

Register Allocation

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Allocate Minimal Number of Registers

exercise

```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Overview today's lecture

Interference graphs

- construction during liveness analysis

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Graph Coloring

- assign registers to local variables and compiler temporaries
- store local variables and temporaries in memory

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Coalescing

- handle move instructions

Overview today's lecture

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- assign registers to local variables and compiler temporaries
- store local variables and temporaries in memory

Coalescing

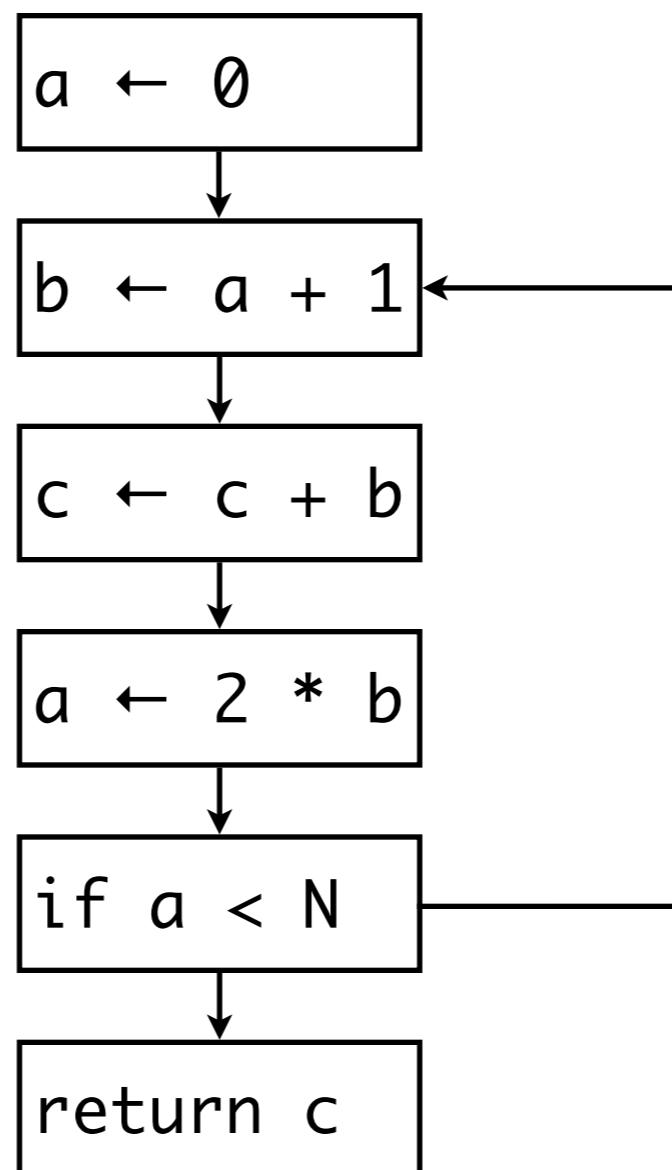
- handle move instructions

Pre-colored nodes

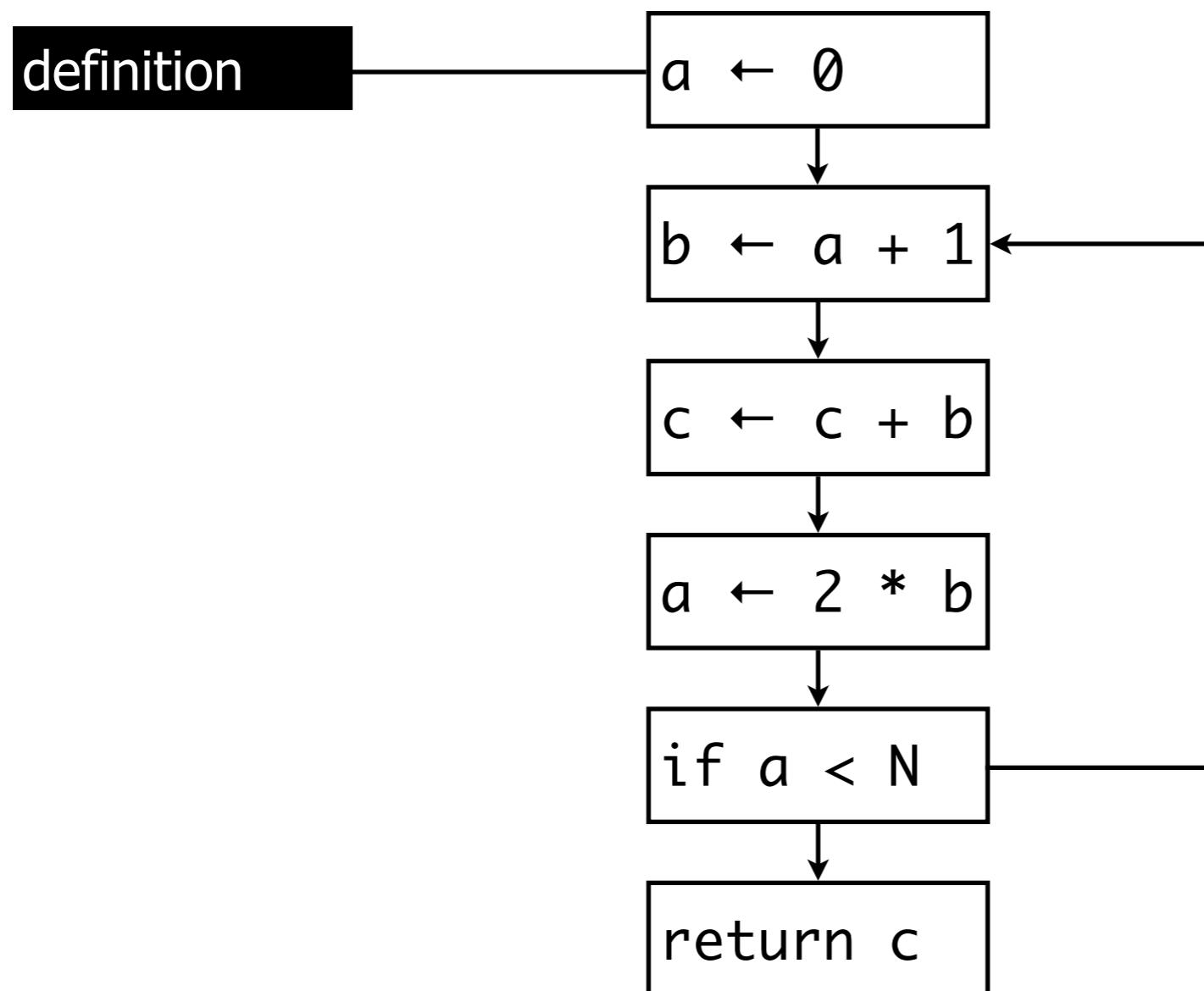
I

Interference Graphs

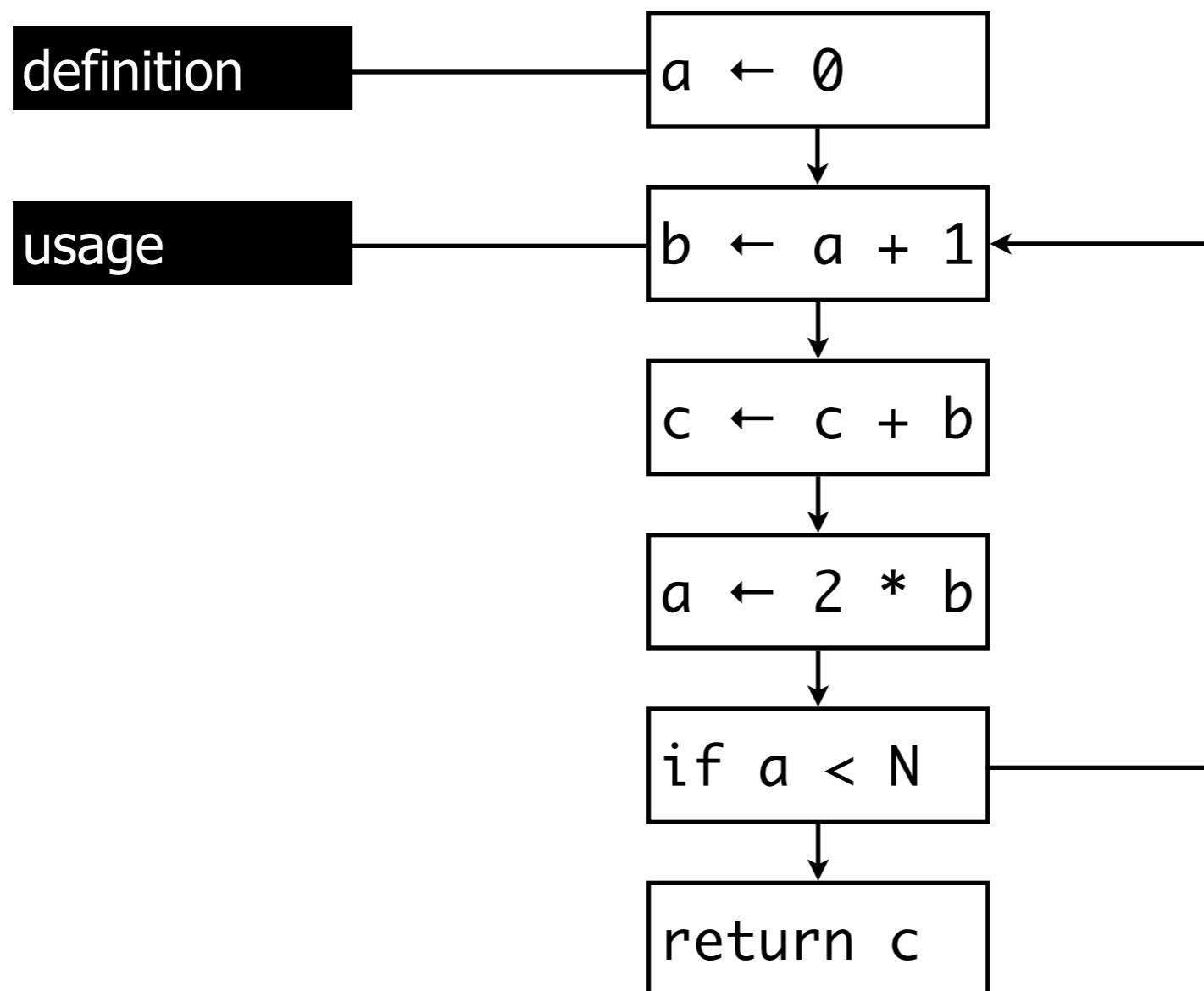
Recap: Liveness Analysis terminology



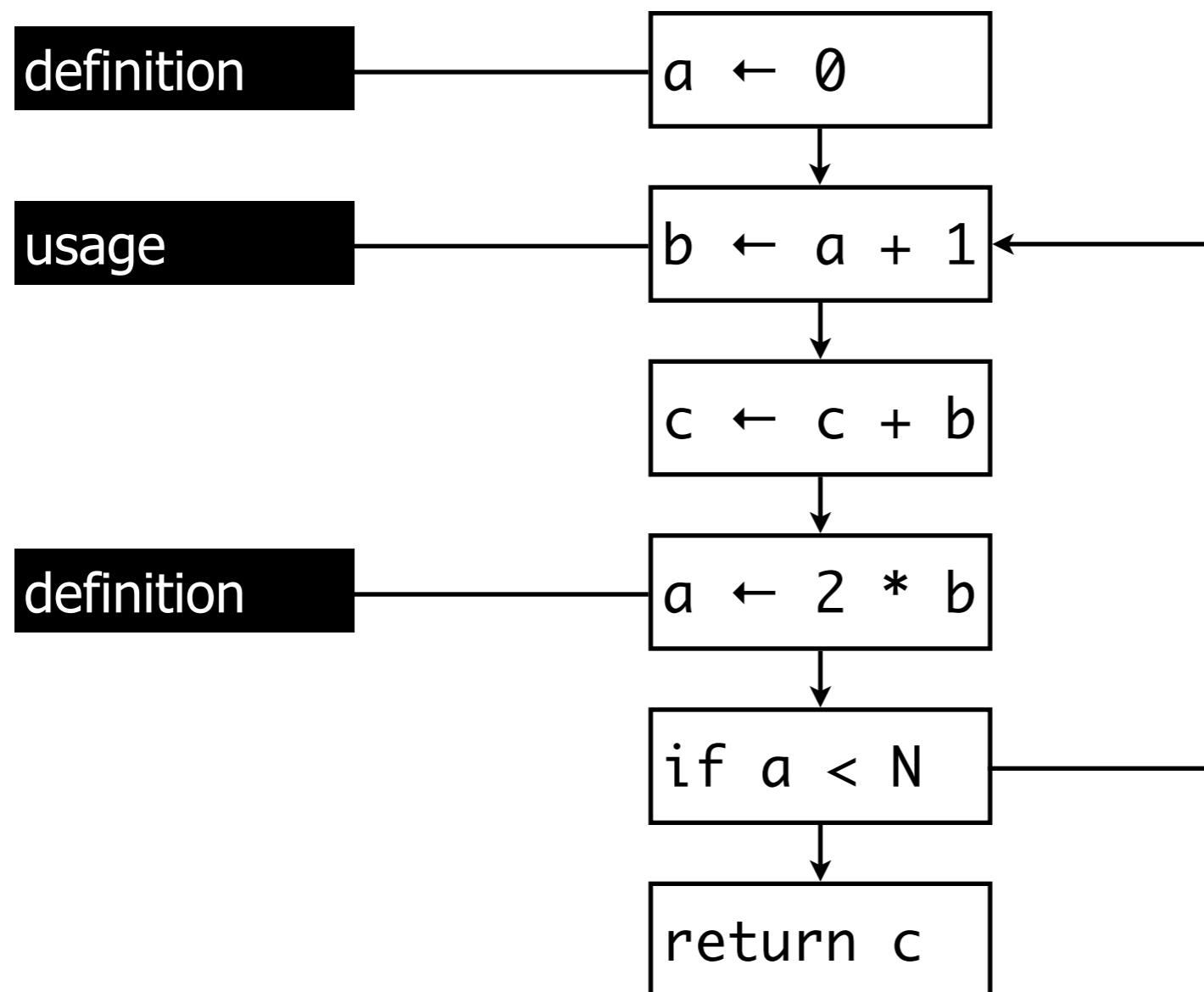
Recap: Liveness Analysis terminology



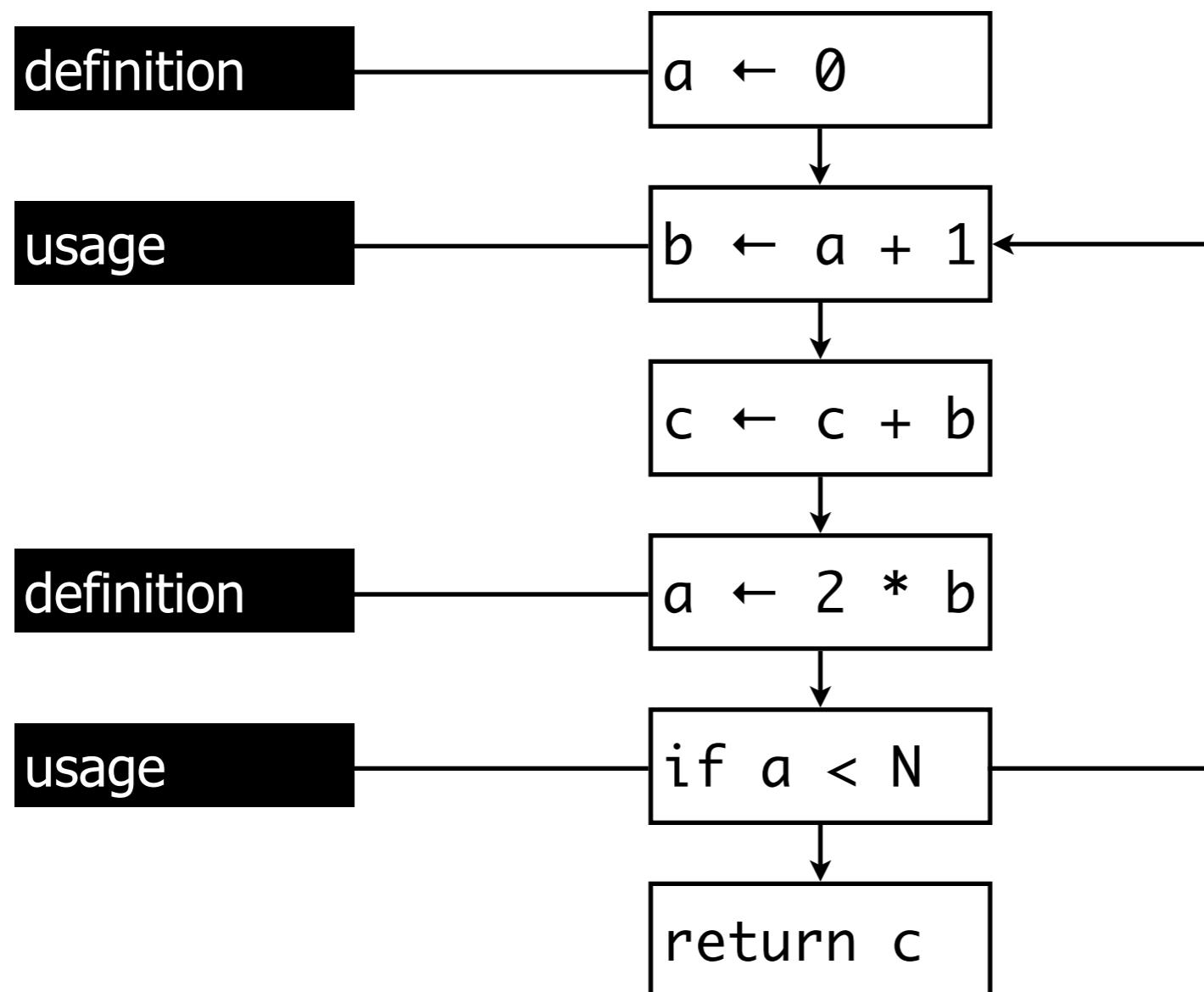
Recap: Liveness Analysis terminology



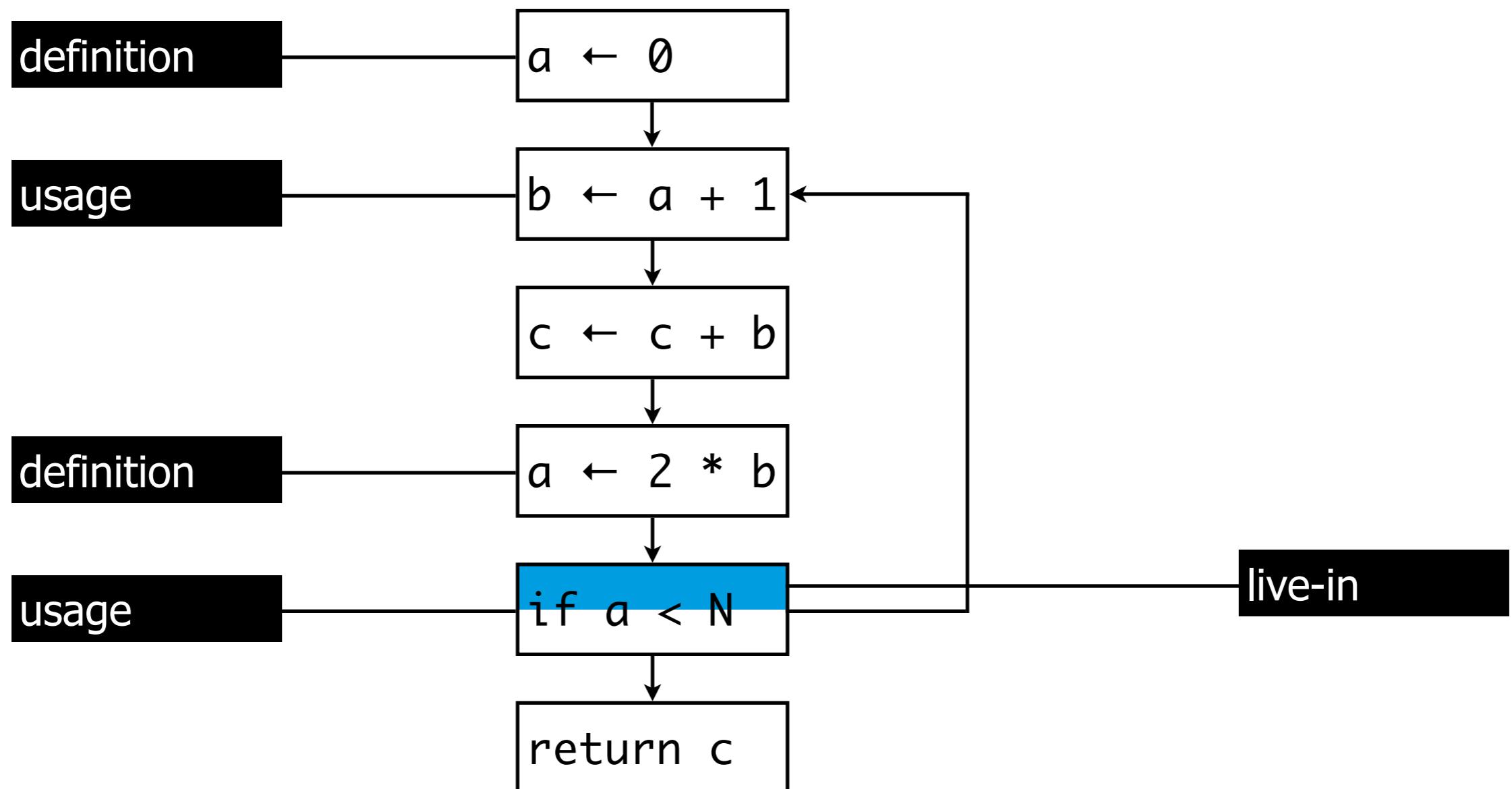
Recap: Liveness Analysis terminology



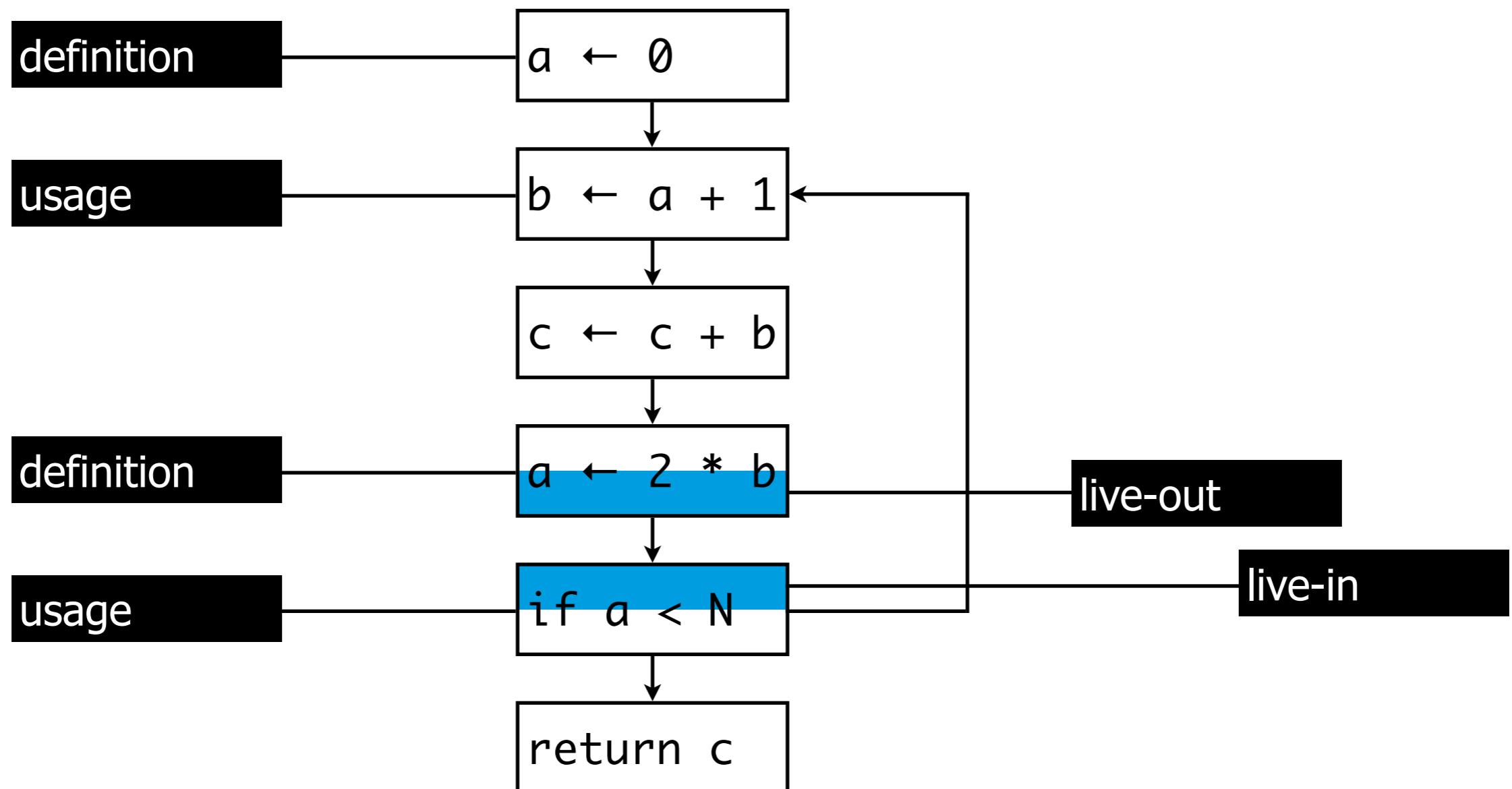
Recap: Liveness Analysis terminology



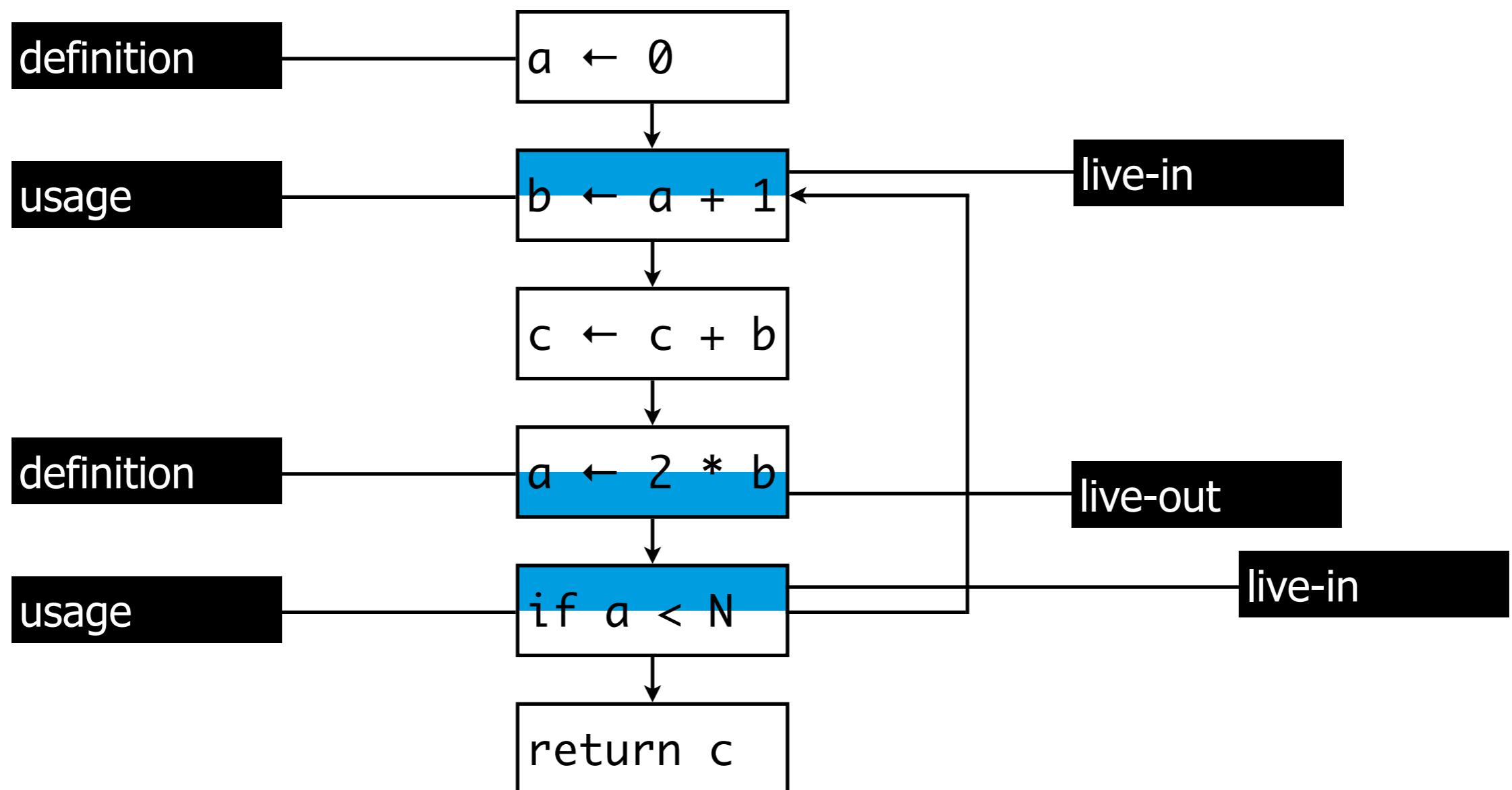
Recap: Liveness Analysis terminology



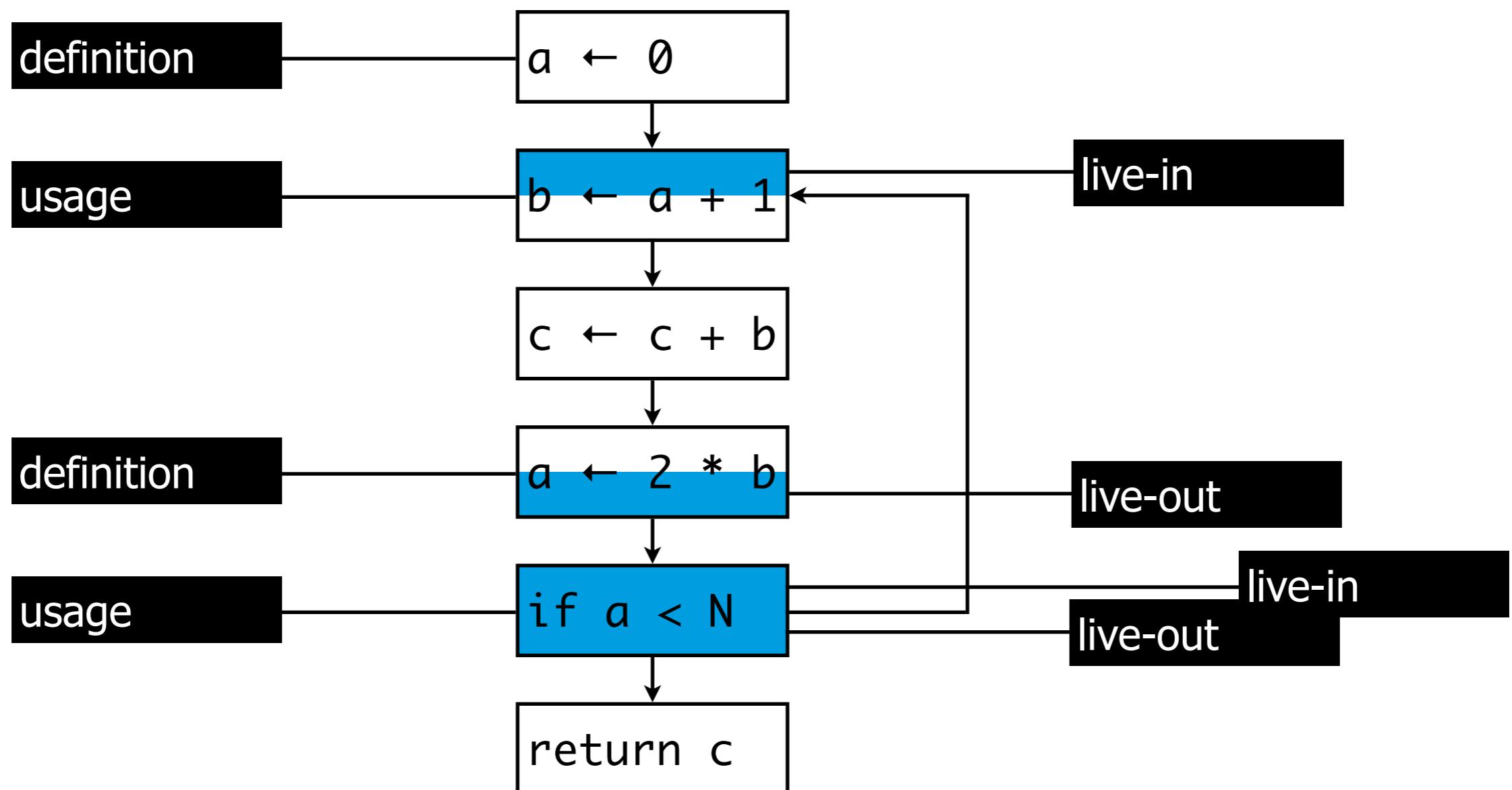
Recap: Liveness Analysis terminology



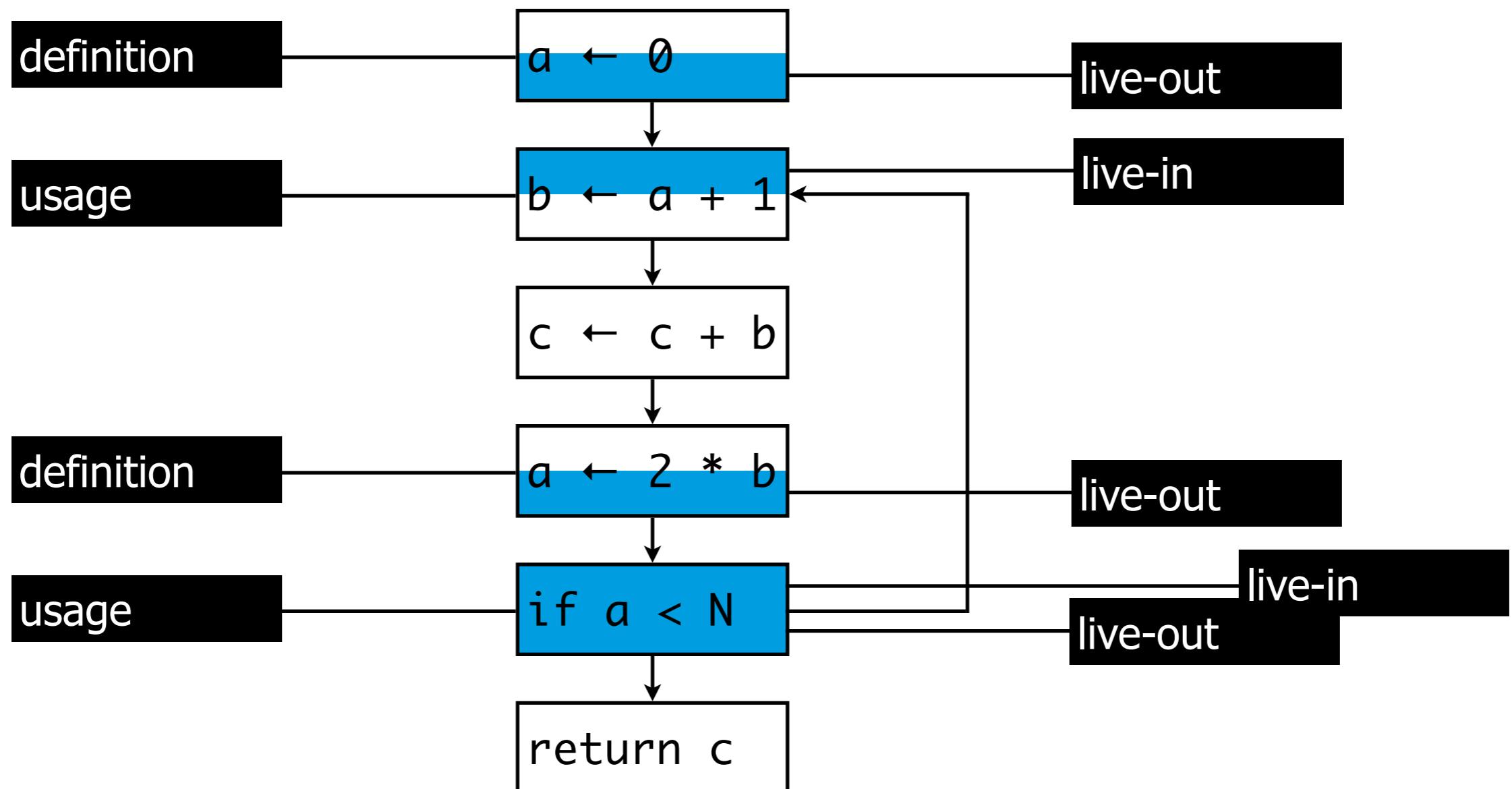
Recap: Liveness Analysis terminology



Recap: Liveness Analysis terminology

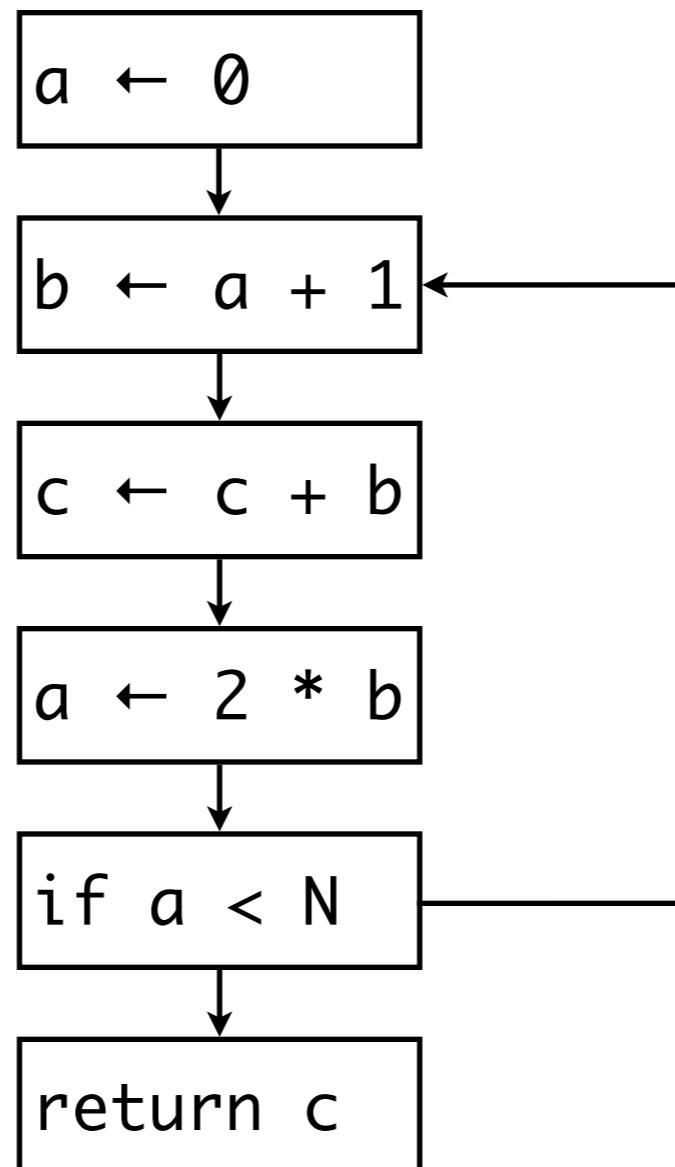


Recap: Liveness Analysis terminology



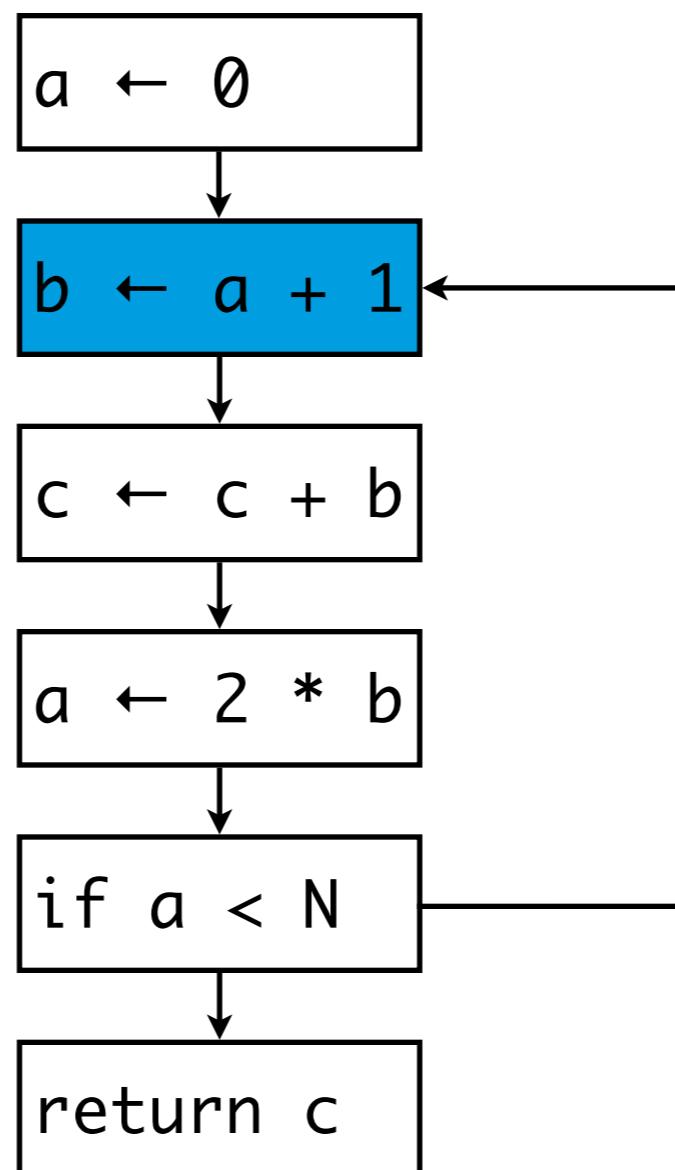
Recap: Liveness Analysis

example



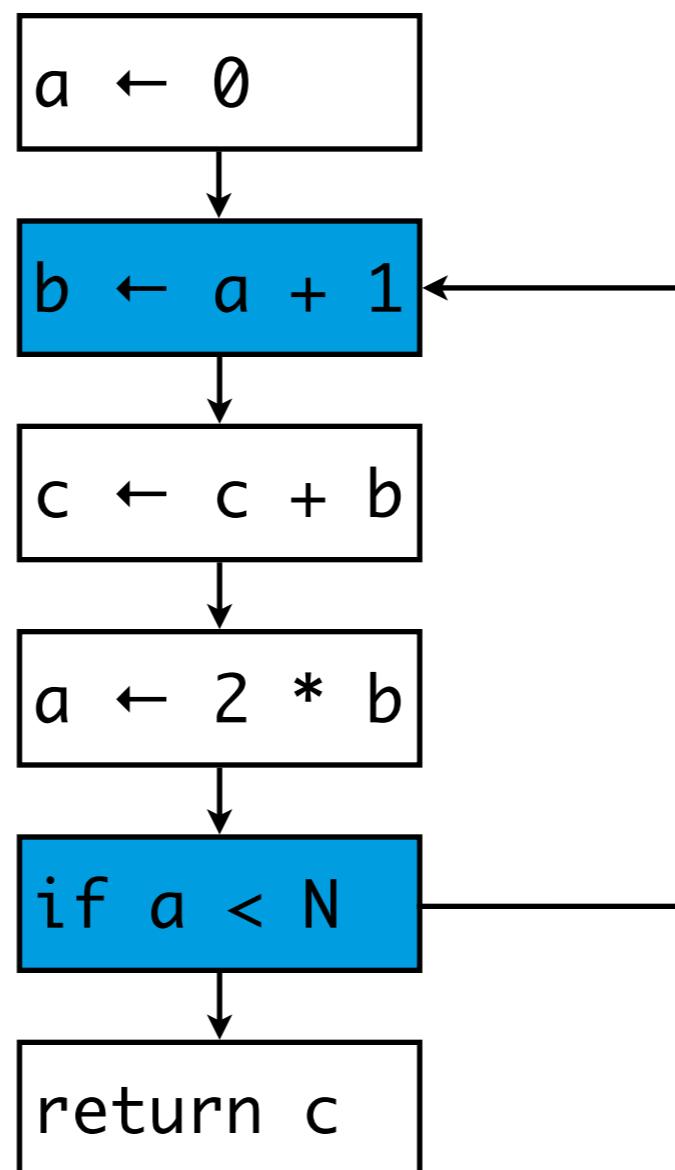
Recap: Liveness Analysis

example

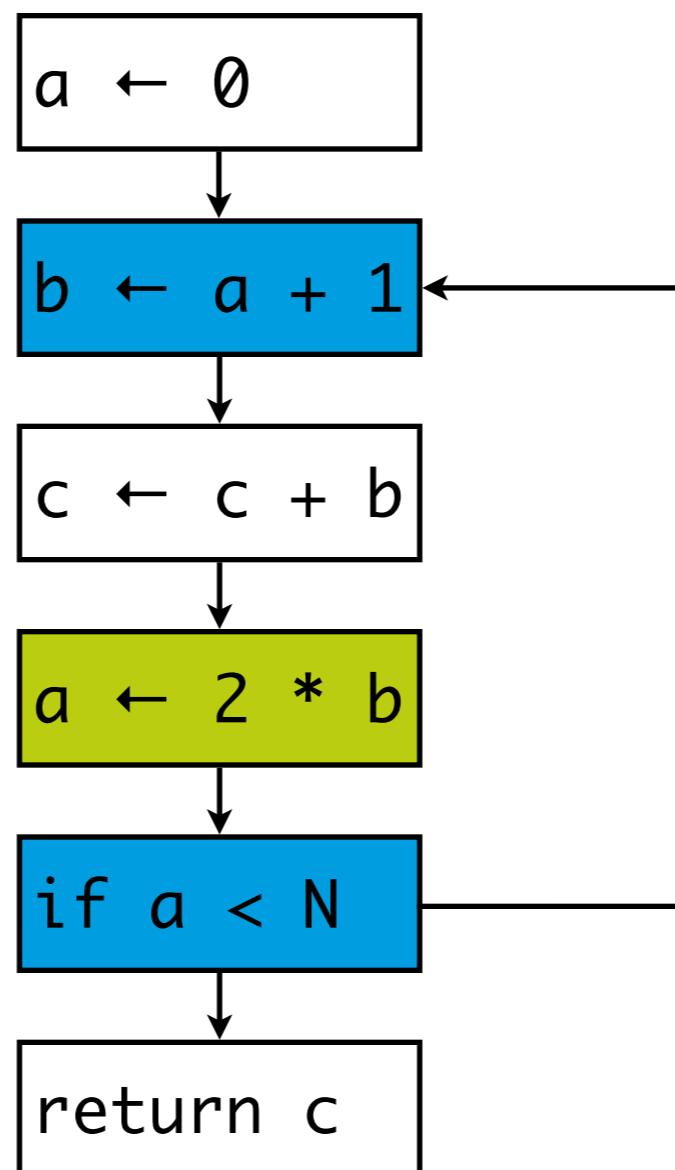


Recap: Liveness Analysis

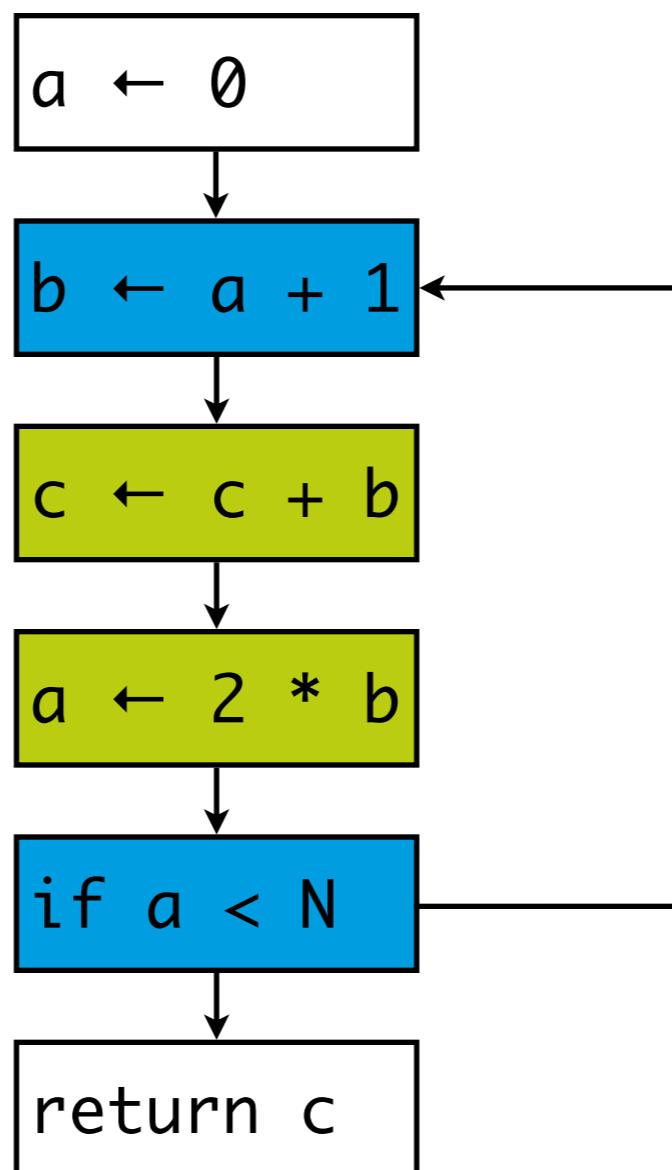
example



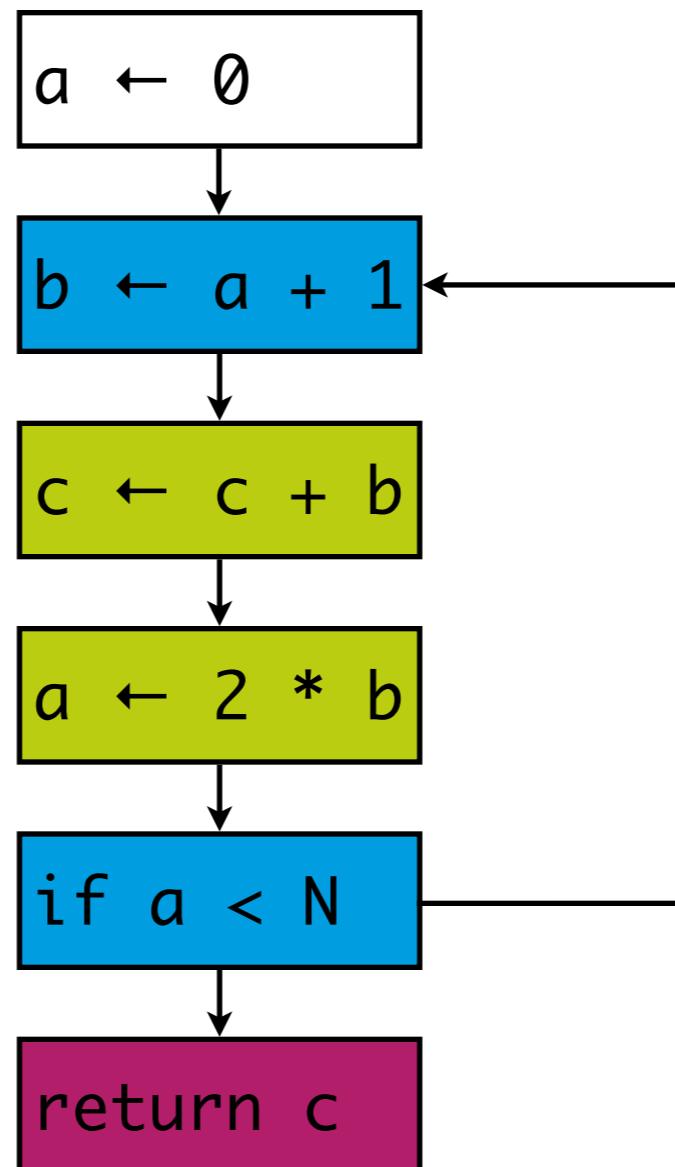
Recap: Liveness Analysis example



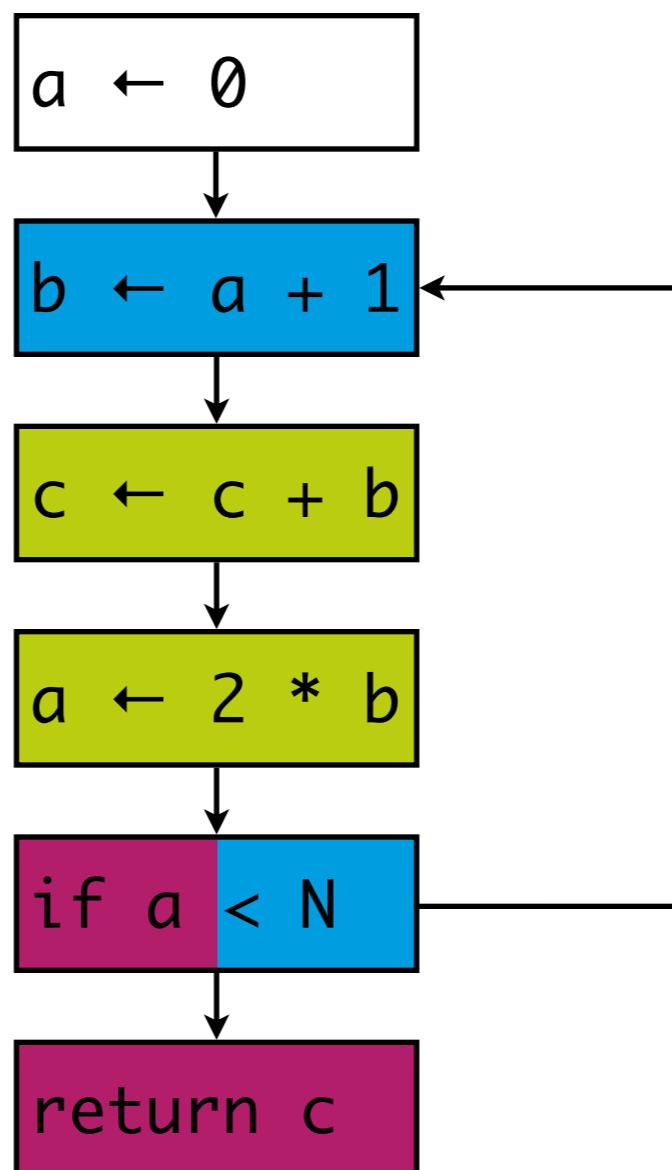
Recap: Liveness Analysis example



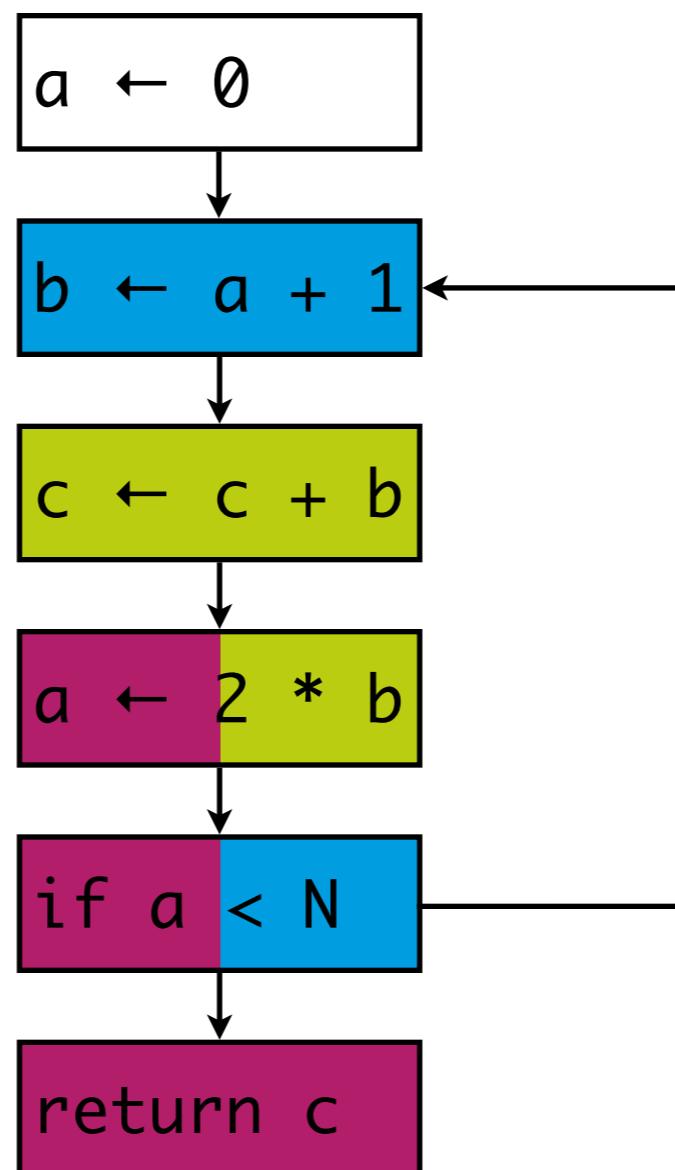
Recap: Liveness Analysis example



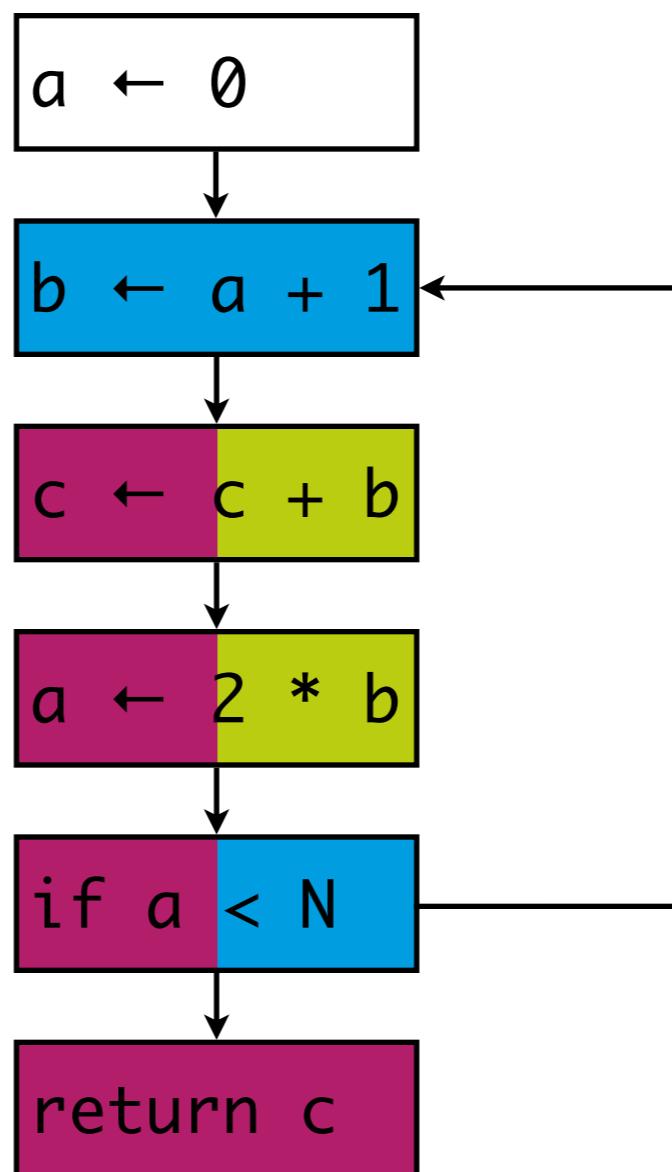
Recap: Liveness Analysis example



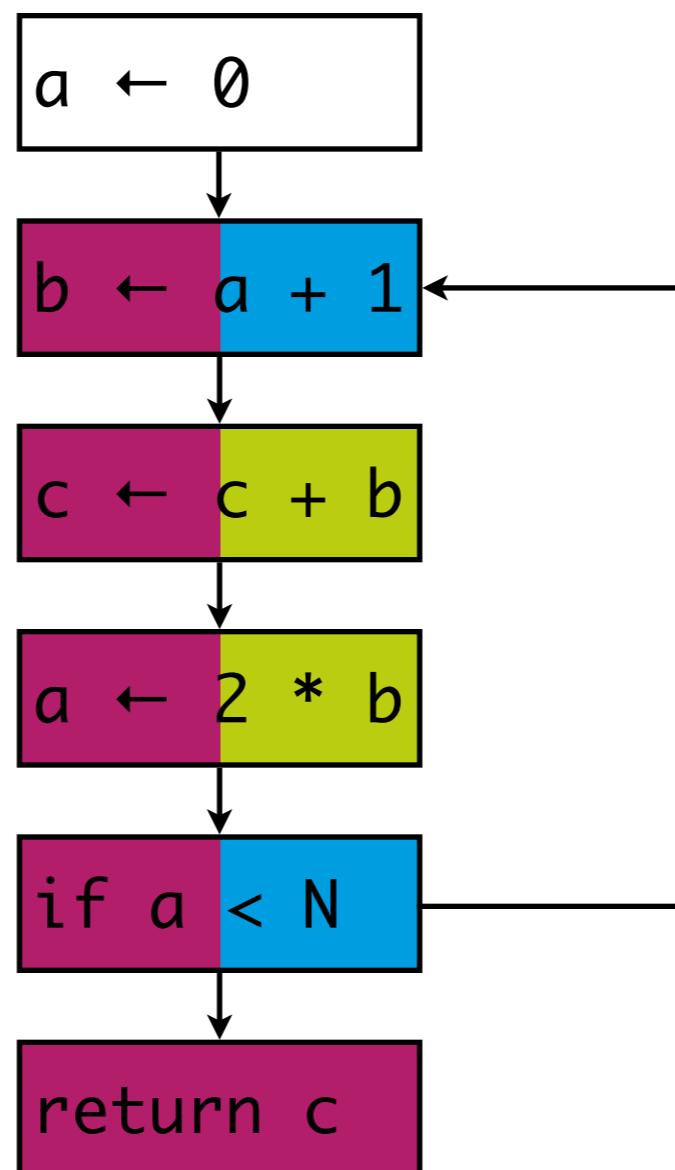
Recap: Liveness Analysis example



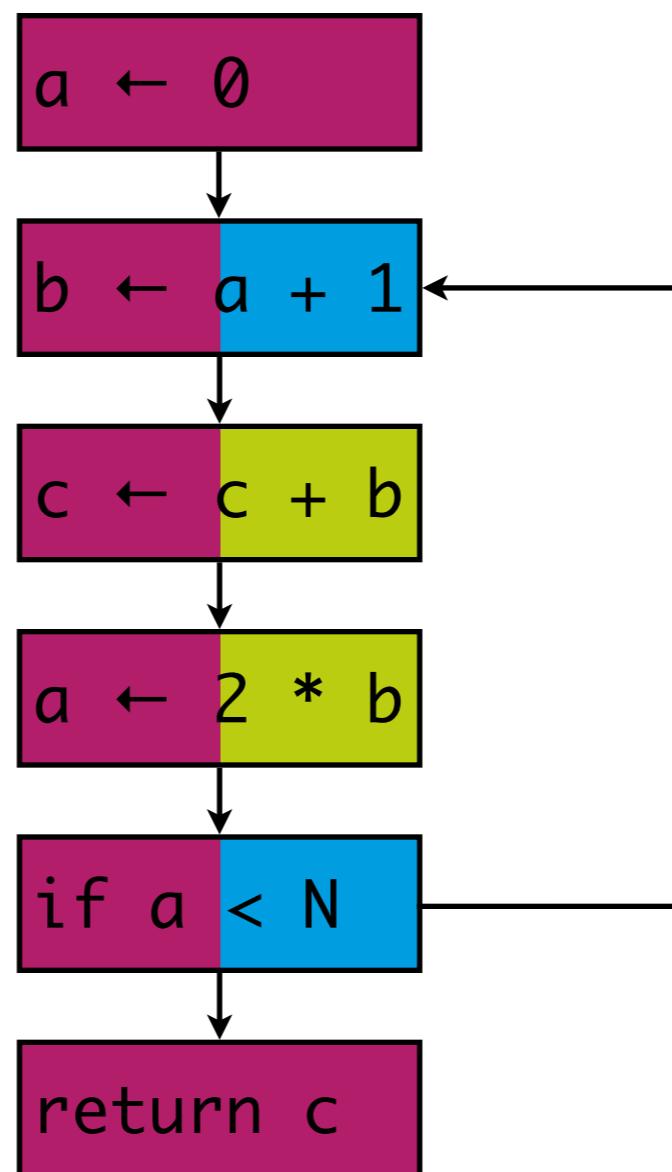
Recap: Liveness Analysis example



Recap: Liveness Analysis example

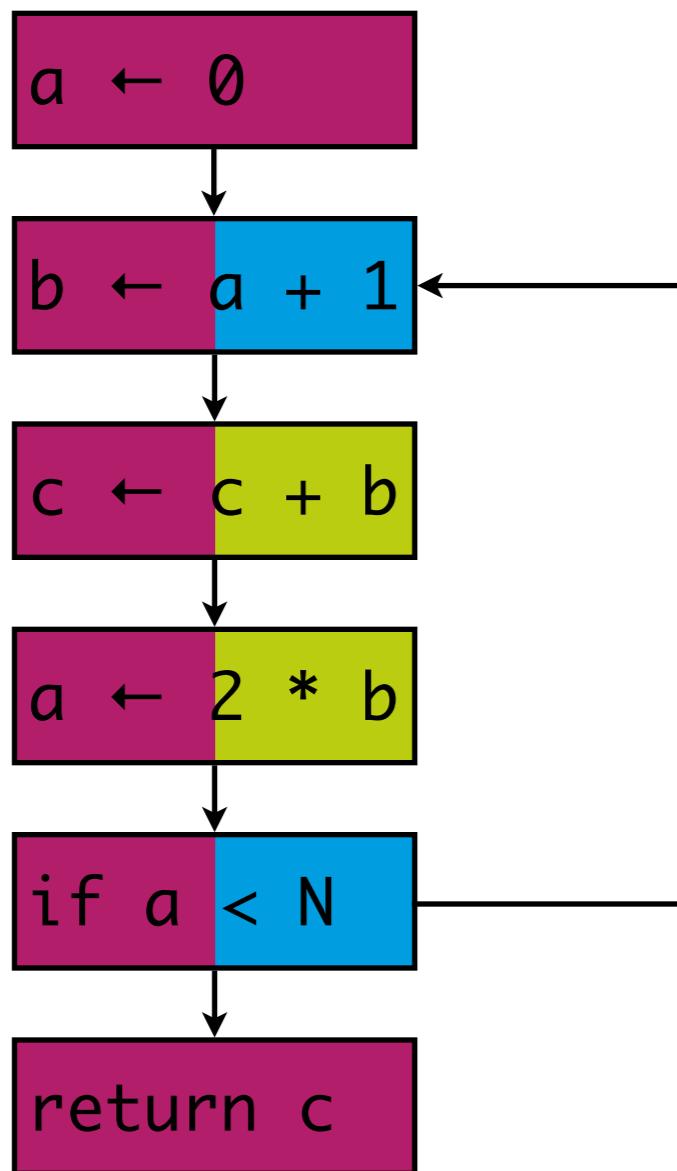


Recap: Liveness Analysis example



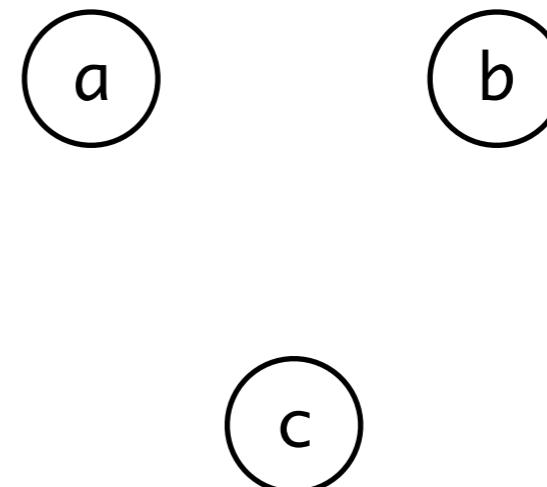
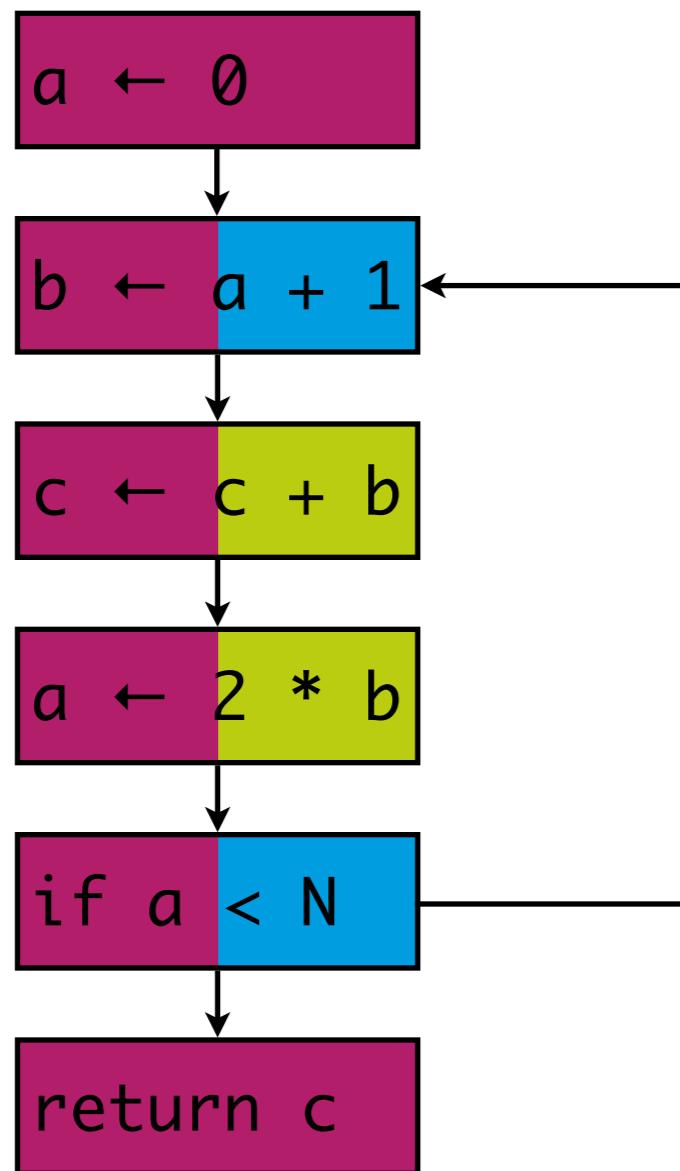
Interference Graphs

example



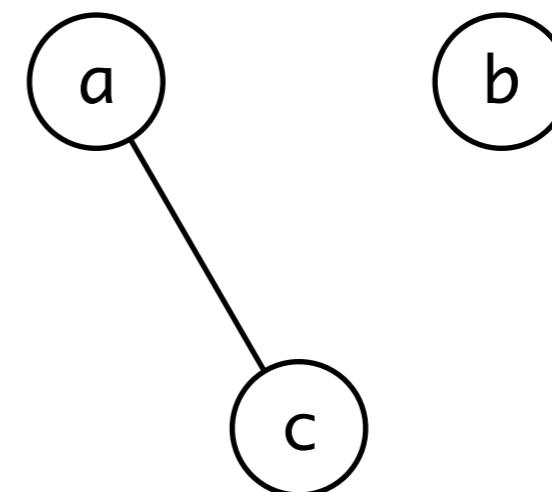
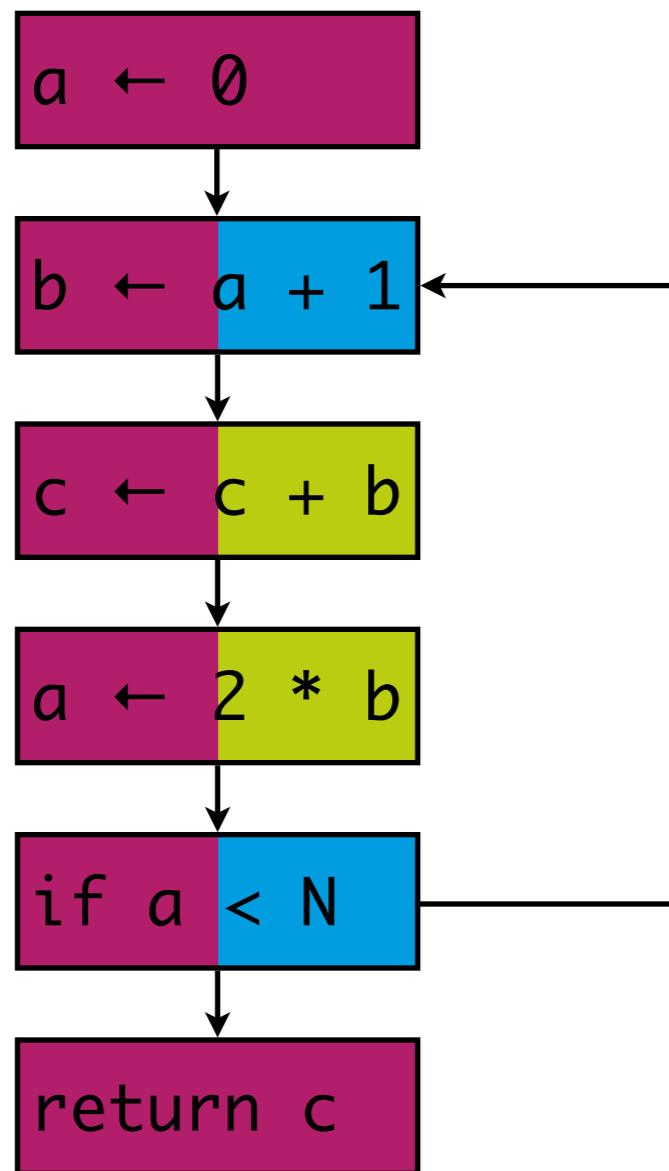
Interference Graphs

example



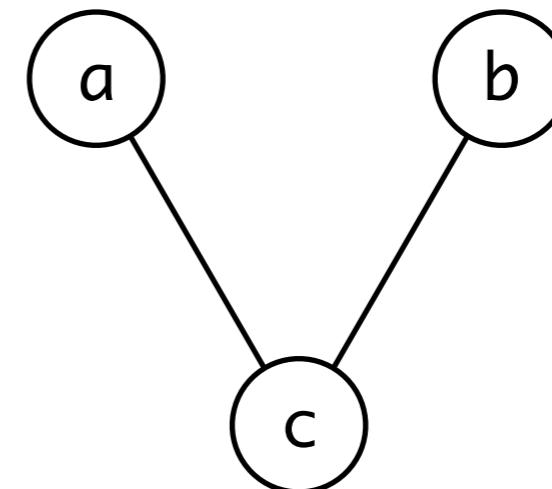
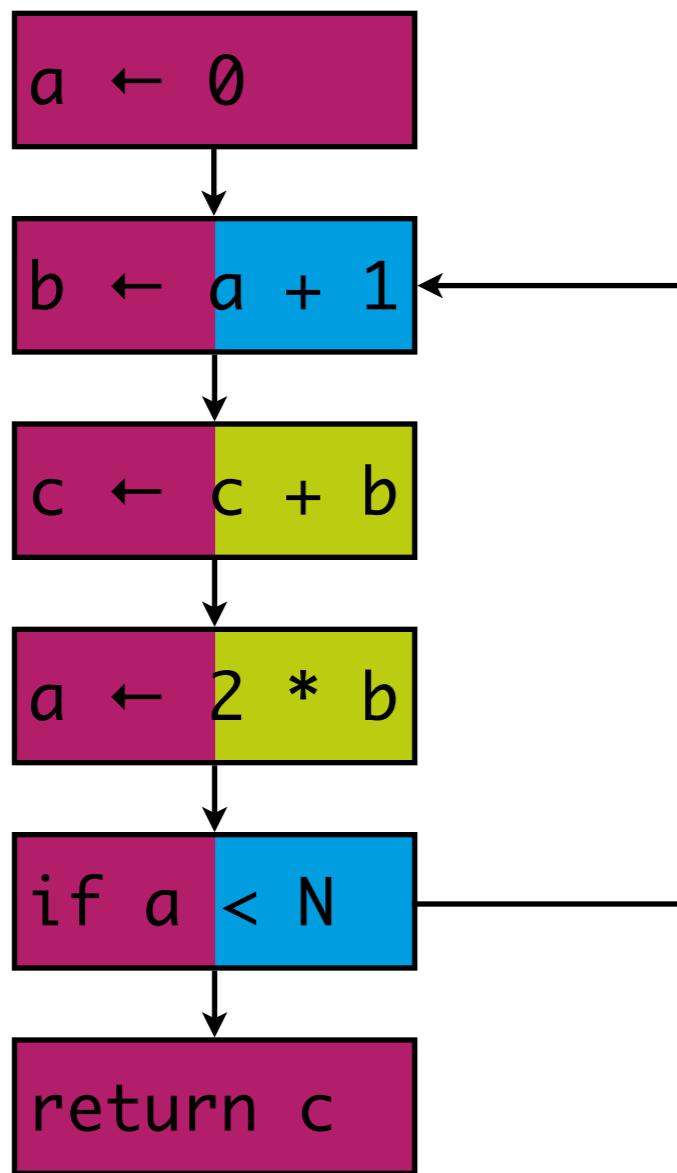
Interference Graphs

example



Interference Graphs

example

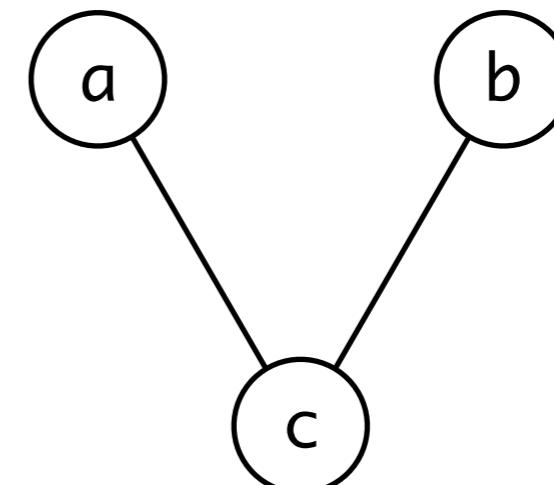
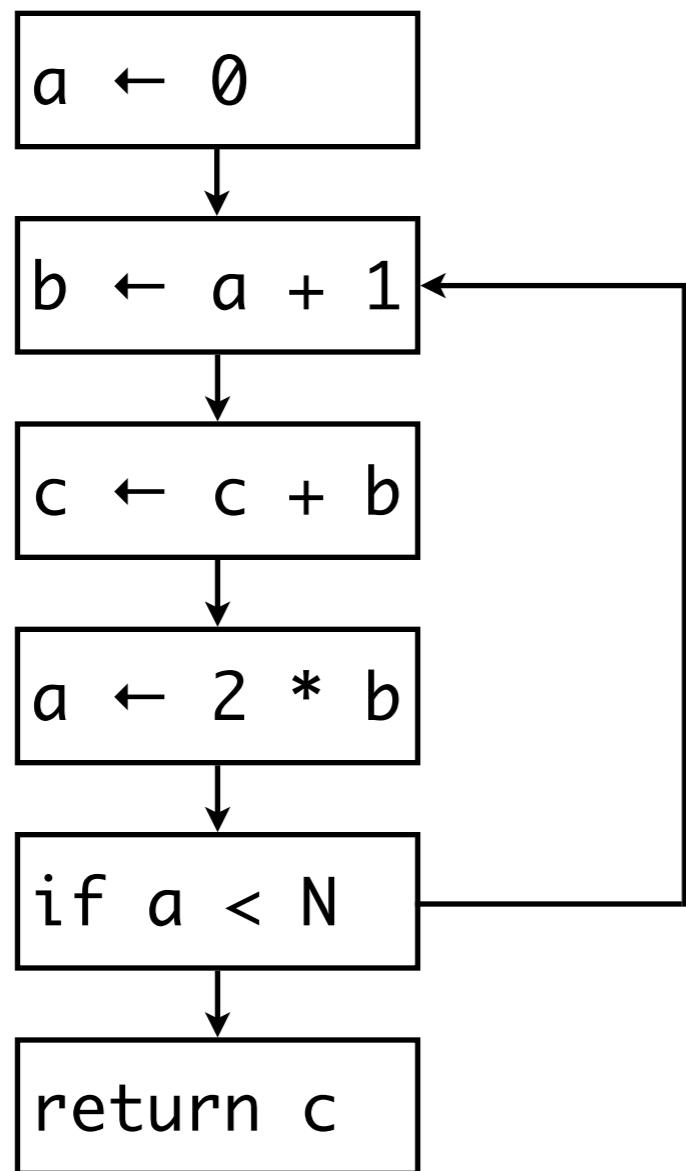


II

Graph Coloring

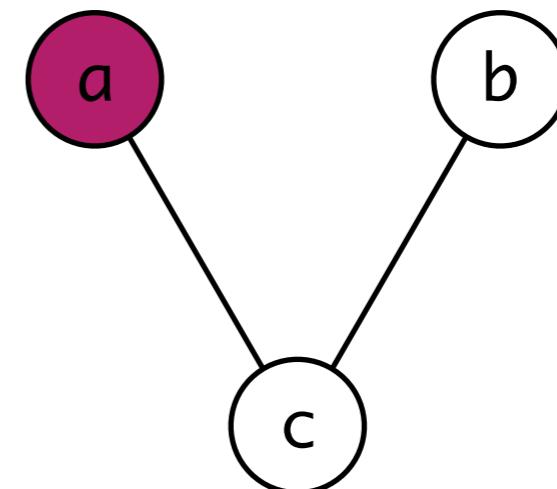
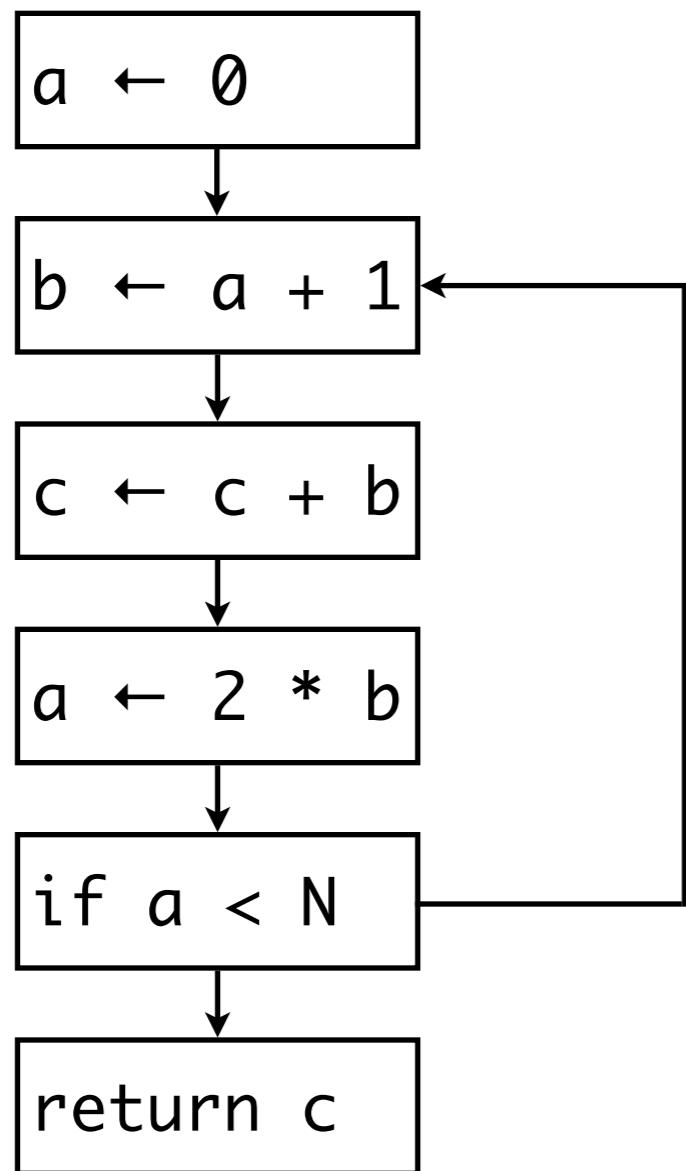
Graph Coloring

example



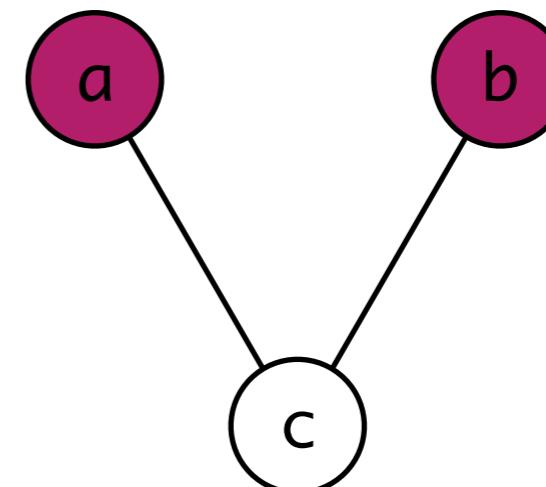
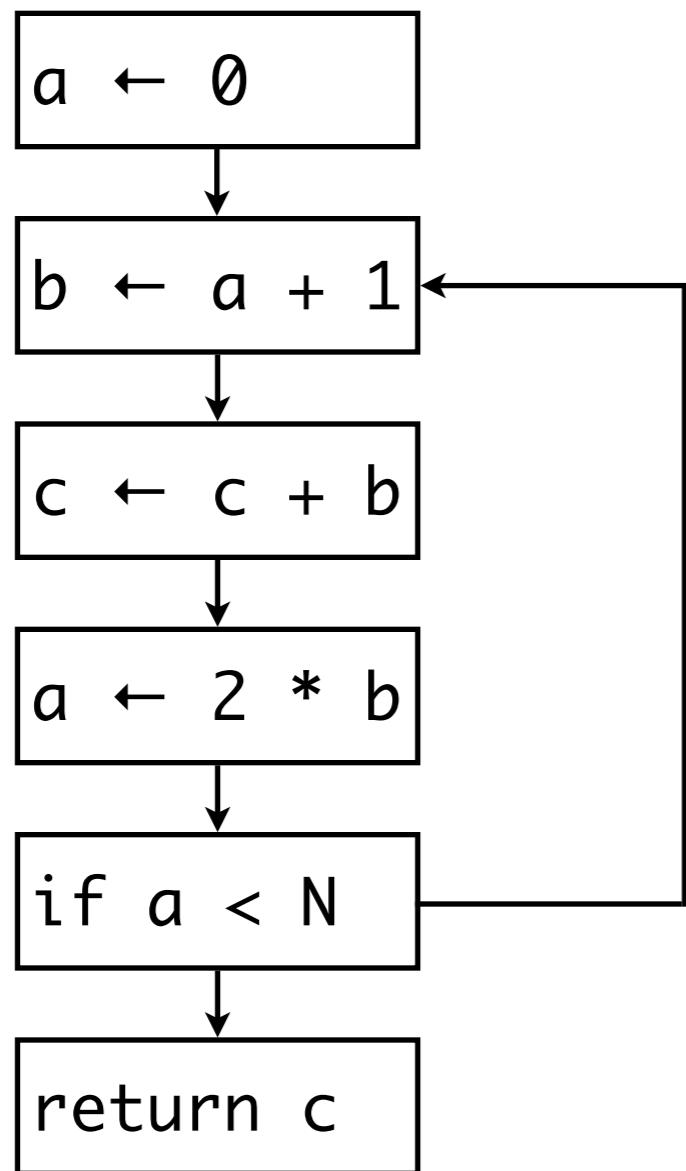
Graph Coloring

example



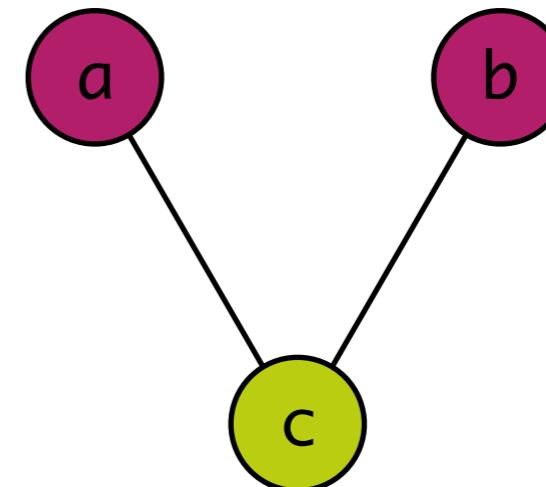
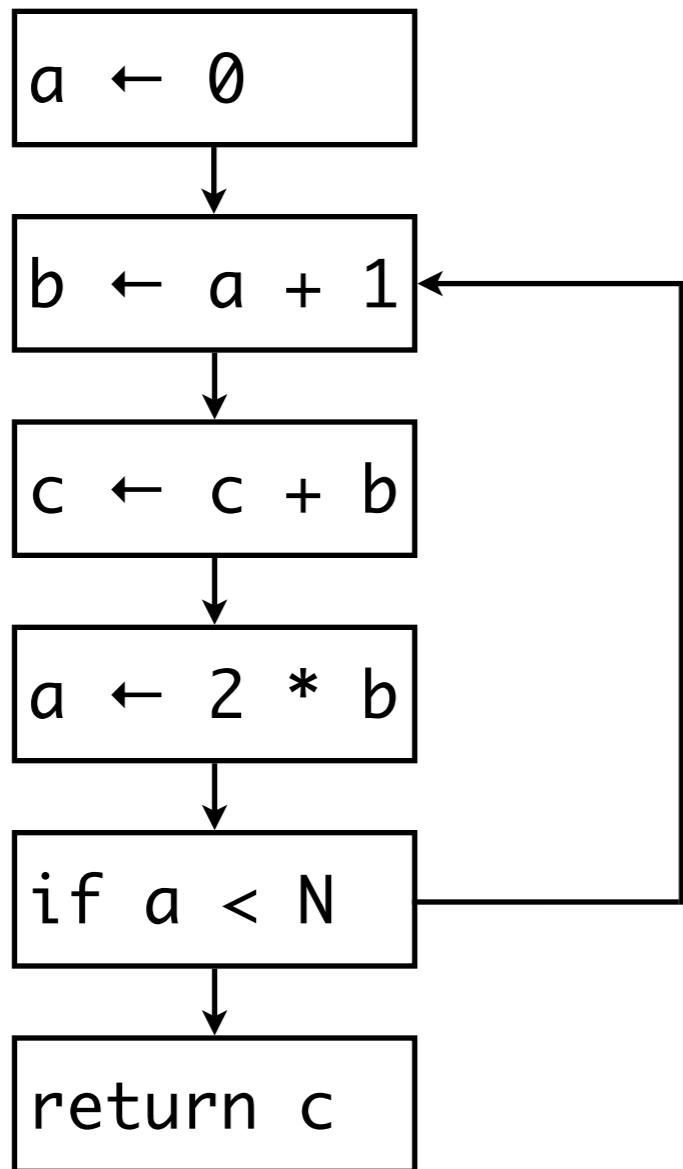
Graph Coloring

example



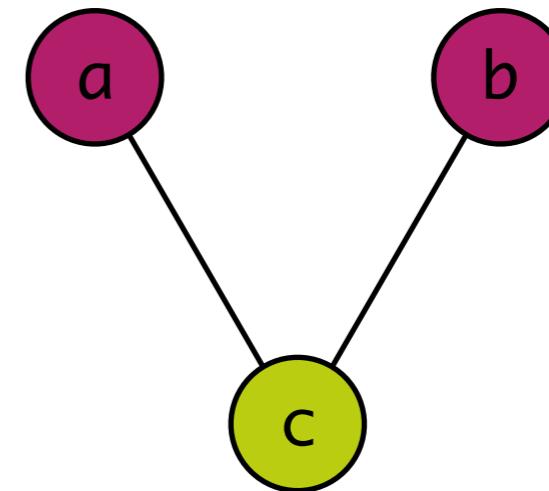
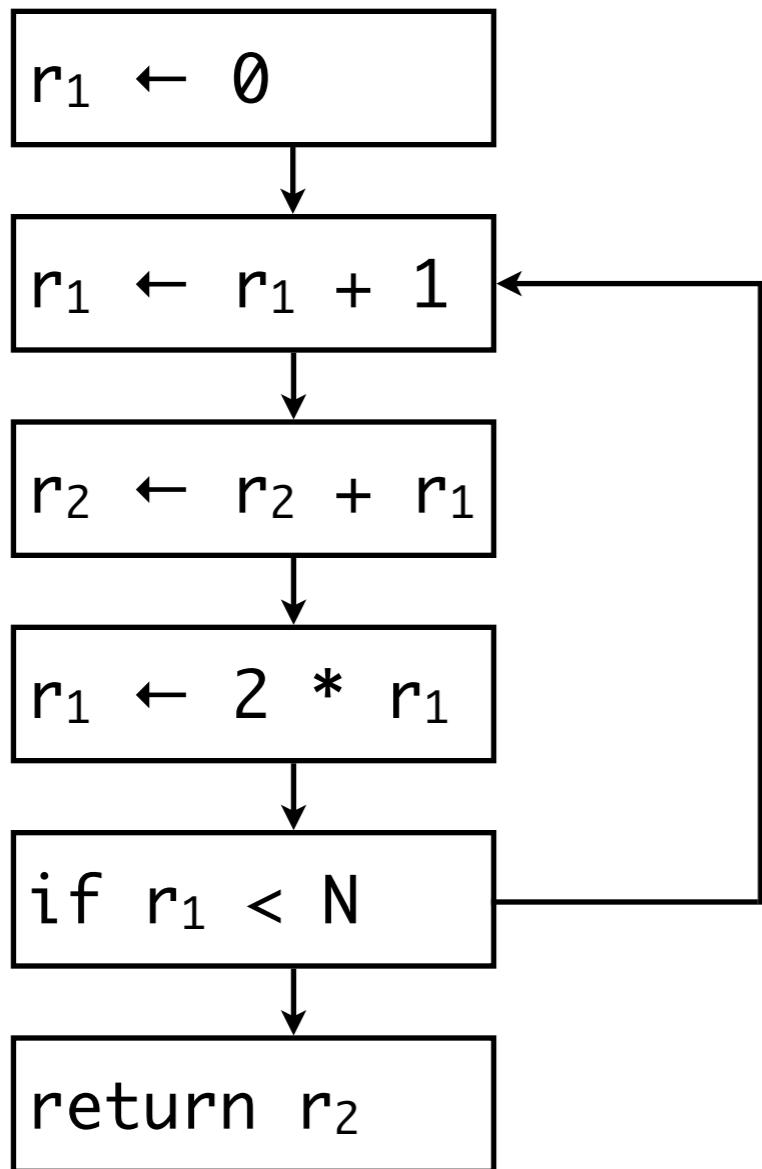
Graph Coloring

example



Graph Coloring

example



Graph Coloring

steps

Simplify

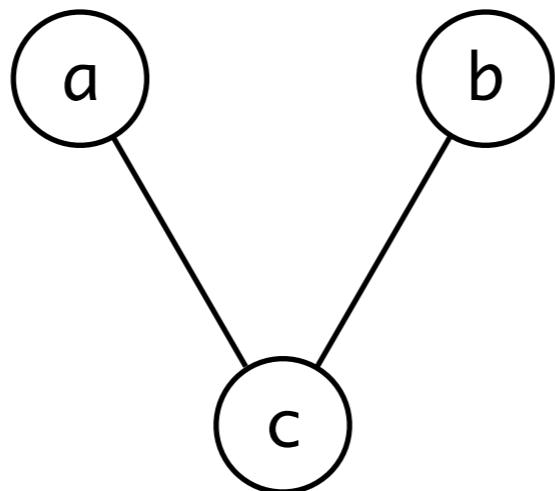
remove node of **insignificant** degree (fewer than k edges)

Select

add node, select color

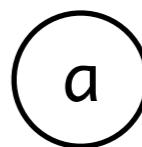
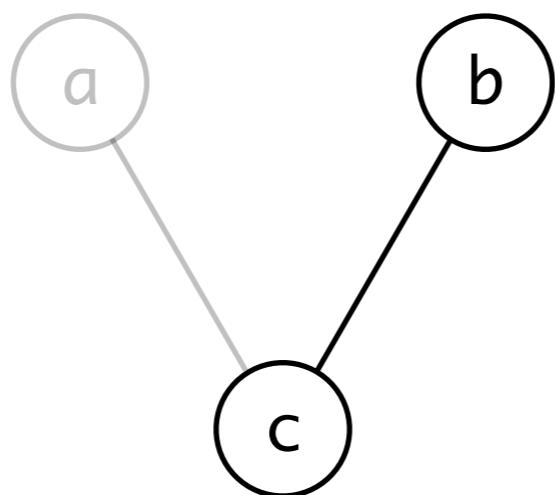
Graph Coloring

example with 2 colors



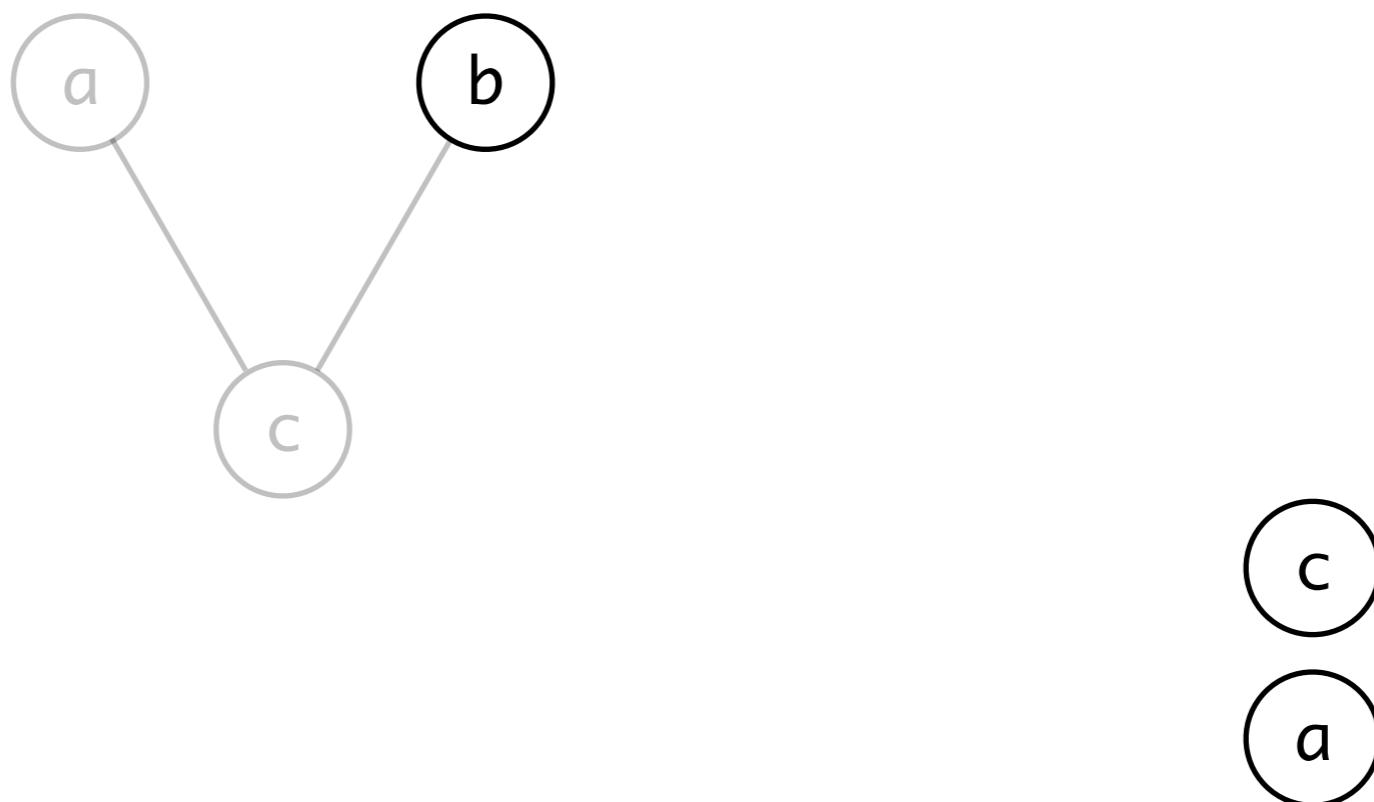
Graph Coloring

example with 2 colors



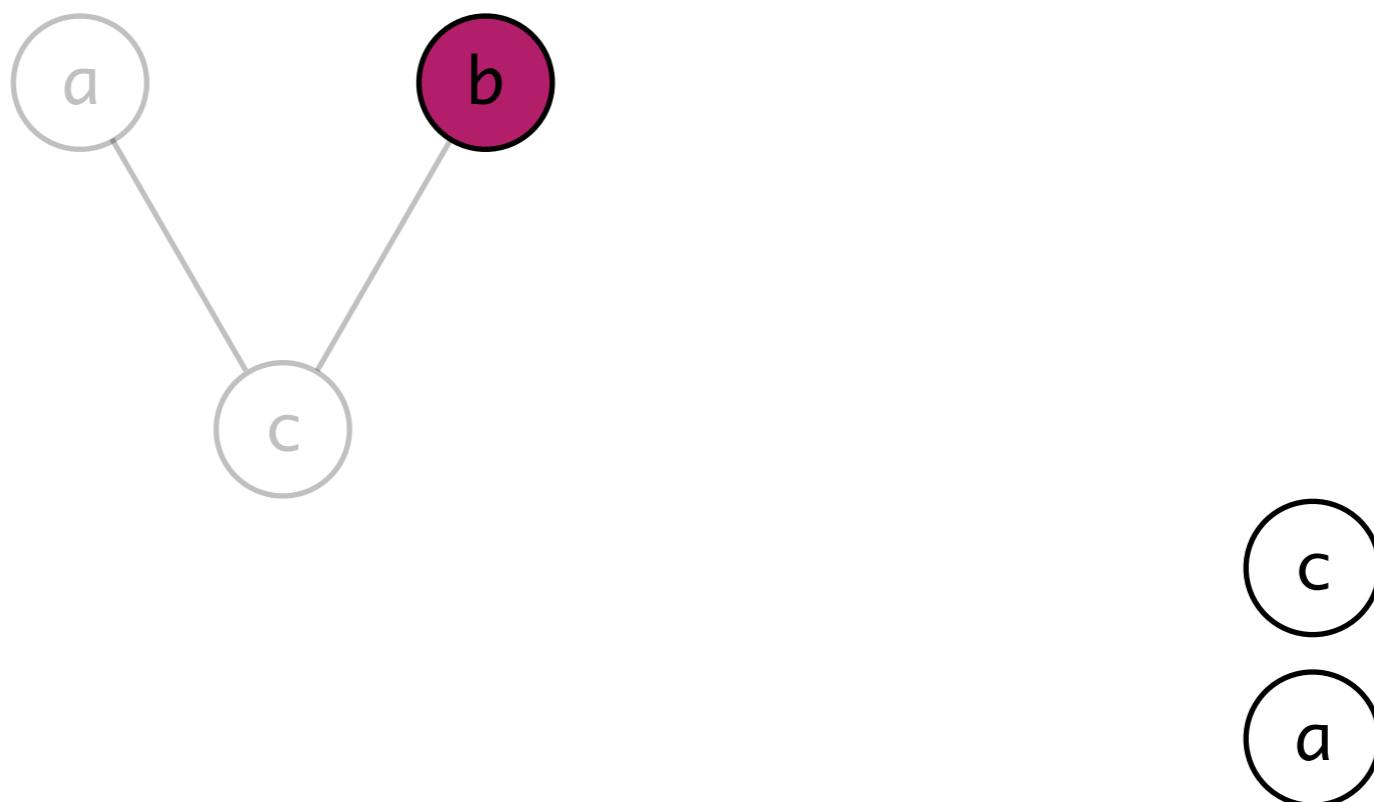
Graph Coloring

example with 2 colors



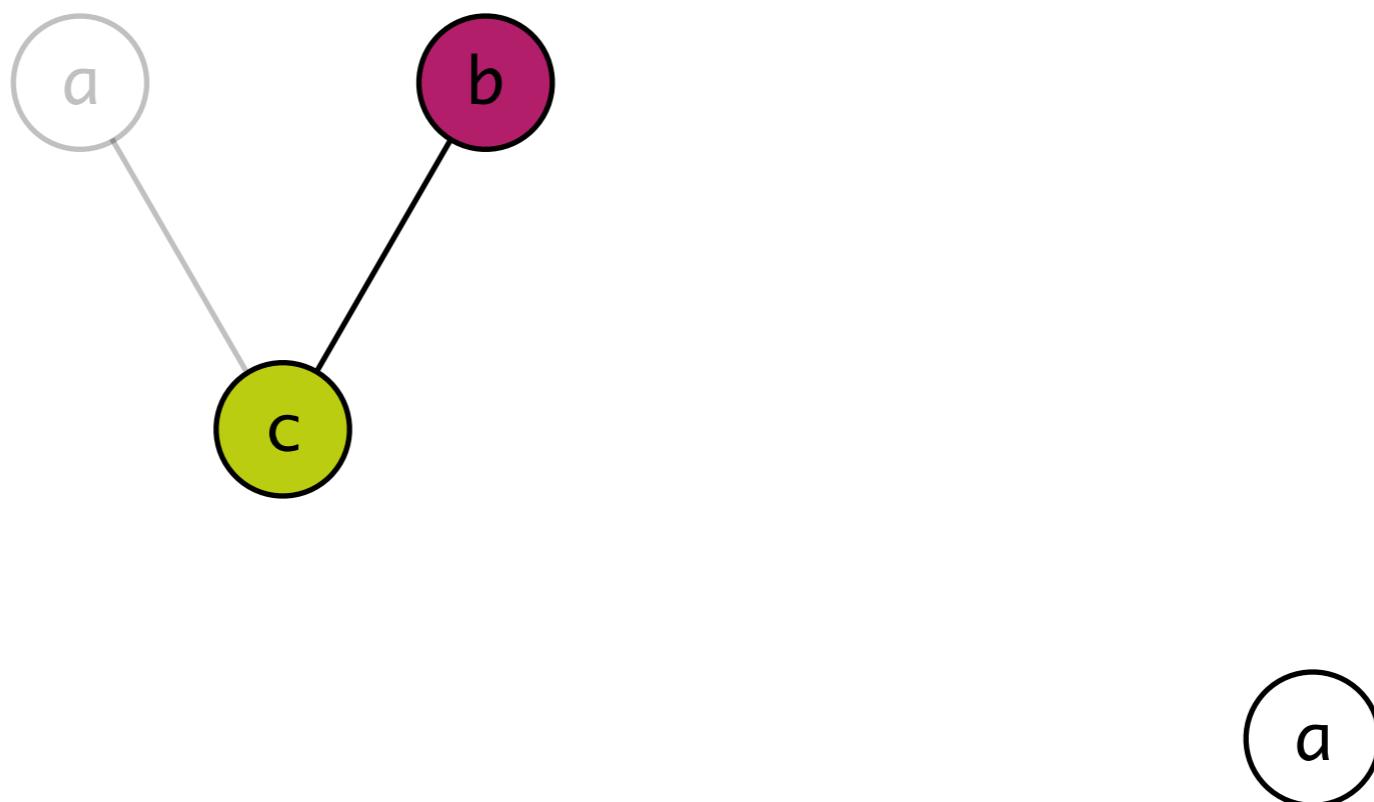
Graph Coloring

example with 2 colors



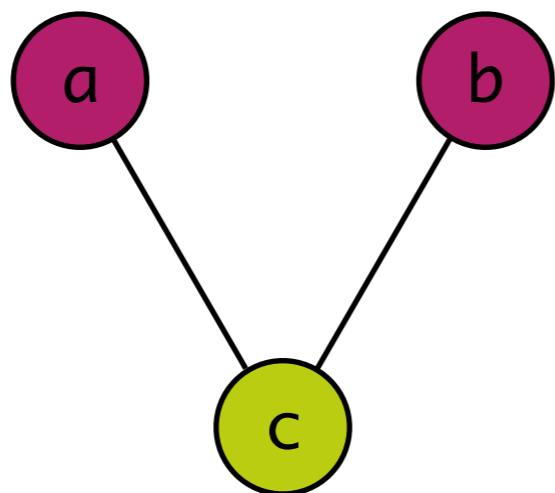
Graph Coloring

example with 2 colors



Graph Coloring

example with 2 colors



Graph Coloring

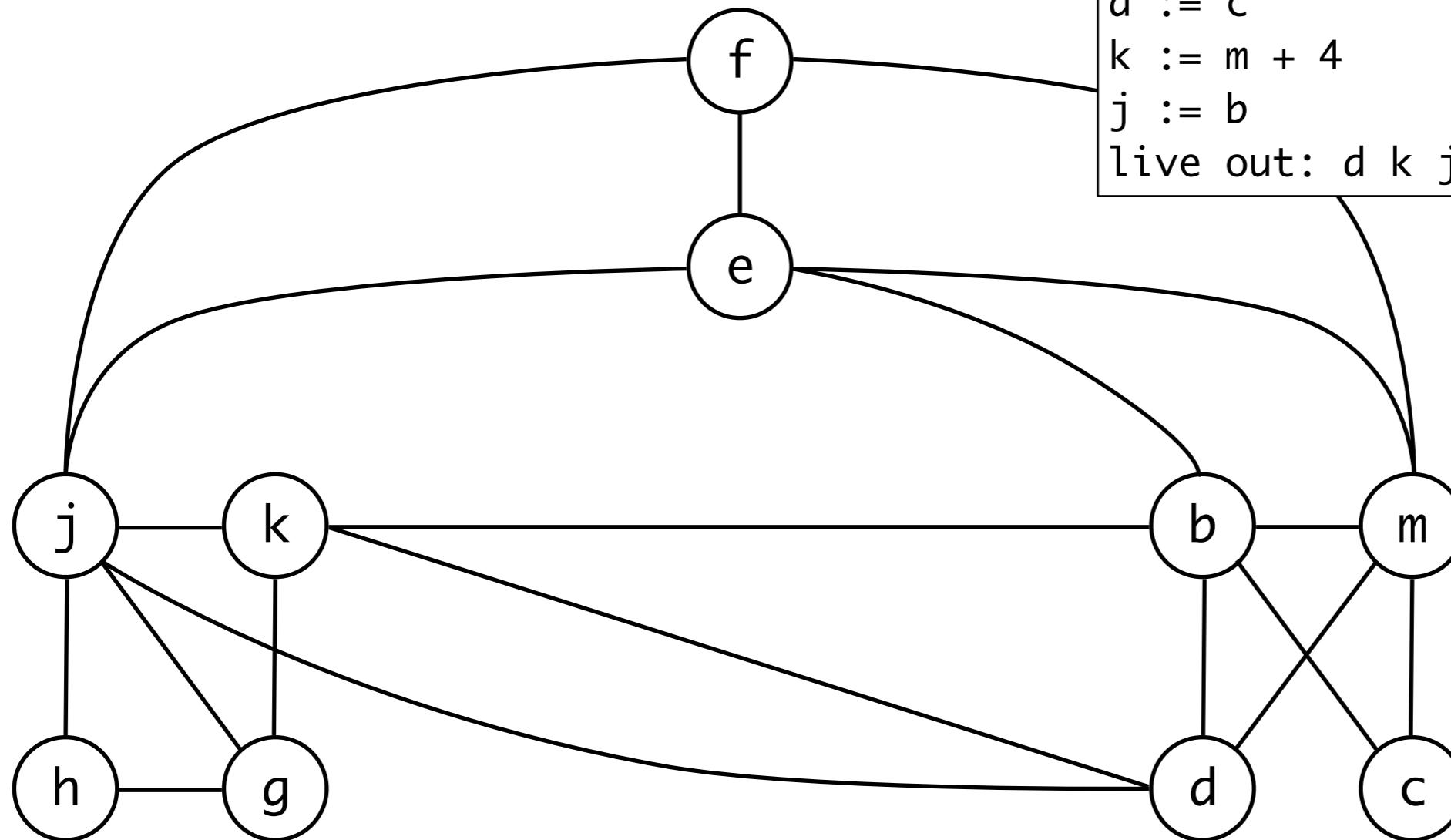
example with 4 colors

