Compiler Components & Generators

Lexical Analysis

Eduardo Souza & Eelco Visser

Overview
today's lecture

lexical analysis

regular languages

- regular grammars
- regular expressions
- finite state automata

equivalence of formalisms

- constructive approach

tool generation

I

regular grammars

Recap: A Theory of Language

formal languages

vocabulary Σ

finite, nonempty set of elements (words, letters)

alphabet

string over Σ

finite sequence of elements chosen from Σ

word, sentence, utterance

formal language λ

set of strings over a vocabulary Σ

$\lambda \subseteq \Sigma^*$

Recap: A Theory of Language
formal grammars

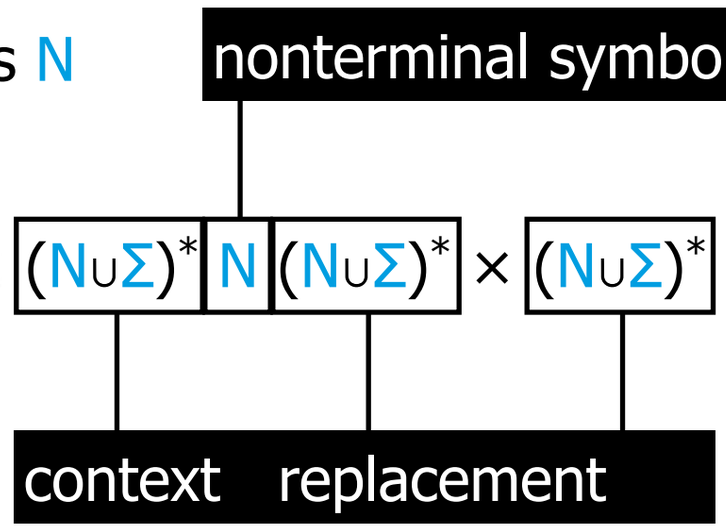
formal grammar $G = (N, \Sigma, P, S)$

nonterminal symbols N

terminal symbols Σ

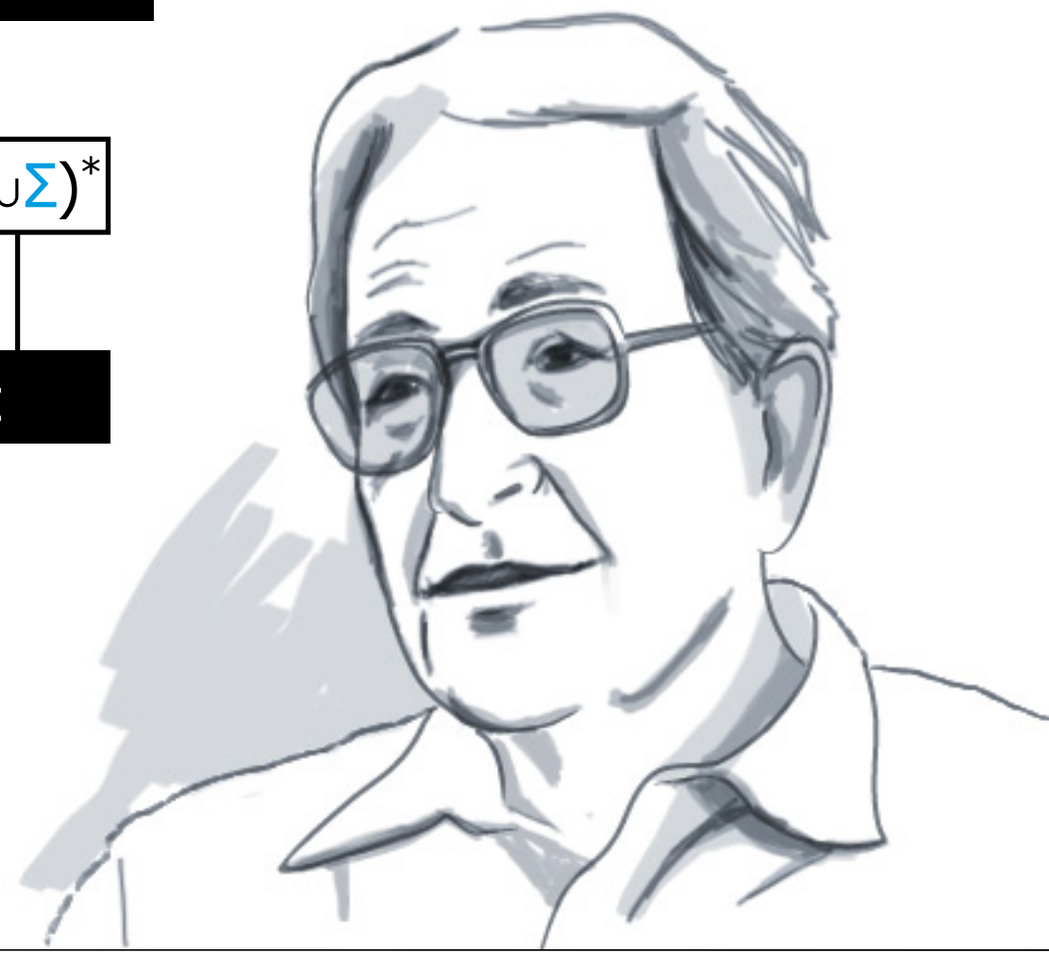
production rules $P \subseteq (N \cup \Sigma)^* N (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

start symbol $S \in N$



grammar classes

- type-0, unrestricted
- type-1, context-sensitive: (a A c, a b c)
- type-2, context-free: $P \subseteq N \times (N \cup \Sigma)^*$
- type-3, regular: (A, x) or (A, xB)



Decimal Numbers

right regular grammar

Num → “0” Num
Num → “1” Num
Num → “2” Num
Num → “3” Num
Num → “4” Num
Num → “5” Num
Num → “6” Num
Num → “7” Num
Num → “8” Num
Num → “9” Num

Num → “0”
Num → “1”
Num → “2”
Num → “3”
Num → “4”
Num → “5”
Num → “6”
Num → “7”
Num → “8”
Num → “9”

Identifiers

right regular grammar

Id → “a” R

…

Id → “z” R

R → “a” R

…

R → “z” R

R → “0” R

…

R → “9” R

Id → “a”

…

Id → “z”

R → “a”

…

R → “z”

R → “0”

…

R → “9”

Recap: A Theory of Language
formal languages

formal grammar $G = (N, \Sigma, P, S)$

derivation relation $\Rightarrow_G \subseteq (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

$w \Rightarrow_G w' \Leftrightarrow$

$\exists (p, q) \in P: \exists u,v \in (N \cup \Sigma)^*:$

$w = u\ p\ v \wedge w' = u\ q\ v$

formal language $L(G) \subseteq \Sigma^*$

$L(G) = \{w \in \Sigma^* \mid S \Rightarrow_G^* w\}$

classes of formal languages

Example

G:

S → a
S → aA
A → aB
B → aC
C → aD
D → aS

Is G regular?
L(G) = ?

