Dataflow Analysis

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Overview

today's lecture

Control flow graphs

- intermediate representation

Non-local optimizations

- dead code elimination
- constant & copy propagation
- common subexpression elimination

Data flow analyses

- liveness analysis
- reaching definitions
- available expressions

Optimizations (again)

I

Optimizations

Optimizing Compilers

optimization

Fully optimizing compiler

- Opt(P) produces smallest program with same I/O behavior
- smallest program for non-terminating programs without I/O:
  Loop = (L : goto L)
- solves halting problem: Opt(Q) = Loop iff Q halts

Optimizing compiler

- produces program with same I/O behavior
- that is smaller or faster

Full employment theorem for compiler writers

- there is always a better optimizing compiler

Local vs Non-Local Optimizations

optimization

Local optimizations

- peephole optimization
- constant folding

- pattern match + rewrite

```
iload 1        iload 1
iload 1   =>   dup
mul            mul
```

```
(3 + 4) * x   =>   7 * x
```

Non-local optimizations

- constant propagation
- dead-code elimination

- require information from different parts of program

Program Optimizations
optimization

Register allocation

- keep non-overlapping temporaries in same register

Common-subexpression elimination

- if expression is computed more than once, eliminate one computation

Dead-code elimination

- delete computation whose result will never be used

Constant folding

- if operands of expression are constant, do computation at compile-time

And many more possible optimizations

Dead Code Elimination

example

```
a ← 0
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← 0
b ← a + 1
c ← c + b

return c
```

delete computation whose result will never be used

Constant Propagation
example

```
a ← 0
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← 0
b ← 0 + 1
c ← c + b
a ← 2 * b
return c
```

substitute reference to constant valued variable

Constant Propagation + Folding

example

```
a ← 1
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

⇒

```
a ← 0
b ← 2
c ← c + 2
a ← 4
return c
```

Copy Propagation
example

```
a ← e
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

⇒

```
a ← e
b ← e + 1
c ← c + b
a ← 2 * b
return c
```

Common Subexpression Elimination

example

c	←	a	+	b
d	←	1		
e	←	a	+	b

⇒

x	←	a	+	b
c	←	x		
d	←	1		
e	←	x		

if expression is computed more than once, eliminate one computation

Intraprocedural Global Optimization

optimization

Terminology

- Intraprocedural: within a single procedure or function
- Global: spans all statements with procedure
- Interprocedural: operating on several procedures at once

Recipe

- Dataflow analysis: traverse flow graph, gather information
- Transformation: modify program using information from analysis

