

LR Parsing

Traditional Parsing Algorithms

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Recap: Traditional Parsing Algorithms

lessons learned

How can we parse context-free languages effectively?

- predictive parsing

Which grammar classes are supported by these algorithms?

- LL(k) grammars, LL(k) languages

How can we generate compiler tools from that?

- implement automaton
- generate parse tables

What are other techniques for implementing top-down parsers?

- Parser Combinators
- PEGs
- ALL(*)

Overview

today's lecture

Overview

today's lecture

efficient parsing algorithms

- LR parsing
- LR parse table generation
- SLR & LALR parse tables
- Generalized LR parsing
- Scannerless Generalized LR parsing

I

LR Parsing

LR parsing

idea

problems with LL parsing

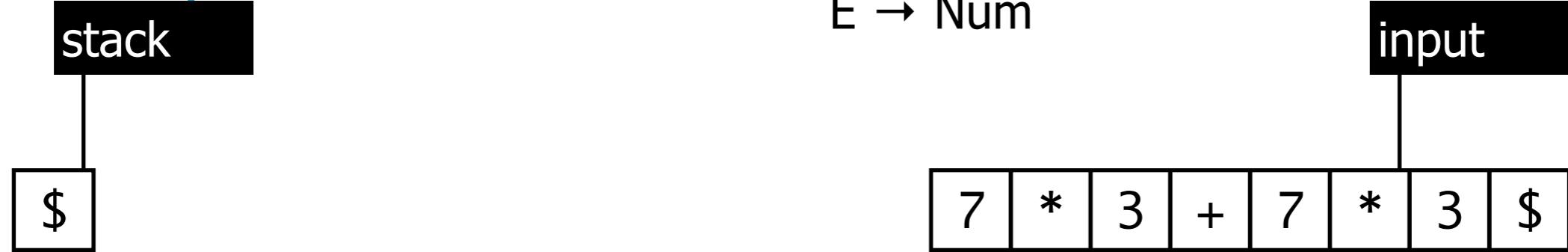
- predicting right rule
- left recursion

LR parsing

- see whole right-hand side of a rule
- look ahead
- shift or reduce

LR parsing

example



LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

$\boxed{\$}$

$\boxed{7 \mid * \mid 3 \mid + \mid 7 \mid * \mid 3 \mid \$}$

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7
----	---

*	3	+	7	*	3	\$
---	---	---	---	---	---	----

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7
\$	E

*	3	+	7	*	3	\$
*	3	+	7	*	3	\$

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7	
\$	E	*

*	3	+	7	*	3	\$
3	+	7	*	3	\$	

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3

*	3	+	7	*	3	\$
+	7	*	3	\$		

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E		

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
+	7	*	3	\$		

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E	+	

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
7 * 3 \$						

LR parsing example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E	+	7

*	3	+	7	*	3	\$	
+	7	*	3	\$			
+	7	*	3	\$			
					*	3	\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7		
\$	E	*	3
\$	E	*	E
\$	E	+	7
\$	E	+	E

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
		*	3	\$		
		*	3	\$		

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7			
\$	E	*	3	
\$	E	*	E	
\$	E	+	7	
\$	E	+	E	*

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
		3	\$			

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7				
\$	E	*	3		
\$	E	*	E		
\$	E	+	7		
\$	E	+	E	*	3

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
				\$		

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7					
\$	E	*	3			
\$	E	*	E			
\$	E	+	7			
\$	E	+	E	*	3	
\$	E	+	E	*	E	

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
				\$		
					\$	

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7					
\$	E	*	3			
\$	E	*	E			
\$	E	+	7			
\$	E	+	E	*	3	
\$	E	+	E	*	E	
\$	E	+	E			

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
				\$		
					\$	
						\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7				
\$	E	*	3		
\$	E	*	E		
\$	E	+	7		
\$	E	+	E	*	3
\$	E	+	E	*	E
\$	E	+	E		
\$	E				

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
				\$		
					\$	
					\$	
						\$

LR parsing

example

$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7				
\$	E	*	3		
\$	E	*	E		
\$	E	+	7		
\$	E	+	E	*	3
\$	E	+	E	*	E
\$	E	+	E		
\$	E	\$			

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
				\$		
					\$	
						\$

LR parsing

example

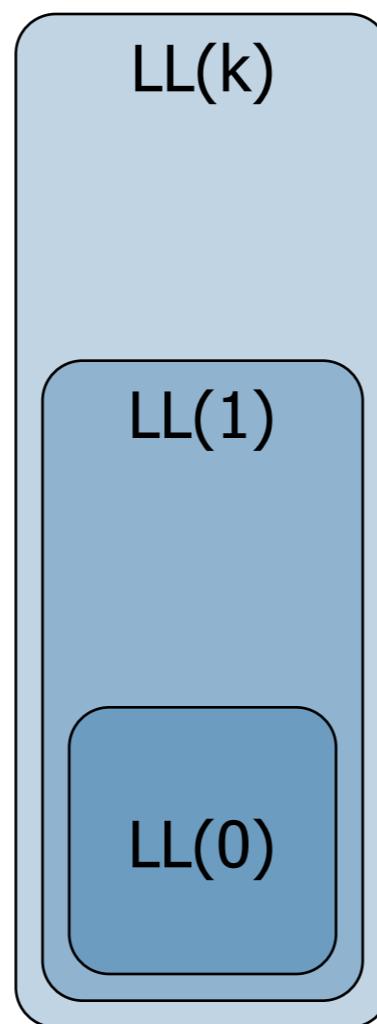
$S \rightarrow E \$$
 $E \rightarrow E * E$
 $E \rightarrow E + E$
 $E \rightarrow \text{Num}$

\$	7				
\$	E	*	3		
\$	E	*	E		
\$	E	+	7		
\$	E	+	E	*	3
\$	E	+	E	*	E
\$	E	+	E		
\$	E	\$			
\$	S				

*	3	+	7	*	3	\$
+	7	*	3	\$		
+	7	*	3	\$		
	*	3	\$			
				\$		
					\$	
						\$

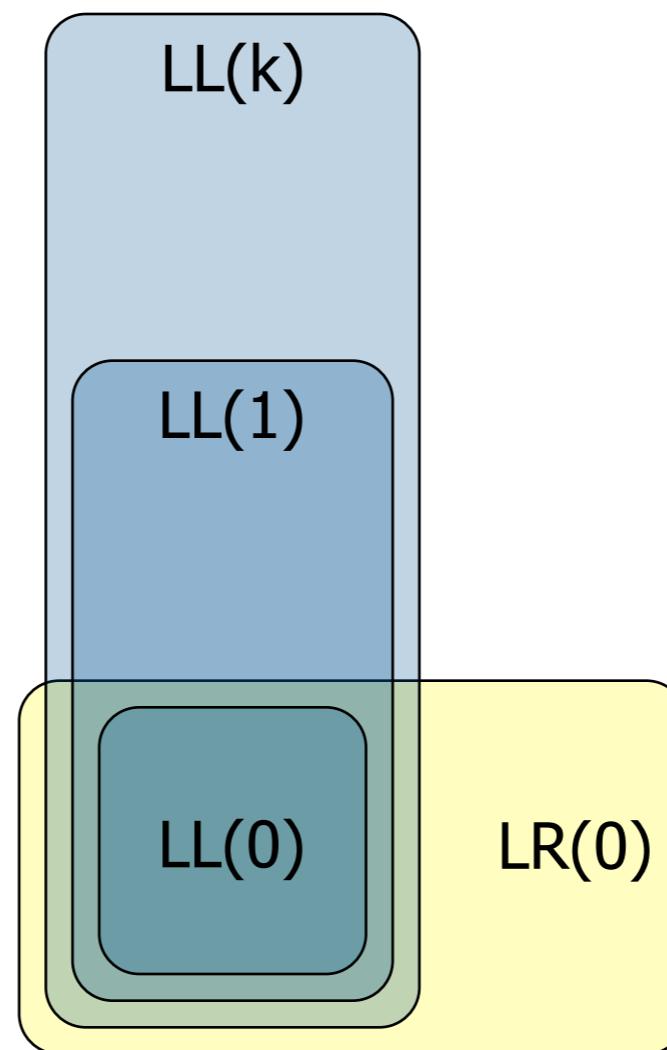
Grammar classes

context-free grammars



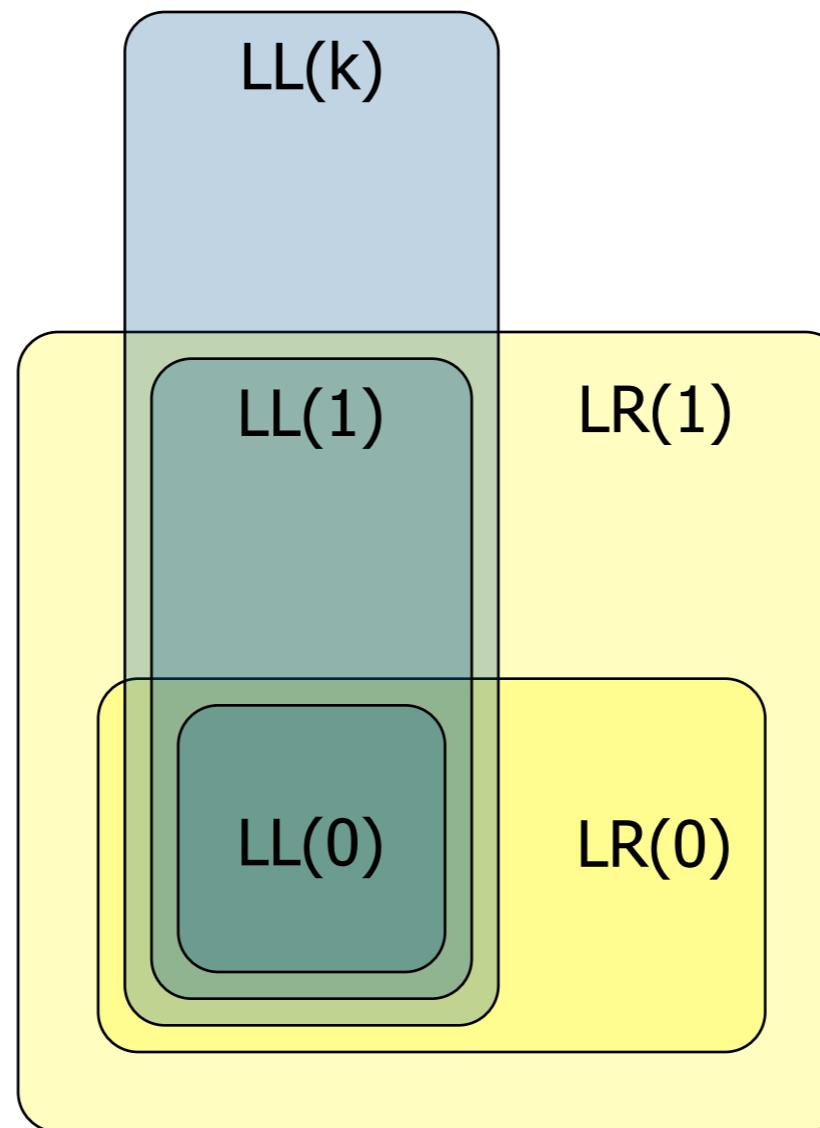
Grammar classes

context-free grammars



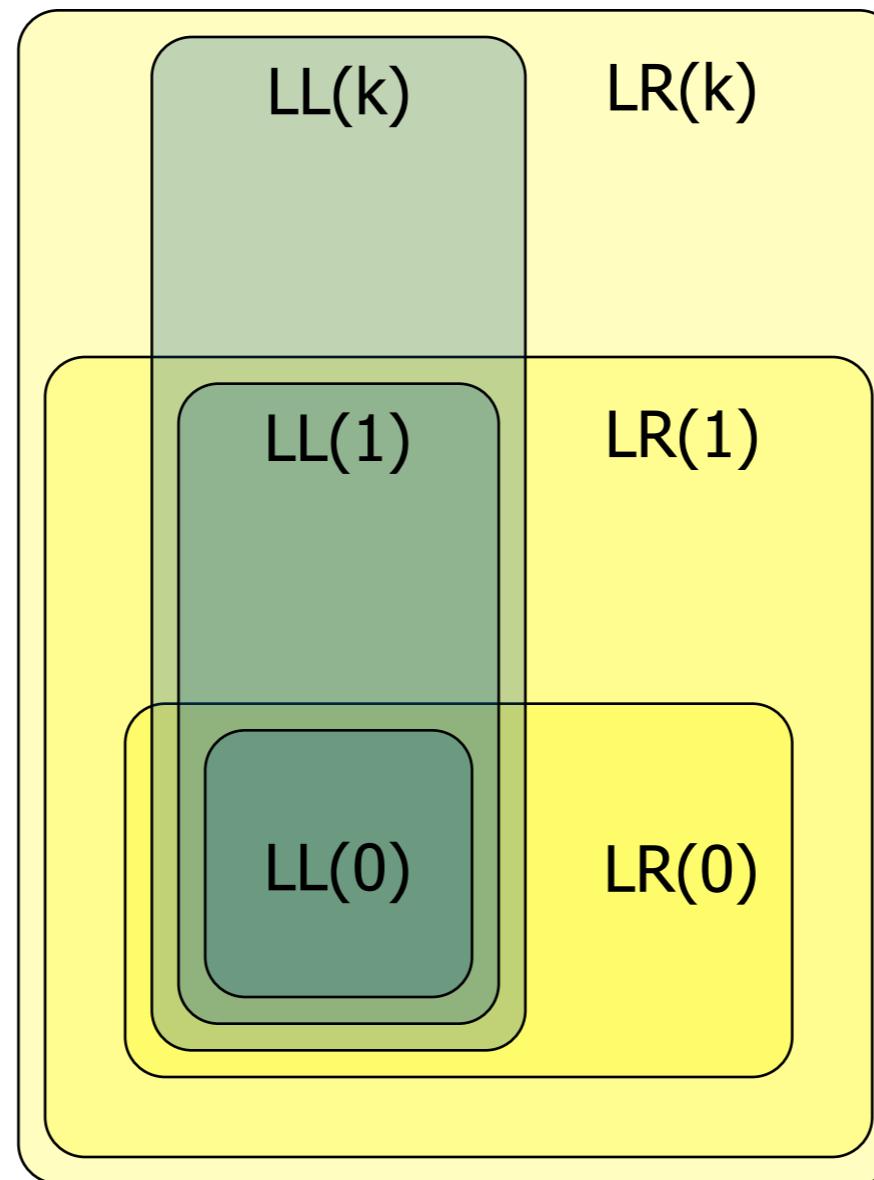
Grammar classes

context-free grammars



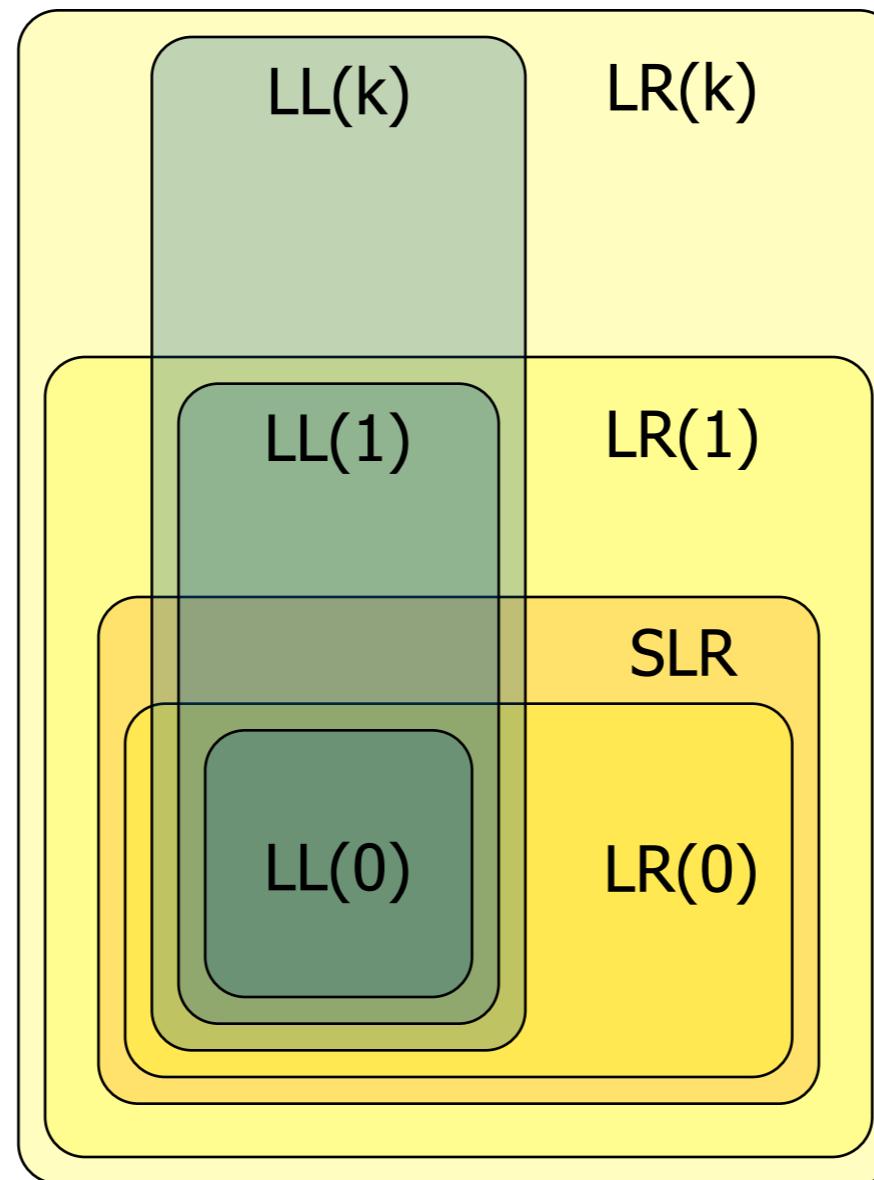
Grammar classes

context-free grammars



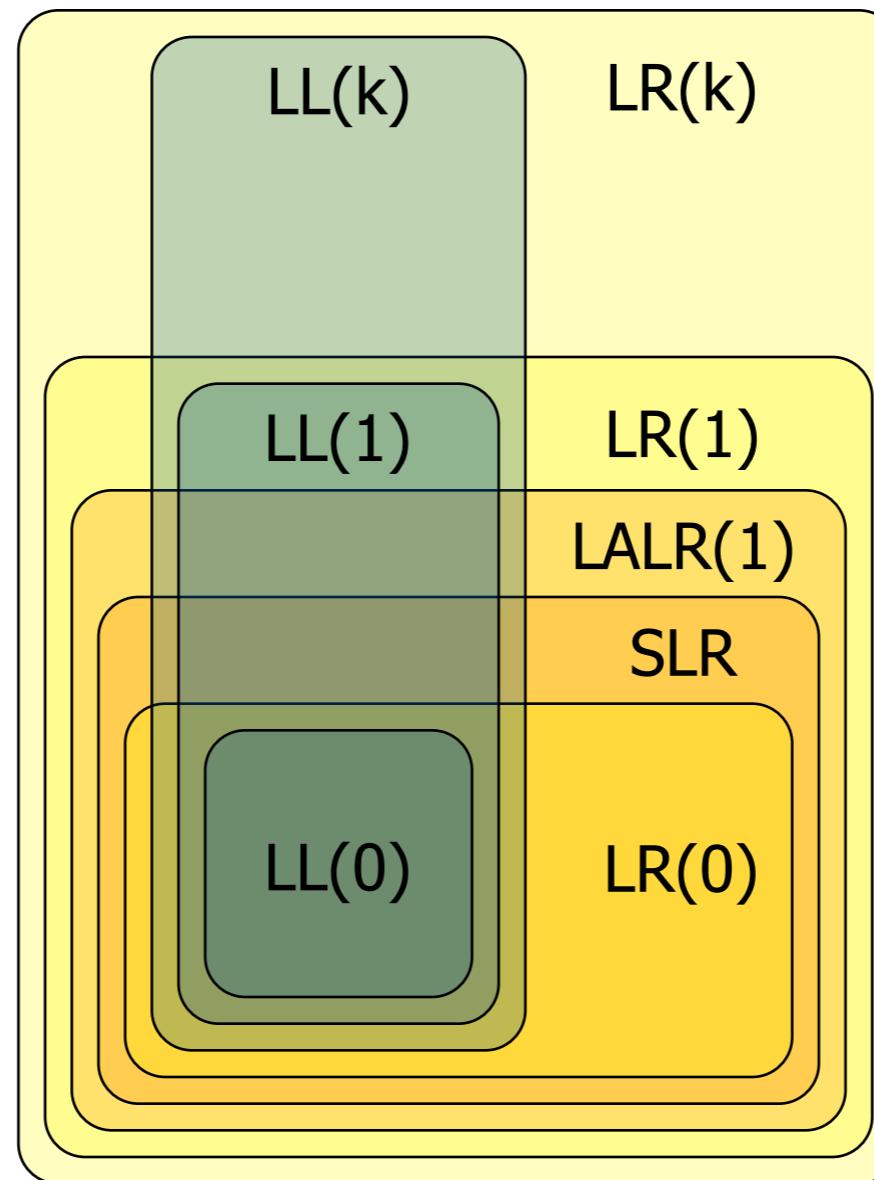
Grammar classes

context-free grammars



Grammar classes

context-free grammars



II

LR Parse Tables

LR parsing

parse table

rows

- states of a DFA

columns

- topmost stack symbol
- Σ, N

entries

- reduce, rule number
- shift, goto state
- goto state
- accept state

	T_1	...	N_1	...
1	$s\ 3$			
2			$g\ 5$	
3	$r\ 1$			
4	$r\ 2$	a		
5				
6			$g\ 1$	
7	$s\ 1$			
8				
...				

LR(0) parse tables

items, closure & goto

$$\begin{aligned} S &\rightarrow x \\ S &\rightarrow (L) \\ L &\rightarrow S \\ L &\rightarrow L , S \end{aligned}$$

LR(0) parse tables

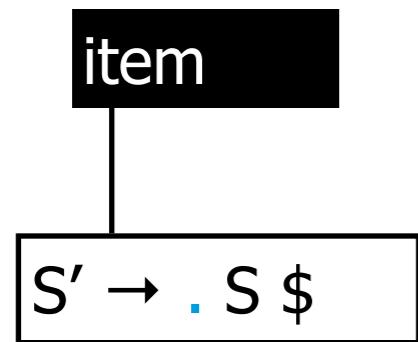
items, closure & goto

$S' \rightarrow . S \$$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

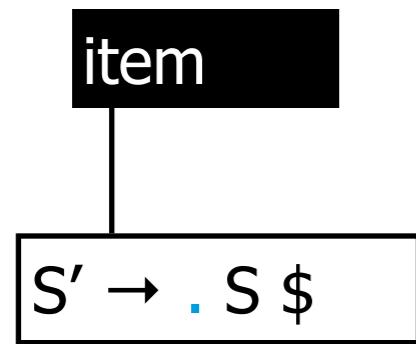
LR(0) parse tables

items, closure & goto


$$\begin{aligned} S &\rightarrow x \\ S &\rightarrow (\ L \) \\ L &\rightarrow S \\ L &\rightarrow L , S \end{aligned}$$

LR(0) parse tables

items, closure & goto



closure

- for every item $A \rightarrow a . X \beta$
- for every rule $X \rightarrow \gamma$
- add item $X \rightarrow . \gamma$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

items, closure & goto

closure

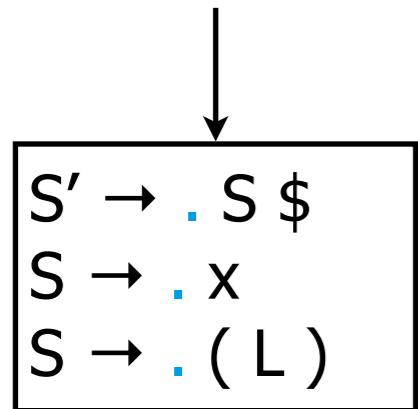
```
S' → . S $  
S → . x  
S → . ( L )
```

- for every item $A \rightarrow a . X \beta$
- for every rule $X \rightarrow \gamma$
- add item $X \rightarrow . \gamma$

```
S → x  
S → ( L )  
L → S  
L → L , S
```

LR(0) parse tables

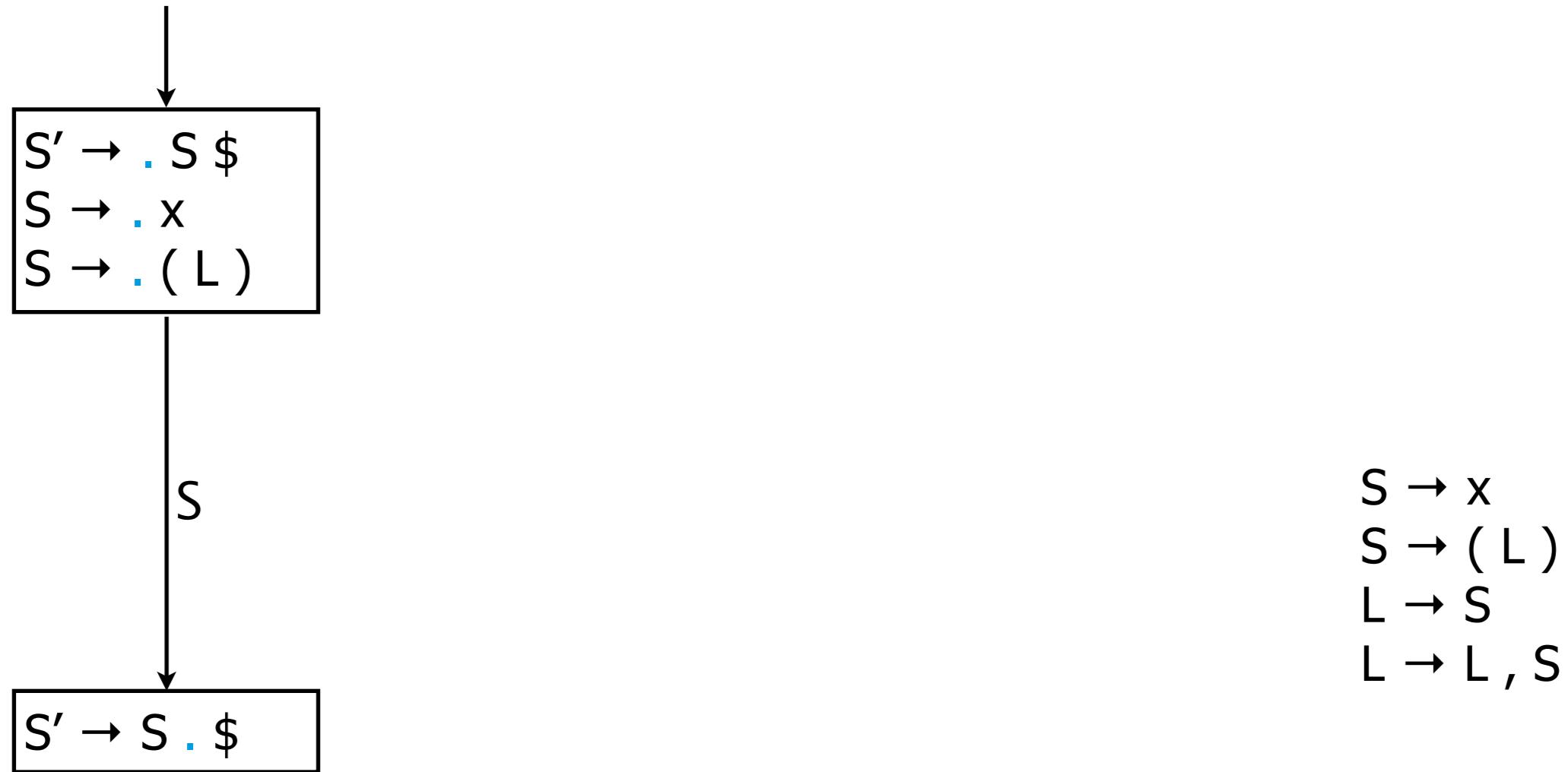
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