II

regular expressions

Recap: Regular Expressions

overview

basics

- symbol from an alphabet
- ε

combinators

- alternation: E1 | E2
- concatenation: E1 E2
- repetition: E*
- optional: E? = E | ε
- one or more: E+ = E E*

Decimal Numbers & Identifiers

regular expressions

Num: (0|1|2|3|4|5|6|7|8|9)+

Id: (a|…|z)(a|…|z|0|…|9)*

Regular Expressions

formal languages

basics

- L(a) = {"a"}
- L(ε) = {""}

combinators

- L(E1 | E2) = L(E1) ∪ L(E2)
- L(E1 E2) = L(E1) · L(E2)
- L(E*) = L(E)*

Decimal Numbers & Identifiers

regular expressions

Num: (0|1|2|3|4|5|6|7|8|9)+

A valid Num is any word that consists of a sequence of one or more digits.

Id: (a|…|z)(a|…|z|0|…|9)*

A valid Id is any word that starts with a lowercase letter followed by sequence of zero or more lowercase letters or digits.

Example

G_r:

S → ('A'= | … | 'Z')*(0 | … | 9)*

$L(G_r)$ = ?

Example

G_r:

S → ('A'= | … | 'Z')*(0 | … | 9)*

$L(G_r)$ = The set of words that start with zero or more capital letters, followed by zero or more decimal digits.

III

finite automata

Finite Automata

formal definition

finite automaton $M = (Q, \Sigma, T, q_0, F)$

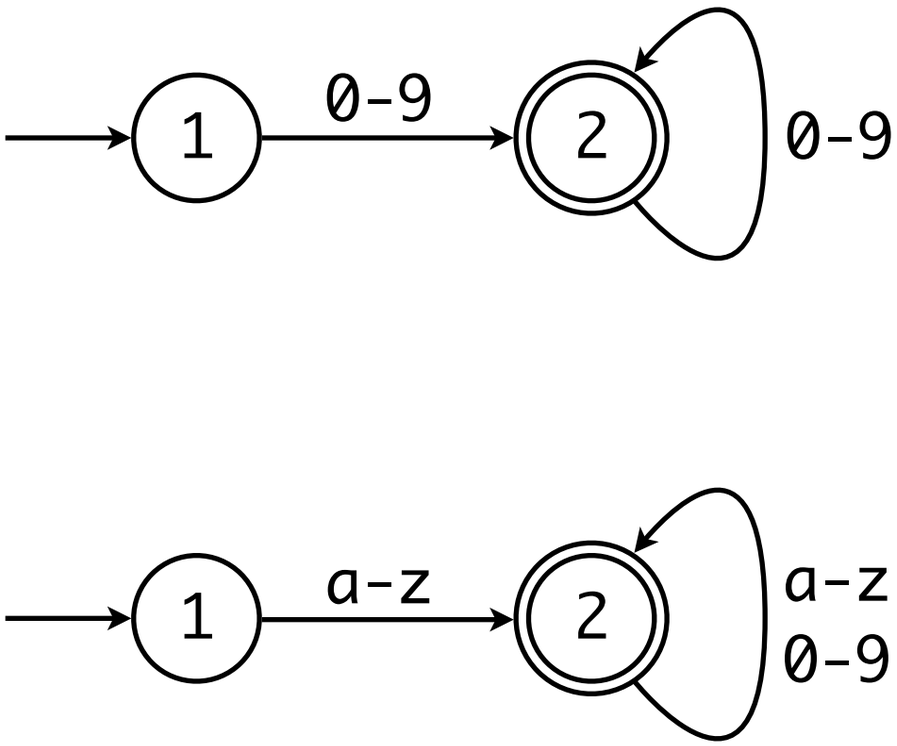
- states Q
- input symbols Σ
- transition function T
- start state $q_0 \in Q$
- final states $F \subseteq Q$

transition function

- nondeterministic FA $T : Q \times \Sigma \rightarrow P(Q)$
- NFA with ε-moves $T : Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow P(Q)$
- deterministic FA $T : Q \times \Sigma \rightarrow Q$

Decimal Numbers & Identifiers

finite automata



Nondeterministic Finite Automata

formal languages

finite automaton $M = (Q, \Sigma, T, q_0, F)$

transition function $T : Q \times \Sigma \rightarrow P(Q)$

$$T(\{q_1,\ldots, q_n\}, x) := T(q_1, x) \cup \ldots \cup T(q_n, x)$$

$$T^*(\{q_1,\ldots, q_n\}, \varepsilon) := \{q_1,\ldots, q_n\}$$

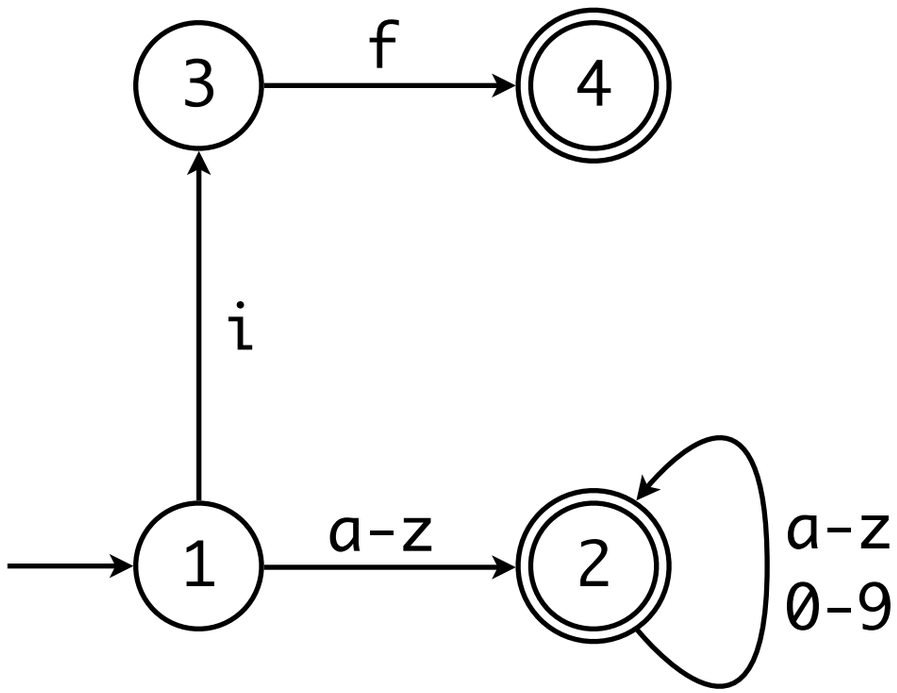
$$T^*(\{q_1,\ldots, q_n\}, xw) := T^*(T(\{q_1,\ldots, q_n\}, x), w)$$