II

Control-flow Graphs

Intermediate Language

quadruples

Store

$a \leftarrow b \oplus c$

$a \leftarrow b$

Memory access

$a \leftarrow M[b]$

$M[a] \leftarrow b$

Functions

$f(a_1, \ldots, a_n)$

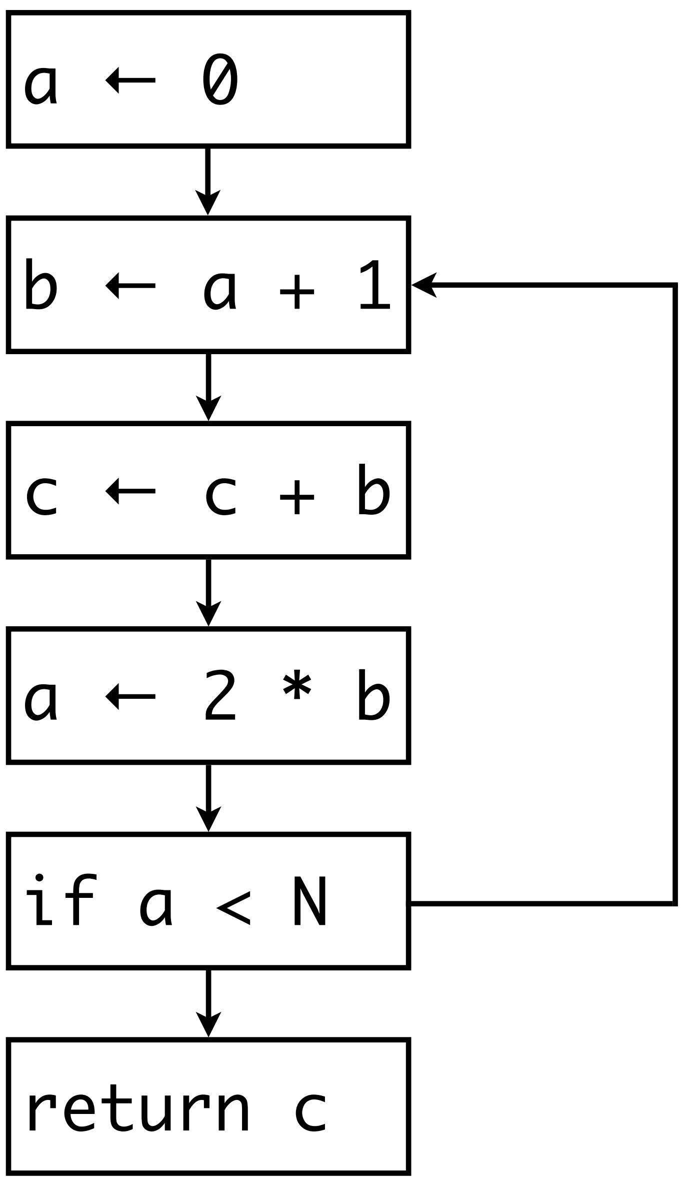
$b \leftarrow f(a_1, \ldots, a_n)$

Jumps

```
L:
goto L
if a ⊗ b
   goto L1
else
   goto L2
```

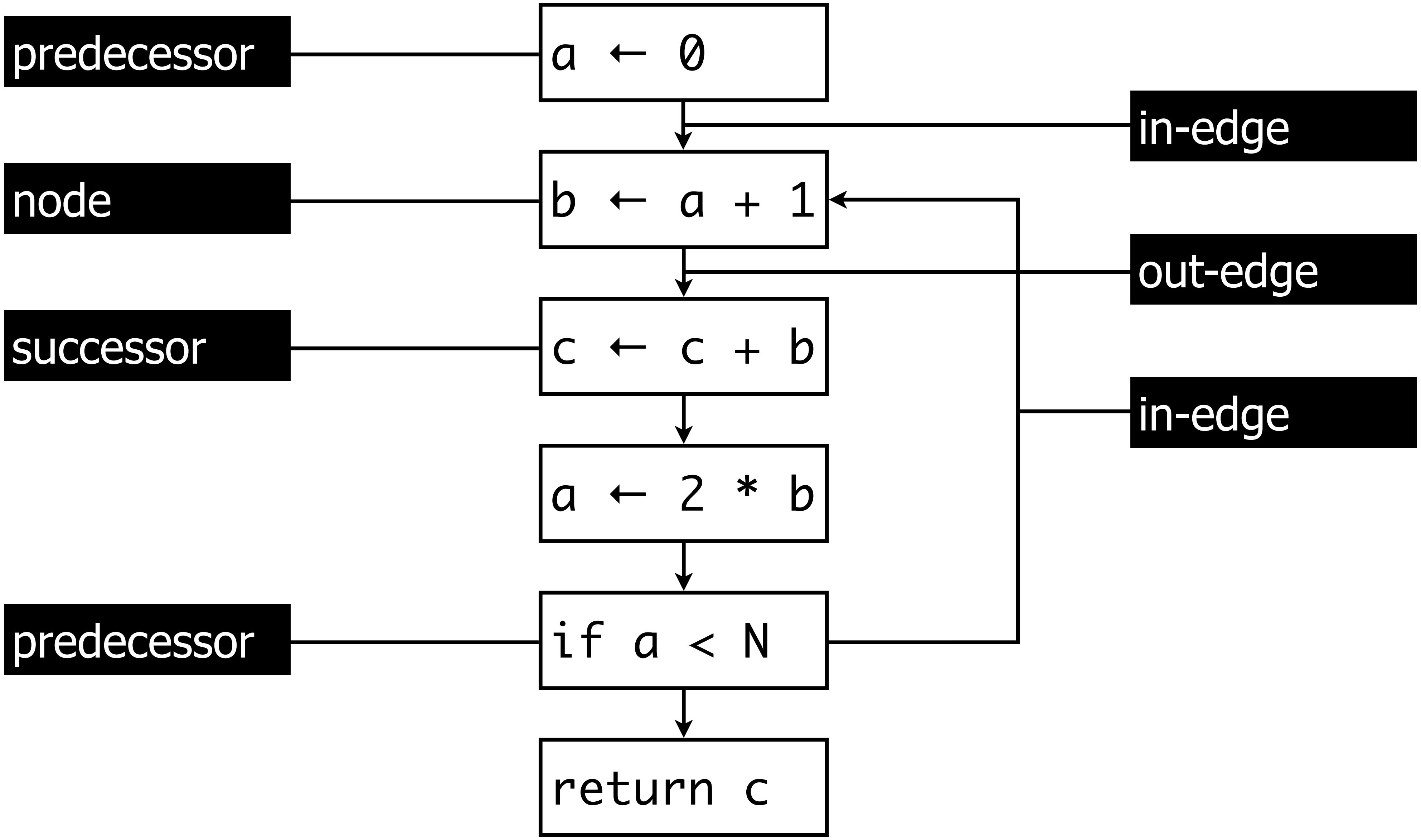
Control-Flow Graphs
example

```
    a ← 0
L1: b ← a + 1
    c ← c + b
    a ← 2 * b
    if a < N
       goto L1
    else
       goto L2
L2: return c
```



