tables

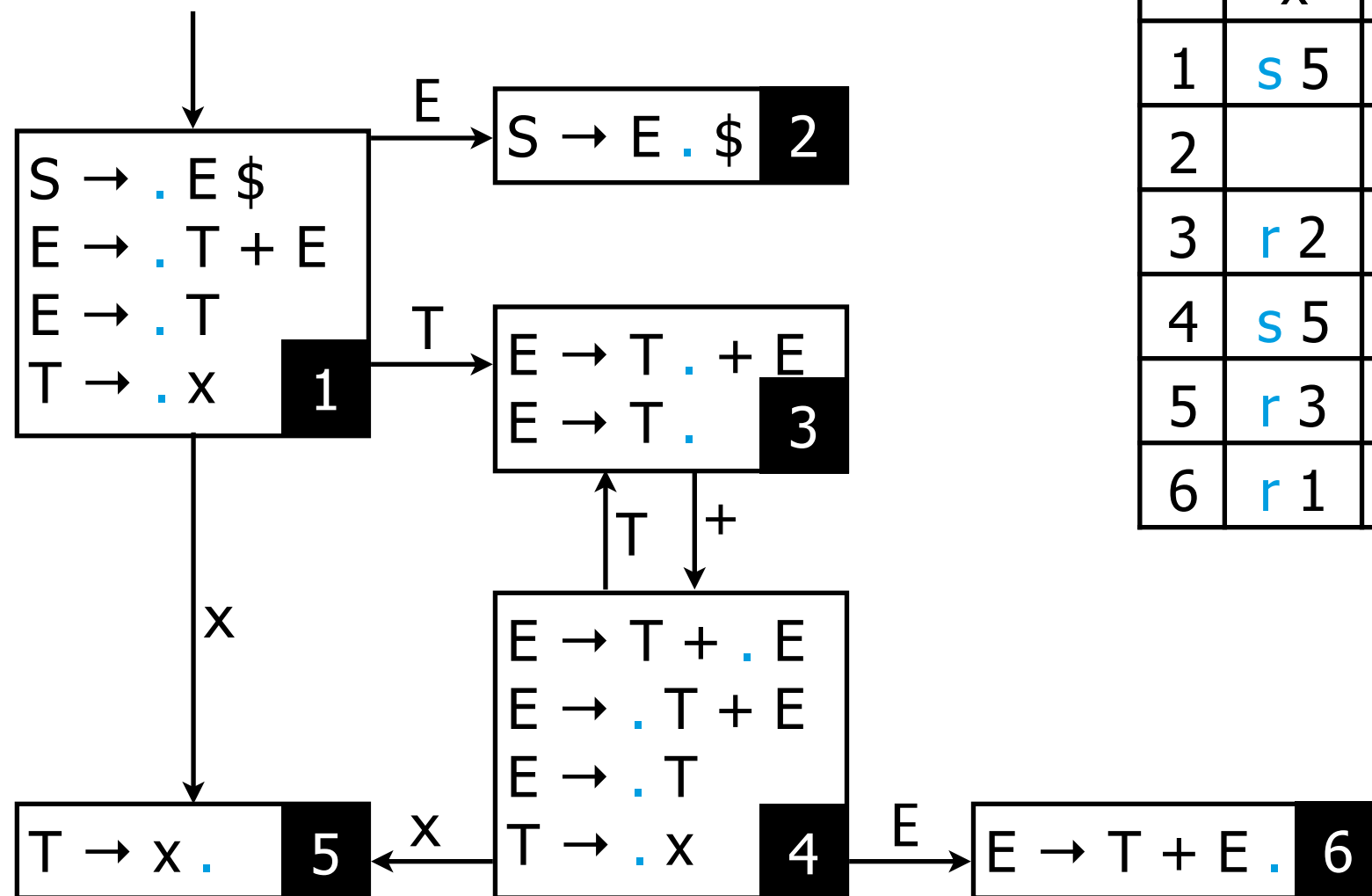
shift-reduce conflicts



$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

SLR parse tables

shift-reduce conflicts



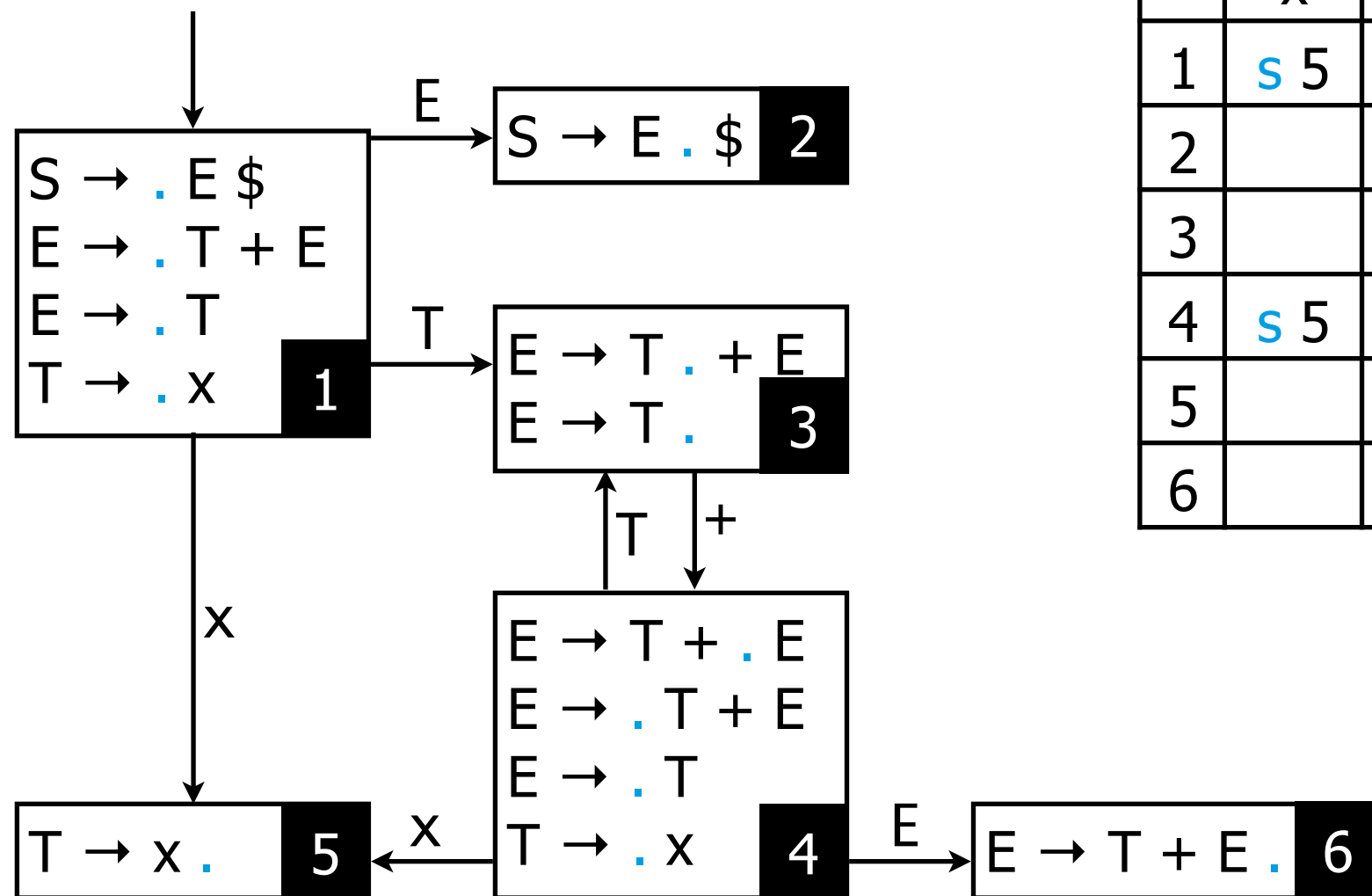
	x	+	\$	E	T
1	s 5			g 2	g 3
2			a		
3	r 2	?	r 2		
4	s 5			g 6	g 3
5	r 3	r 3	r 3		
6	r 1	r 1	r 1		

$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

SLR parse tables

shift-reduce conflicts

Reduce a production $S \rightarrow \dots$
on symbols $k \in \Sigma, k \in \text{Follow}(S)$



	x	+	\$	E	T
1	s 5			g 2	g 3
2			a		
3		s 4	r 2		
4	s 5			g 6	g 3
5		r 3	r 3		
6			r 1		

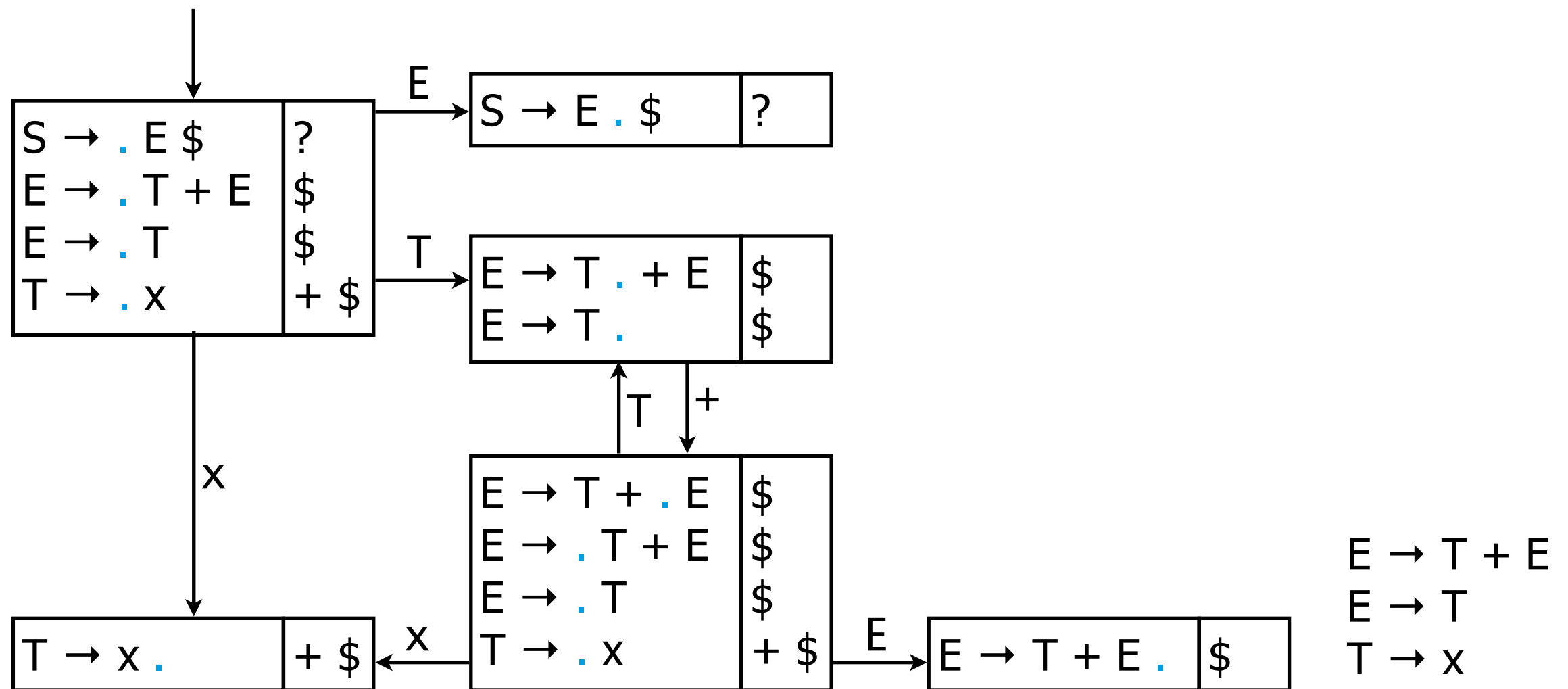
$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

LR(1) parse tables

look-ahead

closure

- for every item $A \rightarrow \alpha \cdot X \beta, z$
- for every rule $X \rightarrow \gamma$
- for every $w \in \text{First}(\beta z)$
- add item $X \rightarrow \cdot \gamma, w$

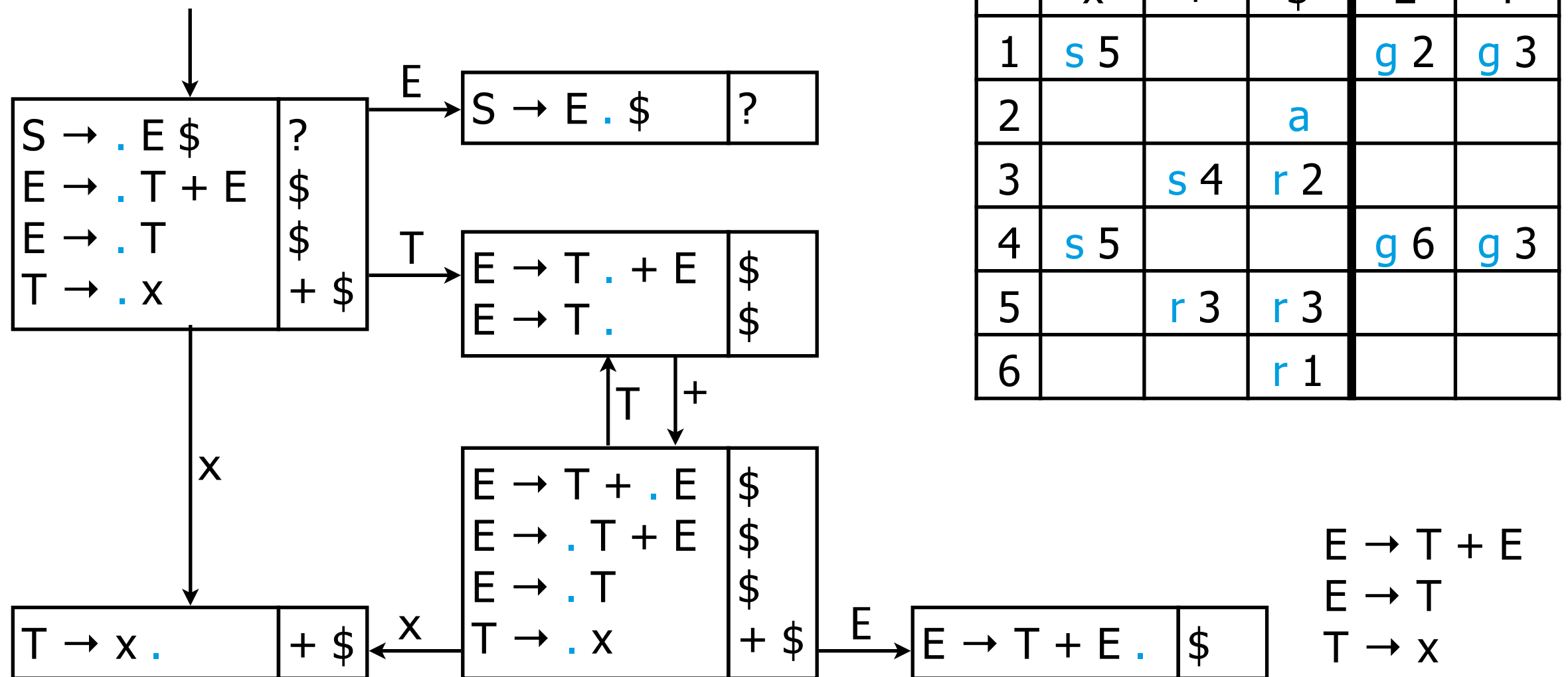


LR(1) parse tables

look-ahead

closure

- for every item $A \rightarrow a \cdot X \beta, z$
- for every rule $X \rightarrow \gamma$
- for every $w \in \text{First}(\beta z)$
- add item $X \rightarrow \cdot \gamma, w$