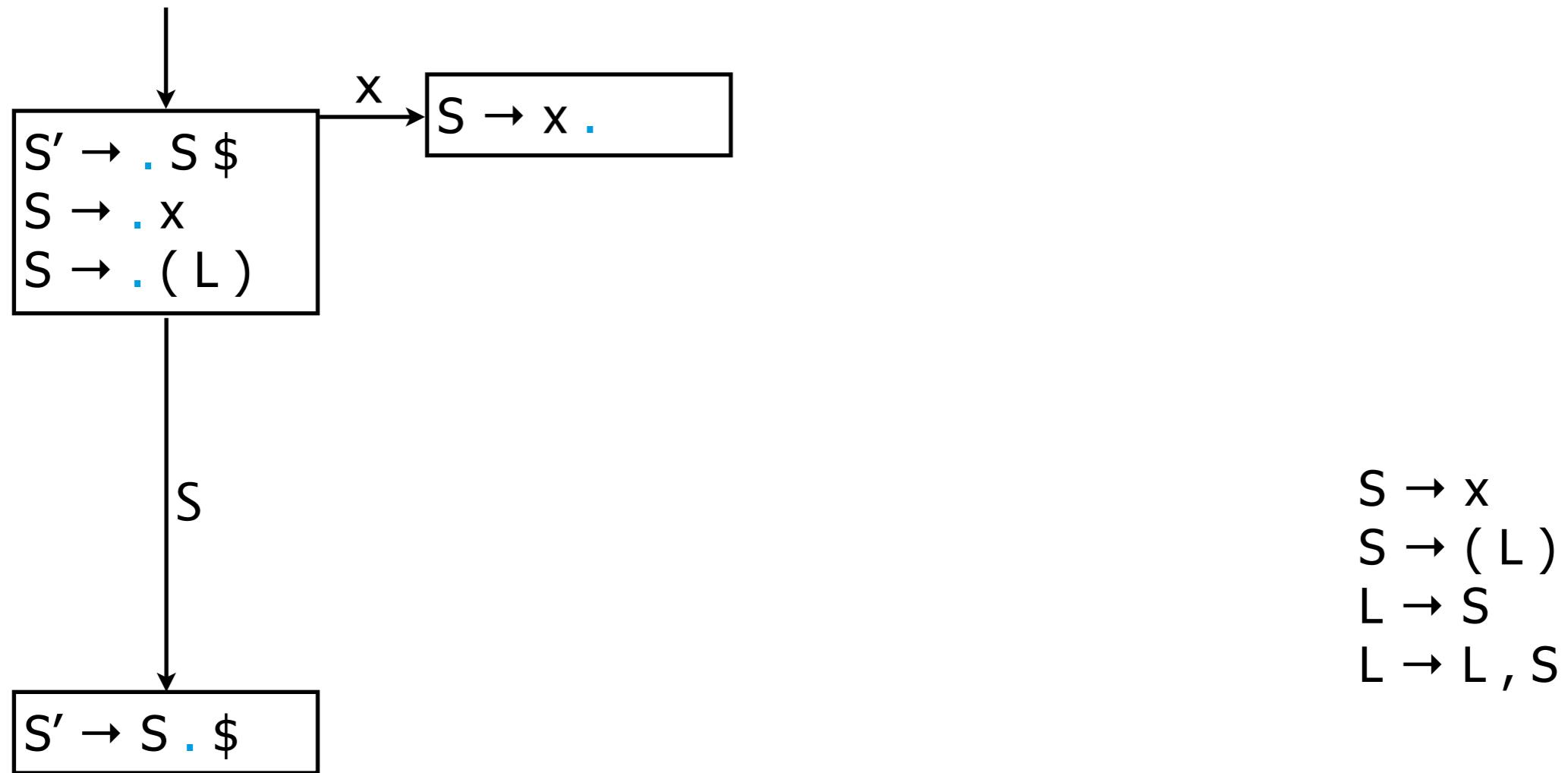
LR(0) parse tables

items, closure & goto



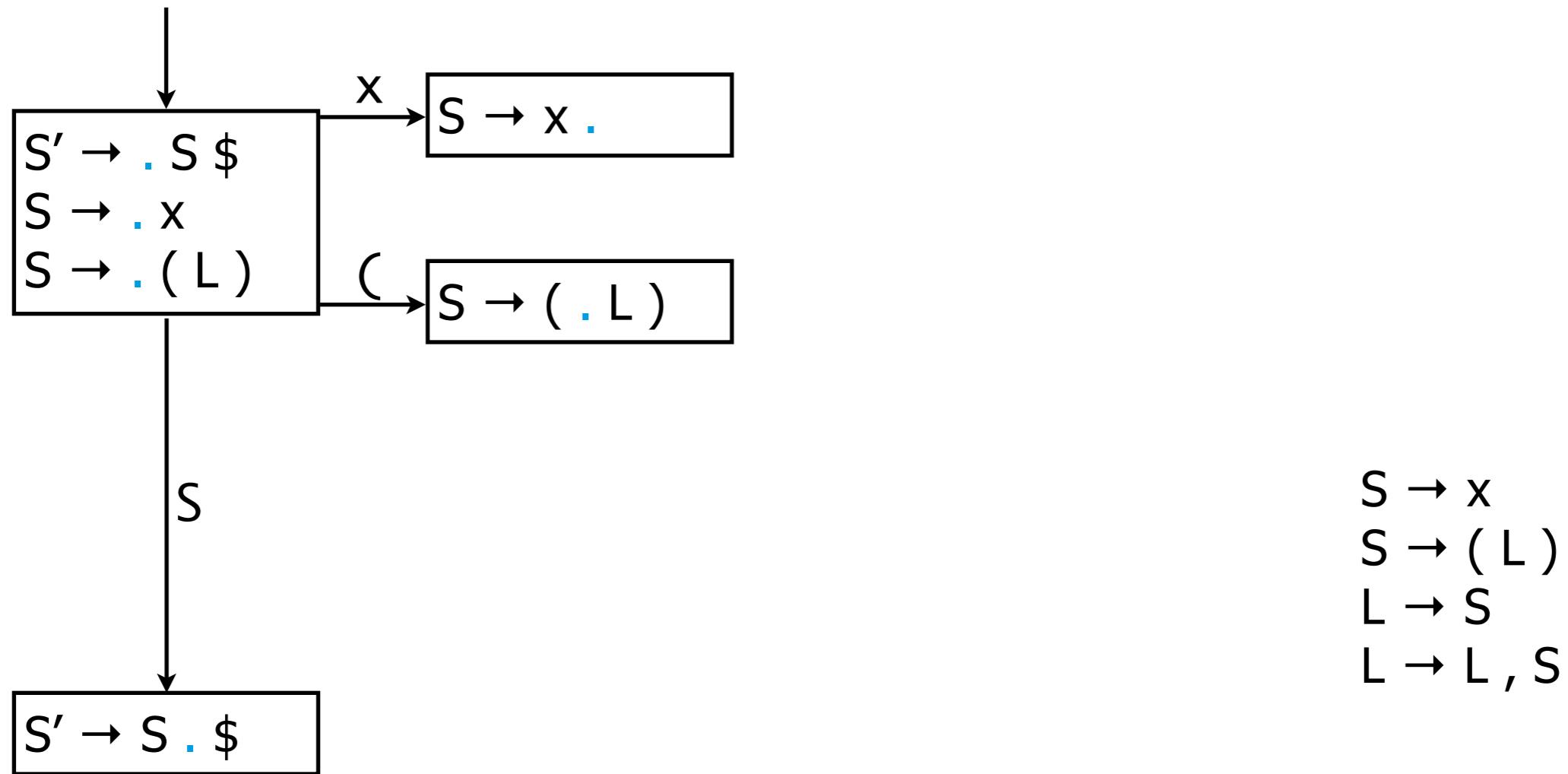
LR(0) parse tables

items, closure & goto



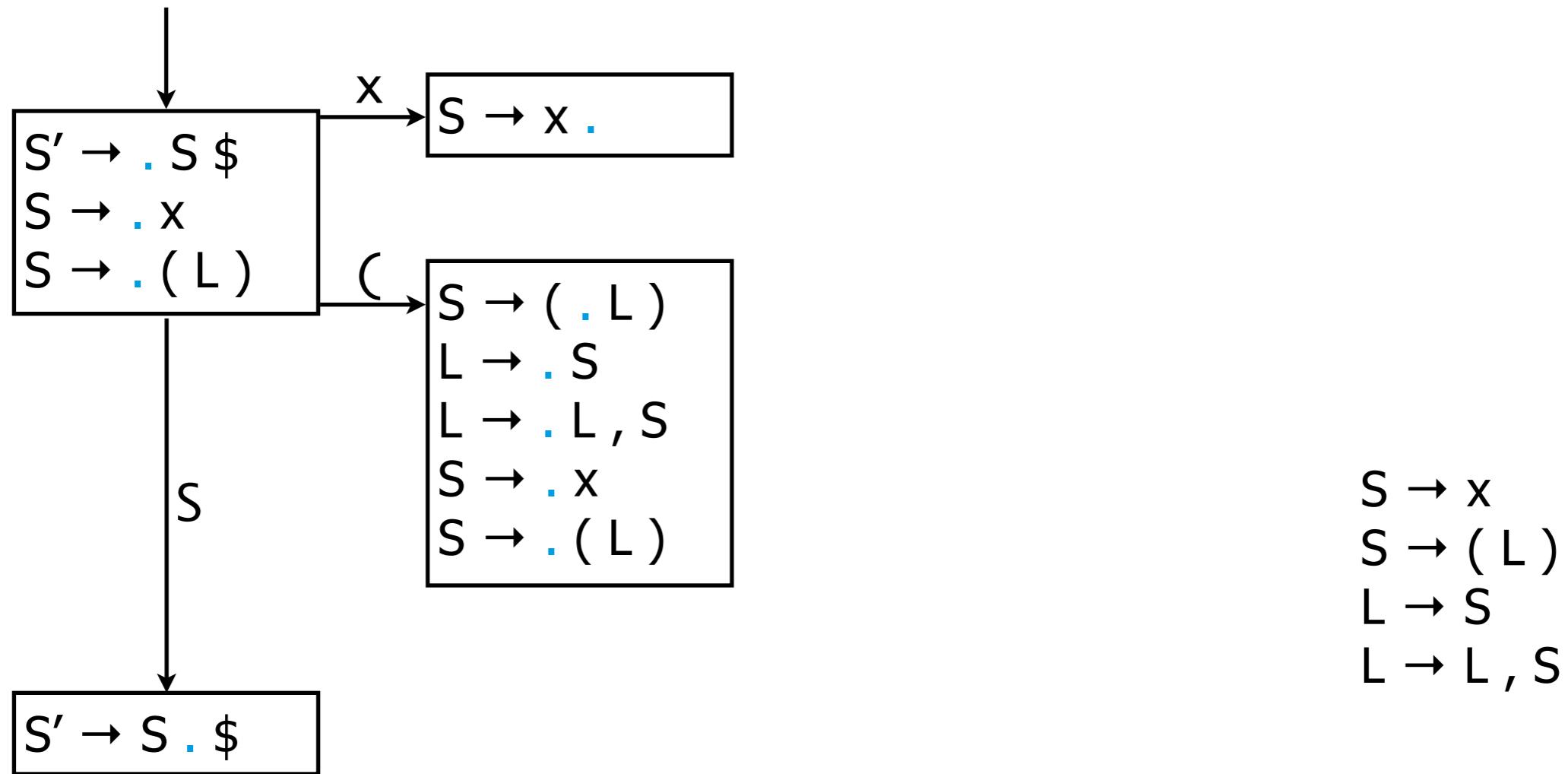
LR(0) parse tables

items, closure & goto



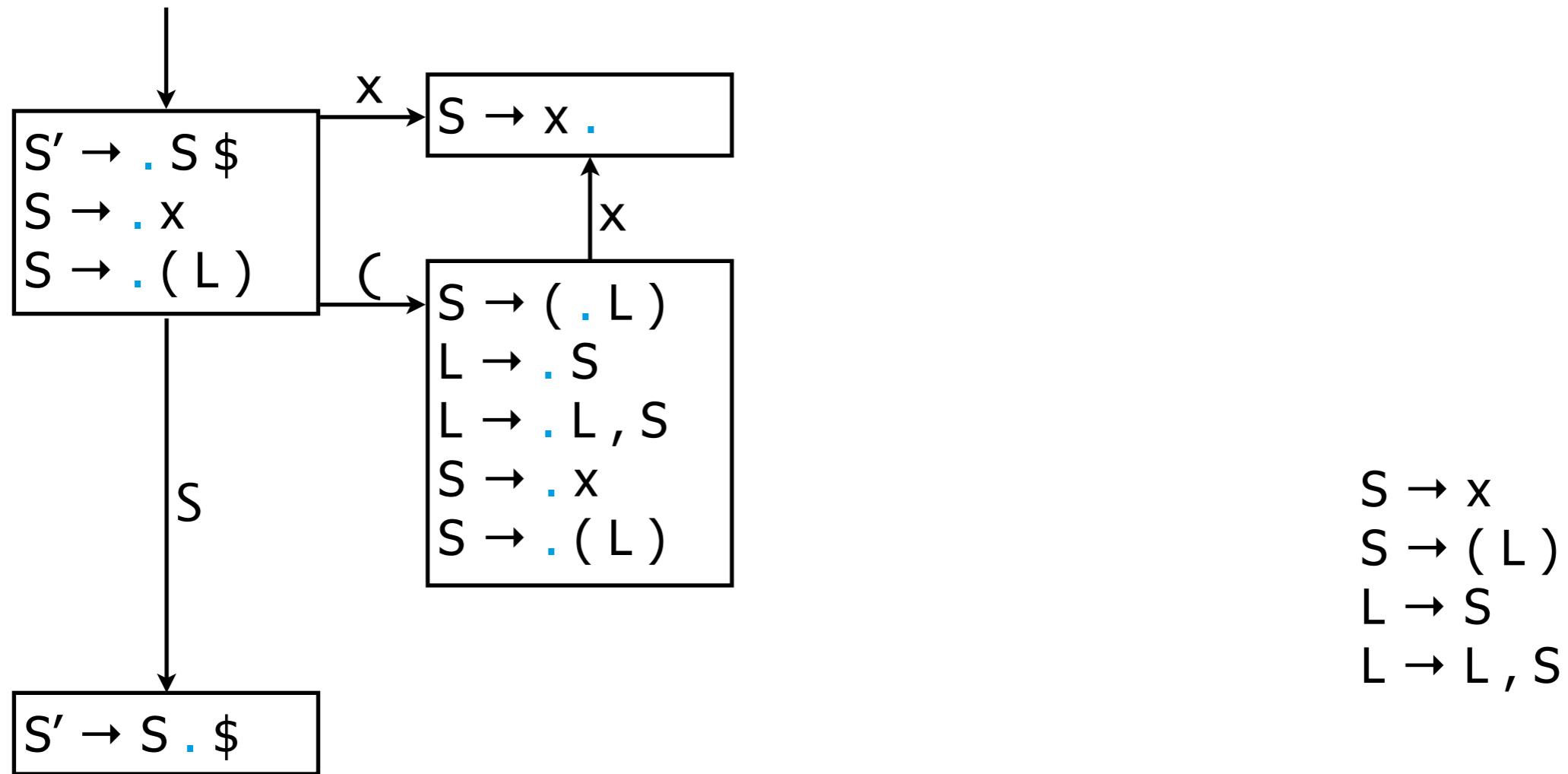
LR(0) parse tables

items, closure & goto



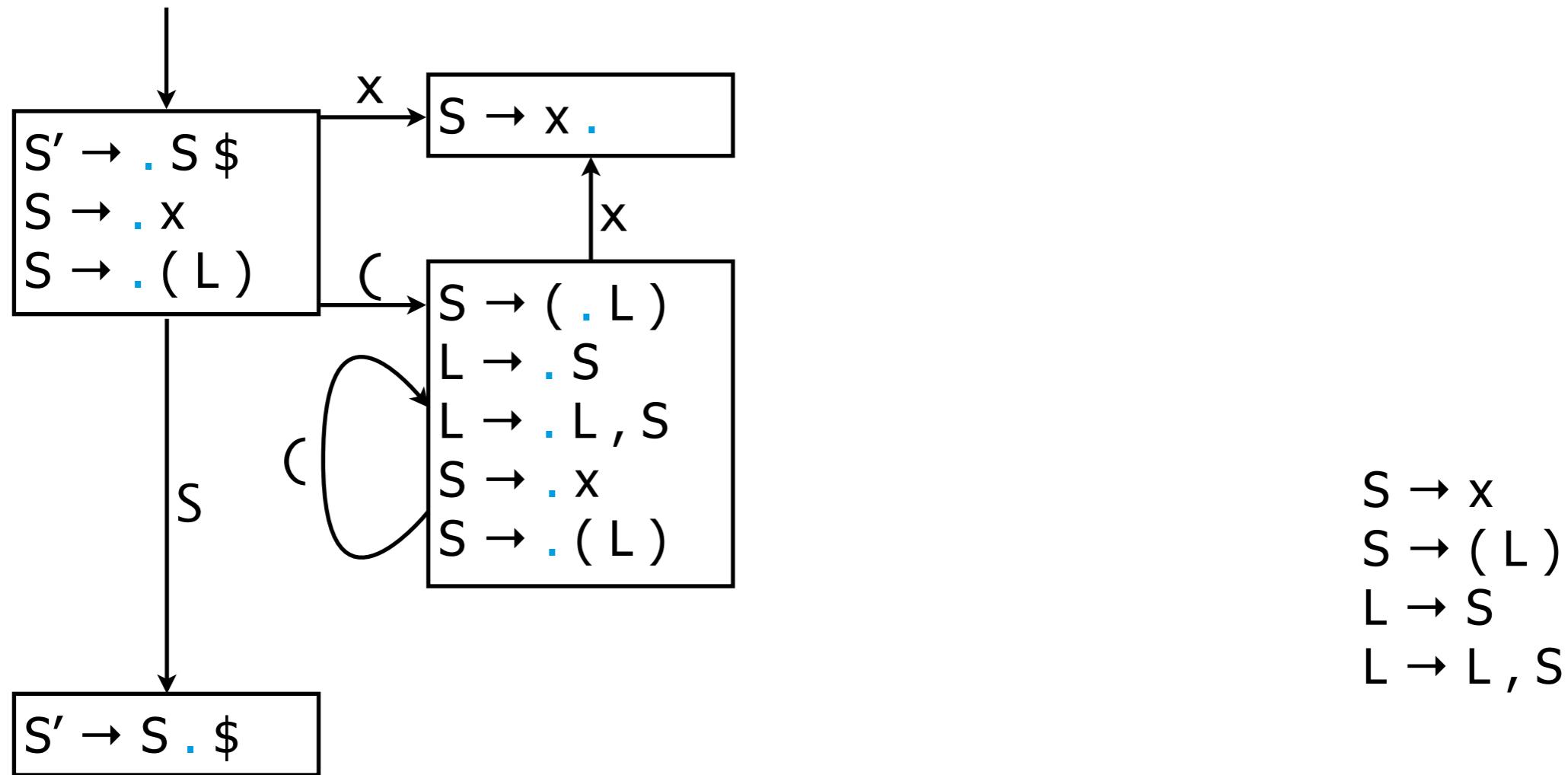
LR(0) parse tables

items, closure & goto



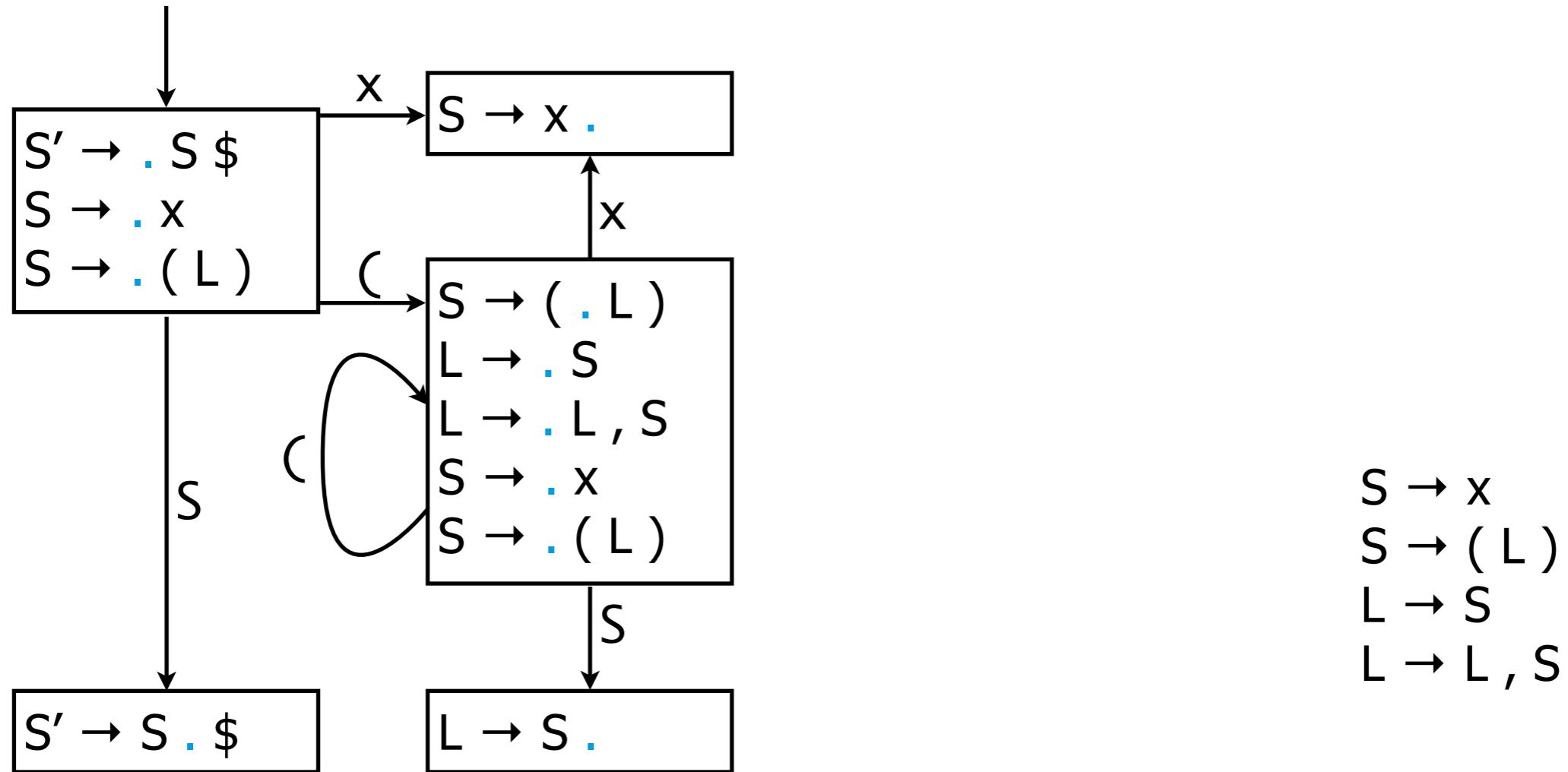
LR(0) parse tables

items, closure & goto



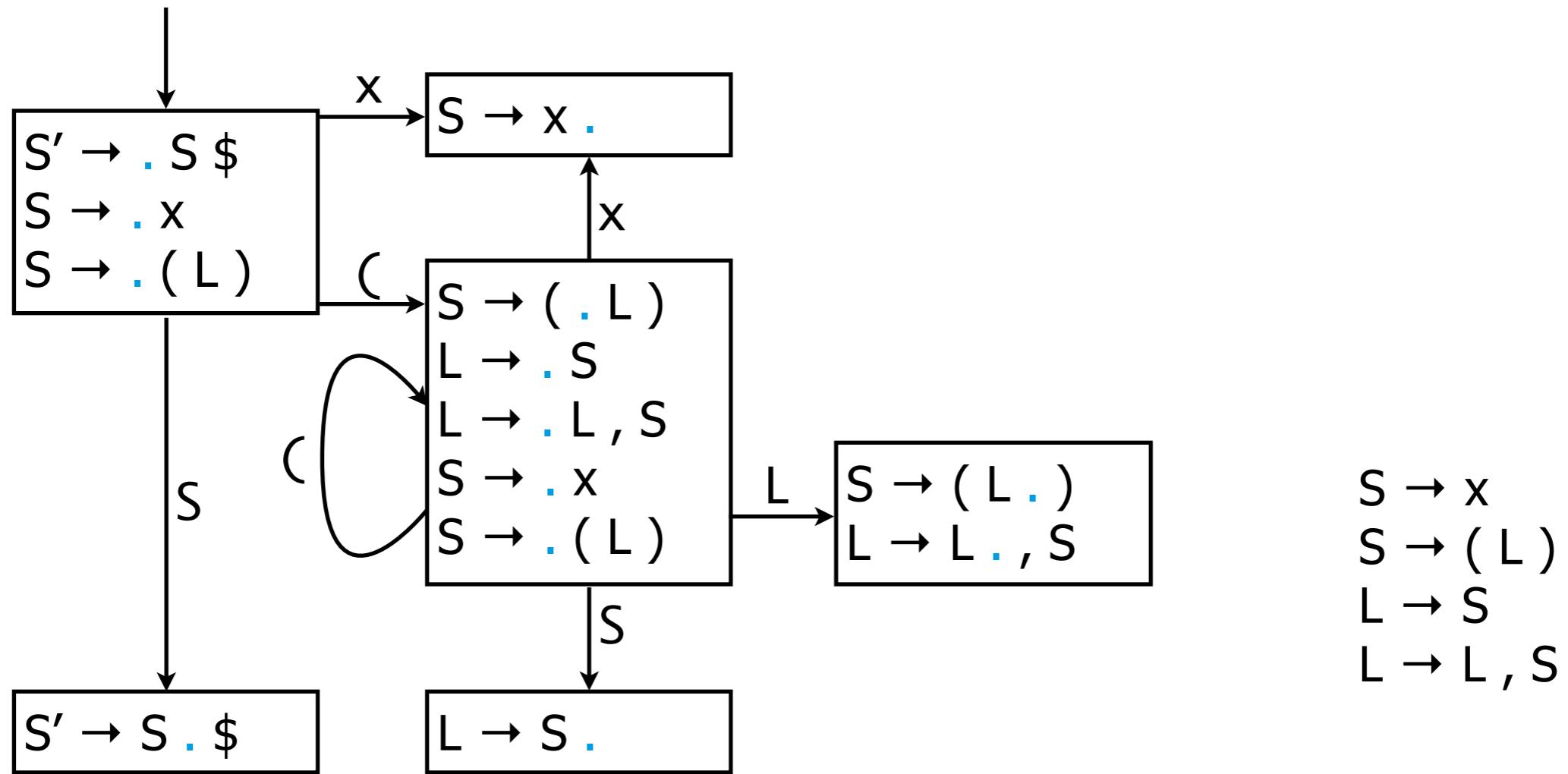
LR(0) parse tables

items, closure & goto



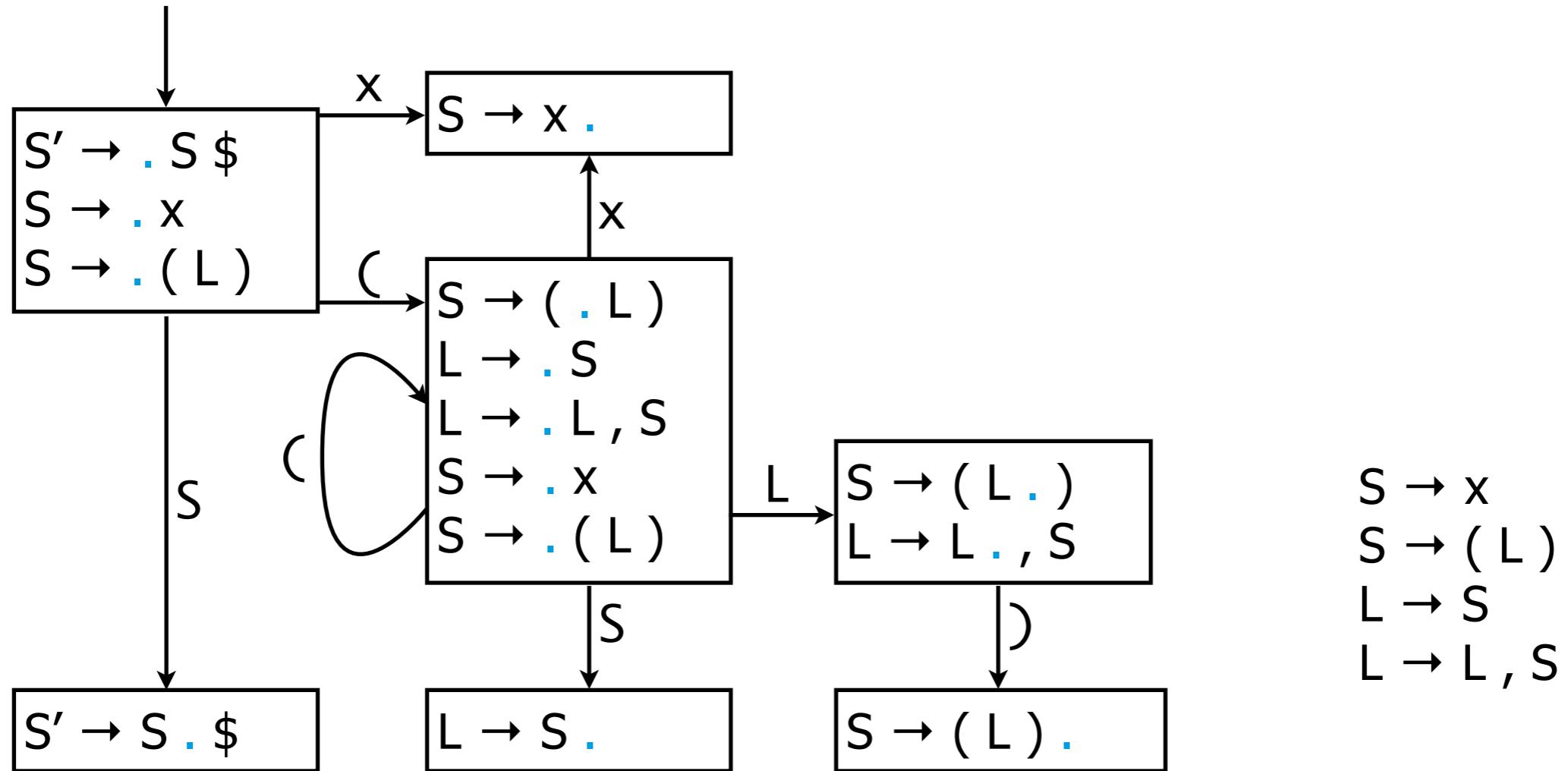
LR(0) parse tables

items, closure & goto



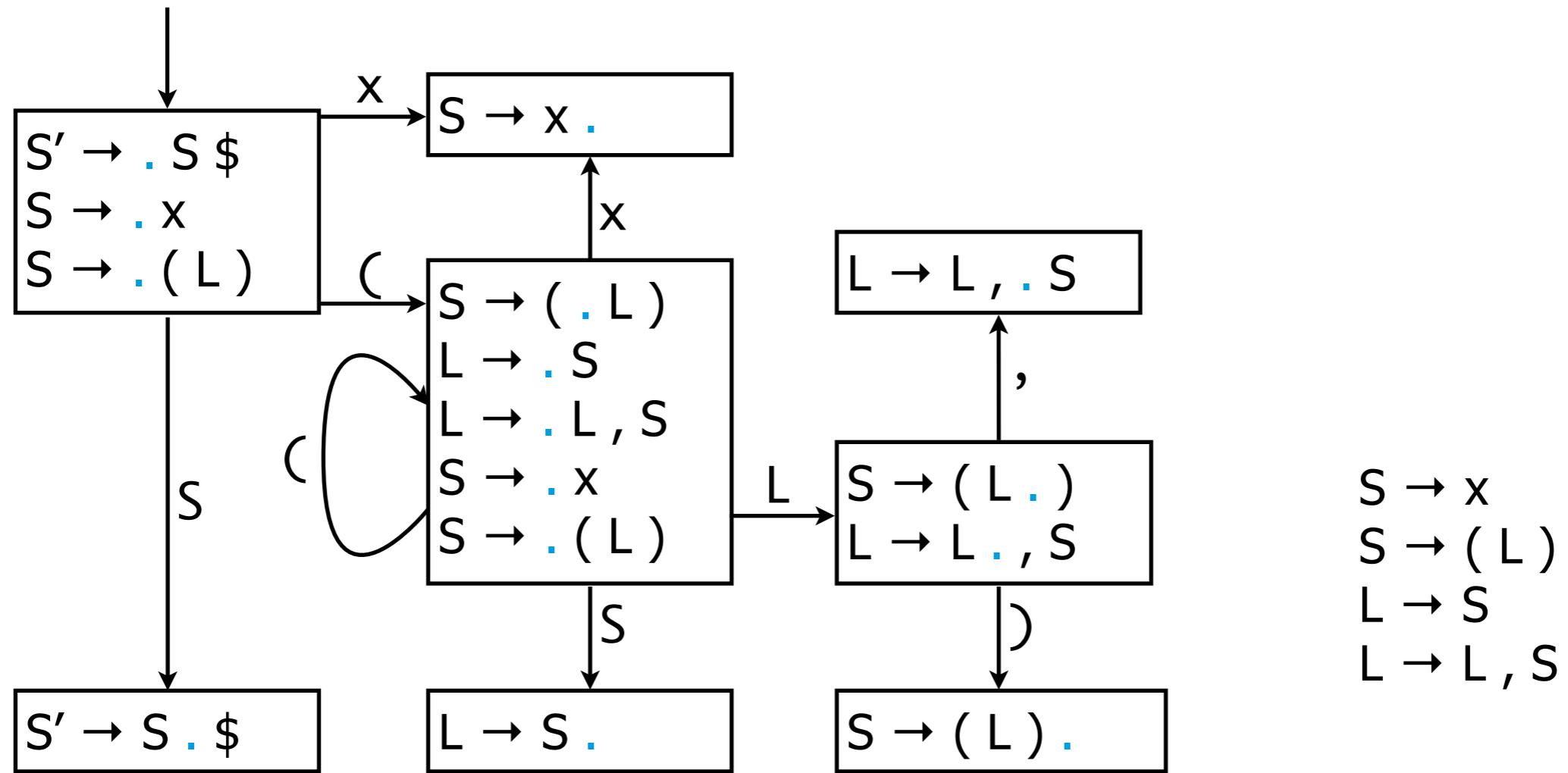
LR(0) parse tables

items, closure & goto



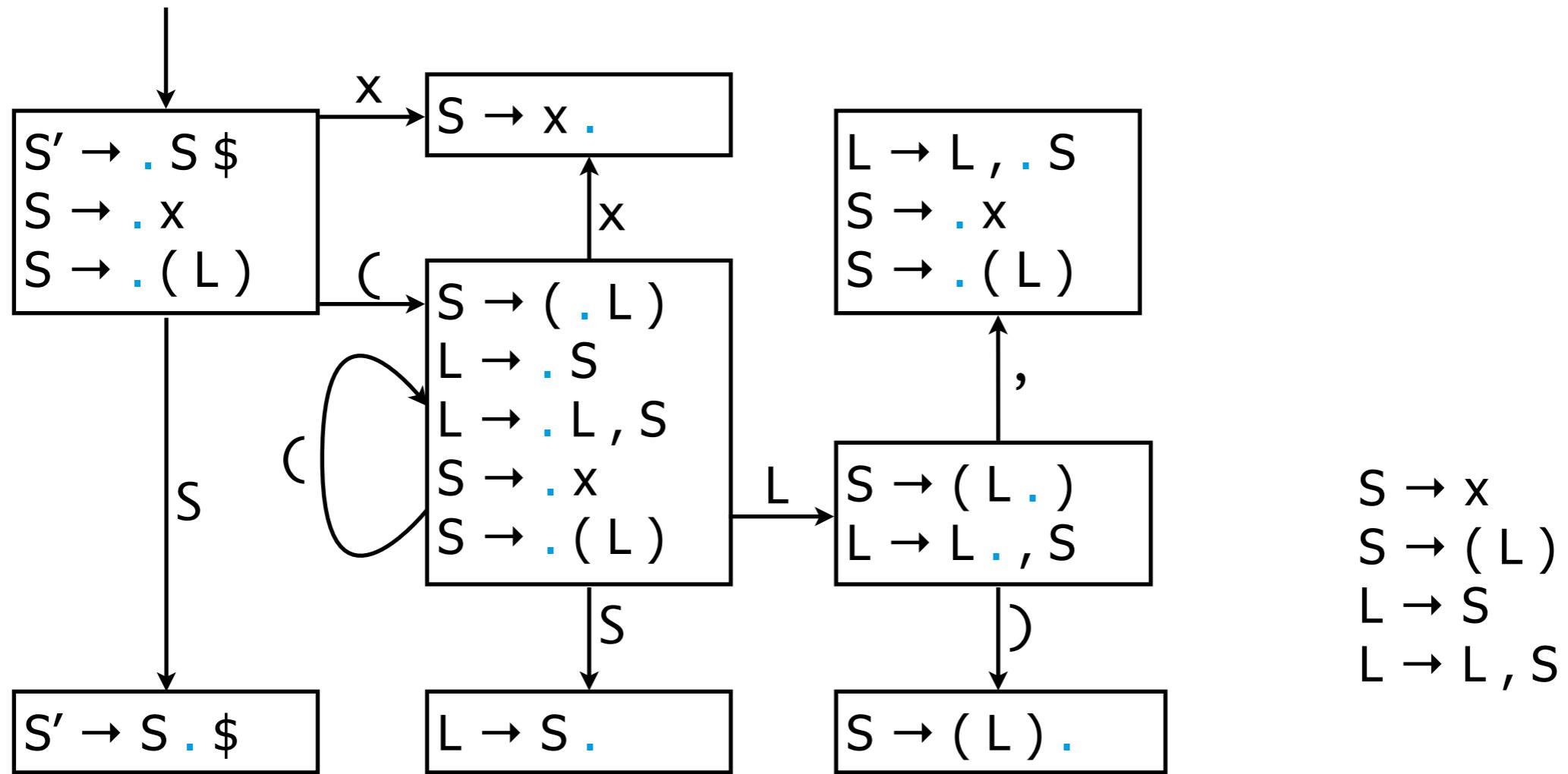
LR(0) parse tables

items, closure & goto



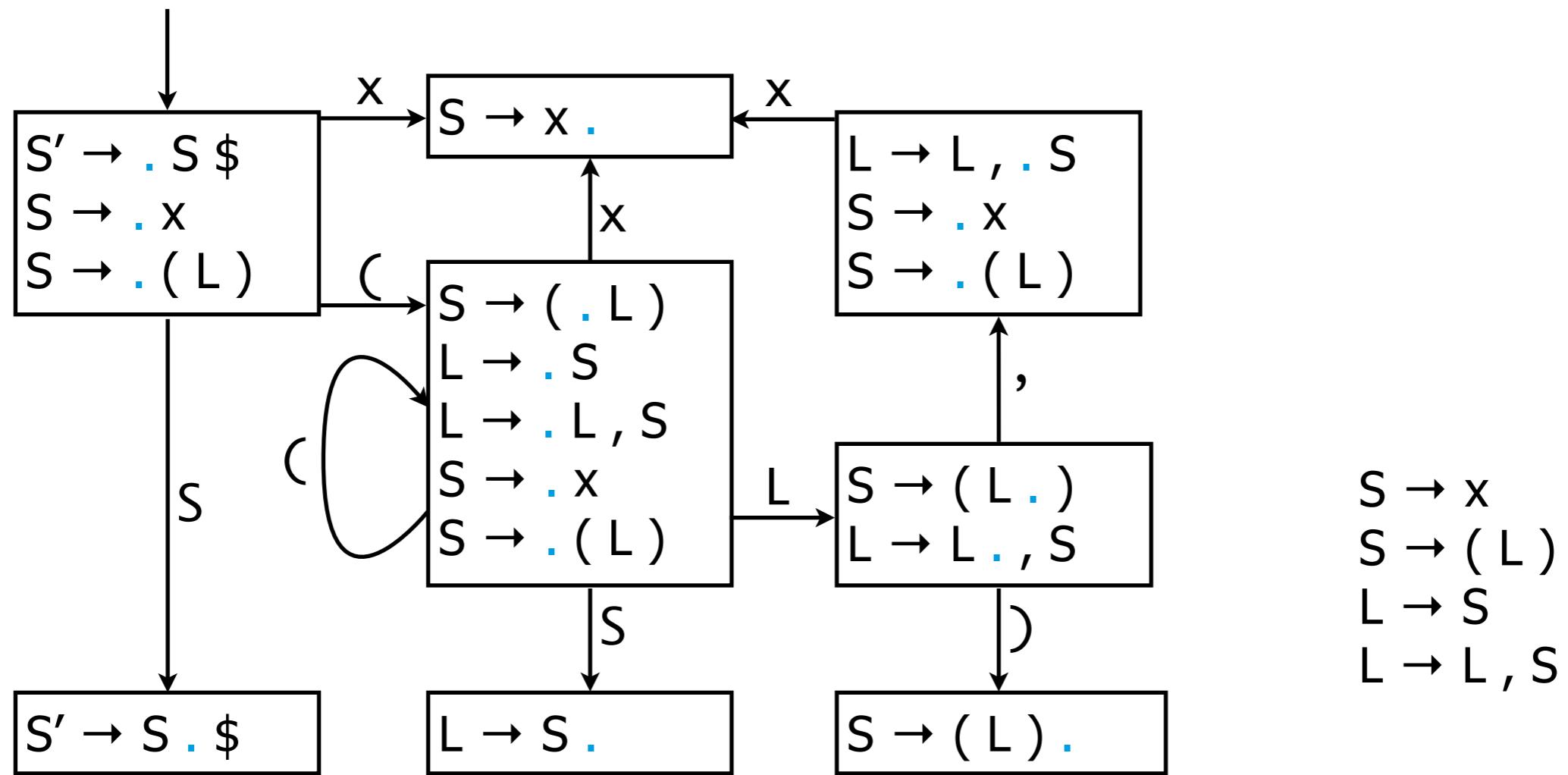
LR(0) parse tables

items, closure & goto



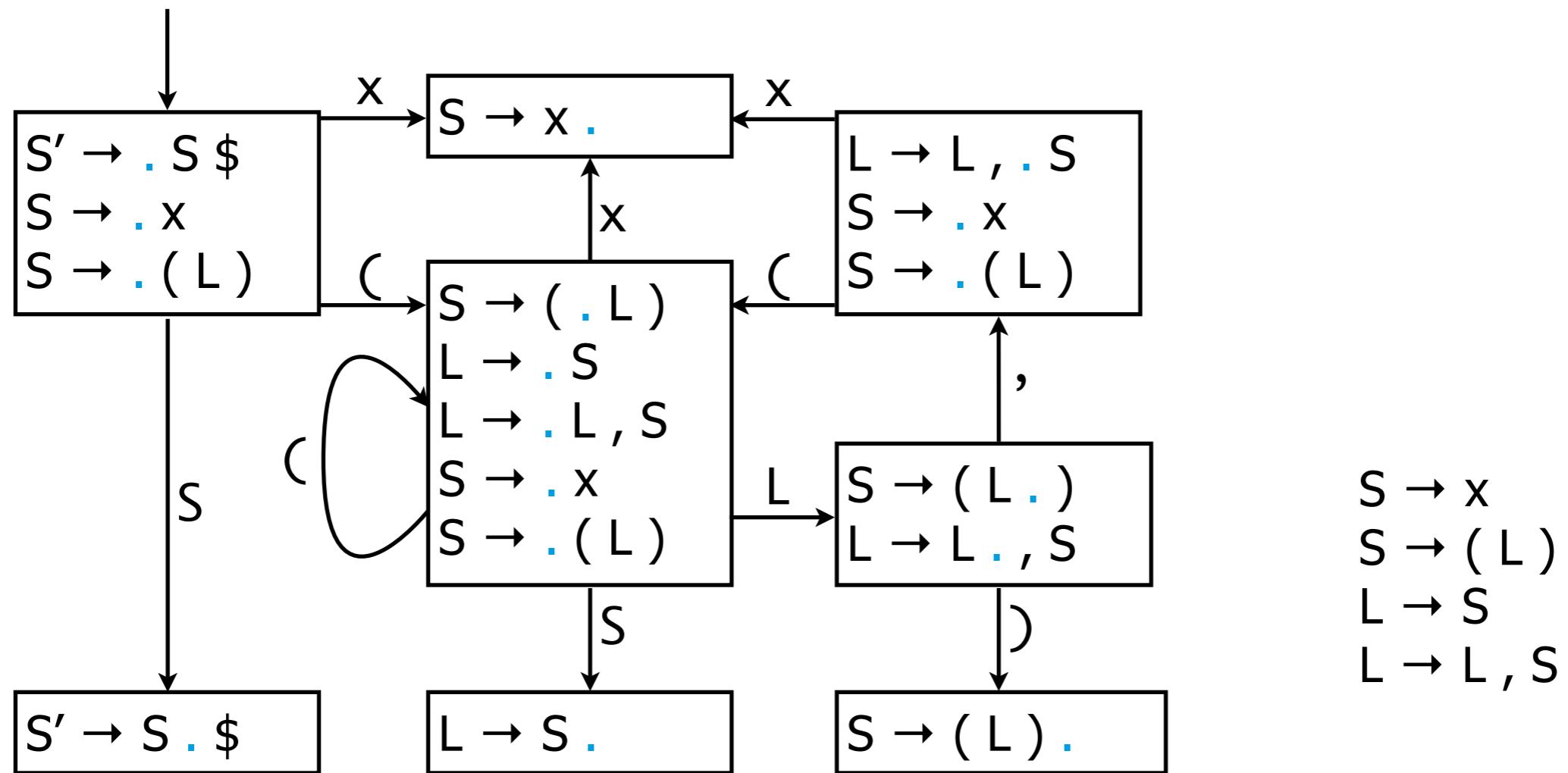
LR(0) parse tables

items, closure & goto



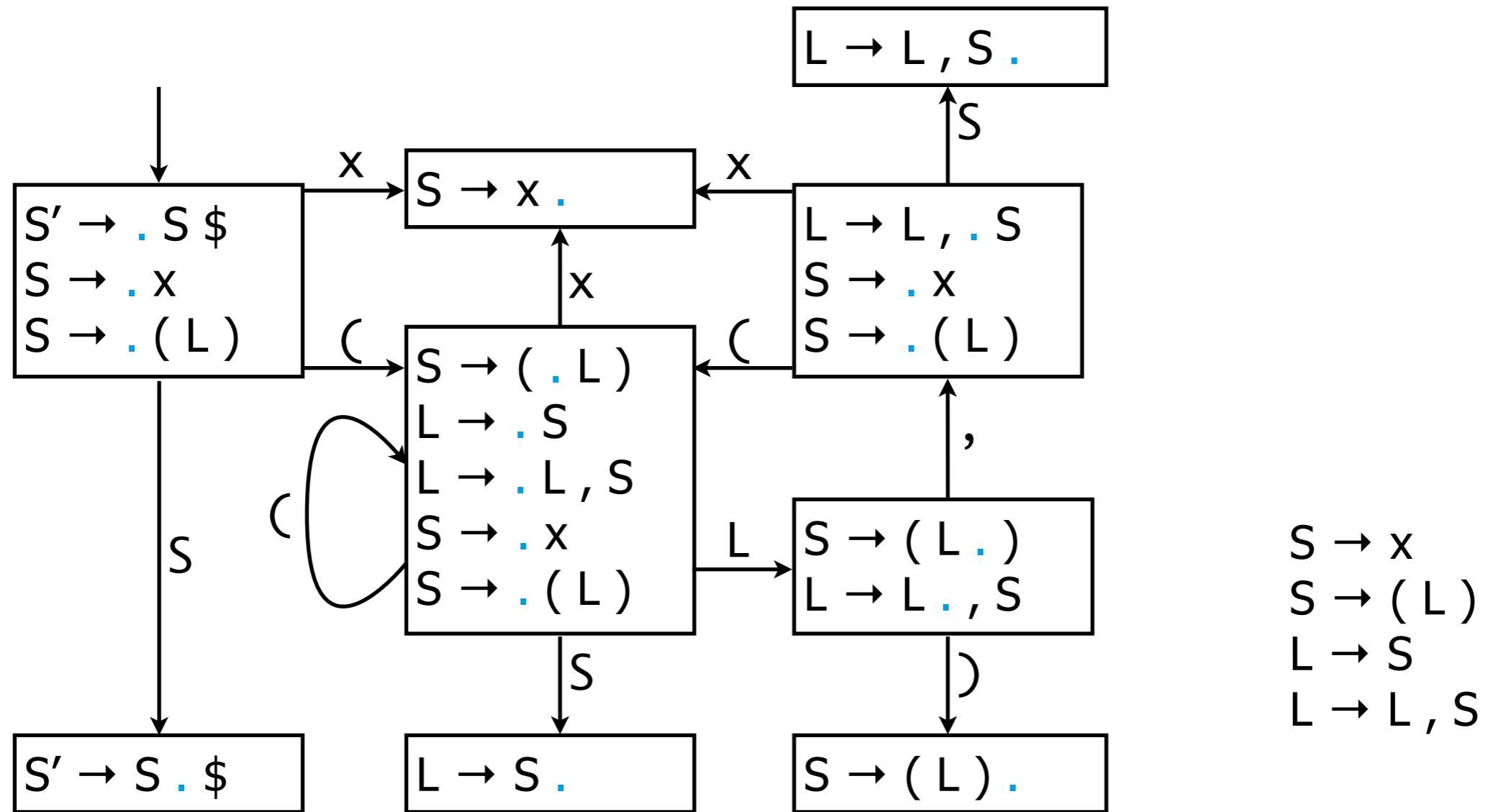