Control-Flow Graphs

terminology



III

Liveness Analysis

Liveness Analysis

definition

Motivation: register allocation

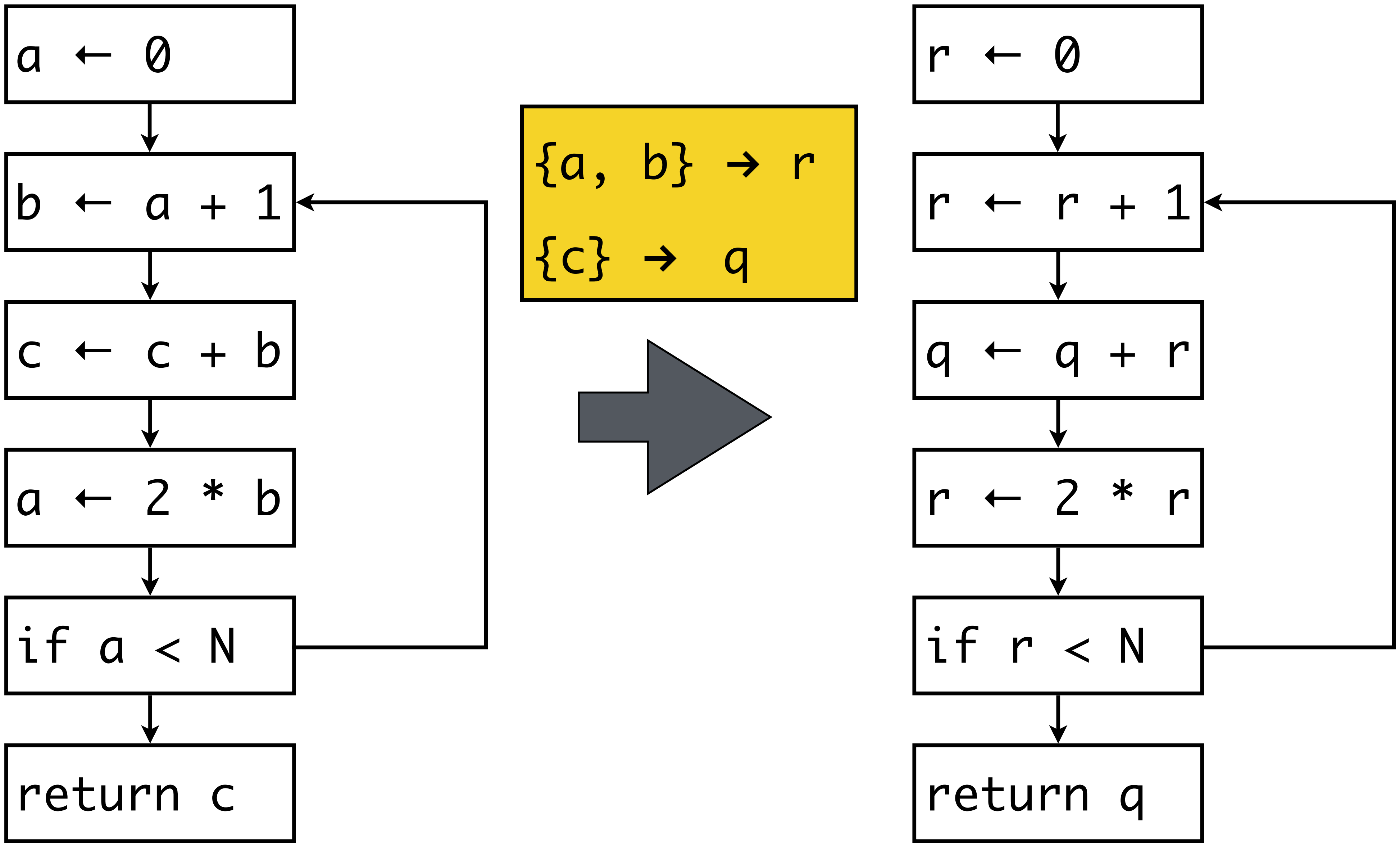
- intermediate code with unbounded number of temporary variables
- map to bounded number of registers
- if two variables are not ‘live’ at the same time, store in same register