```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Graph Coloring

example with 4 colors

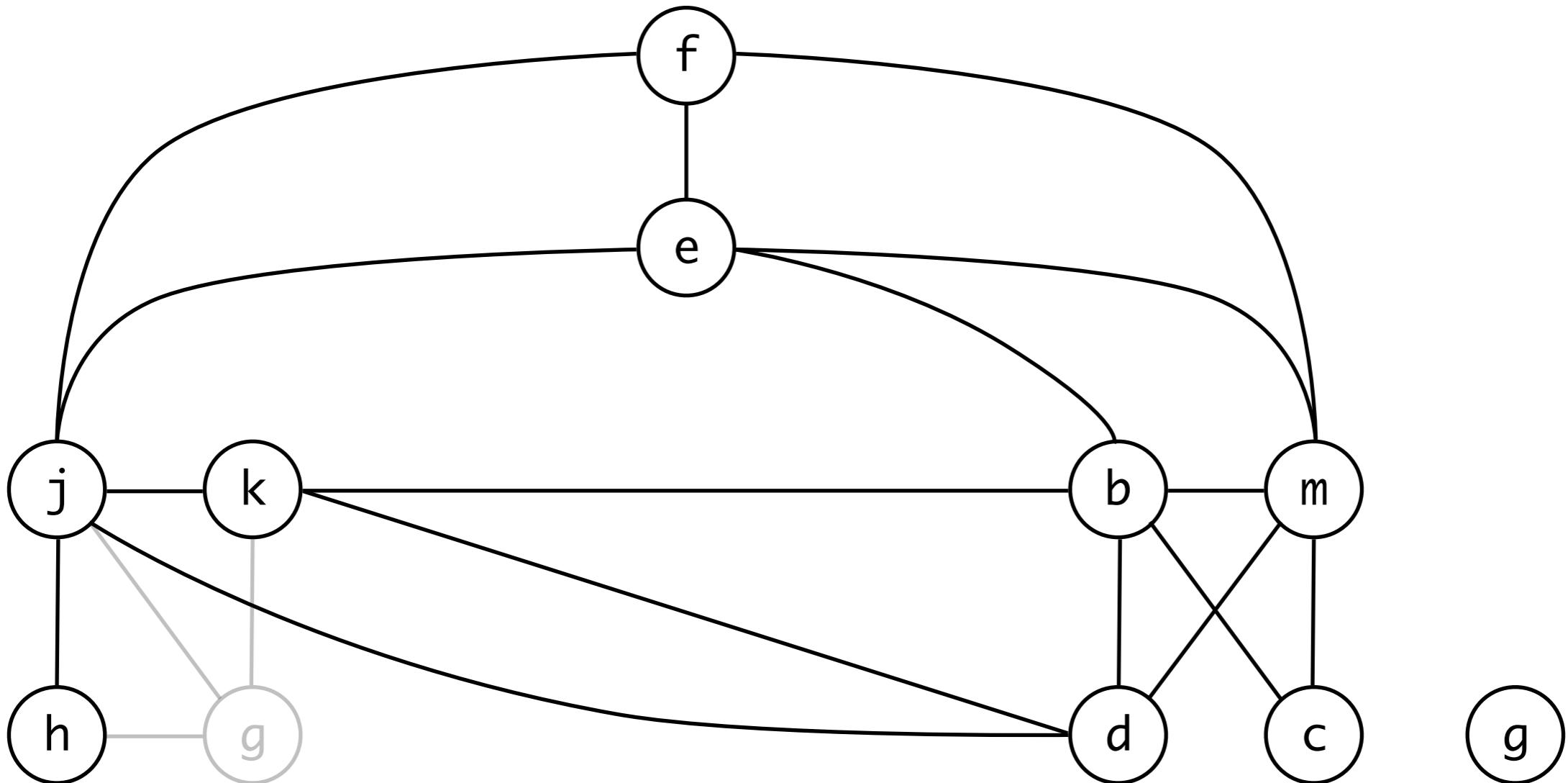
r_1
 r_2
 r_3
 r_4



```
live-in: k j  
g := mem[j + 12]  
h := k - 1  
f := g * h  
e := mem[j + 8]  
m := mem[j + 16]  
b := mem[f]  
c := e + 8  
d := c  
k := m + 4  
j := b  
live out: d k j
```