LR(0) parse tables

items, closure & goto



LR(0) parse tables

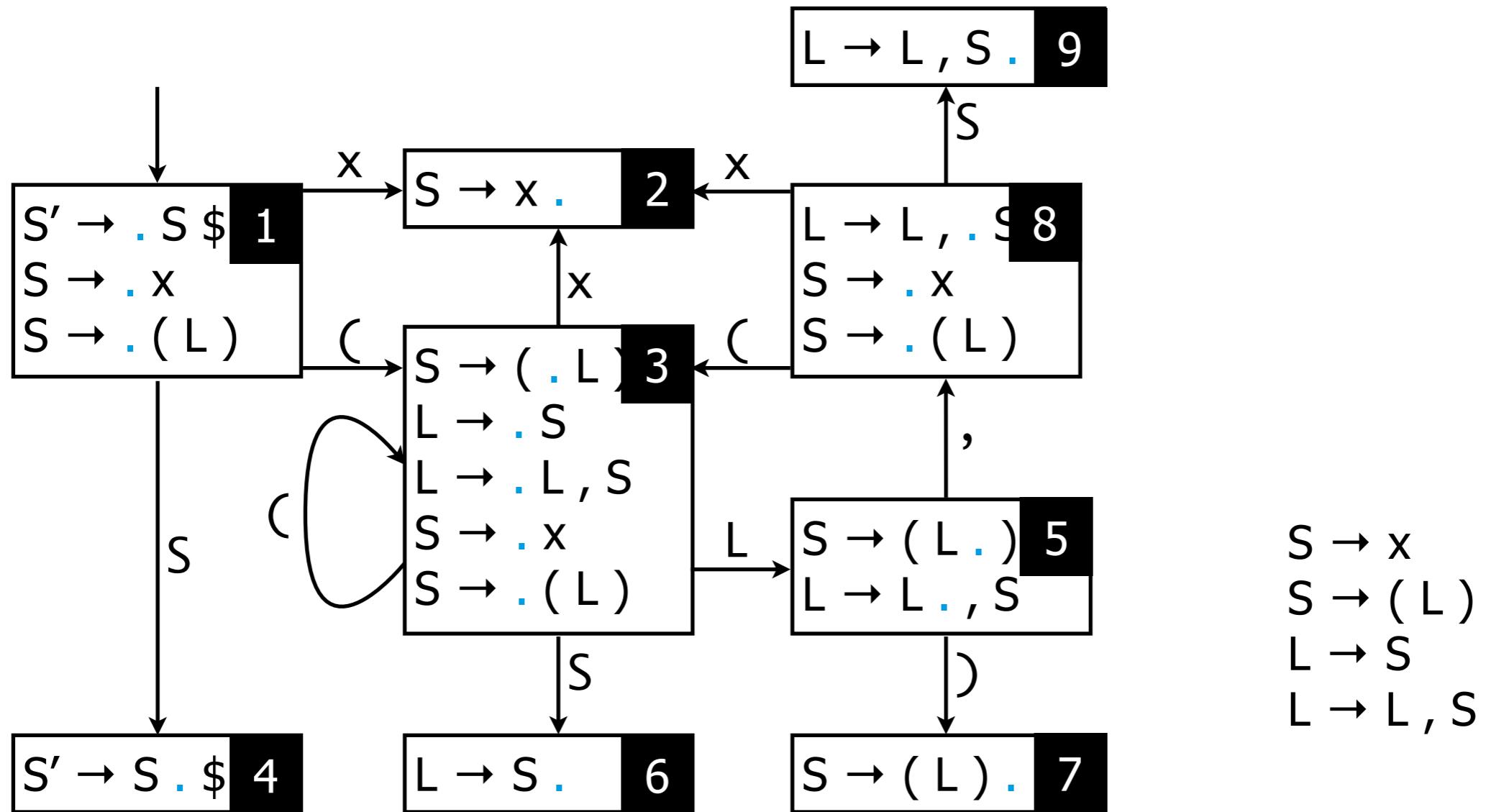
items, closure & goto



$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L , S$

LR(0) parse tables

items, closure & goto



LR(0) parse tables

result

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

(x	,	x)	\$
---	---	---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

x	,	x)	\$
---	---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

2
3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

6
3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

,	x)	\$
---	---	---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5			s 7		s 8		
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

5
3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

8
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

x) \$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

2
8
5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

) \$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5			s 7		s 8		
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