formal language $L(M) \subseteq \Sigma^*$

$$L(M) = \{w \in \Sigma^* \mid T^*(\{q_0\}, w) \cap F \neq \varnothing\}$$

Nondeterministic Finite Automata

formal languages



Deterministic Finite Automata

formal languages

finite automaton $M = (Q, \Sigma, T, q_0, F)$

transition function $T : Q \times \Sigma \rightarrow Q$

$T^*(q, \varepsilon) := q$

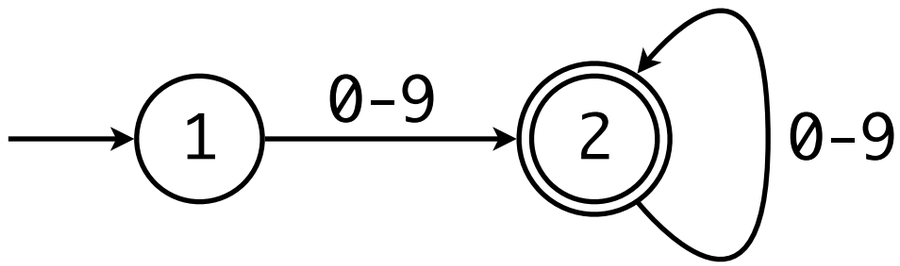
$T^*(q, xw) := T^*(T(q, x), w)$

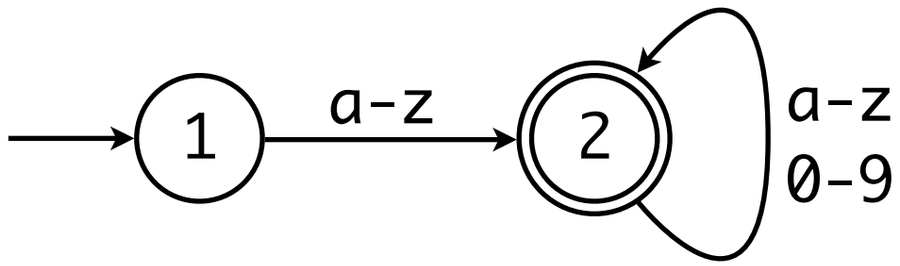
formal language $L(M) \subseteq \Sigma^*$