LALR(1) parse tables

state space reduction

unify states

- with same items
- and same outgoing transitions
- but different look-ahead sets

might introduce new conflicts

LALR(1) parse tables

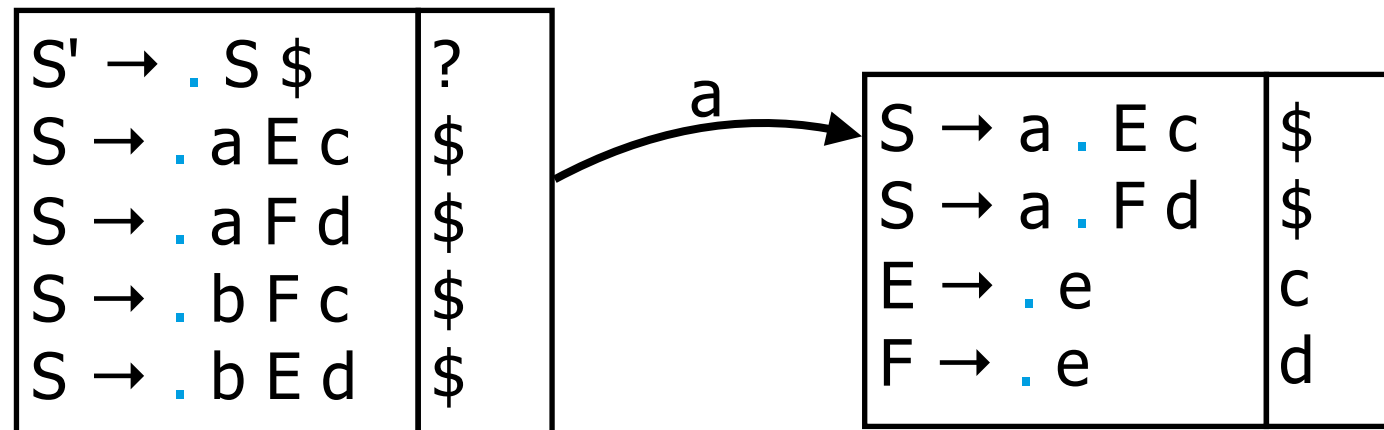
state space reduction

$S' \rightarrow \cdot S \$$	$?$
$S \rightarrow \cdot a E c$	$\$$
$S \rightarrow \cdot a F d$	$\$$
$S \rightarrow \cdot b F c$	$\$$
$S \rightarrow \cdot b E d$	$\$$

$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

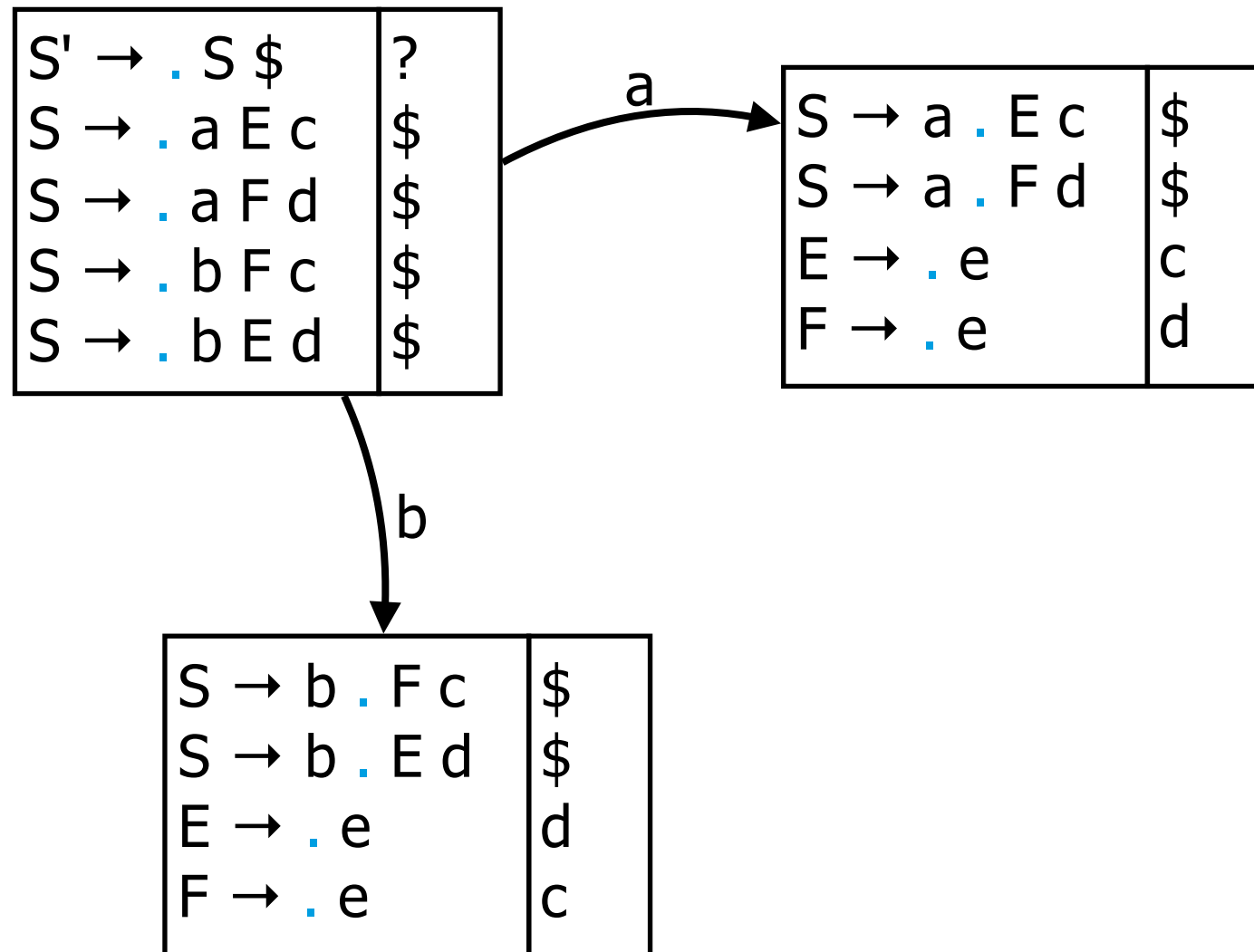
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

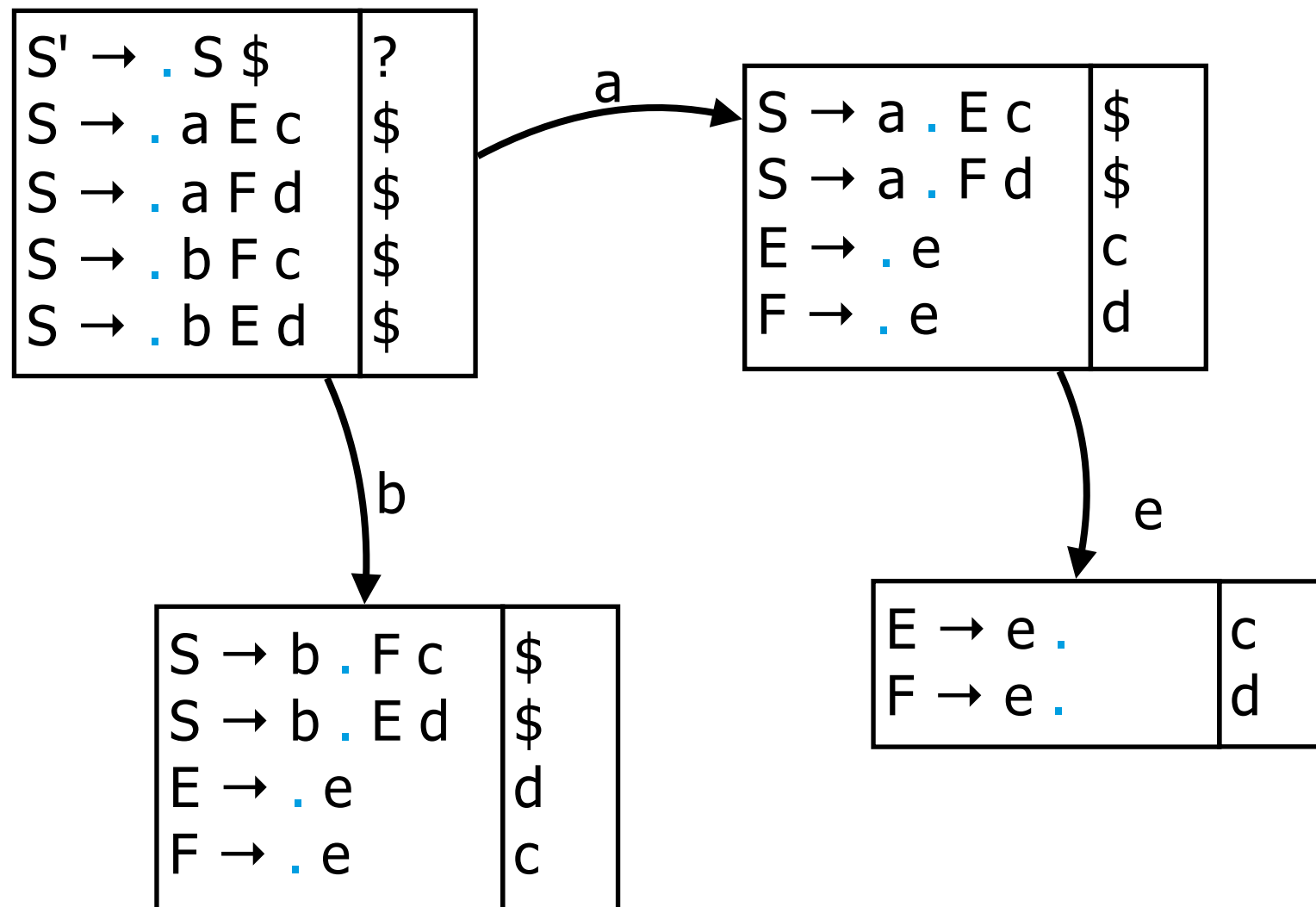
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

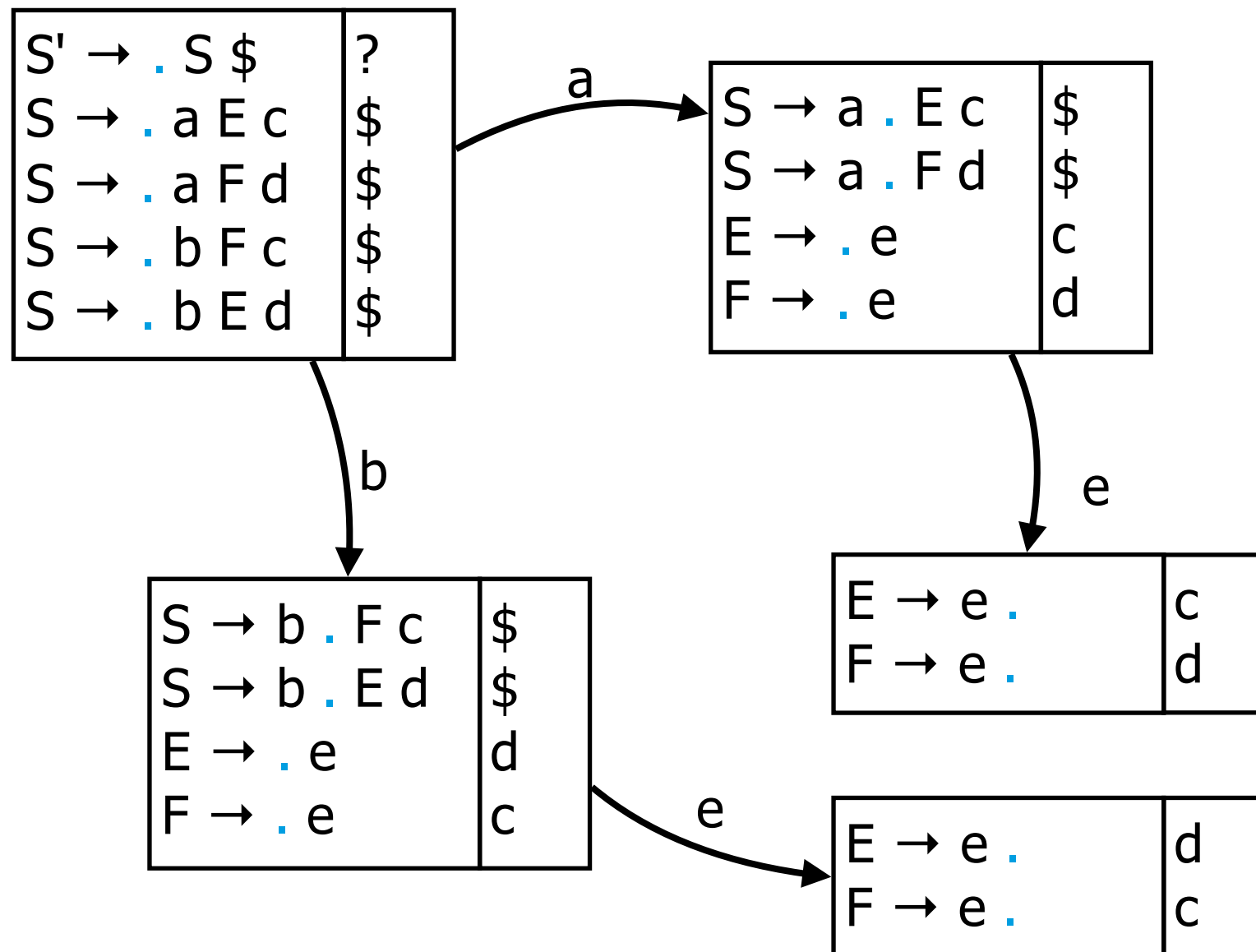
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

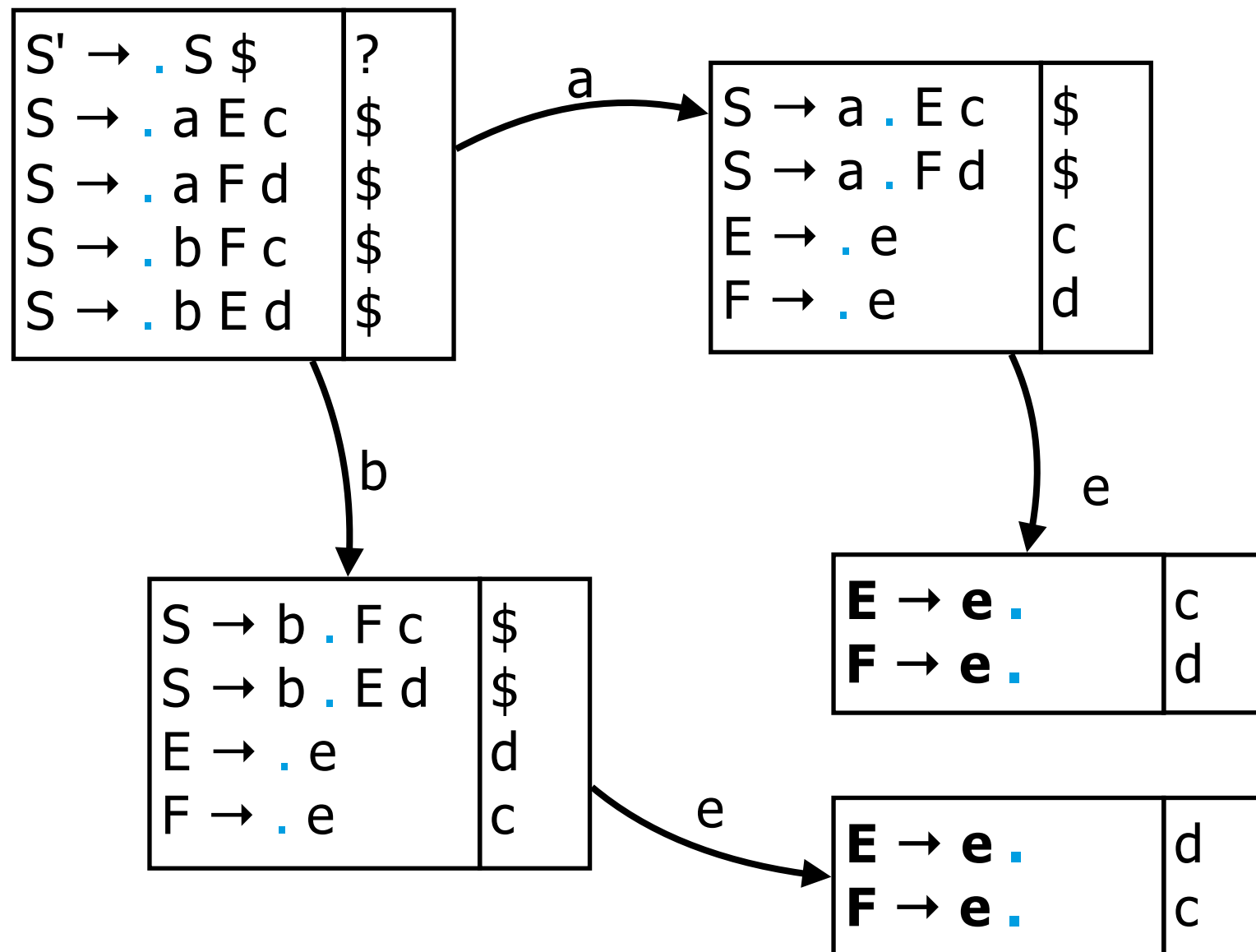
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

