Welcome to...

MUDE

Modelling, Uncertainty, and Data for Engineers

WEEK 3 !

Join the Vevox session

Go to **vevox.app**

Enter the session ID: **130-474-361**

Or scan the QR code

MUDE experience so far

No (big) issues	0%
I'm struggling with probability	0%
I'm struggling with the programming	0%
I'm struggling with linear algebra	0%

##/##

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Results slide

MDE experience so far

No (big) issues

##.##%

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##.##%

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##.##%

I'm struggling with linear algebra

##.##%

RESULTS SLIDE

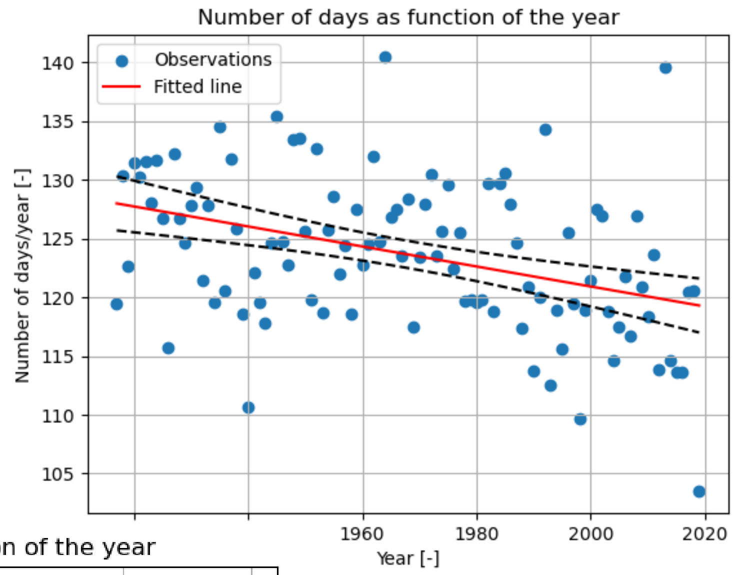
Modelling, Uncertainty and Data for Engineers (MUDE)

Week 1.3-1.4 : Sensing and Observation Theory

Sandra Verhagen

TU Delft

Where we have been, and where we are going



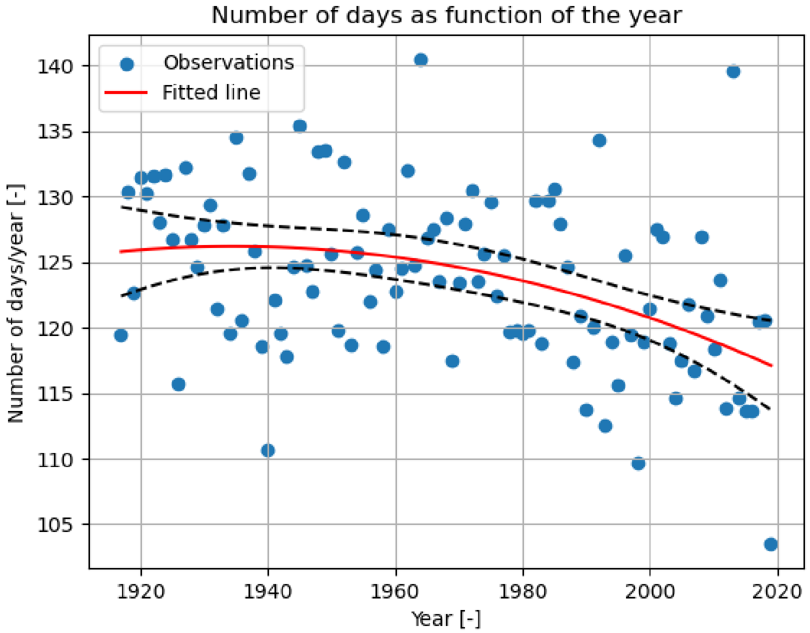
Identify, create, validate simple models

Estimate uncertainty in model output given uncertain inputs

→ Covariance matrix, Σ_X and Σ_Y

In weeks 3 and 4:

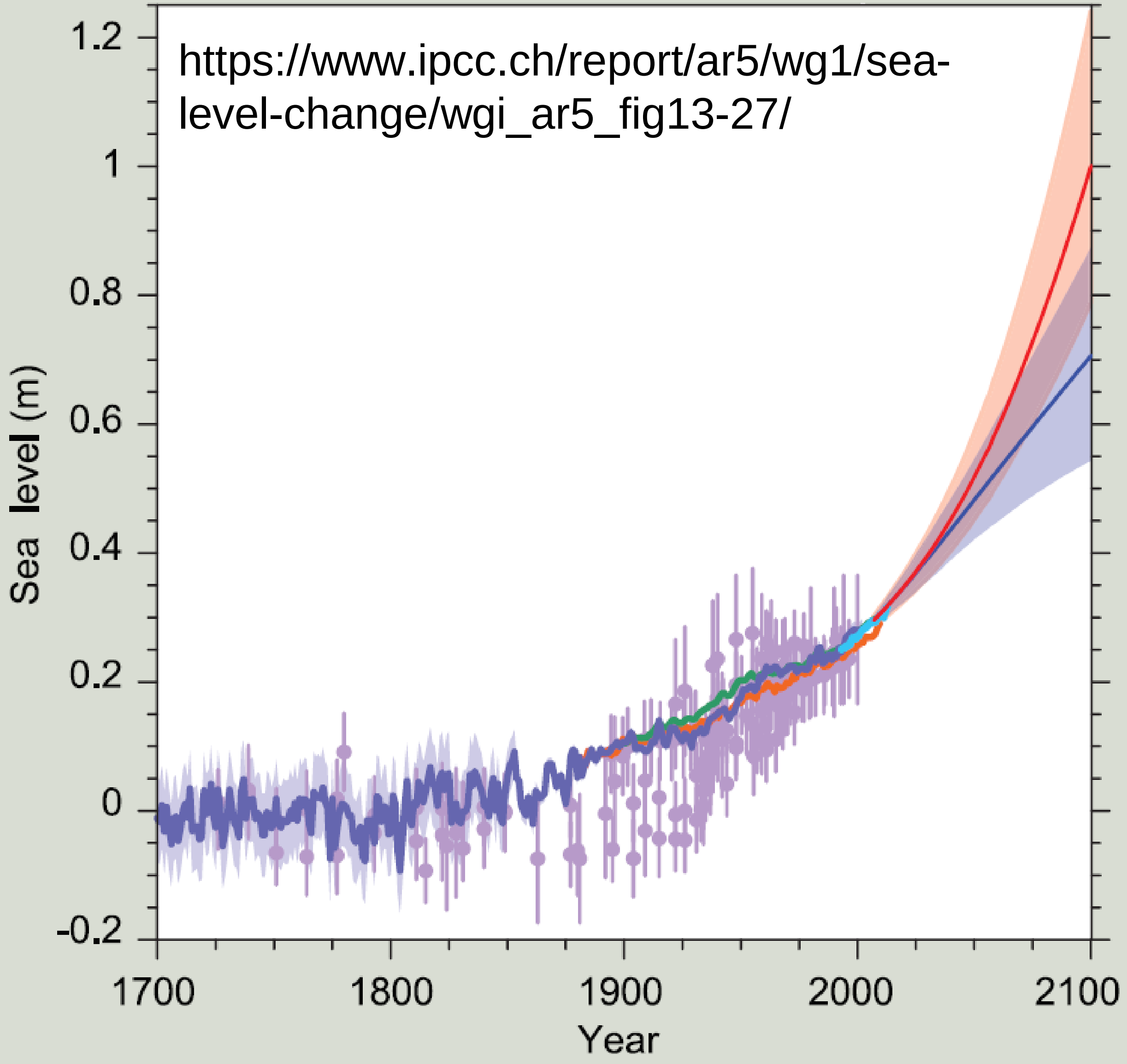
- Models to describe process/phenomenon of interest
- Build functions more complex than a line / polynomial
- Fit the model to data, taking into account uncertainty
- Construct confidence intervals
- Use statistical techniques to validate models