$L(M) = \{w \in \Sigma^* \mid T^*(q_0, w) \in F\}$

Deterministic Finite Automata

formal languages



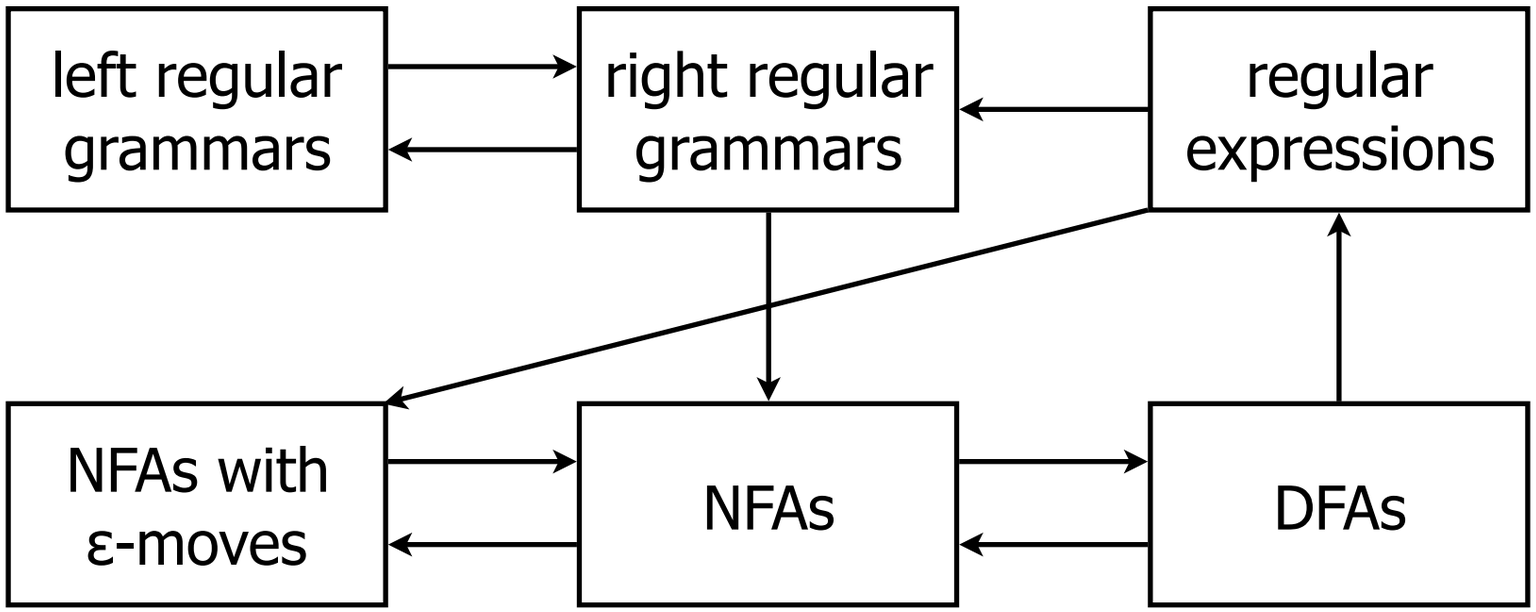


coffee break

IV

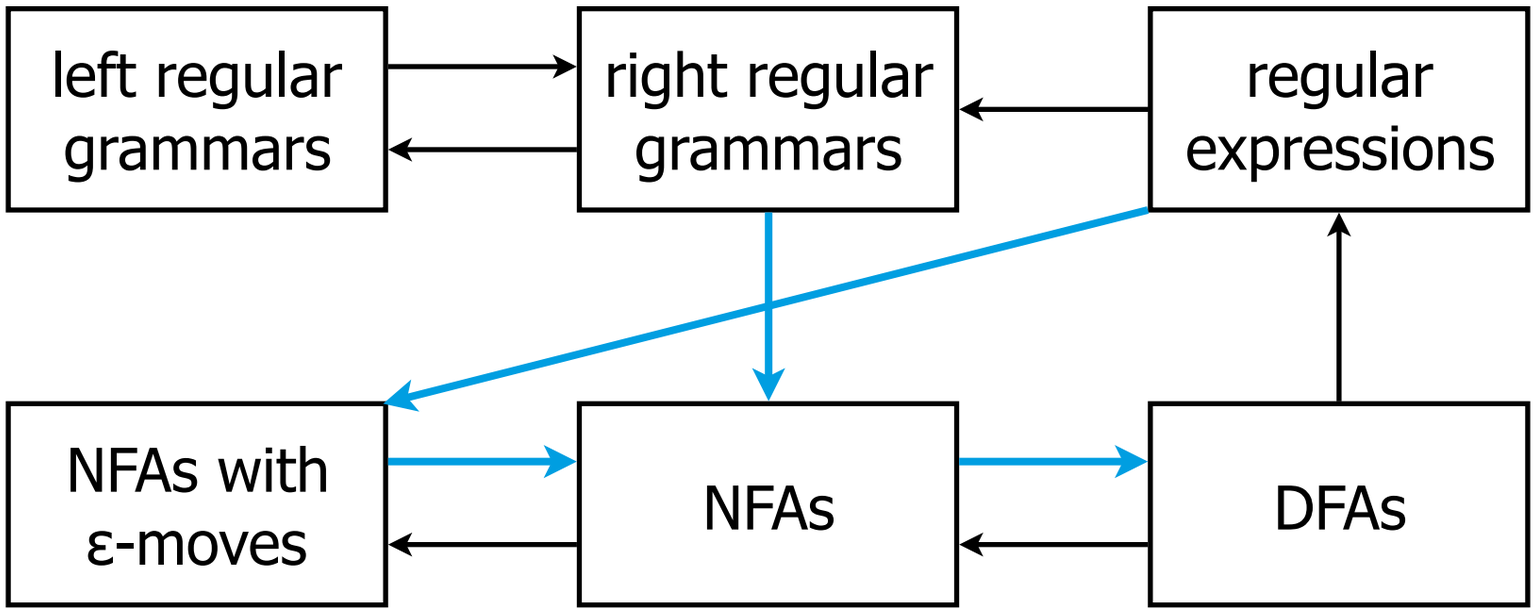
equivalence

Regular Languages
formalisms

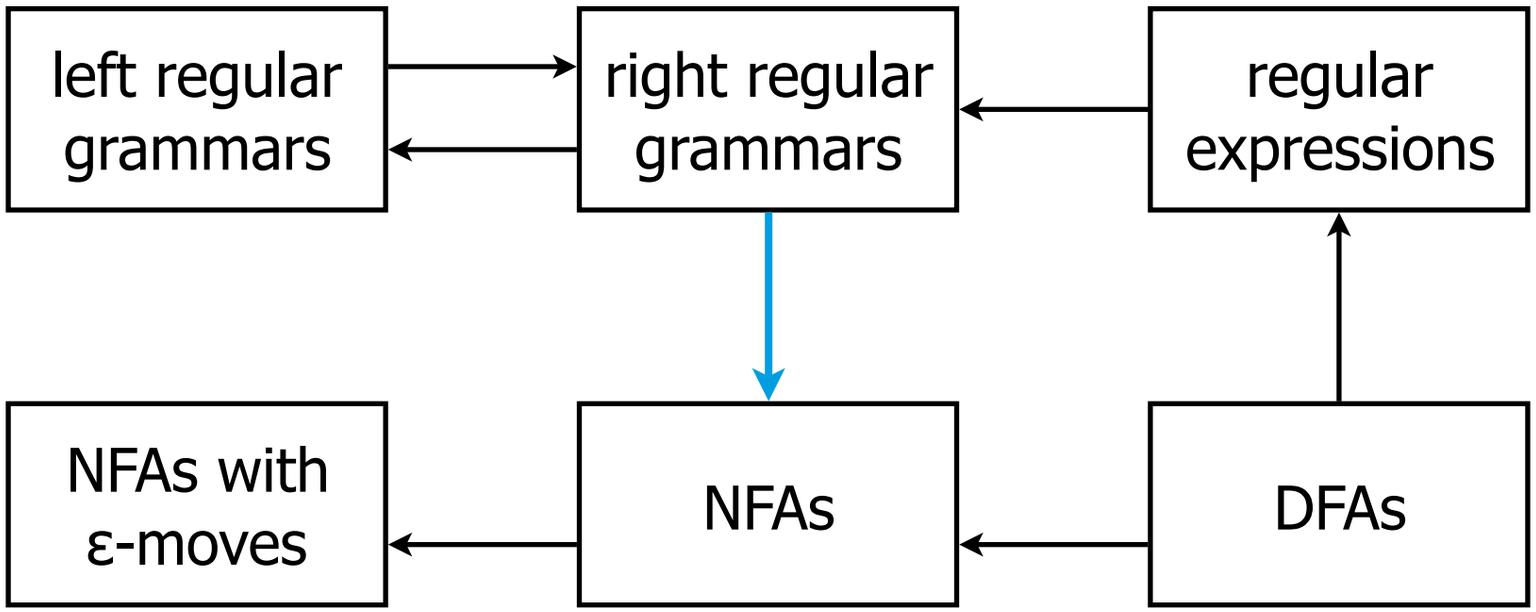


Regular Languages

formalisms



Regular Languages
formalisms



NFA construction

right regular grammar

formal grammar $G = (N, \Sigma, P, S)$

finite automaton $M = (N \cup \{f\}, \Sigma, T, S, F)$

transition function T

$(X, aY) \in P : (X, a, Y) \in T$

$(X, a) \in P : (X, a, f) \in T$

final states F

$(S, \varepsilon) \in P : F = \{S, f\}$

else: $F = \{f\}$

NFA construction

Example

Num → “0” Num
Num → “1” Num
Num → “2” Num
Num → “3” Num
Num → “4” Num
Num → “5” Num
Num → “6” Num
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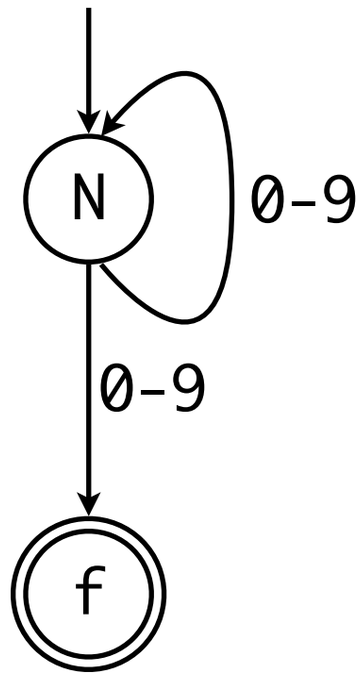