Graph Coloring

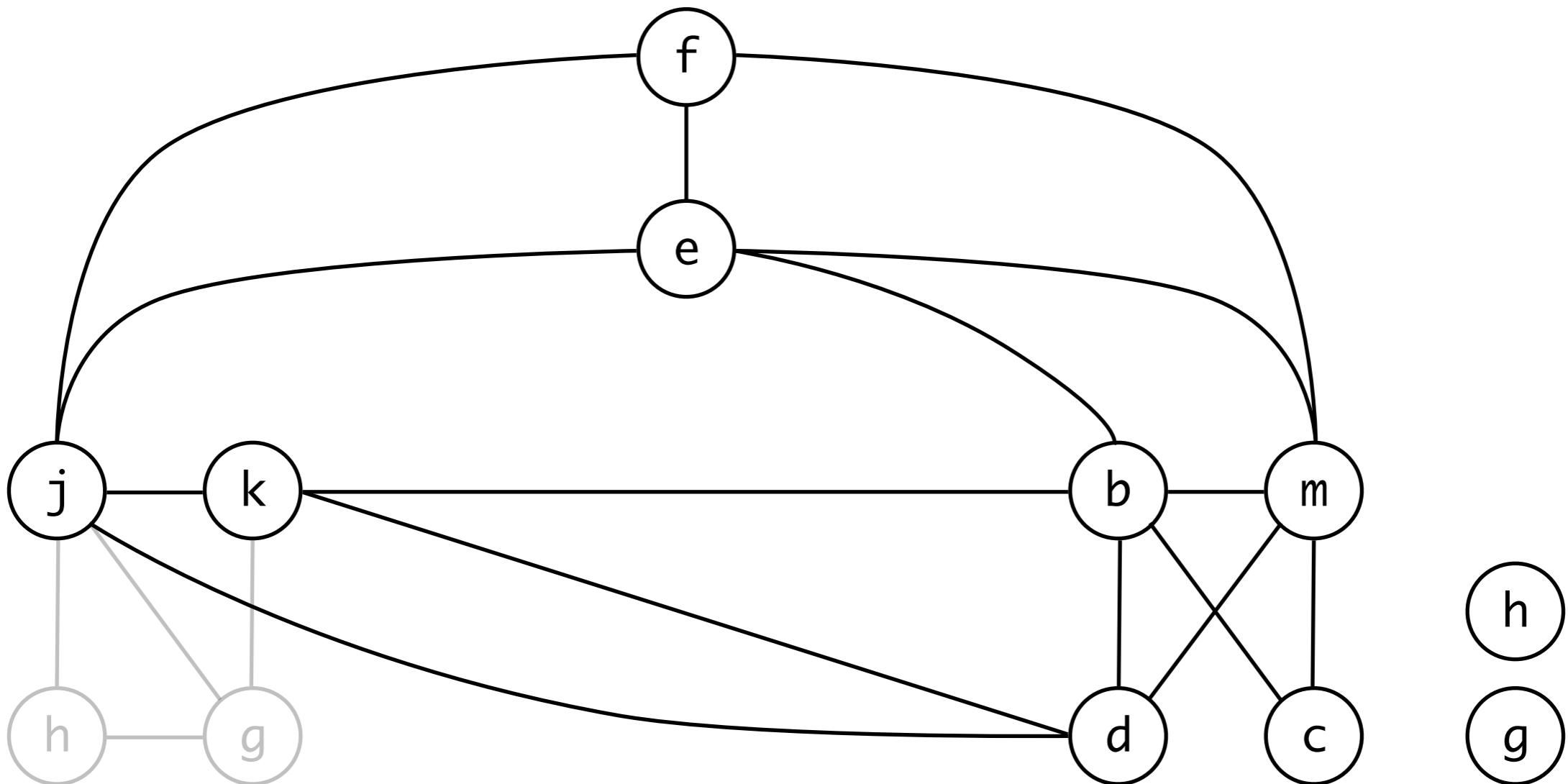
example with 4 colors



Graph Coloring

example with 4 colors

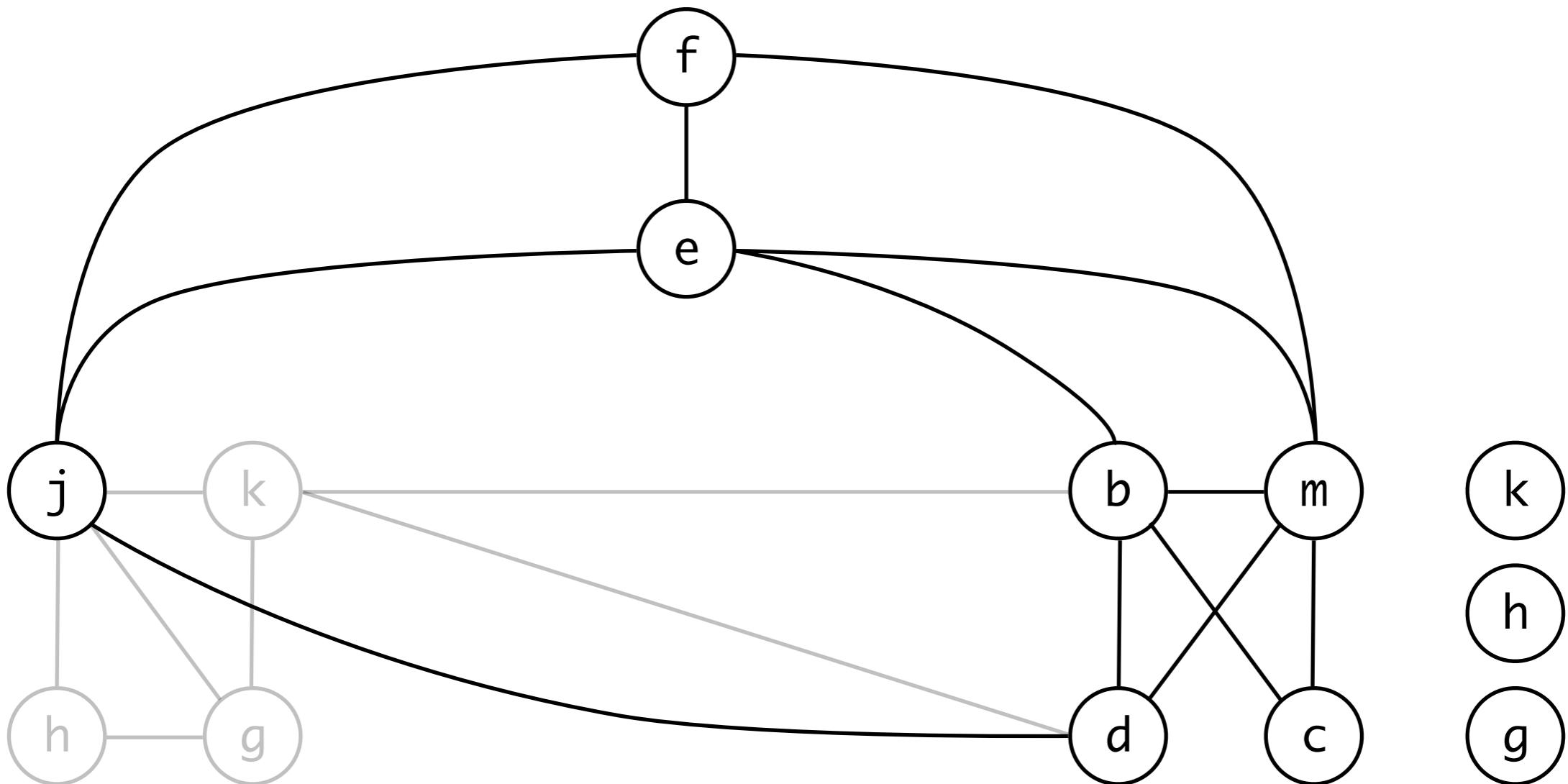
r_1
 r_2
 r_3
 r_4



Graph Coloring

example with 4 colors

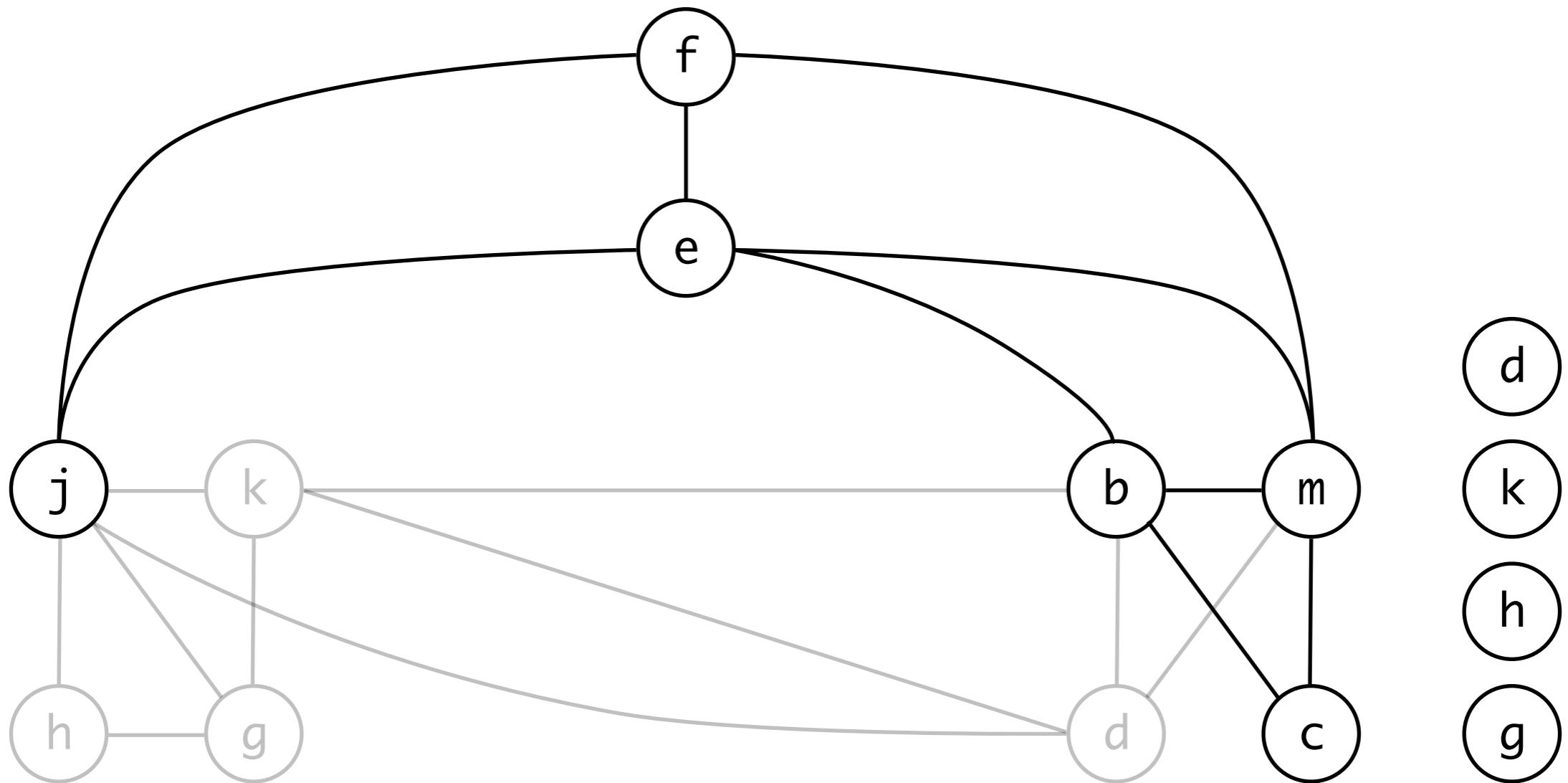
r_1
 r_2
 r_3
 r_4



Graph Coloring

example with 4 colors

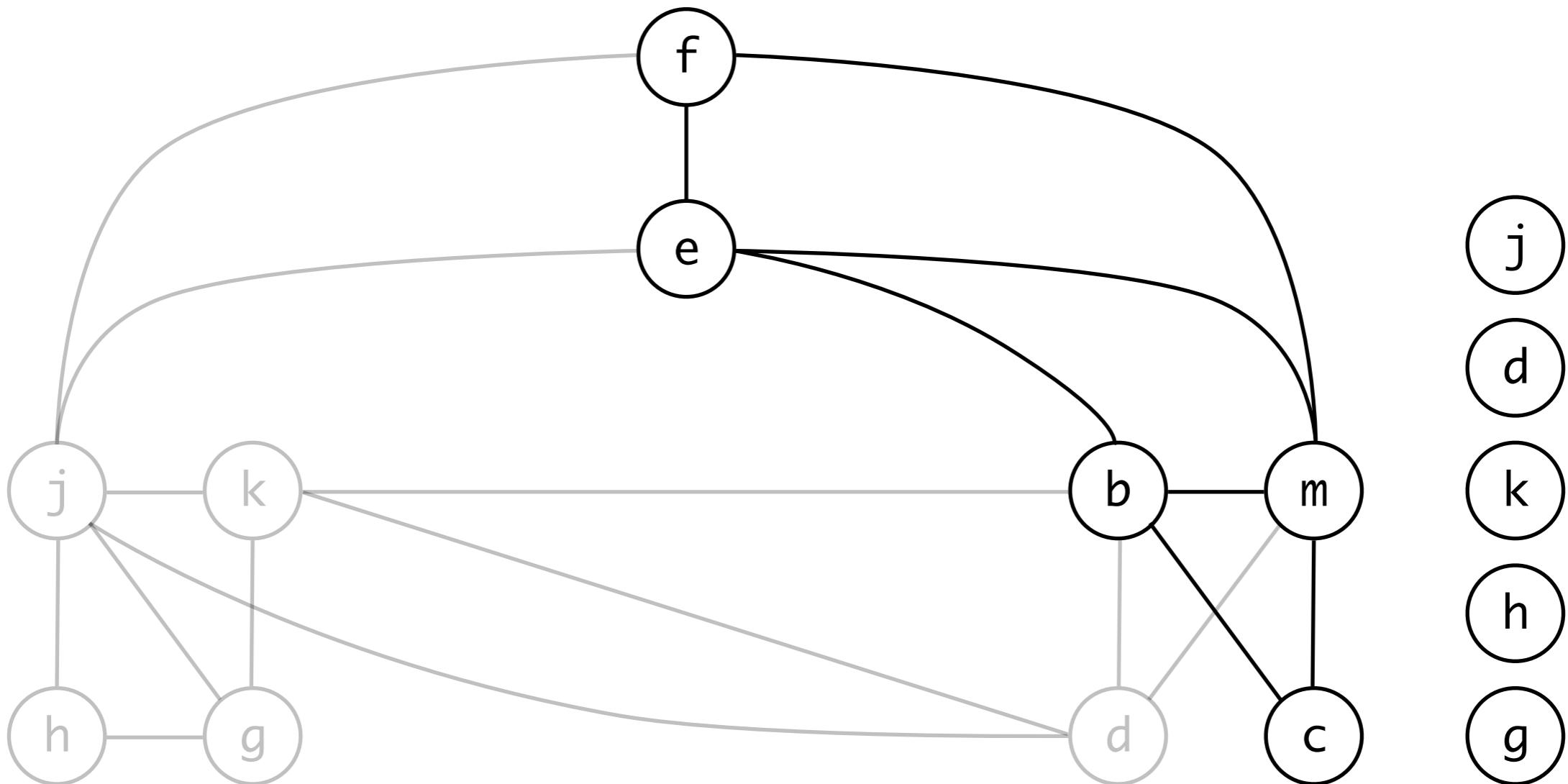
r_1
 r_2
 r_3
 r_4



Graph Coloring

example with 4 colors

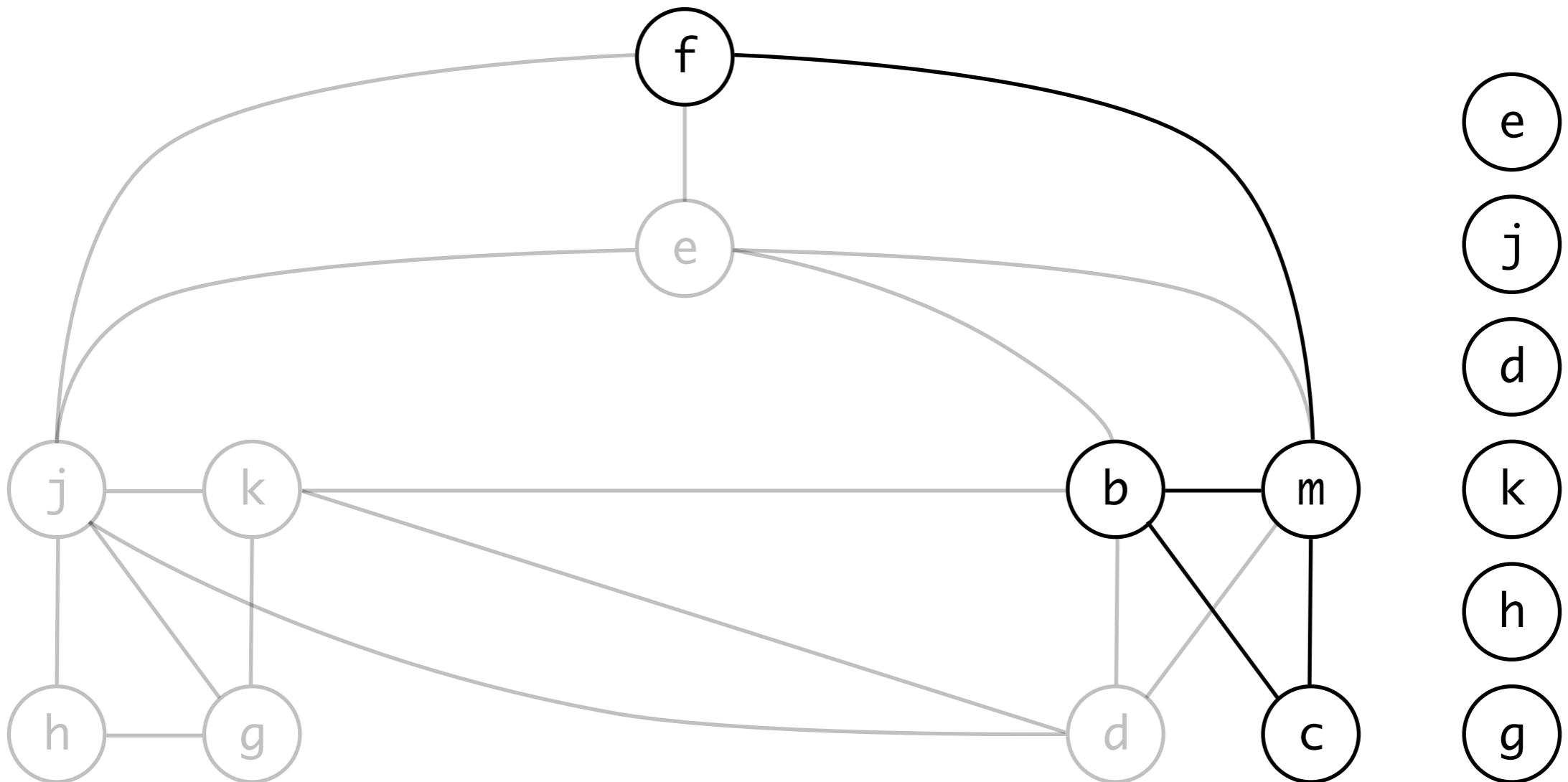
r_1
 r_2
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Graph Coloring

example with 4 colors

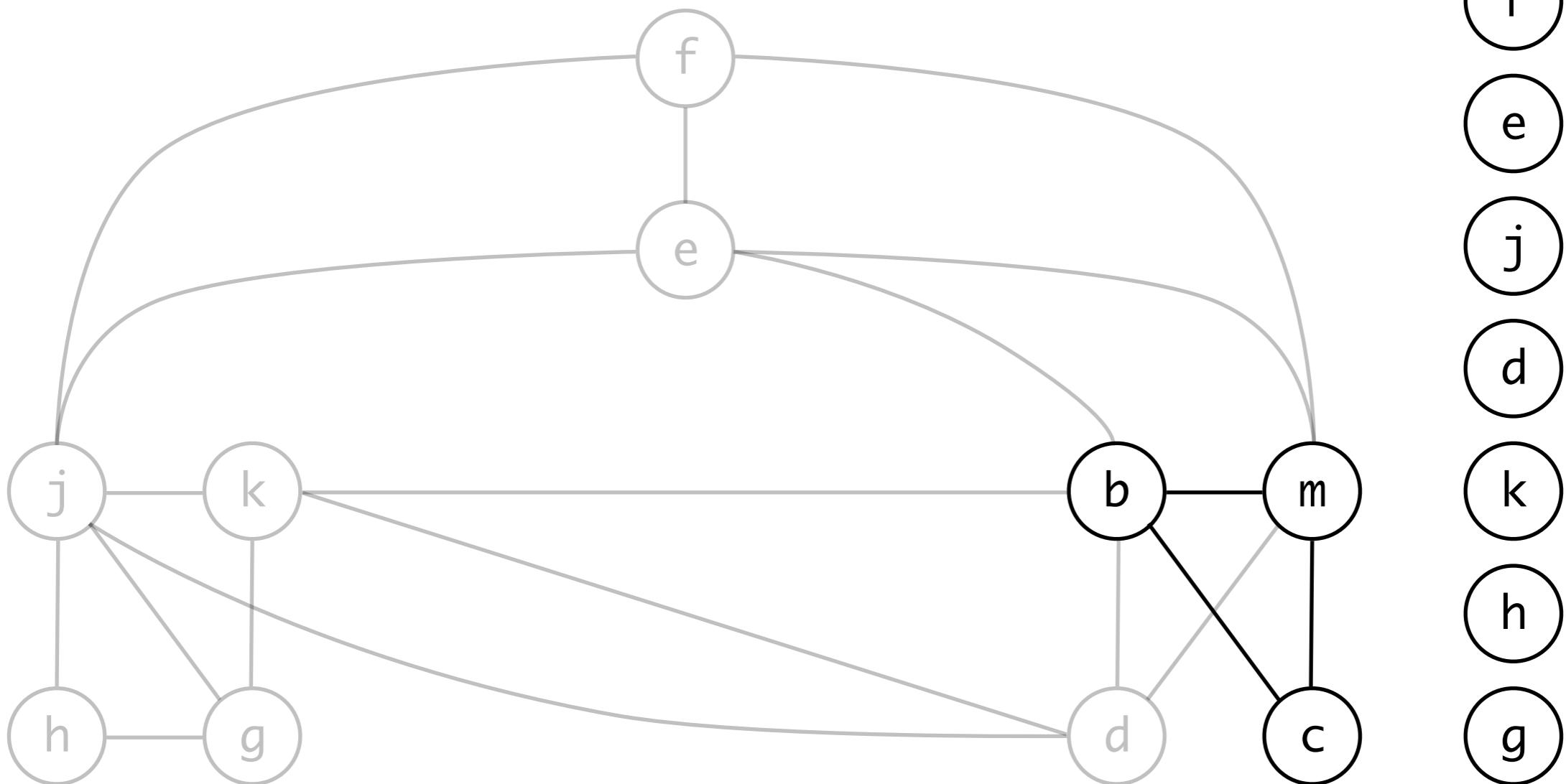
r_1
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Graph Coloring

example with 4 colors

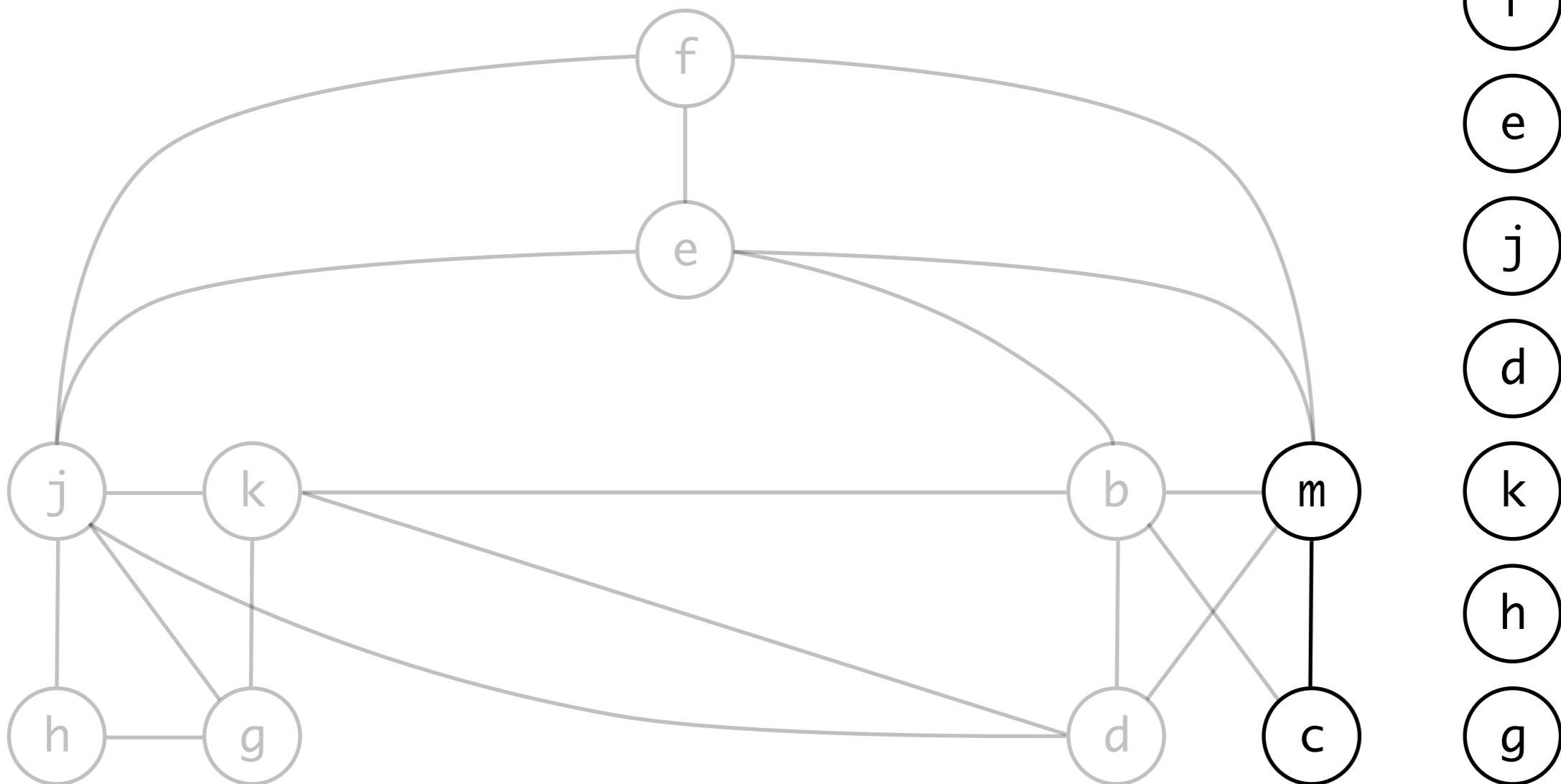
r_1
 r_2
 r_3
 r_4



Graph Coloring

example with 4 colors

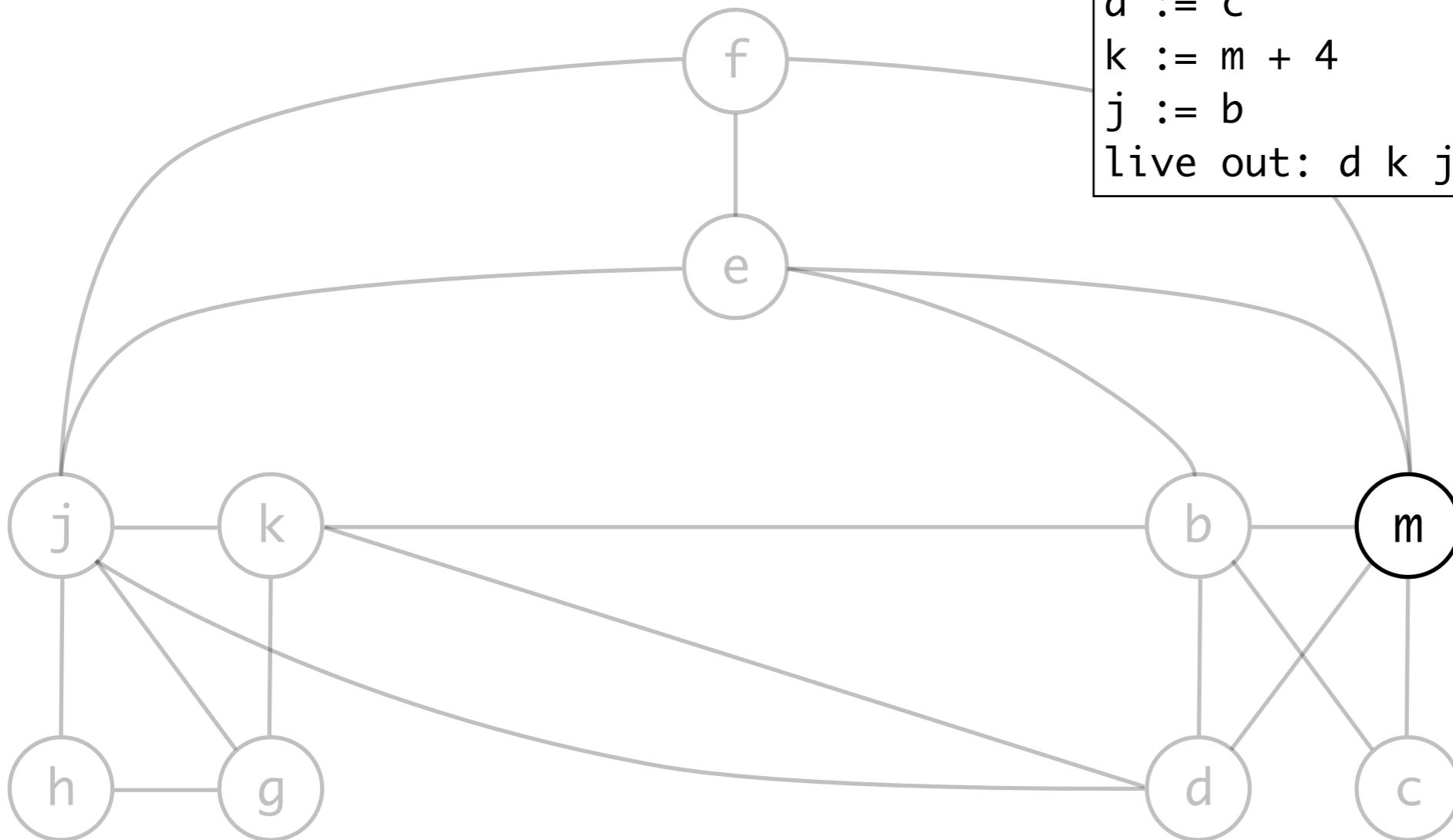
r_1
 r_2
 r_3
 r_4



Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4



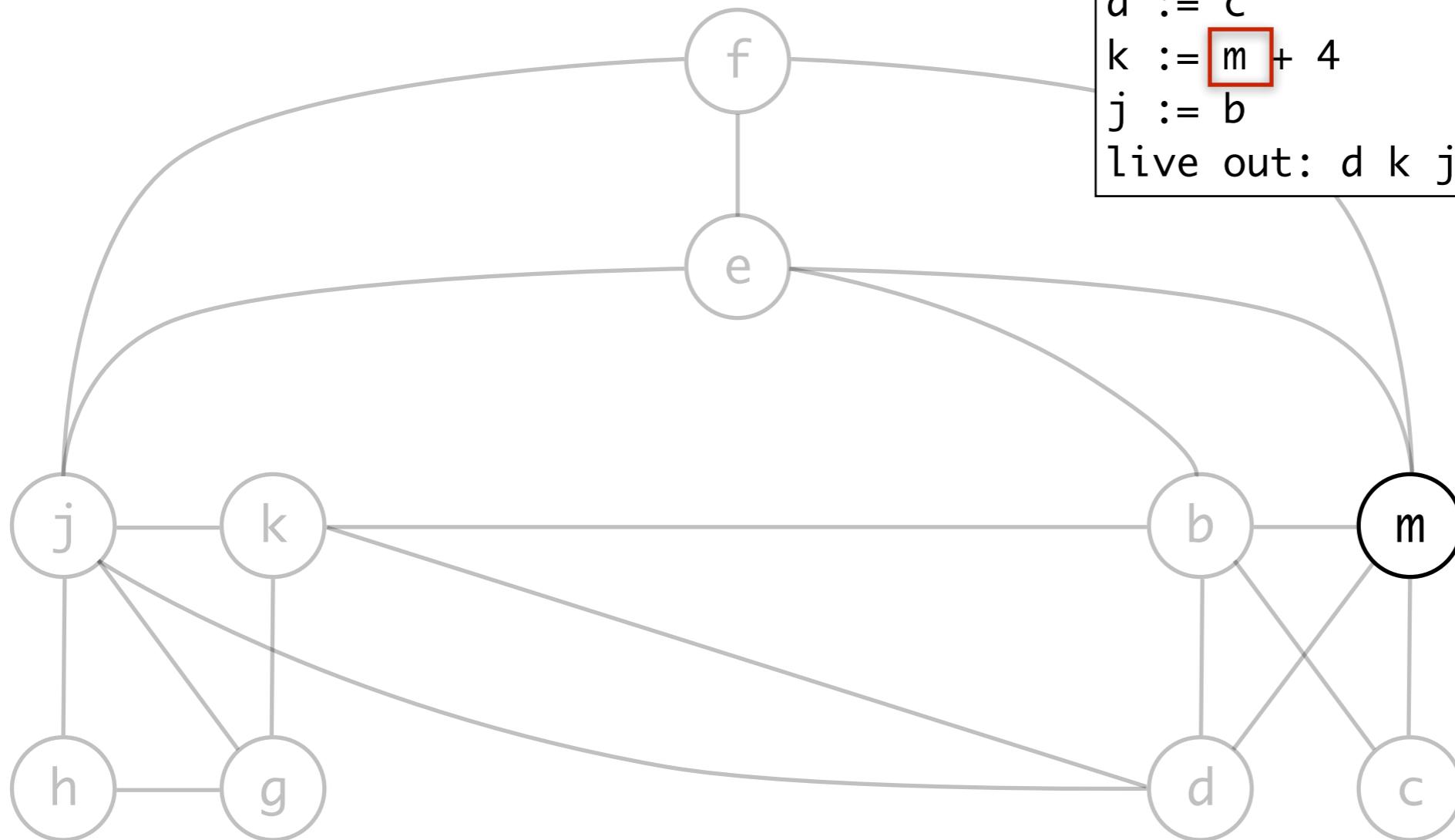
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live-in: k j  
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h := k - 1  
f := g * h  
e := mem[j + 8]  
m := mem[j + 16]  
b := mem[f]  
c := e + 8  
d := c  
k := m + 4  
j := b  
live out: d k j
```

- (c)
- (b)
- (f)
- (e)
- (j)
- (d)
- (k)
- (h)
- (g)

Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4

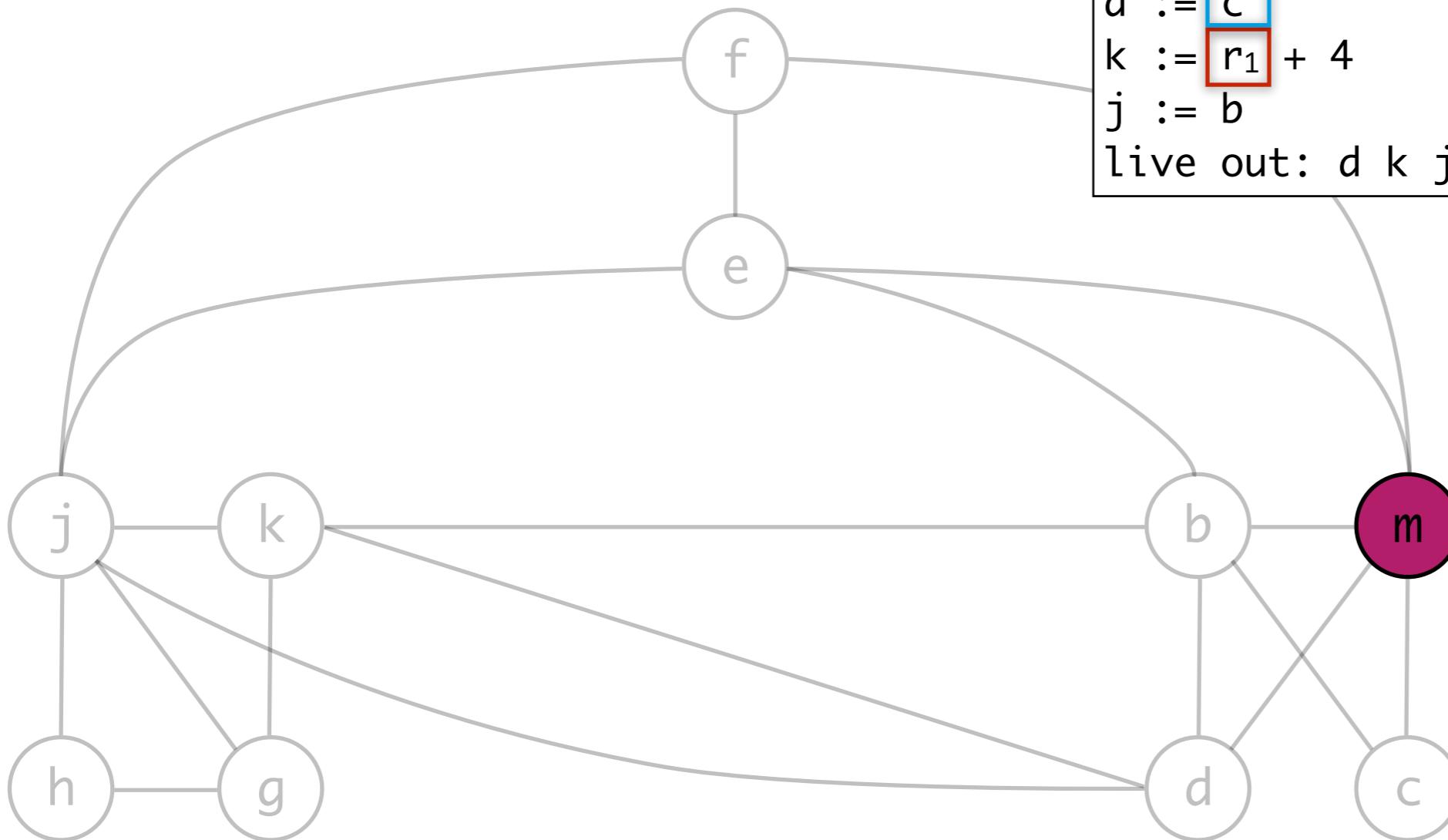


live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16] (highlighted)
b := mem[f]
c := e + 8
d := c
k := m + 4 (highlighted)
j := b
live out: d k j

Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4



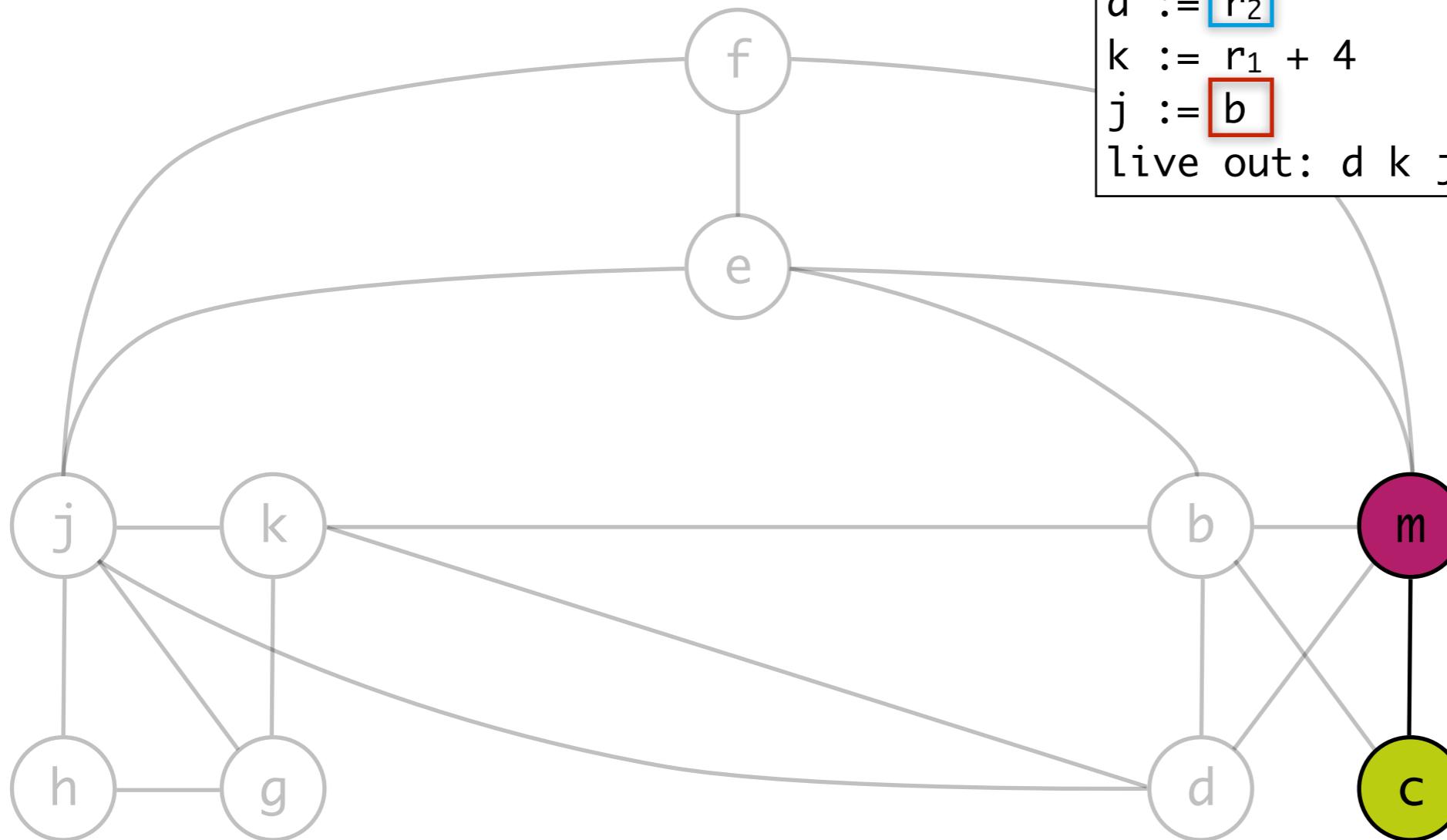
```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
r1 := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := r1 + 4
j := b
live out: d k j
```

- (c)
- (b)
- (f)
- (e)
- (j)
- (d)
- (k)
- (h)
- (g)

Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4

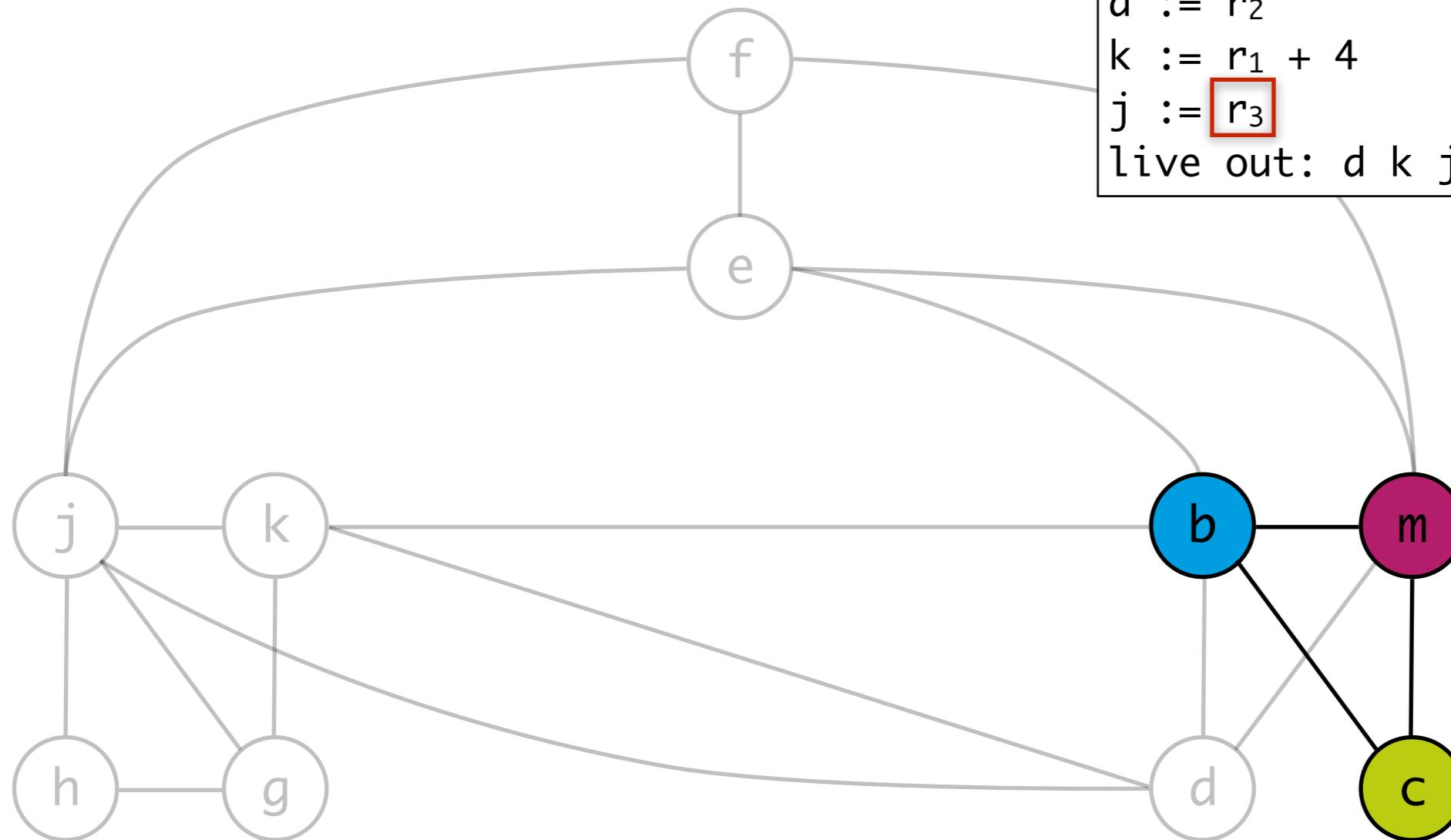


```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
r1 := mem[j + 16]
b := mem[f]
r2 := e + 8
d := r2
k := r1 + 4
j := b
live out: d k j
```

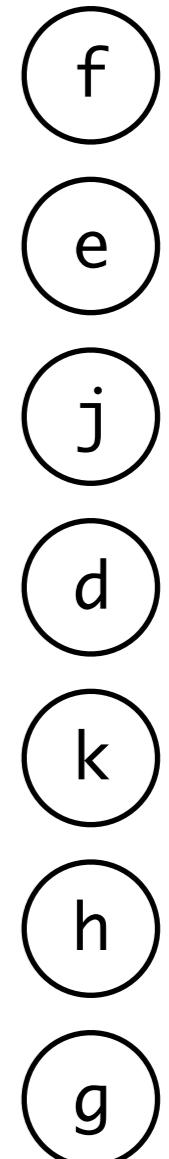
Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4



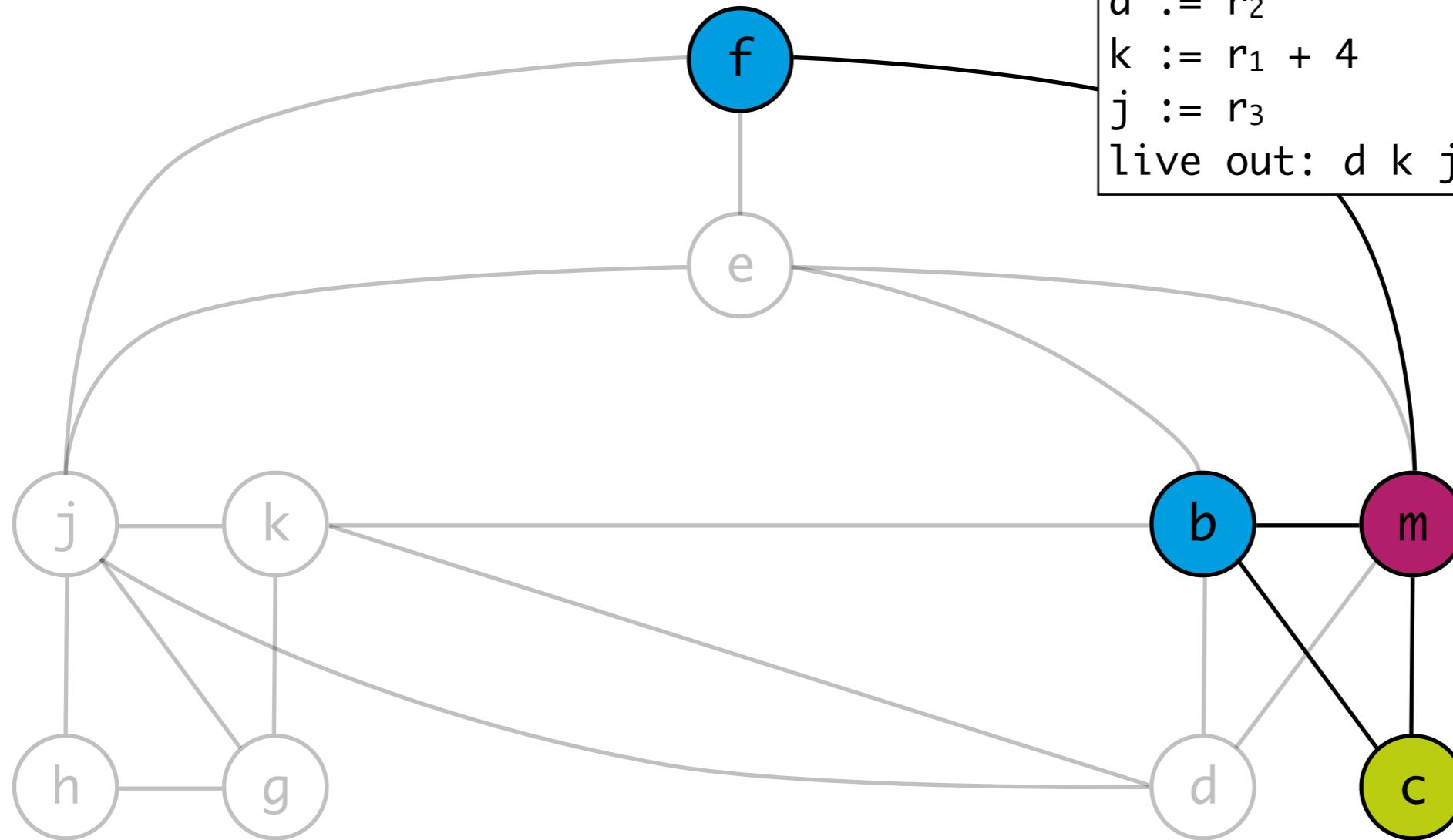
```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
r1 := mem[j + 16]
r3 := mem[f]
r2 := e + 8
d := r2
k := r1 + 4
j := r3
live out: d k j
```



Graph Coloring

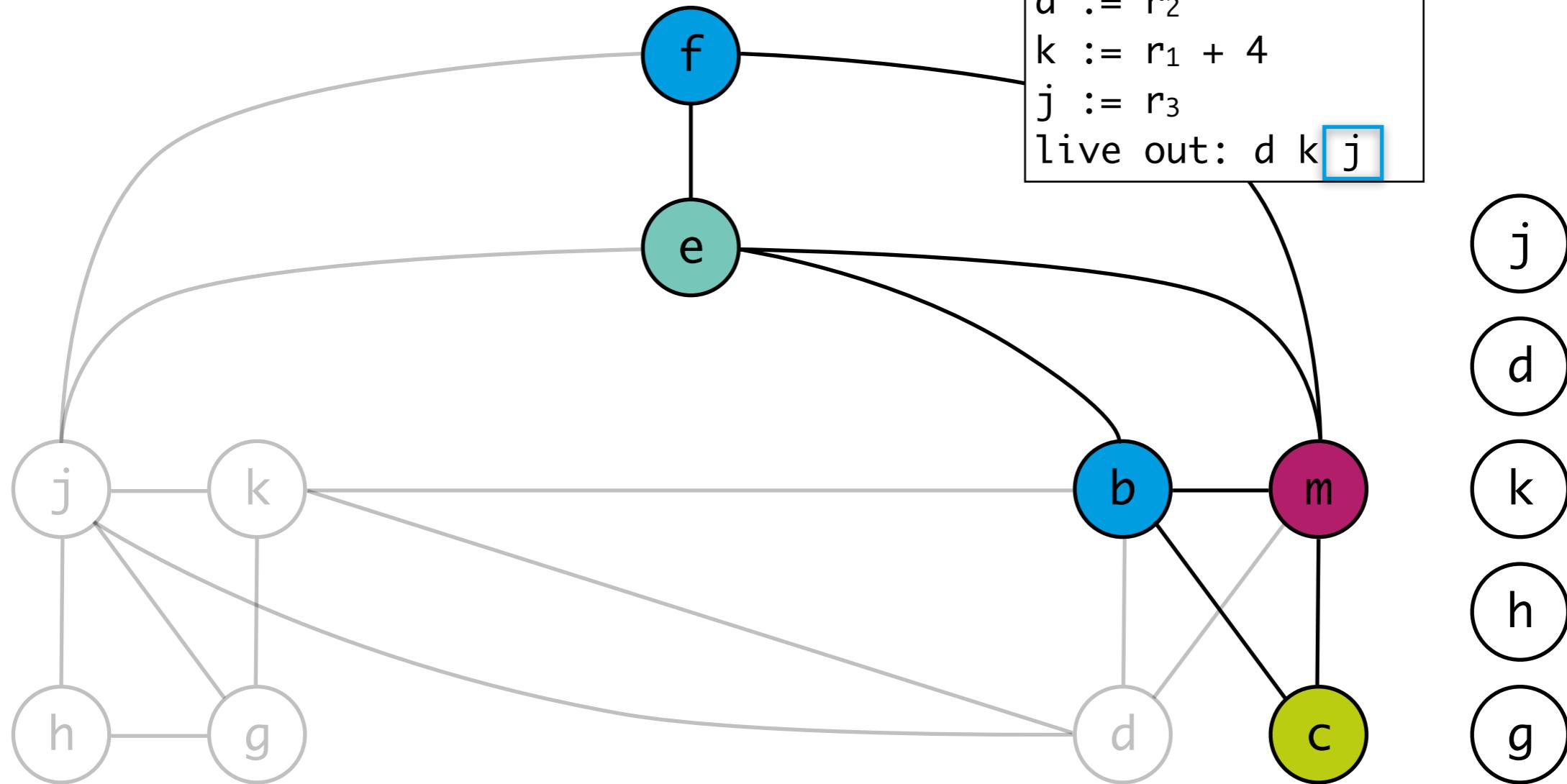
example with 4 colors

r_1
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 r_3
 r_4



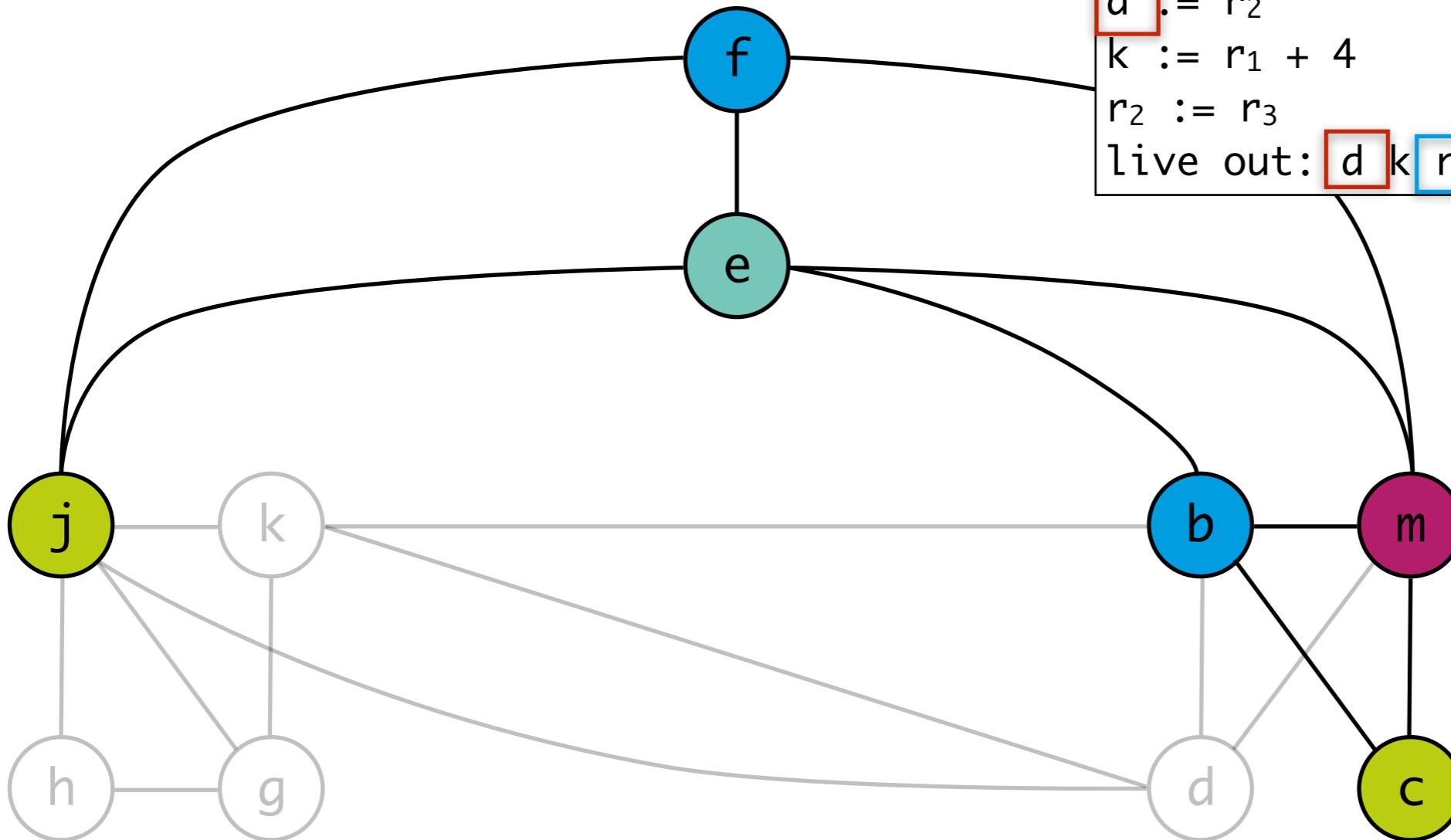
Graph Coloring example with 4 colors

r_1
 r_2
 r_3
 r_4



Graph Coloring example with 4 colors

r_1
 r_2
 r_3
 r_4

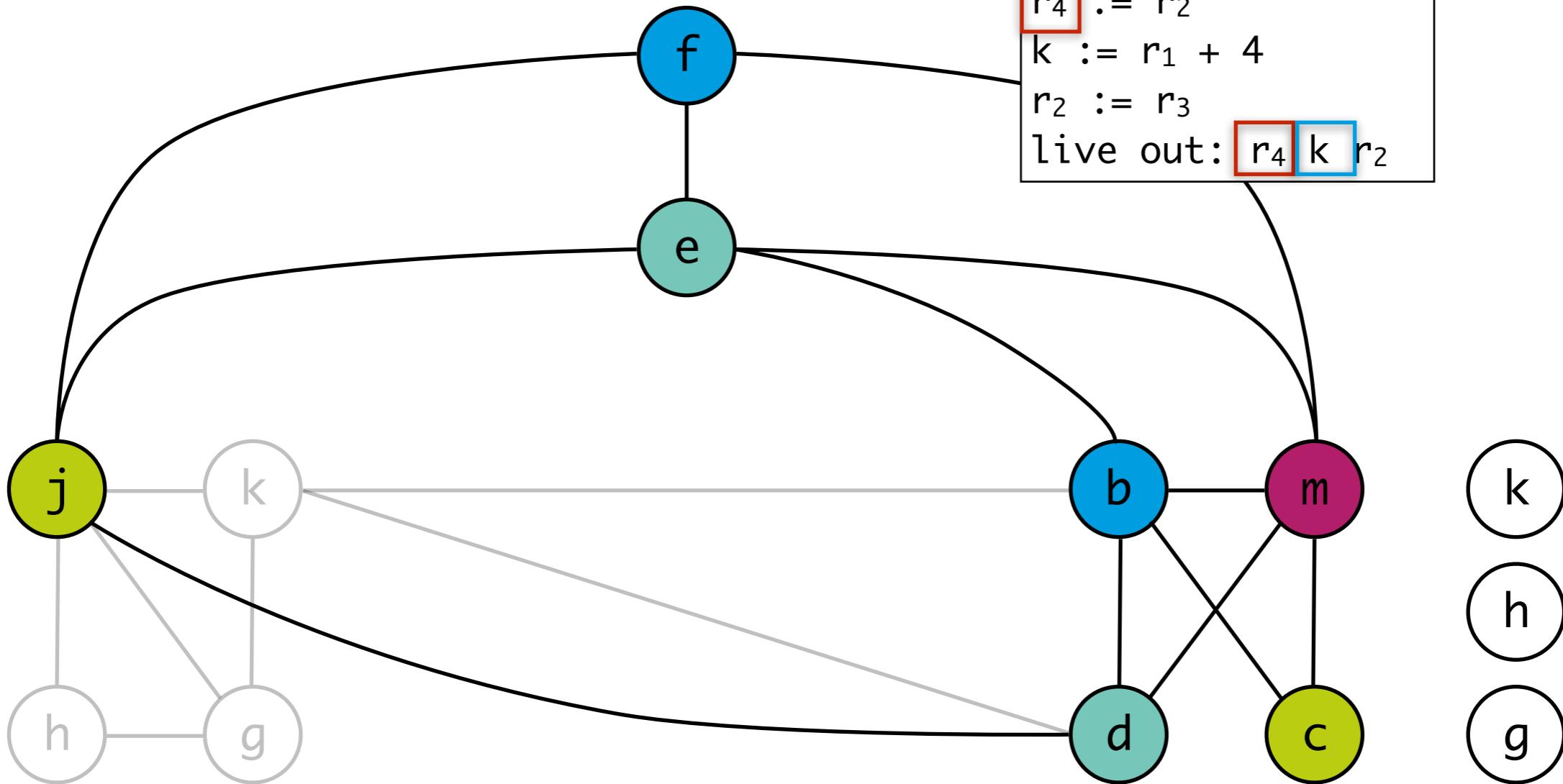