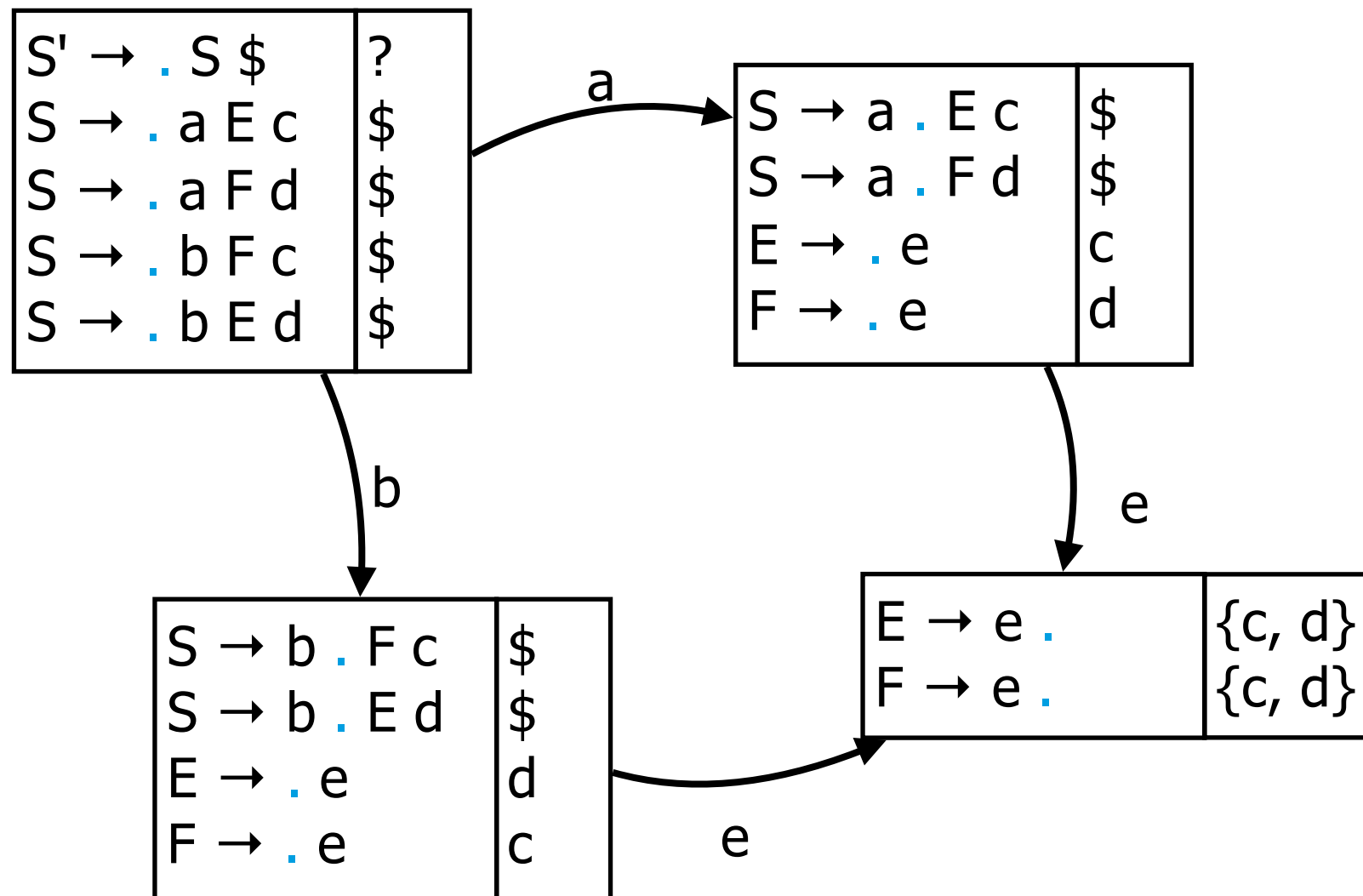
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

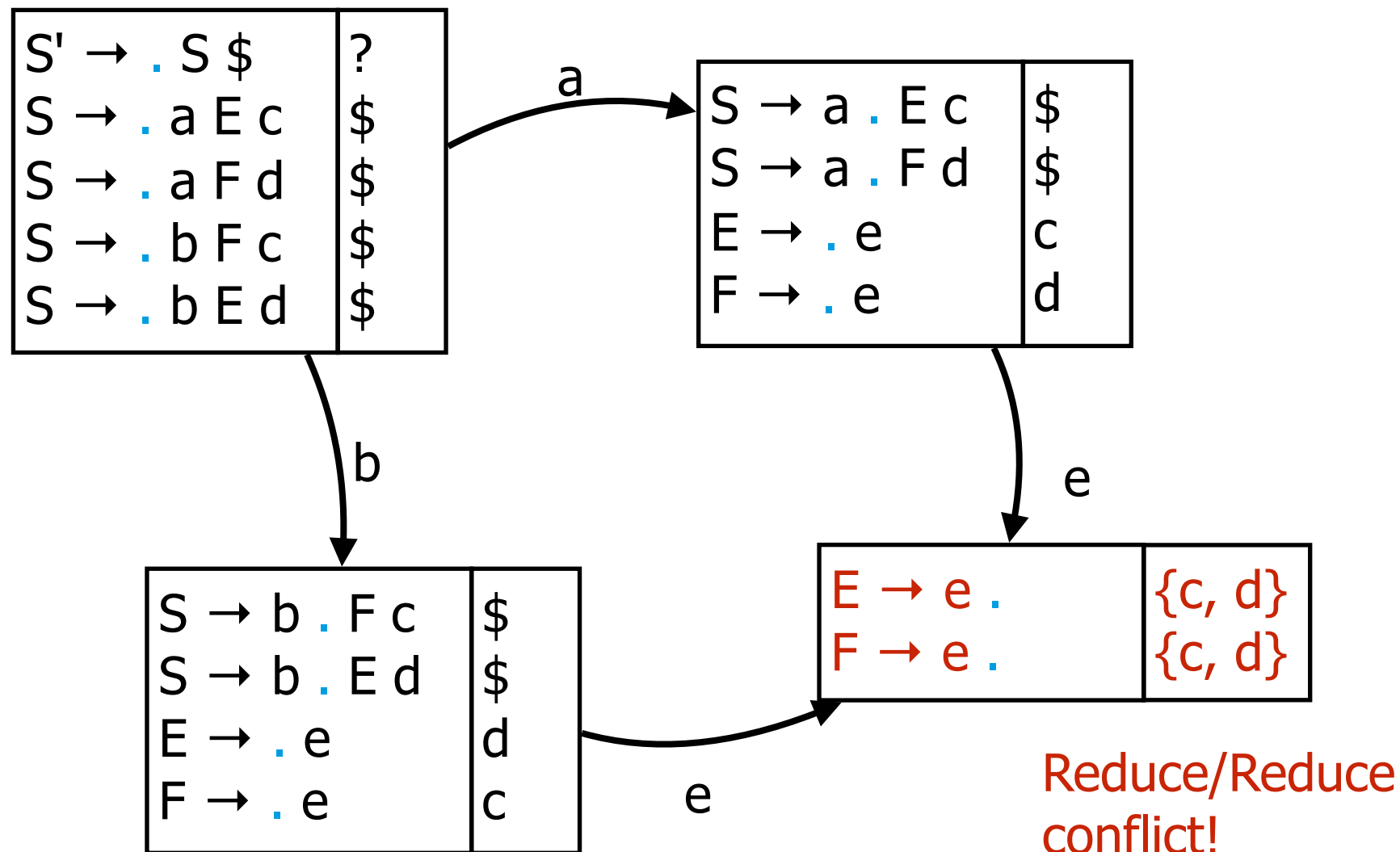
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

IV

Generalized-LR Parsing

Generalized Parsing

- Parse all interpretations of the input, therefore it can handle ambiguous grammars.
- Parsers split whenever finding an ambiguous interpretation and act in (pseudo) parallel.
- Multiple parsers can join whenever they finish parsing an ambiguous fragment of the input.
- Some parsers may "die", if the ambiguity was caused by a lack of lookahead.

Generalized LR

- Multiple parsers are synchronized on shift actions.
- Each parser has its own stack, and as they share states, the overall structure becomes a graph (GSS).
- If two parsers have the same state on top of their stack, they are joined into a single parser.
- Reduce actions affect all possible paths from the top of the stack.

Generalized LR

SLR table

$$\begin{aligned} S &\rightarrow E \$ \\ E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow a \end{aligned}$$

Generalized LR

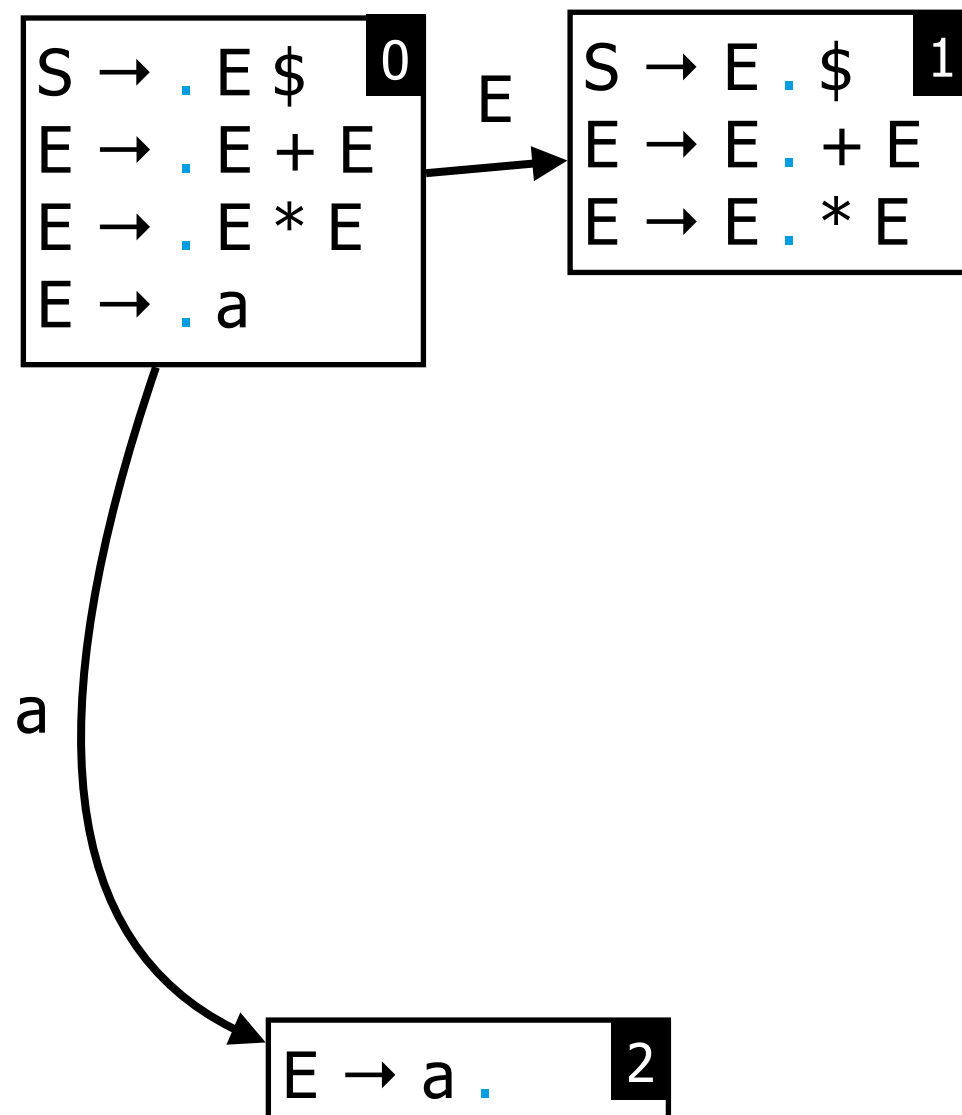
SLR table

$S \rightarrow \cdot E \$$	0
$E \rightarrow \cdot E + E$	
$E \rightarrow \cdot E * E$	
$E \rightarrow \cdot a$	

$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

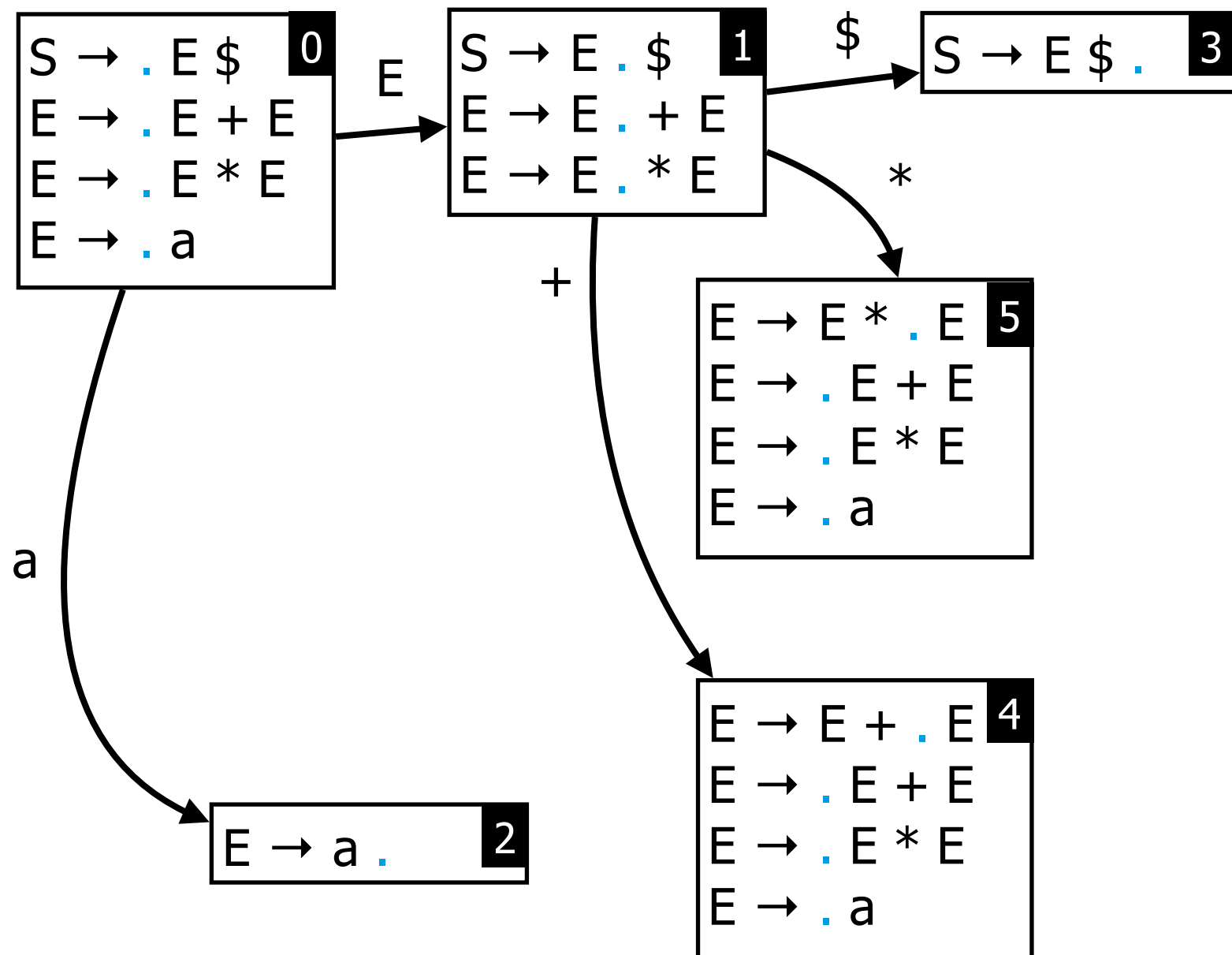
SLR table



$$\begin{array}{l} S \rightarrow E \$ \\ E \rightarrow E + E \\ E \rightarrow E * E \\ E \rightarrow a \end{array}$$