NFA construction

Example

formal grammar $G = (N, \Sigma, P, S)$

finite automaton $M = (N \cup \{f\}, \Sigma, T, S, F)$

transition function T

$(X, aY) \in P : (X, a, Y) \in T$

$(X, a) \in P : (X, a, f) \in T$

final states F

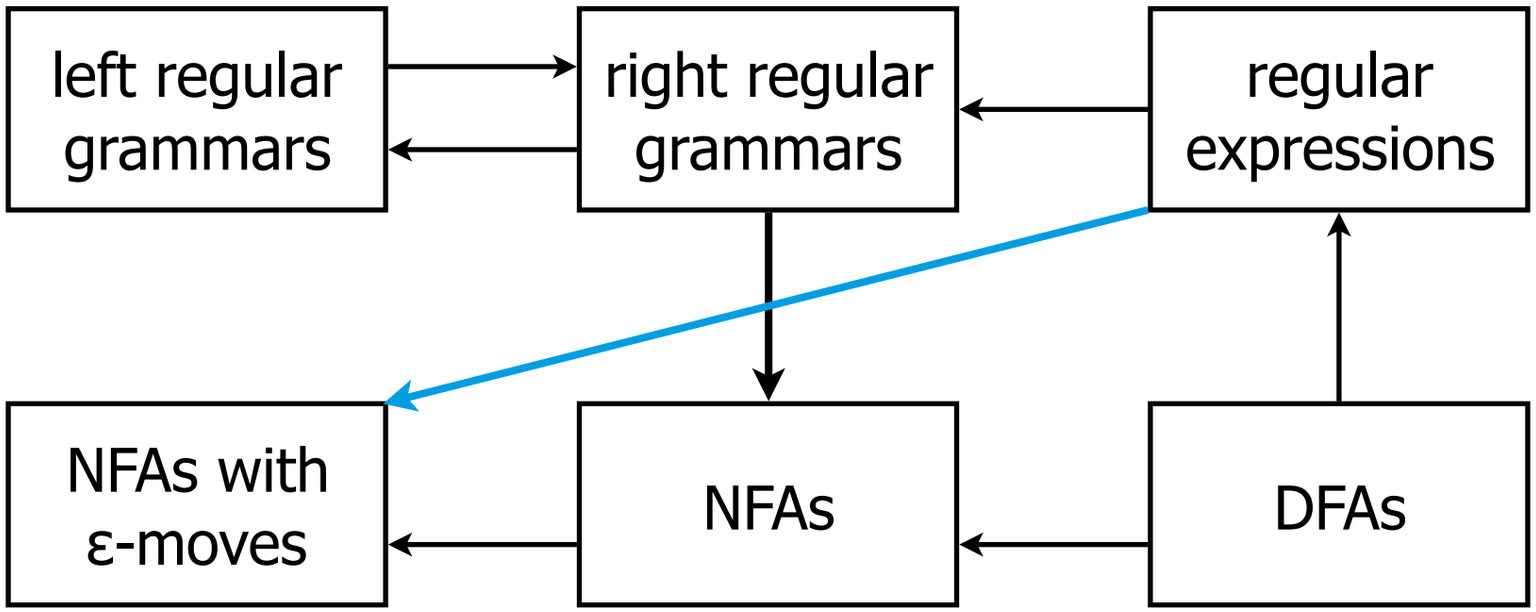
$(S, \varepsilon) \in P : F = \{S, f\}$

else: $F = \{f\}$

G:

$S \rightarrow a$
$S \rightarrow aA$
$A \rightarrow aB$
$B \rightarrow aC$
$C \rightarrow aD$
$D \rightarrow aS$

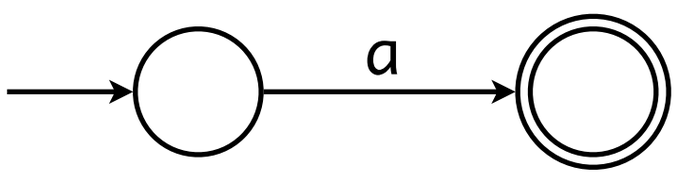
Regular Languages
formalisms



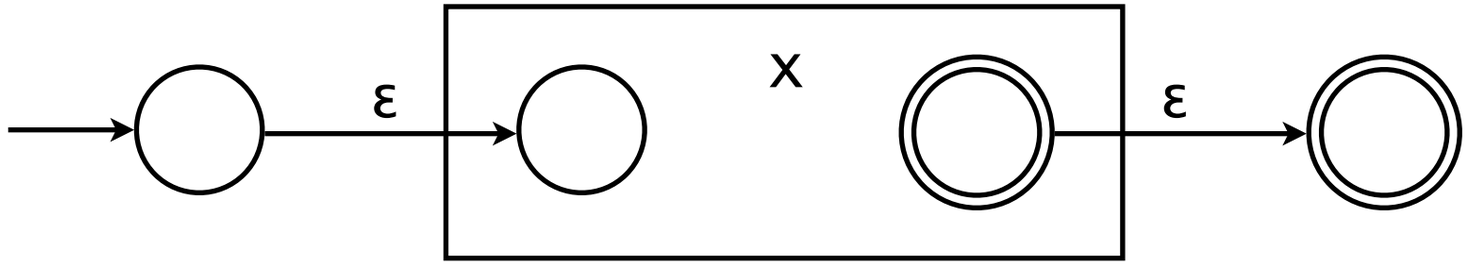
NFA construction

regular expressions

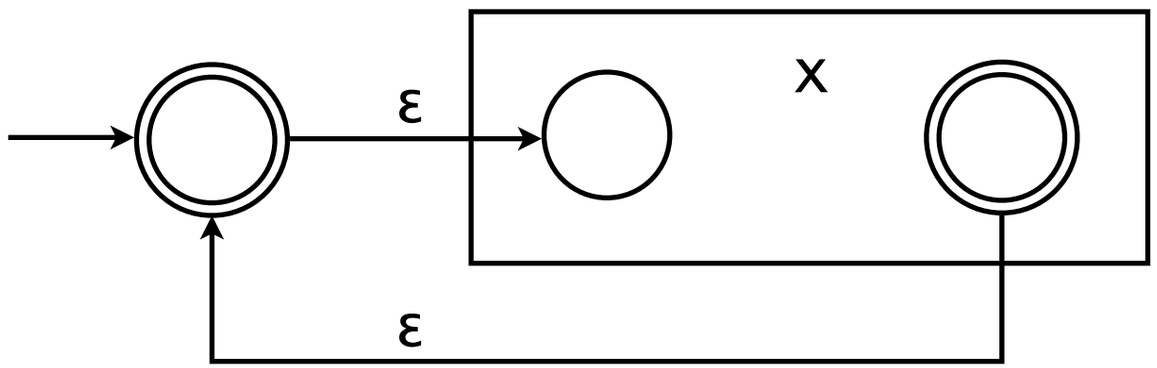
a



(x)