```
live-in: k r2
g := mem[r2] + 12
h := k - 1
r3 := g * h
r4 := mem[r2] + 8
r1 := mem[r2] + 16
r3 := mem[r3]
r2 := r4 + 8
d := r2
k := r1 + 4
r2 := r3
live out: d k r2
```

d
k
h
g

Graph Coloring example with 4 colors

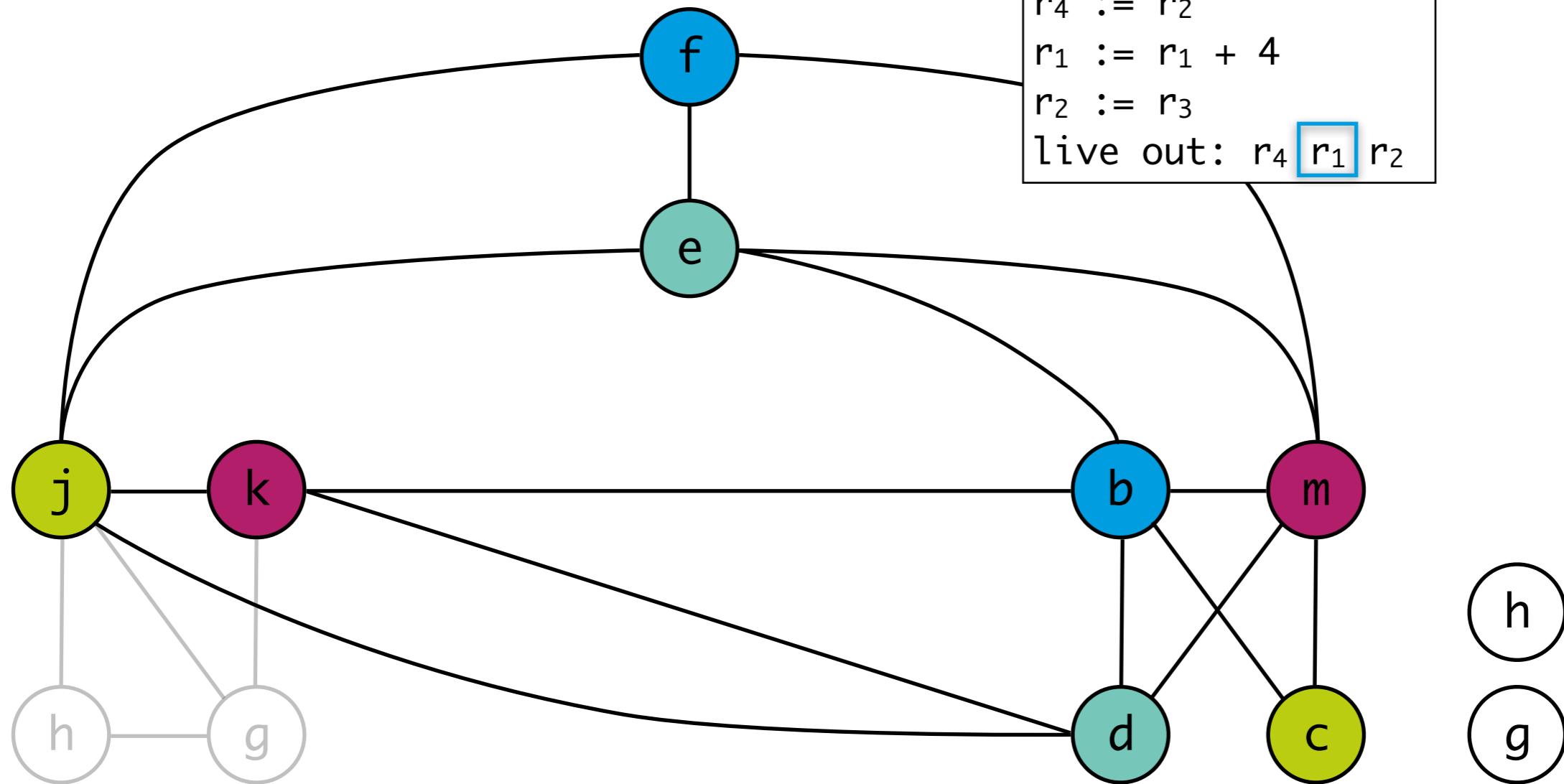
r_1
 r_2
 r_3
 r_4



Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4

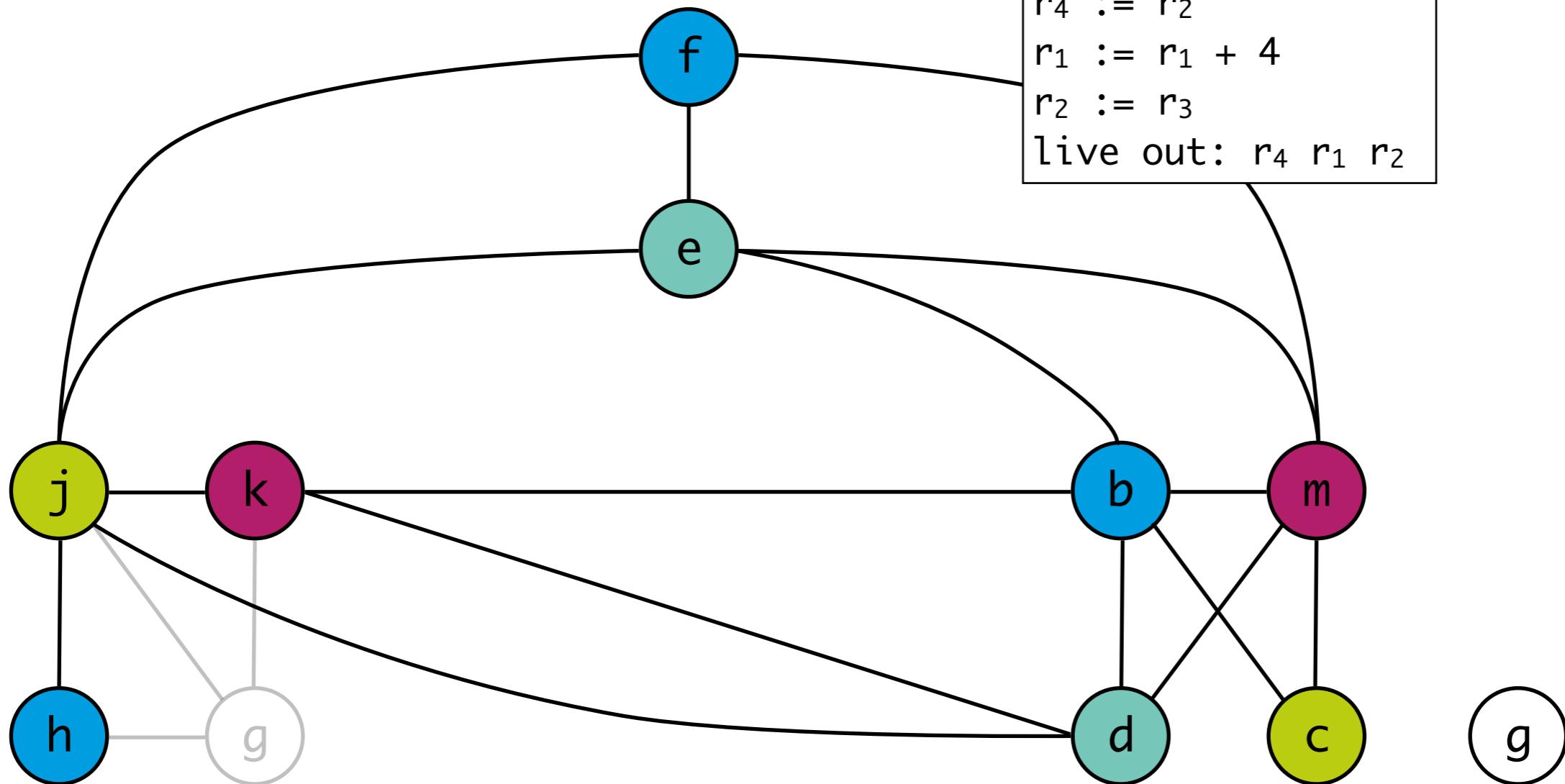


```
live-in: r1 r2
g := mem[r2 + 12]
h := r1 - 1
r3 := g * h
r4 := mem[r2 + 8]
r1 := mem[r2 + 16]
r3 := mem[r3]
r2 := r4 + 8
r4 := r2
r1 := r1 + 4
r2 := r3
live out: r4 r1 r2
```

Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4

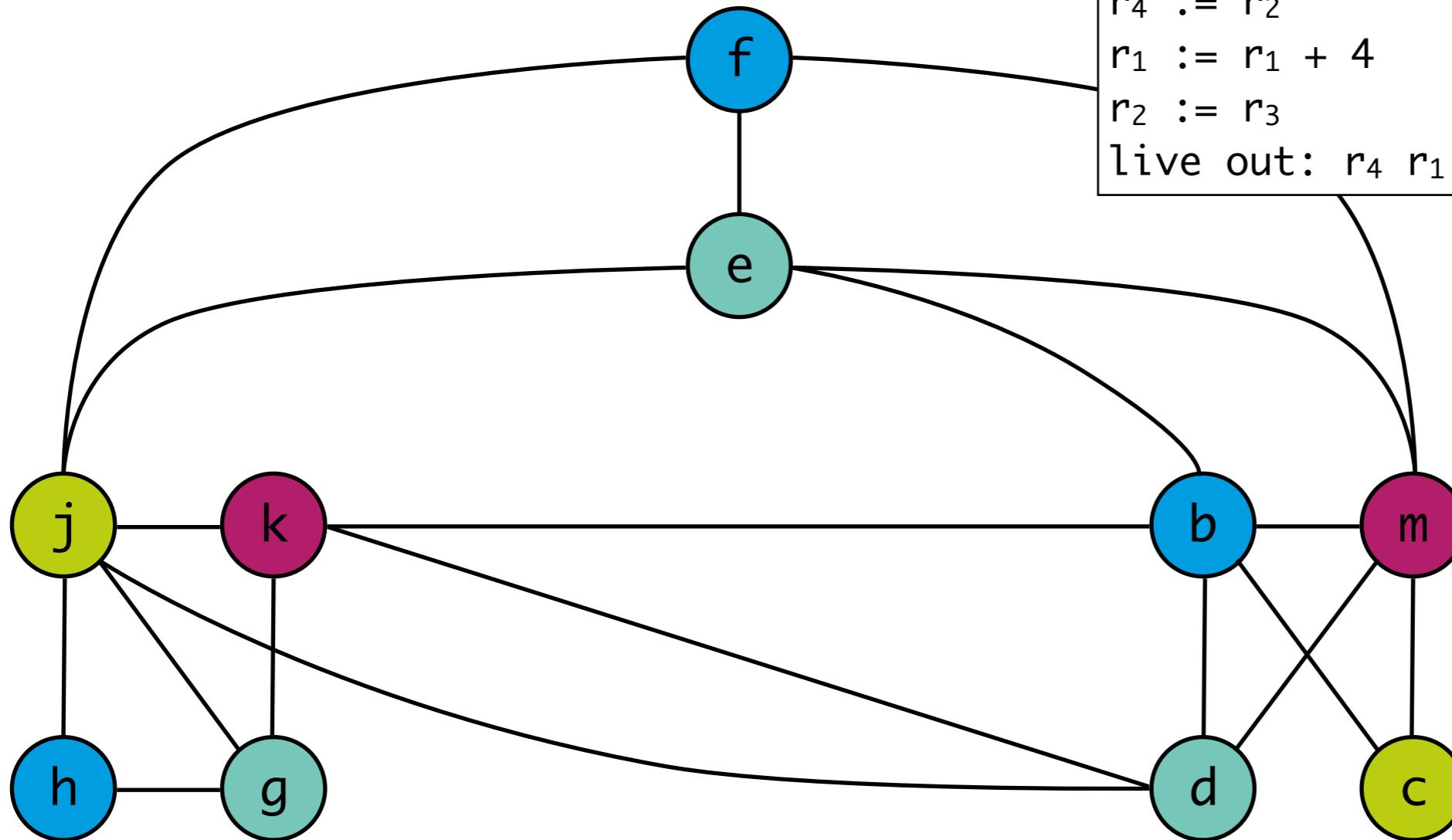


```
live-in: r1 r2
g := mem[r2 + 12]
r3 := r1 - 1
r3 := g * r3
r4 := mem[r2 + 8]
r1 := mem[r2 + 16]
r3 := mem[r3]
r2 := r4 + 8
r4 := r2
r1 := r1 + 4
r2 := r3
live out: r4 r1 r2
```

Graph Coloring

example with 4 colors

r_1
 r_2
 r_3
 r_4