)	\$
---	----

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

3
1

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

5
3
1

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

) \$

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

1

\$

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

LR(0) parse tables

result

	()	x	,	\$	S	L
1	s 3		s 2			g 4	
2	r 1	r 1	r 1	r 1	r 1		
3	s 3		s 2			g 6	g 5
4					a		
5		s 7		s 8			
6	r 3	r 3	r 3	r 3	r 3		
7	r 2	r 2	r 2	r 2	r 2		
8	s 3		s 2			g 9	
9	r 4	r 4	r 4	r 4	r 4		

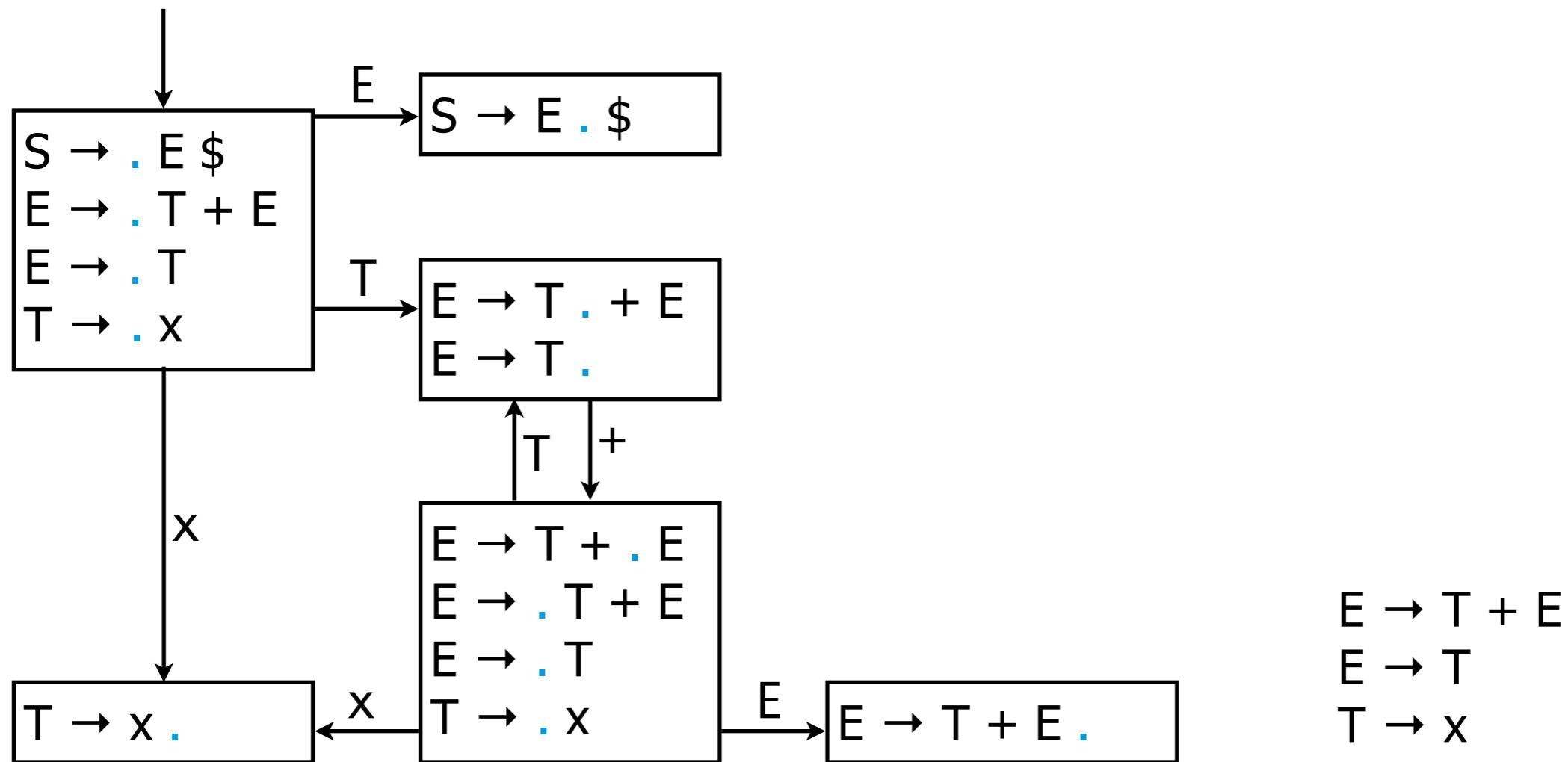
$S \rightarrow x$
 $S \rightarrow (L)$
 $L \rightarrow S$
 $L \rightarrow L, S$

III

Conflict Resolution

SLR parse tables

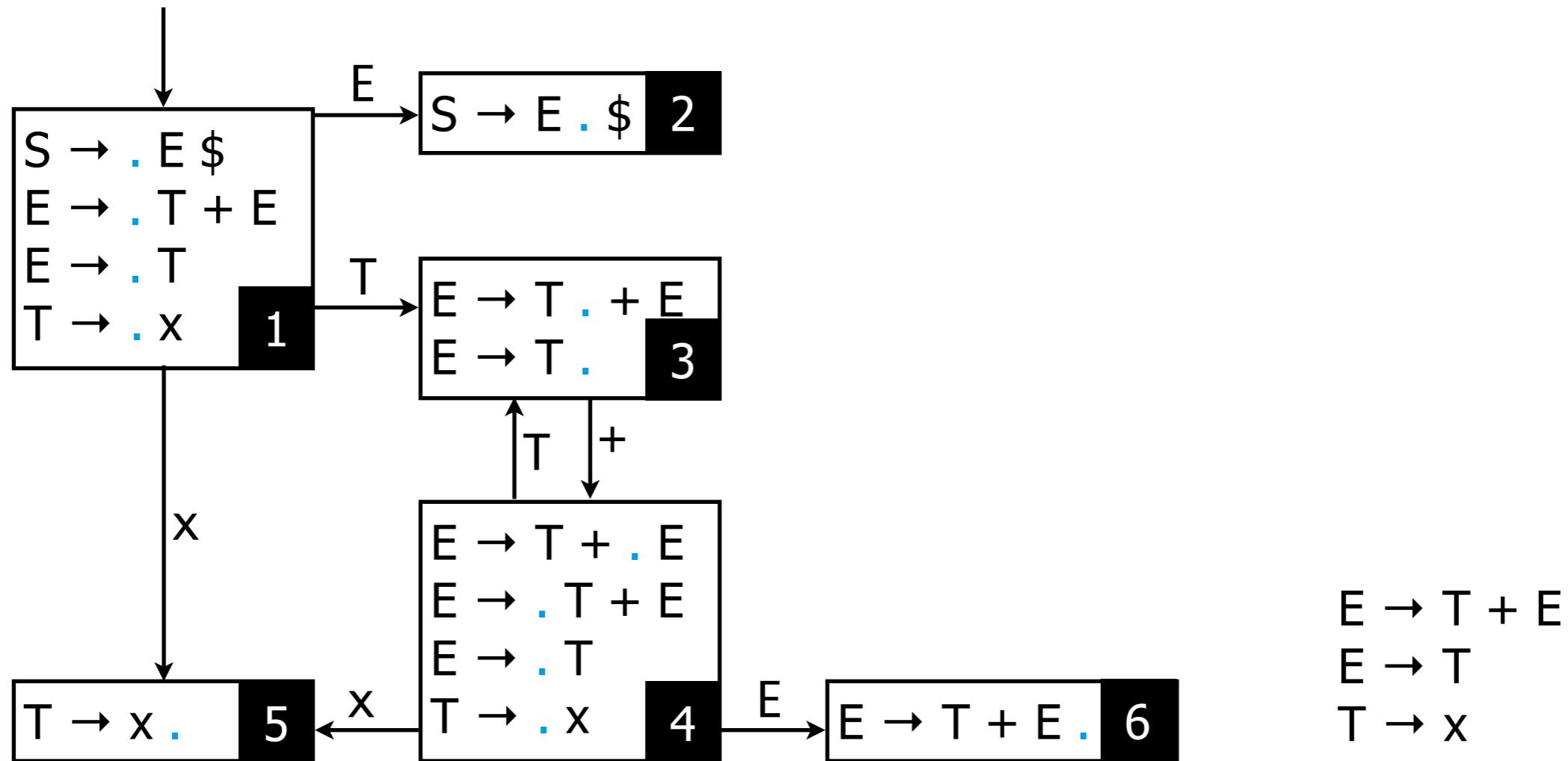
shift-reduce conflicts



$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

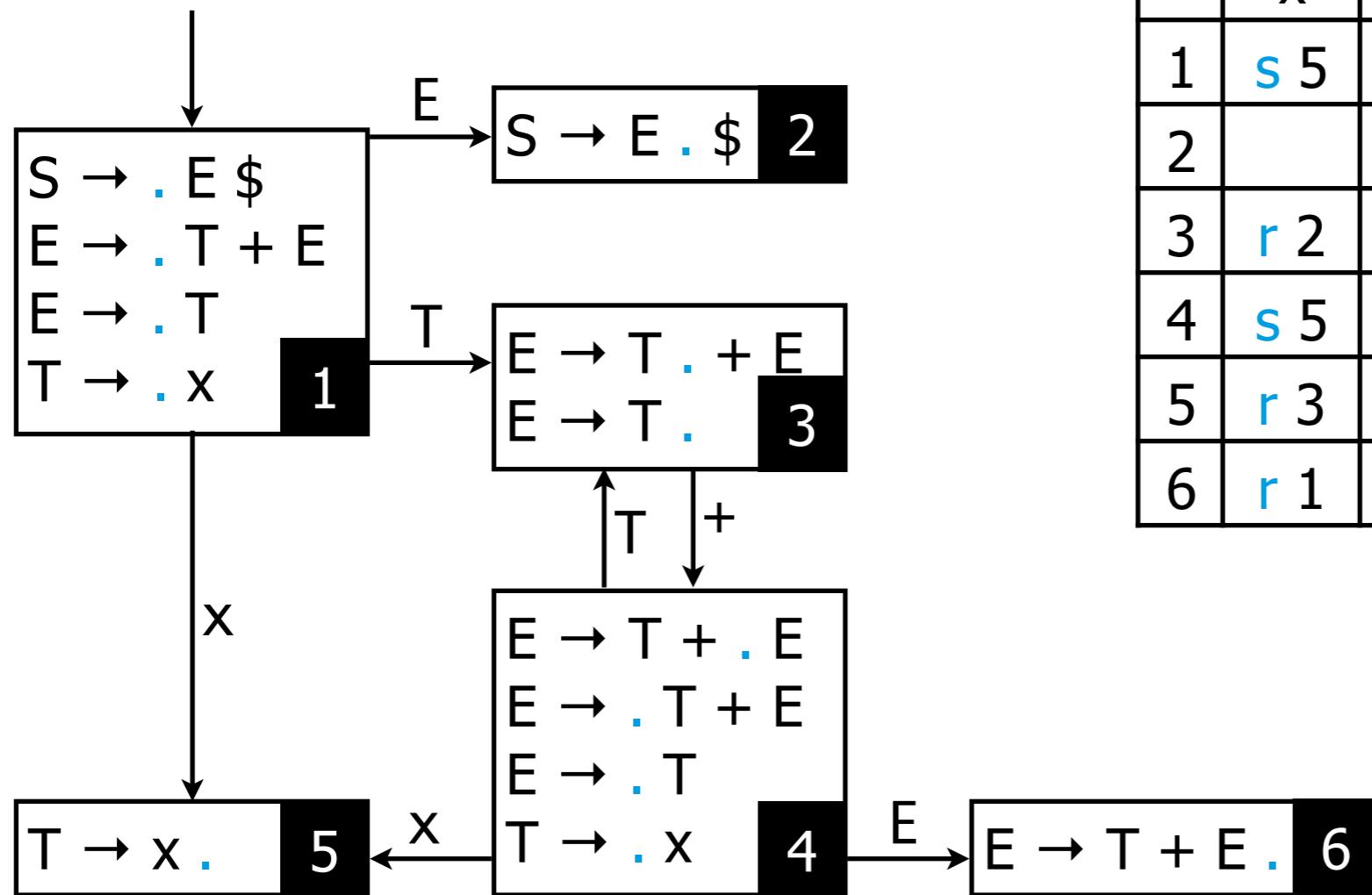
SLR parse tables

shift-reduce conflicts



SLR parse tables

shift-reduce conflicts



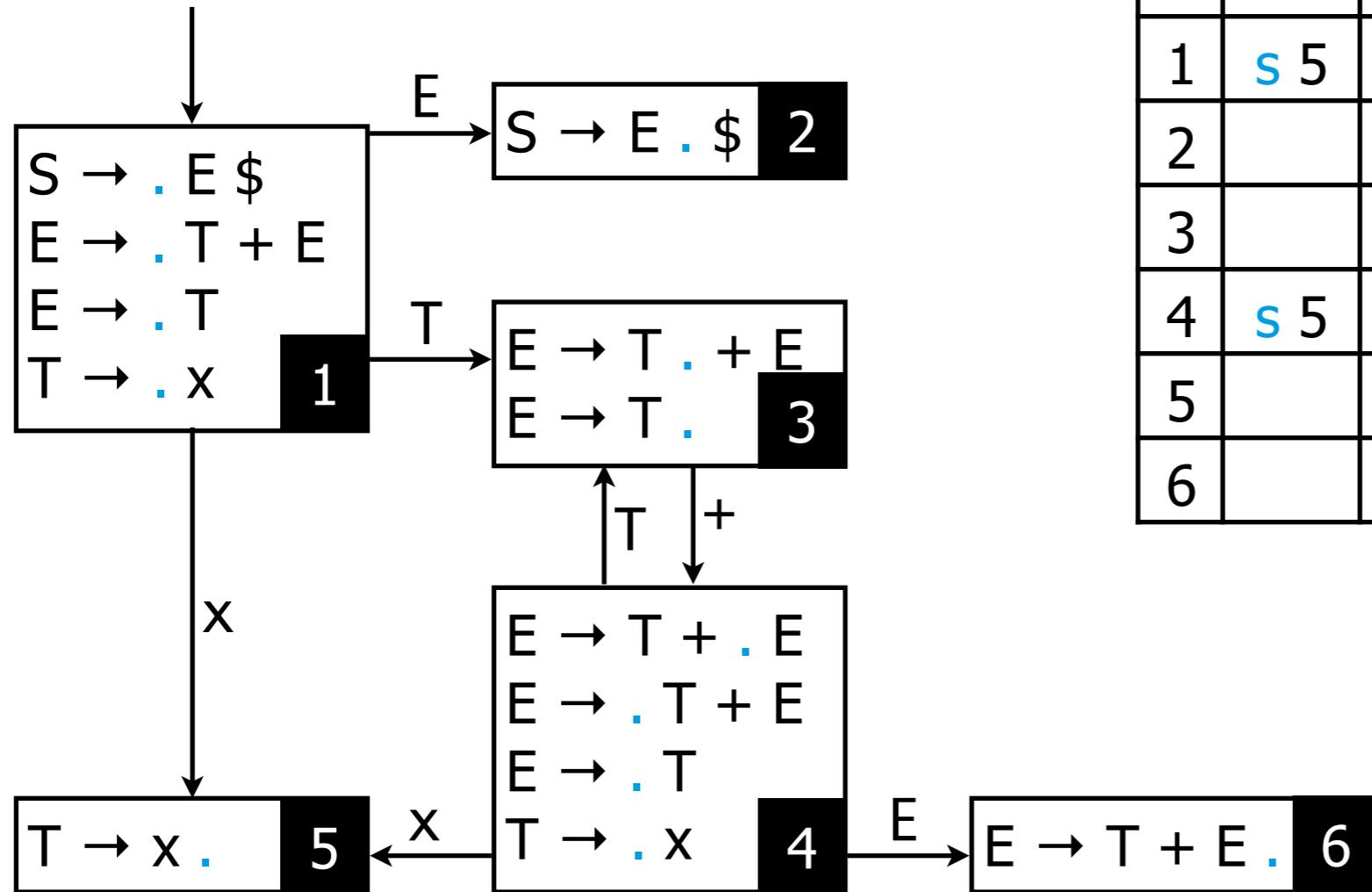
	x	+	\$	E	T
1	s 5			g 2	g 3
2				a	
3	r 2	?	r 2		
4	s 5			g 6	g 3
5	r 3	r 3	r 3		
6	r 1	r 1	r 1		

$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

SLR parse tables

shift-reduce conflicts

Reduce a production $S \rightarrow \dots$
on symbols $k \in \Sigma, k \in \text{Follow}(S)$



	x	+	\$	E	T
1	s 5			g 2	g 3
2				a	
3		s 4	r 2		
4	s 5			g 6	g 3
5		r 3	r 3		
6			r 1		

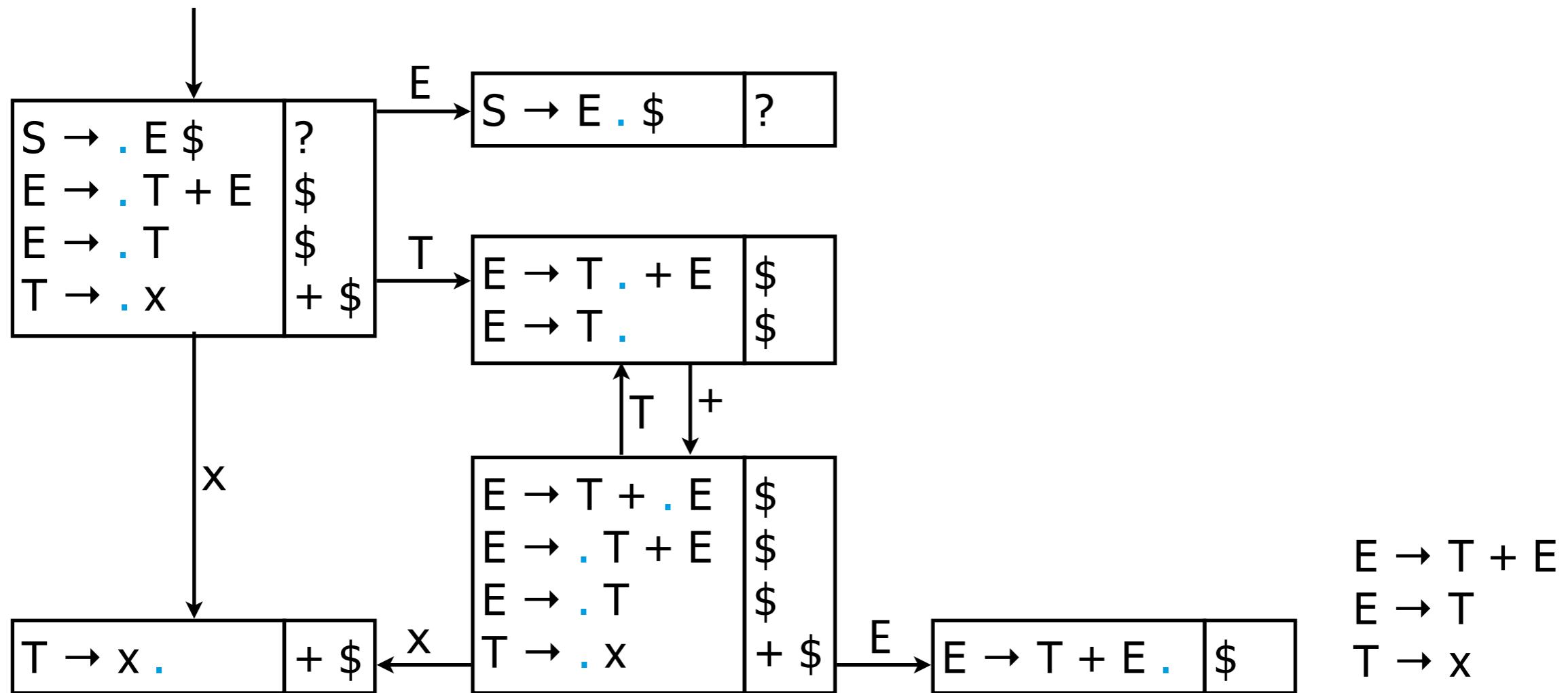
$E \rightarrow T + E$
 $E \rightarrow T$
 $T \rightarrow x$

LR(1) parse tables

look-ahead

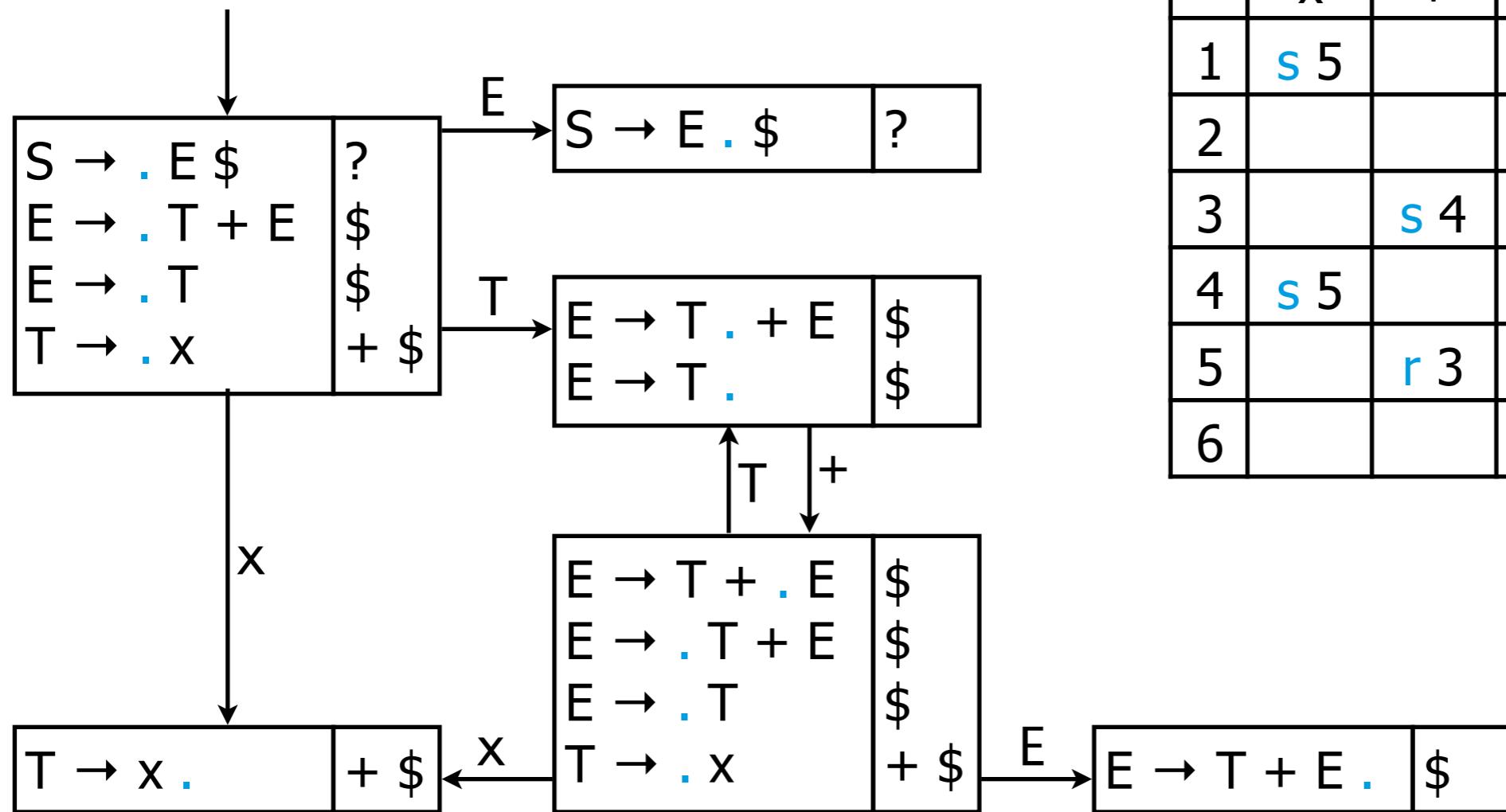
closure

- for every item $A \rightarrow a . X \beta, z$
 - for every rule $X \rightarrow \gamma$
 - for every $w \in \text{First}(\beta z)$
 - add item $X \rightarrow . \gamma, w$



closure

- for every item $A \rightarrow a \cdot X \beta, z$
- for every rule $X \rightarrow \gamma$
- for every $w \in \text{First}(\beta z)$
- add item $X \rightarrow \cdot \gamma, w$



LALR(1) parse tables

state space reduction

unify states

- with same items
- and same outgoing transitions
- but different look-ahead sets

might introduce new conflicts

LALR(1) parse tables

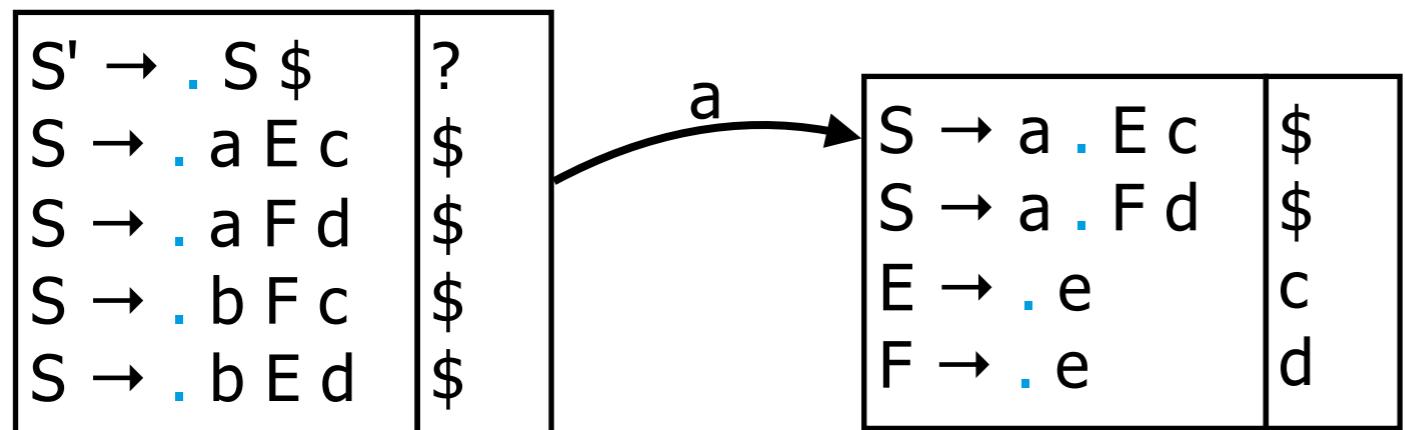
state space reduction

$S' \rightarrow . S \$$?
$S \rightarrow . a E c$	\$
$S \rightarrow . a F d$	\$
$S \rightarrow . b F c$	\$
$S \rightarrow . b E d$	\$

$$\begin{aligned}S' &\rightarrow S \$ \\S &\rightarrow a E c \\S &\rightarrow a F d \\S &\rightarrow b F c \\S &\rightarrow b E d \\E &\rightarrow e \\F &\rightarrow e\end{aligned}$$