Sensing and observation theory

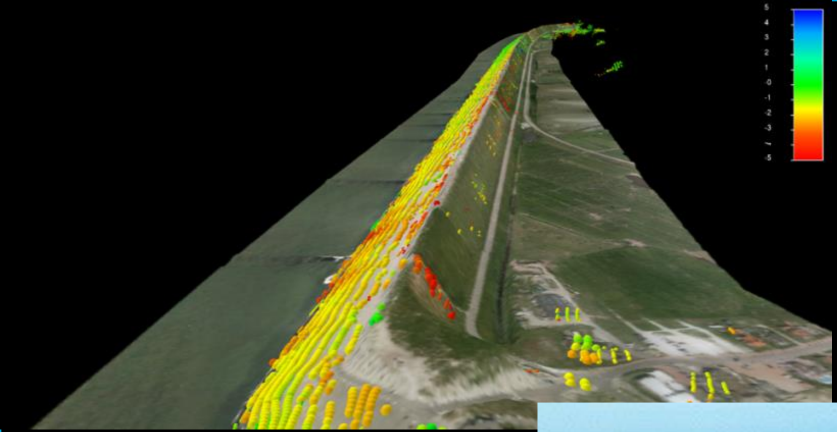
- Science and engineering: need observations!
- Observations → parameters of interest?
- Estimation results: interpretation & uncertainty

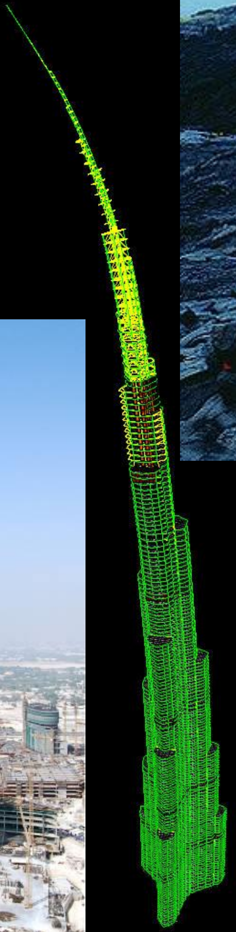
→ Input for other engineers, decision makers, …



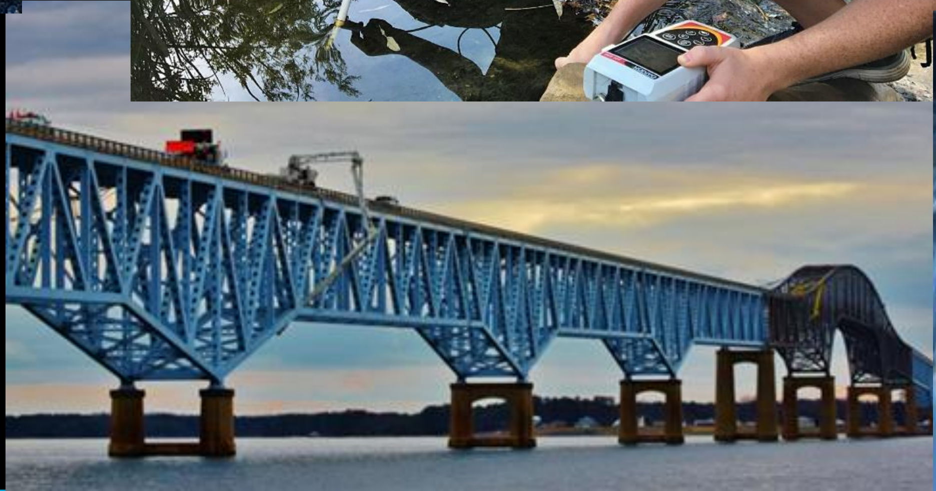
Do we need higher dikes?

Monitoring and Sensing: why?