```
live-in: r1 r2
r4 := mem[r2 + 12]
r3 := r1 - 1
r3 := r4 * r3
r4 := mem[r2 + 8]
r1 := mem[r2 + 16]
r3 := mem[r3]
r2 := r4 + 8
r4 := r2
r1 := r1 + 4
r2 := r3
live out: r4 r1 r2
```

III

Spilling

Optimistic Coloring

steps

Simplify

remove node of **insignificant** degree (fewer than k edges)

Spill

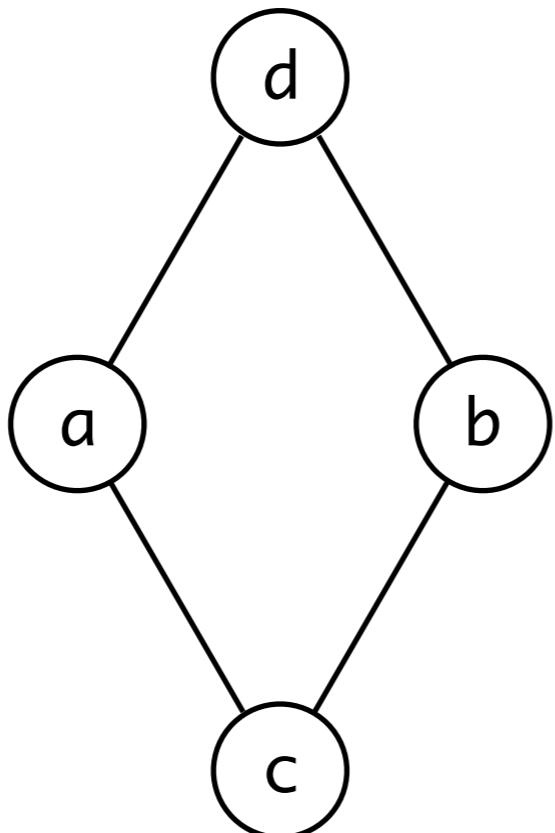
remove node of **significant** degree (k or more edges)

Select

add node, select color

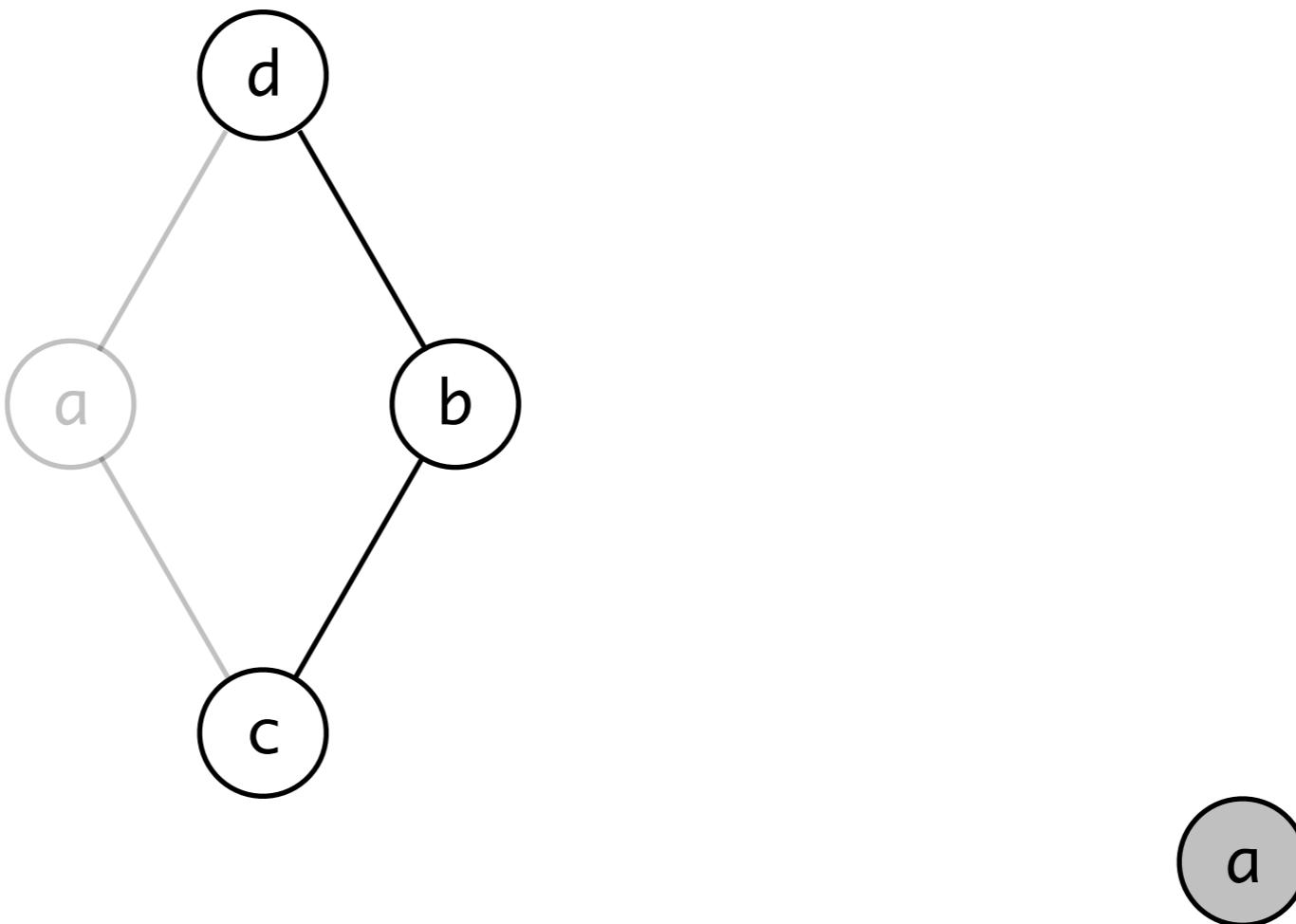
Optimistic Coloring

example with 2 colors



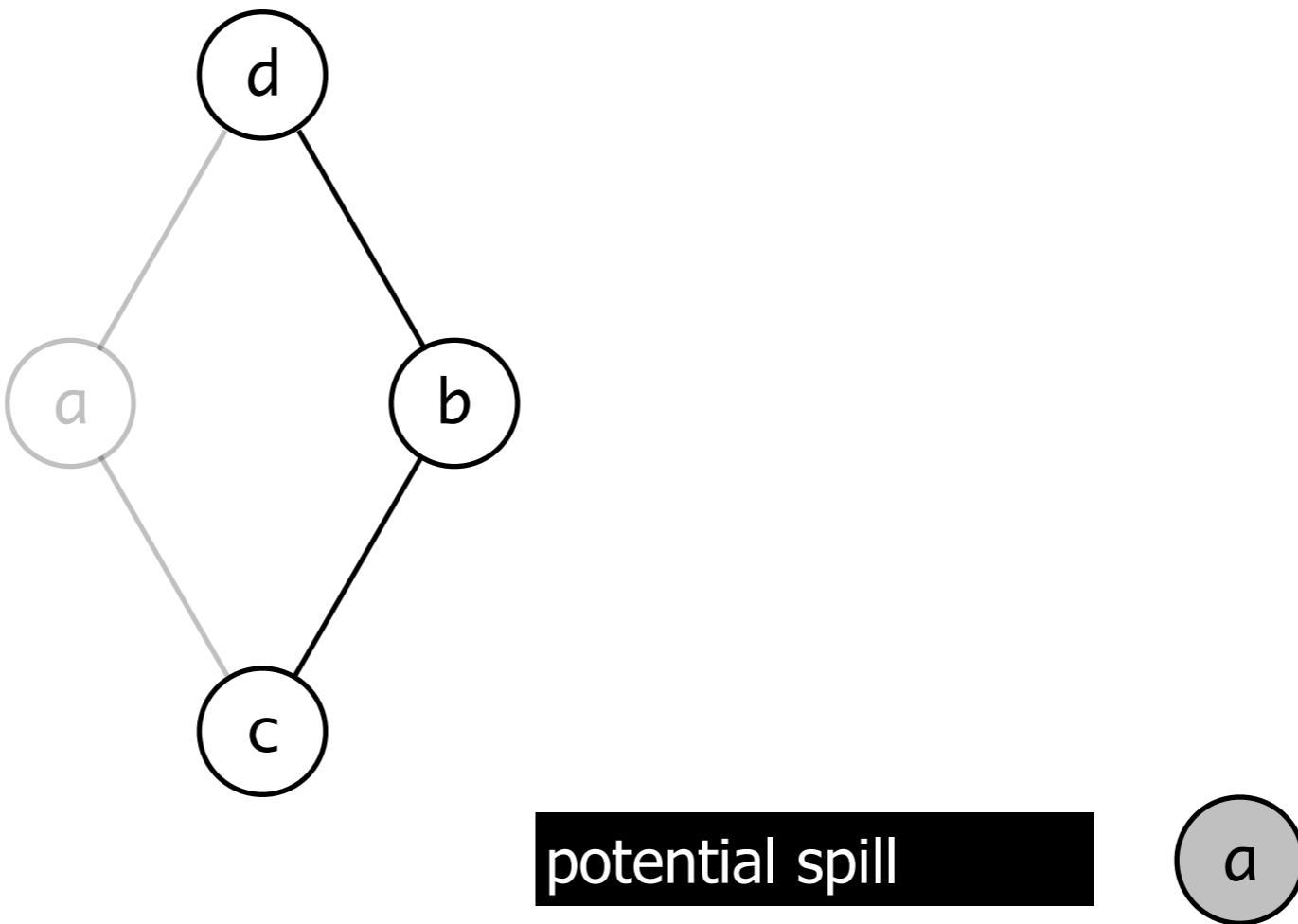
Optimistic Coloring

example with 2 colors



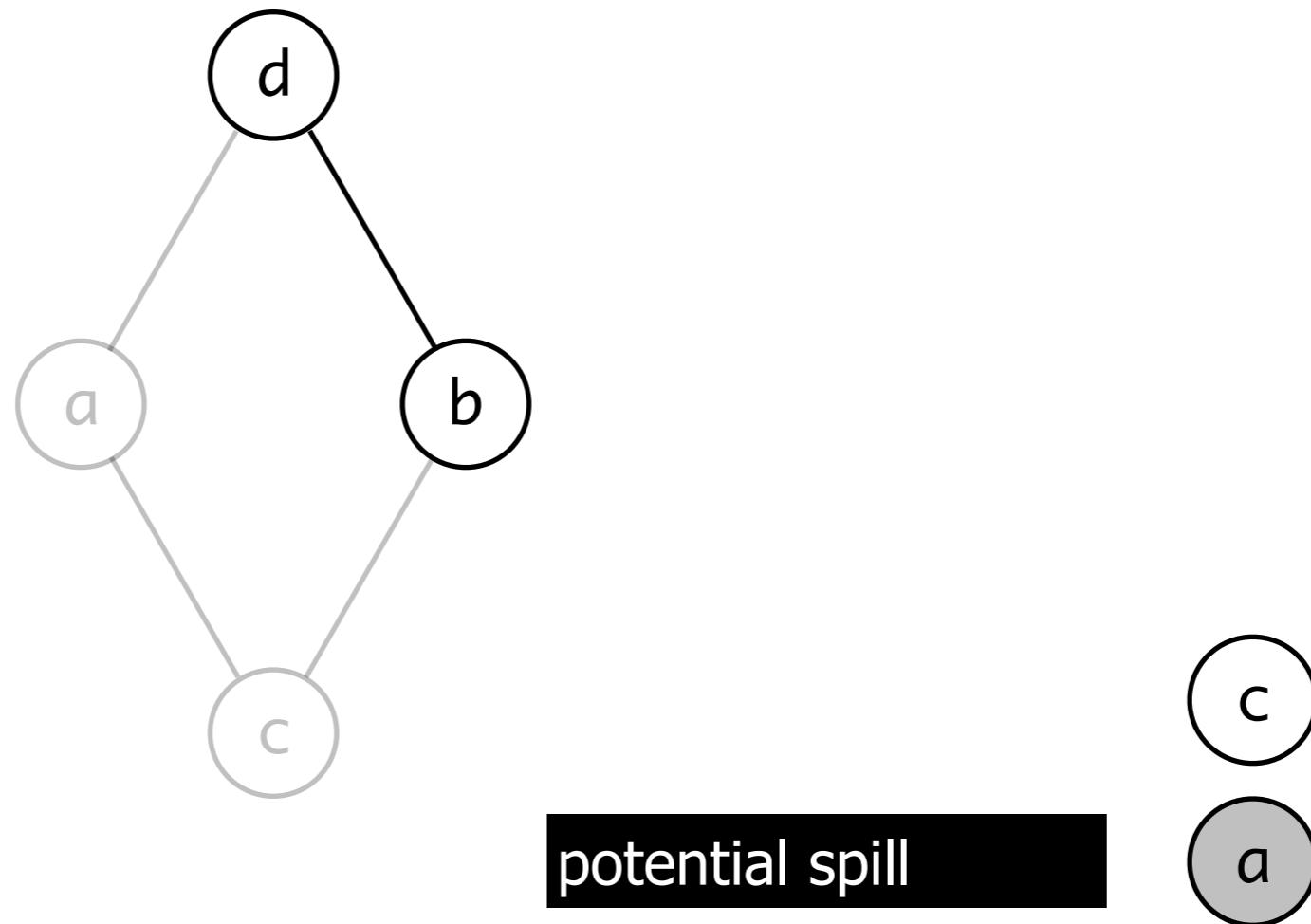
Optimistic Coloring

example with 2 colors



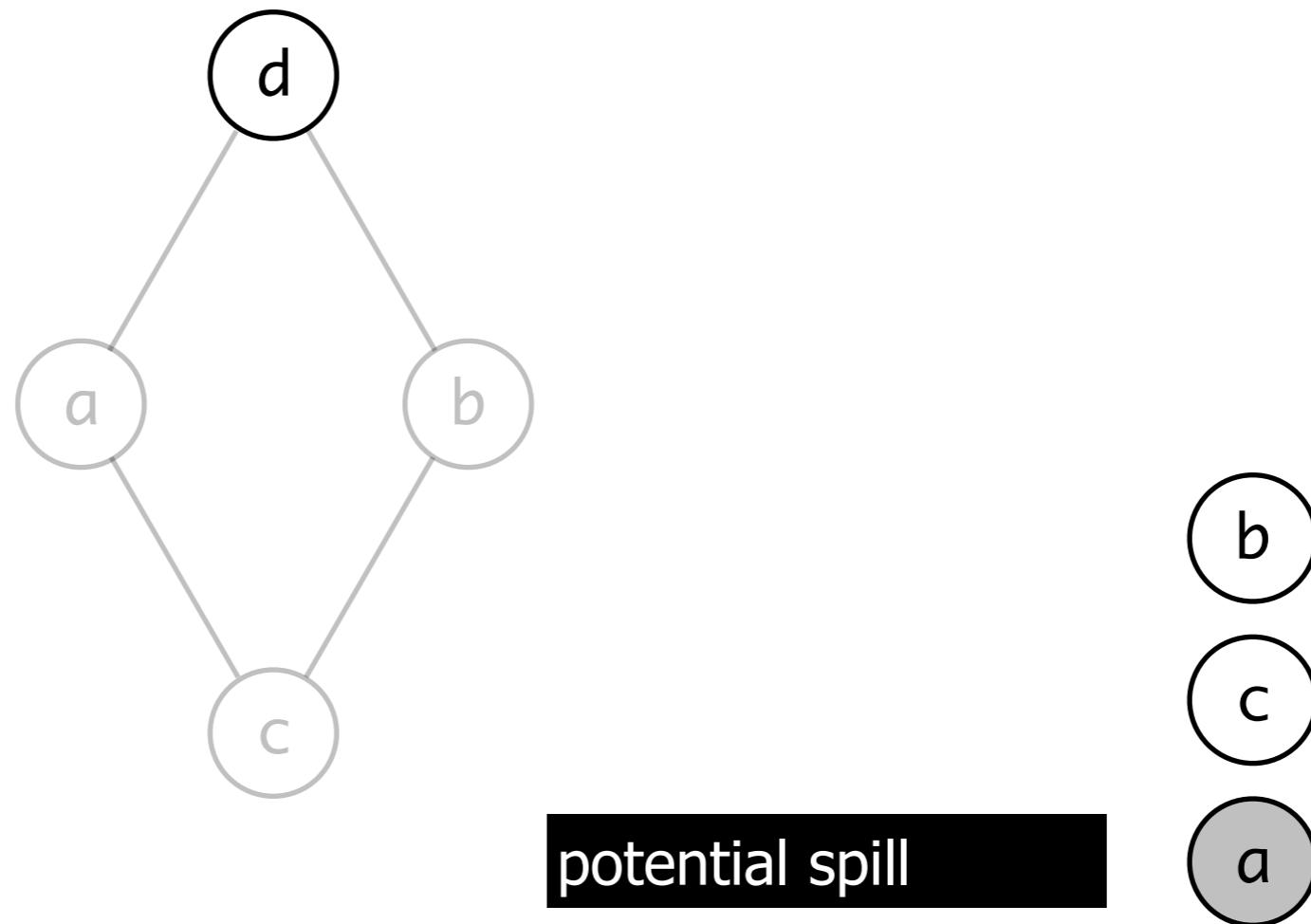
Optimistic Coloring

example with 2 colors



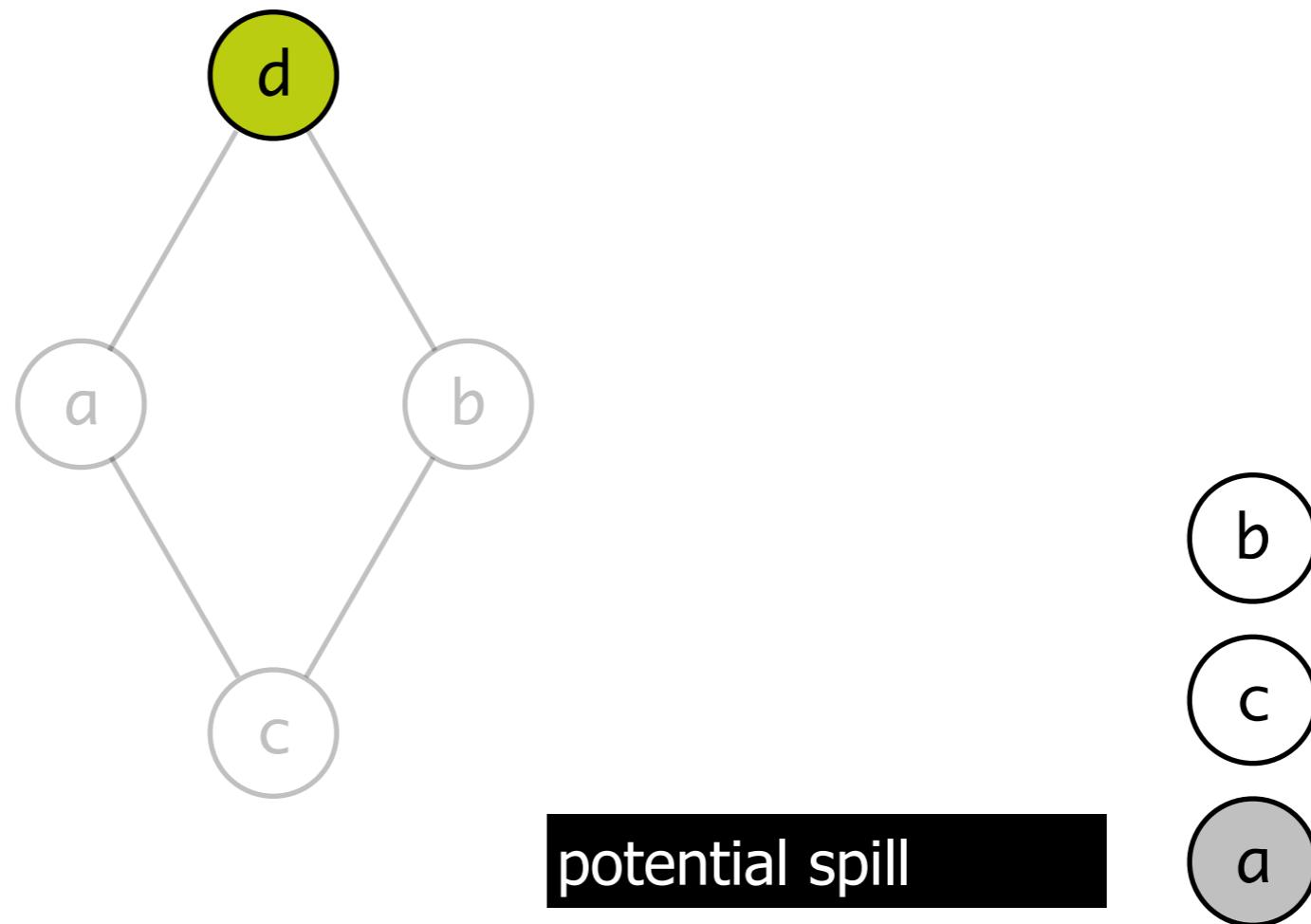
Optimistic Coloring

example with 2 colors



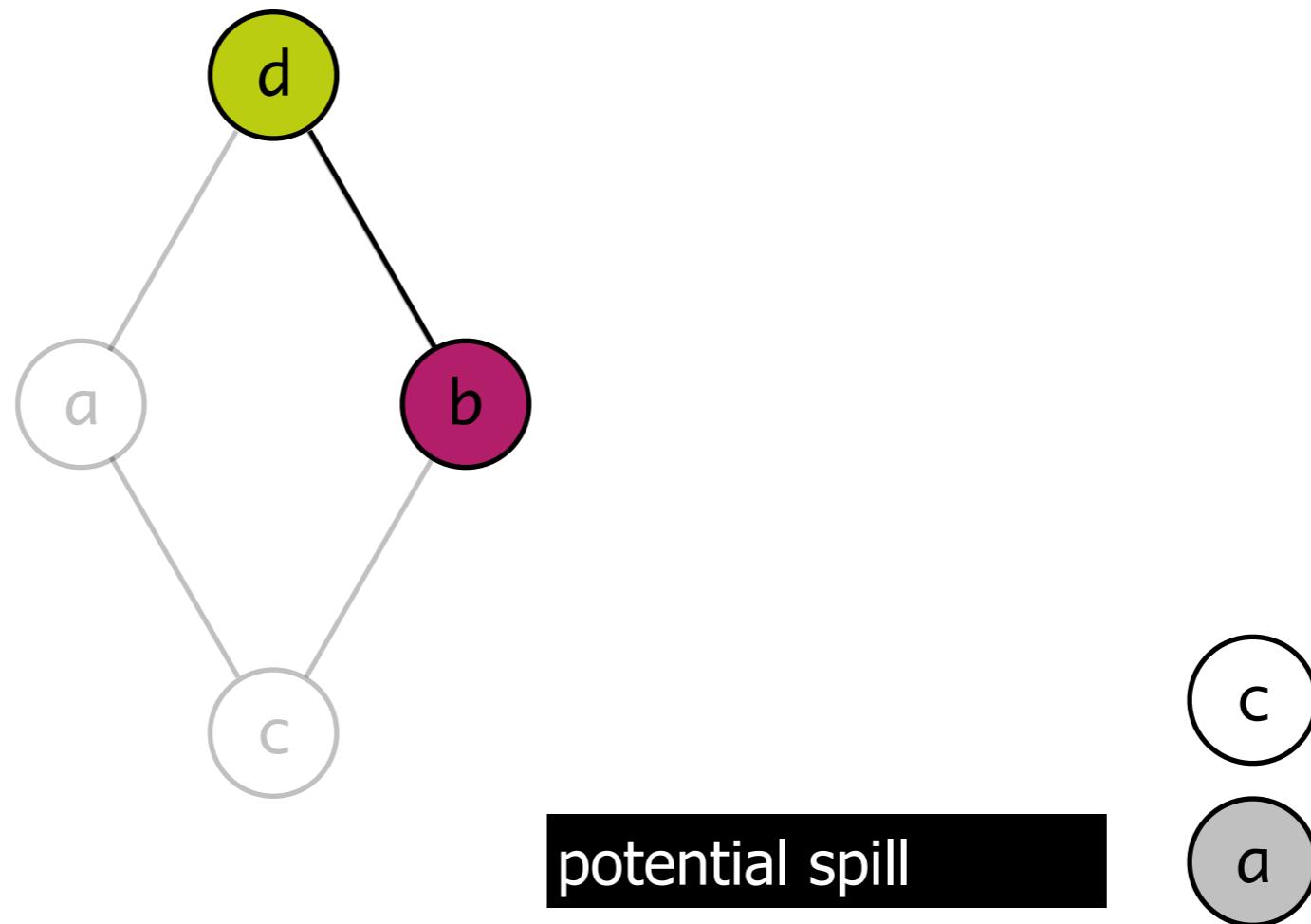
Optimistic Coloring

example with 2 colors



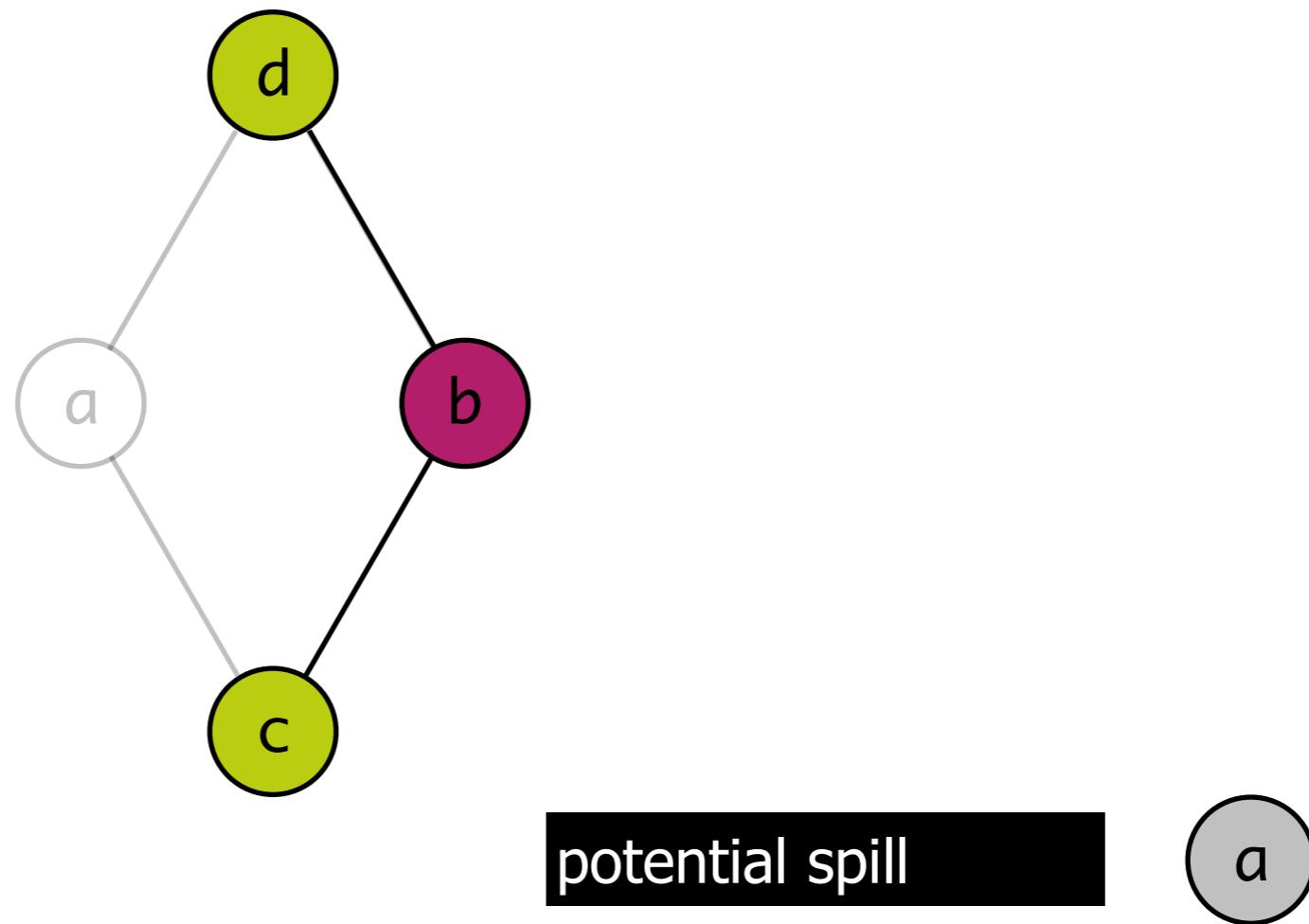
Optimistic Coloring

example with 2 colors



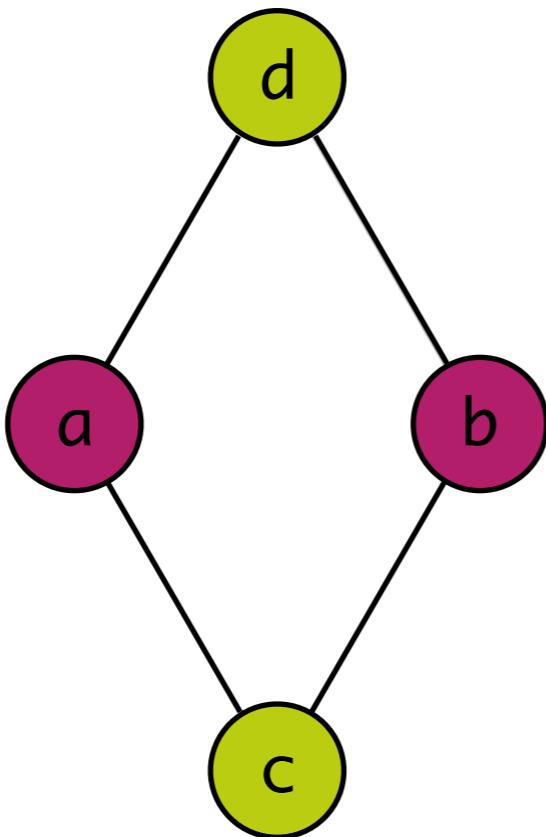
Optimistic Coloring

example with 2 colors



Optimistic Coloring

example with 2 colors



Spilling steps

Simplify

remove node of **insignificant** degree (less than k edges)

Spill

remove node of **significant** degree (k or more edges)

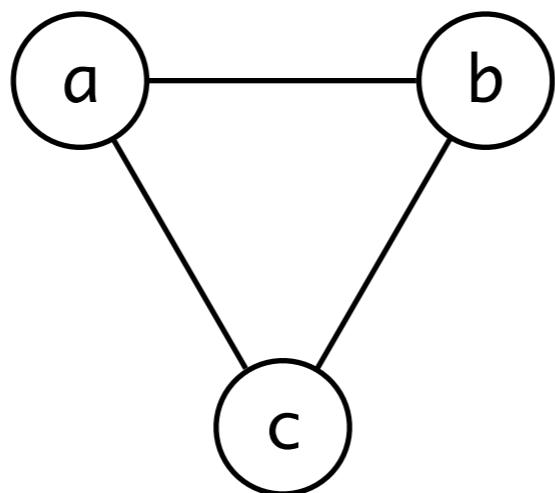
Select

add node, select color

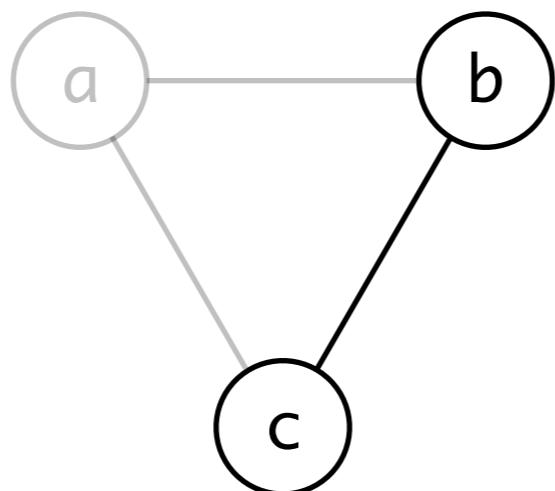
Actual spill

Start over

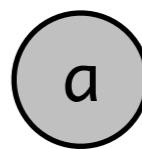
Spilling example with 2 colors



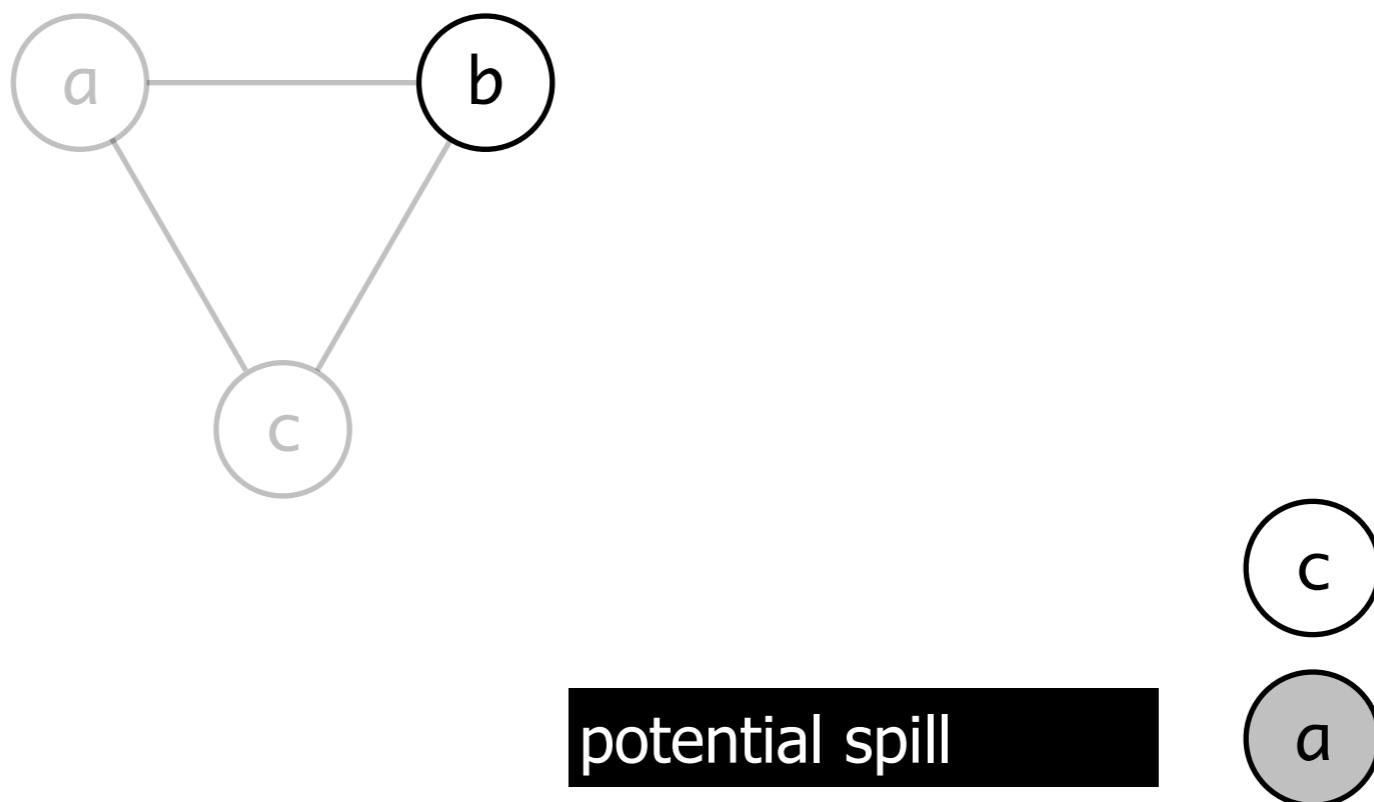
Spilling example with 2 colors



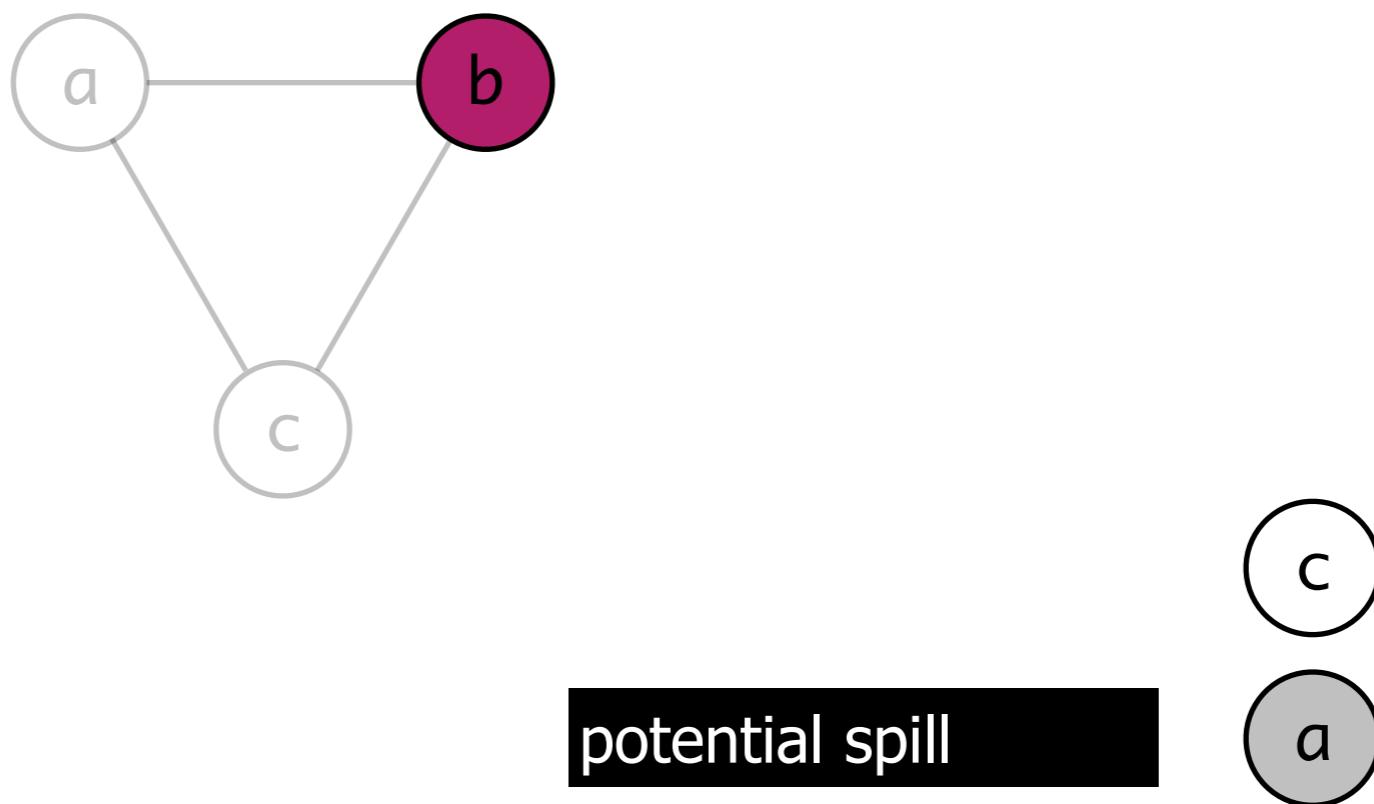
potential spill



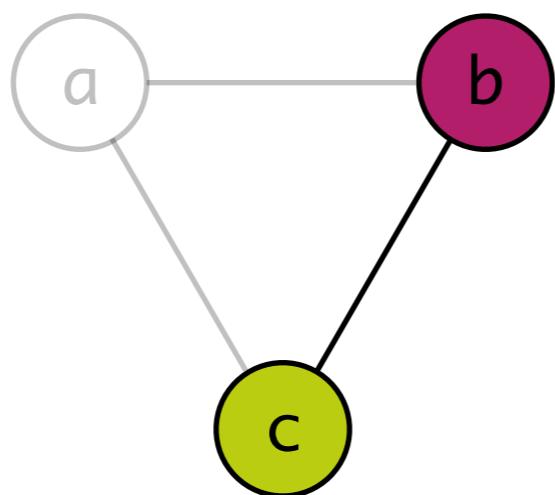
Spilling example with 2 colors



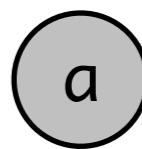
Spilling example with 2 colors



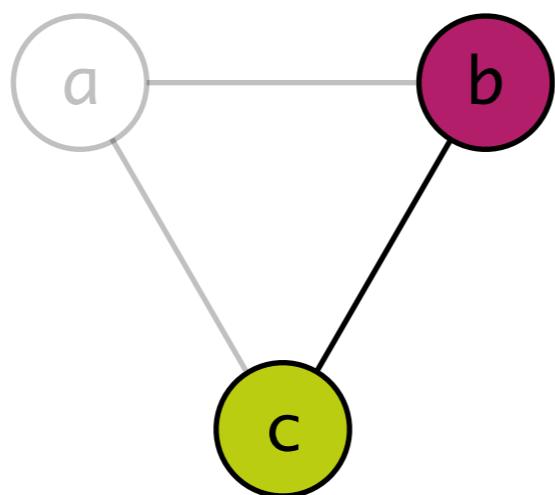
Spilling example with 2 colors



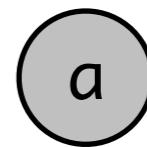
potential spill



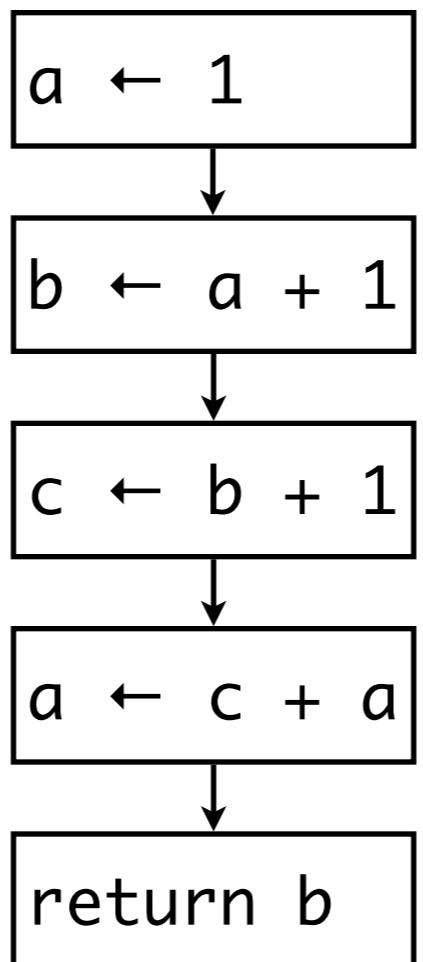
Spilling example with 2 colors



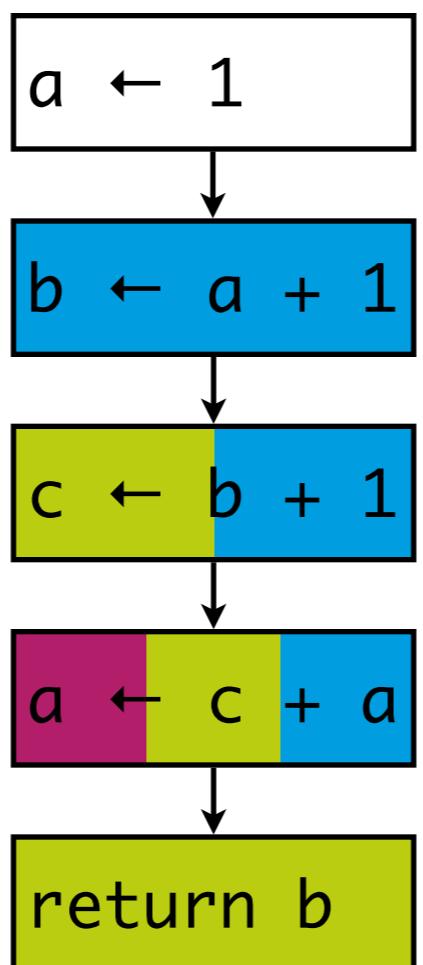
actual spill



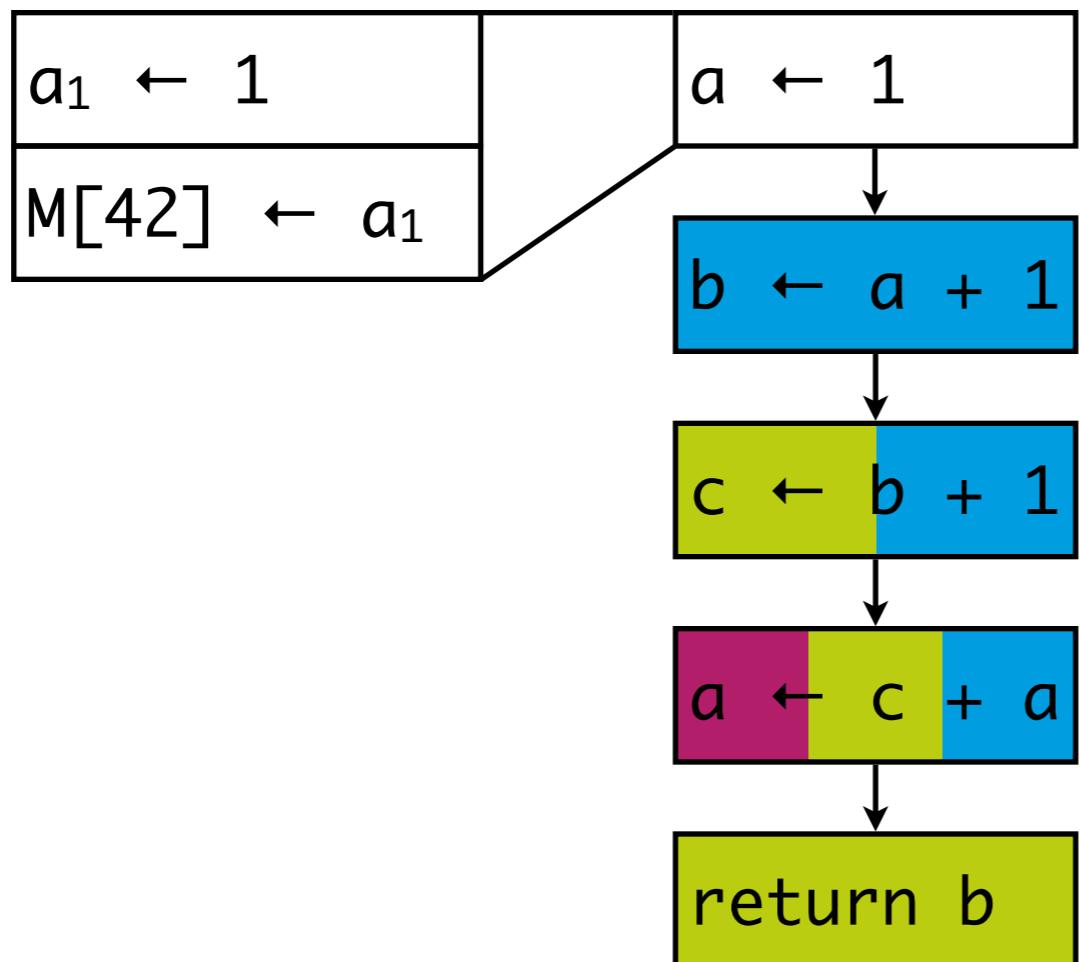
Spilling example



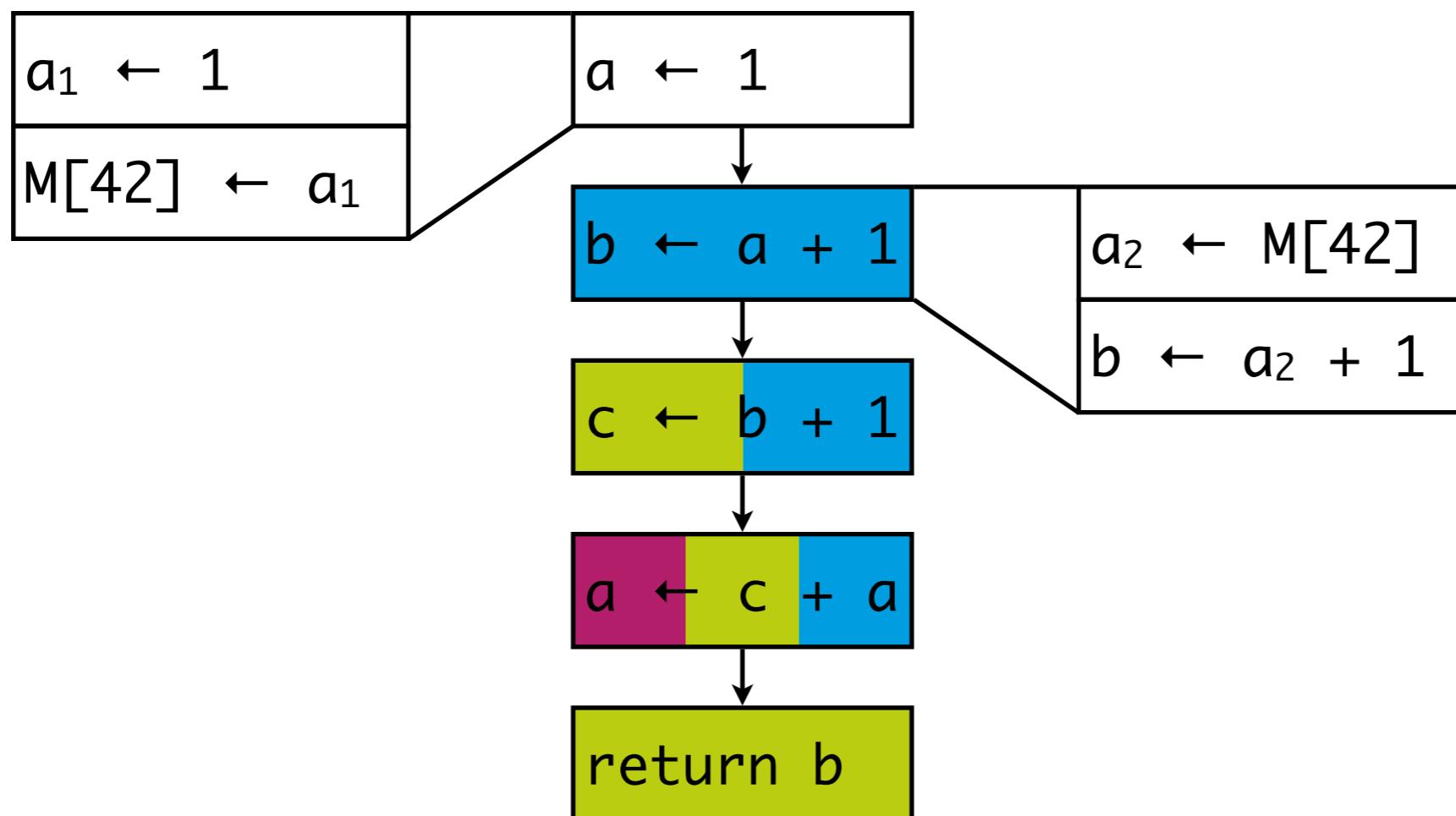
Spilling example



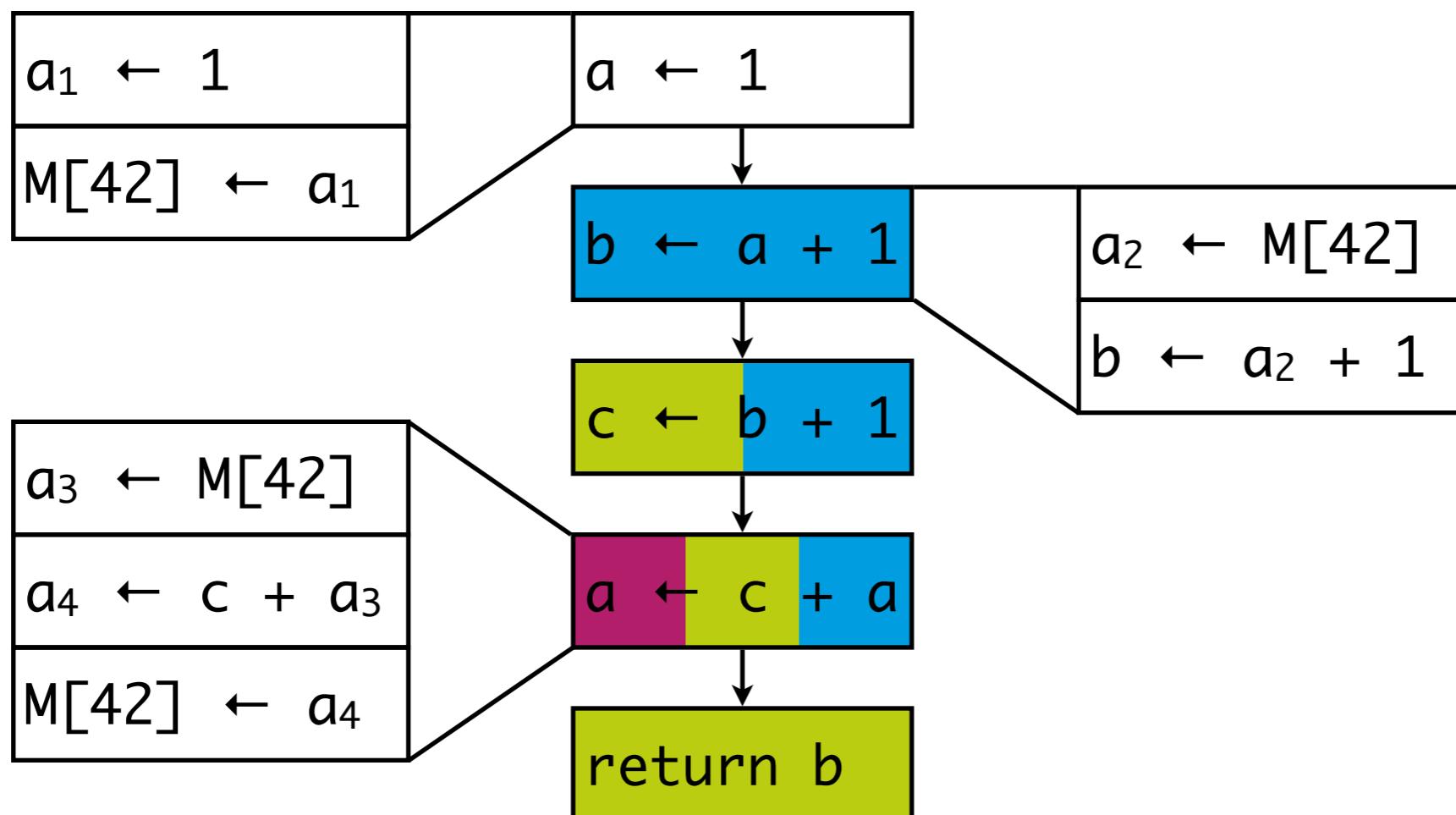
Spilling example



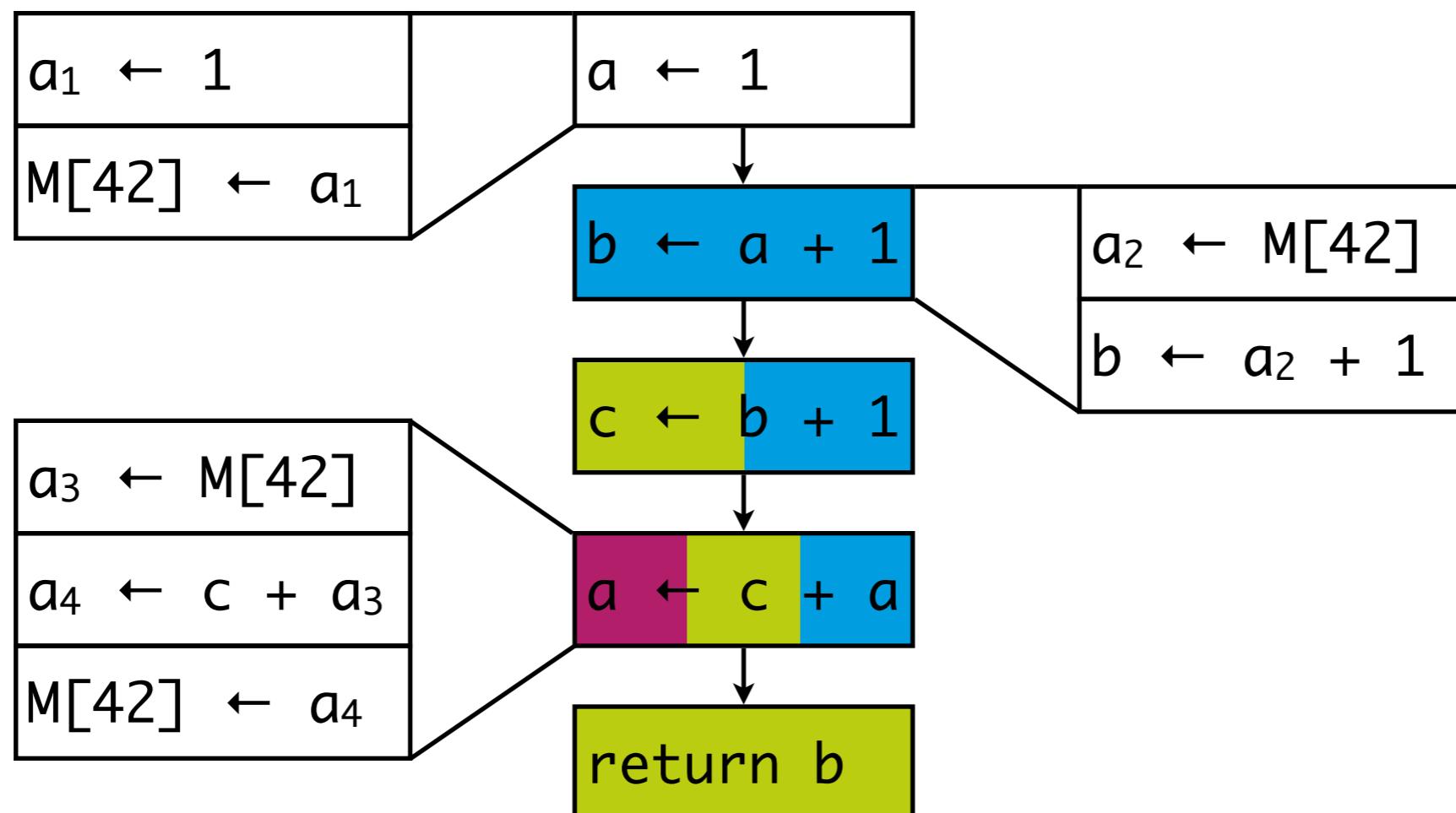
Spilling example



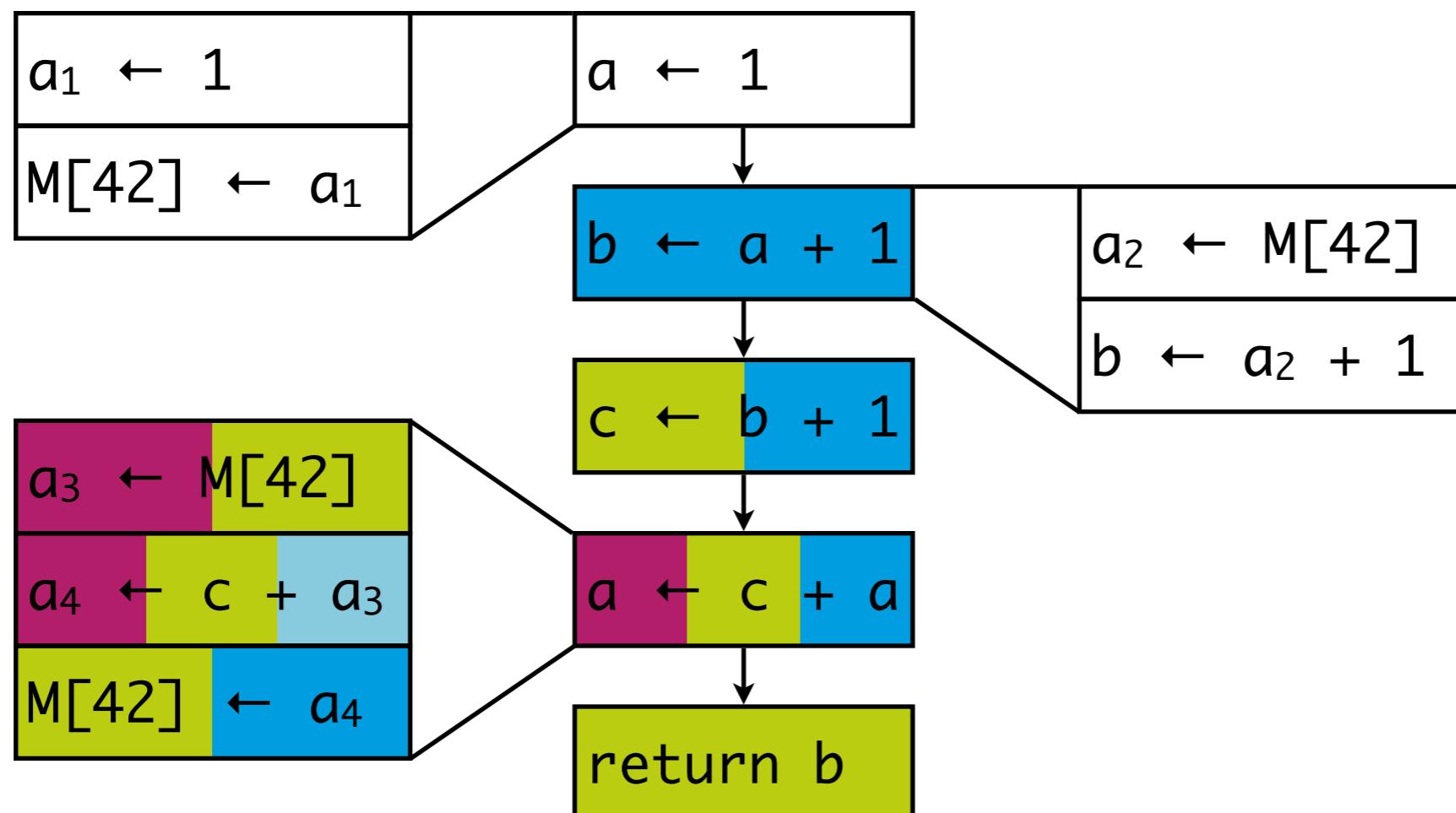
Spilling example



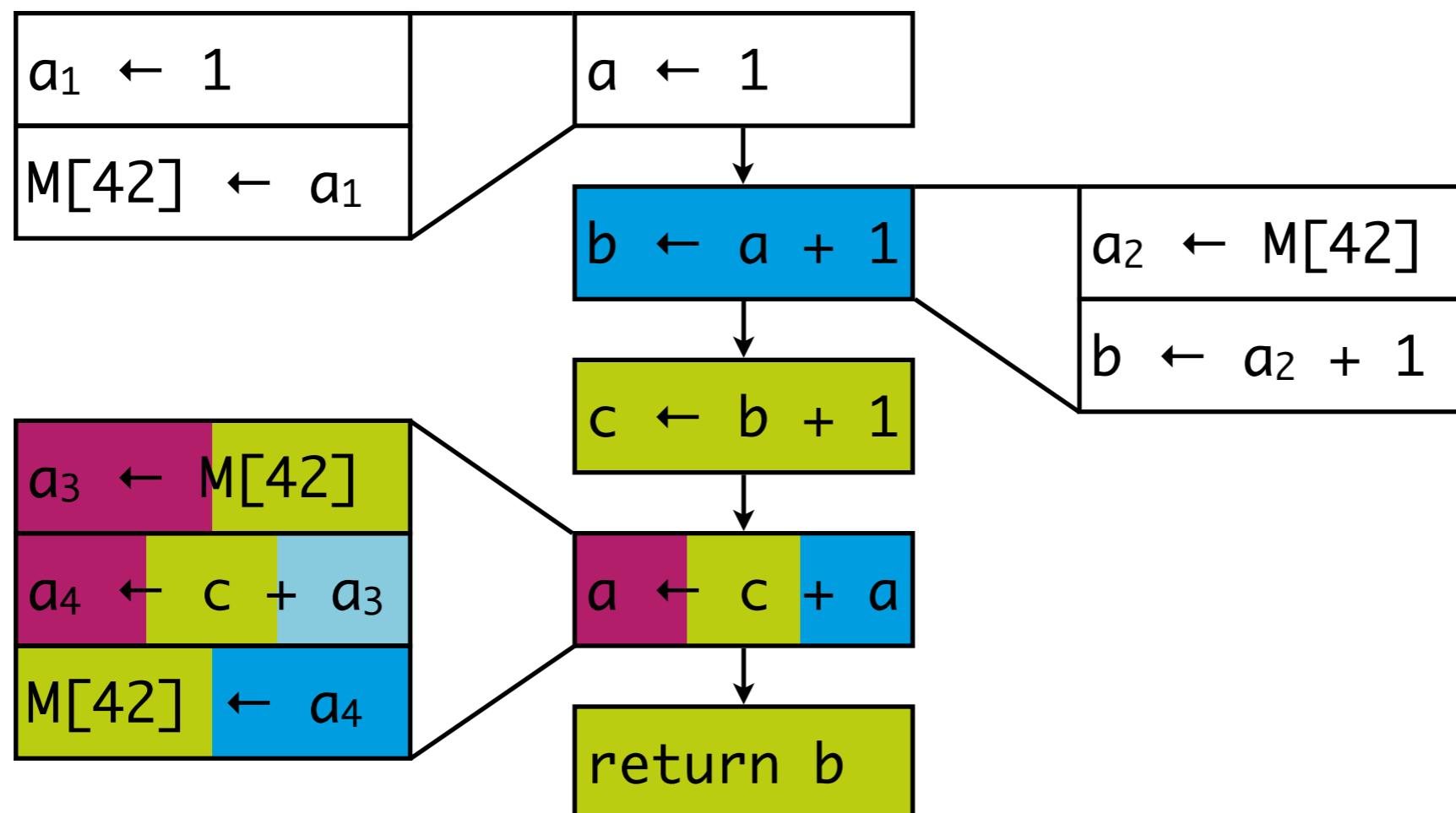
Spilling example



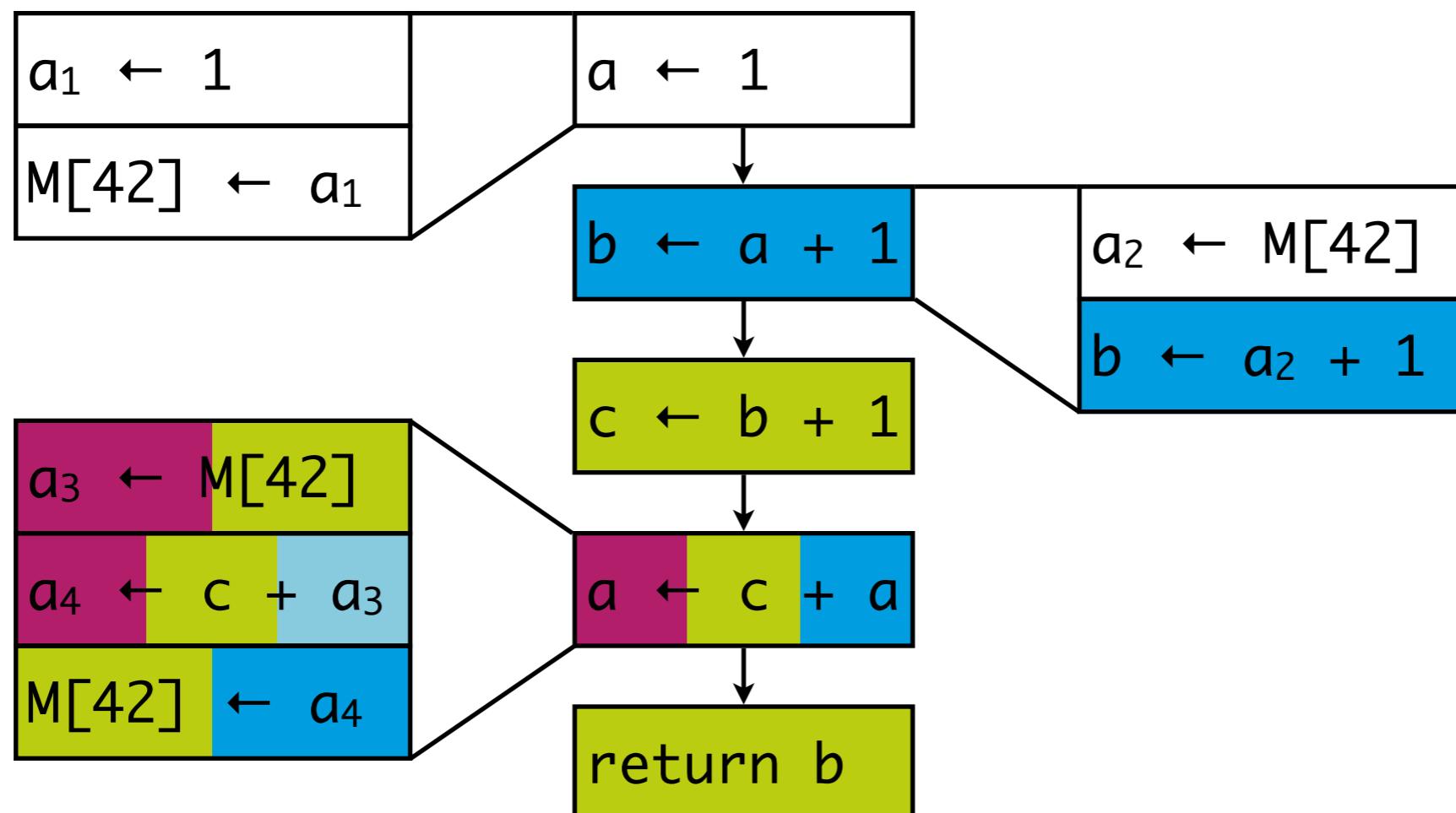
Spilling example



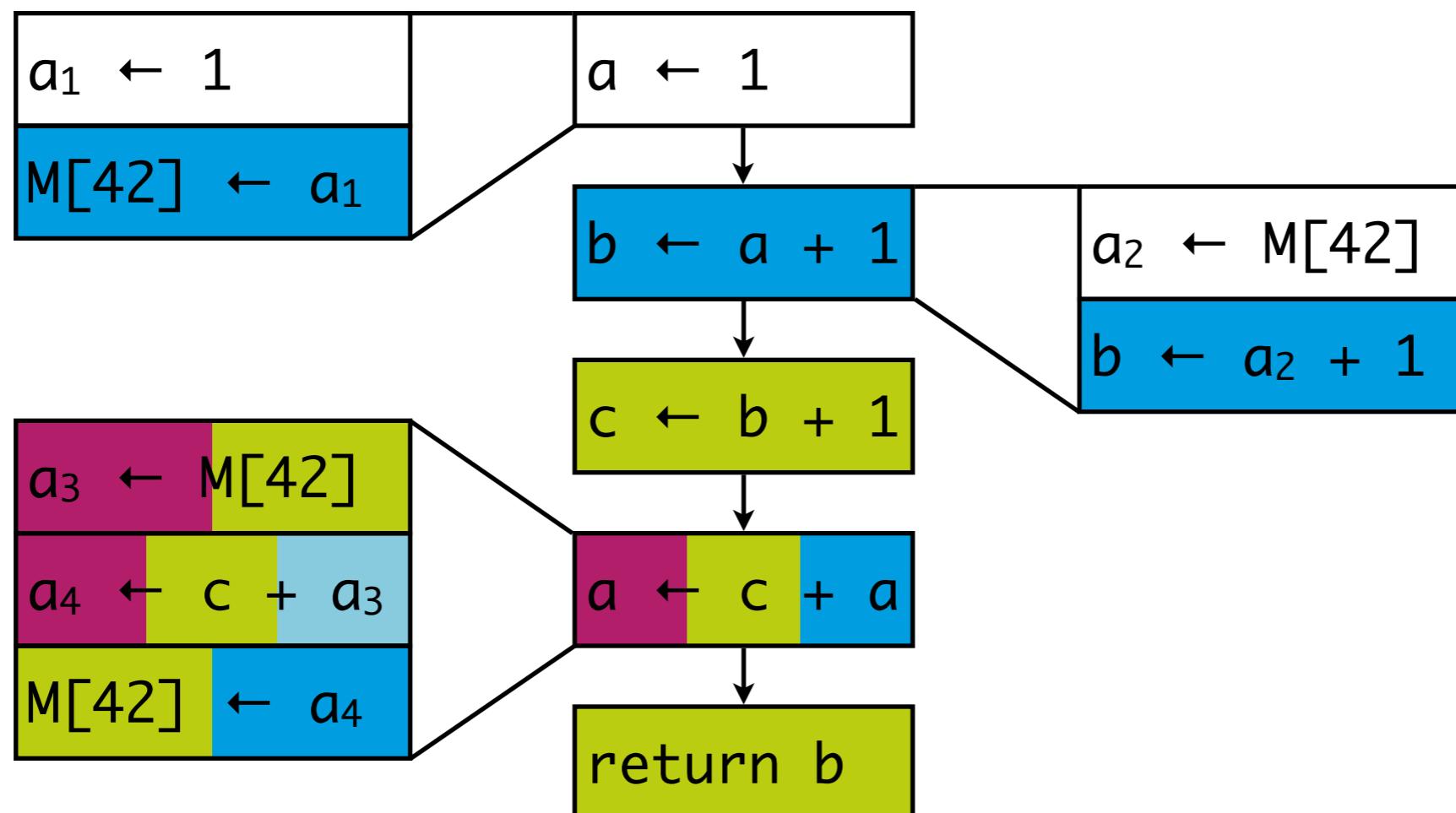
Spilling example



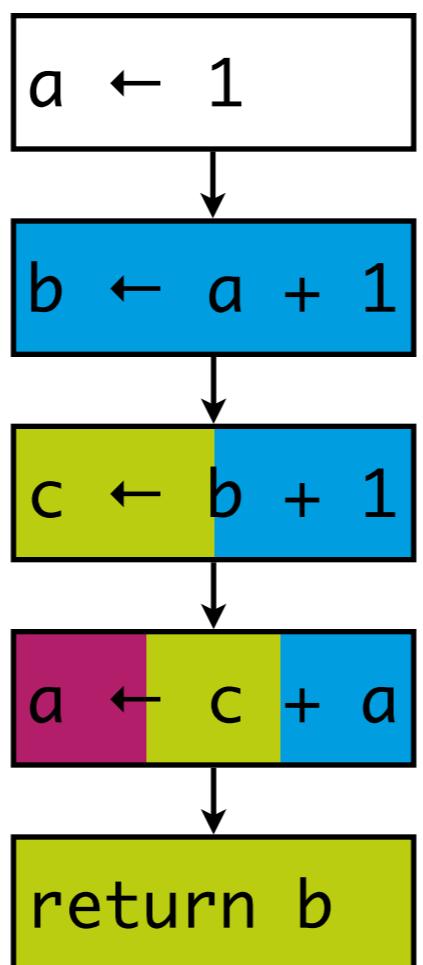
Spilling example



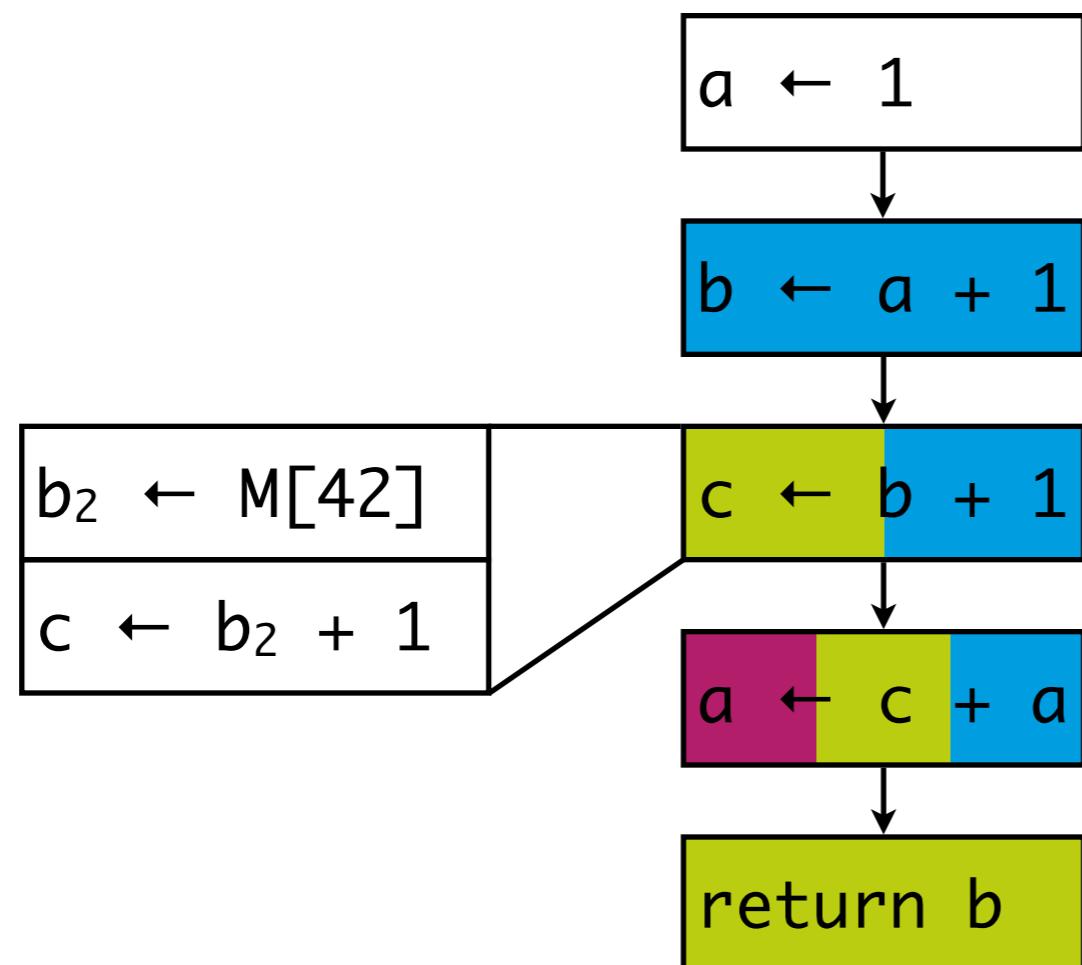
Spilling example



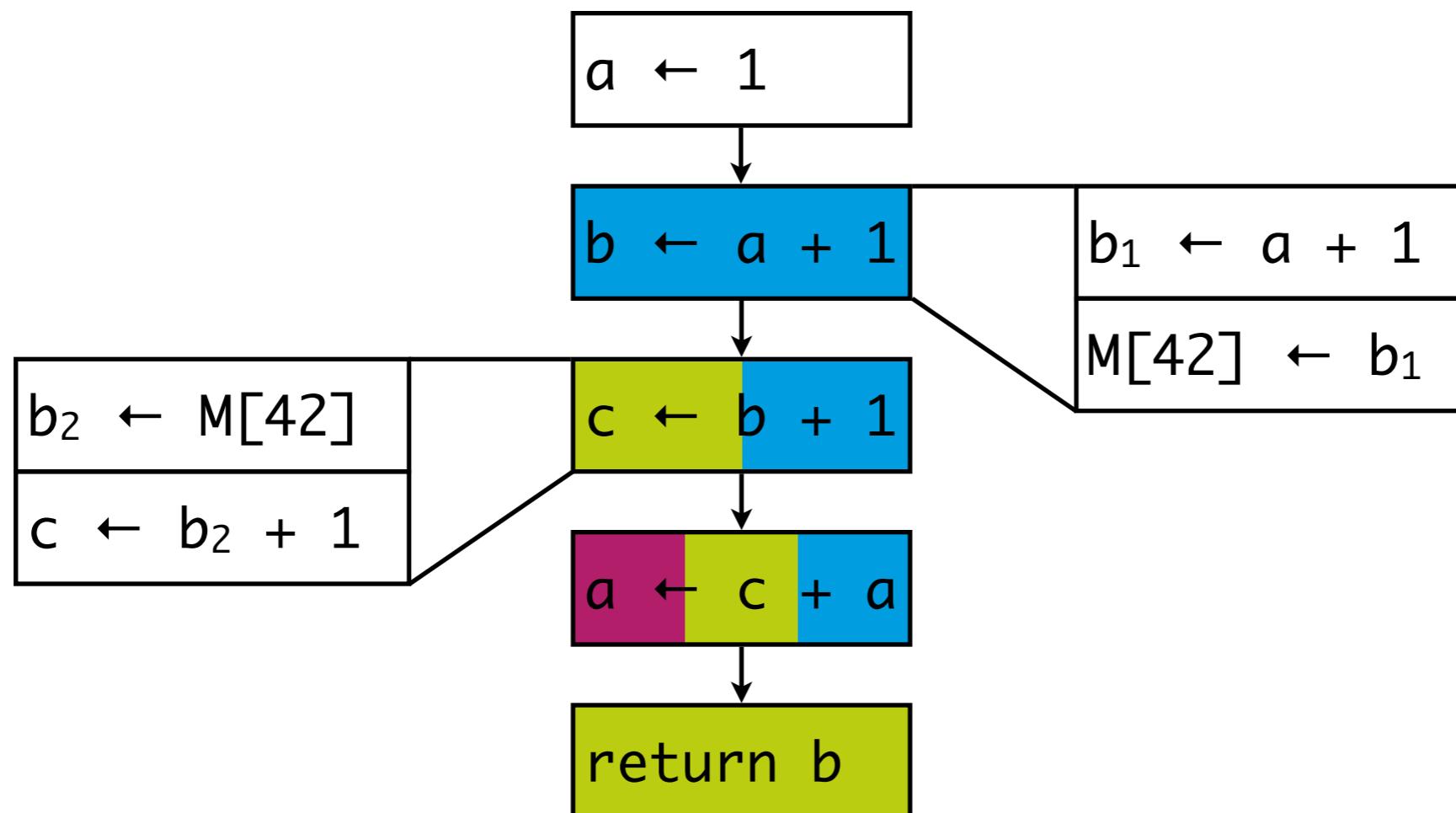
Spilling example



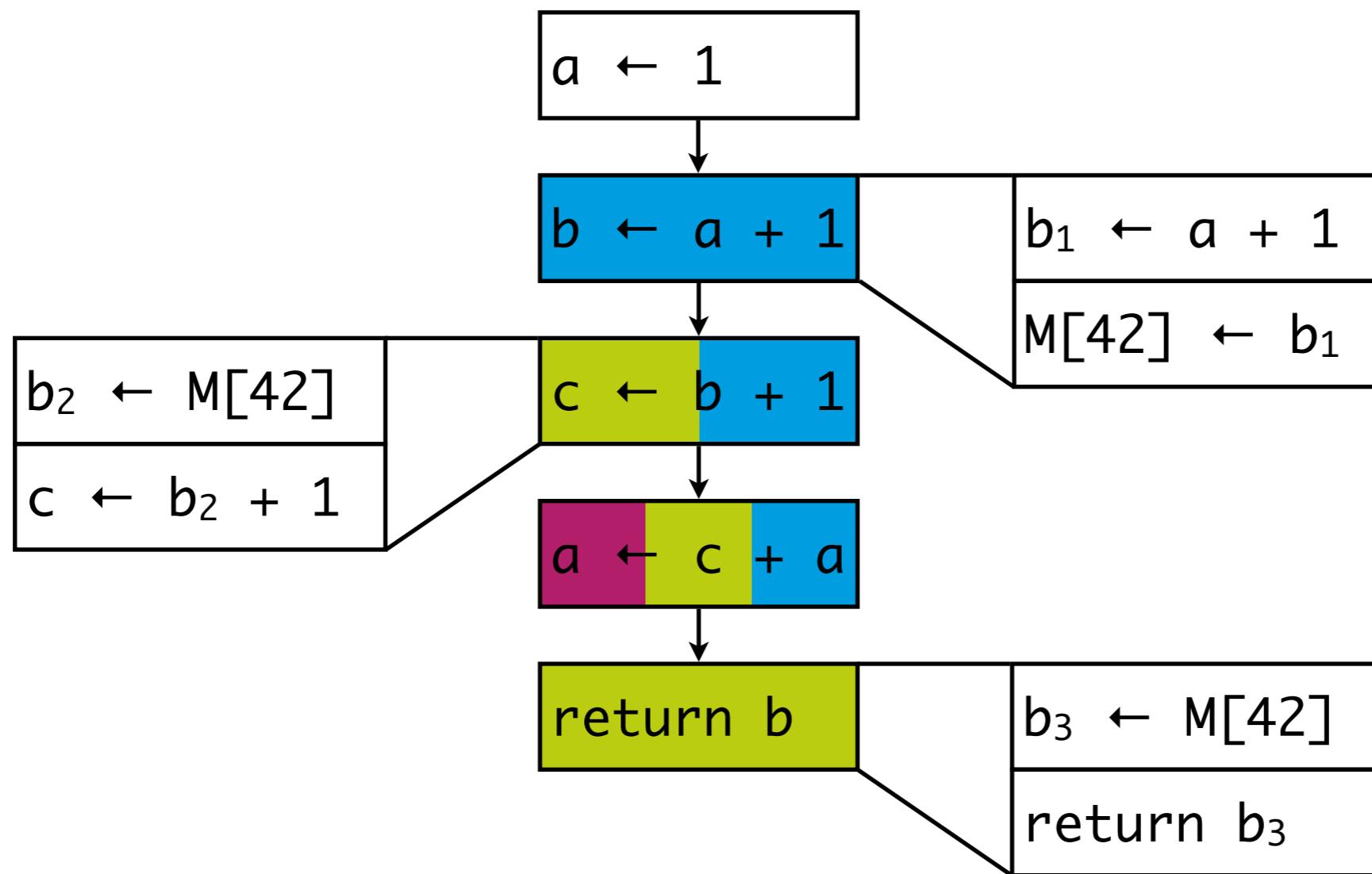
Spilling example



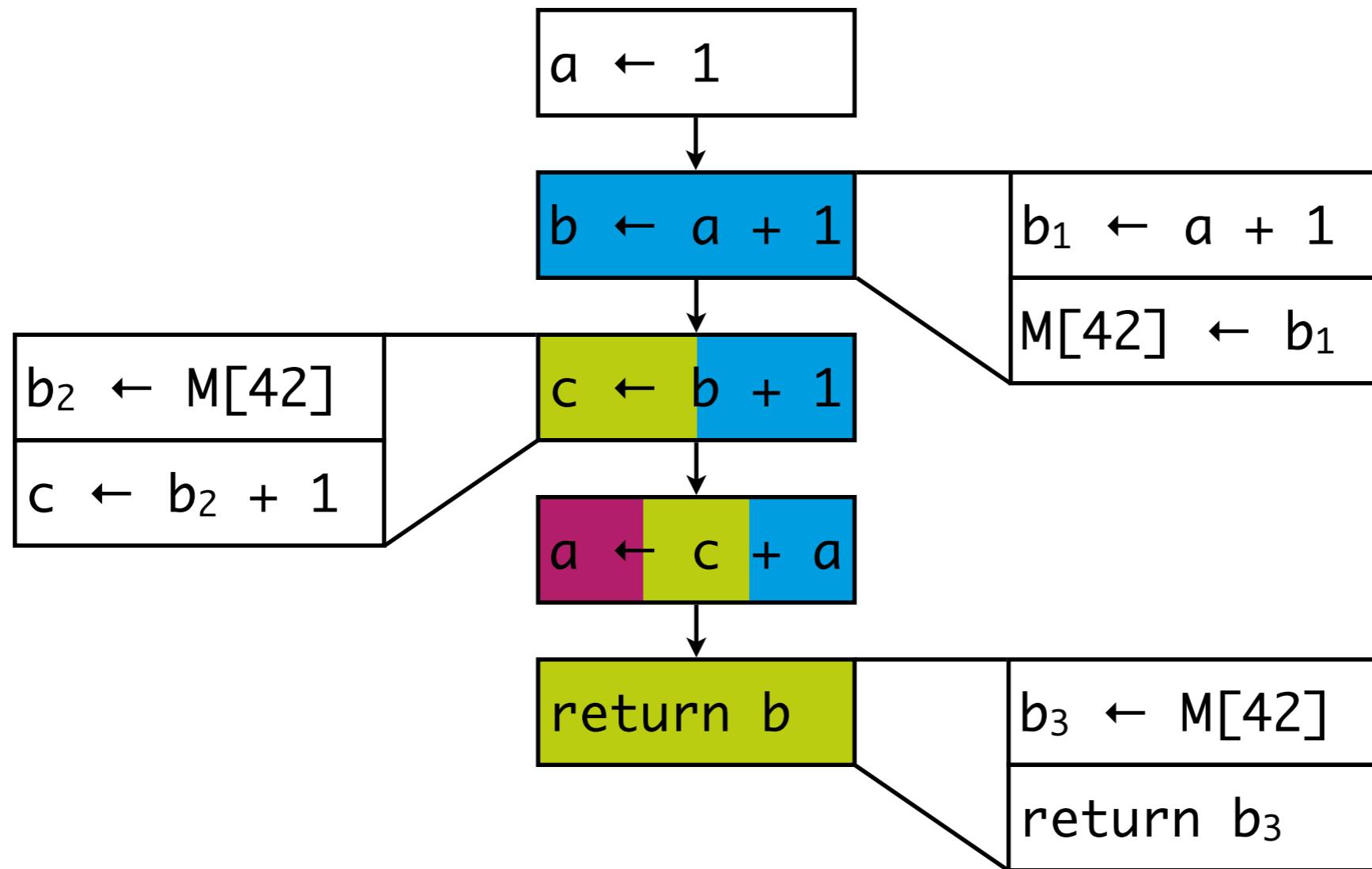
Spilling example



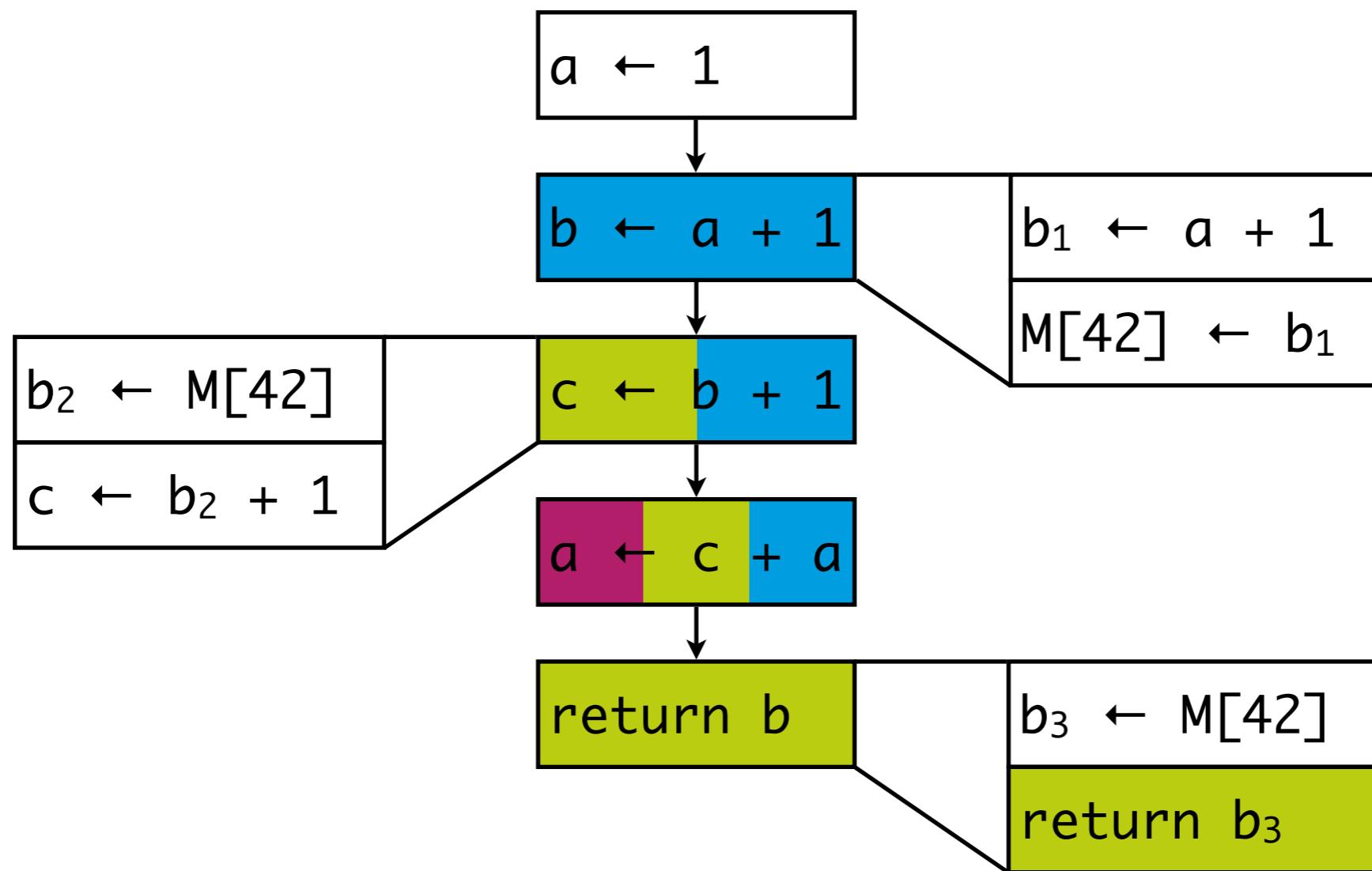
Spilling example



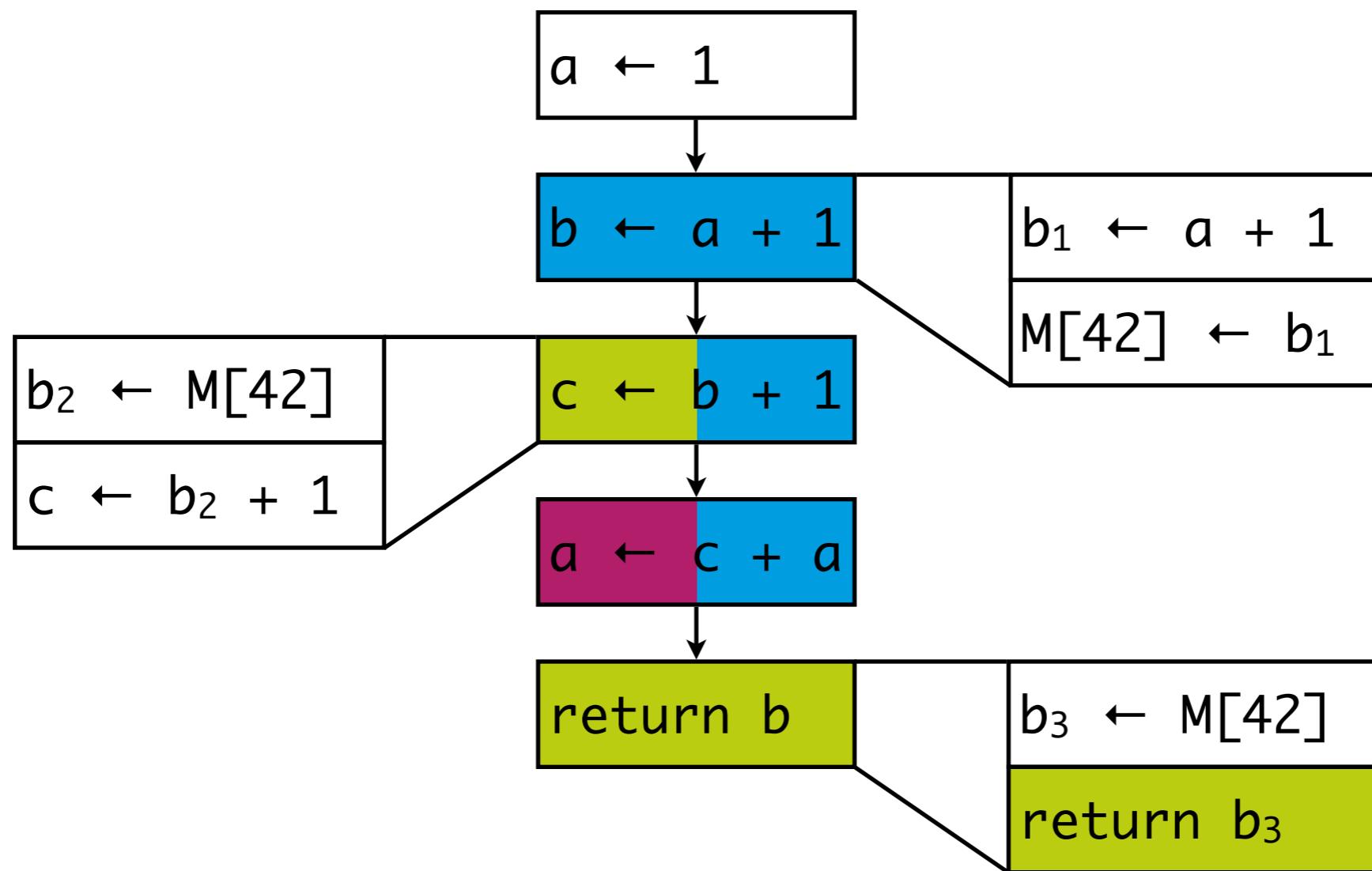
Spilling example



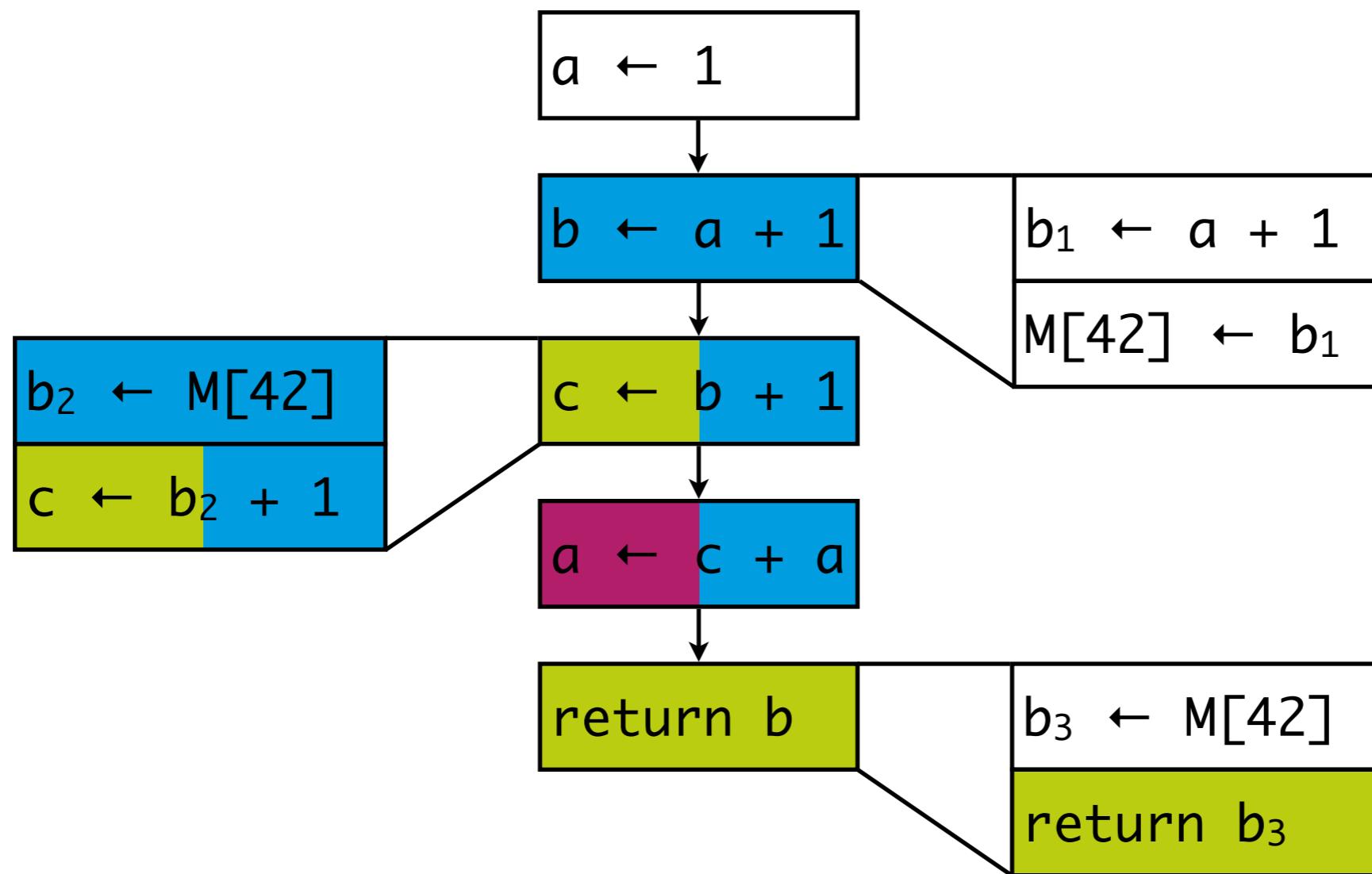
Spilling example



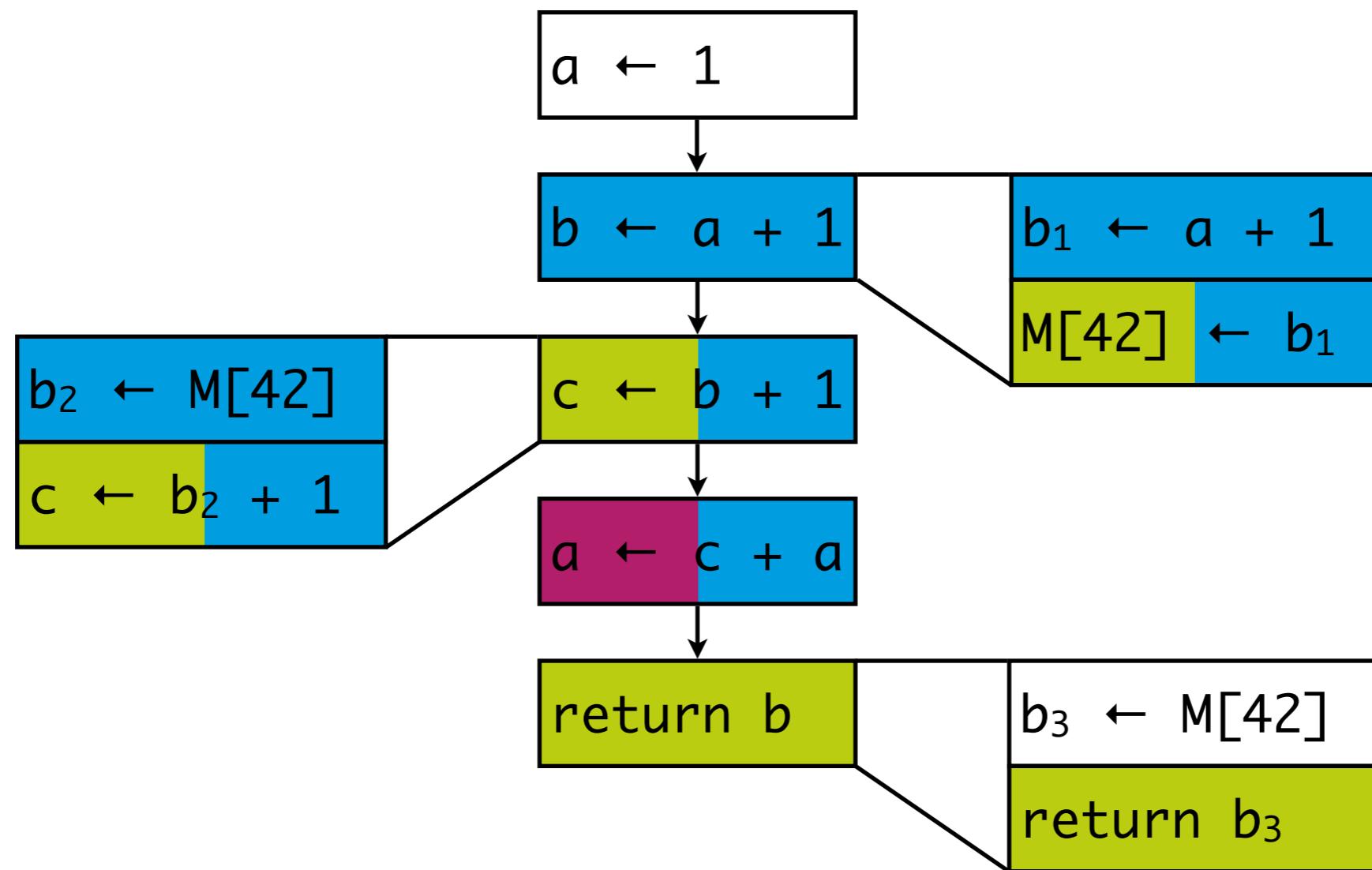
Spilling example



Spilling example



Spilling example



IV

Coalescing

Eliminating Move Instructions

coalescing