LALR(1) parse tables

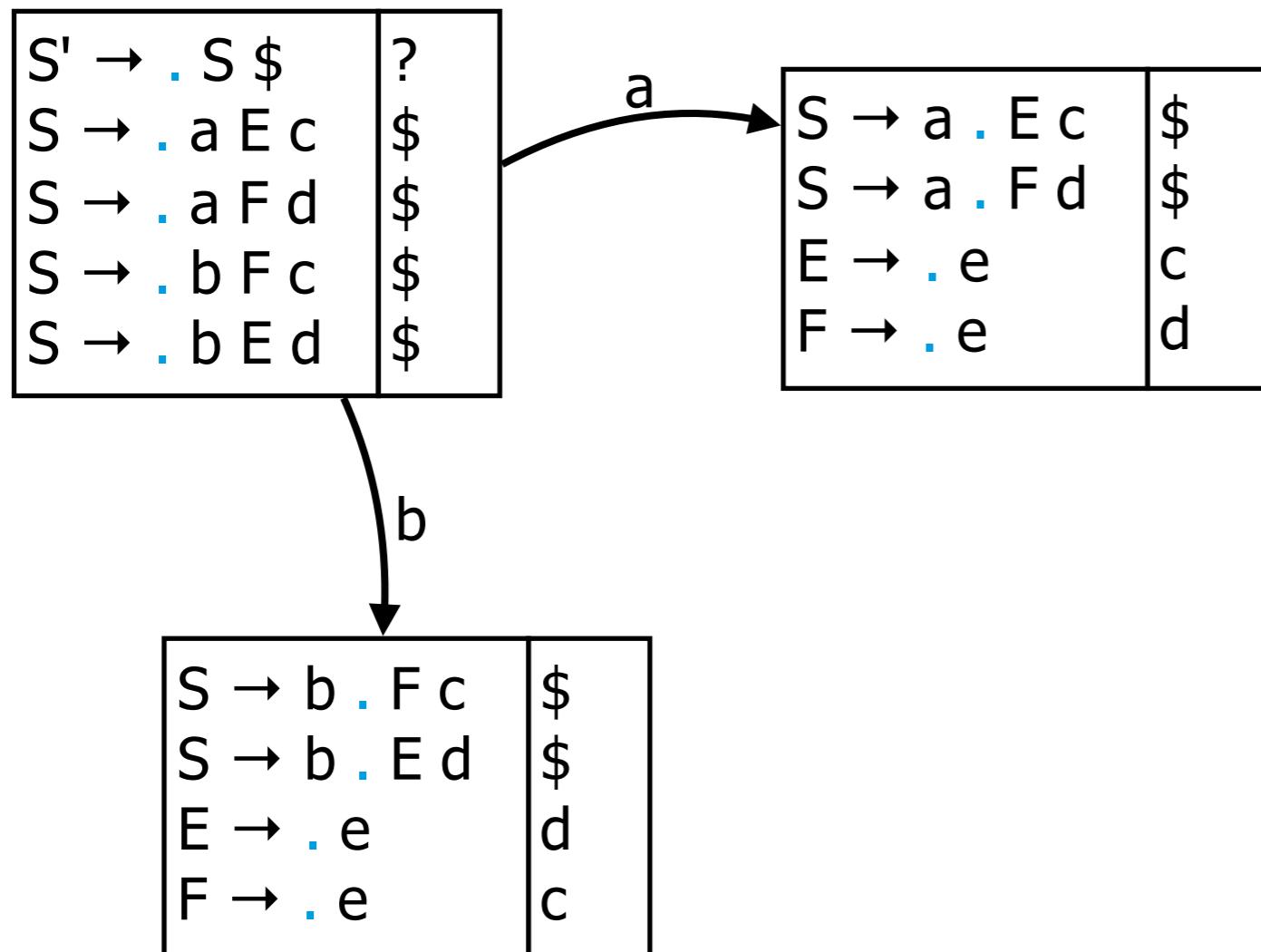
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

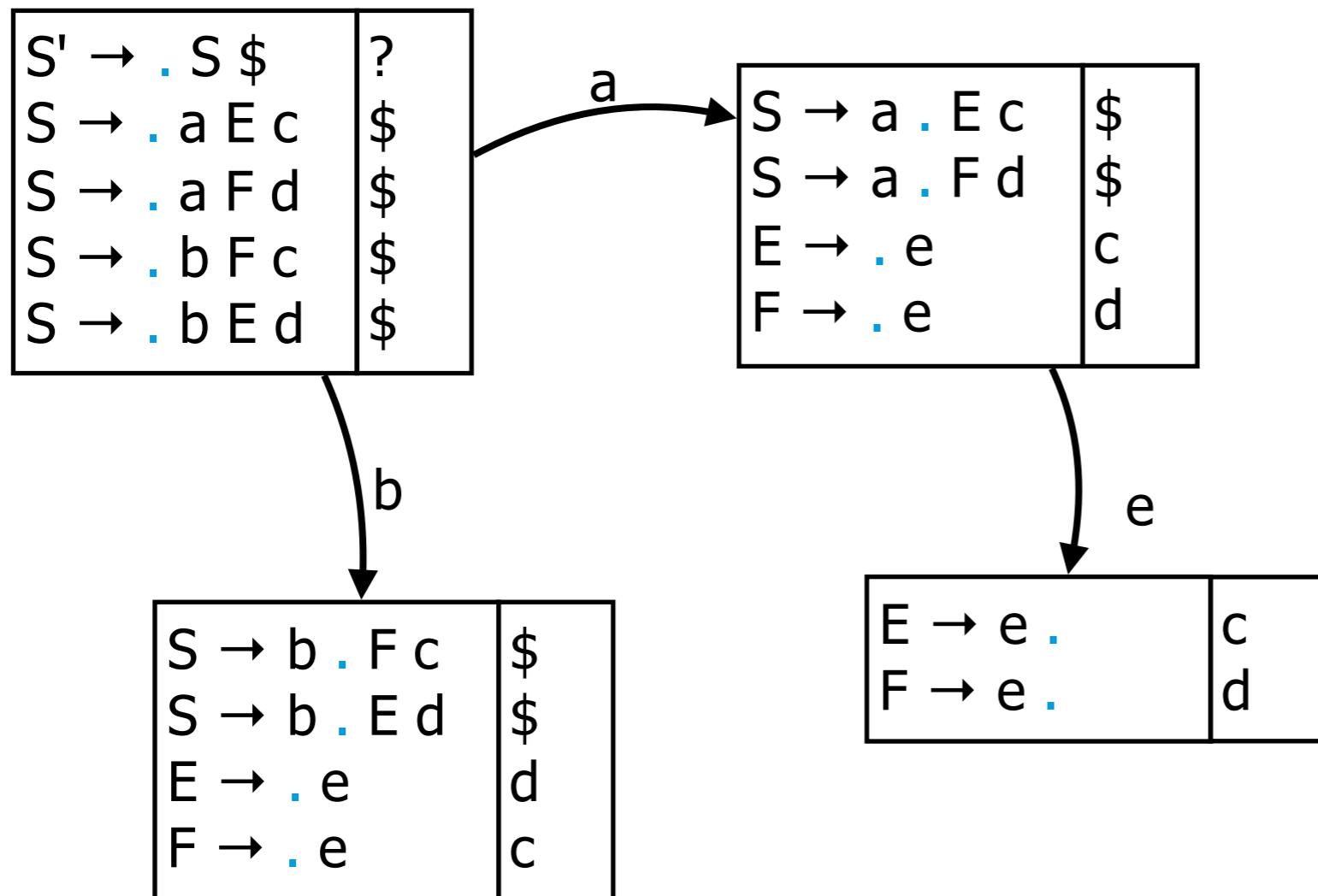
LALR(1) parse tables

state space reduction



LALR(1) parse tables

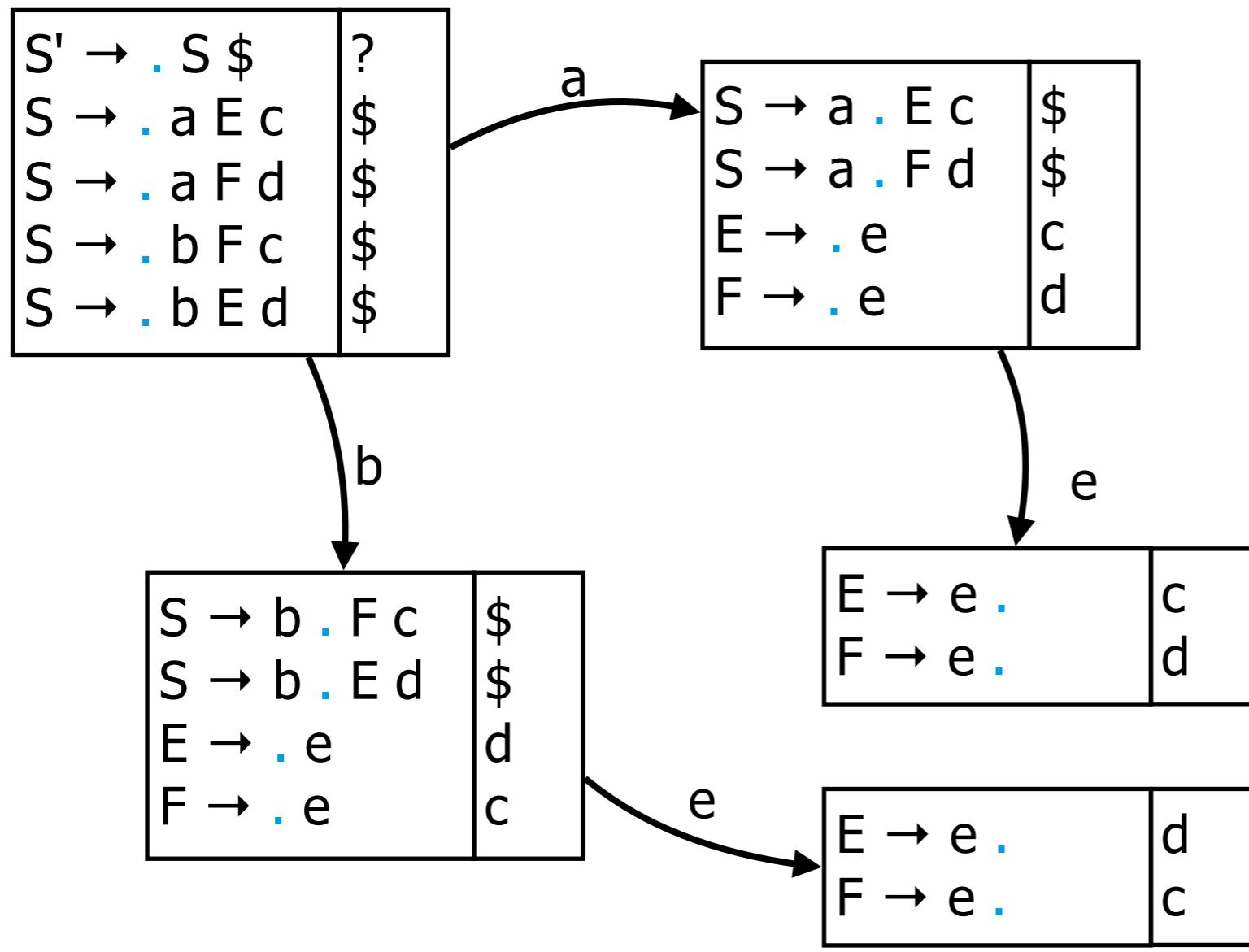
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

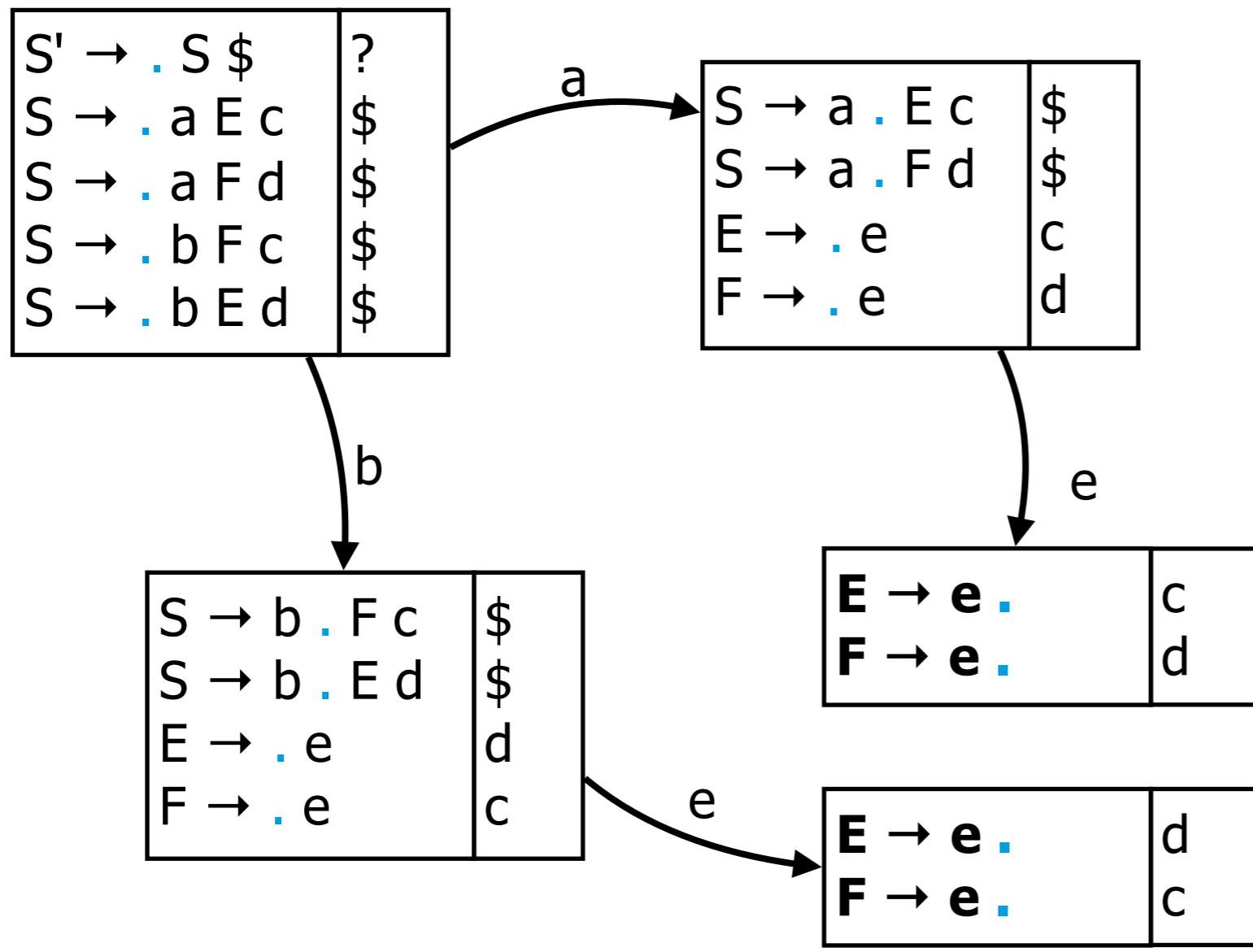
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

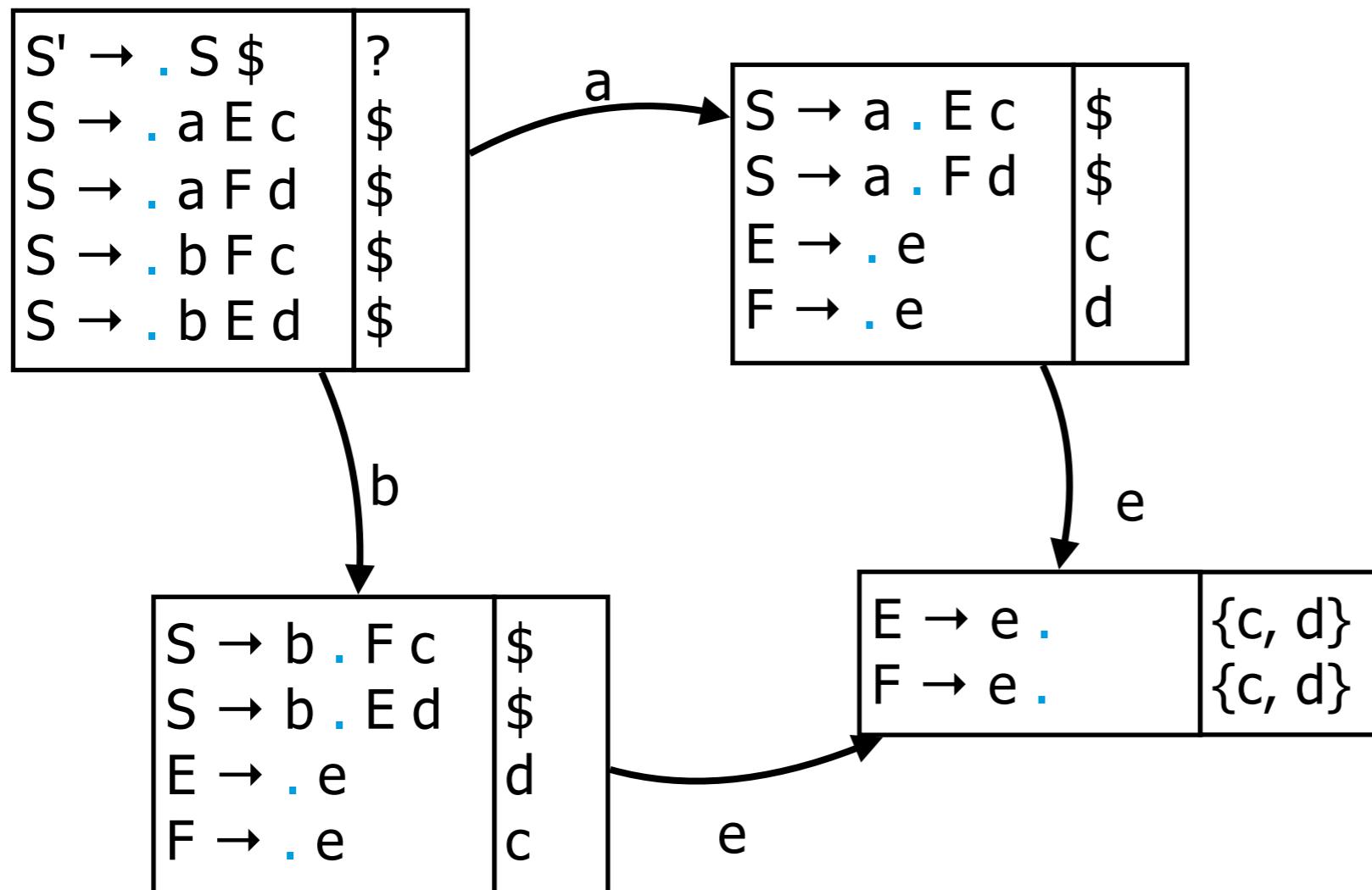
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

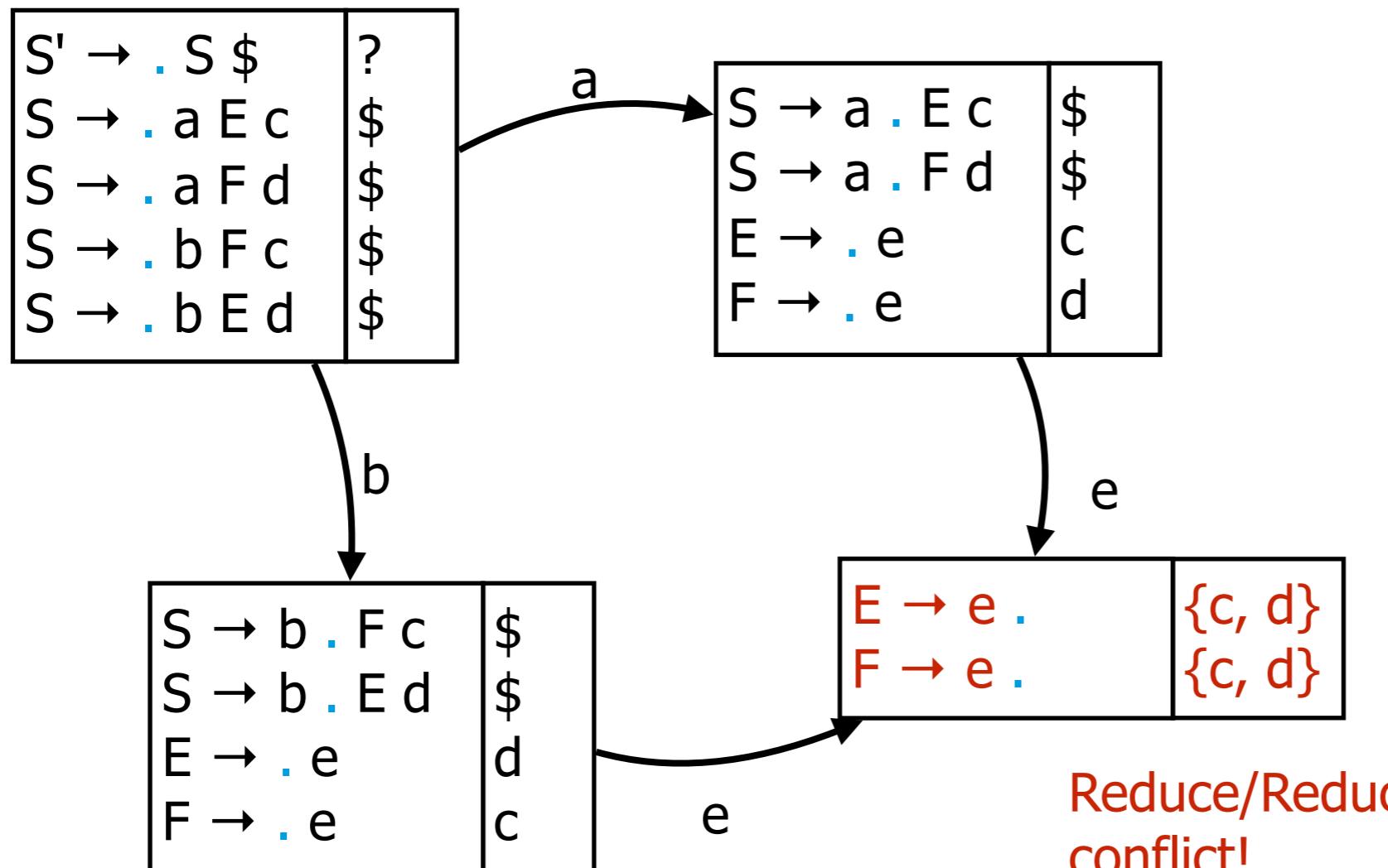
state space reduction



$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

LALR(1) parse tables

state space reduction



Reduce/Reduce
conflict!

$S' \rightarrow S \$$
 $S \rightarrow a E c$
 $S \rightarrow a F d$
 $S \rightarrow b F c$
 $S \rightarrow b E d$
 $E \rightarrow e$
 $F \rightarrow e$

IV

Generalized-LR Parsing

Generalized Parsing

- Parse all interpretations of the input, therefore it can handle ambiguous grammars.
- Parsers split whenever finding an ambiguous interpretation and act in (pseudo) parallel.
- Multiple parsers can join whenever they finish parsing an ambiguous fragment of the input.
- Some parsers may "die", if the ambiguity was caused by a lack of lookahead.

Generalized LR

- Multiple parsers are synchronized on shift actions.
- Each parser has its own stack, and as they share states, the overall structure becomes a graph (GSS).
- If two parsers have the same state on top of their stack, they are joined into a single parser.
- Reduce actions affect all possible paths from the top of the stack.

Generalized LR

SLR table

$$\begin{aligned} S &\rightarrow E \$ \\ E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow a \end{aligned}$$

Generalized LR

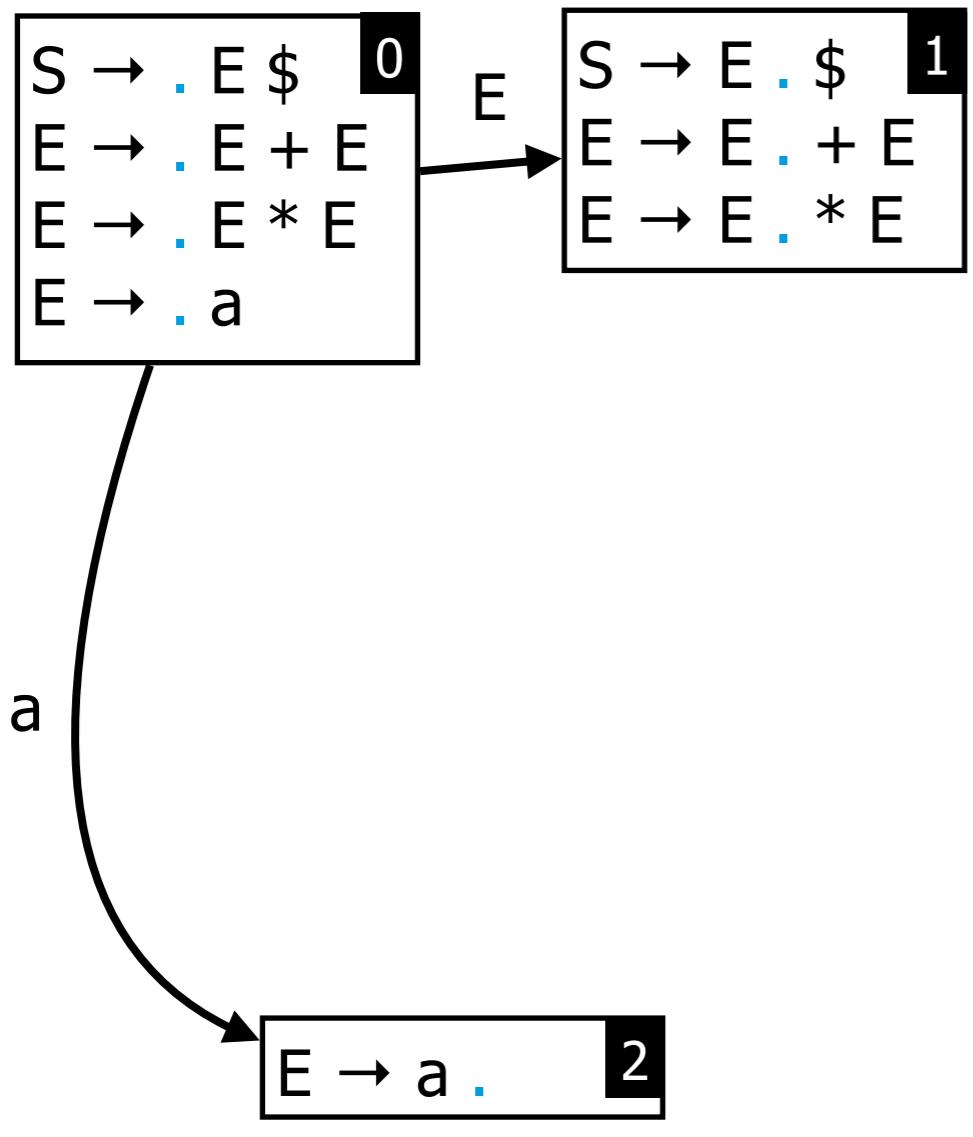
SLR table

$S \rightarrow \cdot E \$$	0
$E \rightarrow \cdot E + E$	
$E \rightarrow \cdot E * E$	
$E \rightarrow \cdot a$	

$$\begin{aligned} S &\rightarrow E \$ \\ E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow a \end{aligned}$$