Generalized LR

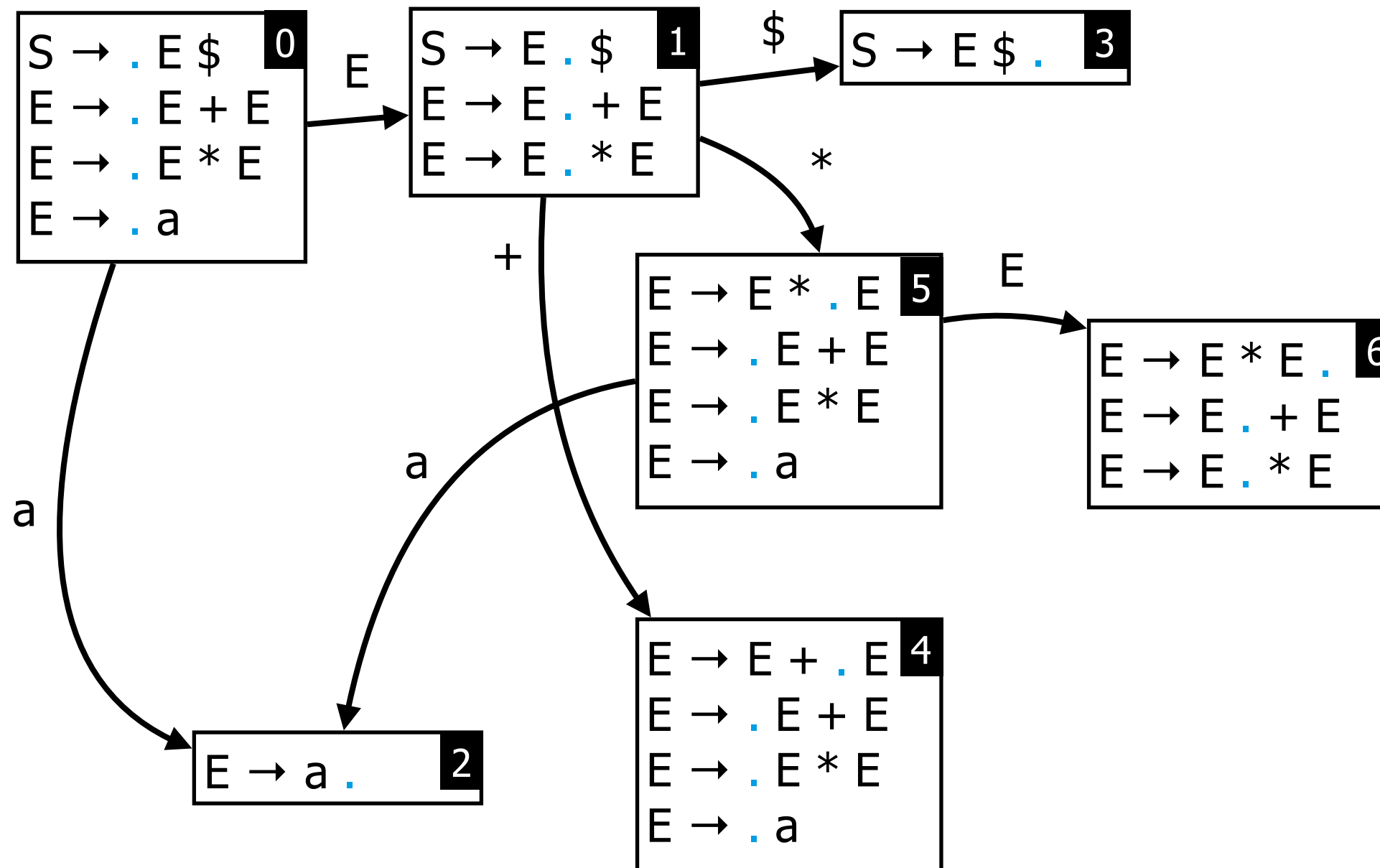
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

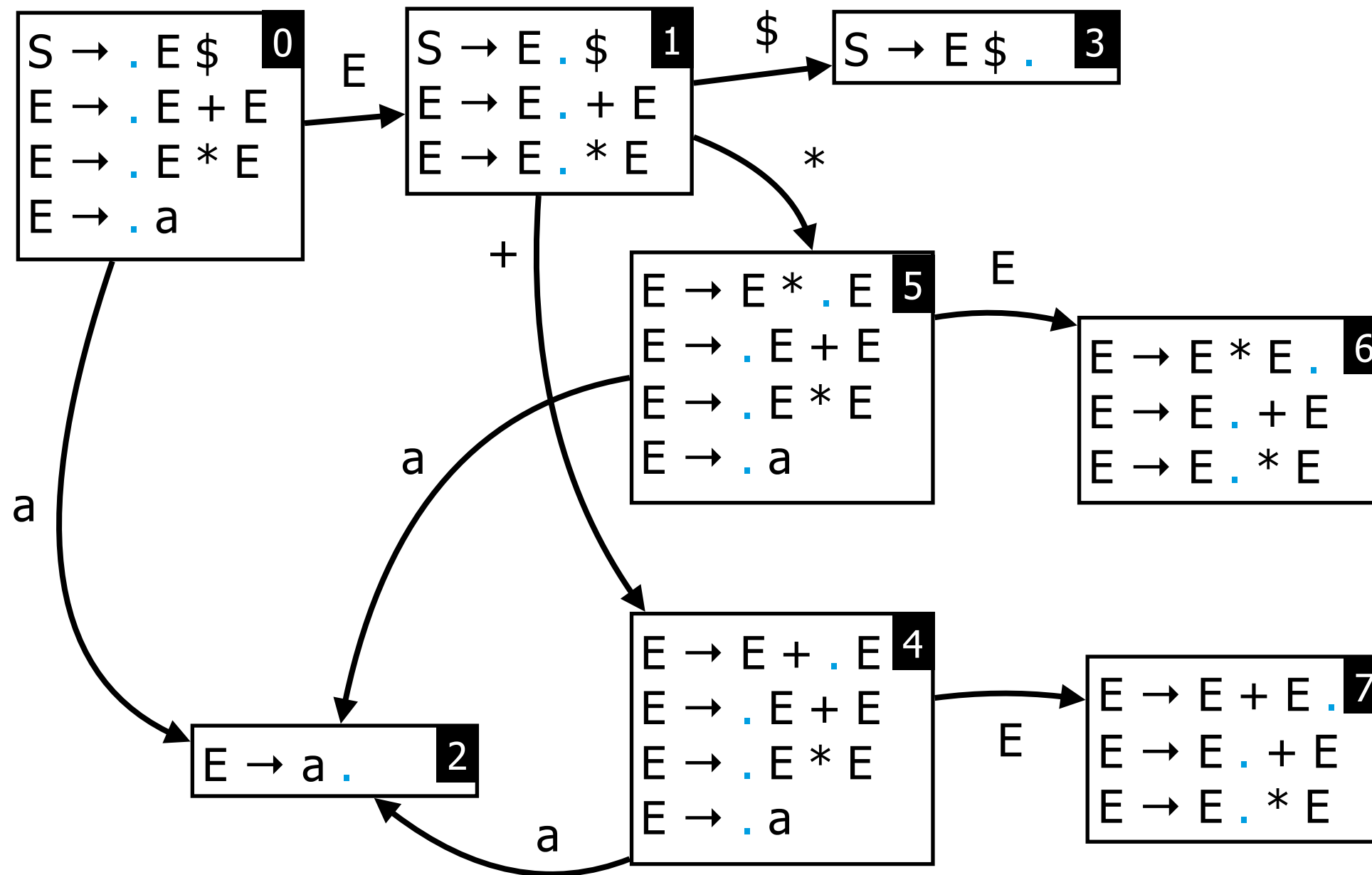
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

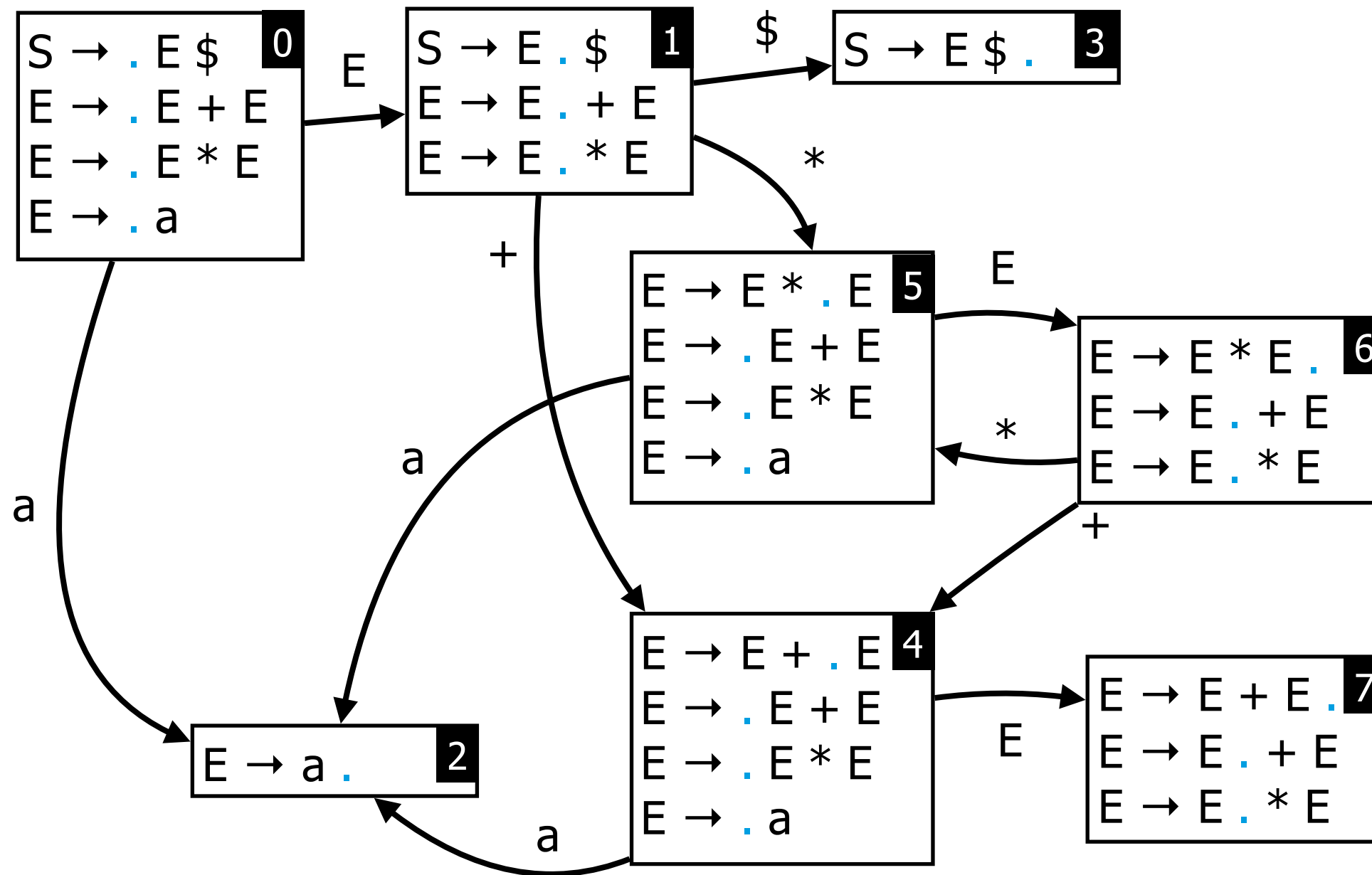
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

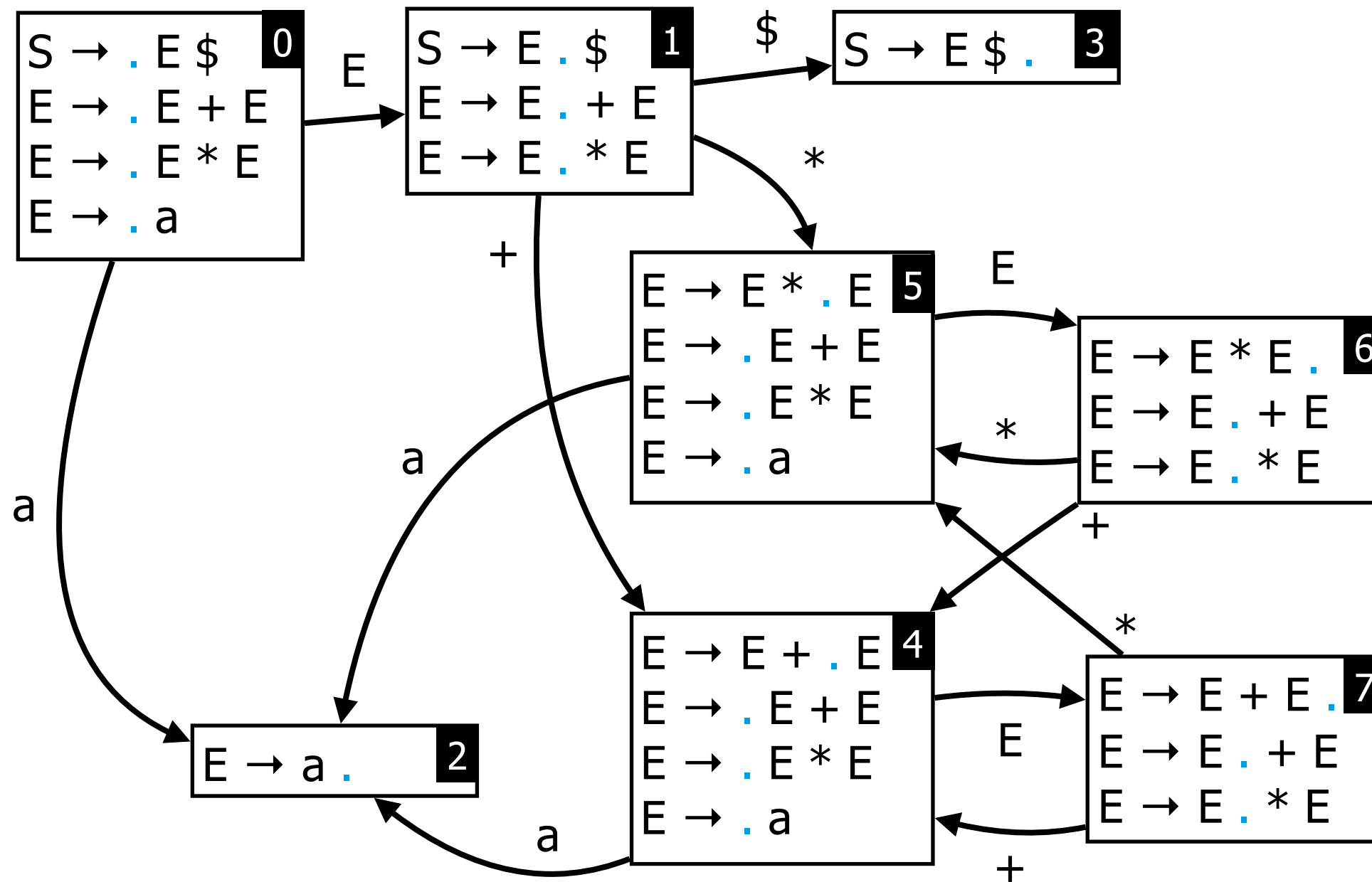
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