Liveness

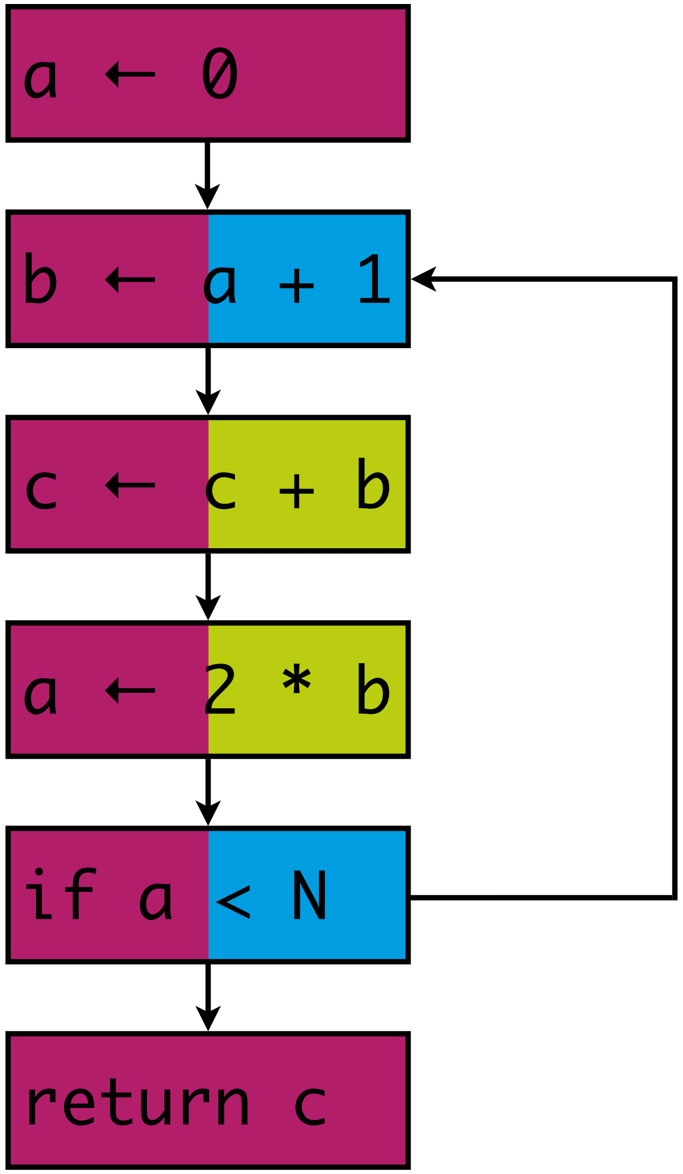
- a variable is **live** if it holds a value that may be needed in the future

Register Allocation

example



Liveness Analysis
example



Liveness Analysis
definitions

Def and **use**

- an assignment to a variable **defines** a variable
- an occurrence of a variable in an expression **uses** that variable
- def[x] = {n | n defines x}, def[n] = {x | n defines x}
- use[x] = {n | n uses x}, use[n] = {x | n uses x}