Generalized LR

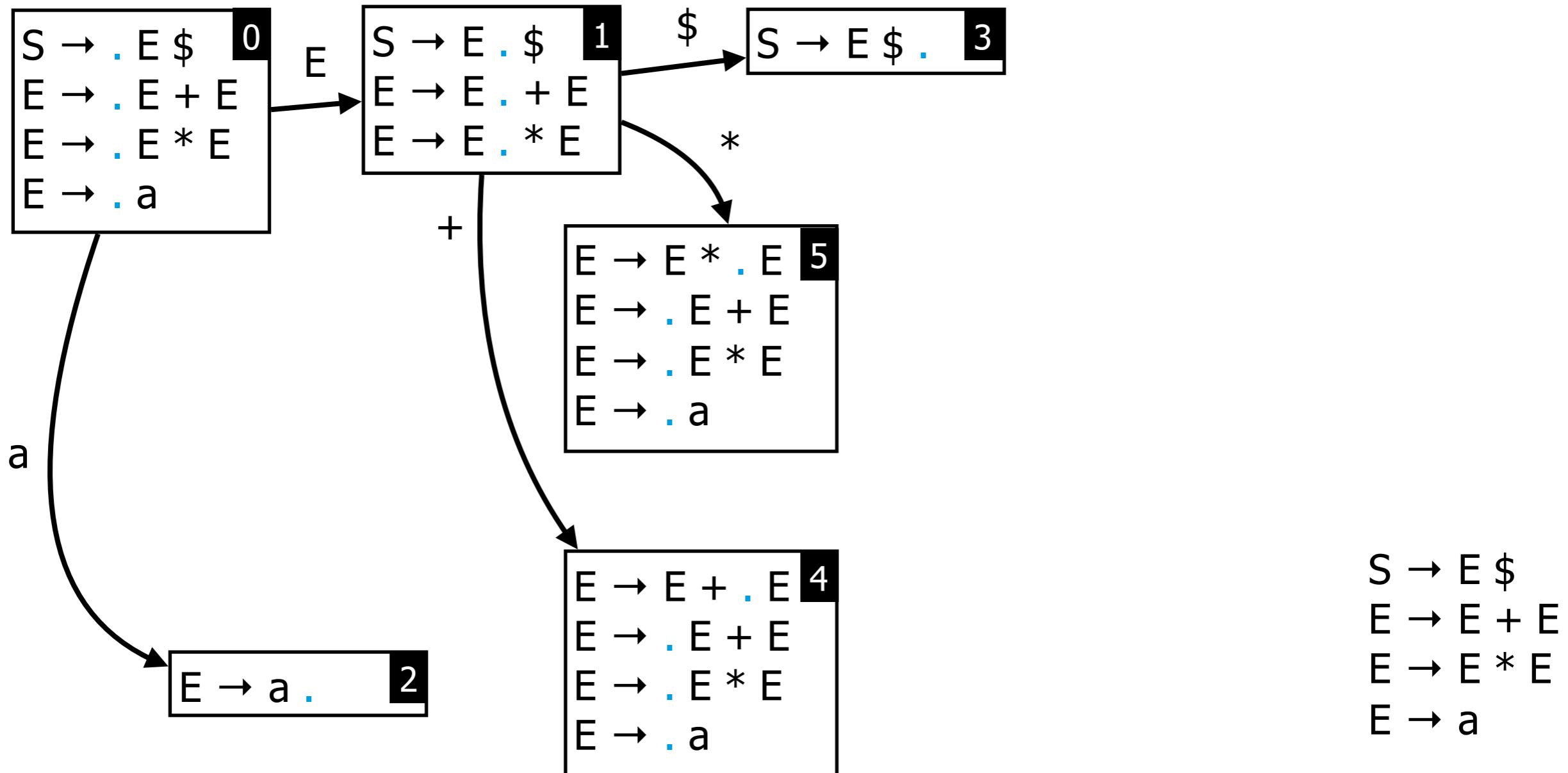
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

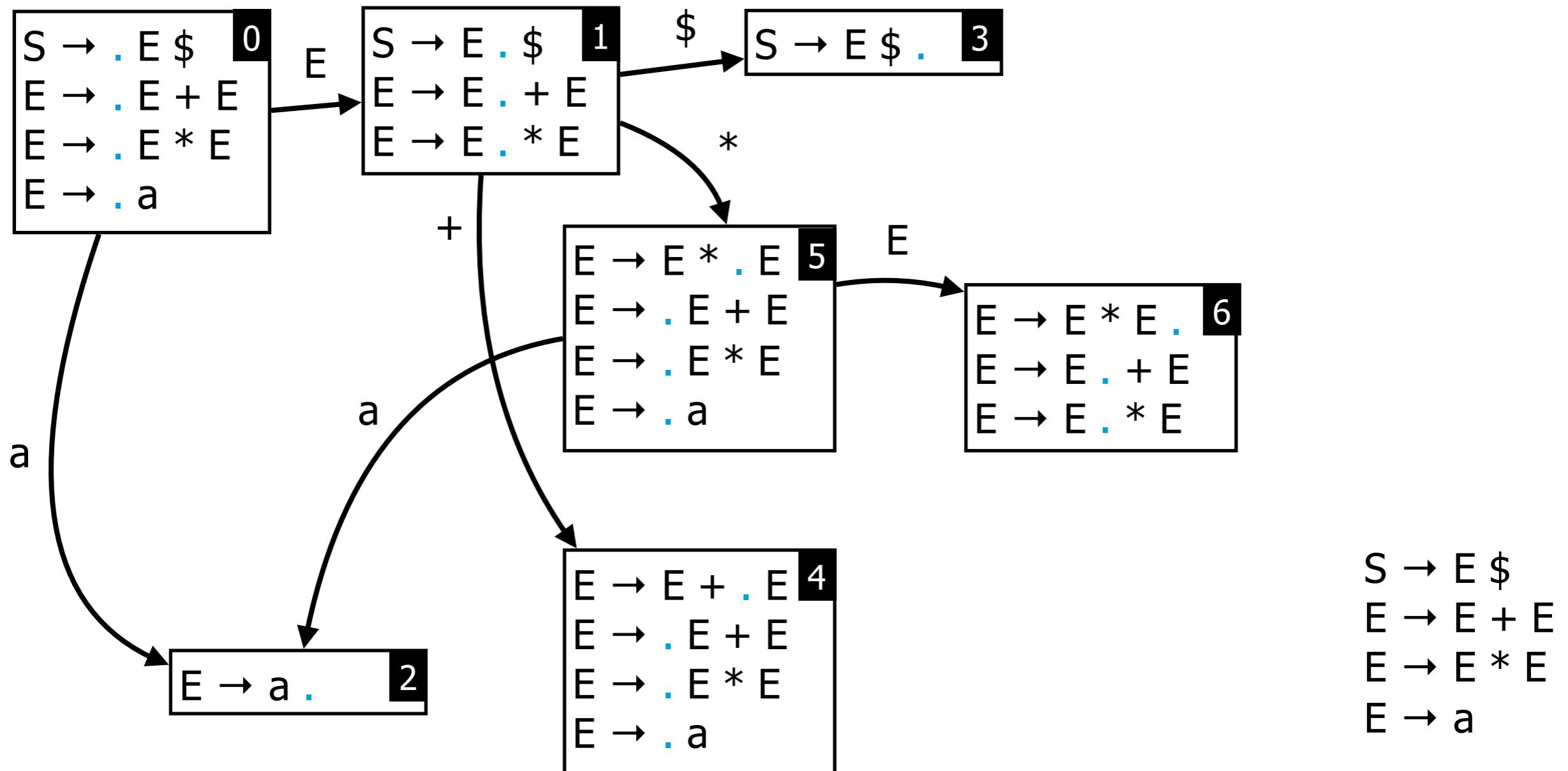
Generalized LR

SLR table


$$\begin{array}{l} S \rightarrow E \$ \\ E \rightarrow E + E \\ E \rightarrow E * E \\ E \rightarrow a \end{array}$$

Generalized LR

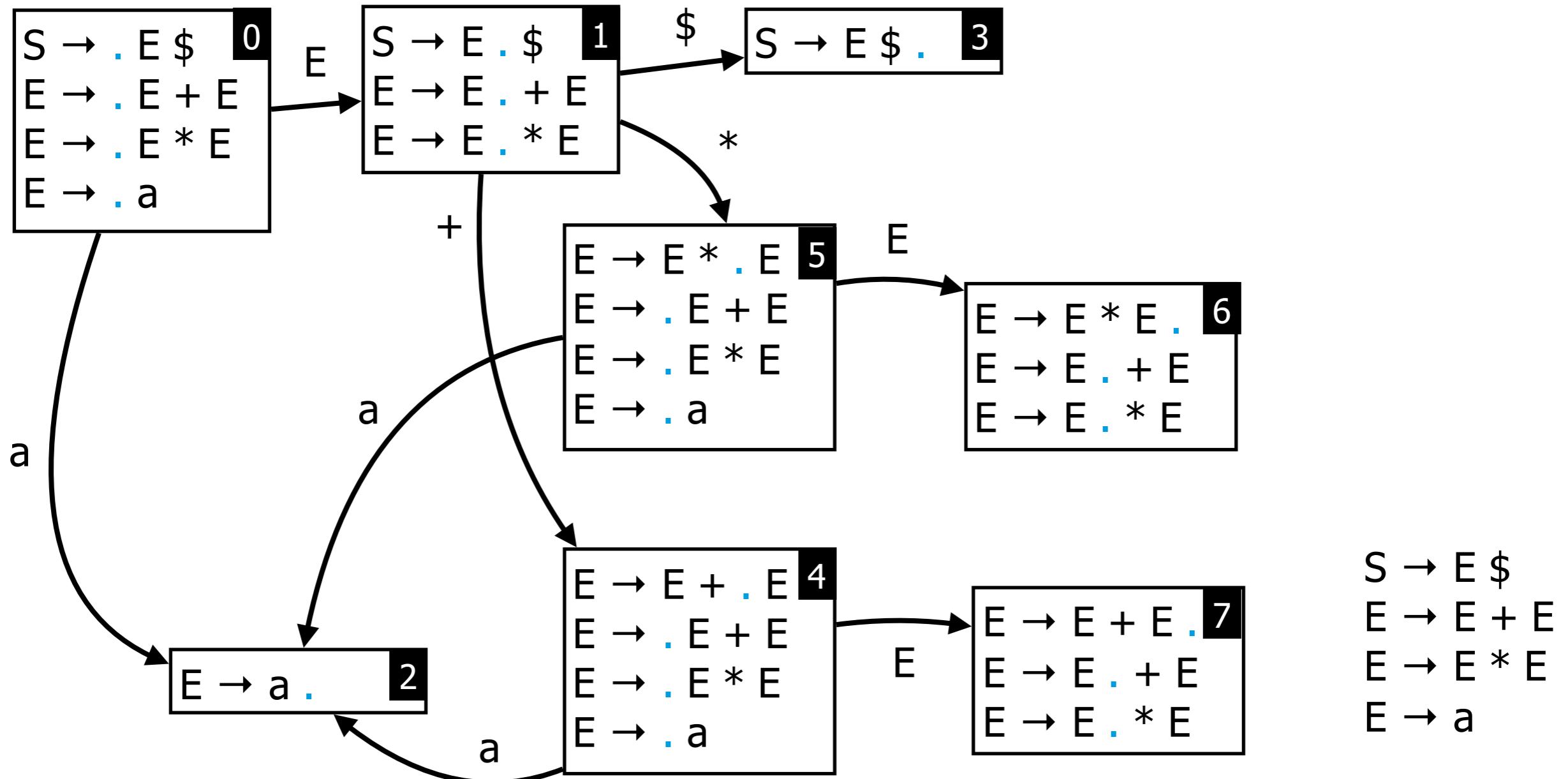
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

Generalized LR

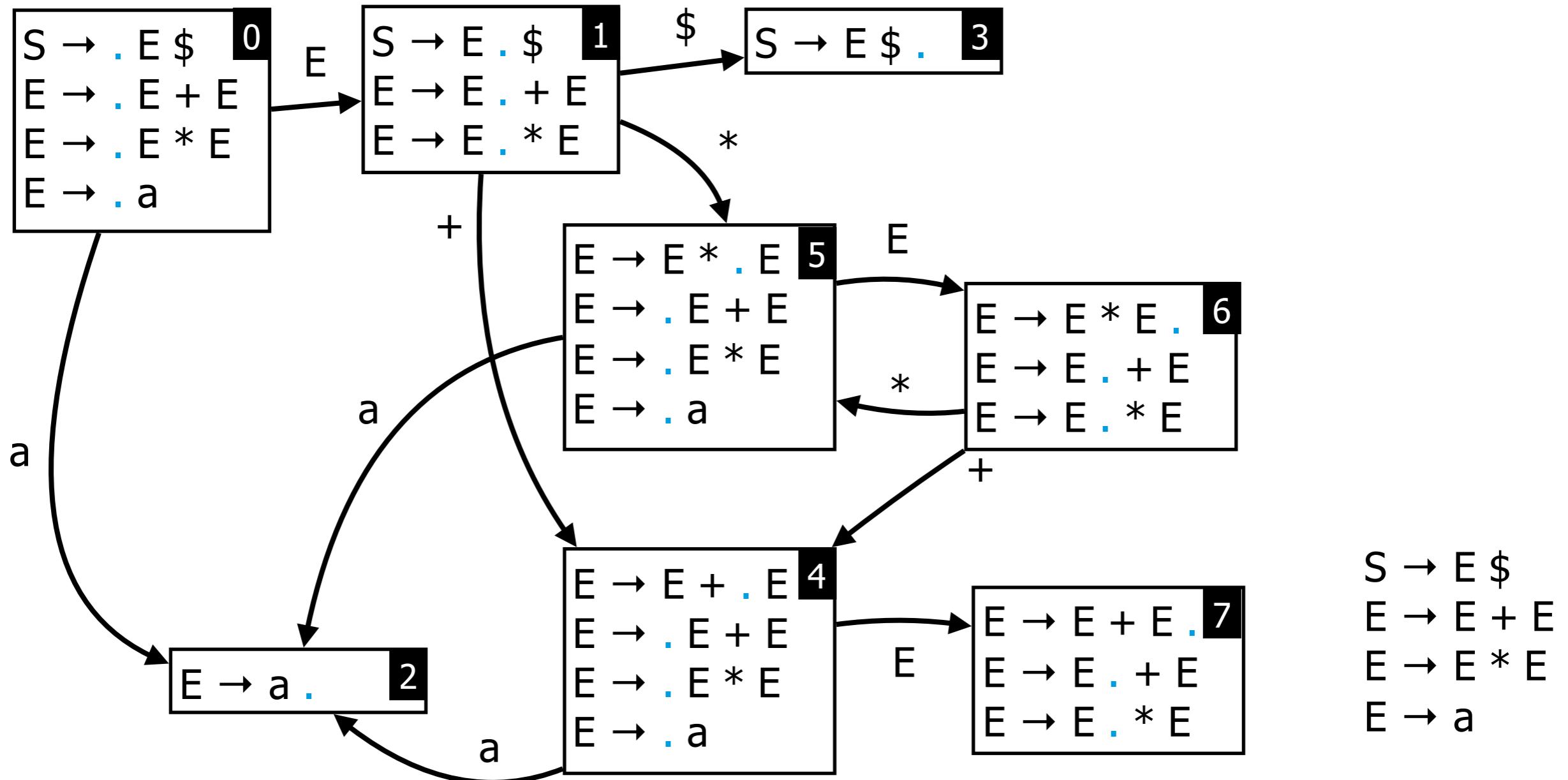
SLR table



$S \rightarrow E \$$
 $E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow a$

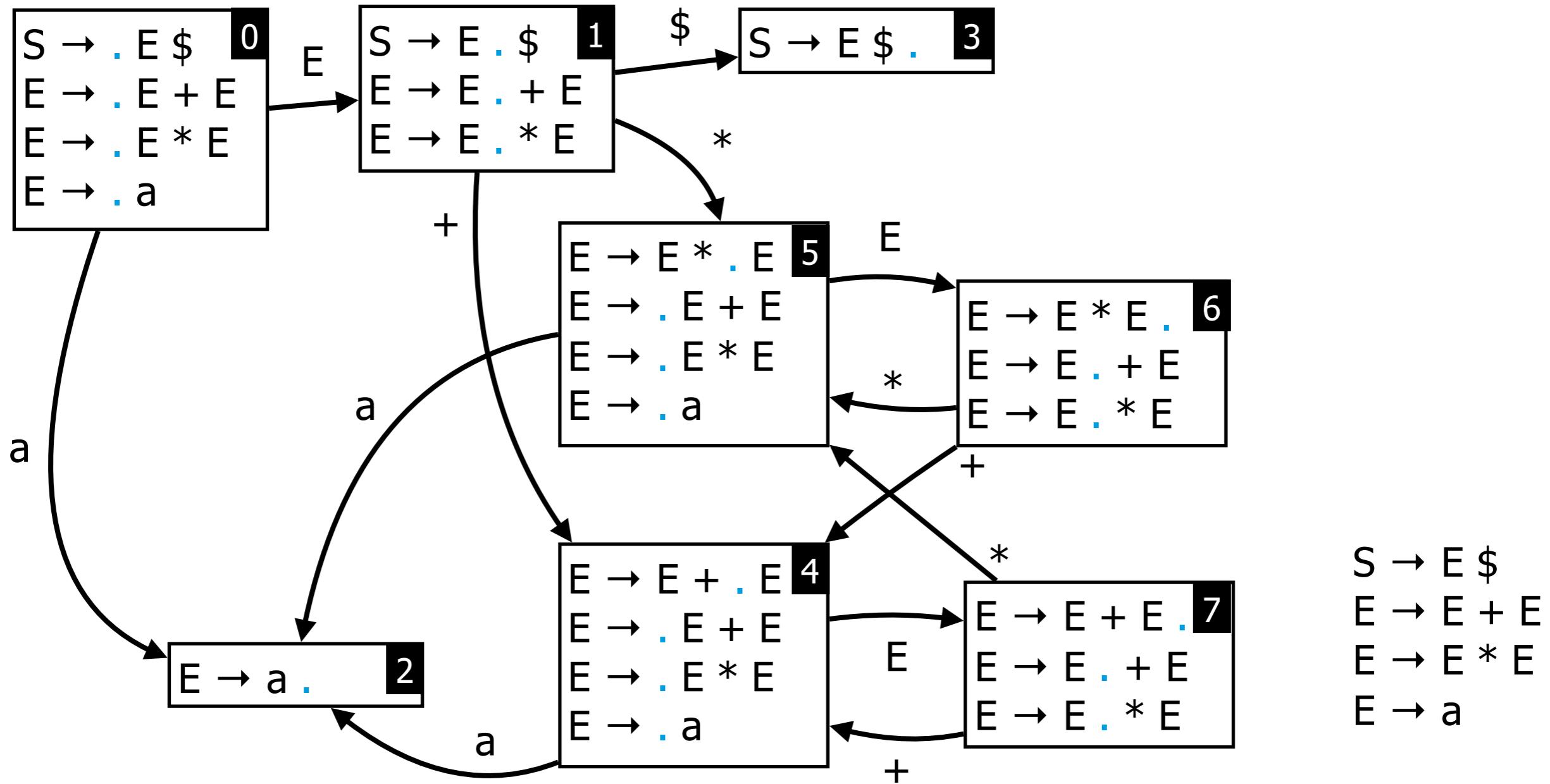
Generalized LR

SLR table



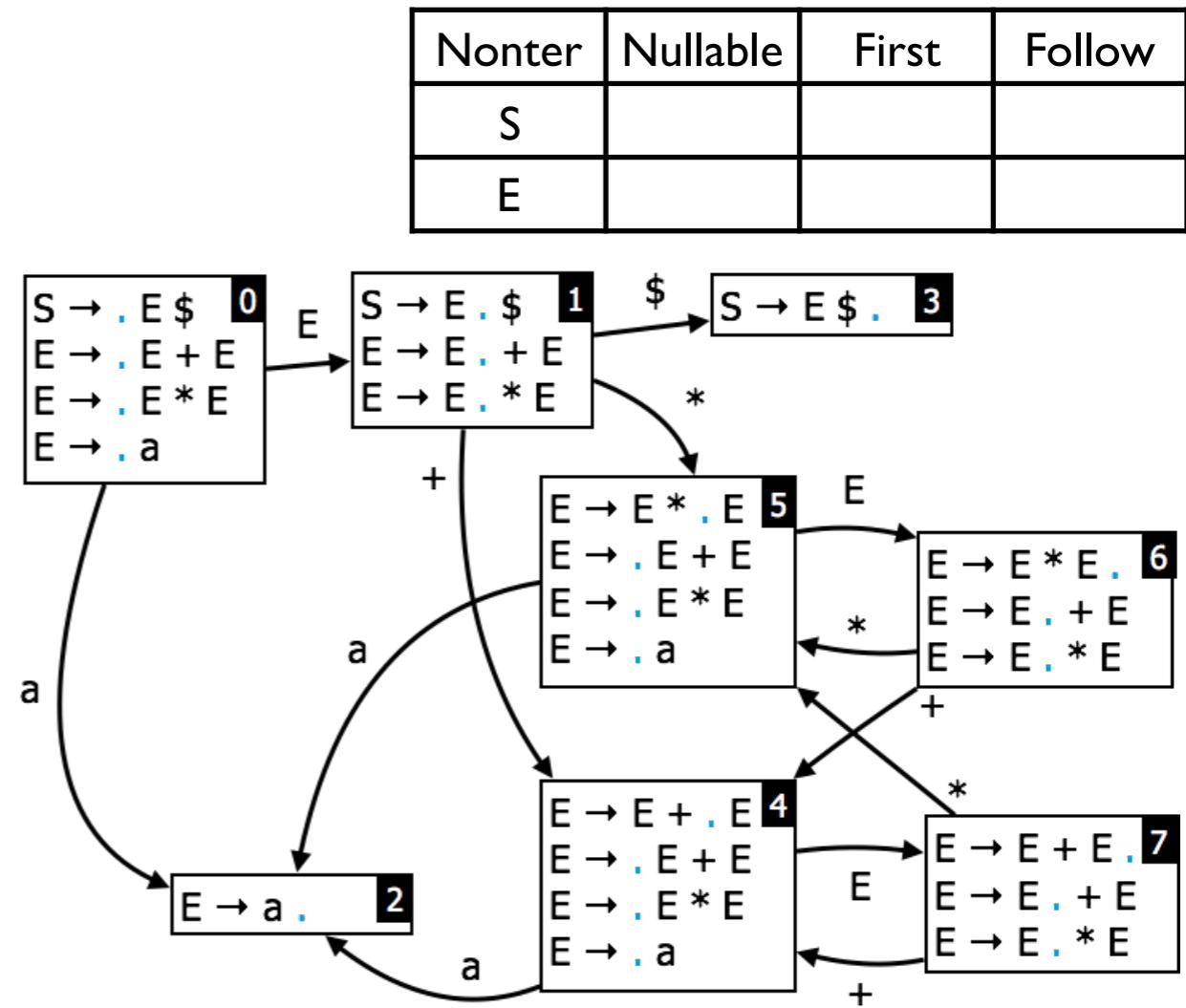
Generalized LR

SLR table



Generalized LR SLR table

State	Action				Goto	
	a	+	*	\$	S	E
0						
1						
2						
3						
4						
5						
6						
7						

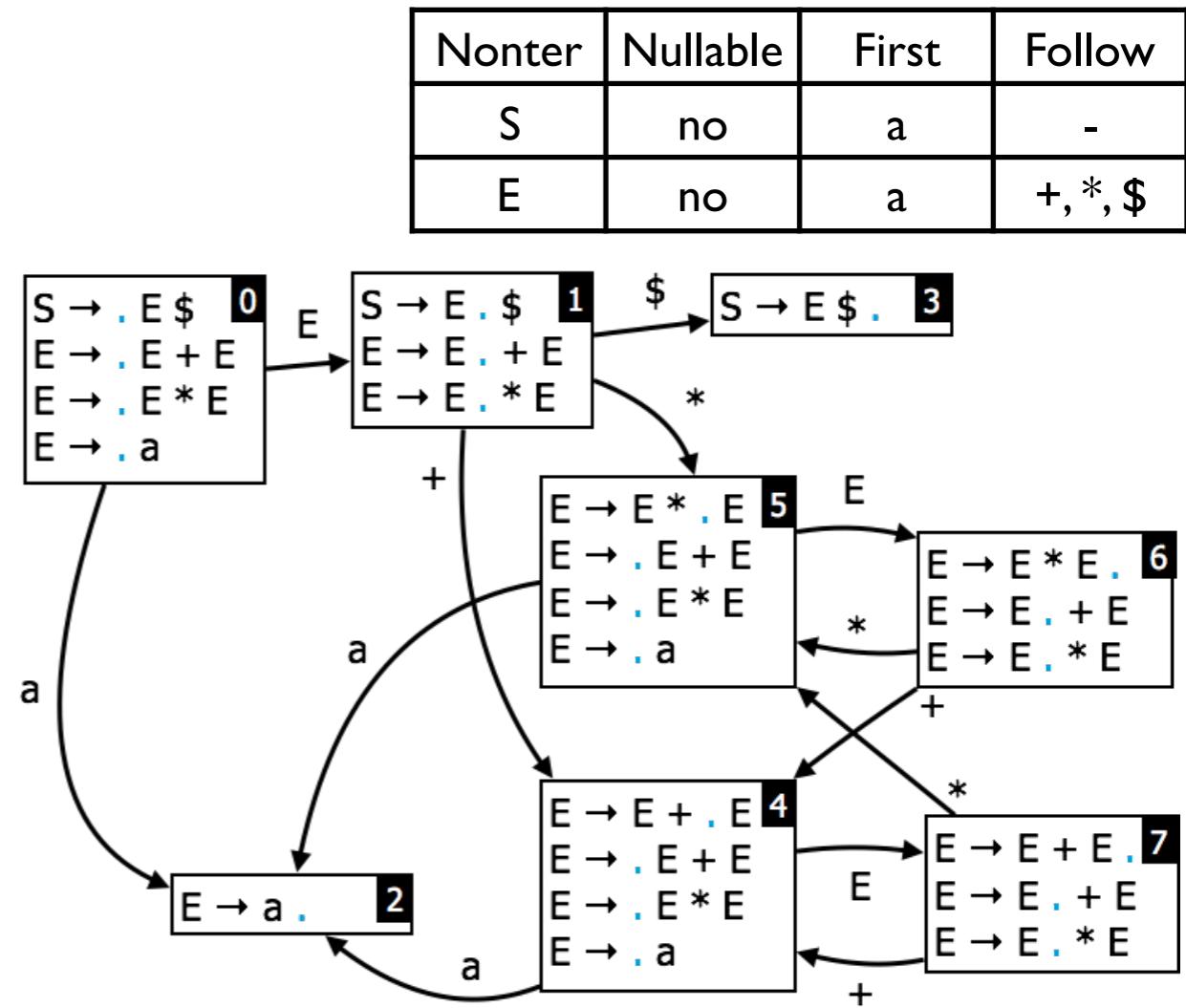


- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

Generalized LR

SLR table

State	Action				Goto	
	a	+	*	\$	S	E
0						
1						
2						
3						
4						
5						
6						
7						