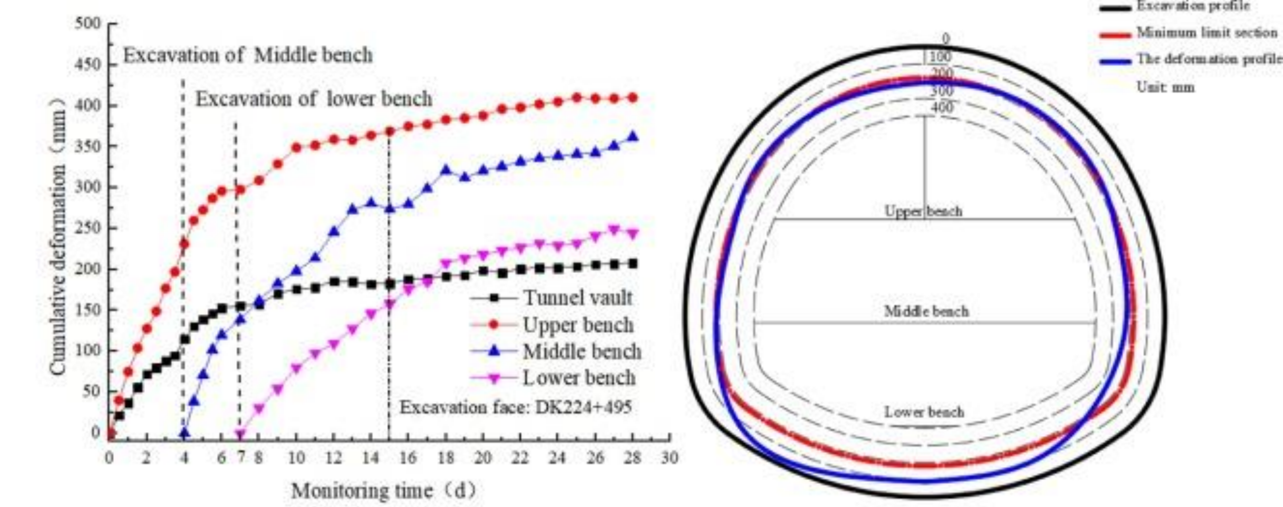





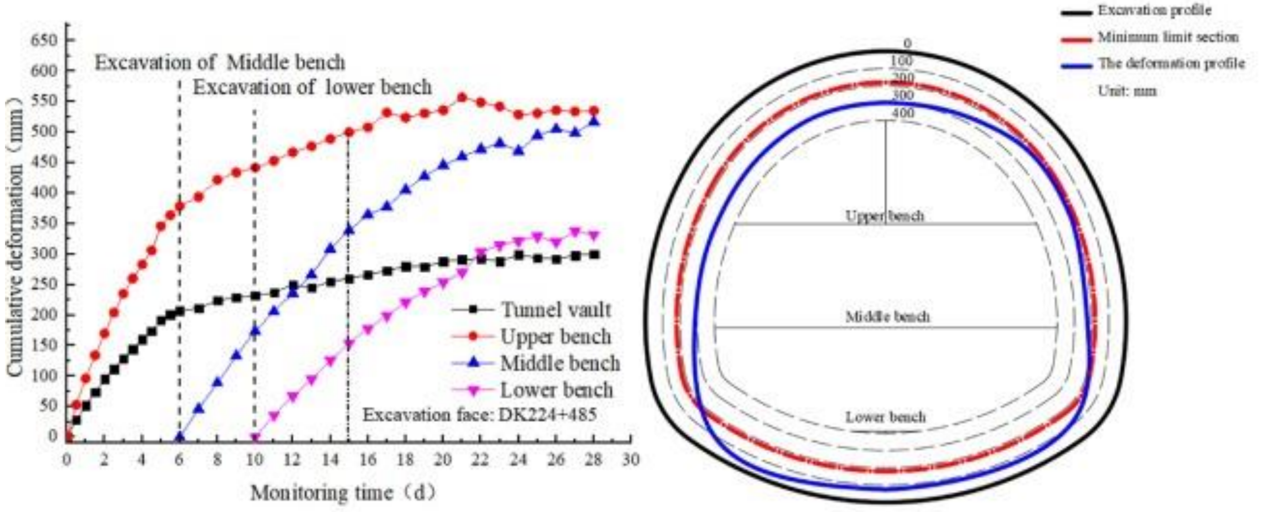



Sensing and observation theory: applications

- Sea level rise
- Subsidence / uplift
- Air quality modelling
- Settlement of soils
- Tunnel deformation
- Bridge motions
- Traffic flow rate
- Water vapor content
- Ground water level