SLR table



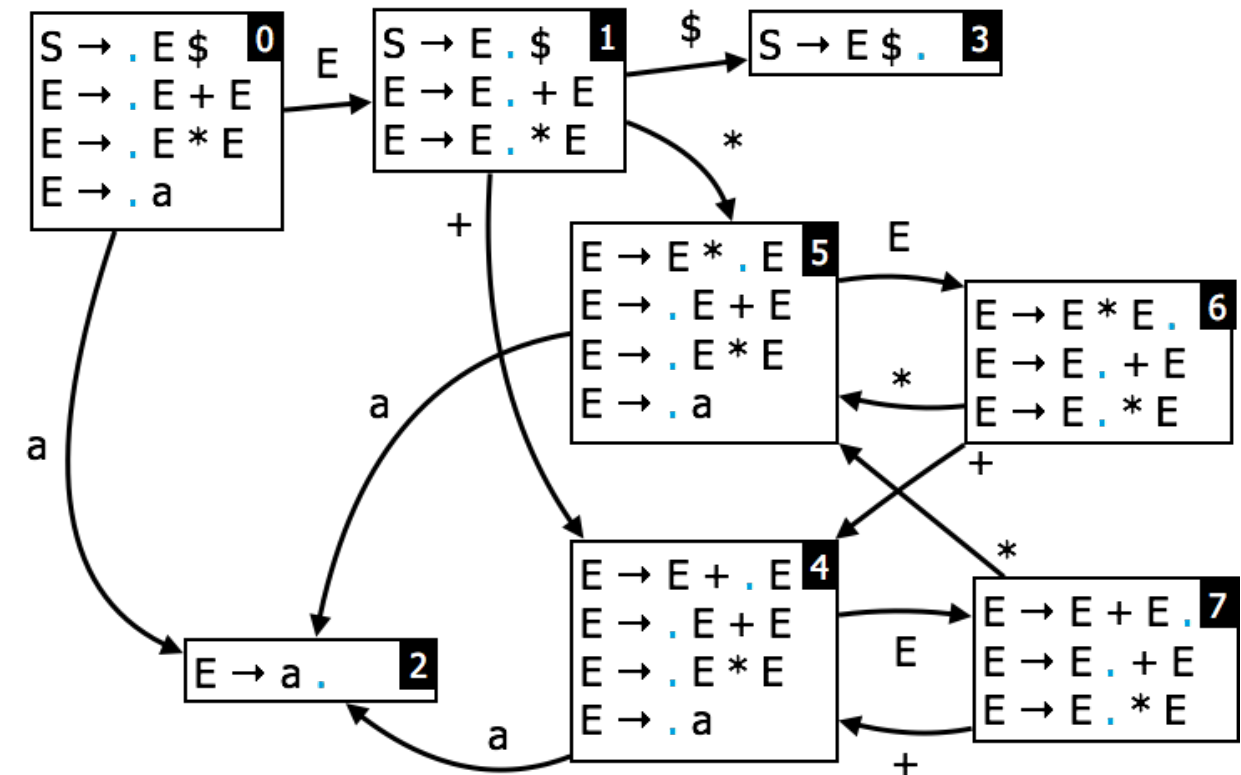
$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

SLR table

State	Action				Goto	
	a	+	*	\$	S	E
0						
1						
2						
3						
4						
5						
6						
7						

Nonter	Nullable	First	Follow
S			
E			



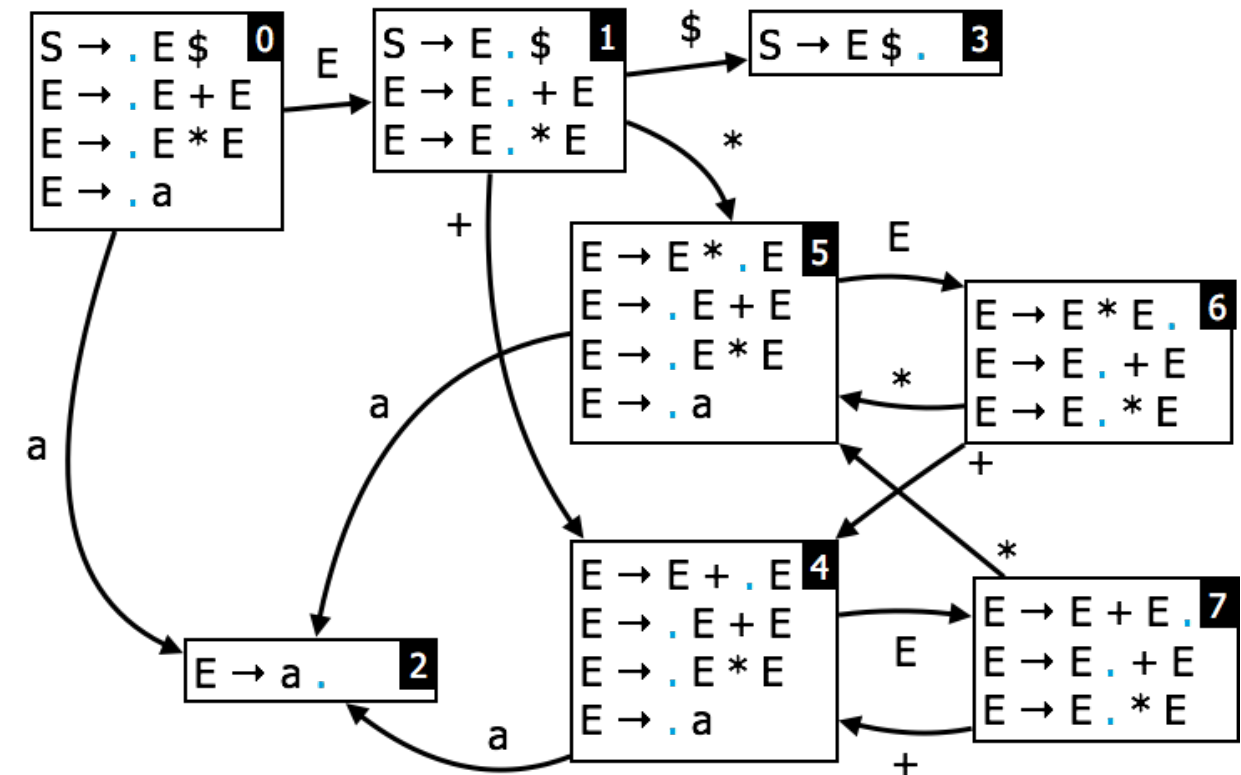
- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

Generalized LR

SLR table

State	Action				Goto	
	a	+	*	\$	S	E
0						
1						
2						
3						
4						
5						
6						
7						

Nonter	Nullable	First	Follow
S	no	a	-
E	no	a	+, *, \$



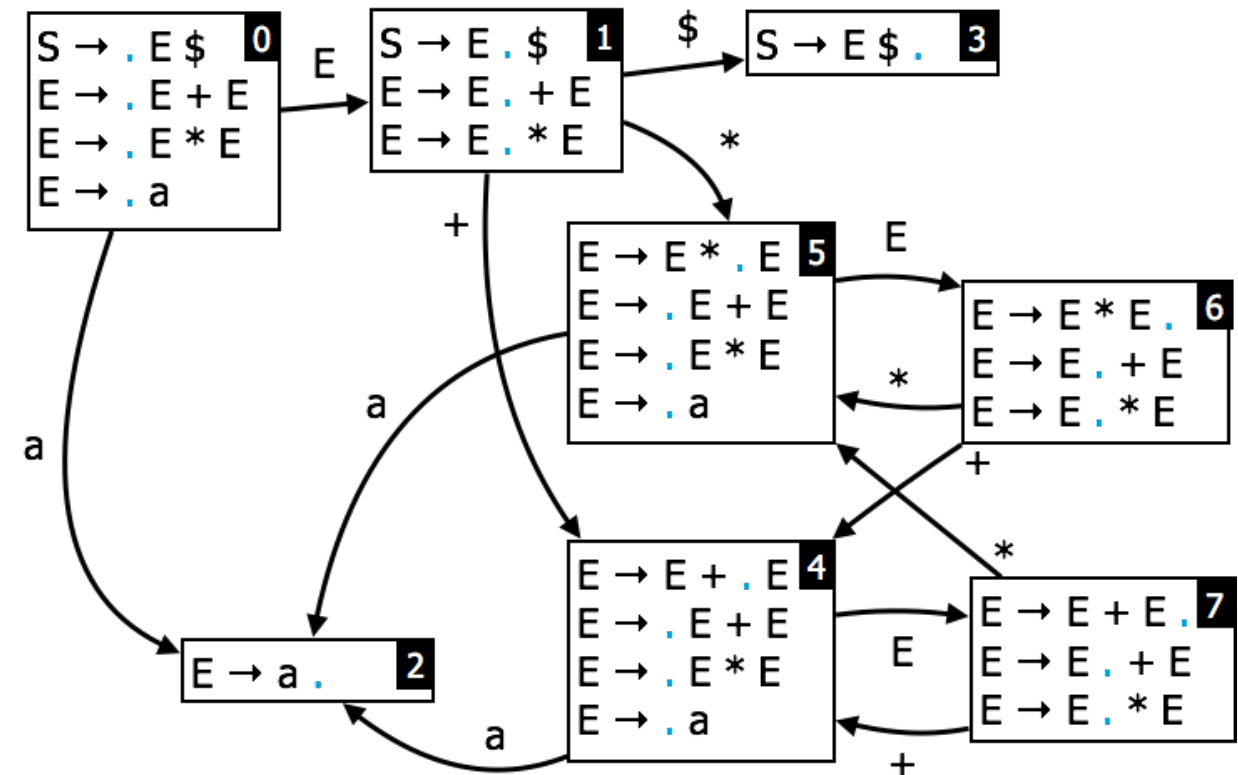
- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