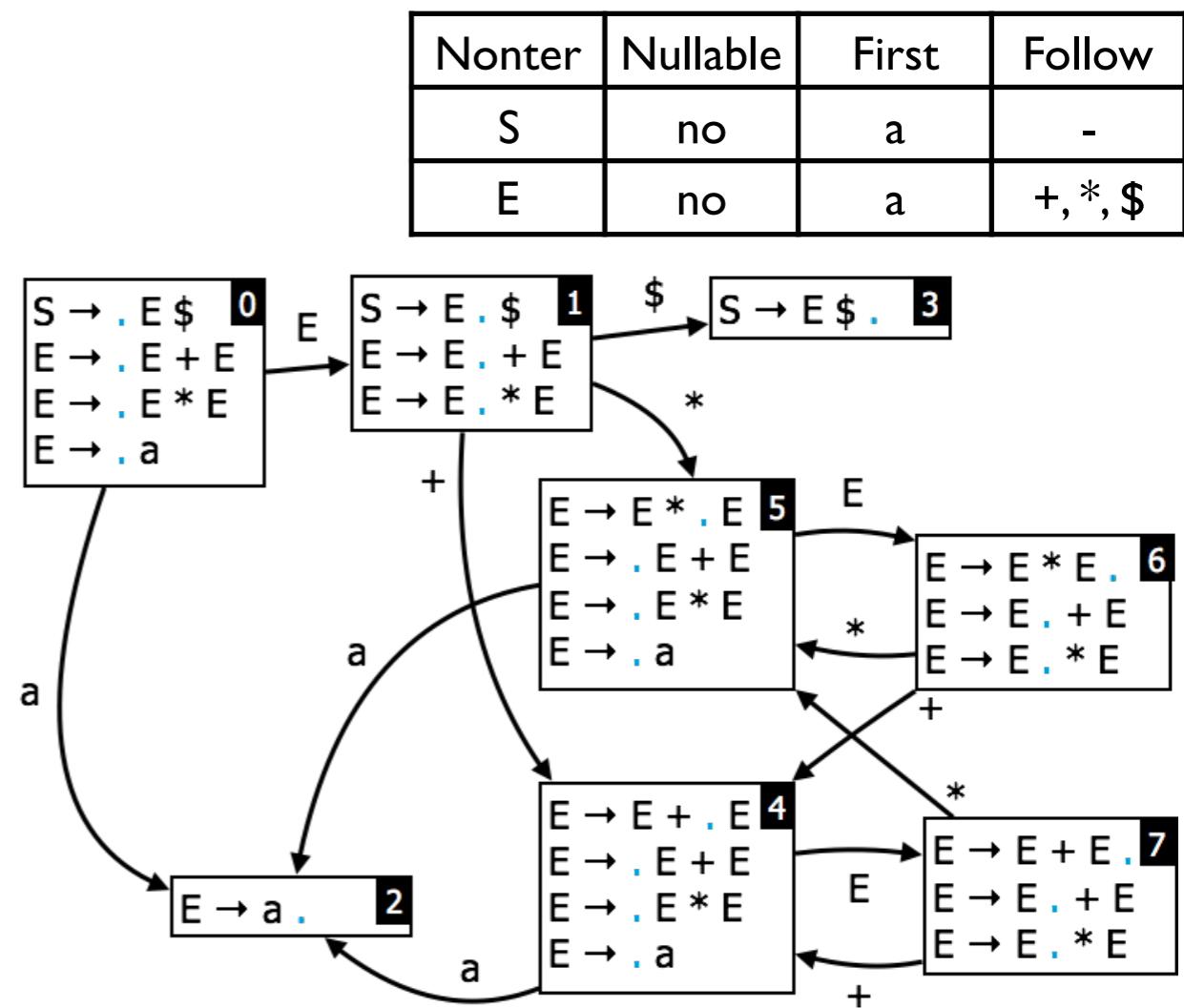


- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

Generalized LR

SLR table

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
I		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		



- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

Generalized LR Parsing

a + a * a

- (0) S → E \$
- (1) E → E + E
- (2) E → E * E
- (3) E → a

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

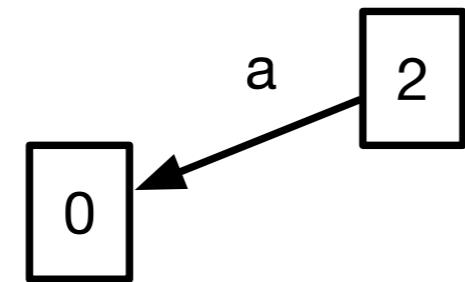
0

Generalized LR Parsing

+ a * a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		



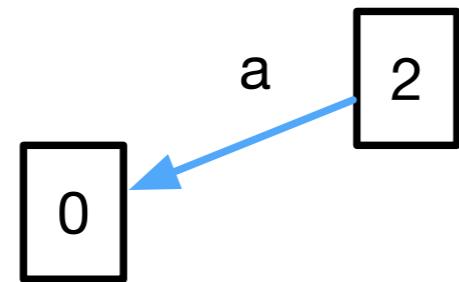
synchronize on shifts

Generalized LR Parsing

+ a * a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

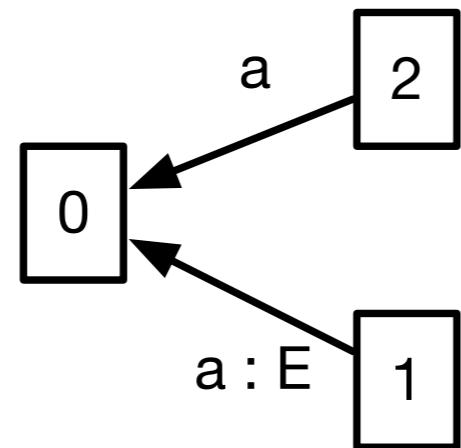


Generalized LR Parsing

+ a * a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

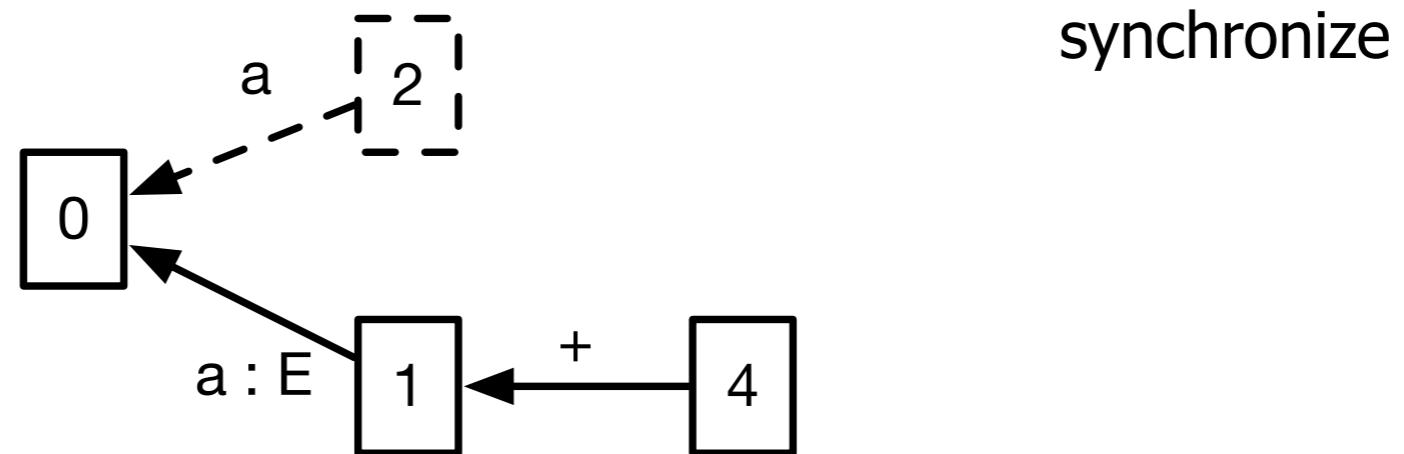


Generalized LR Parsing

$a * a$

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

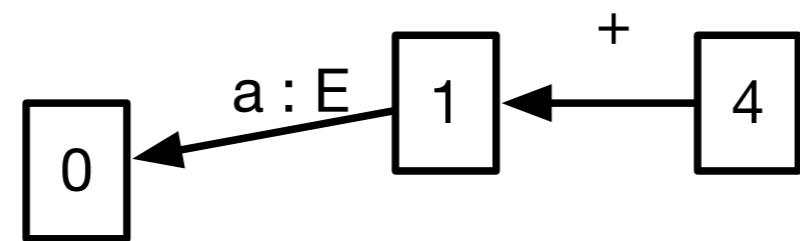


Generalized LR Parsing

a * a

- (0) S → E \$
- (1) E → E + E
- (2) E → E * E
- (3) E → a

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

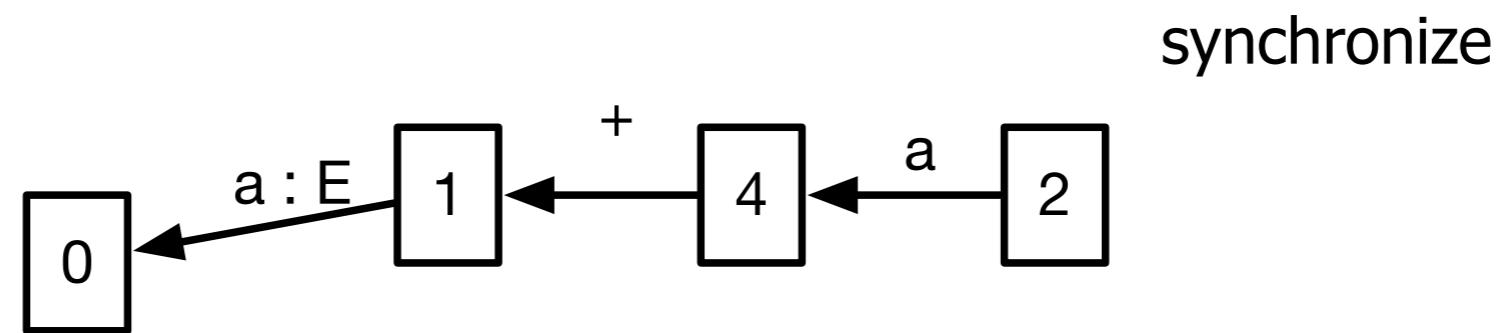


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

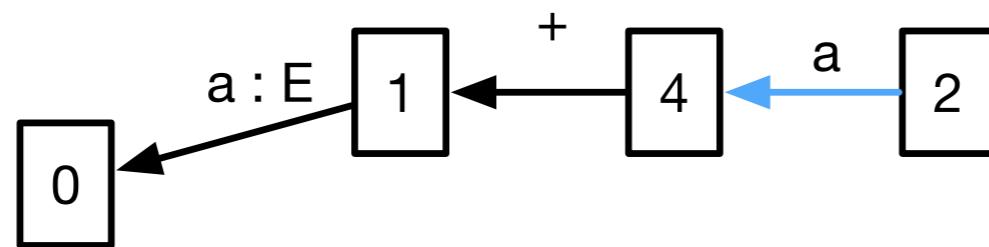


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

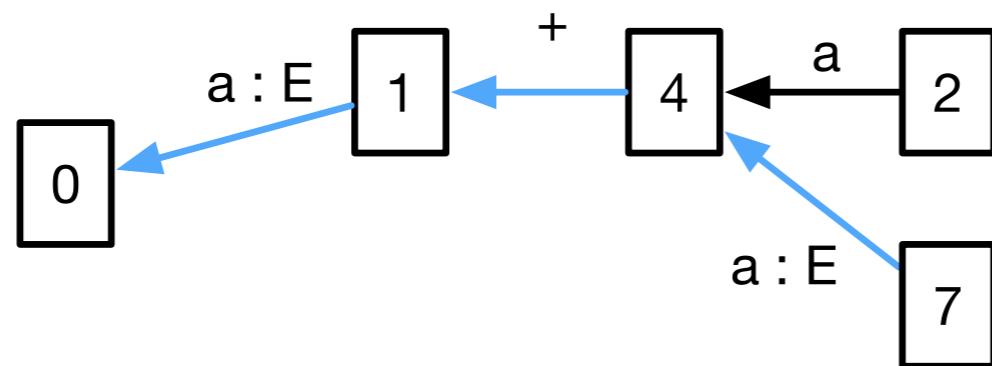


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

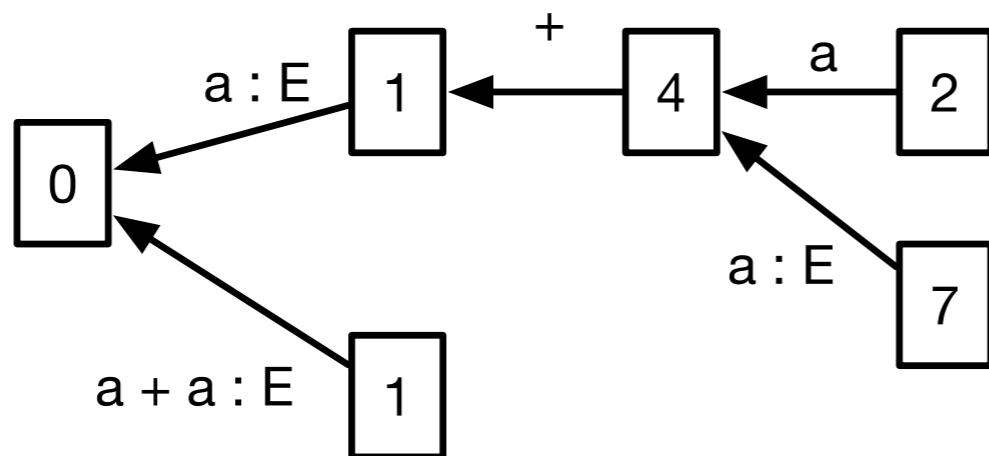


Generalized LR Parsing

* a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		

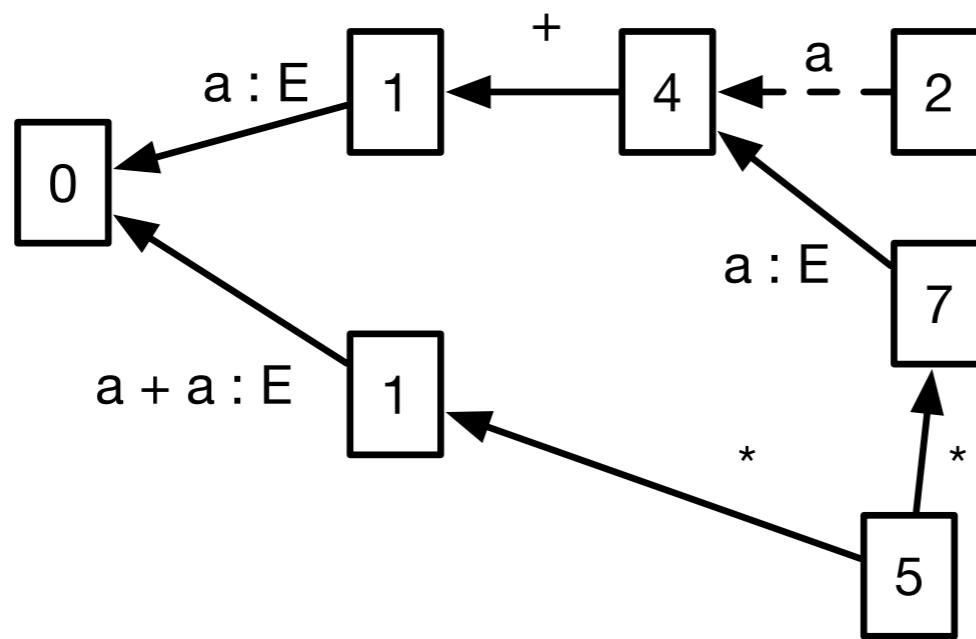


Generalized LR Parsing

a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
- (2) $E \rightarrow E * E$
- (3) $E \rightarrow a$

State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		



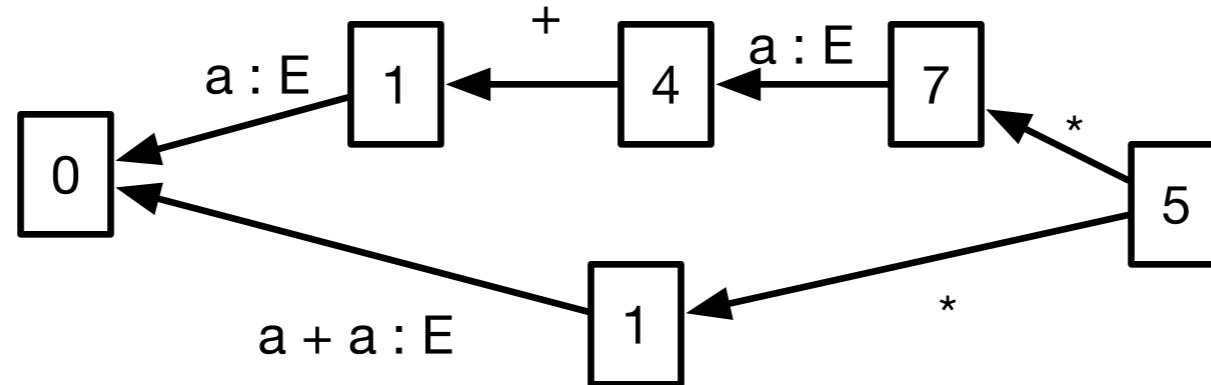
synchronize

Generalized LR Parsing

a

- (0) $S \rightarrow E \$$
- (1) $E \rightarrow E + E$
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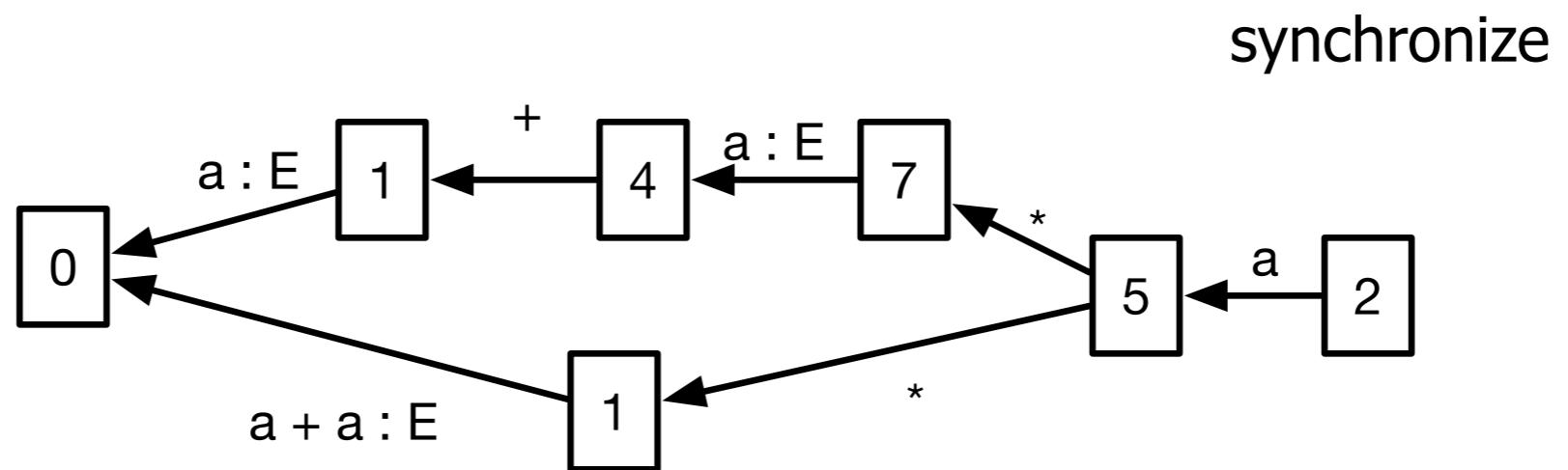
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
1		s4	s5	s3		
2		r3	r3	r3		
3				acc		
4	s2					7
5	s2					6
6		s4/r2	s5/r2	r2		
7		s4/r1	s5/r1	r1		