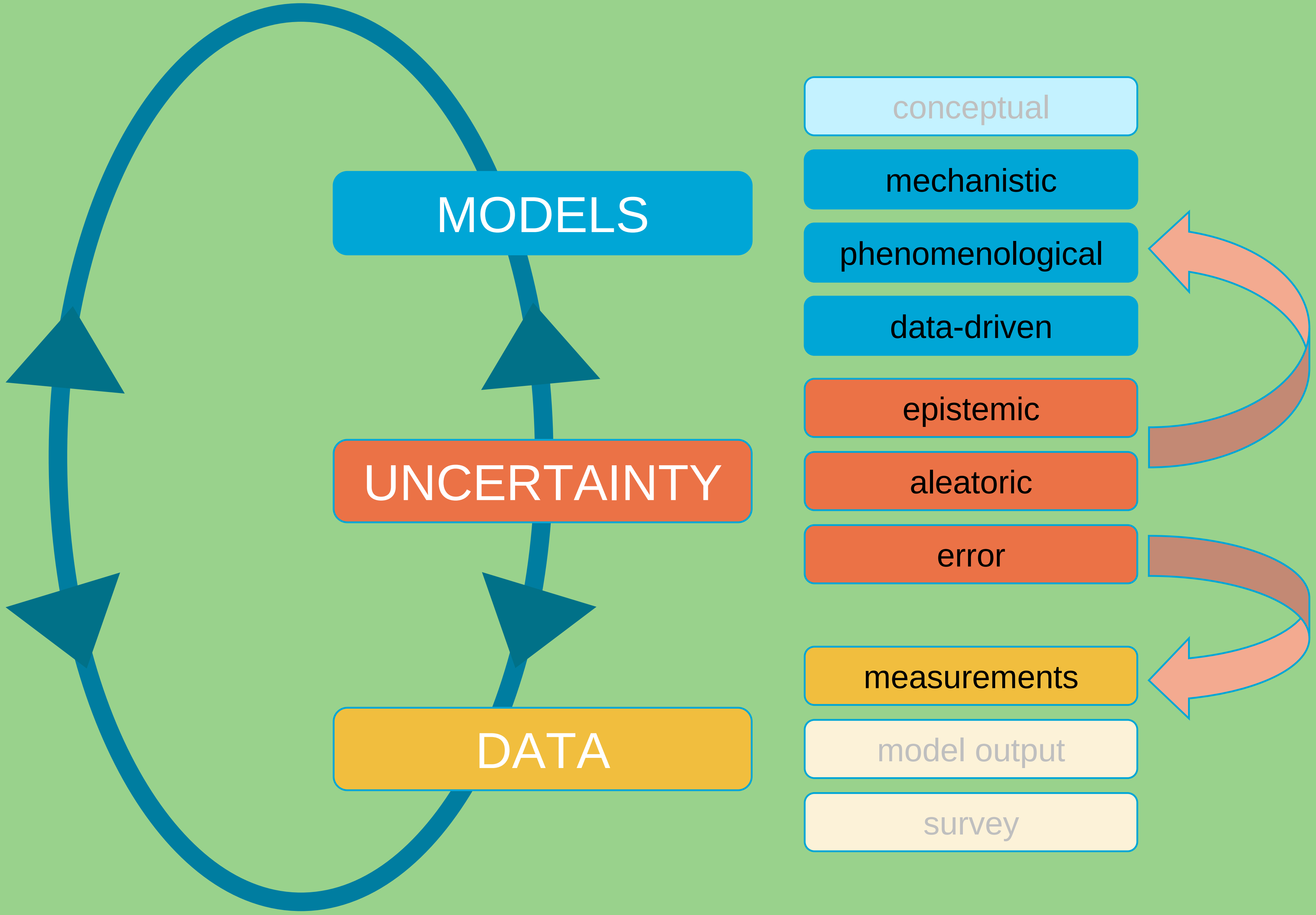


(a) DK224+530 Section monitoring data and deformation profile



(b) DK224+520 Section monitoring data and deformation profile

Input data Y → model & estimate parameters of interest x → Output data $\hat{X} = q(Y)$



MODELS
UNCERTAINTY
DATA
conceptual
mechanistic
phenomenological
data-driven
epistemic
aleatoric
error
measurements
model output
survey

What sensor / observation types are used in your discipline?