```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Eliminating Move Instructions

coalescing

```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing

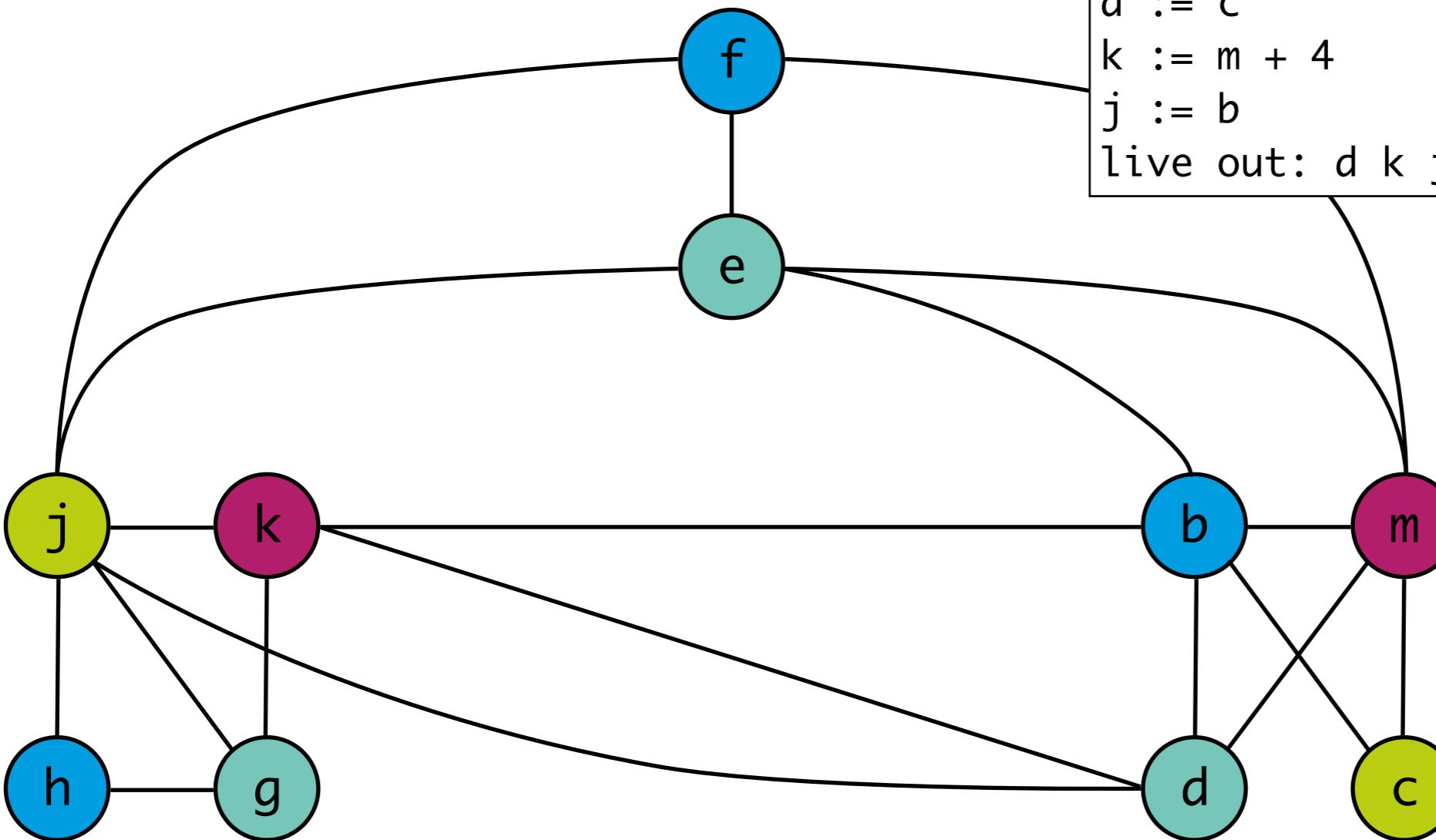
coalesce |kəʊə'les|

verb [no object]

come together to form one mass or whole: *the puddles had coalesced into shallow streams.*

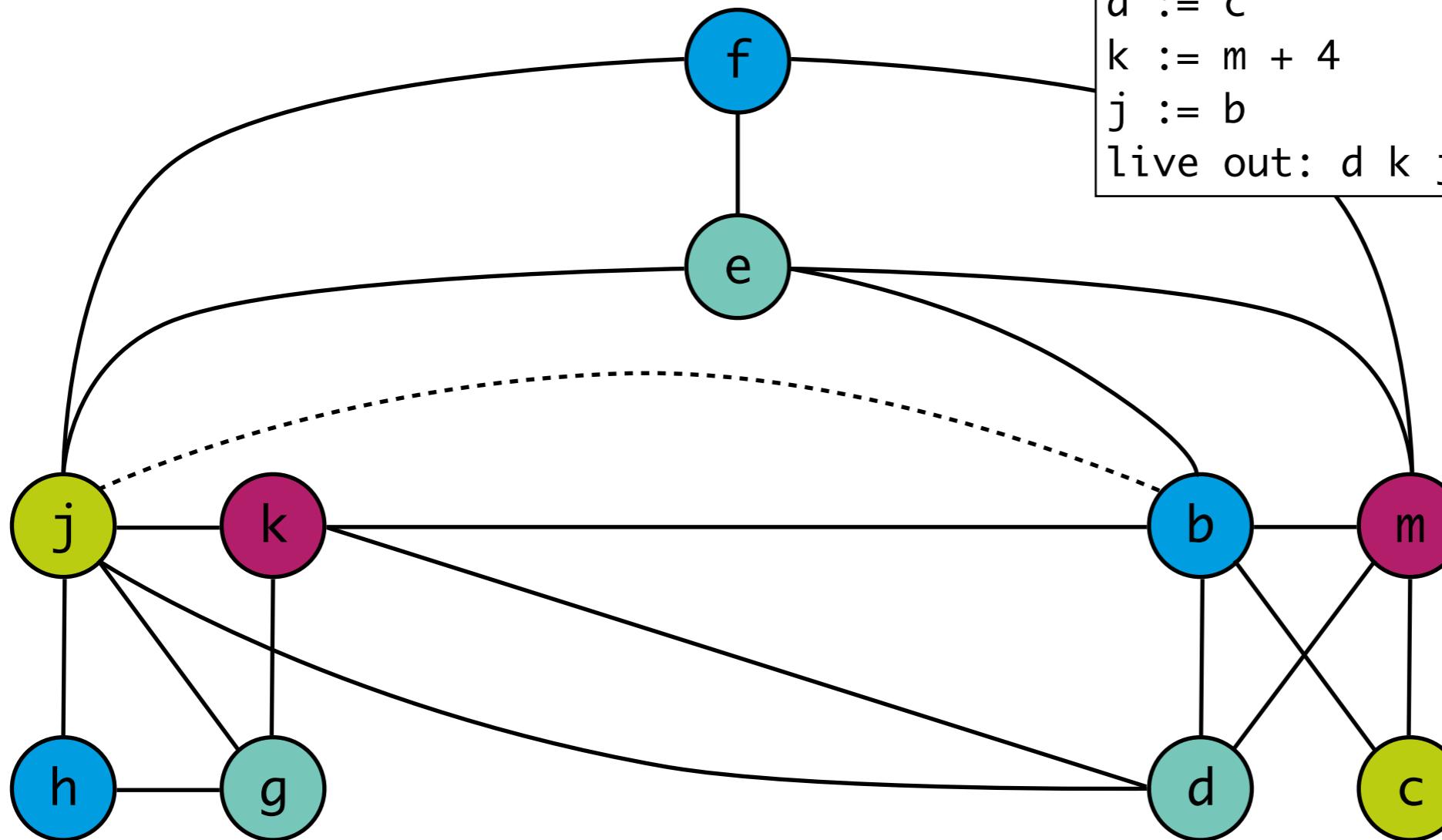
• [with object] combine (elements) in a mass or whole: *his idea served to coalesce all that happened into one connected whole.*

Recap: Graph Coloring example



Recap: Graph Coloring example

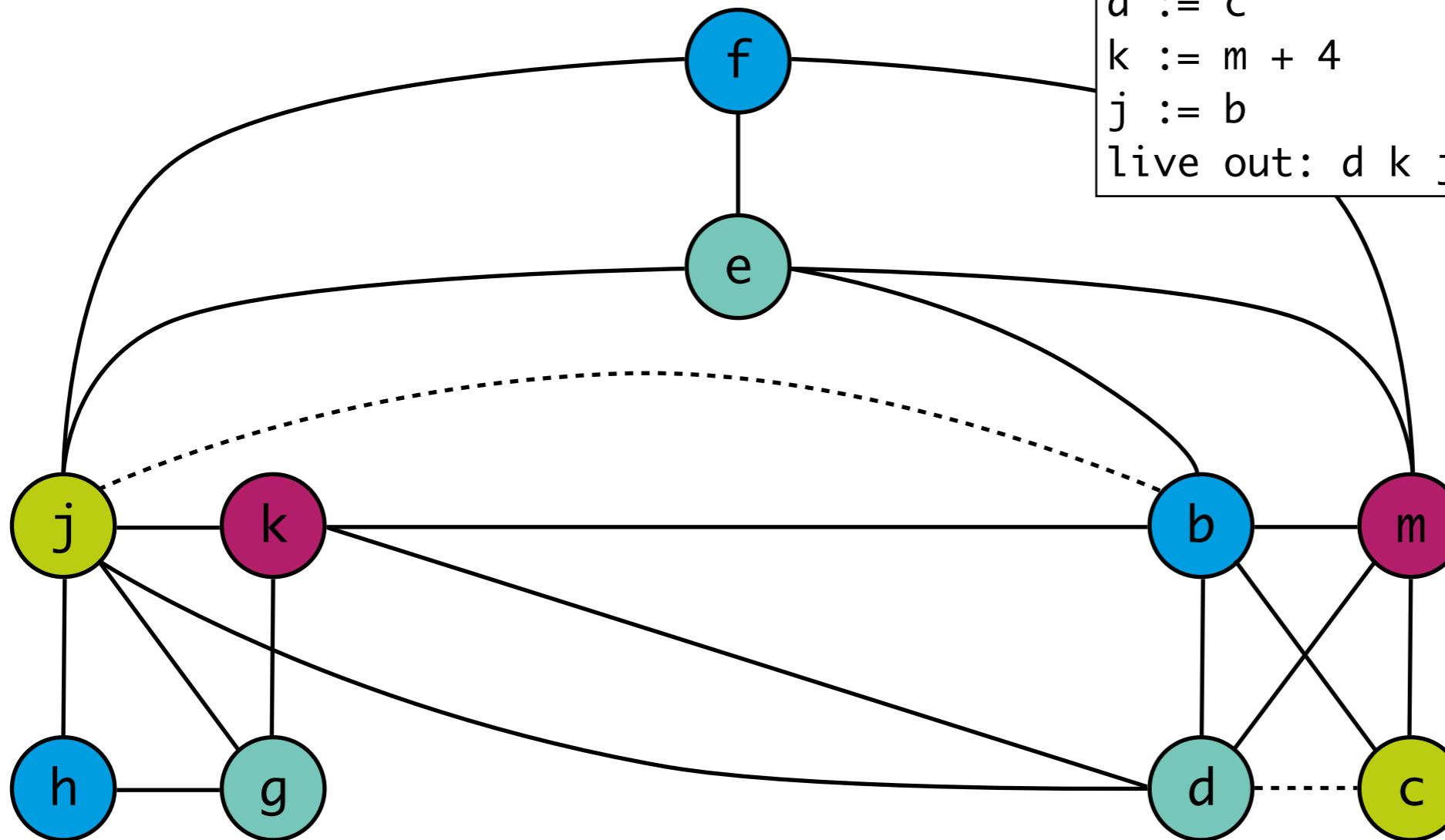
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Recap: Graph Coloring example

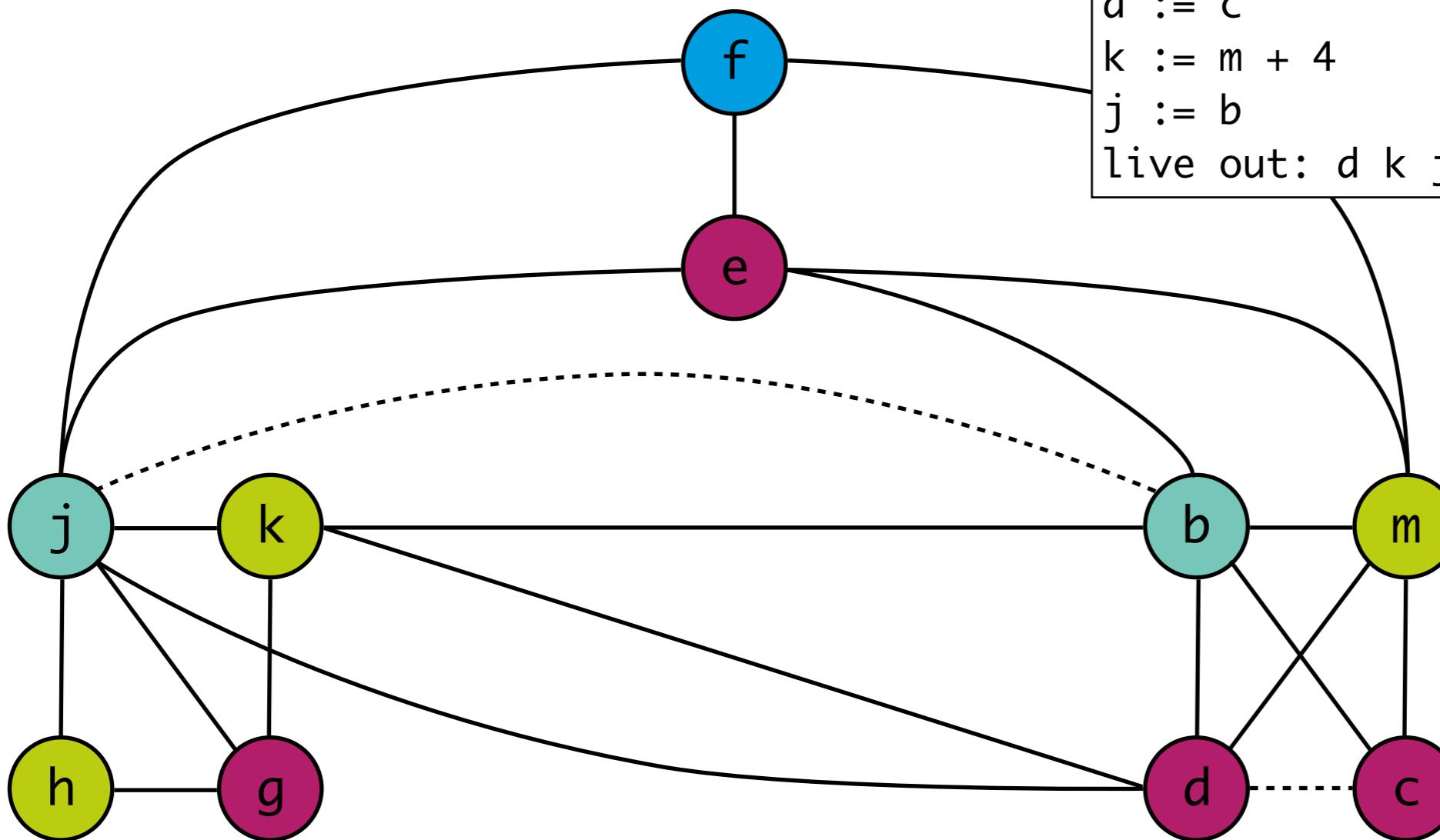
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing better solution

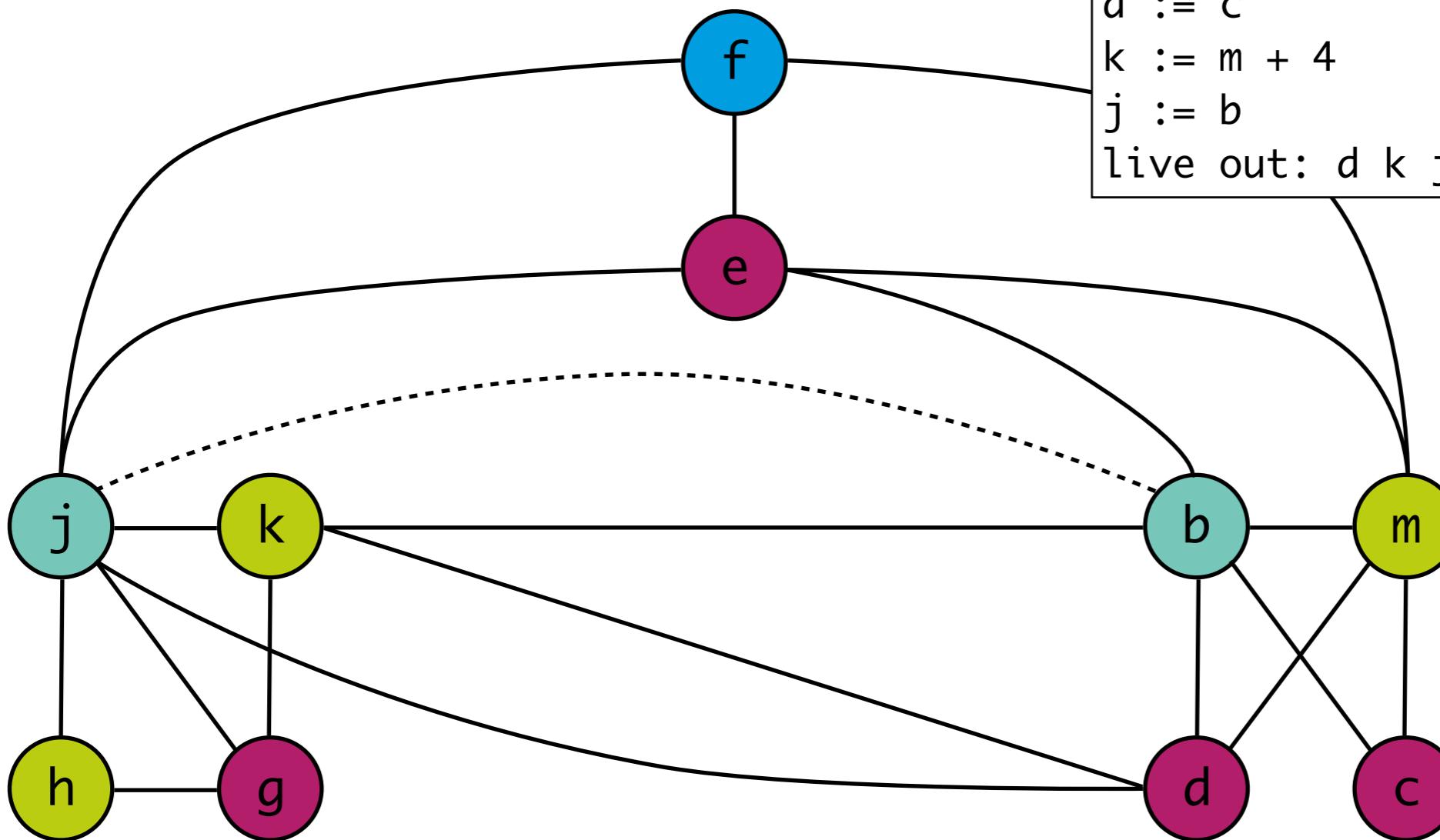
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing better solution

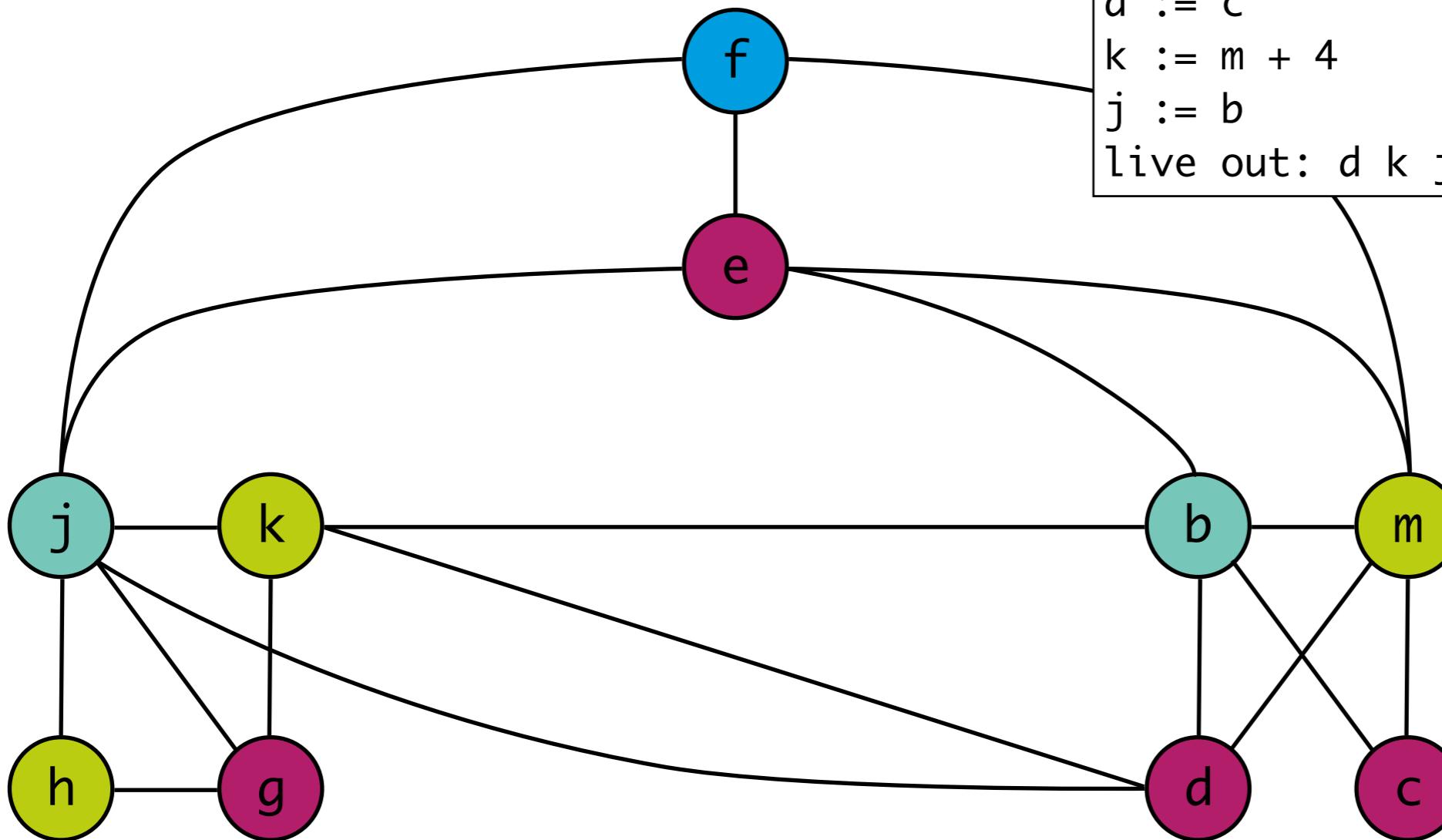
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing better solution

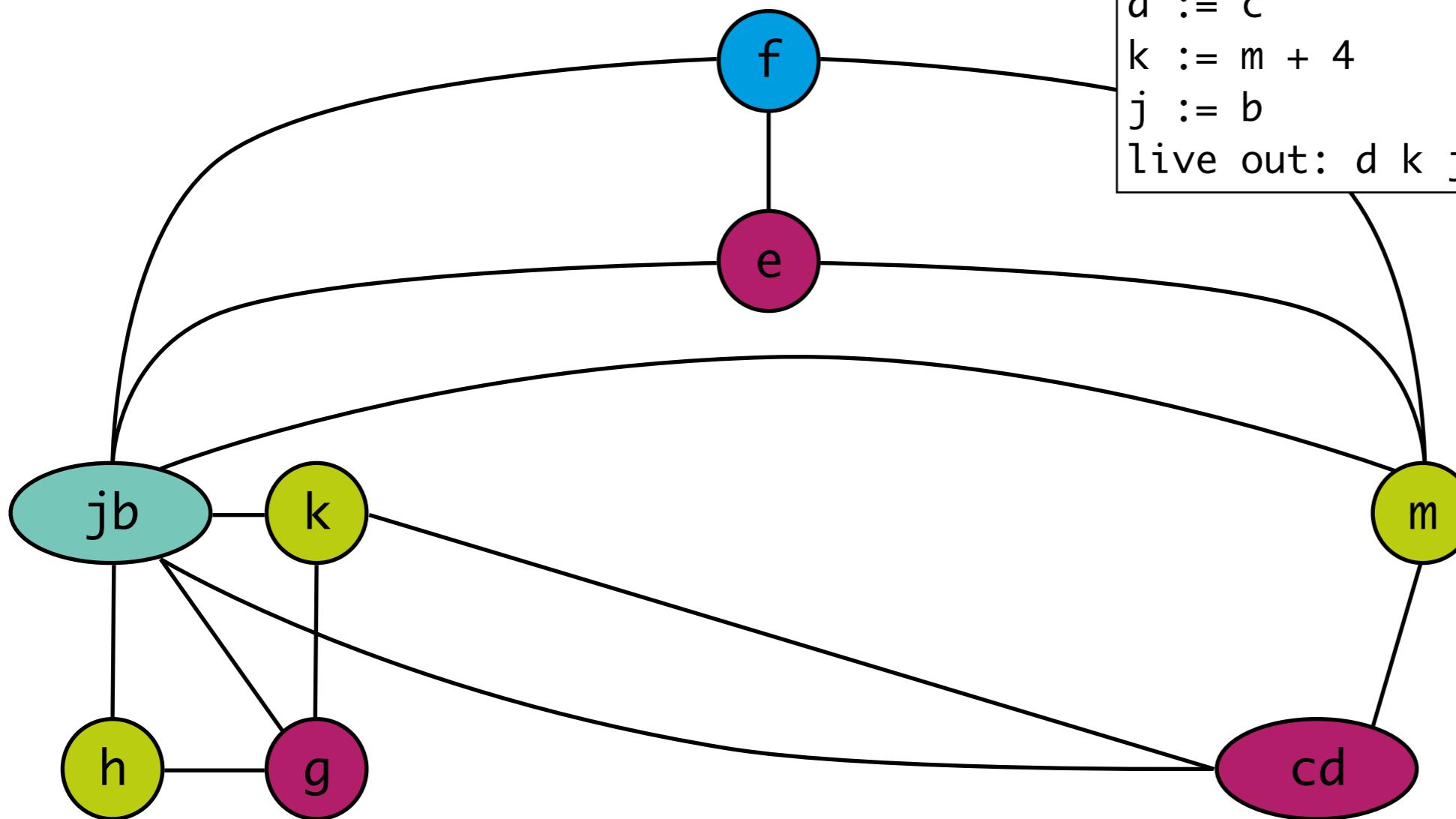
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing coalescing nodes

```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```



Coalescing conservative strategies

Briggs

- a/b has fewer than k neighbours of **significant** degree
- nodes of **insignificant** degree and a/b can be simplified
- remaining graph is colorable

George

- all neighbours of a of significant degree interfere also with b
- neighbours of a of **insignificant** degree can be simplified
- subgraph of original graph is colorable

Graph Coloring

steps

Simplify

remove **non-move-related** node of **insignificant** degree

Coalesce

Freeze

turn **move-related** node of **insignificant** degree into **non-move-related**

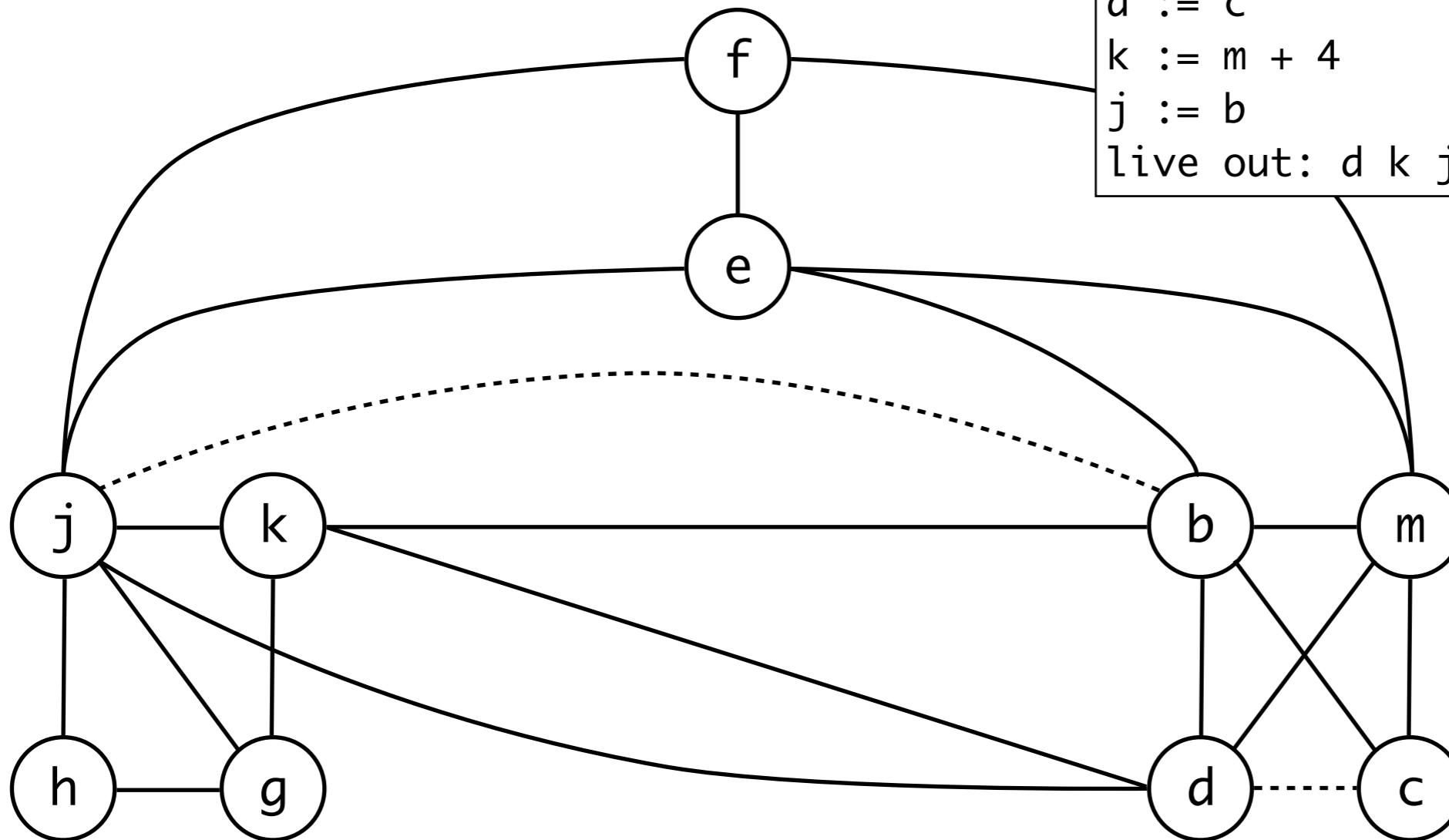
Spill

Select

Start over

Coalescing example

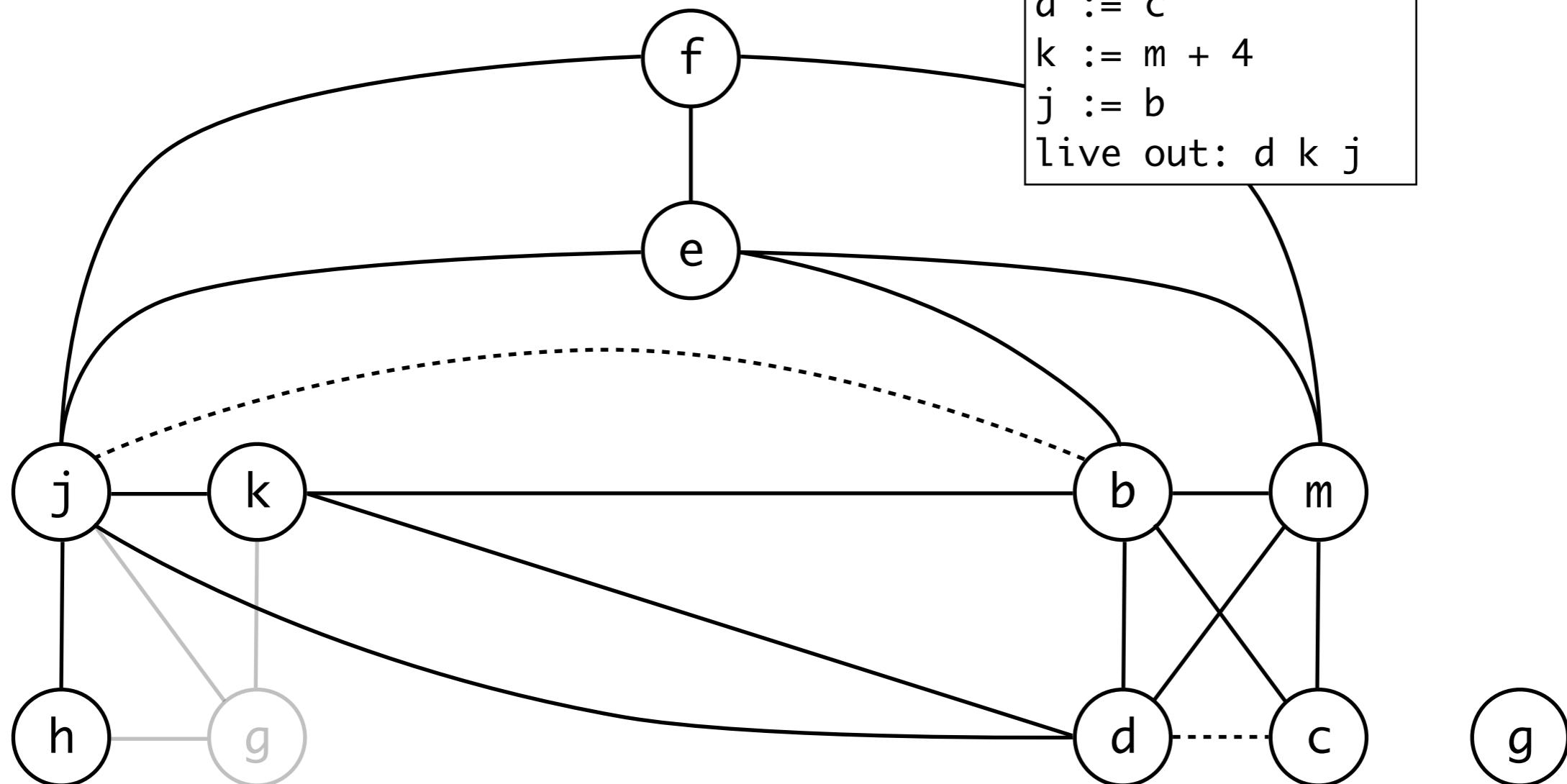
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing example

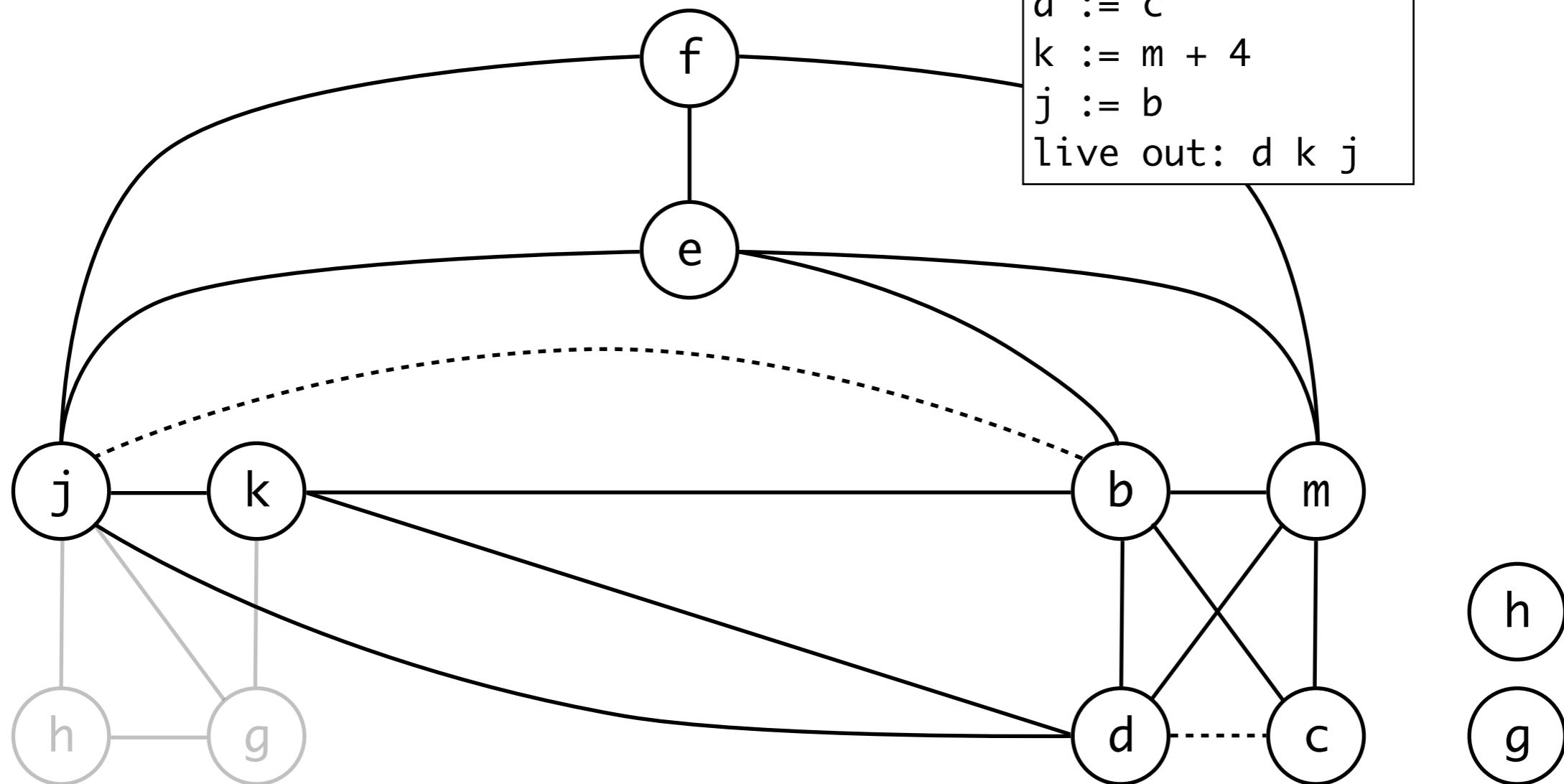
r_1
 r_2
 r_3
 r_4



```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

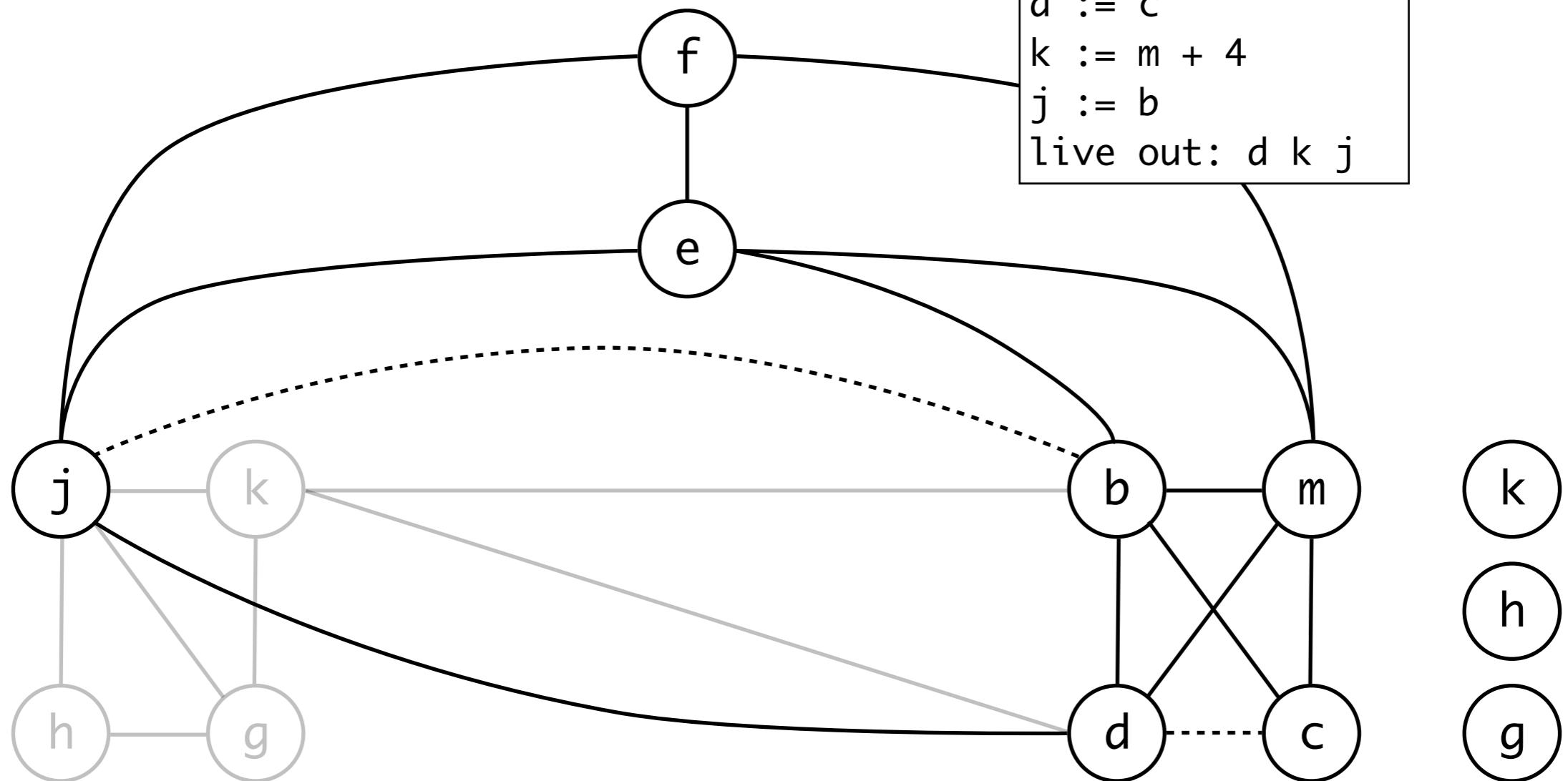
Coalescing example

r_1
 r_2
 r_3
 r_4



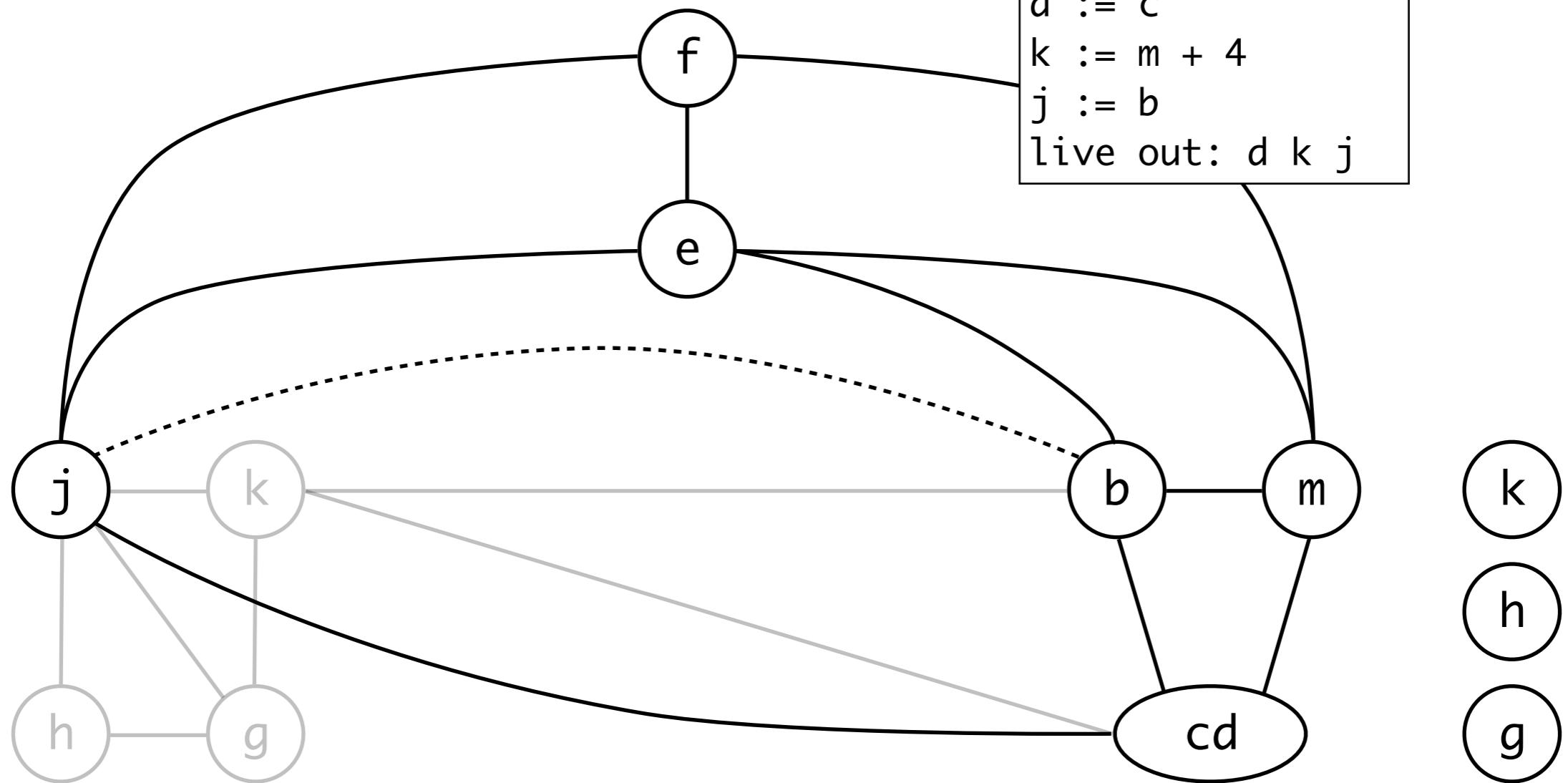
Coalescing example

r_1
 r_2
 r_3
 r_4



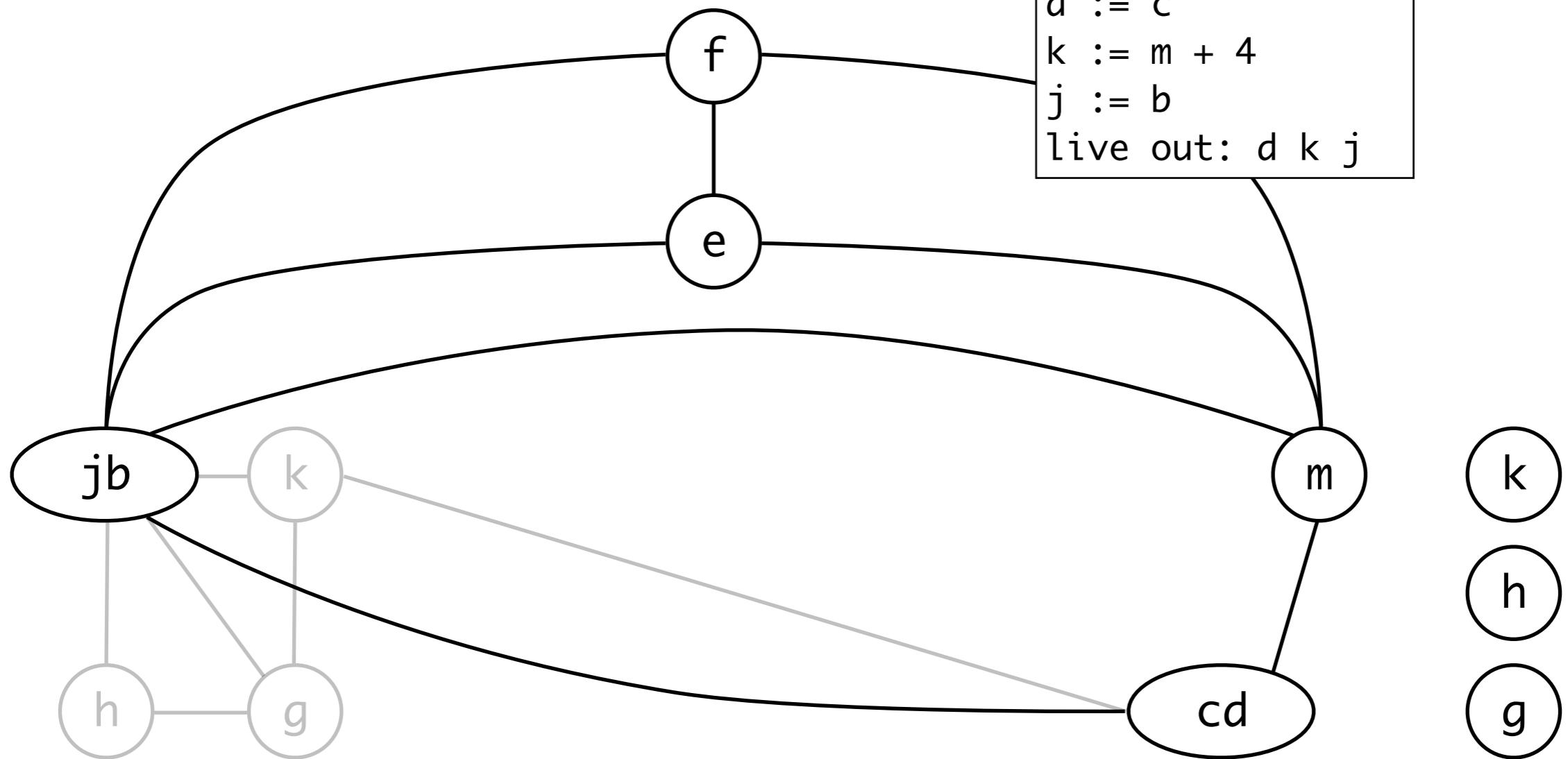
Coalescing example

r_1
 r_2
 r_3
 r_4



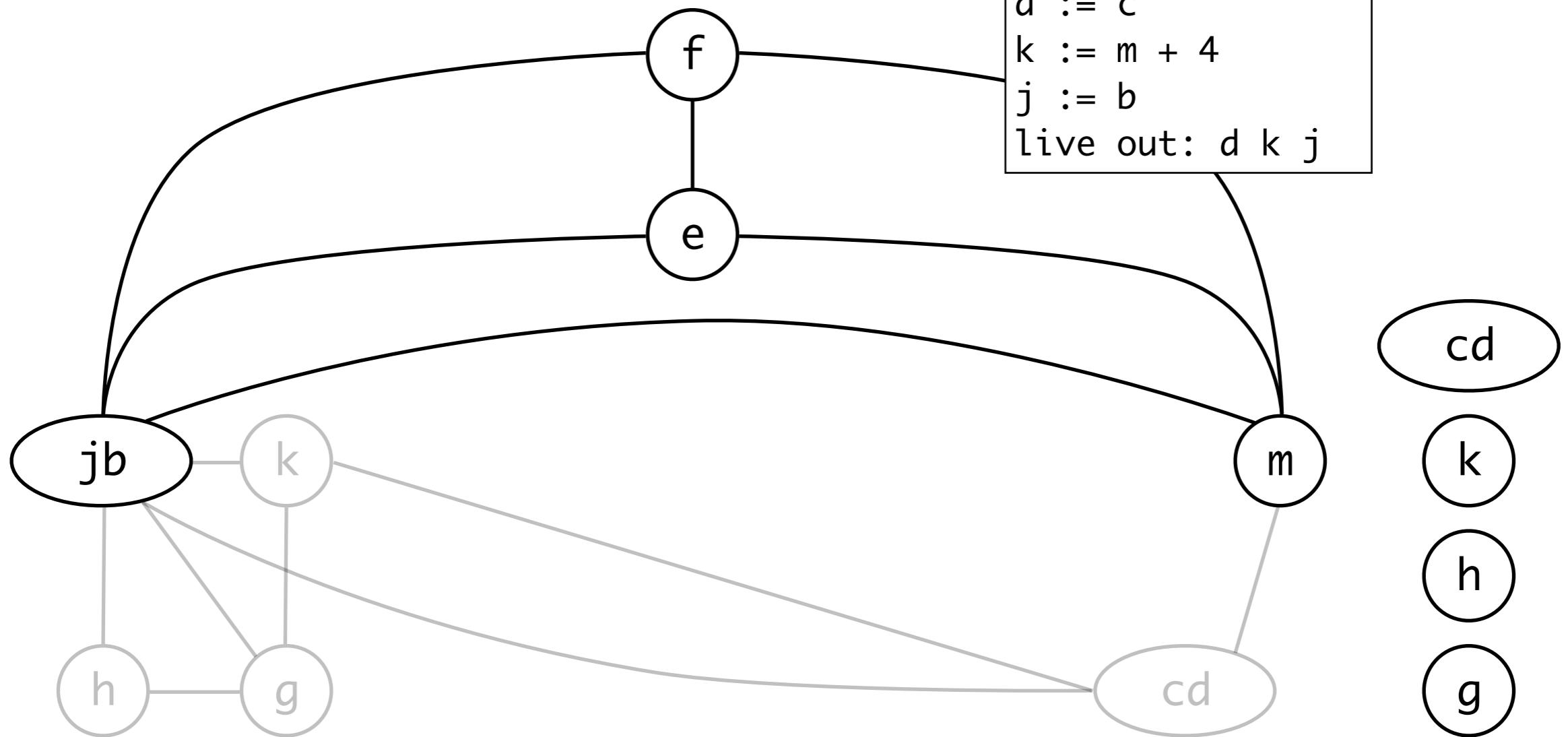
Coalescing example

r_1
 r_2
 r_3
 r_4



Coalescing example

r_1
 r_2
 r_3
 r_4

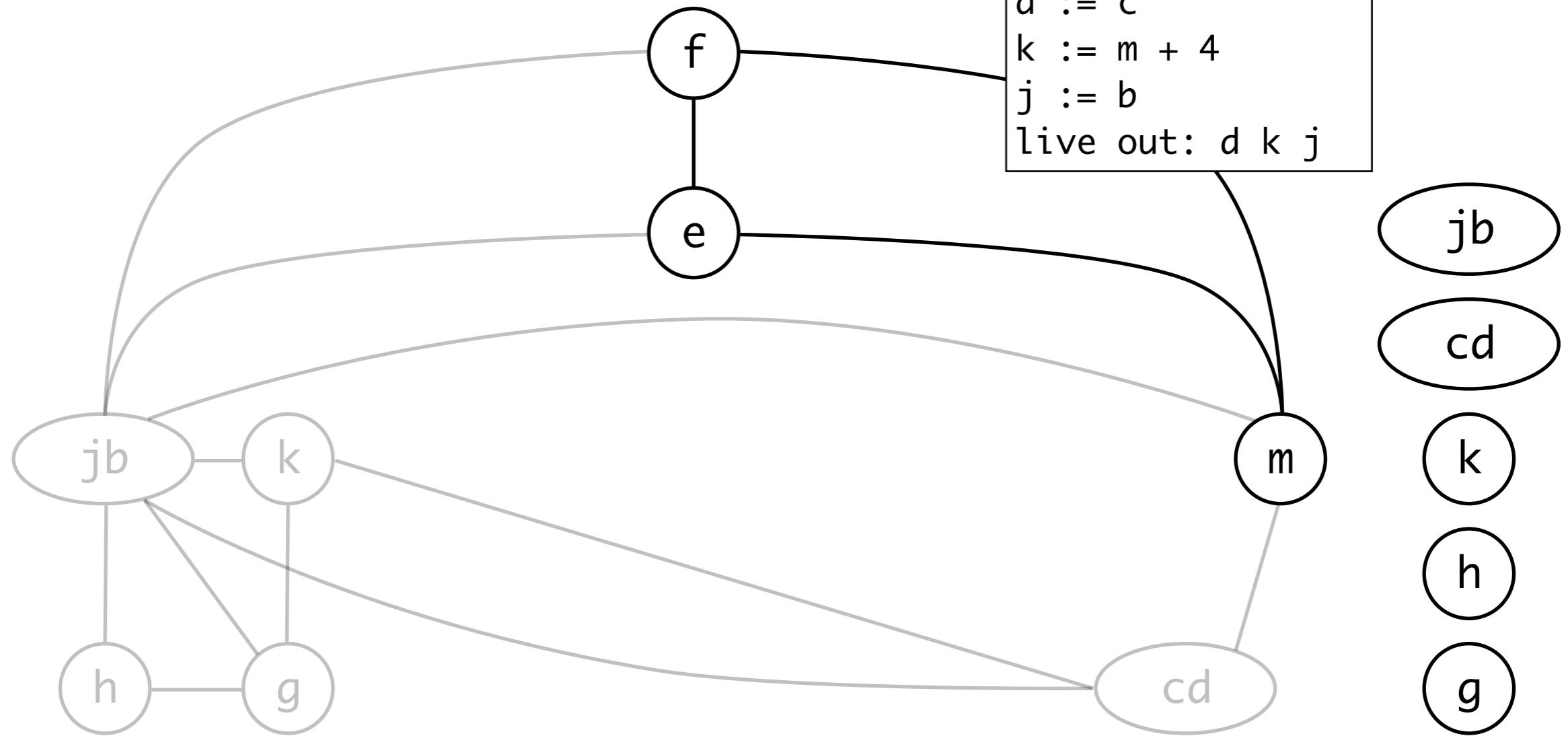