Generalized LR

SLR table

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

Nonter	Nullable	First	Follow
S	no	a	-
E	no	a	+, *, \$



- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

Generalized LR Parsing

$a + a * a$

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

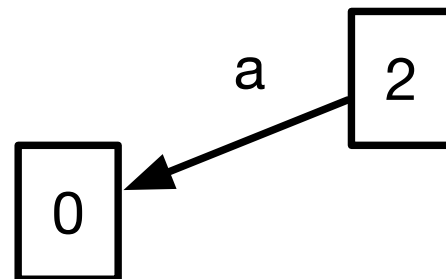
0

Generalized LR Parsing

+ a * a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		



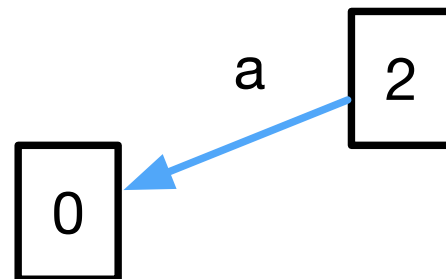
synchronize on shifts

Generalized LR Parsing

+ a * a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

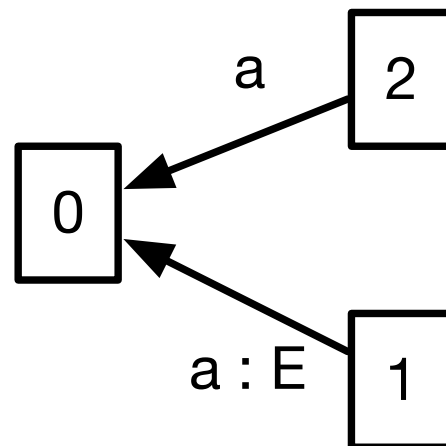


Generalized LR Parsing

+ a * a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

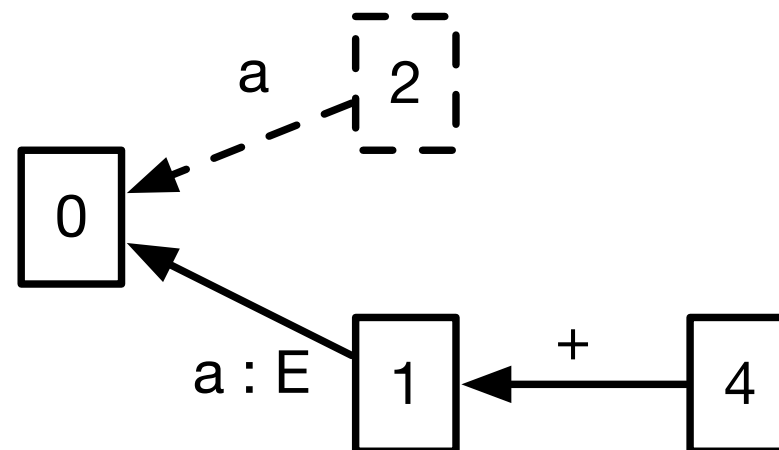


Generalized LR Parsing

$a * a$

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		



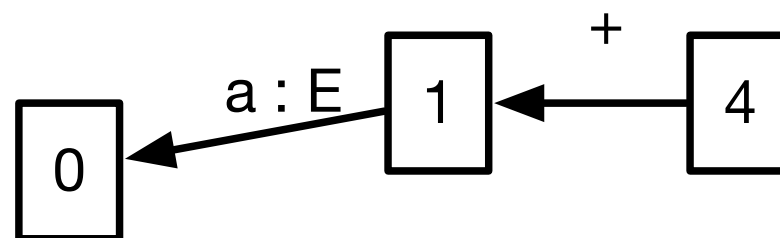
synchronize

Generalized LR Parsing

$a * a$

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

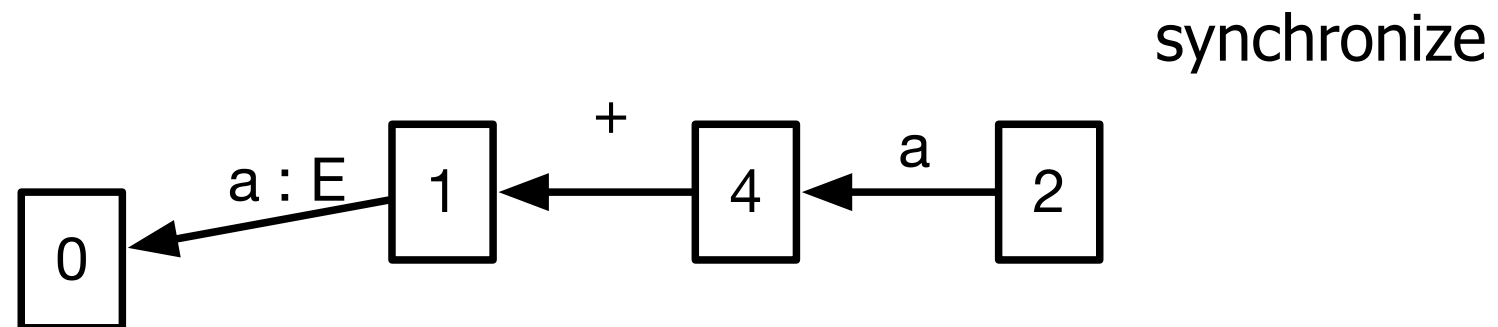


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

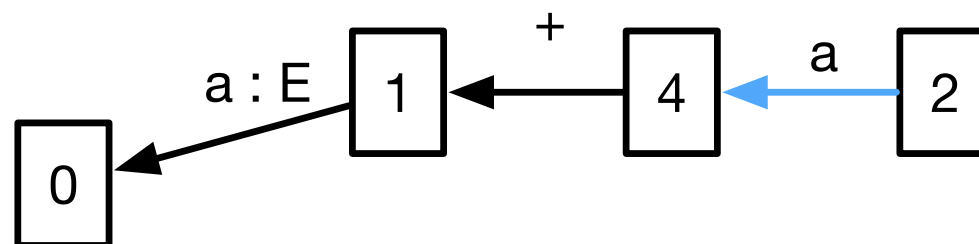


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

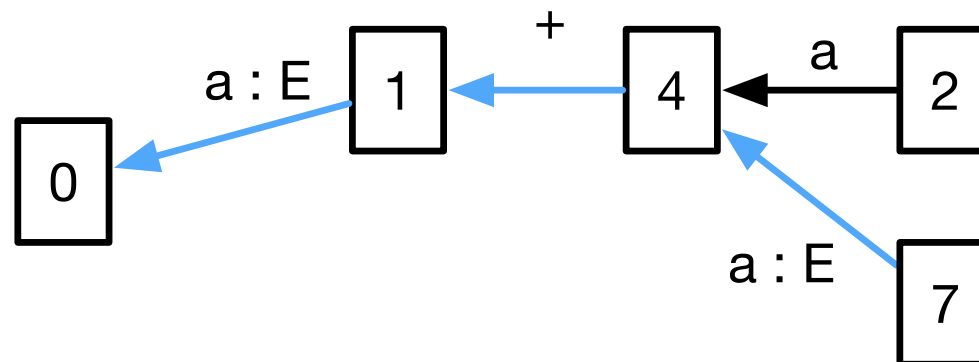


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

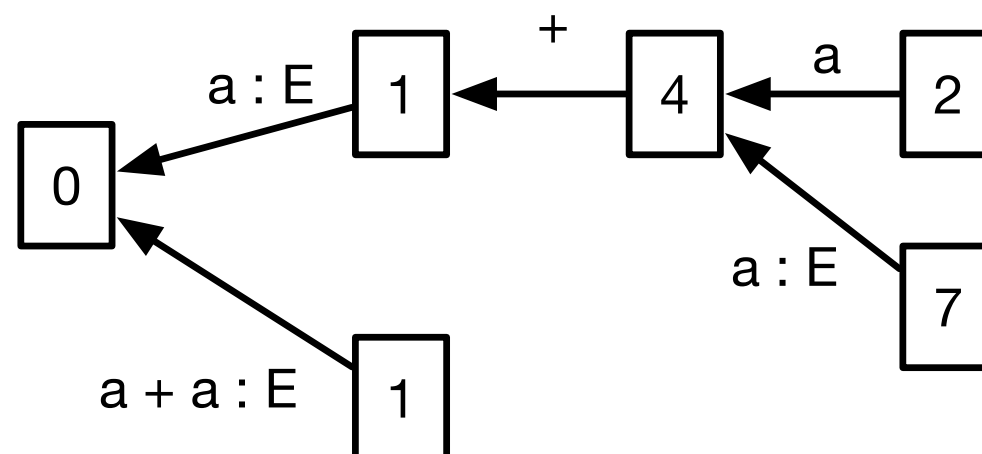


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
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6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

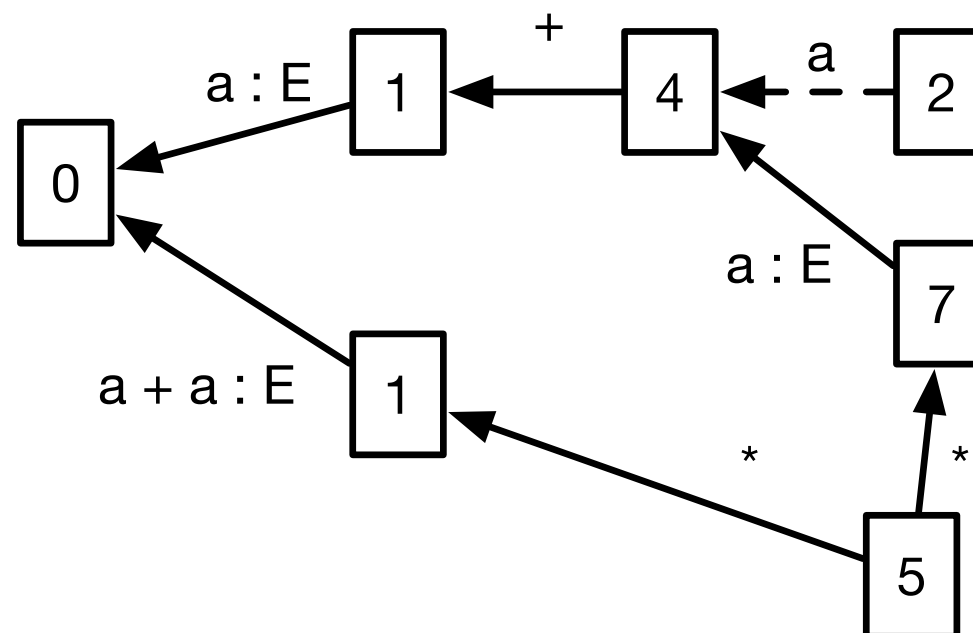


Generalized LR Parsing

a

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	a	+	*	\$	S	E
0	s2					1
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4	s2					7
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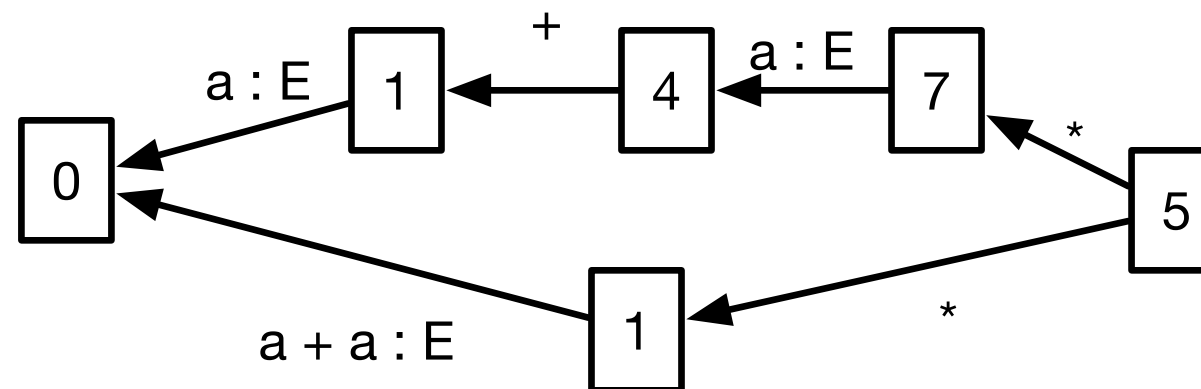
synchronize