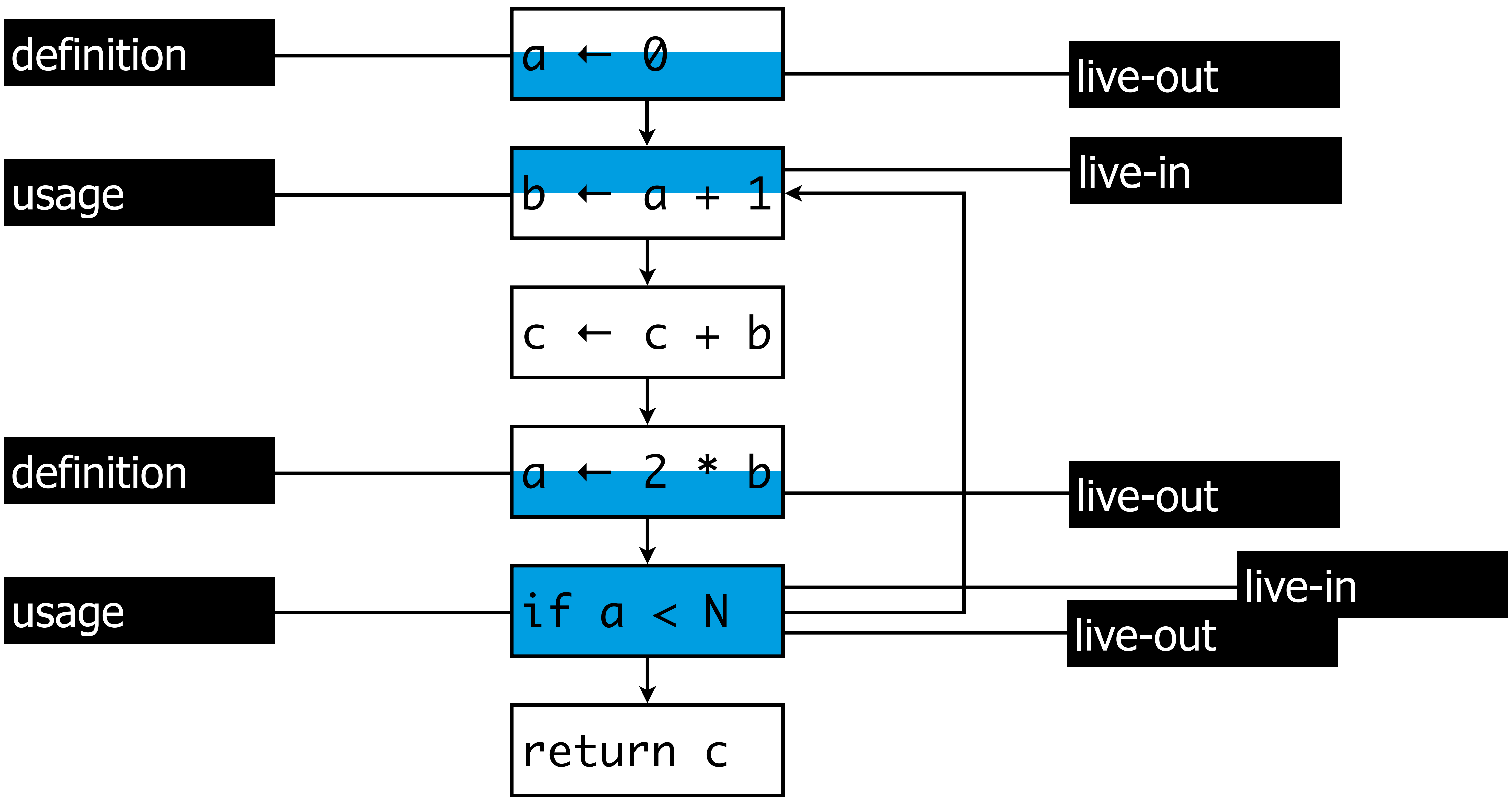
A variable is **live on an edge** if

- there is a **directed path** from that edge **to** a **use** of the variable
- that does **not** go **through** any **def**

A variable is **live in/out at a node**

- **live-in**: it is live on any of the in-edges of that node
- **live-out**: it is live on any of the out-edges of the node

Liveness Analysis
terminology



Liveness Analysis

formalization

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

1. If a variable is used at node n, then it is live-in at n

2. If a variable is live-out but not defined at node n, it is live-in at n

3. If a variable is live-in at node n, then it is live-out at its predecessors

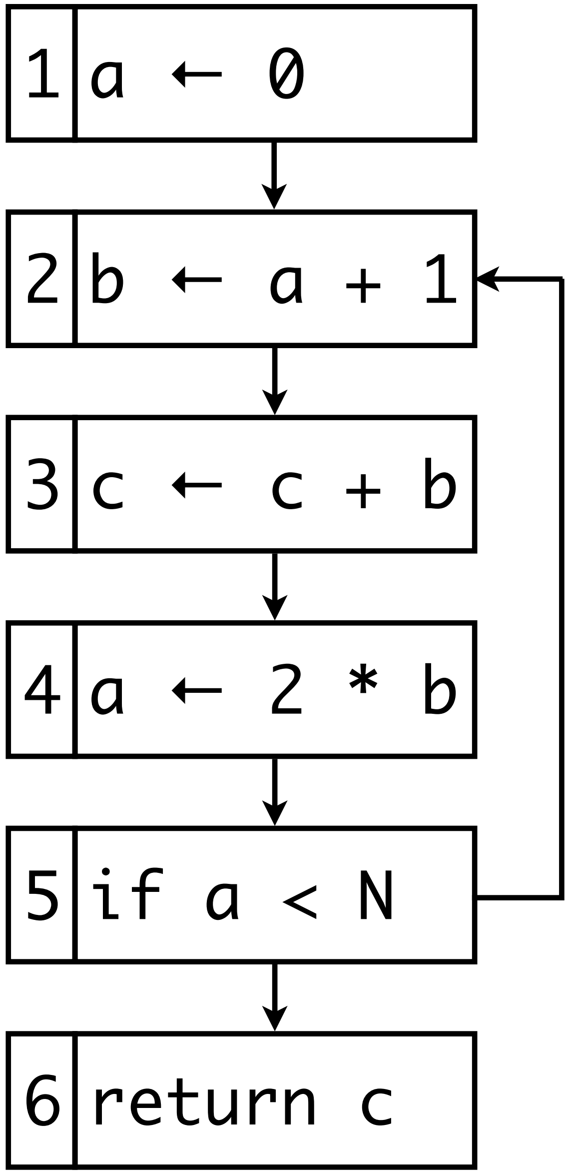
Liveness Analysis

algorithm

```
for each n
    in[n] ← {} ; out[n] ← {}
repeat
    for each n
        in'[n] = in[n] ; out'[n] = out[n]
        in[n] = use[n] ∪ (out[n] - def[n])
        for each s in succ[n]
            out[n] = out[n] ∪ in[s]
until
    for all n: in[n] = in'[n] and out[n] = out'[n]
```