```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```

Coalescing example

r_1
 r_2
 r_3
 r_4

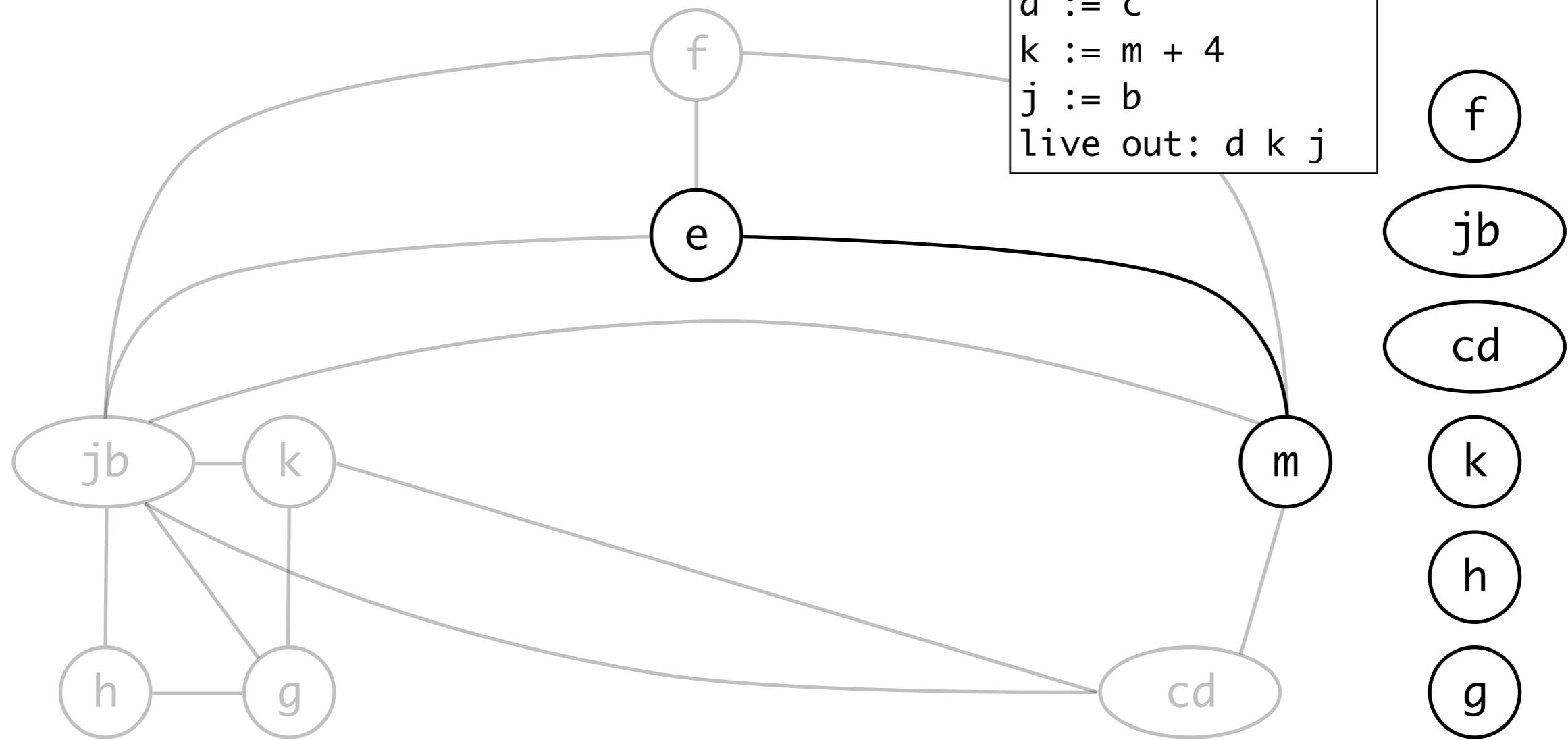
```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```



Coalescing example

r_1
 r_2
 r_3
 r_4

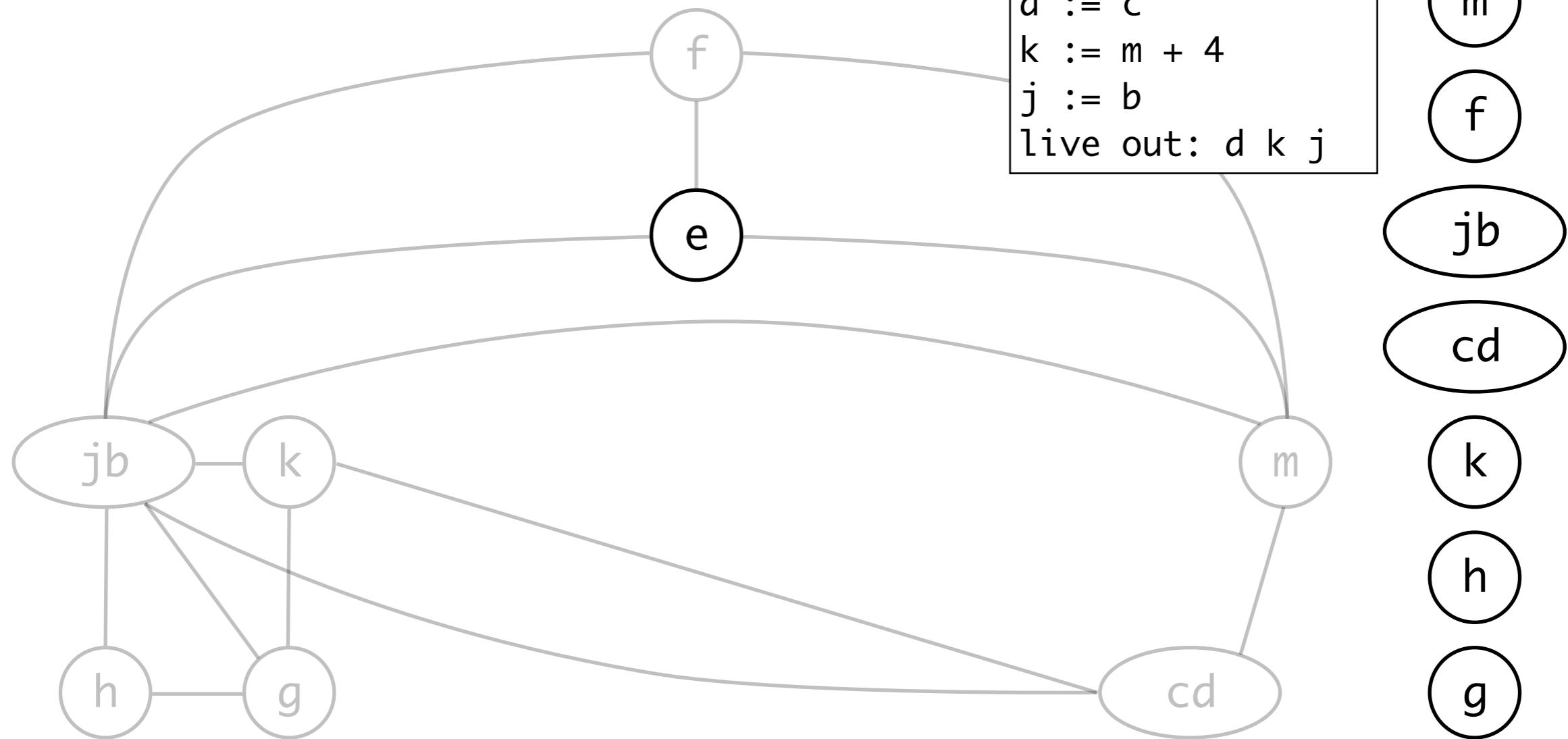
```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```



Coalescing example

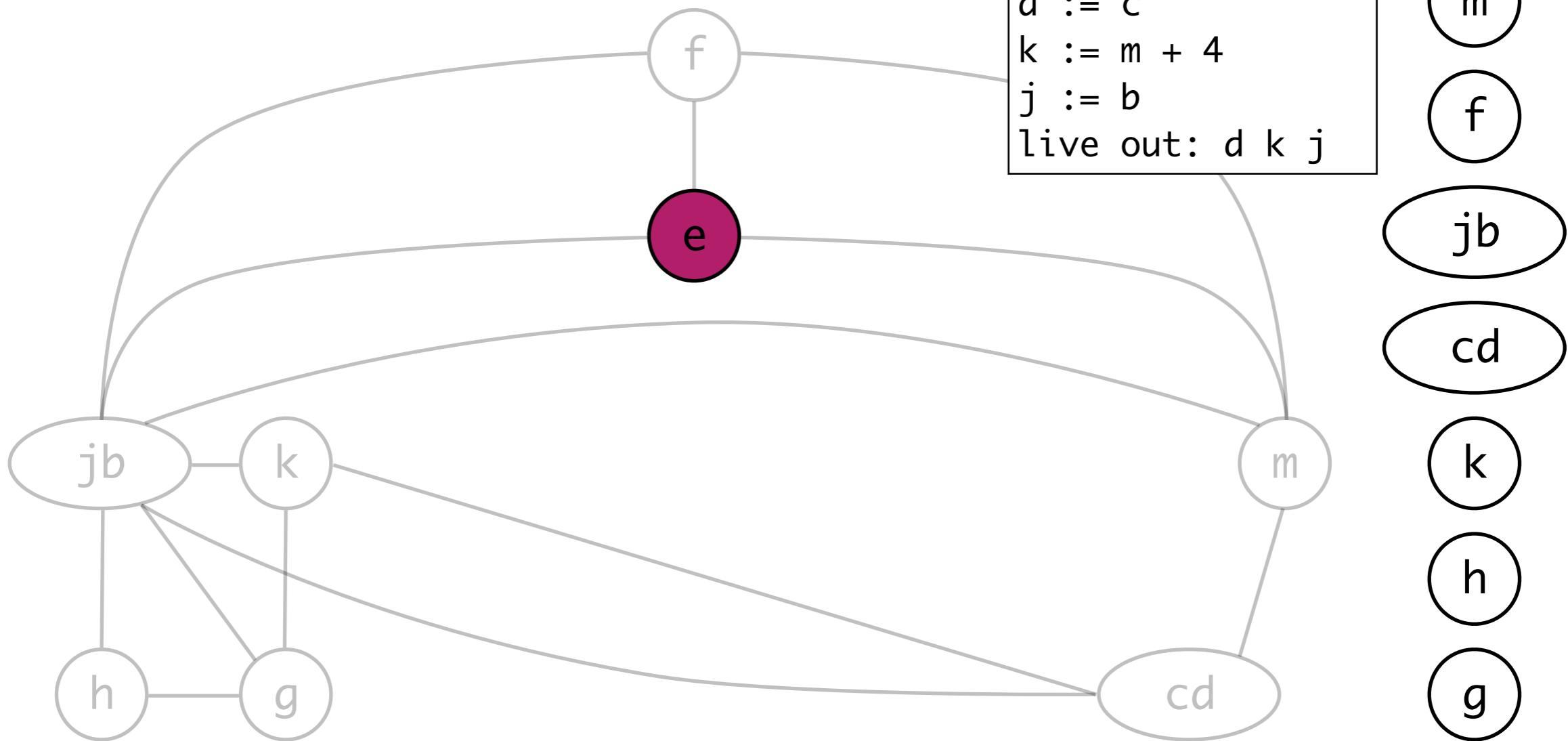
r_1
 r_2
 r_3
 r_4

```
live-in: k j
g := mem[j + 12]
h := k - 1
f := g * h
e := mem[j + 8]
m := mem[j + 16]
b := mem[f]
c := e + 8
d := c
k := m + 4
j := b
live out: d k j
```



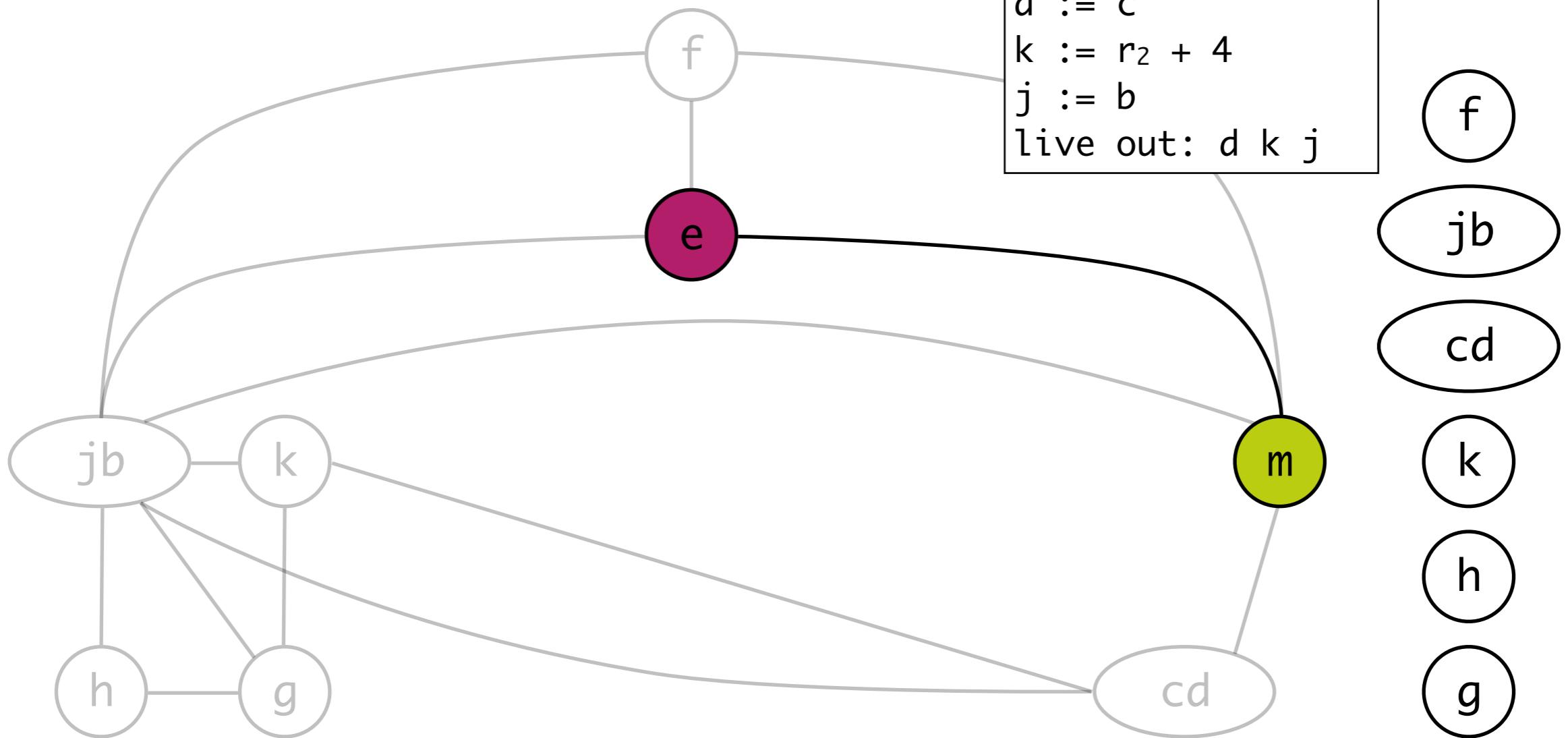
Coalescing example

r_1
 r_2
 r_3
 r_4



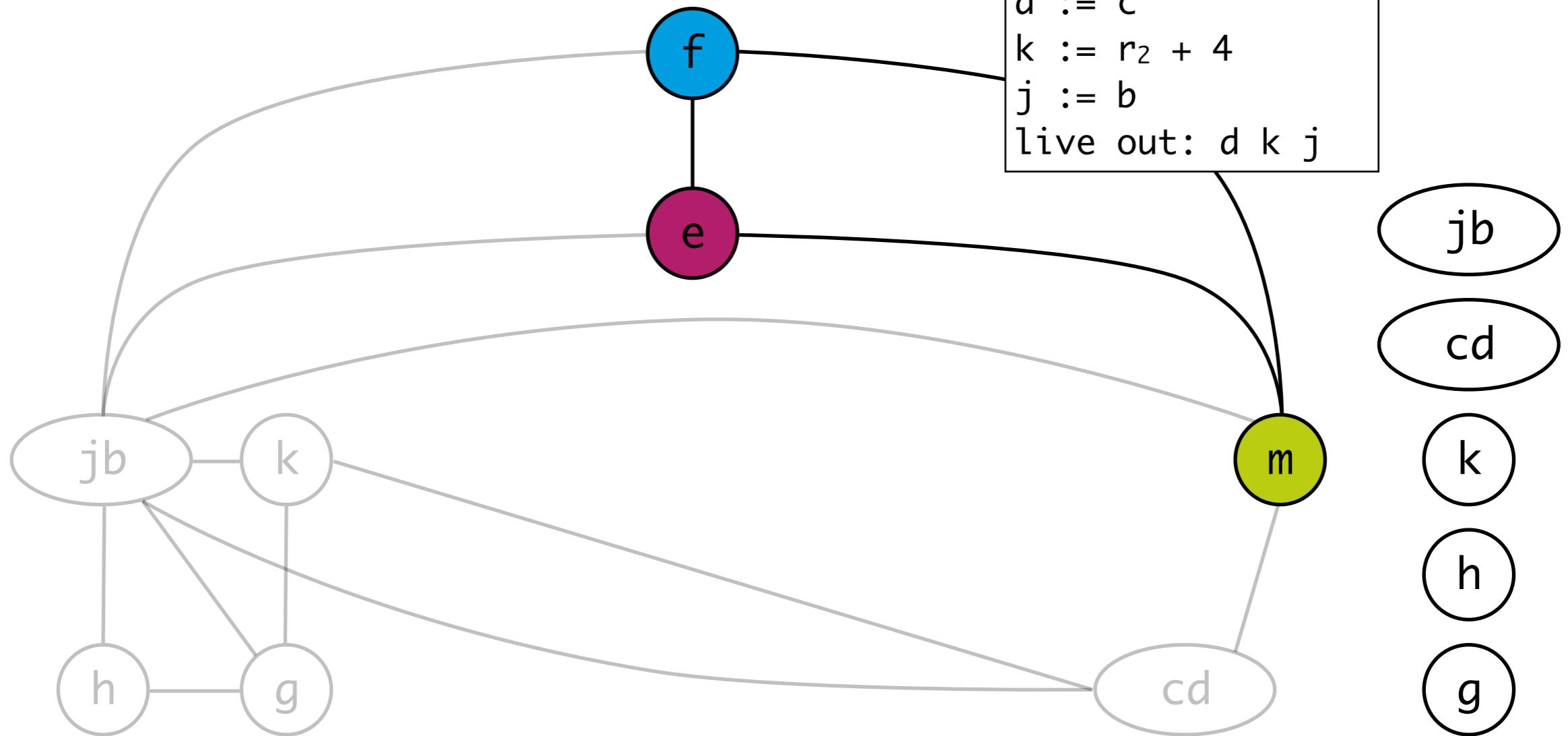
Coalescing example

r_1
 r_2
 r_3
 r_4



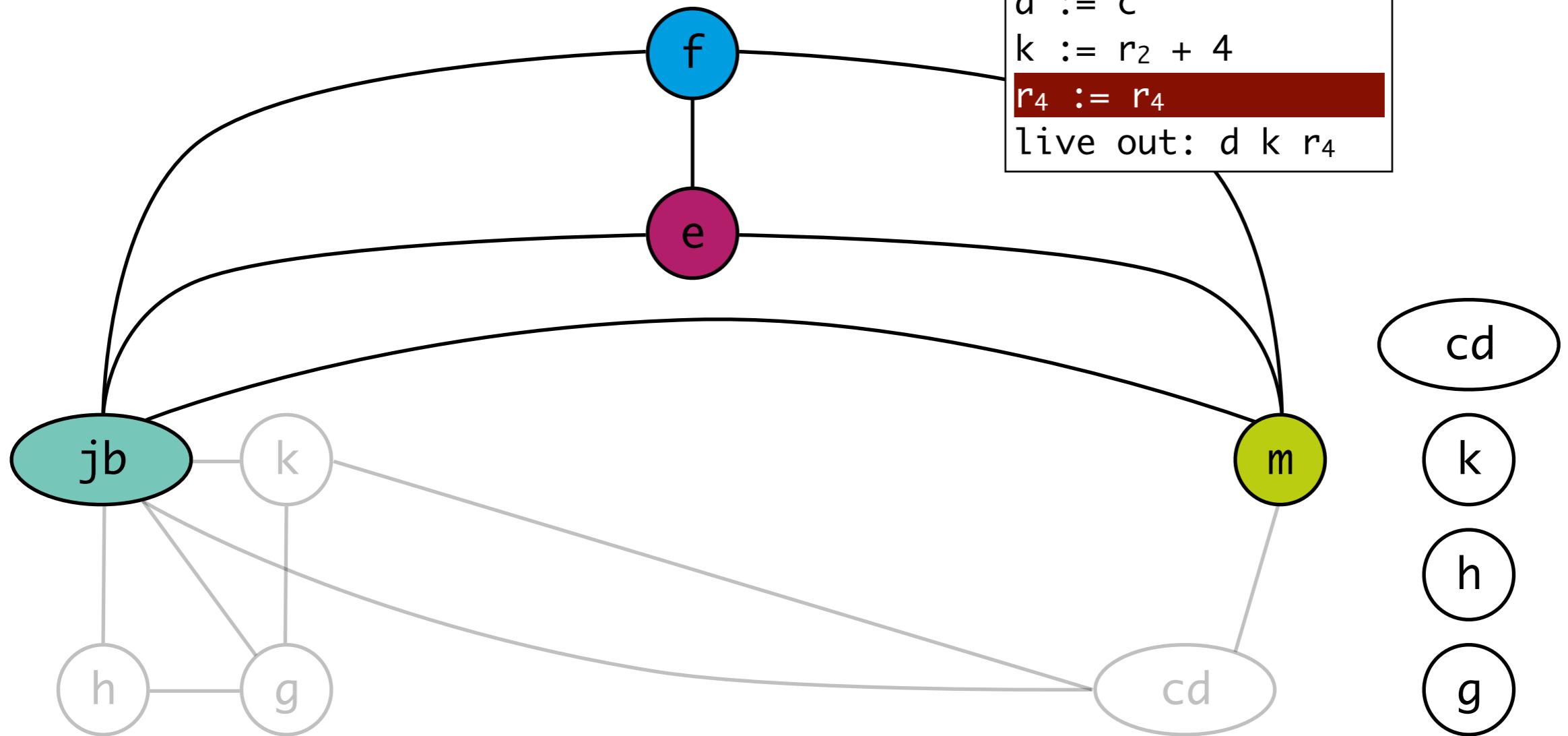
Coalescing example

r_1
 r_2
 r_3
 r_4



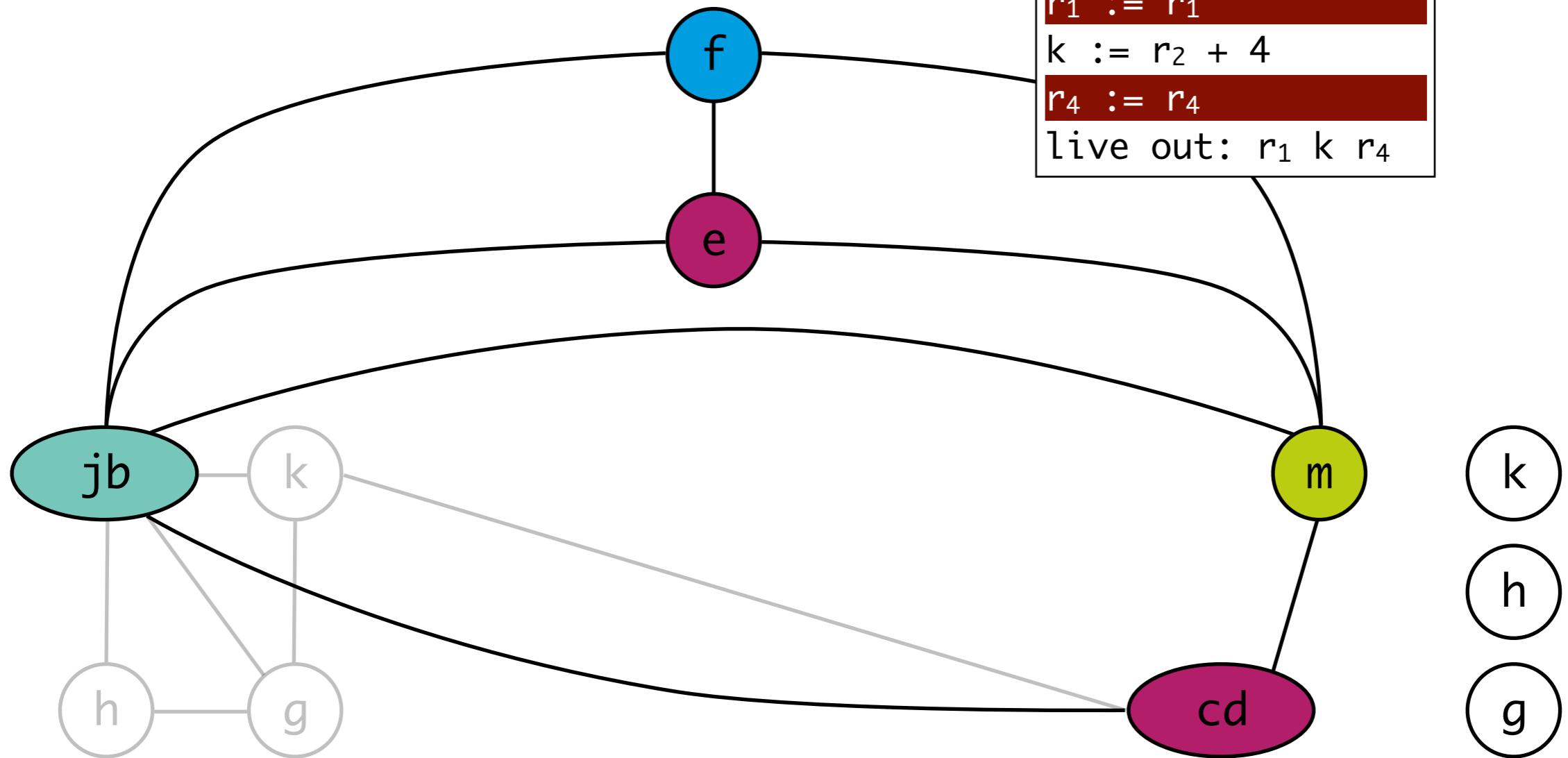
Coalescing example

r_1
 r_2
 r_3
 r_4



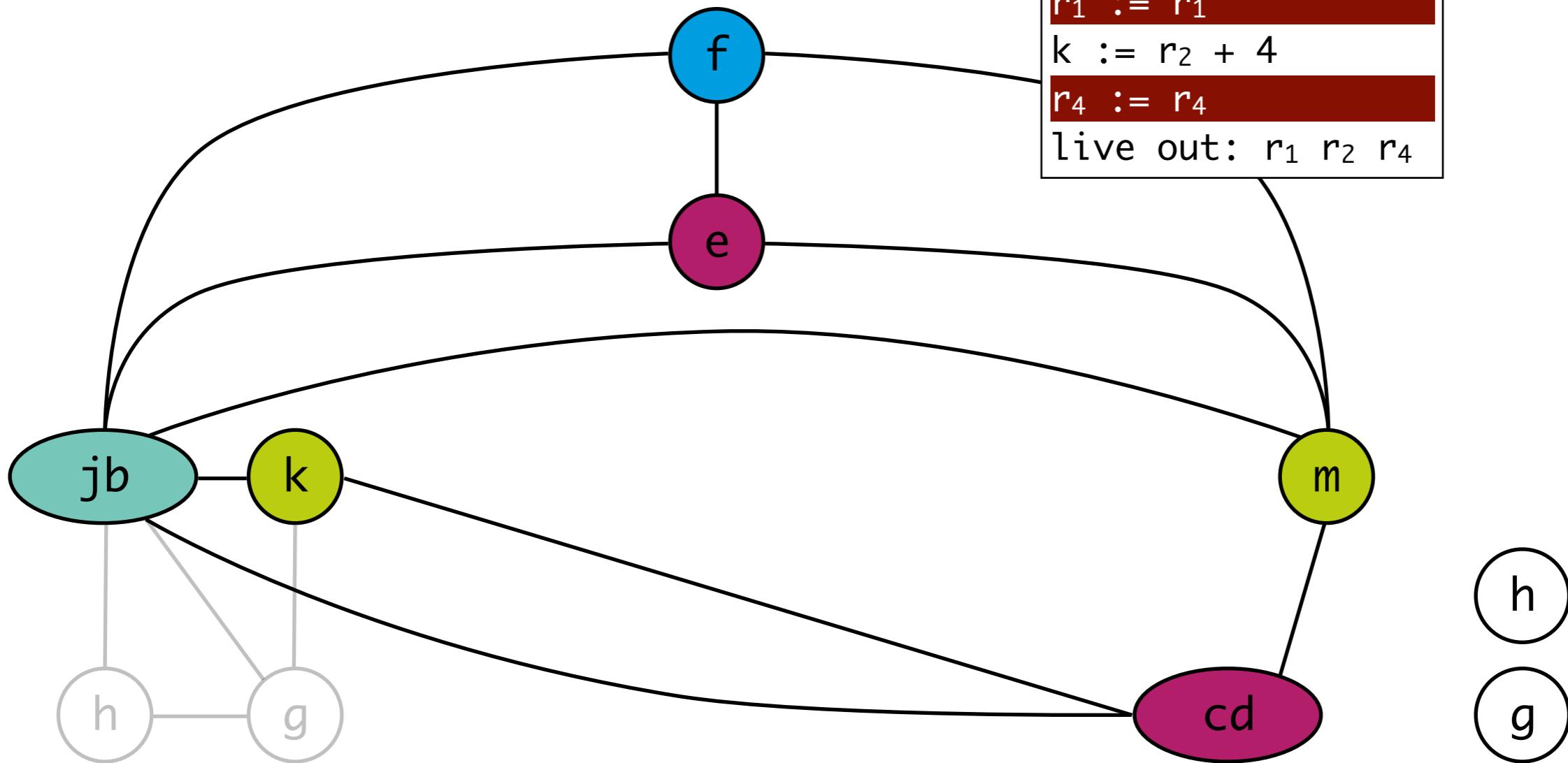
Coalescing example

r_1
 r_2
 r_3
 r_4



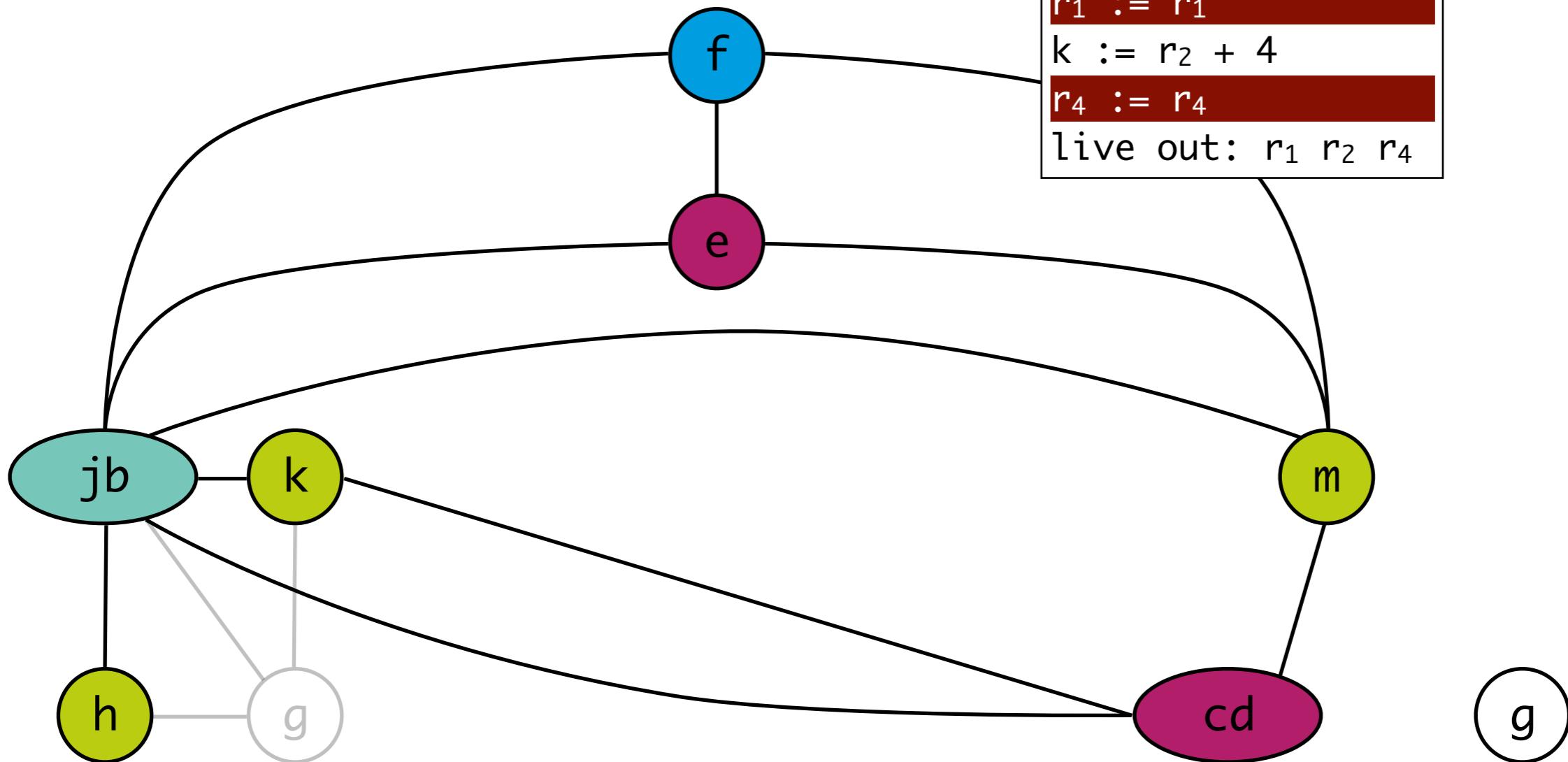
Coalescing example

r_1
 r_2
 r_3
 r_4



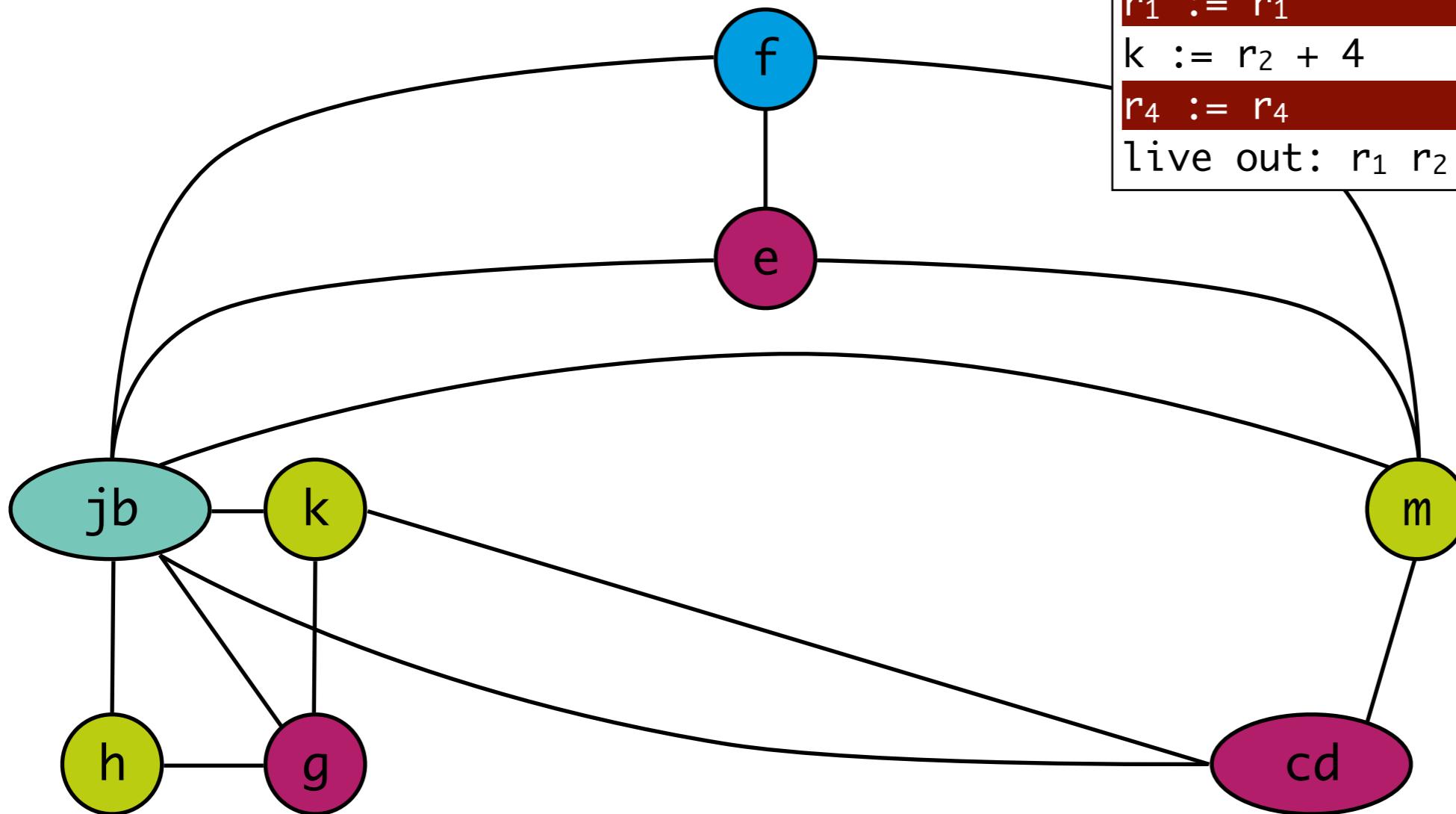
Coalescing example

r_1
 r_2
 r_3
 r_4



Coalescing coalescing nodes

r_1
 r_2
 r_3
 r_4



```
live-in: r2 r4
r1 := mem[r4 + 12]
r2 := r2 - 1
r3 := r1 * r2
r1 := mem[r4 + 8]
r2 := mem[r4 + 16]
b := mem[r3]
r1 := r1 + 8
r1 := r1
k := r2 + 4
r4 := r4
live out: r1 r2 r4
```

V

Pre-Colored Nodes

Recap: Calling Conventions

CDECL

Caller

- push parameters right-to-left on the stack
- clean-up stack after call

```
push 21  
push 42  
call _f  
add ESP 8
```

Callee

- save old BP
- initialise new BP
- save registers
- return result in AX
- restore registers
- restore BP

```
push EBP  
mov EBP ESP  
mov EAX [EBP + 8]  
mov EDX [EBP + 12]  
add EAX EDX  
pop EBP  
ret
```

Recap: Calling Conventions

STDCALL

Caller

- push parameters right-to-left on the stack

```
push 21  
push 42  
call _f@8
```

Callee

- save old BP
- initialise new BP
- save registers
- return result in AX
- restore registers
- restore BP

```
push EBP  
mov EBP ESP  
mov EAX [EBP + 8]  
mov EDX [EBP + 12]  
add EAX EDX  
pop EBP  
ret 8
```

Recap: Calling Conventions

FASTCALL

Caller

- passes parameters in registers
- pushes additional parameters right-to-left on the stack
- cleans up the stack

```
mov ECX 21  
mov EDX 42  
call @f@8
```

Callee

- save old **BP**, initialise new **BP**
- save registers
- return result in **AX**
- restore registers
- restore **BP**

```
push EBP  
mov EBP ESP  
mov EAX ECX  
add EAX EDX  
pop EBP  
ret
```

Recap: Calling Conventions

saving registers

Not enough registers for all local variables across life time

- save register to memory to free for other use

Caller-save registers

- Caller is responsible for saving and restoring register

Callee-save registers

- Callee is responsible for saving and restoring register

Use callee-save registers to pass parameters

Pre-Colored Nodes representing registers

Nodes

- register = pre-colored node
- no simplify, no spill
- coalesce possible

Edges

- all registers interfere with each other
- explicit usage of registers
- call and return instructions influence liveness

Callee-Save Register in Temporary pre-colored nodes

```
enter: def(r7)  
...  
exit: use(r7)
```

```
enter: def(r7)  
t ← r7  
...  
r7 ← t  
exit: use(r7)
```

Pre-Colored Nodes

example

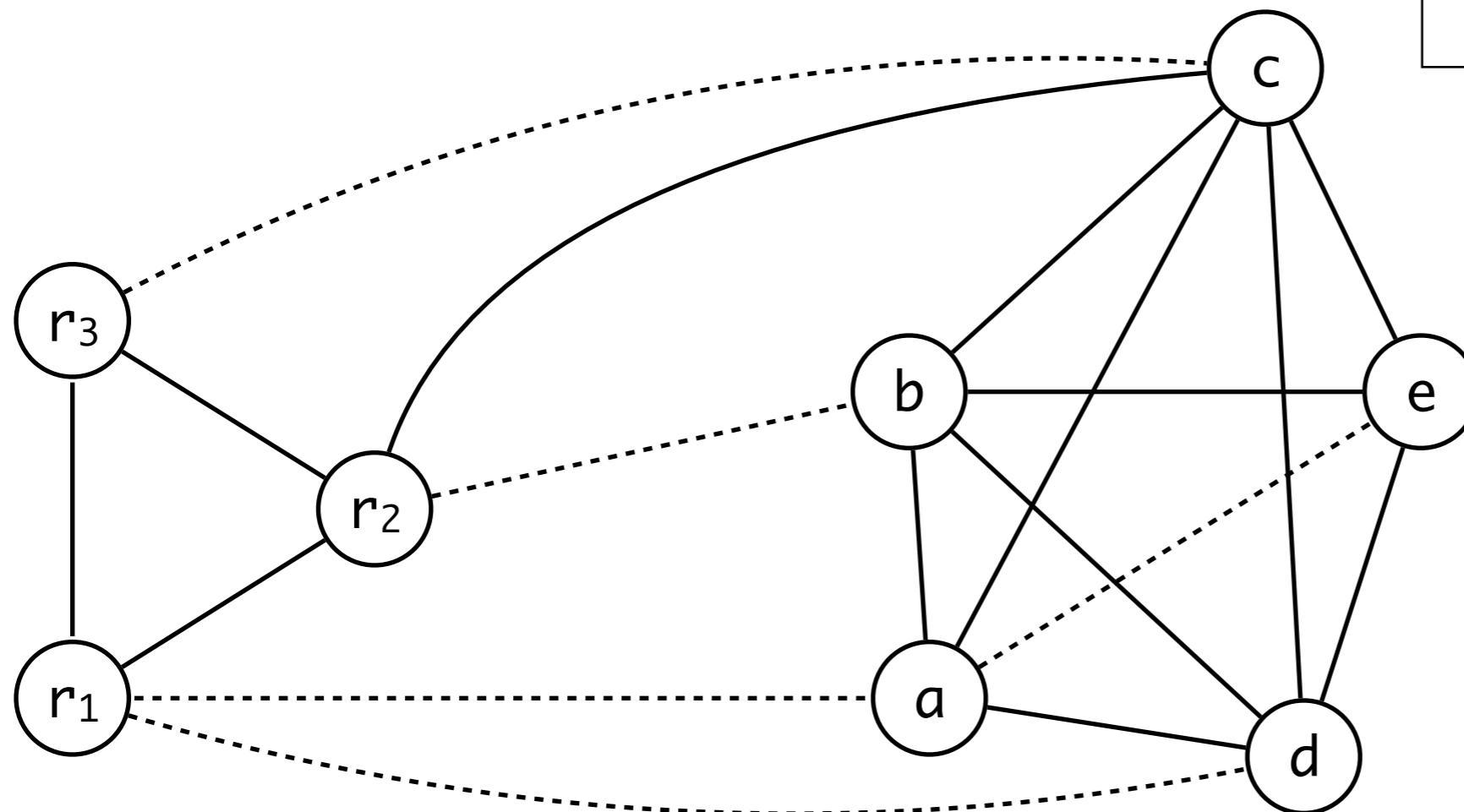
```
int f(int a, int b) {  
    int d = 0;  
    int e = a;  
    do {  
        d = d + b;  
        e = e - 1;  
    } while (e > 0);  
    return d;  
}
```

```
enter : c ← r3 // callee-save  
         a ← r1 // caller-save  
         b ← r2 // caller-save  
         d ← 0  
         e ← a  
loop :  d ← d + b  
         e ← e - 1  
         if e > 0 goto loop  
         r1 ← d  
         r3 ← c  
         return (r1, r3 live out)
```

machine has 3 registers

Pre-Colored Nodes

example

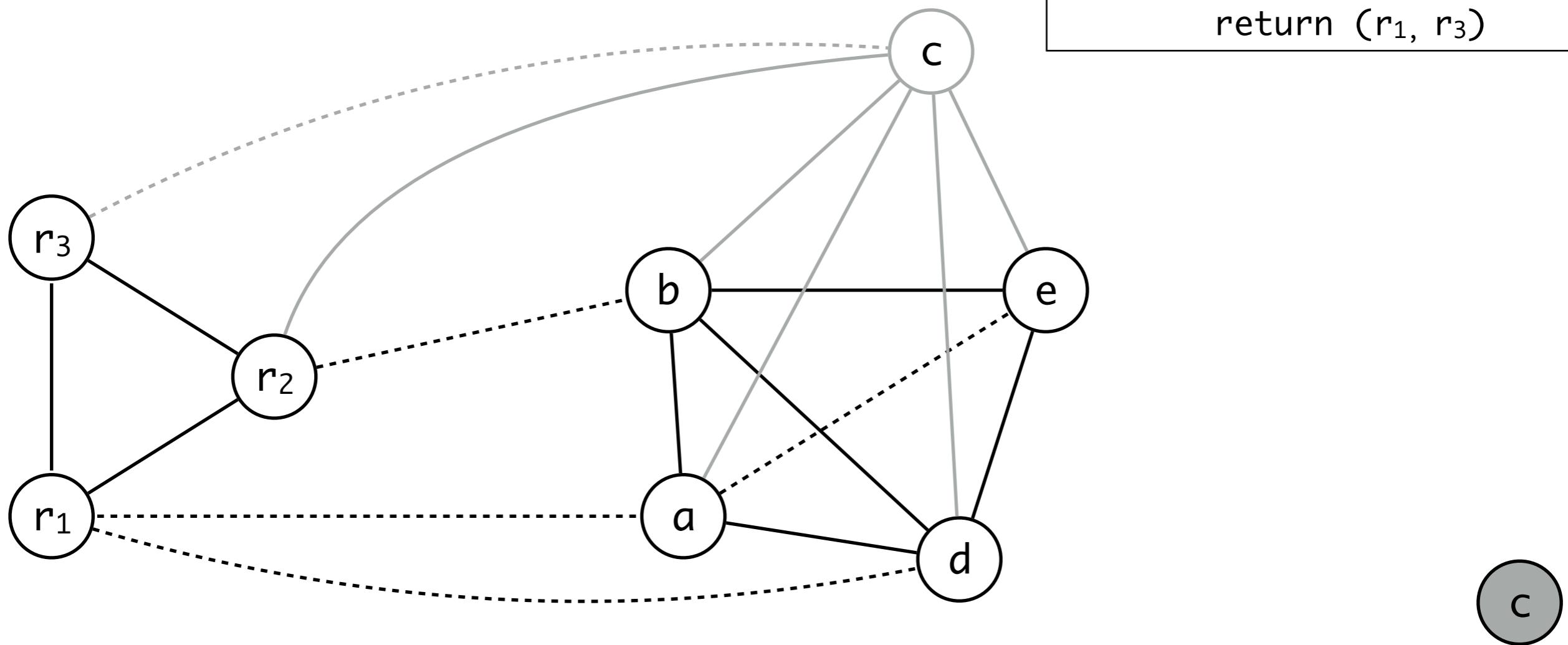


```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop  : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```

Pre-Colored Nodes

example

spill c

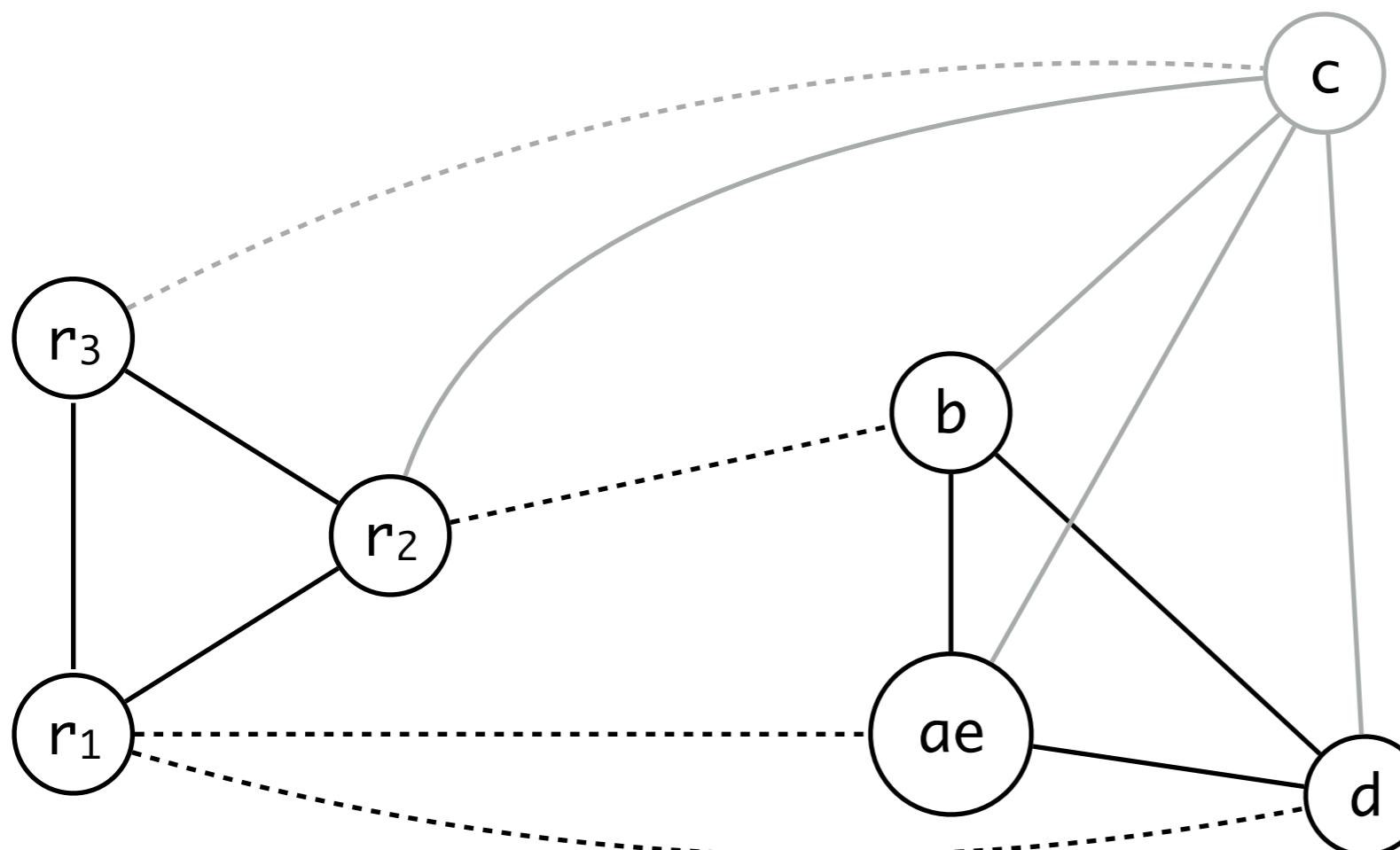


```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```

Pre-Colored Nodes

example

coalesce a and e

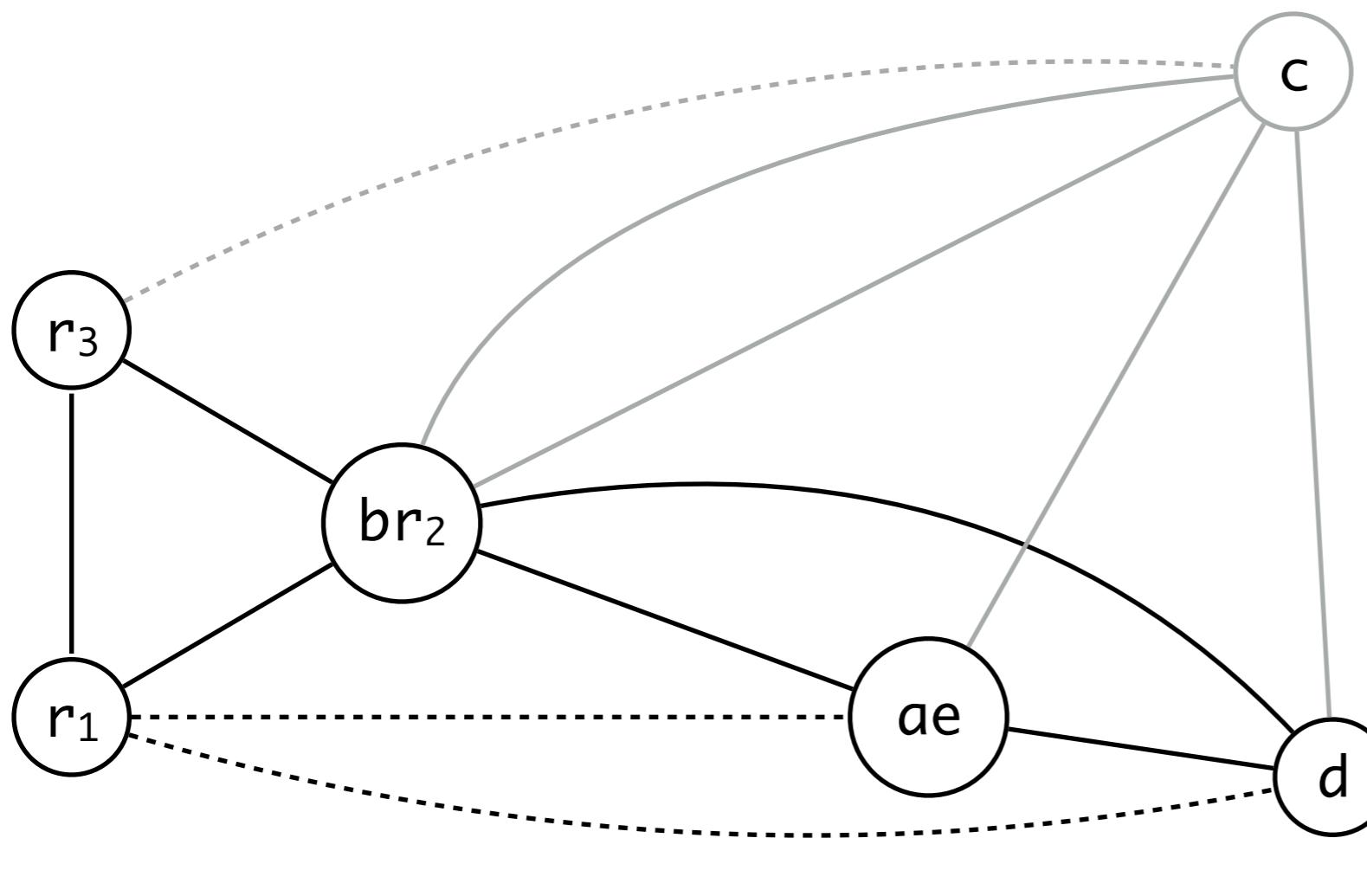


```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```

Pre-Colored Nodes

example

coalesce r_2 and b

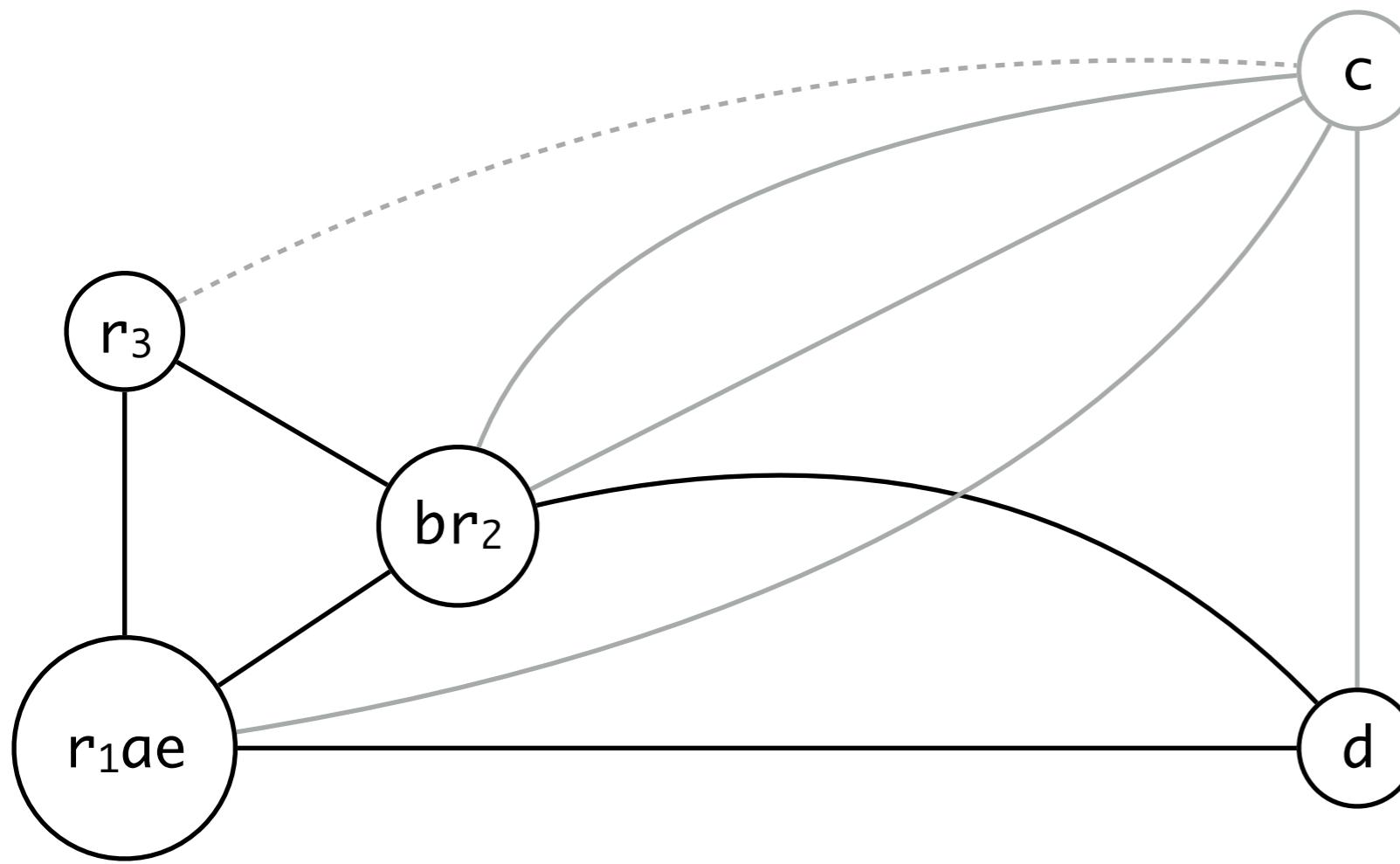