Generalized LR Parsing

a

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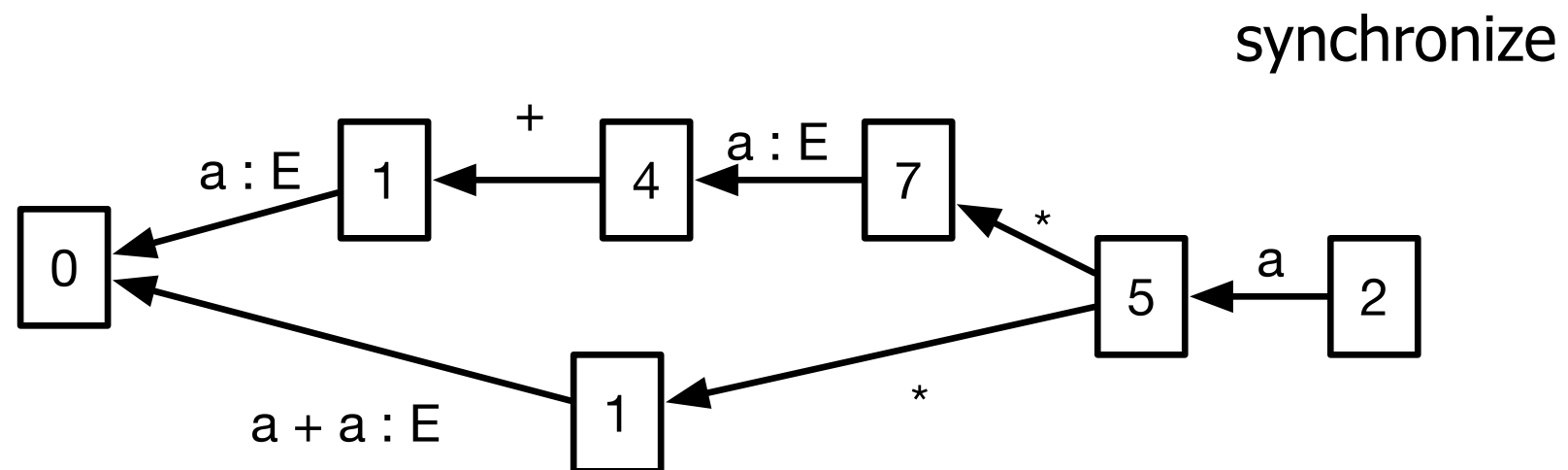
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
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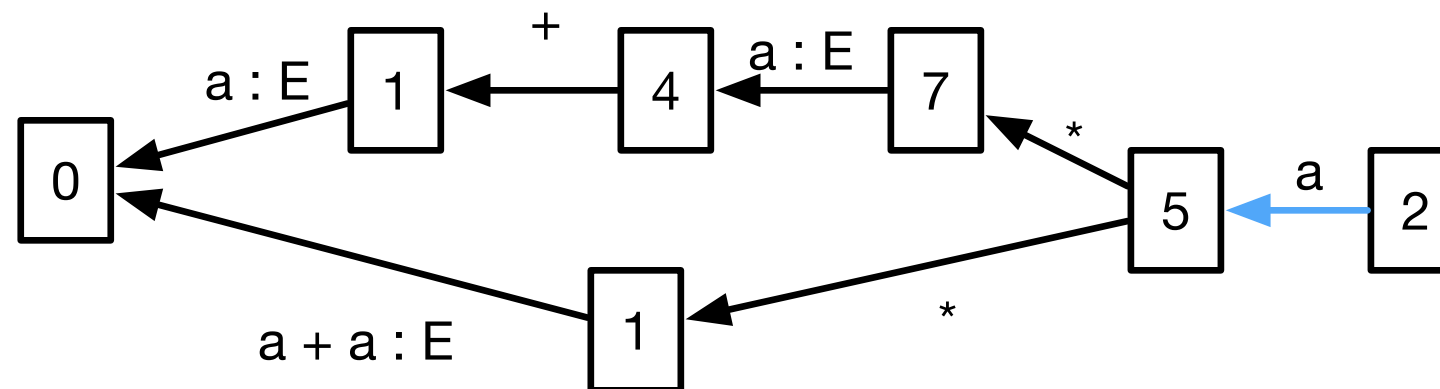
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
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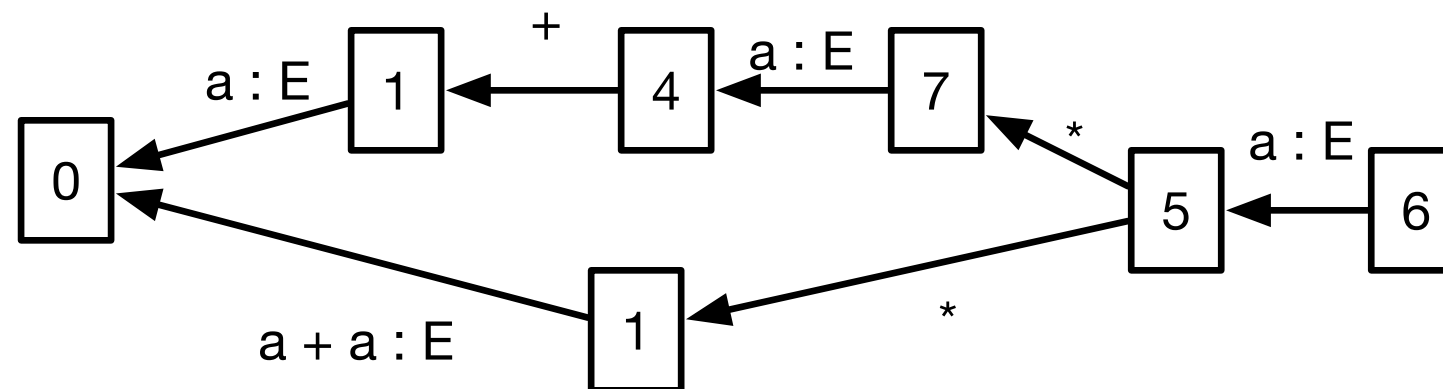
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
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2		r3	r3	r3		
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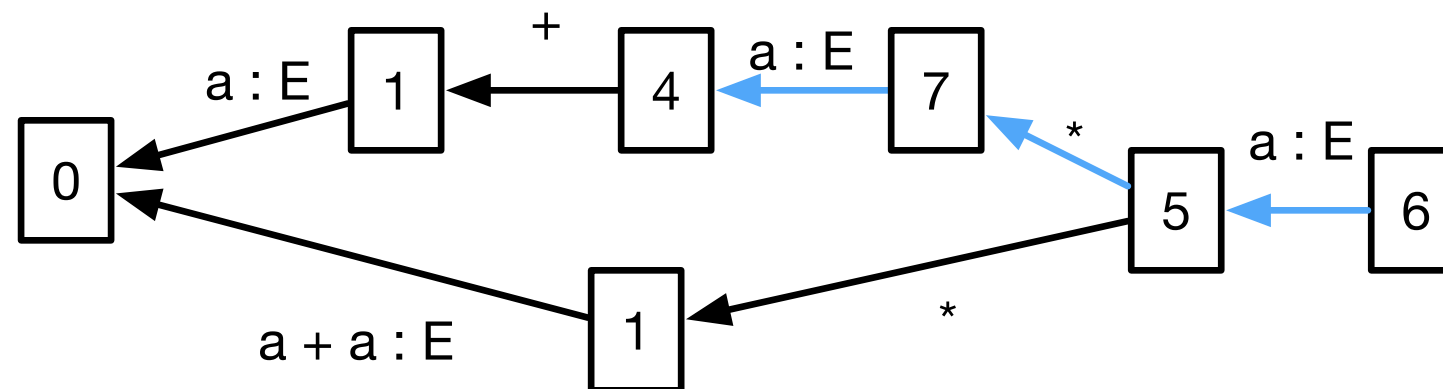
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
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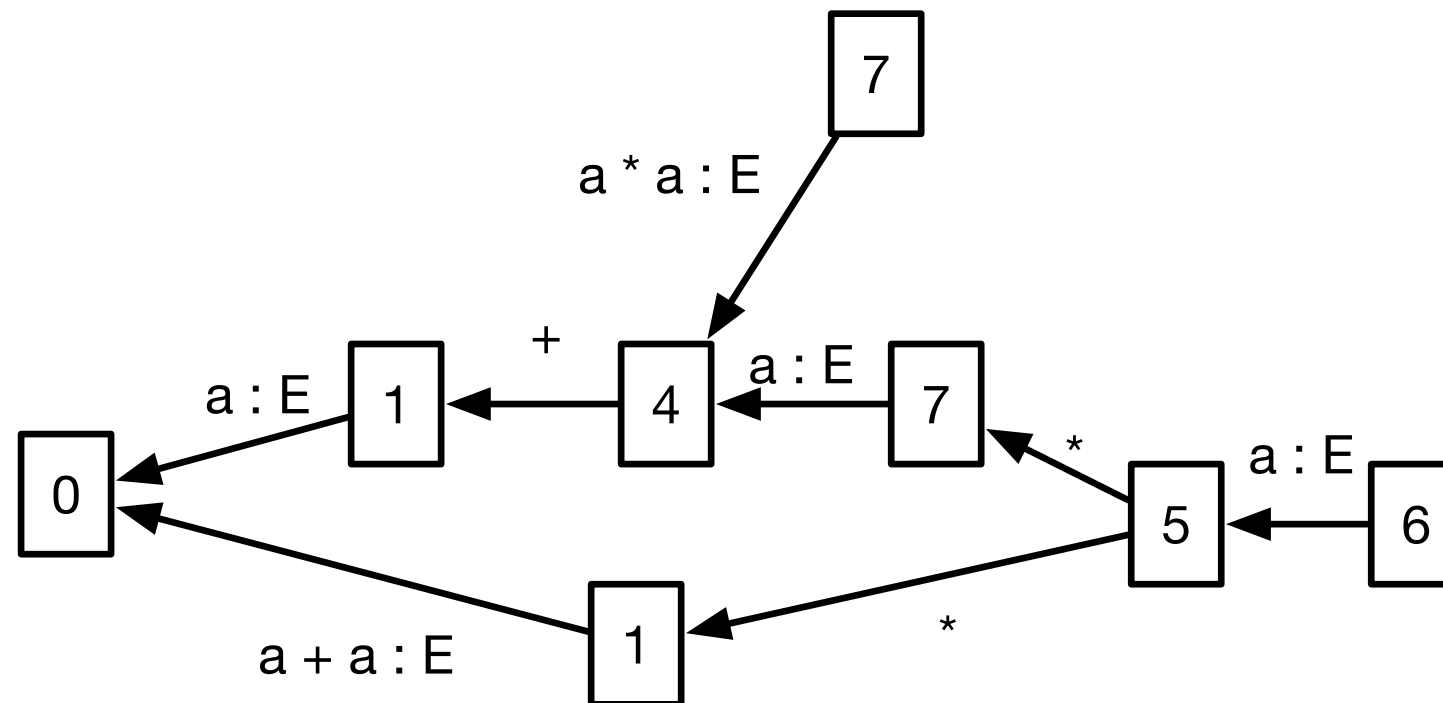
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
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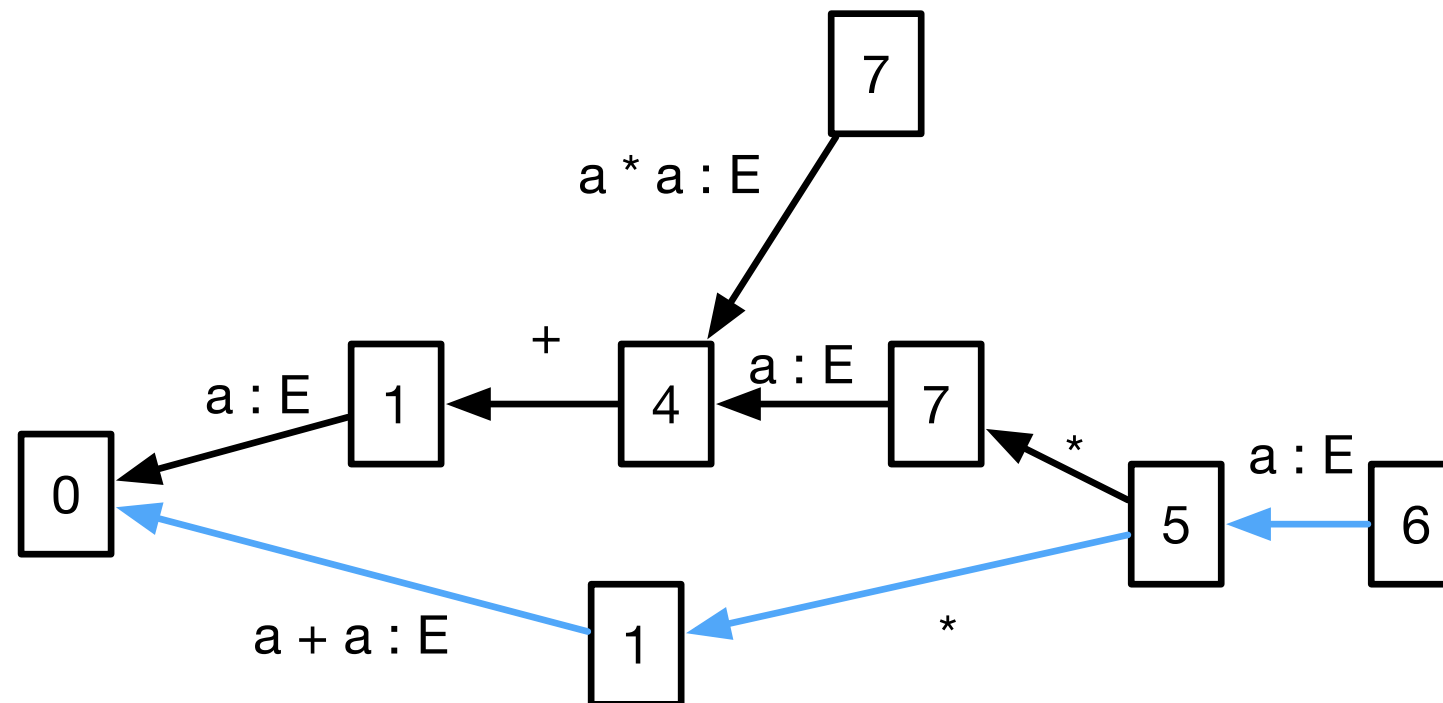
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
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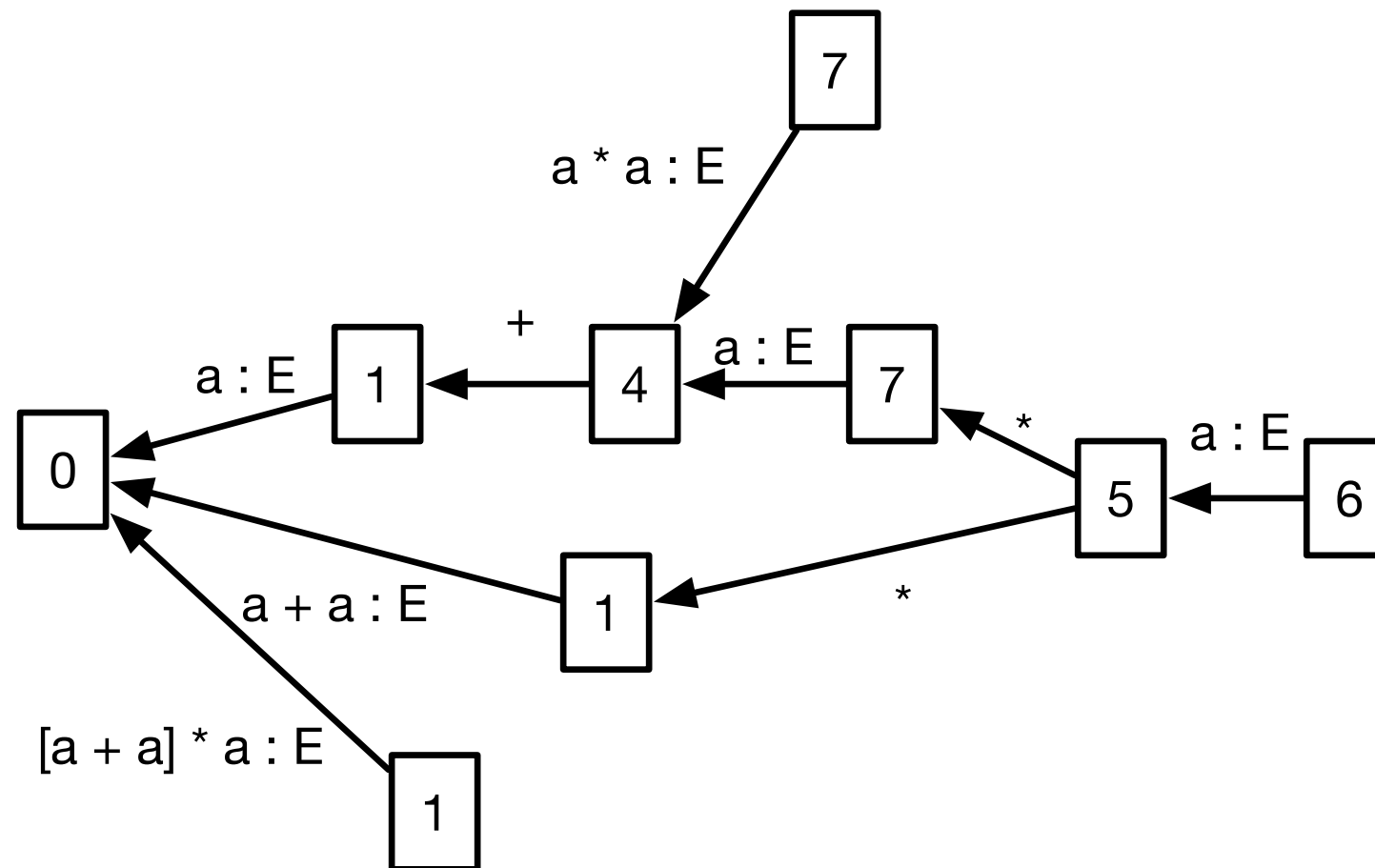
State	Action				Goto	
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0	s2					1
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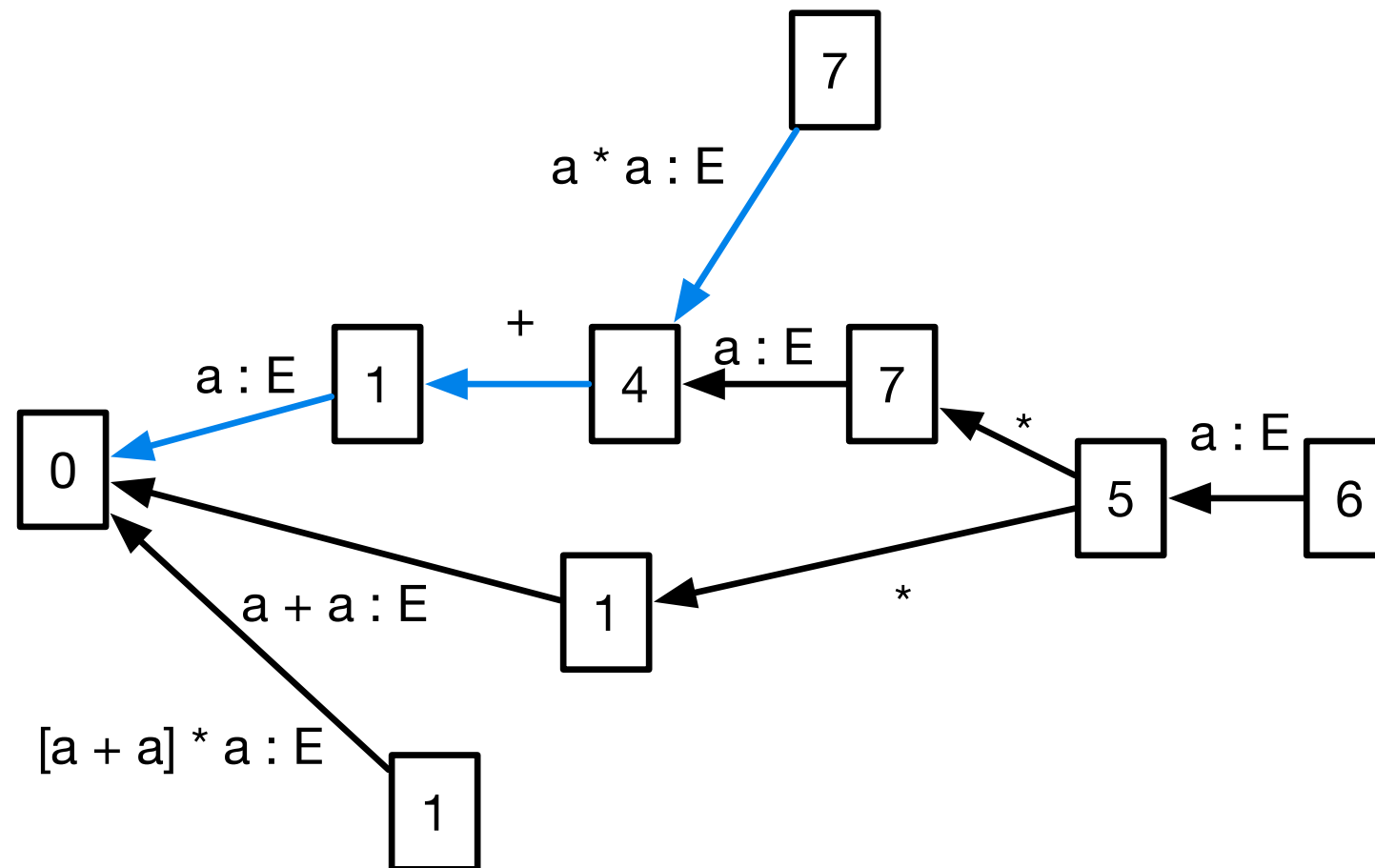
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
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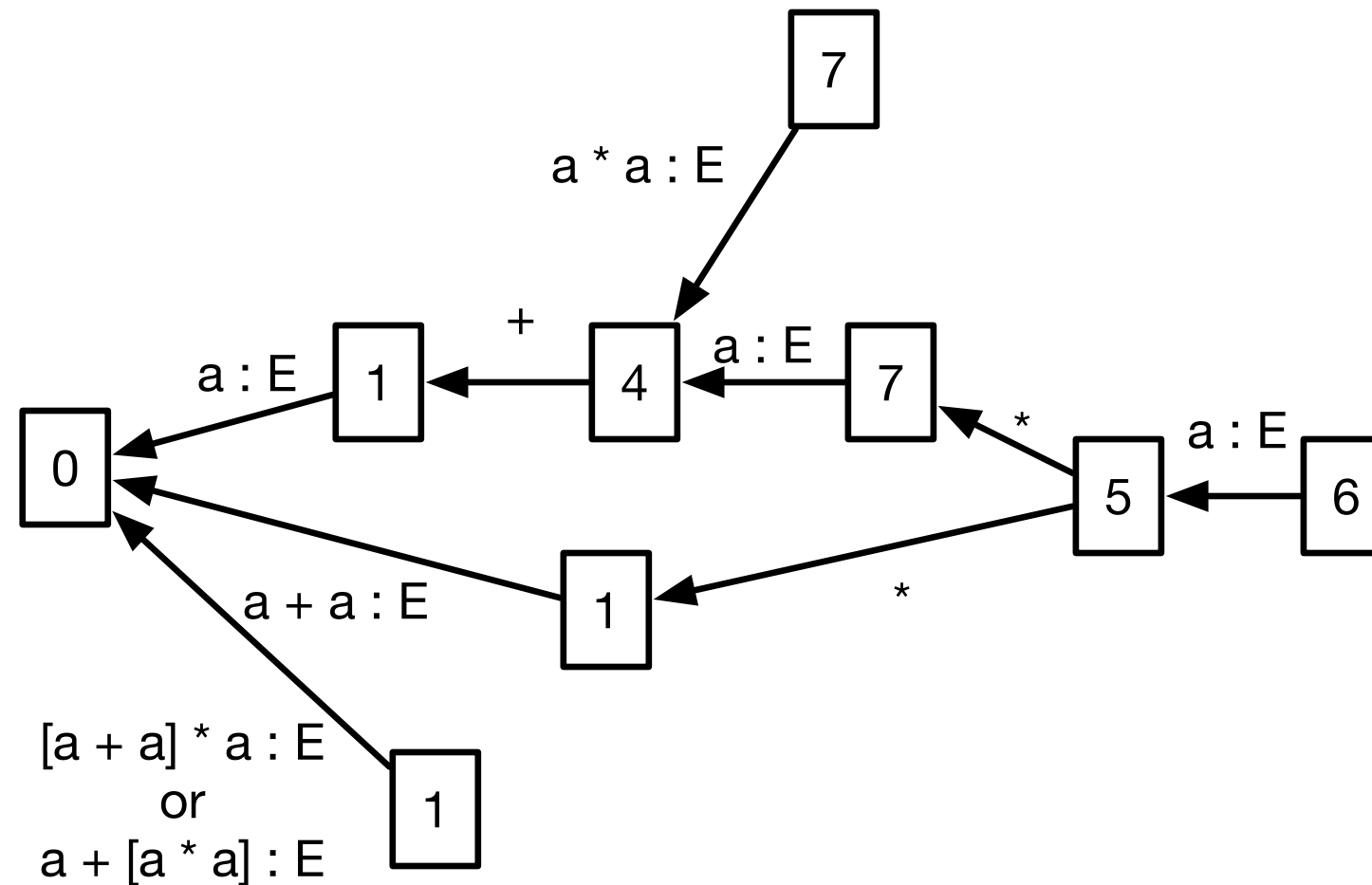
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					1
1		s4	s5	s3		
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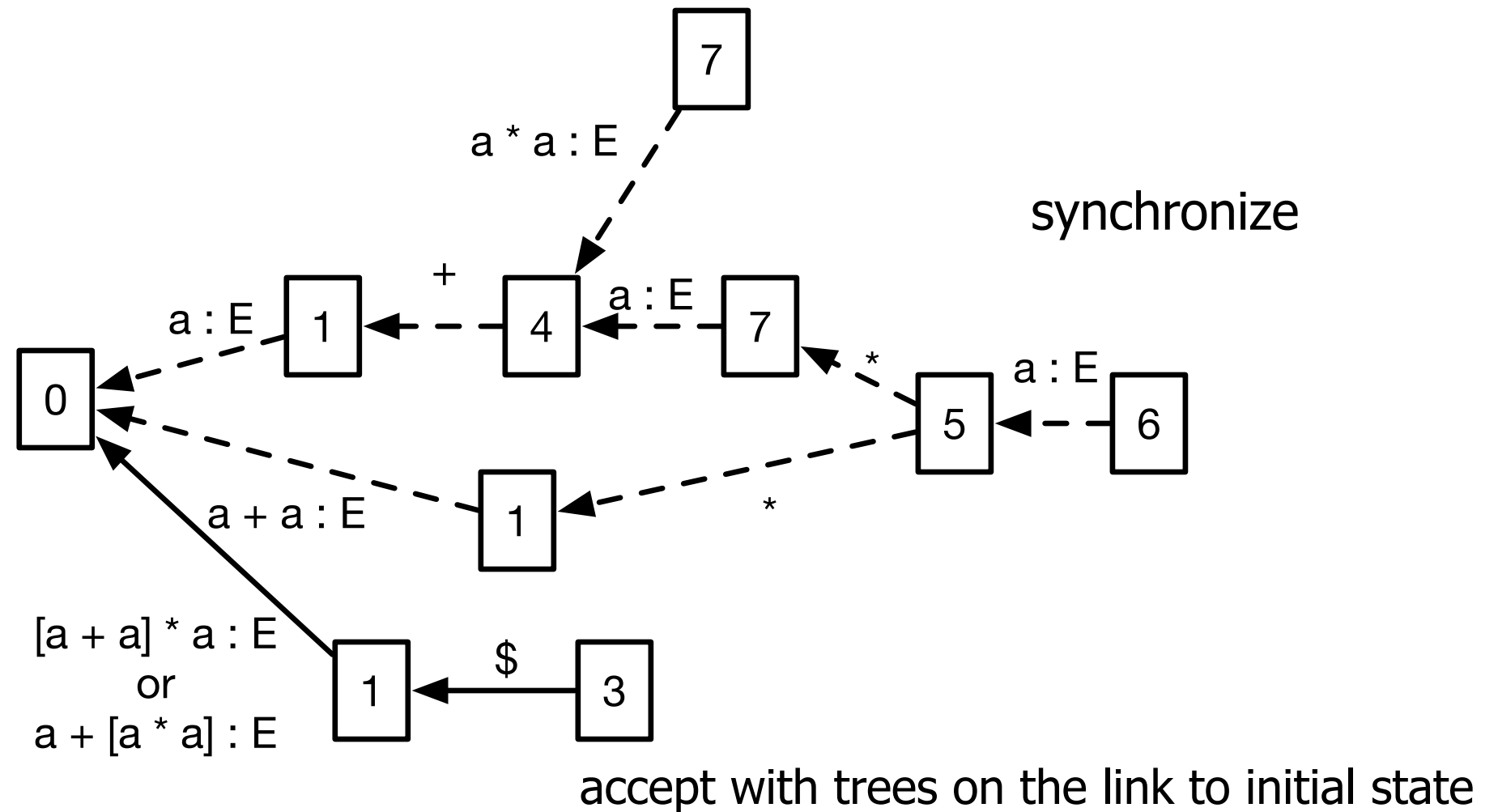
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0	s2					1
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V