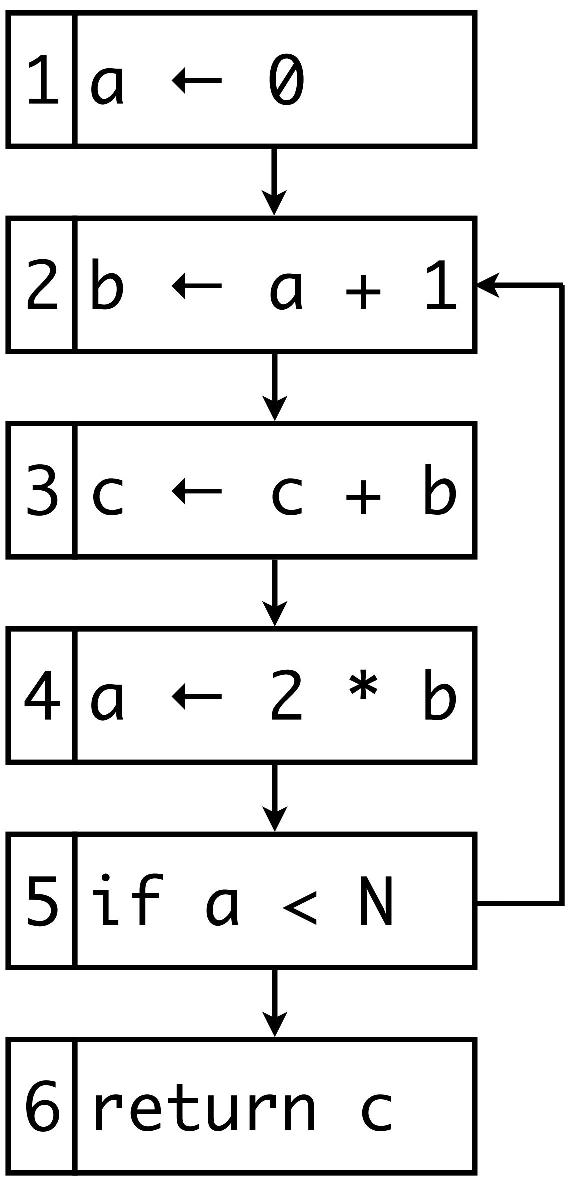
Liveness Analysis

example



			1		2		3		4		5		6		7	
	u	d	i	o	i	o	i	o	i	o	i	o	i	o	i	o
1		a				a		a		ac	c	ac	c	ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b	bc	bc	bc	bc
4	b	a	b		b	a	b	a	b	ac	bc	ac	bc	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c		c		c	