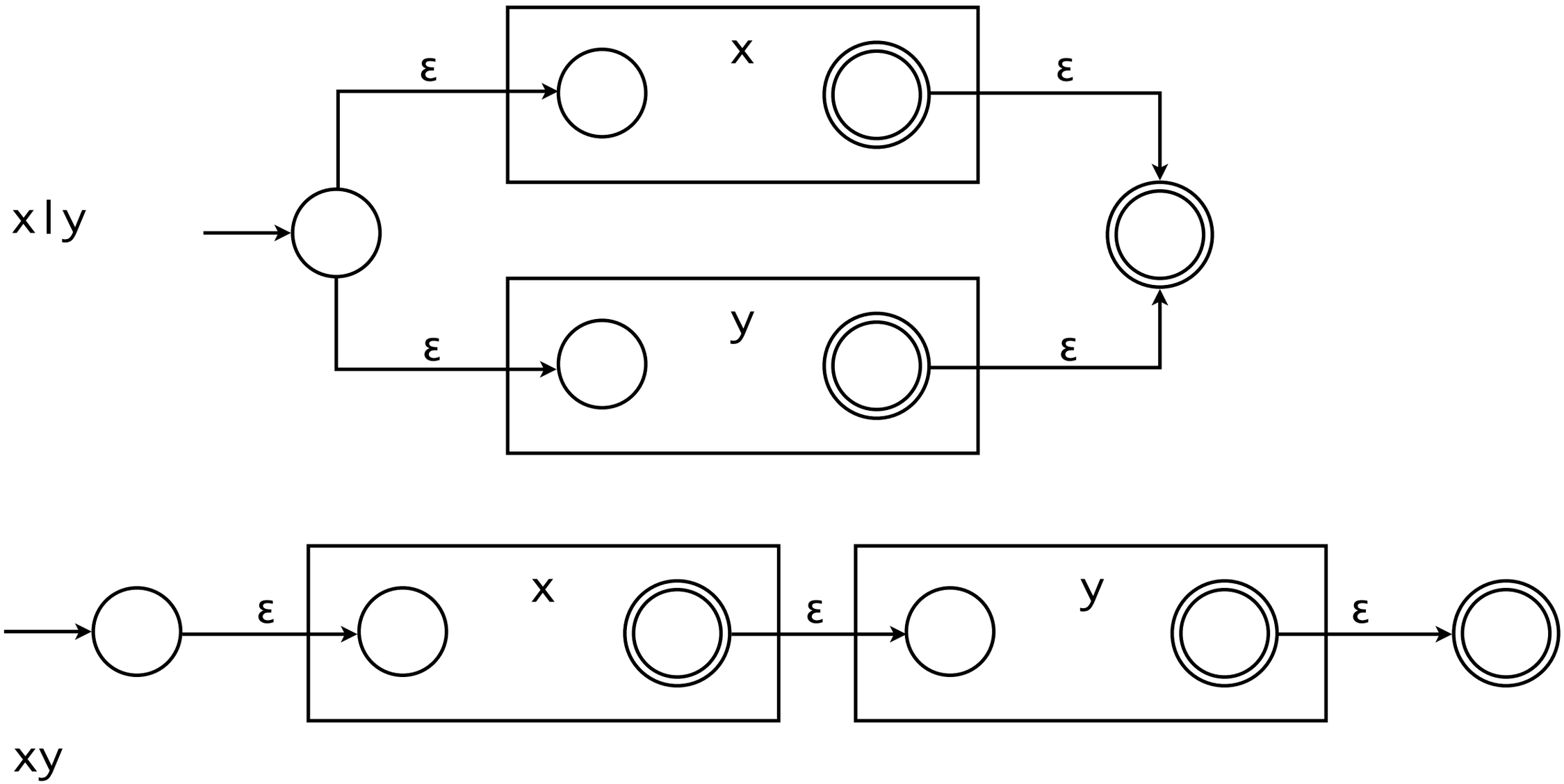


x*



NFA construction

regular expressions



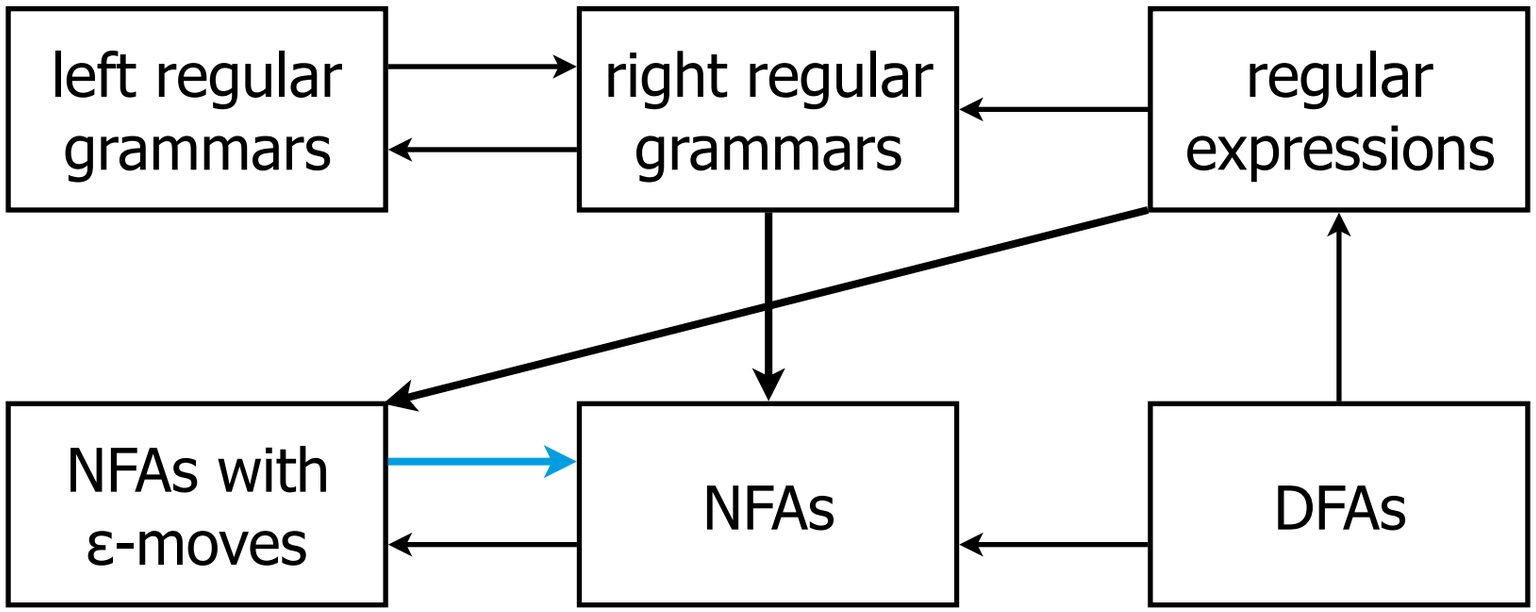
NFA construction

regular expressions

G_r:

S → ('A'= | … | 'Z')*(0 | … | 9)*

Regular Languages
formalisms



NFA construction

ε elimination

additional final states

- states with ε-moves into final states
- become final states themselves

additional transitions

- ε-move from source to target state
- transitions from target state
- add these transitions to the source state

NFA construction

ε elimination

G_r:

$$S \rightarrow (\text{'A'}= \mid \ldots \mid \text{'Z'})^*(0 \mid \ldots \mid 9)^*$$