Scannerless Generalized-LR Parsing

Scannerless Generalized LR

- Integrates scanning + parsing into a single GLR algorithm.
- Normalization separates lexical and context-free symbols.
- Crucial for avoiding conflicts when composing languages.
- Introduces lexical ambiguities. Solution:
 - Follow Restrictions (implement longest match).
 - Reject Rules (implement reserved keywords).

Scannerless Generalized LR

Normalization

context-free syntax

Exp.Add = <<Exp> + <Exp>> {left}
Exp.Inc = <<Exp>++>
Exp.ID = ID

lexical syntax

ID = [a-zA-Z] IDRest*
IDRest = [a-zA-Z0-9]
LAYOUT = [\ \t\n\r]
ID = "let" {reject}

Scannerless Generalized LR

Normalization

Separate lexical from context-free symbols and separate symbols in context-free syntax by optional layout.

context-free syntax

```
Exp.Add = <<Exp> + <Exp>> {left}  
Exp.Inc = <<Exp>++>  
Exp.ID = ID
```

lexical syntax

```
ID = [a-zA-Z] IDRest*  
IDRest = [a-zA-Z0-9]  
LAYOUT = [\ \t\n\r]  
ID = "let" {reject}
```

```
Exp-CF.Add = Exp-CF LAYOUT?-CF "+" LAYOUT?-CF Exp-CF {left}  
Exp-CF.Inc = Exp-CF LAYOUT?-CF "++"  
Exp-CF.ID = ID-CF  
ID-LEX = [a-zA-Z] IDRest*-LEX  
IDRest*-LEX = [a-zA-Z0-9]  
ID-LEX = "let" {reject}  
LAYOUT-CF = LAYOUT-LEX
```

Scannerless Generalized LR

Normalization

Create injections from lexical to context-free symbols

context-free syntax

```
Exp.Add    = <<Exp> + <Exp>> {left}  
Exp.Inc    = <<Exp>++>  
Exp.ID     = ID
```

lexical syntax

```
ID          = [a-zA-Z] IDRest*  
IDRest      = [a-zA-Z0-9]  
LAYOUT      = [\ \t\n\r]  
ID          = "let" {reject}
```

```
LAYOUT-CF = LAYOUT-LEX  
IDRest-CF = IDRest-LEX  
IDRest*-CF = IDRest*-LEX  
ID-CF = ID-LEX
```

Scannerless Generalized LR

Normalization

Normalize literals symbols and character classes.

context-free syntax

```
Exp.Add    = <<Exp> + <Exp>> {left}  
Exp.Inc    = <<Exp>++>  
Exp.ID     = ID
```

lexical syntax

```
ID          = [a-zA-Z] IDRest*  
IDRest      = [a-zA-Z0-9]  
LAYOUT      = [\ \t\n\r]  
ID          = "let" {reject}
```

"+" = [\43]

"++" = [\43] [\43]

"let" = [\108] [\101] [\116]

ID-LEX = [\65-\90\97-\122] IDRest*-LEX

IDRest*-LEX = [\48-\57\65-\90\97-\122]

LAYOUT-LEX = [\9-\10\13\32]

Scannerless Generalized LR

Normalization

Normalize regular expressions.

context-free syntax

```
Exp.Add    = <<Exp> + <Exp>> {left}  
Exp.Inc    = <<Exp>++>  
Exp.ID     = ID
```

lexical syntax

```
ID          = [a-zA-Z] IDRest*  
IDRest      = [a-zA-Z0-9]  
LAYOUT      = [\ \t\n\r]  
ID          = "let" {reject}
```

LAYOUT?-CF = LAYOUT-CF

LAYOUT?-CF =

IDRest+-LEX = IDRest-LEX

IDRest+-LEX = IDRest+-LEX IDRest-LEX

IDRest*-LEX =

IDRest*-LEX = IDRest+-LEX

IDRest+-CF = IDRest+-LEX

Scannerless Generalized LR

Normalization

Define extra rules for the start symbols.

context-free start-symbols

Exp

$\langle \text{START} \rangle = \text{LAYOUT?} \text{-CF Exp-CF LAYOUT?} \text{-CF}$
 $\langle \text{Start} \rangle = \langle \text{START} \rangle [\backslash 256]$

Scannerless Generalized LR

Parse Table Generation

- Generation is based on SLR(1) item-sets.
- Uses character classes instead of tokens.
- Follow sets and goto actions are calculated for productions instead of non-terminals.

Scannerless Generalized LR

Lexical Disambiguation

- A reject rule is of the form:

`ID = "let" {reject}`

- When SGLR does a reduction with a reject rule, it marks the link as rejected.
- Further action on a stack is forbidden whenever all links to it are rejected.
- Follow restrictions are implemented as filters on the follow sets of productions.

Generalized LR

Context-free Disambiguation

- Priority rules: applied at parse table generation.

$E \rightarrow E * . E$
$E \rightarrow . E + E$
$E \rightarrow . E * E$
$E \rightarrow . a$

multiplication has higher
priority than addition!

Generalized LR

Context-free Disambiguation

- Priority rules: applied at parse table generation.

$E \rightarrow E * . E$
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$E \rightarrow . E * E$
$E \rightarrow . a$

multiplication is left
associative!

Generalized LR

Context-free Disambiguation

- Priority rules: applied at parse table generation.

$E \rightarrow E * . E$
$E \rightarrow . E + E$
$E \rightarrow . E * E$
$E \rightarrow . a$

multiplication is left
associative!

- Disambiguation filters: applied after parsing (prefer and avoid).

VI

Summary

Summary

lessons learned

Summary

lessons learned

How can we generate LR parse tables?

- items, closure, goto

Summary

lessons learned

How can we generate LR parse tables?

- items, closure, goto

How can we improve LR(0) parse table generation?

- SLR: consider FOLLOW sets to avoid shift-reduce conflicts
- LR(1): consider look-ahead in states
- LALR(1): unify LR(1) states to reduce state space

Summary

lessons learned

How can we generate LR parse tables?

- items, closure, goto

How can we improve LR(0) parse table generation?

- SLR: consider FOLLOW sets to avoid shift-reduce conflicts
- LR(1): consider look-ahead in states
- LALR(1): unify LR(1) states to reduce state space

How can we handle conflicts in the parse table?

- generalized parsing - supports all class of context free grammars
- scannerless generalized LR - allow for proper language composition.

Literature

[learn more](#)

Literature

[learn more](#)

LR parsing

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

Alfred V. Aho, Ravi Sethi, Jeffrey D. Ullman, Monica S. Lam: Compilers: Principles, Techniques, and Tools, 2nd edition. 2006

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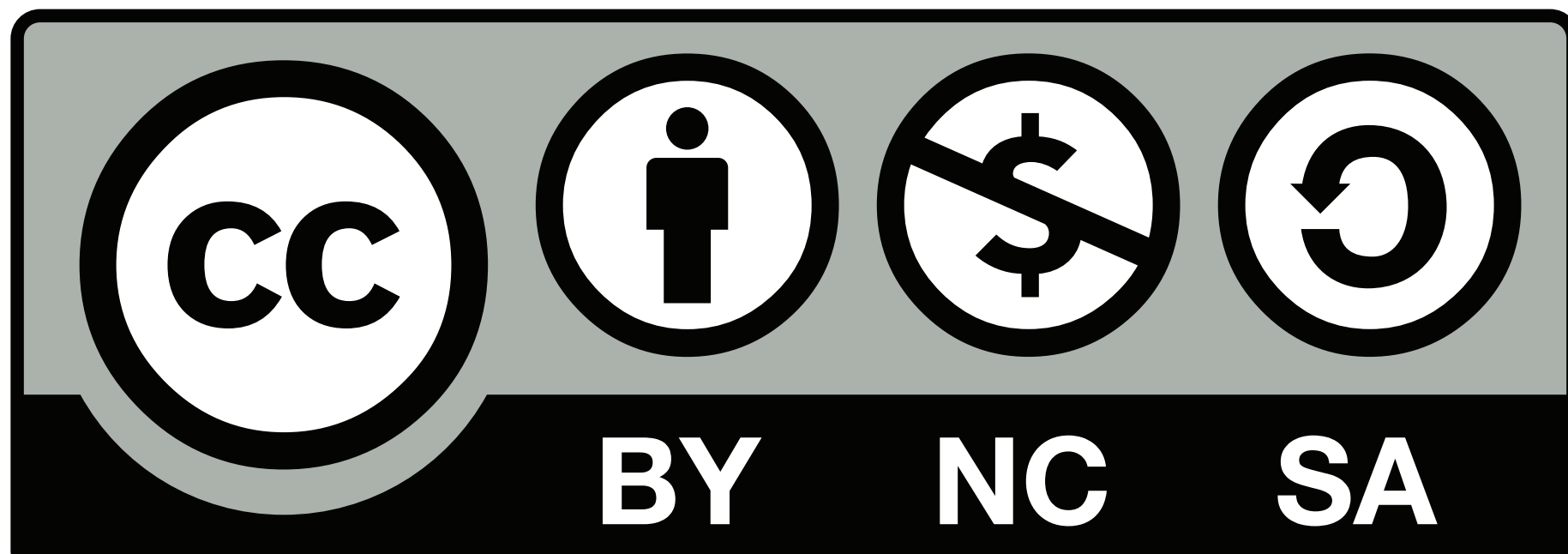
Alfred V. Aho, Ravi Sethi, Jeffrey D. Ullman, Monica S. Lam: Compilers: Principles, Techniques, and Tools, 2nd edition. 2006

Generalised LR parsing

Eelco Visser: Syntax Definition for Language Prototyping. PhD thesis 1997

M.G.J. van den Brand, J. Scheerder, J.J. Vinju, and E. Visser: Disambiguation Filters for Scannerless Generalized LR Parsers. CC 2002

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