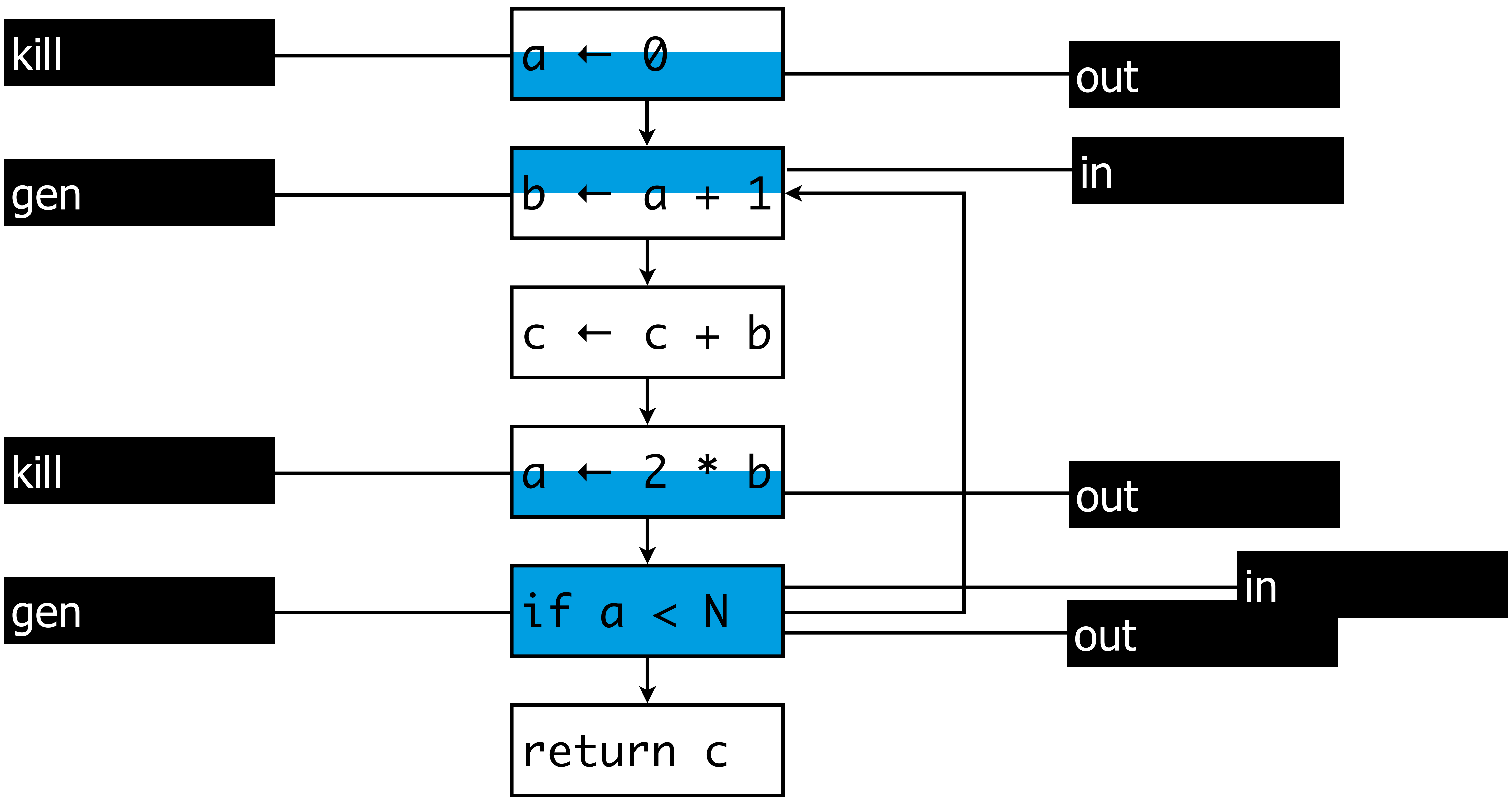
Liveness Analysis
optimization



			1		2		3	
	u	d	o	i	o	i	o	i
6	c			c		c		c
5	a		c	ac	ac	ac	ac	ac
4	b	a	ac	bc	ac	bc	ac	bc
3	bc	c	bc	bc	bc	bc	bc	bc
2	a	b	bc	ac	bc	ac	bc	ac
1		a	ac	a	ac	c	ac	c

Liveness Analysis

generalisation



Liveness Analysis

gen & kill

	gen	kill
$a \leftarrow b \oplus c$	$\{b,c\}$	$\{a\}$
$a \leftarrow b$	$\{b\}$	$\{a\}$
$a \leftarrow M[b]$	$\{b\}$	$\{a\}$
$M[a] \leftarrow b$	$\{a,b\}$	
$f(a_1, \ldots, a_n)$	$\{a_1,\ldots,a_n\}$	
$a \leftarrow f(a_1, \ldots, a_n)$	$\{a_1,\ldots,a_n\}$	$\{a\}$
goto L		
if $a \otimes b$	$\{a,b\}$	

Liveness Analysis

generalisation

$$in[n] = gen[n] \cup (out[n] - kill[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

IV

More Analyses

Reaching Definitions
definition

Unambiguous definition of a

- a statement of the form (d : a ← b ⊕ c) or (d : a ← M[x])

Definition d reaches statement u if

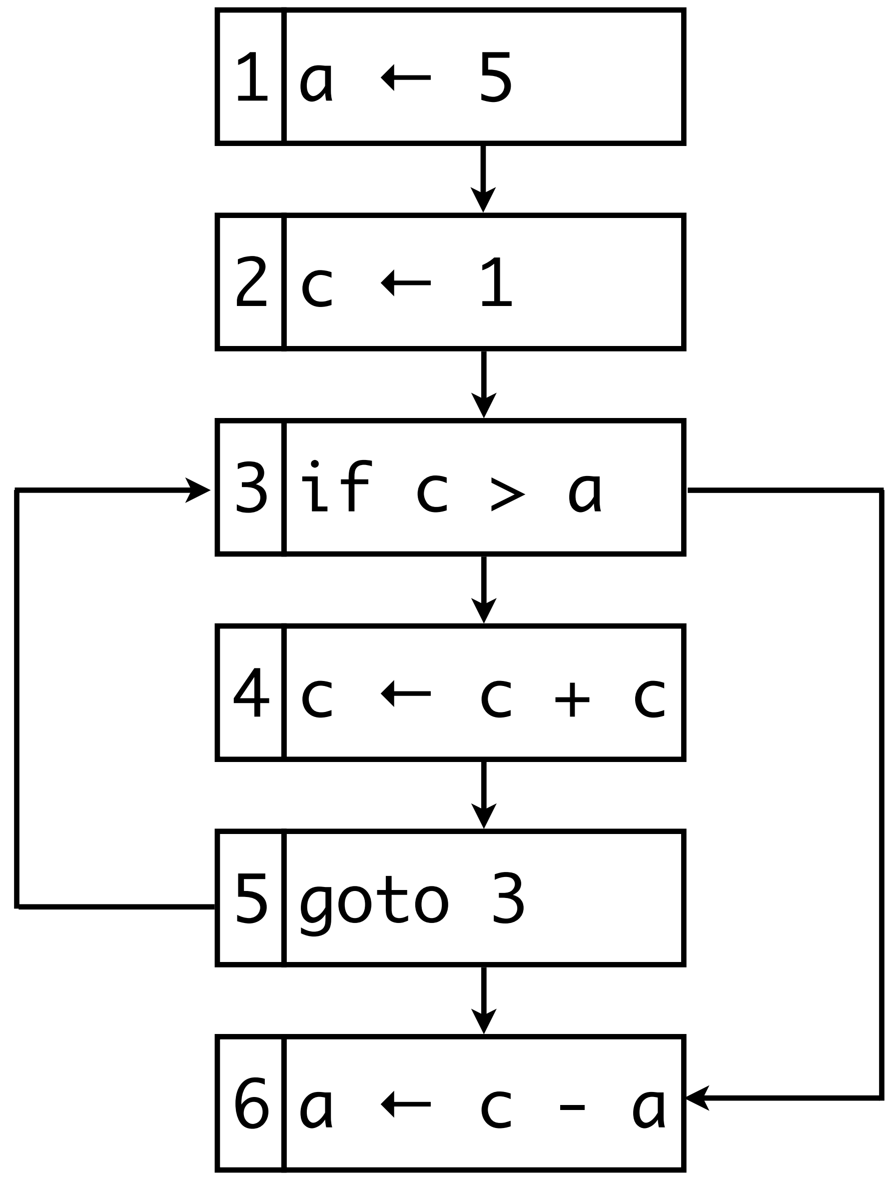
- there is some path in control-flow graph from d to u
- that does not contain an unambiguous definition of a