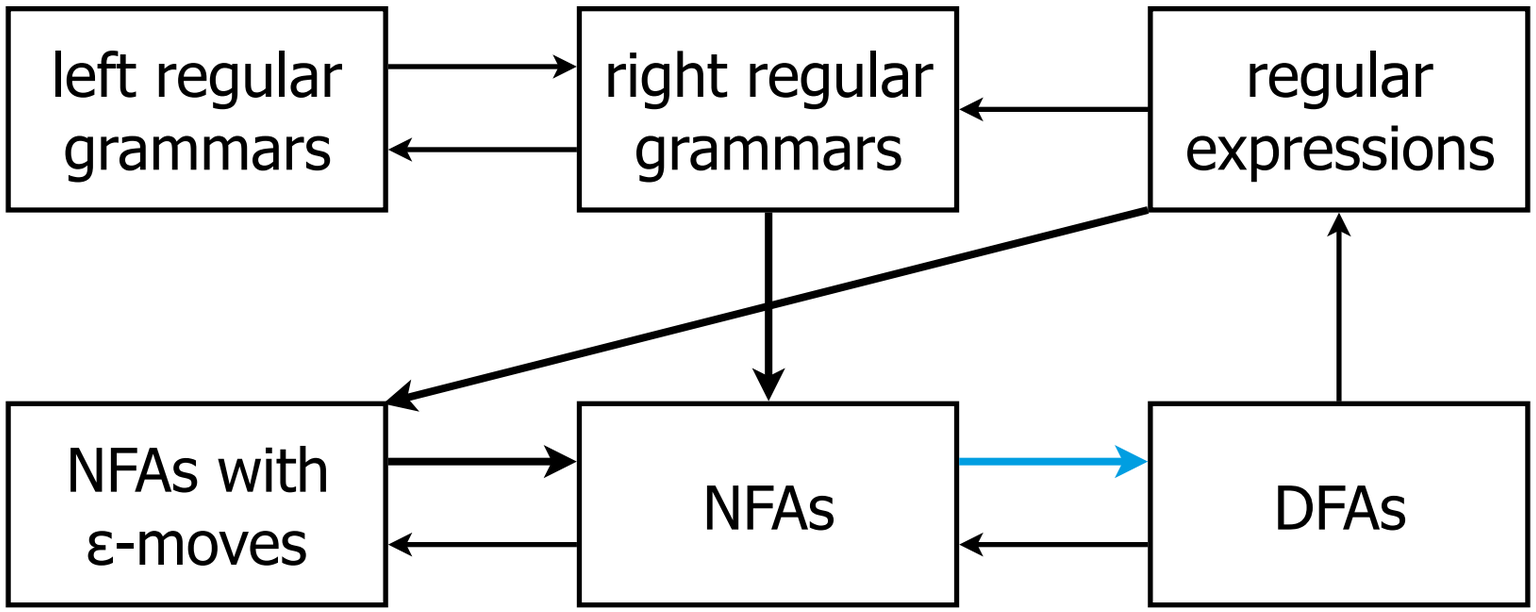
additional final states

- states with ε-moves into final states
- become final states themselves

additional transitions

- ε-move from source to target state
- transitions from target state
- add these transitions to the source state

Regular Languages
formalisms



Powerset construction

eliminating nondeterminism

nondeterministic finite automaton $M = (Q, \Sigma, T, q_0, F)$

deterministic finite automaton $M' = (P(Q), \Sigma, T', \{q_0\}, F')$

transition function T'

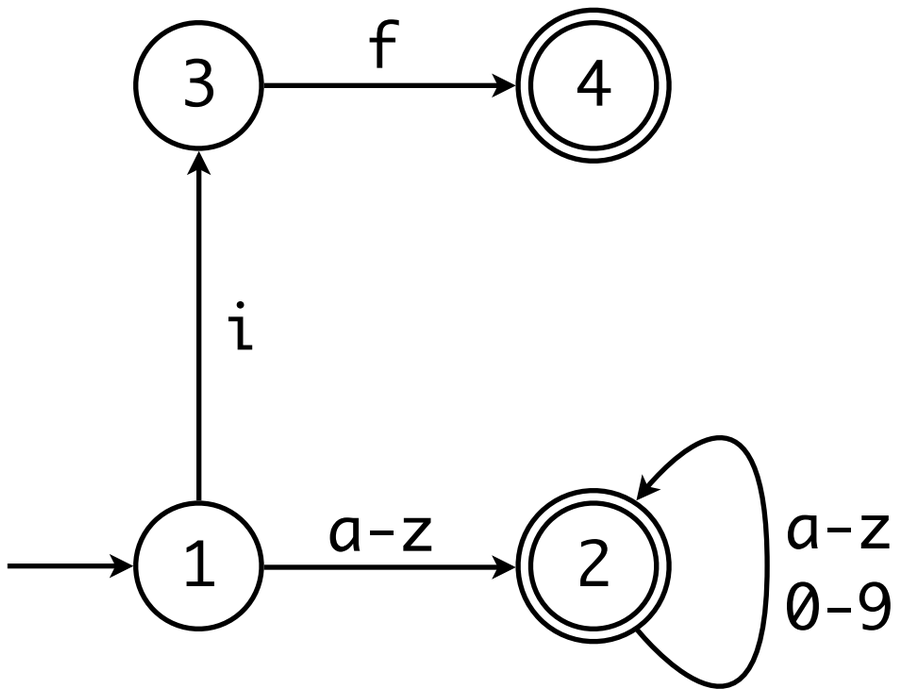
- $T'(\{q_1,..., q_n\}, x) = T(\{q_1,..., q_n\}, x) = T(q_1, x) \cup ... \cup T(q_n, x)$