Generalized LR Parsing

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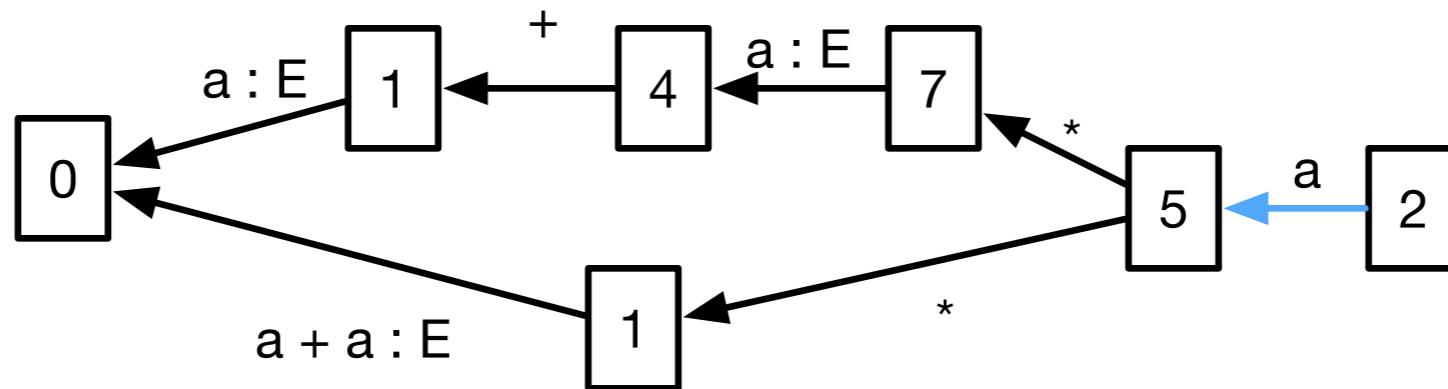
State	Action				Goto	
	a	+	*	\$	S	E
0	s2					I
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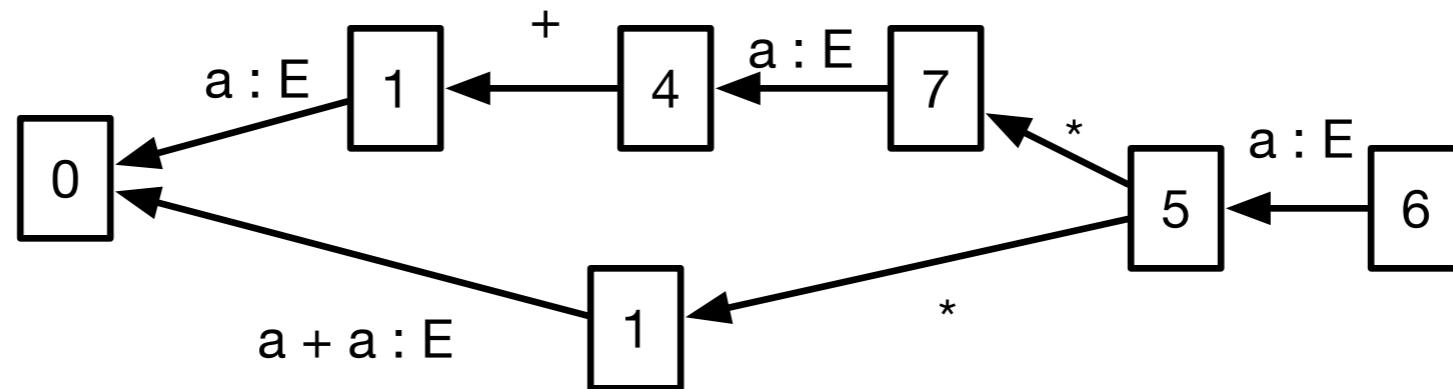
State	Action				Goto	
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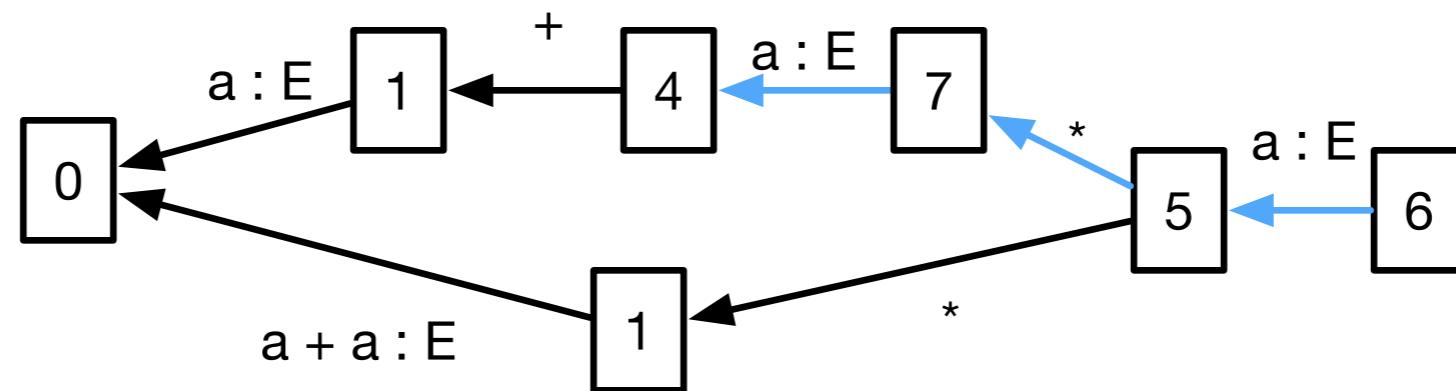
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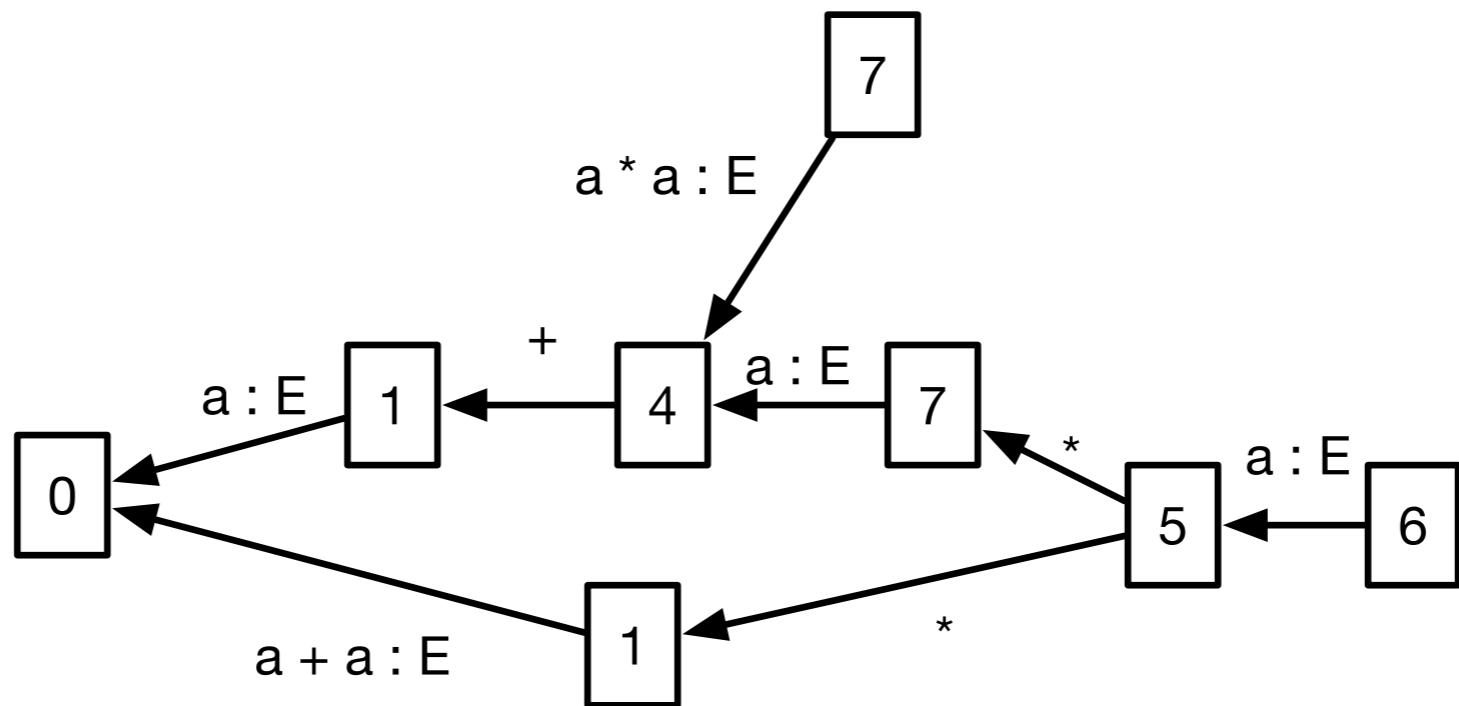
State	Action				Goto	
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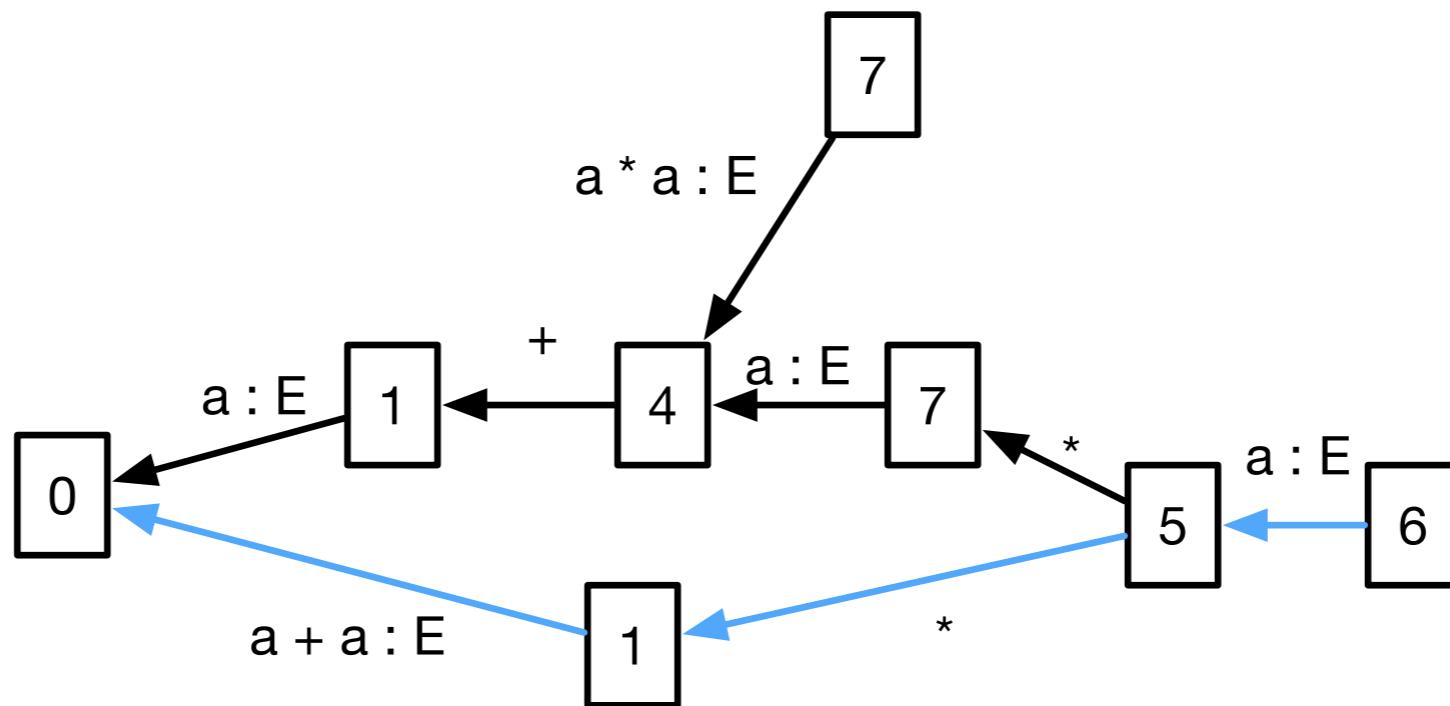
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	a	+	*	\$	S	E
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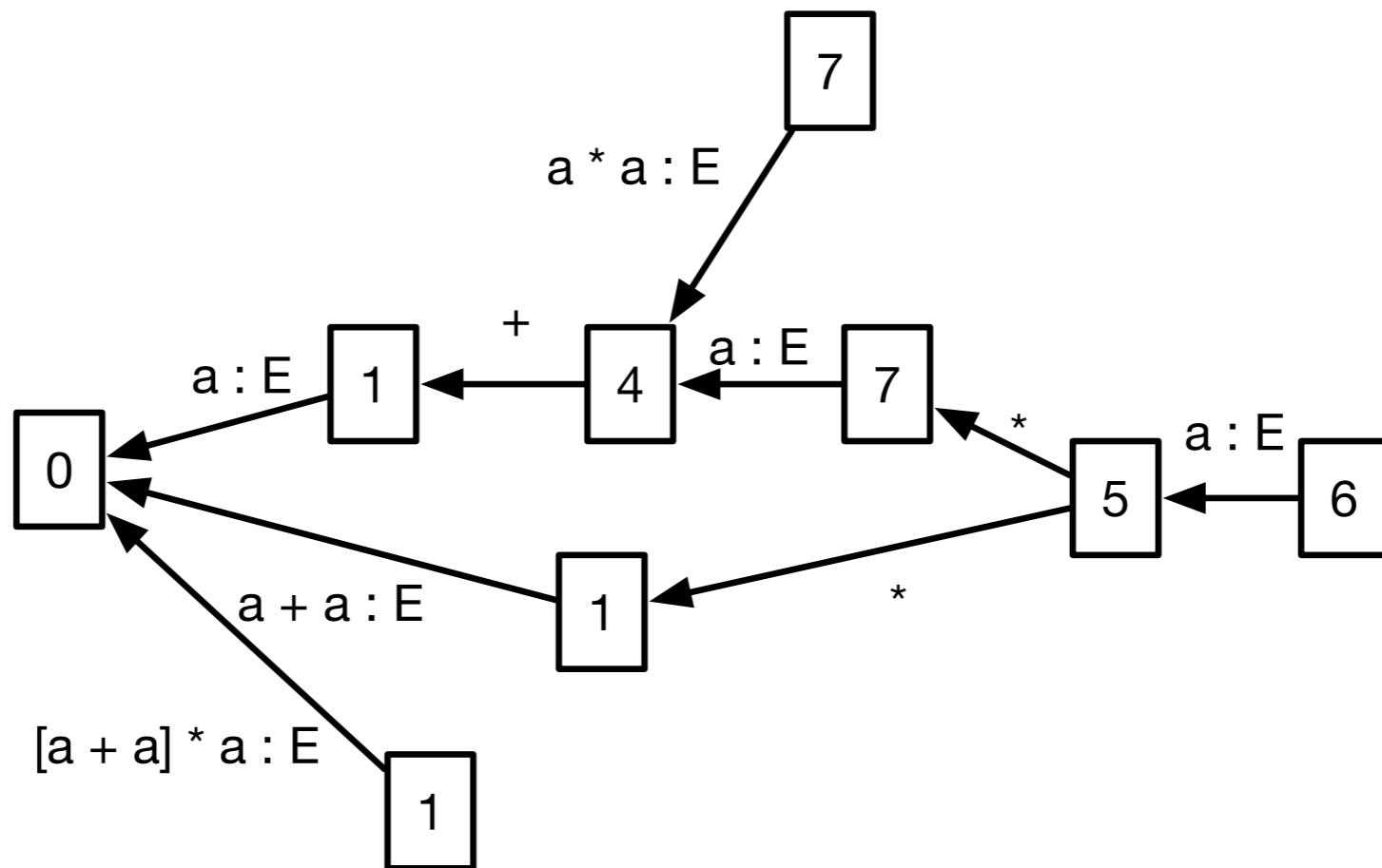
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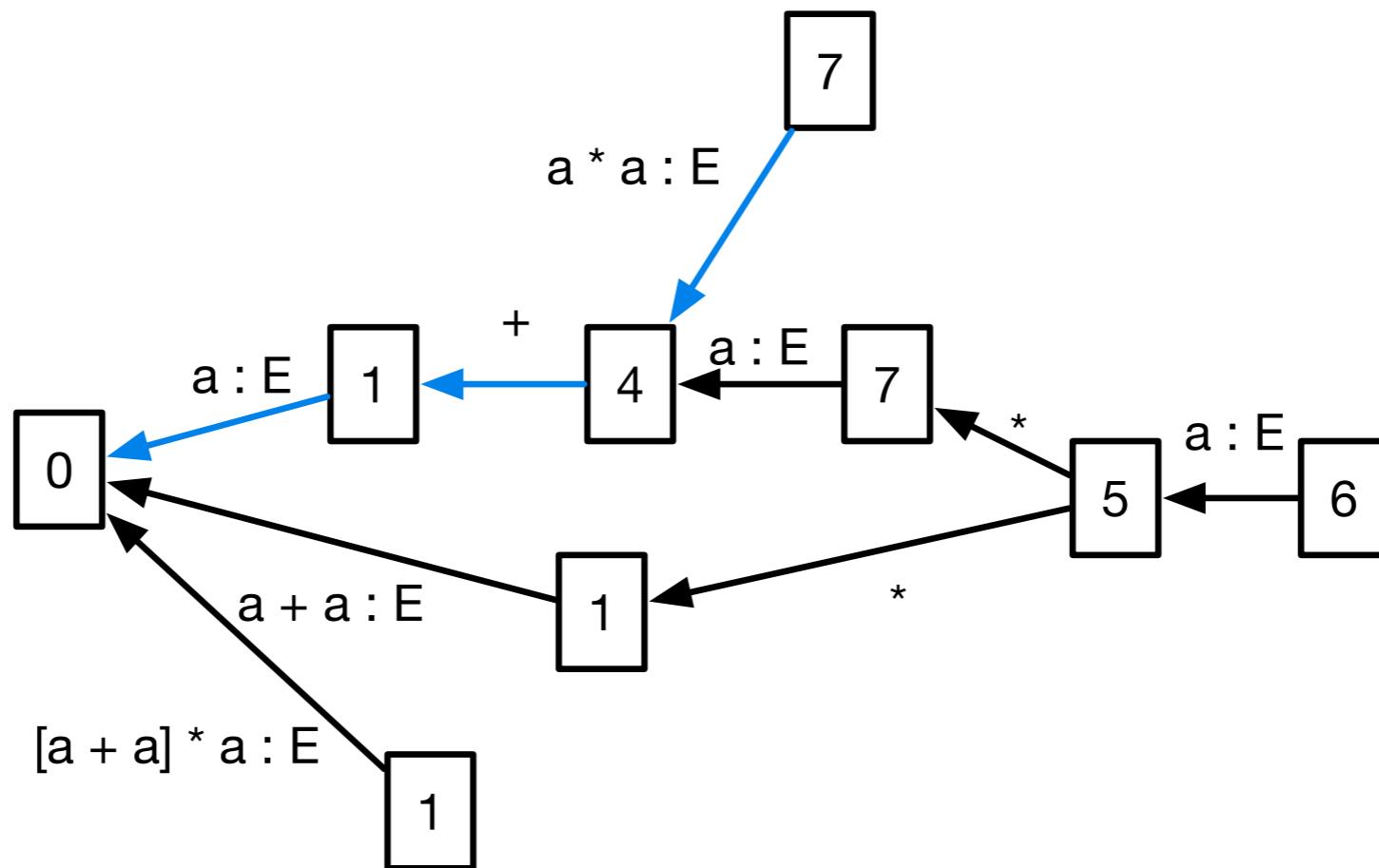
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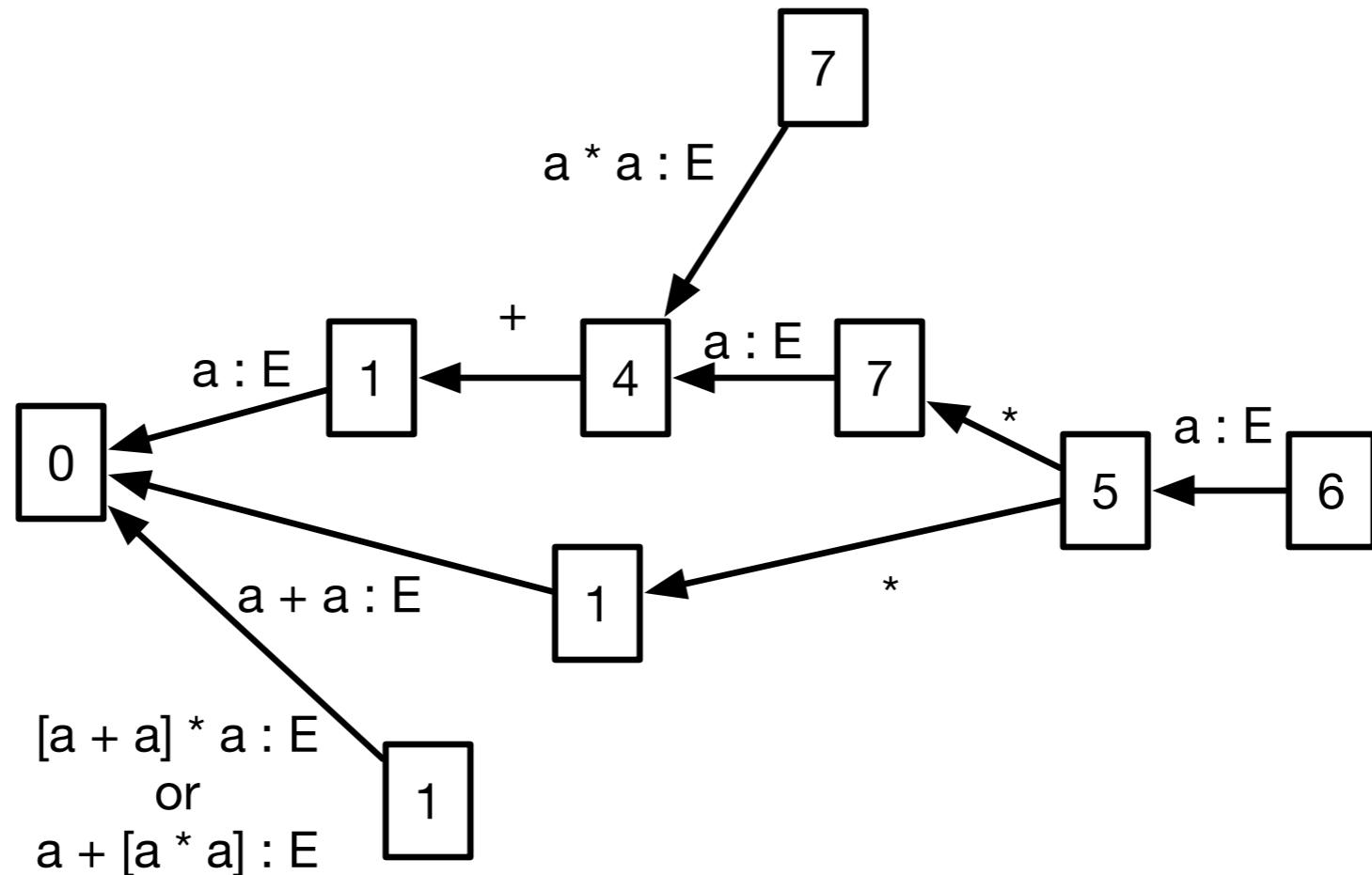
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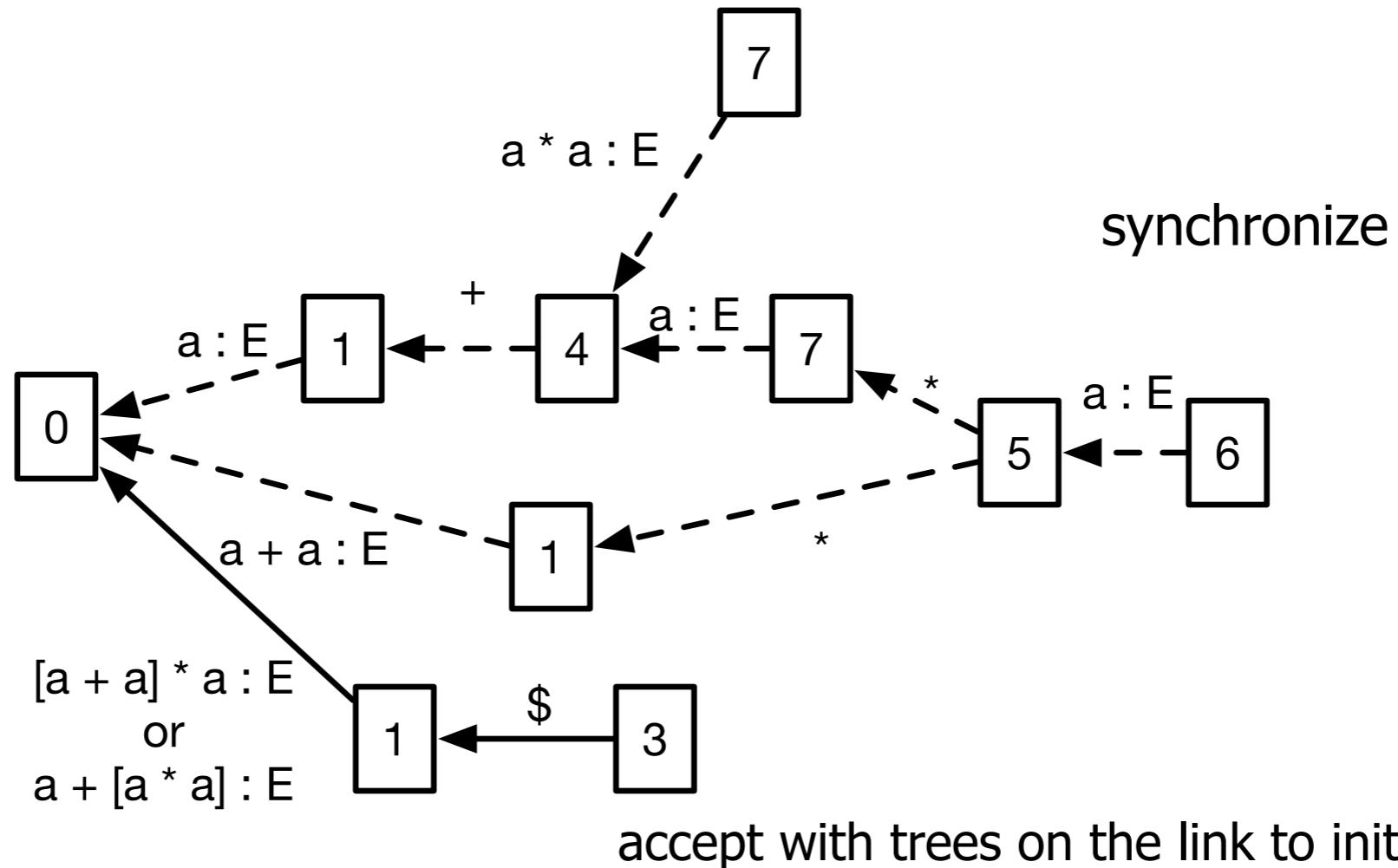
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V

Scannerless Generalized-LR Parsing

Scannerless Generalized LR

- Integrates scanning + parsing into a single GLR algorithm.
- Normalization separates lexical and context-free symbols.
- Crucial for avoiding conflicts when composing languages.
- Introduces lexical ambiguities. Solution:
 - Follow Restrictions (implement longest match).
 - Reject Rules (implement reserved keywords).

Scannerless Generalized LR Normalization

context-free syntax

```
Exp.Add    = <<Exp> + <Exp>> {left}
Exp.Inc    = <<Exp>++>
Exp.ID     = ID
```

lexical syntax

```
ID          = [a-zA-Z] IDRest*
IDRest     = [a-zA-Z0-9]
LAYOUT      = [\t\n\r]
ID          = "let" {reject}
```

Scannerless Generalized LR Normalization

Separate lexical from context-free symbols
and separate symbols in context-free syntax
by optional layout.

context-free syntax

Exp.Add	= <<Exp> + <Exp>> {left}
Exp.Inc	= <<Exp>++>
Exp.ID	= ID

lexical syntax

ID	= [a-zA-Z] IDRest*
IDRest	= [a-zA-Z0-9]
LAYOUT	= [\t\n\r]
ID	= "let" {reject}

Exp-CF.Add = Exp-CF LAYOUT?-CF "+" LAYOUT?-CF Exp-CF {left}
Exp-CF.Inc = Exp-CF LAYOUT?-CF "++"
Exp-CF.ID = ID-CF
ID-LEX = [a-zA-Z] IDRest*-LEX
IDRest*-LEX = [a-zA-Z0-9]
ID-LEX = "let" {reject}
LAYOUT-CF = LAYOUT-LEX

Scannerless Generalized LR Normalization

context-free syntax

Exp.Add	= <<Exp> + <Exp>> {left}
Exp.Inc	= <<Exp>++>
Exp.ID	= ID

Create injections from lexical to
context-free symbols

lexical syntax

ID	= [a-zA-Z] IDRest*
IDRest	= [a-zA-Z0-9]
LAYOUT	= [\t\n\r]
ID	= "let" {reject}

LAYOUT-CF = LAYOUT-LEX
IDRest-CF = IDRest-LEX
IDRest*-CF = IDRest*-LEX
ID-CF = ID-LEX

Scannerless Generalized LR Normalization

Normalize literals symbols and character classes.

context-free syntax

```
Exp.Add    = <<Exp> + <Exp>> {left}
Exp.Inc   = <<Exp>++>
Exp.ID    = ID
```

lexical syntax

```
  ID          = [a-zA-Z] IDRest*  
  IDRest     = [a-zA-Z0-9]  
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  ID          = "let" {reject}
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Scannerless Generalized LR Normalization

context-free syntax

Exp.Add	= <<Exp> + <Exp>> {left}
Exp.Inc	= <<Exp>>>
Exp.ID	= ID

Normalize regular expressions.

lexical syntax

ID	= [a-zA-Z] IDRest*
IDRest	= [a-zA-Z0-9]
LAYOUT	= [\t\n\r]
ID	= "let" {reject}

LAYOUT?-CF = LAYOUT-CF

LAYOUT?-CF =

IDRest+-LEX = IDRest-LEX

IDRest+-LEX = IDRest+-LEX IDRest-LEX

IDRest*-LEX =

IDRest*-LEX = IDRest+-LEX

IDRest+-CF = IDRest+-LEX

Scannerless Generalized LR Normalization

Define extra rules for the start symbols.

context-free start-symbols

Exp

$\langle \text{START} \rangle = \text{LAYOUT?}-\text{CF} \text{ Exp-CF LAYOUT?}-\text{CF}$
 $\langle \text{Start} \rangle = \langle \text{START} \rangle \text{ [\text{\\256}]}$

Scannerless Generalized LR

Parse Table Generation

- Generation is based on SLR(1) item-sets.
- Uses character classes instead of tokens.
- Follow sets and goto actions are calculated for productions instead of non-terminals.

Scannerless Generalized LR

Lexical Disambiguation

- A reject rule is of the form:

```
ID = "let" {reject}
```

- When SGLR does a reduction with a reject rule, it marks the link as rejected.
- Further action on a stack is forbidden whenever all links to it are rejected.
- Follow restrictions are implemented as filters on the follow sets of productions.

Generalized LR

Context-free Disambiguation

- Priority rules: applied at parse table generation.

$$\begin{array}{l} E \rightarrow E * . E \\ E \rightarrow . E + E \\ E \rightarrow . E * E \\ E \rightarrow . a \end{array}$$

multiplication has higher priority than addition!

Generalized LR

Context-free Disambiguation

- Priority rules: applied at parse table generation.

$$\begin{array}{l} E \rightarrow E * . E \\ E \rightarrow . E + E \\ E \rightarrow . E * E \\ E \rightarrow . a \end{array}$$

multiplication is left
associative!

Generalized LR

Context-free Disambiguation

- Priority rules: applied at parse table generation.

$$\begin{array}{l} E \rightarrow E * . E \\ E \rightarrow . E + E \\ E \rightarrow . E * E \\ E \rightarrow . a \end{array}$$

multiplication is left
associative!

- Disambiguation filters: applied after parsing (prefer and avoid).

VI

Summary

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lessons learned

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How can we generate LR parse tables?

- items, closure, goto

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- items, closure, goto

How can we improve LR(0) parse table generation?

- SLR: consider FOLLOW sets to avoid shift-reduce conflicts
- LR(1): consider look-ahead in states
- LALR(1): unify LR(1) states to reduce state space

Summary lessons learned

How can we generate LR parse tables?

- items, closure, goto

How can we improve LR(0) parse table generation?

- SLR: consider FOLLOW sets to avoid shift-reduce conflicts
- LR(1): consider look-ahead in states
- LALR(1): unify LR(1) states to reduce state space

How can we handle conflicts in the parse table?

- generalized parsing - supports all class of context free grammars
- scannerless generalized LR - allow for proper language composition.

Literature

[learn more](#)

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LR parsing

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

Alfred V. Aho, Ravi Sethi, Jeffrey D. Ullman, Monica S. Lam: Compilers: Principles, Techniques, and Tools, 2nd edition. 2006

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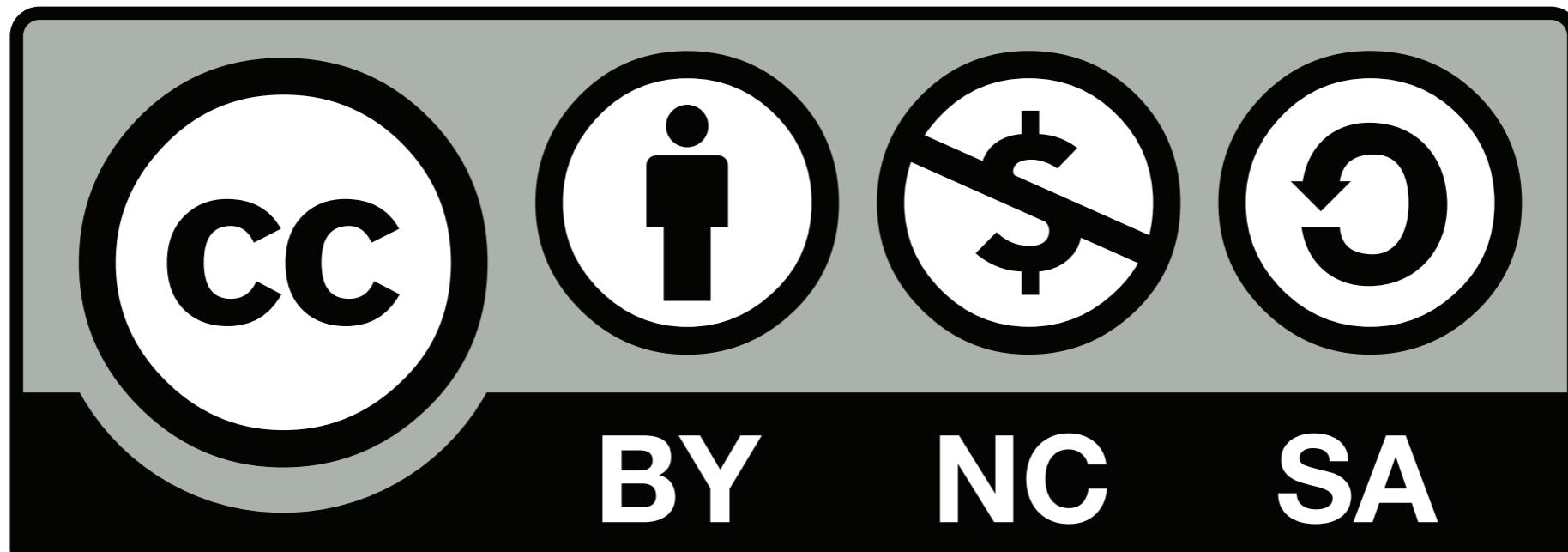
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