Used in

- constant propagation

Reaching Definitions

example



Reaching Definitions

gen & kill

	gen	kill
d: a ← b ⊕ c	{d}	defs(a)-{d}
d: a ← b	{d}	defs(a)-{d}
d: a ← M[b]	{d}	defs(a)-{d}
M[a] ← b		
f(a_1, …, a_n)		
d: a ← f(a_1, …, a_n)	{d}	defs(a)-{d}
goto L		
if a ⊗ b		

defs(a) : all definitions of a

Reaching Definitions

formalisation

$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

Available Expressions
definition

An expression $(b \oplus c)$ is available at node n if

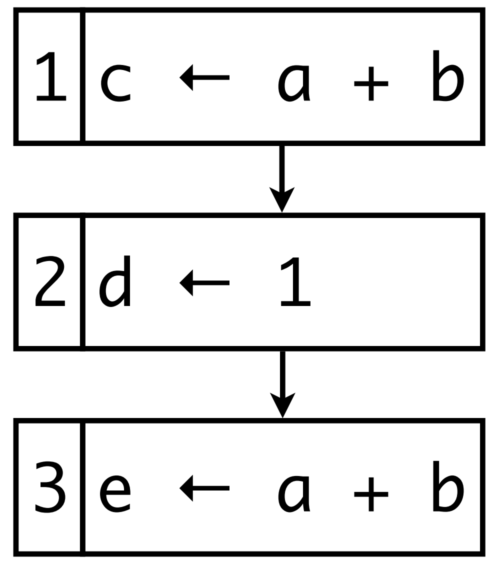
- on every path from the entry node to node n
- $(b \oplus c)$ is computed at least once, and
- there are no definitions of b or c since most recent occurrence of $(b \oplus c)$ on that path

Used in

- common-subexpression elimination

Available Expressions

example



Available Expressions

gen & kill

	gen	kill
d: a ← b ⊕ c	{b⊕c}-kill	exps(a)
d: a ← b		
d: a ← M[b]	{M[b]}-kill	exps(a)
M[a] ← b		exps(M[_])
$f(a_1, …, a_n)$		exps(M[_])
d: a ← $f(a_1, …, a_n)$		exps(M[_]) ∪ exps(a)
goto L		
if a ⊗ b		

Available Expressions

formalisation

$$in[n] = \bigcap_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

Reaching Expressions
definition

An expression (s : a ← b ⊕ c) reaches node n if

- there is a path from s to n
- that does not go through assignment to b or c
- or through any computation of (b ⊕ c)

Used in

- common-subexpression elimination

V

Optimizations

Dead Code Elimination

example

```
a ← 0
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← 0
b ← a + 1
c ← c + b

return c
```

```
consider:
  (s : a ← b ⊕ c) or (s : a ← M[x])
  if a is not live-out at s
transform:
  remove s
```

Common Subexpression Elimination

example

```
c ← a + b
d ← 1
e ← a + b
```

⇒

```
x ← a + b
c ← x
d ← 1
e ← x
```

```
consider:
  (n : a ← b ⊕ c) reaches (s : d ← b ⊕ c)
  path from n to s does not compute b ⊕ c or define b or c
  e is a fresh variable
transform:
  n  : a ← b ⊕ c
  n' : e ← a
  …
  s : d ← e
```

Constant Propagation

example

```
a ← 0
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

⇒

```
a ← 0
b ← 0 + 1
c ← c + b
a ← 2 * b
return c
```

```
consider: (d : a ← c) & (n : e ← a ⊕ b)
if c is constant
   & (d reaches n)
   & (no other def of a reaches n)
transform:
  (n : e ← a ⊕ b) => (n : e ← c ⊕ b)
```

Copy Propagation

example

```
a ← e
b ← a + 1
c ← c + b
a ← 2 * b
return c
```

```
a ← e
b ← e + 1
c ← c + b
a ← 2 * b
return c
```

```
consider: (d : a ← z) & (n : e ← a ⊕ b)
if z is a variable
   & (d reaches n)
   & (no other def of a reaches n)
   & (no def of z on path from d to n)
transform:
  (n : e ← a ⊕ b) => (n : e ← z ⊕ b)
```

V

Summary

Summary

Liveness analysis

- intermediate language
- control-flow graphs
- definition & algorithm

More dataflow analyses

- reaching definitions
- available expressions

Optimizations

Literature

learn more

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

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