```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop  : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```

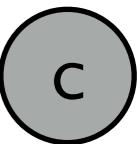
Pre-Colored Nodes

example

coalesce r_1 and ae



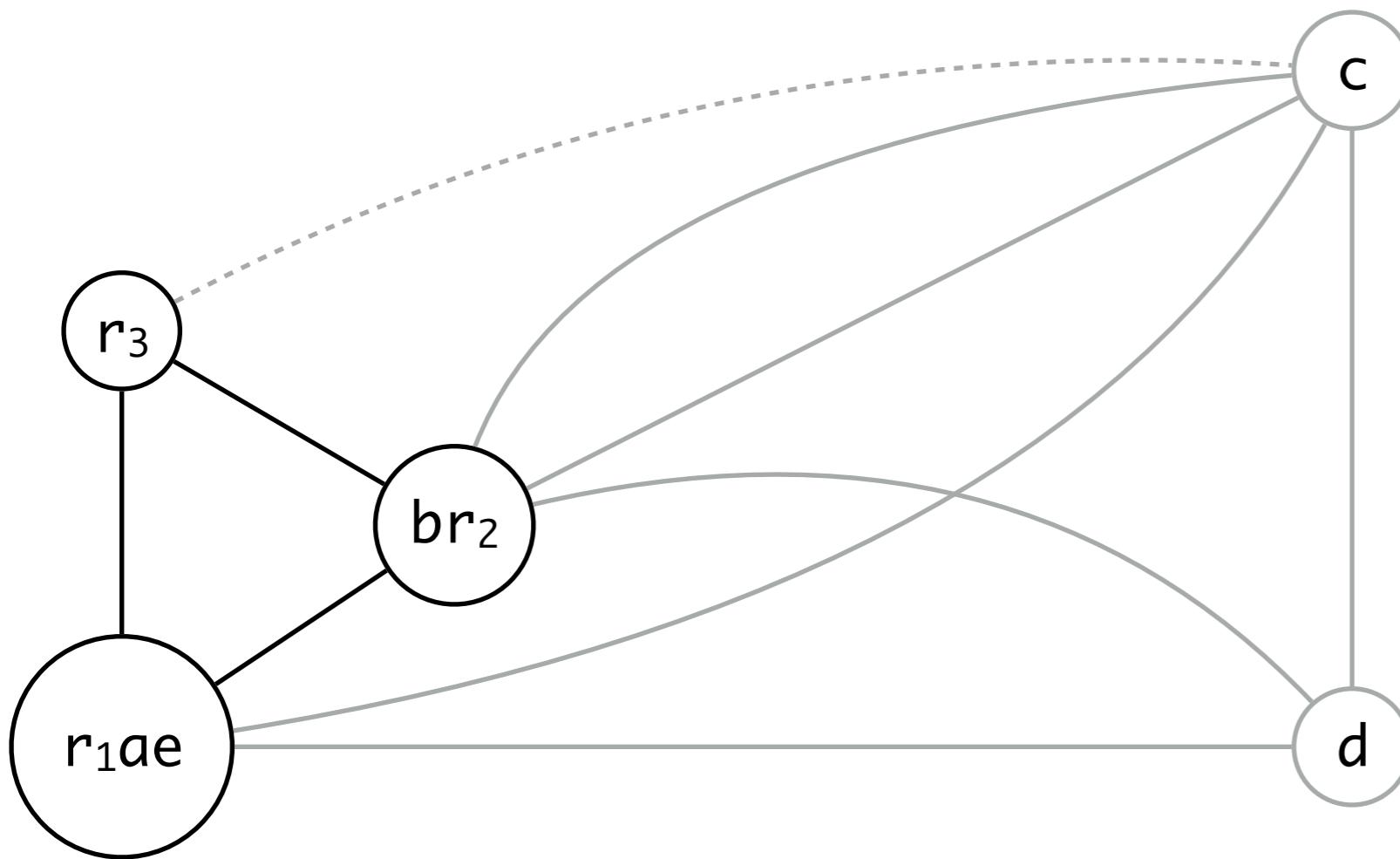
```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```



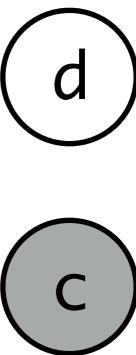
Pre-Colored Nodes

example

simplify d



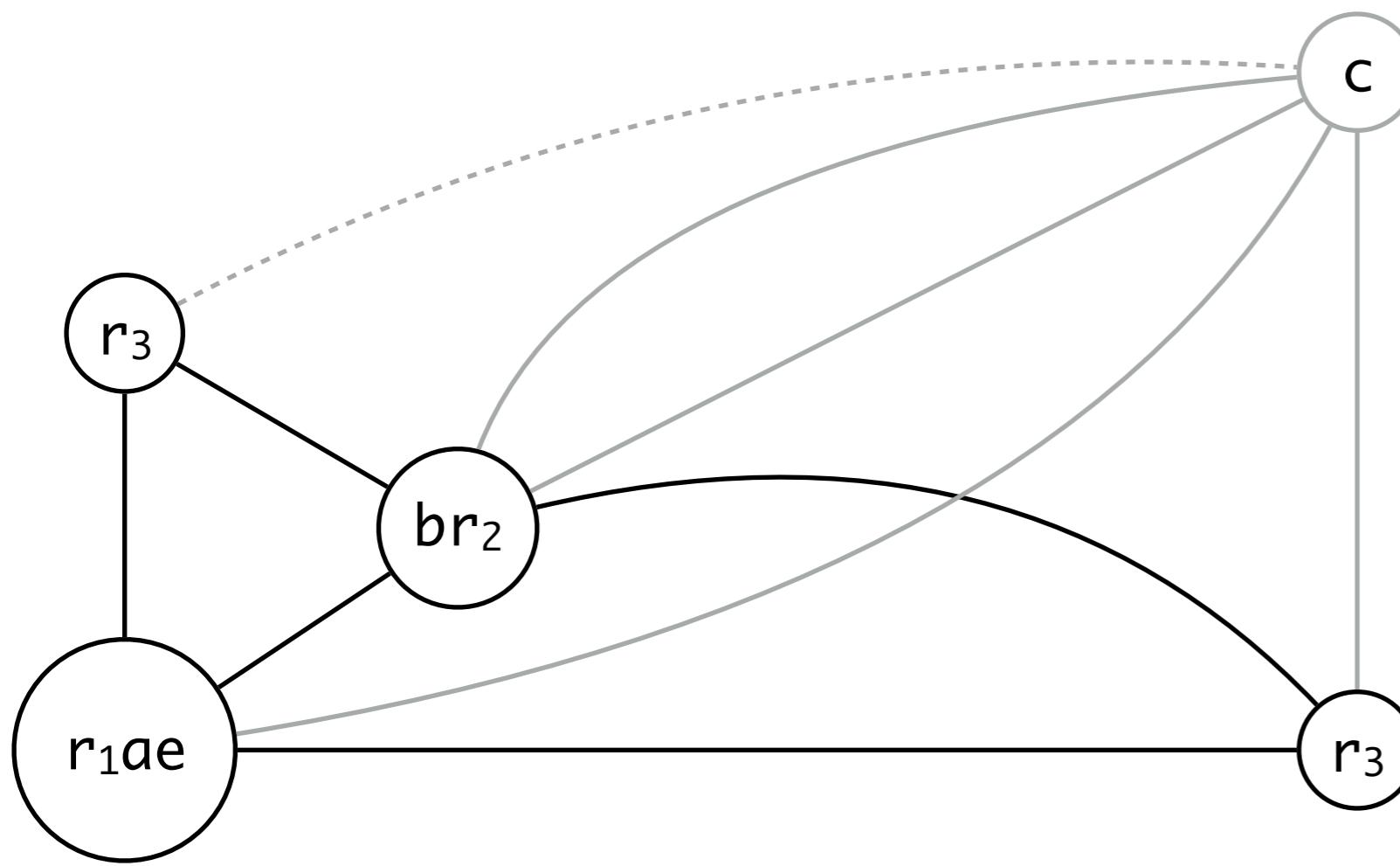
```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop  : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```



Pre-Colored Nodes

example

color d as r₃

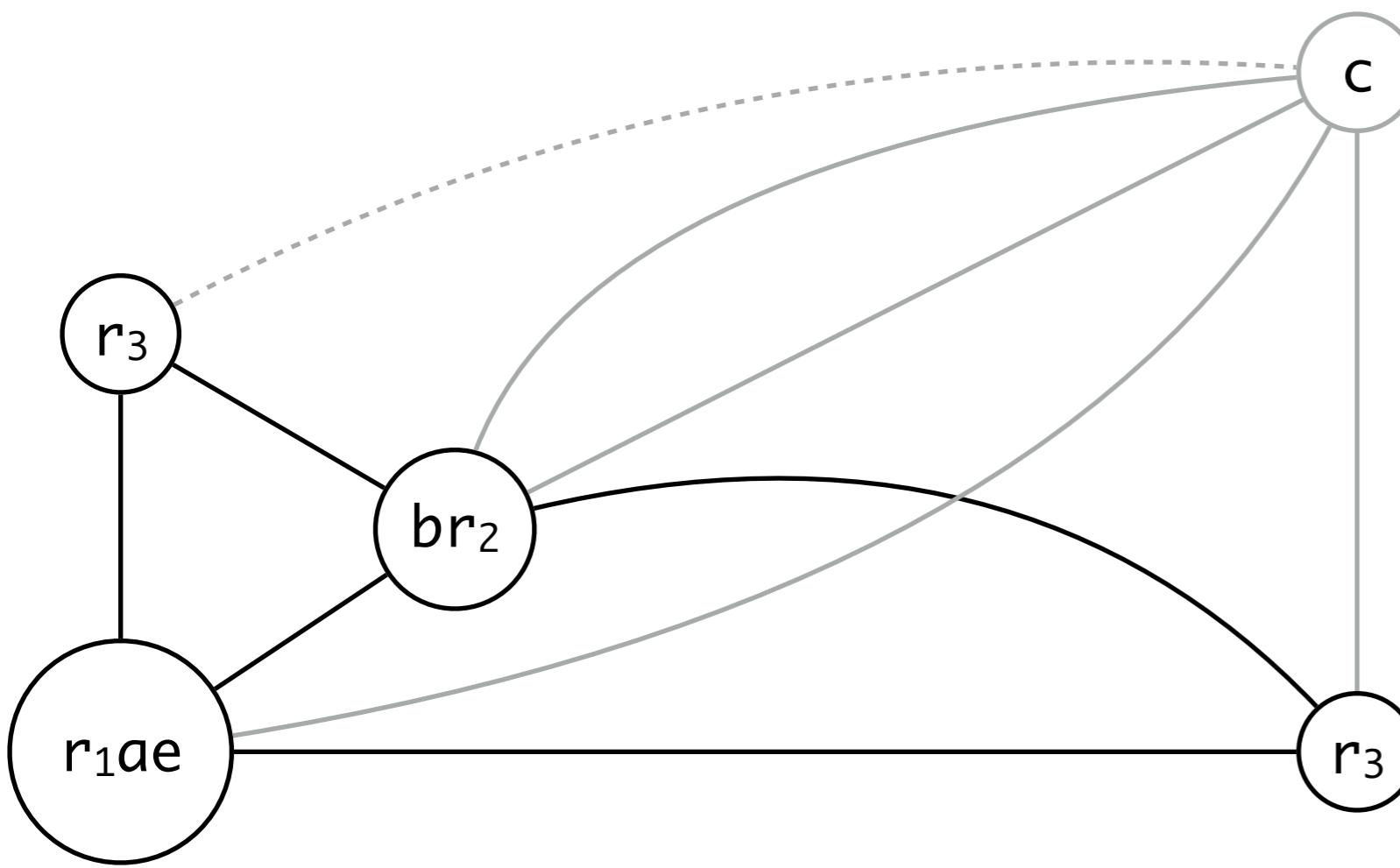


```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
return (r1, r3)
```

Pre-Colored Nodes

example

spill c

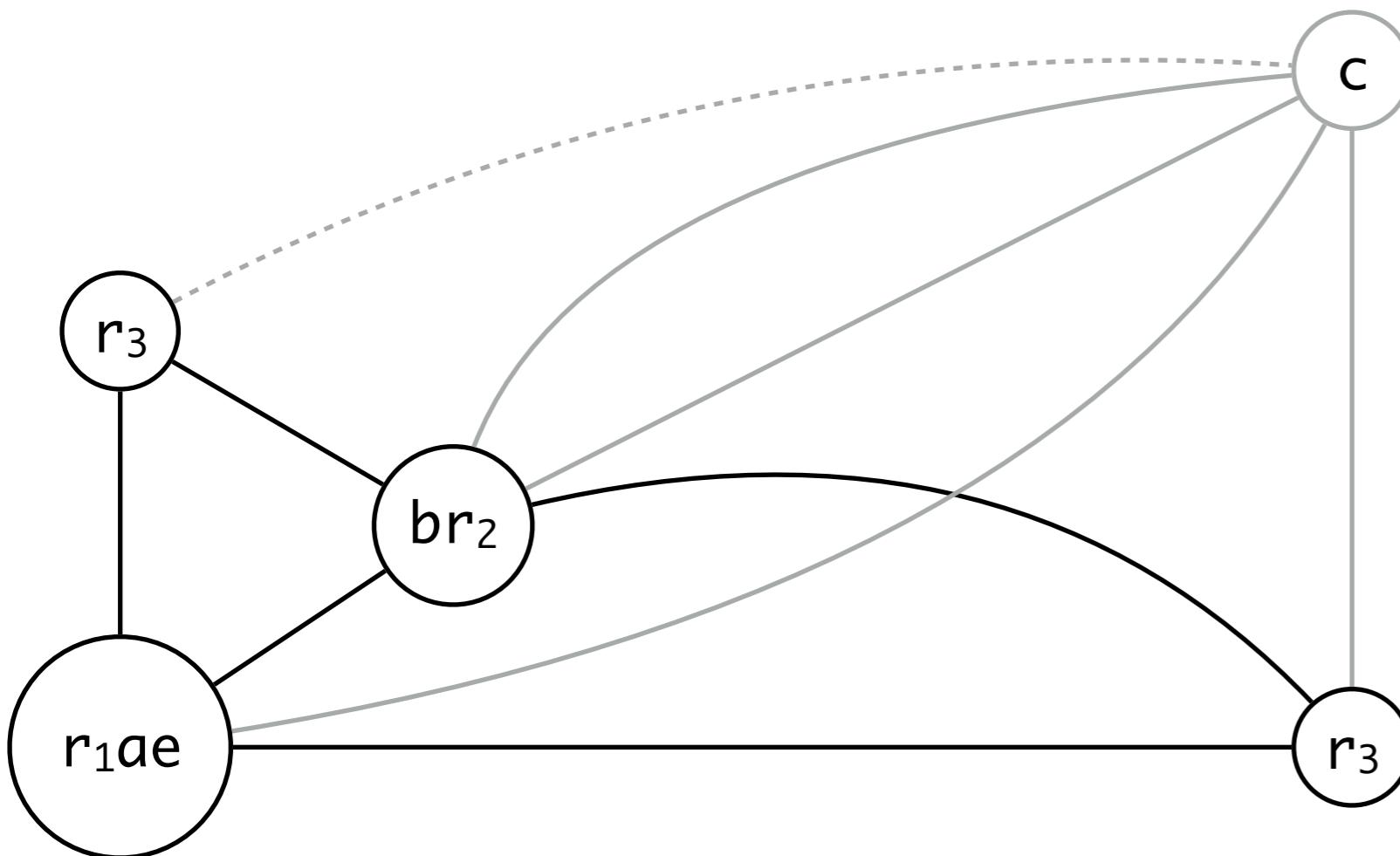


```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop  :  $d \leftarrow d + b$ 
         $e \leftarrow e - 1$ 
        if  $e > 0$  goto loop
 $r_1 \leftarrow d$ 
 $r_3 \leftarrow c_2$ 
 $c_2 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```

Pre-Colored Nodes

example

spill c

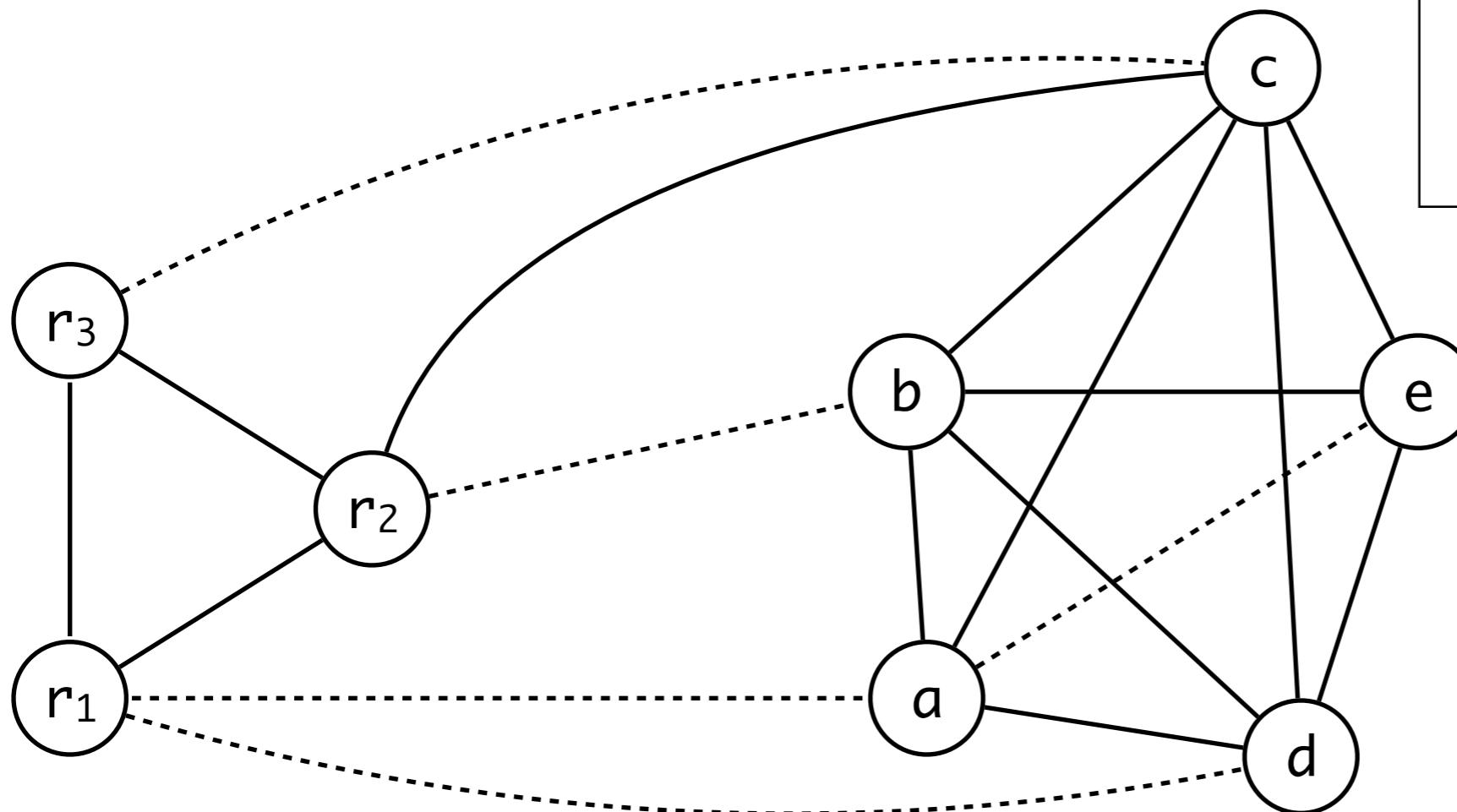


```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop  :  $d \leftarrow d + b$ 
         $e \leftarrow e - 1$ 
        if  $e > 0$  goto loop
 $r_1 \leftarrow d$ 
 $r_3 \leftarrow c_2$ 
 $c_2 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```

Pre-Colored Nodes

examples

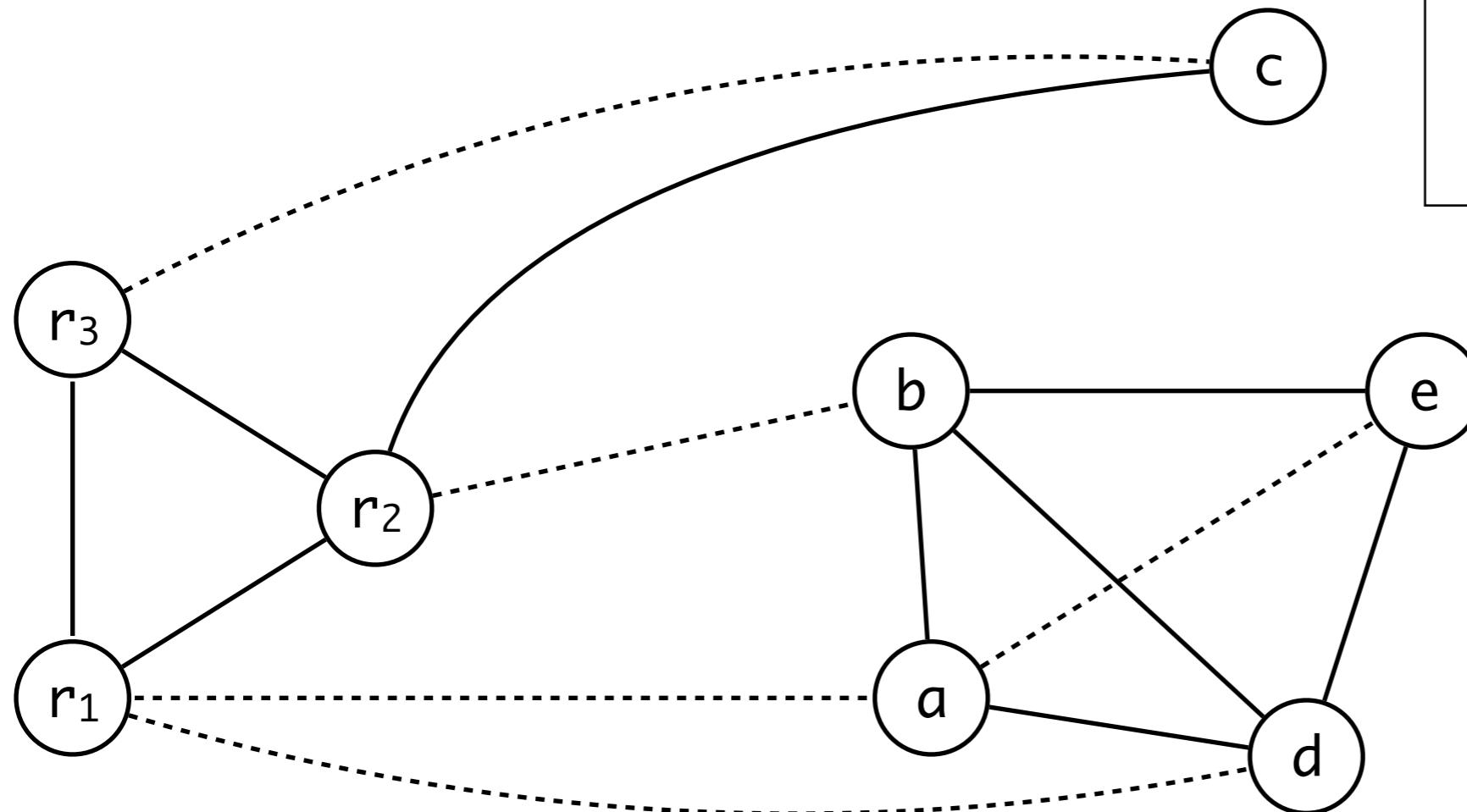
start over



```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop :  $d \leftarrow d + b$ 
         $e \leftarrow e - 1$ 
        if  $e > 0$  goto loop
 $r_1 \leftarrow d$ 
 $r_3 \leftarrow c_2$ 
 $c_2 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```

Pre-Colored Nodes

examples



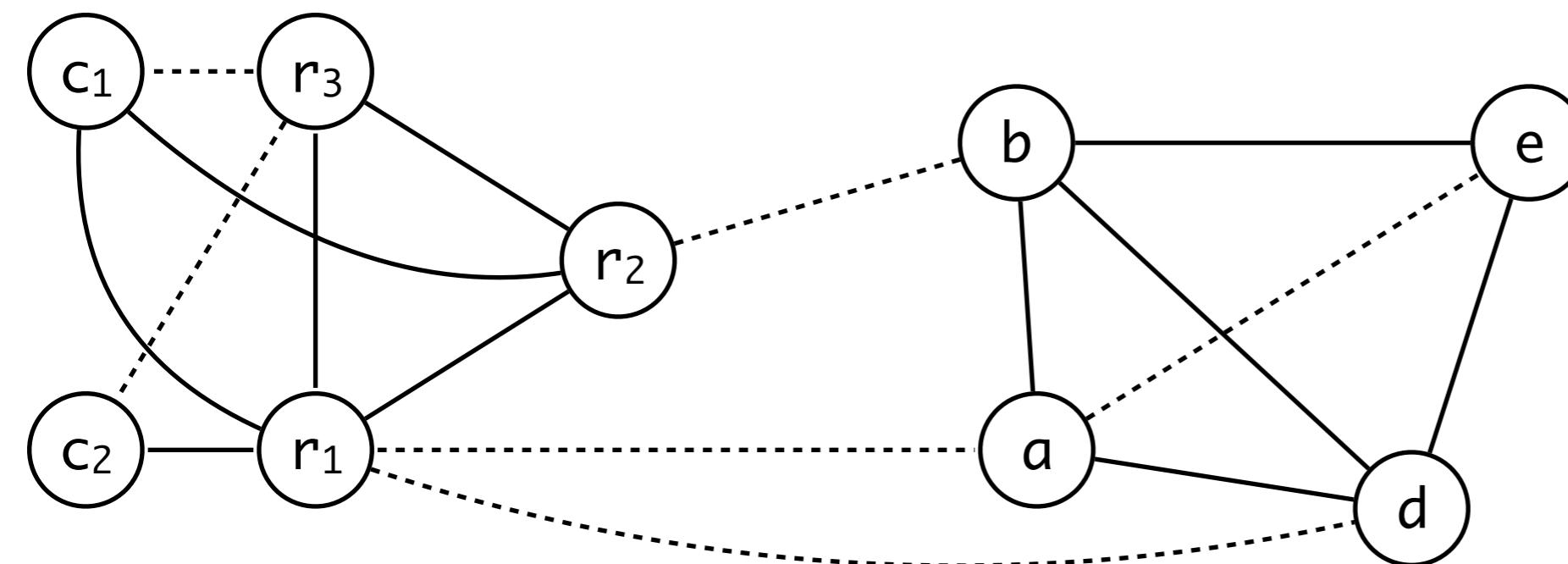
```
enter :  $c_1 \leftarrow r_3$   
 $M[c_{loc}] \leftarrow c_1$   
 $a \leftarrow r_1$   
 $b \leftarrow r_2$   
 $d \leftarrow 0$   
 $e \leftarrow a$   
loop :  $d \leftarrow d + b$   
 $e \leftarrow e - 1$   
if  $e > 0$  goto loop  
 $r_1 \leftarrow d$   
 $r_3 \leftarrow c_2$   
 $c_2 \leftarrow M[c_{loc}]$   
return ( $r_1, r_3$ )
```

Pre-Colored Nodes

examples

new graph

```
enter : c1 ← r3
        M[cloc] ← c1
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c2
        c2 ← M[cloc]
        return (r1, r3)
```

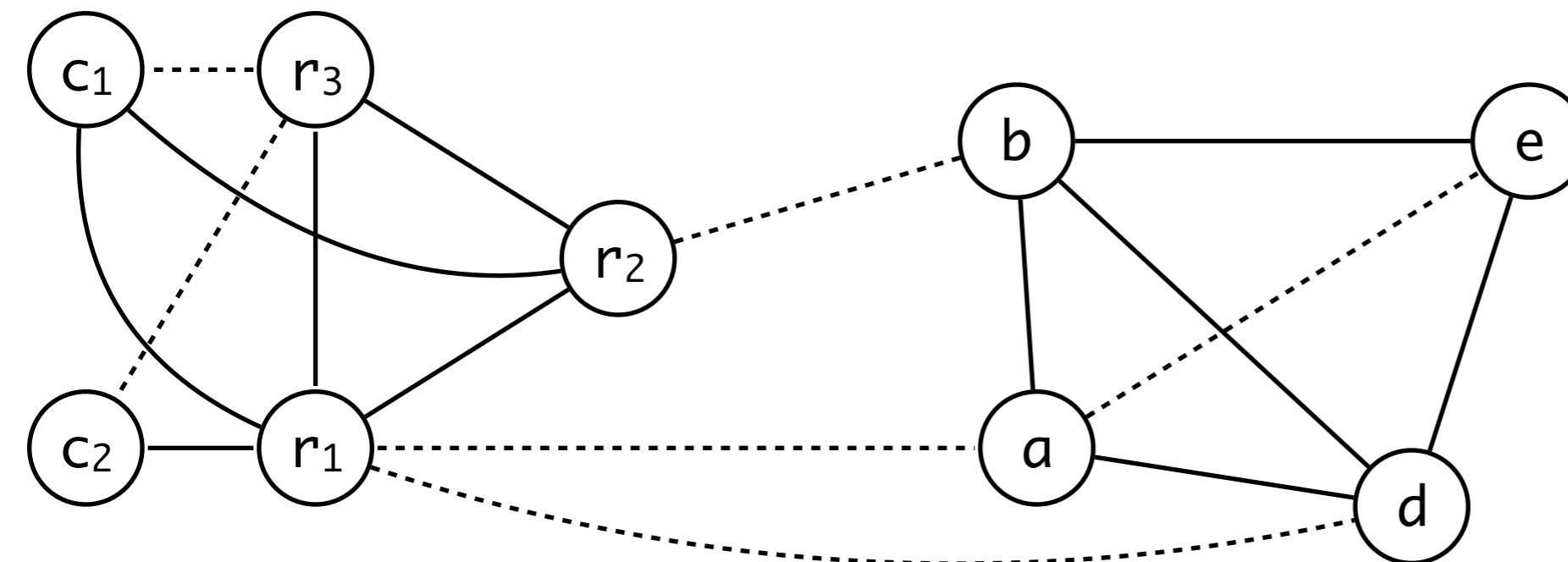


Pre-Colored Nodes

examples

coalesce c_1, c_2, r_3

```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop :  $d \leftarrow d + b$ 
        $e \leftarrow e - 1$ 
       if  $e > 0$  goto loop
        $r_1 \leftarrow d$ 
        $r_3 \leftarrow c_2$ 
        $c_2 \leftarrow M[c_{loc}]$ 
       return ( $r_1, r_3$ )
```

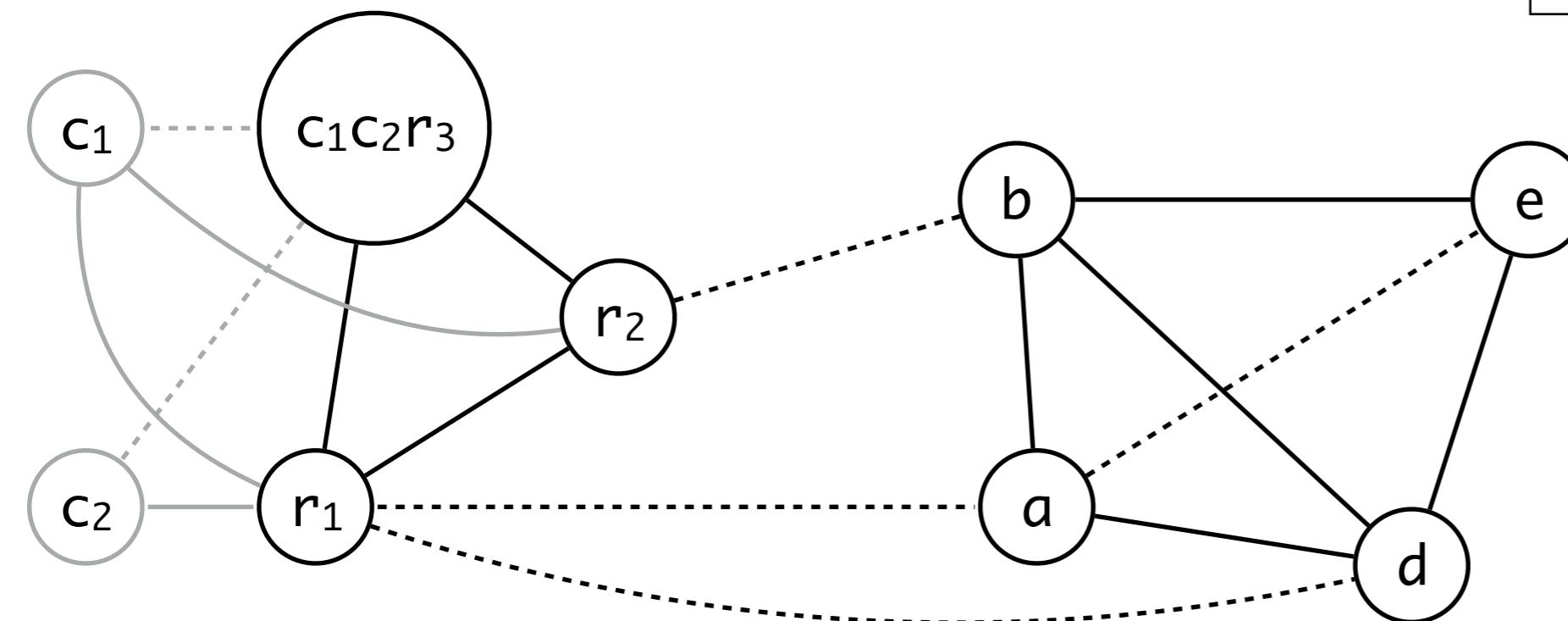


Pre-Colored Nodes

examples

coalesce c_1, c_2, r_3

```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop :  $d \leftarrow d + b$ 
        $e \leftarrow e - 1$ 
       if  $e > 0$  goto loop
        $r_1 \leftarrow d$ 
        $r_3 \leftarrow c_2$ 
        $c_2 \leftarrow M[c_{loc}]$ 
       return ( $r_1, r_3$ )
```

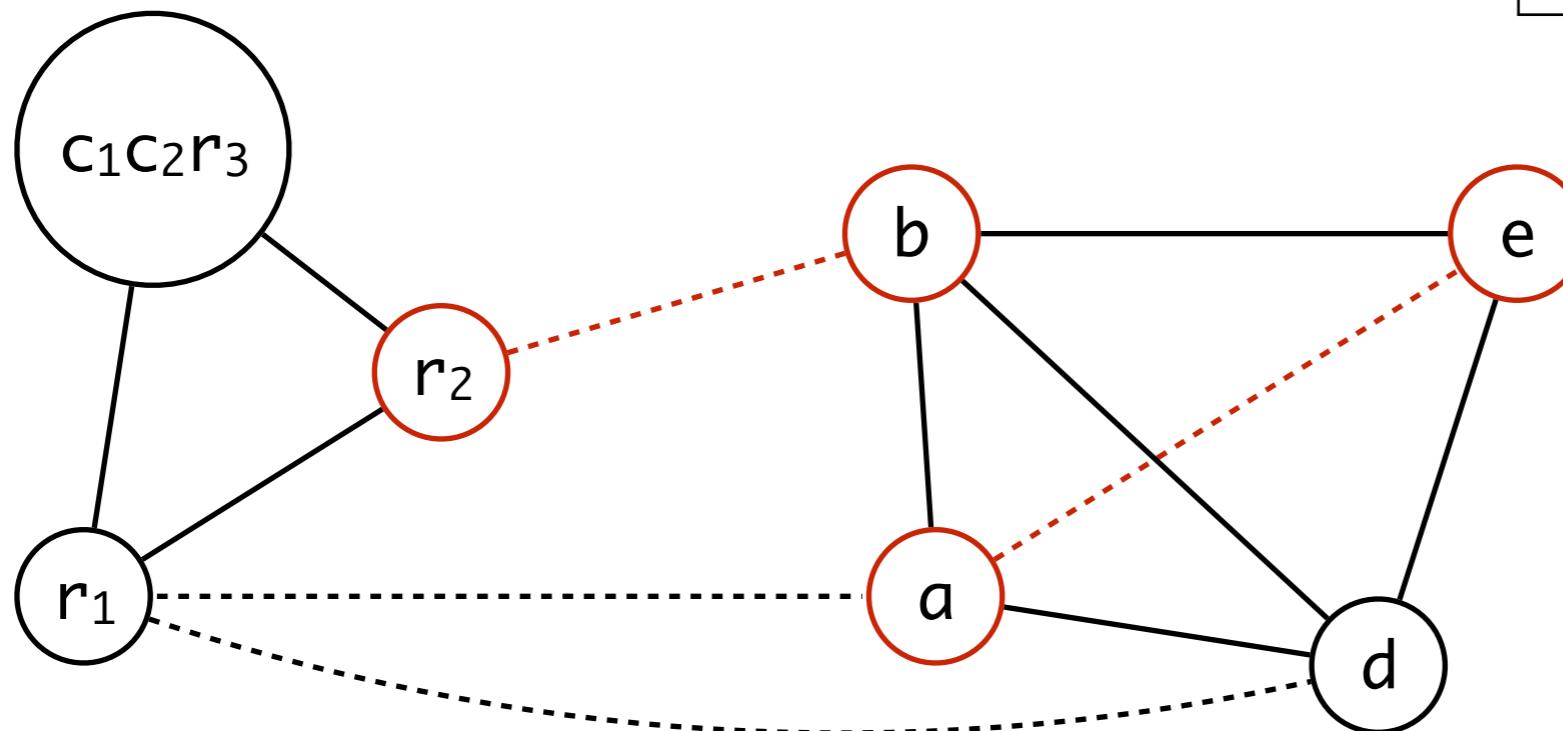


Pre-Colored Nodes

examples

coalesce (b, r_2) and (a, e)

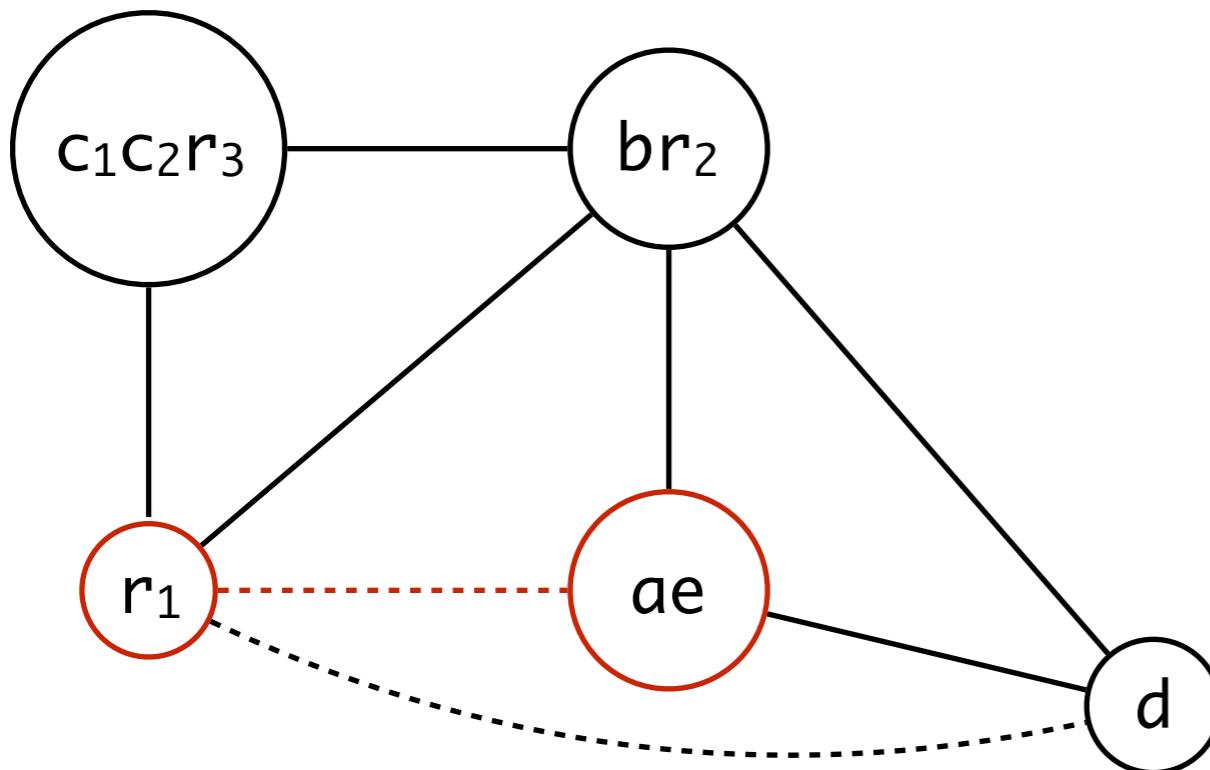
```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop :  $d \leftarrow d + b$ 
         $e \leftarrow e - 1$ 
        if  $e > 0$  goto loop
 $r_1 \leftarrow d$ 
 $r_3 \leftarrow c_2$ 
 $c_2 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```



Pre-Colored Nodes

examples

coalesce (ae , r_1)

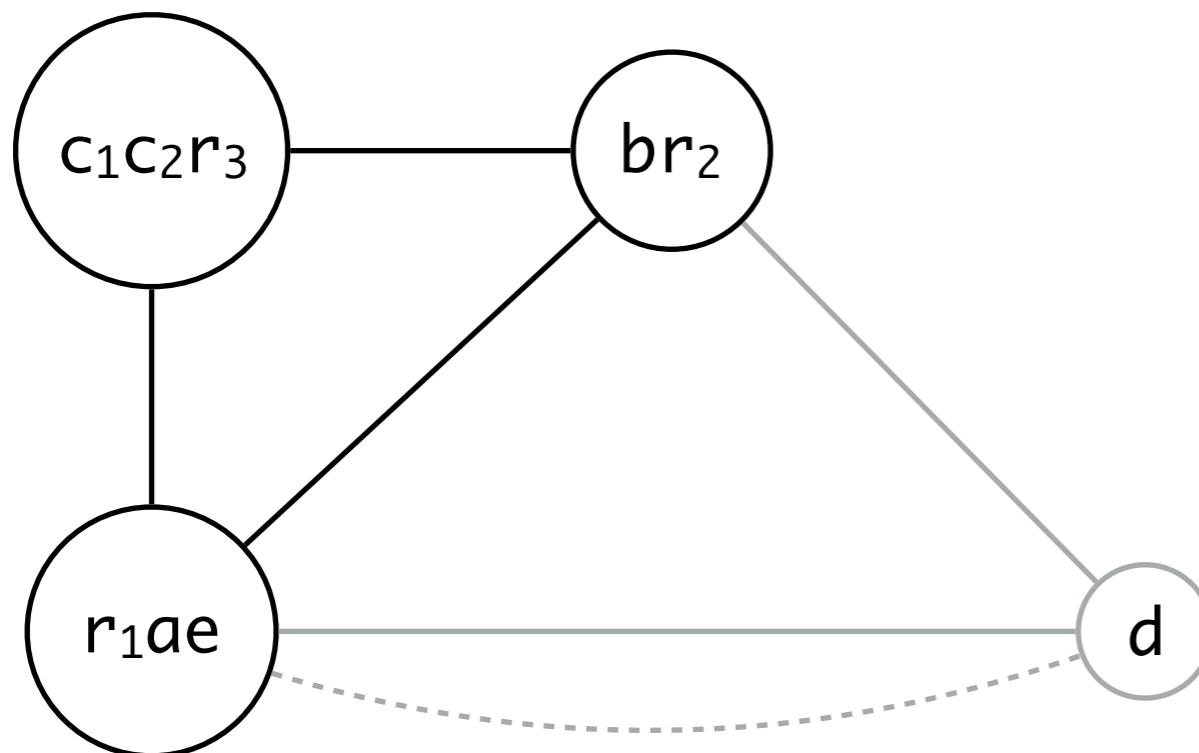


```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop :  $d \leftarrow d + b$ 
         $e \leftarrow e - 1$ 
        if  $e > 0$  goto loop
 $r_1 \leftarrow d$ 
 $r_3 \leftarrow c_2$ 
 $c_2 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```

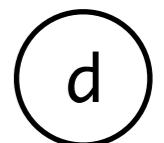
Pre-Colored Nodes

examples

simplify d



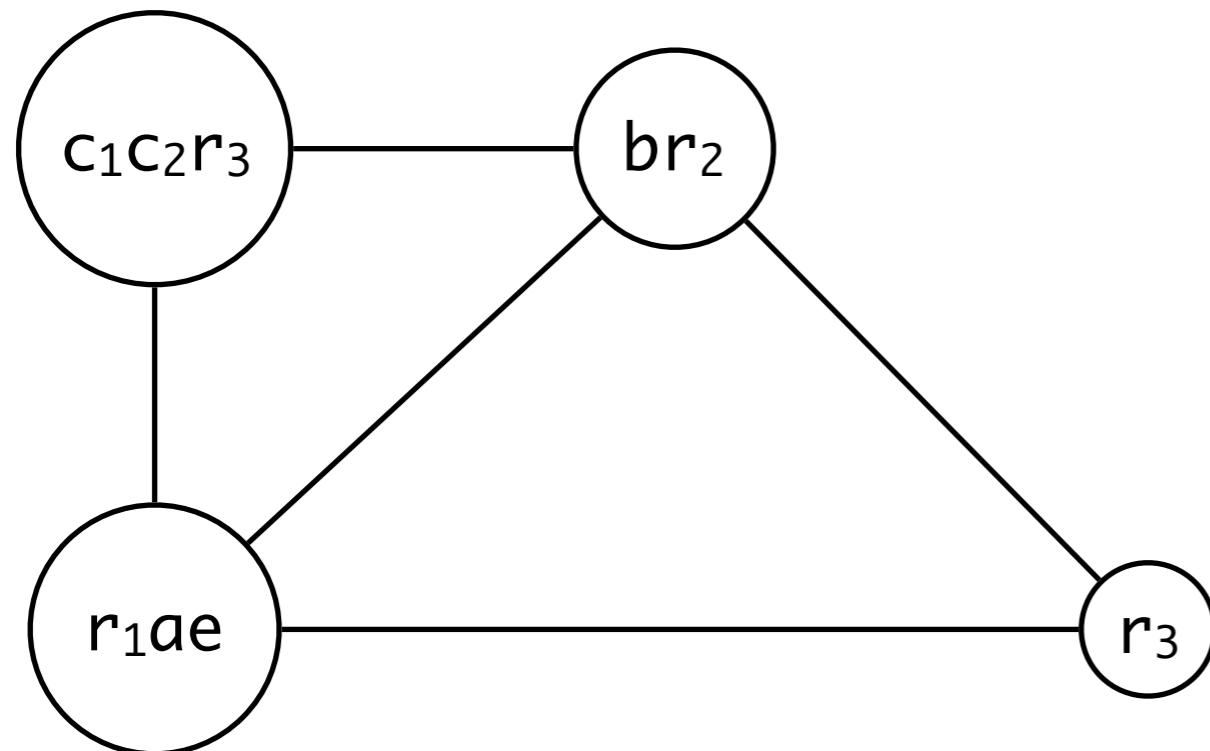
```
enter :  $c_1 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow c_1$ 
         $a \leftarrow r_1$ 
         $b \leftarrow r_2$ 
         $d \leftarrow 0$ 
         $e \leftarrow a$ 
loop :  $d \leftarrow d + b$ 
         $e \leftarrow e - 1$ 
        if  $e > 0$  goto loop
 $r_1 \leftarrow d$ 
 $r_3 \leftarrow c_2$ 
 $c_2 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```



Pre-Colored Nodes

examples

color d as r₃

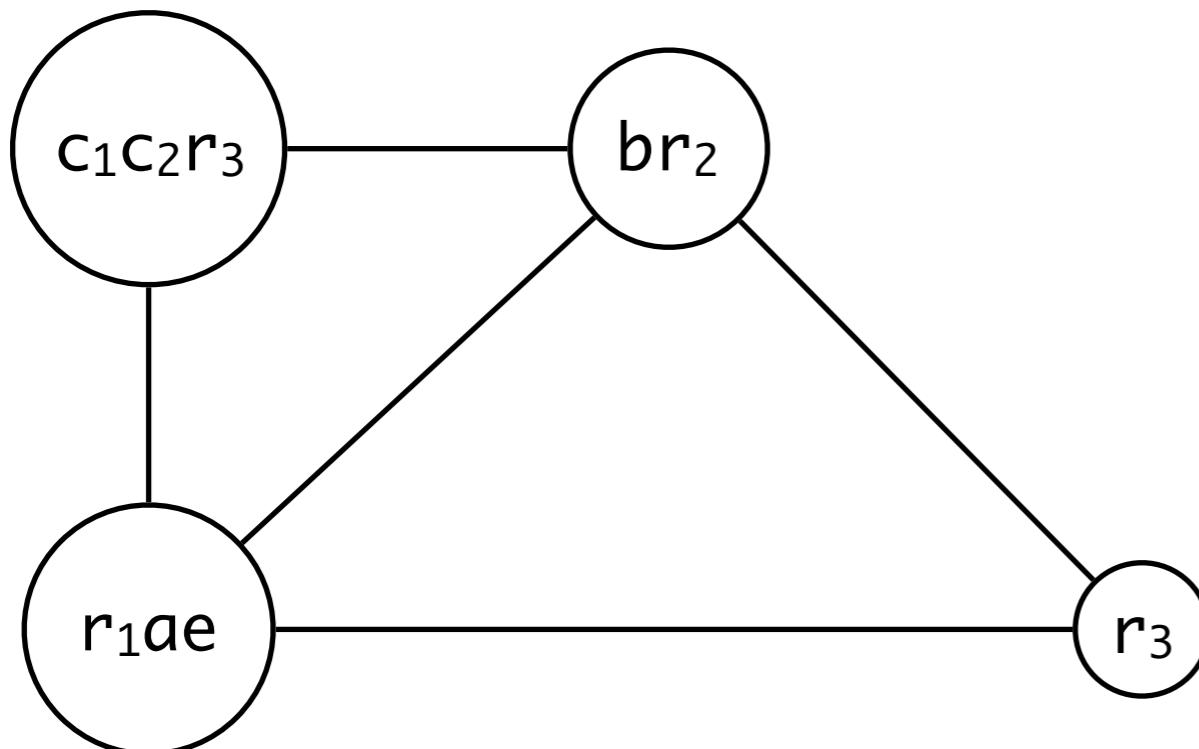


```
enter : c1 ← r3
        M[cloc] ← c1
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop : d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c2
        c2 ← M[cloc]
        return (r1, r3)
```

Pre-Colored Nodes

examples

apply register assignment



```
enter :  $r_3 \leftarrow r_3$ 
         $M[c_{loc}] \leftarrow r_3$ 
         $r_1 \leftarrow r_1$ 
         $r_2 \leftarrow r_2$ 
         $r_3 \leftarrow 0$ 
         $r_1 \leftarrow r_1$ 
loop :  $r_3 \leftarrow r_3 + r_2$ 
         $r_1 \leftarrow r_1 - 1$ 
        if  $r_1 > 0$  goto loop
         $r_1 \leftarrow r_3$ 
         $r_3 \leftarrow r_3$ 
         $r_3 \leftarrow M[c_{loc}]$ 
return ( $r_1, r_3$ )
```

Pre-Colored Nodes

example

```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop :  d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
        return (r1, r3)
```

```
enter : r3 ← r3
        M[cloc] ← r3
        r1 ← r1
        r2 ← r2
        r3 ← 0
        r1 ← r1
loop : r3 ← r3 + r2
        r1 ← r1 - 1
        if r1 > 0 goto loop
        r1 ← r3
        r3 ← r3
        r3 ← M[cloc]
        return (r1, r3)
```

Pre-Colored Nodes

example

```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop :  d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
        return (r1, r3)
```

```
enter : r3 ← r3
        M[cloc] ← r3
        r1 ← r1
        r2 ← r2
        r3 ← 0
        r1 ← r1
loop : r3 ← r3 + r2
        r1 ← r1 - 1
        if r1 > 0 goto loop
        r1 ← r3
        r3 ← r3
        r3 ← M[cloc]
        return (r1, r3)
```

Pre-Colored Nodes

example

```
enter : c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop :  d ← d + b
        e ← e - 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
        return (r1, r3)
```

```
enter : M[cloc] ← r3
        r3 ← 0
loop :  r3 ← r3 + r2
        r1 ← r1 - 1
        if r1 > 0 goto loop
        r1 ← r3
        r3 ← M[cloc]
        return (r1, r3)
```

Pre-Colored Nodes

example

```
int f(int a, int b) {  
    int d = 0;  
    int e = a;  
    do {  
        d = d + b;  
        e = e - 1;  
    } while (e > 0);  
    return d;  
}
```

```
enter : M[cloc] ← r3  
        r3 ← 0  
loop :  r3 ← r3 + r2  
        r1 ← r1 - 1  
        if r1 > 0 goto loop  
        r3 ← M[cloc]  
        return (r1, r3)
```

VI

Summary

Summary lessons learned

How can we assign registers to local variables and temporaries?

- perform liveness analysis
- build interference graph
- color interference graph

What to do if the graph is not colorable?

- keep local variables in memory

How to handle move instructions efficiently?

- coalesce nodes safely

Literature

[learn more](#)

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

Lal George, Andrew W. Appel: Iterative Register Coalescing.
POPL 1996

Lal George, Andrew W. Appel: Iterative Register Coalescing.
TOPLAS 18(3), 1996

Outlook

coming next

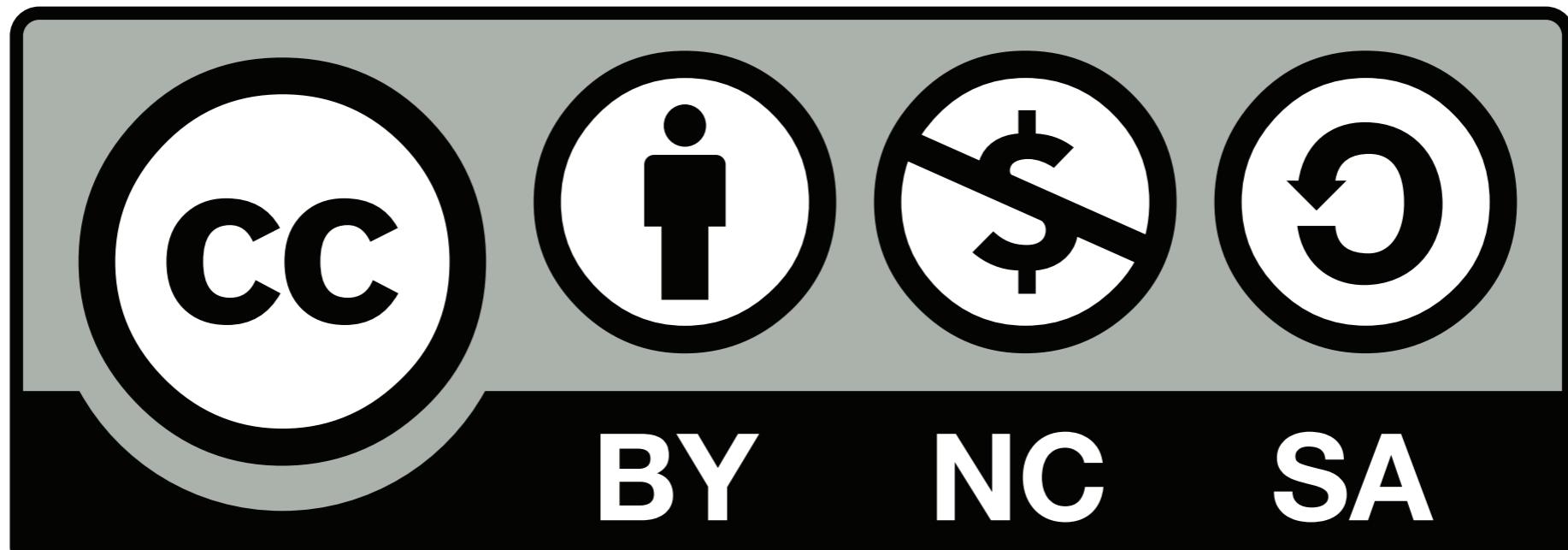
Compiler components & their generators

- Lecture 12: Data-Flow Analysis **Dec 6**
- Lecture 13: Register Allocation **Dec 13**
- Lecture 14: LL Parsing **Dec 20**
- Lecture 15: LR Parsing **Jan 10**

Exam preparation

- Question & Answer & Outlook **Jan 24**
- Exam **Jan 31**

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