final states $F' = \{S \mid S \subseteq Q, S \cap F \neq \varnothing\}$

- all states that include a final state of the original NFA

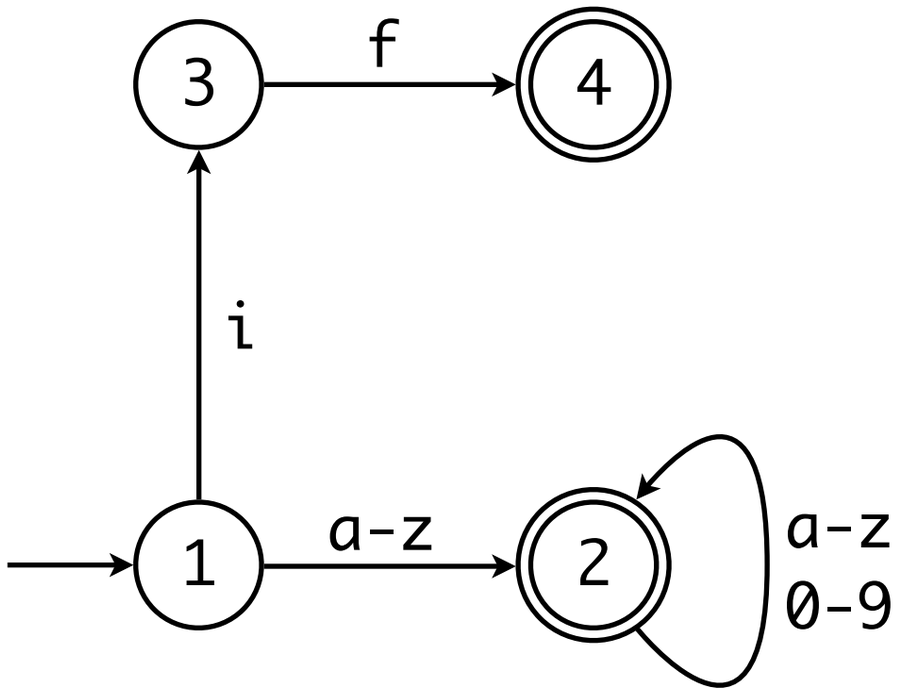
Powerset construction

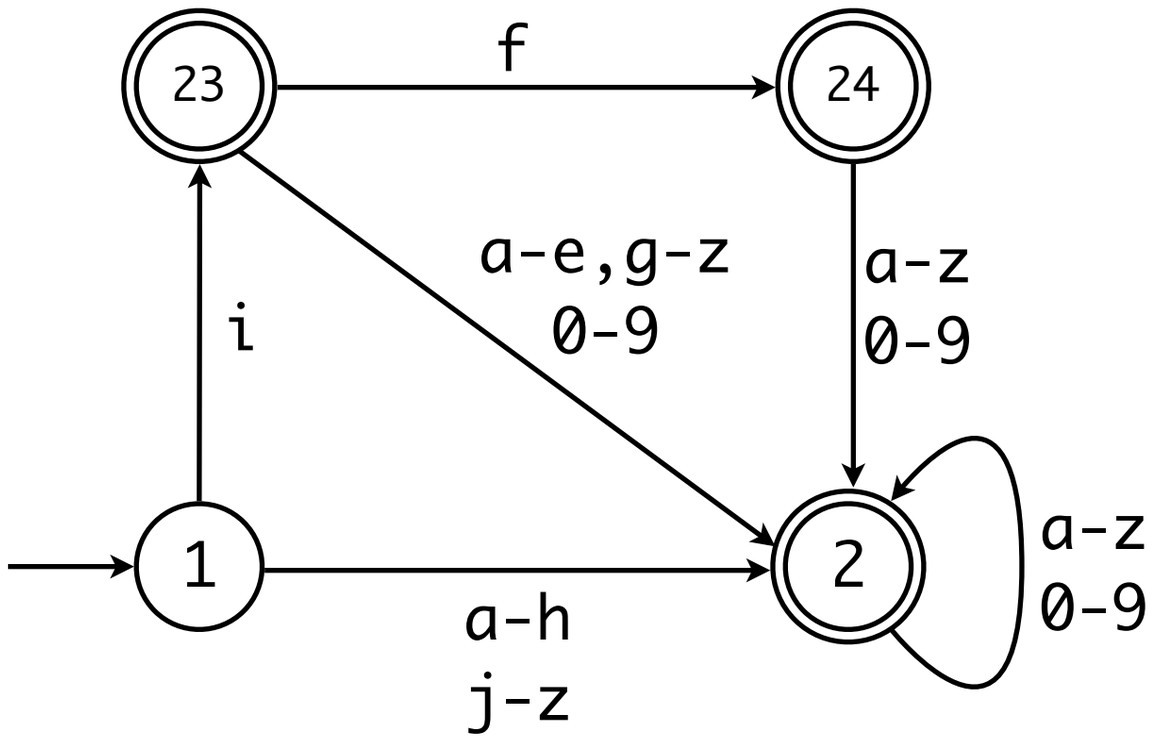
example



Powerset construction

example





V

summary

Summary
lessons learned

What are the formalisms to describe regular languages?

- regular grammars
- regular expressions
- finite state automata

Why are these formalisms equivalent?

- constructive proofs

How can we generate compiler tools from that?

- implement DFAs
- generate transition tables

Literature

learn more

formal languages

Noam Chomsky: Three models for the description of language. 1956

J. E. Hopcroft, R. Motwani, J. D. Ullman: Introduction to Automata Theory, Languages, and Computation. 2006

lexical analysis

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

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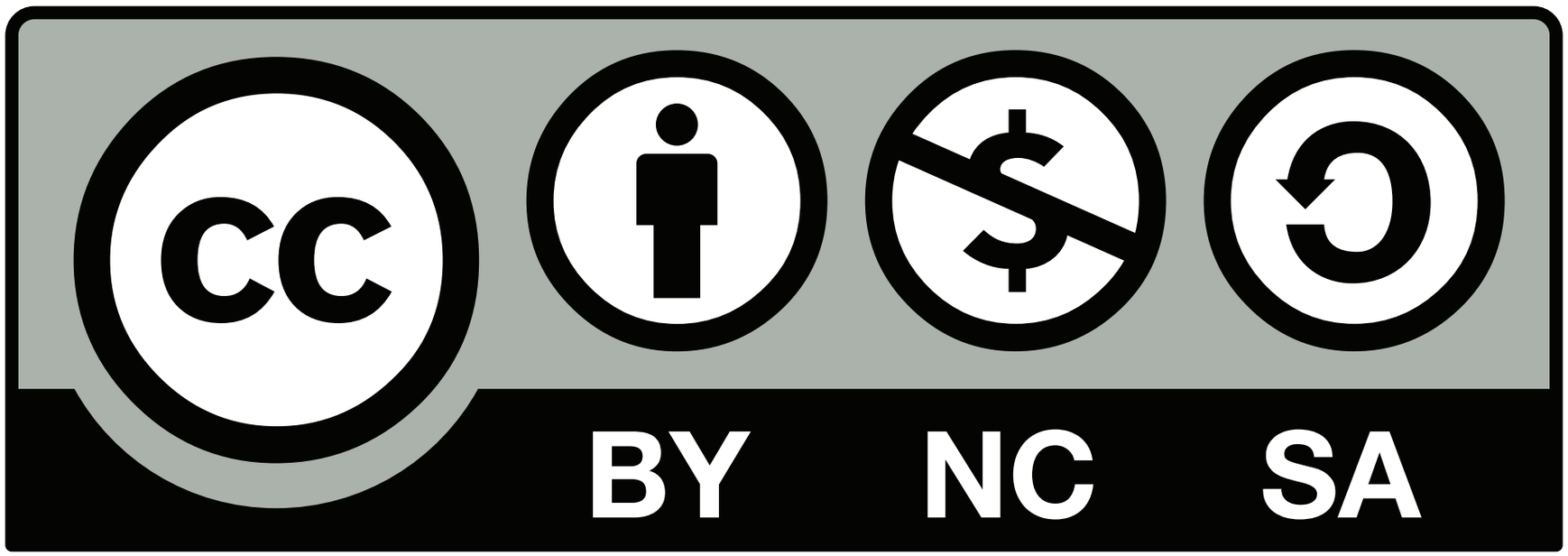
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