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What sensor / observation types are used in your discipline?

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What sensor / observation types are used in your discipline?

RESULTS SLIDE

Sensor/observation types

- camera: visible, IR, UV, hyperspectral
- radar
- radio signals
- rain gauges
- tide gauges
- stress / strain sensors
- acoustic sensors
- accelerometers
- gyroscopes
- temperature
- pressure
- ….

Ingredients

Input data Y → model & estimate parameters of interest x → Output data $\hat{X} = q(Y)$

You wil need …

- … a model to describe relation between Y and x
- … to select and apply an appropriate estimation method
- … to apply uncertainty propagation to assess the precision of $\hat{X}$
- … to apply tests to assess validity of our model

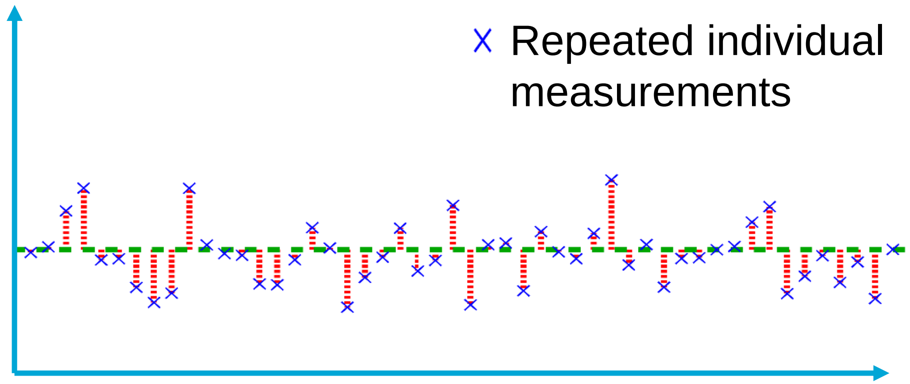
Ingredients

Input data Y → model & estimate parameters of interest x → Output data $\hat{X} = q(Y)$

You wil need …

- … a model to describe relation between Y and x
- … to select and apply an appropriate estimation method
- … to apply uncertainty propagation to assess the precision of $\hat{X}$
- … to apply tests to assess validity of our model

random errors



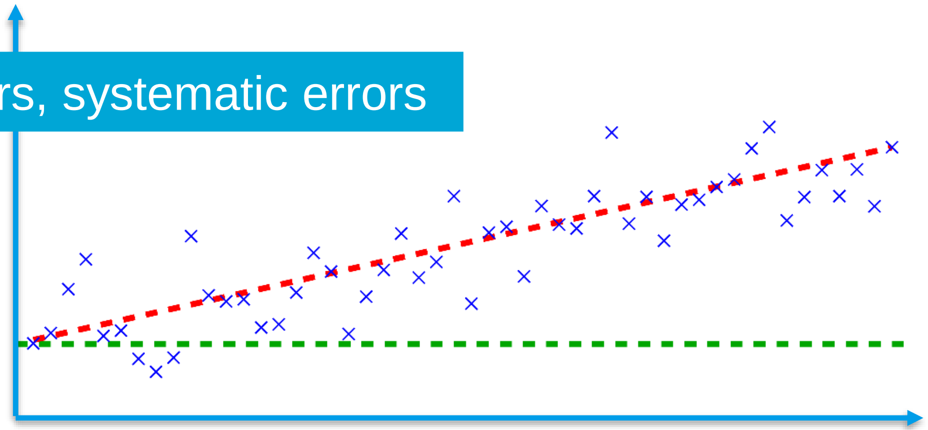
Ingredients

Input data Y → estimate parameters of interest x → Output data $\hat{X} = q(Y)$

You wil need …

- … a model to describe relation between Y and x
- … to select and apply an appropriate estimation method
- … to apply uncertainty propagation to assess the precision of $\hat{X}$
- … to apply tests to assess validity our model
  - to account for errors in data

outliers, systematic errors



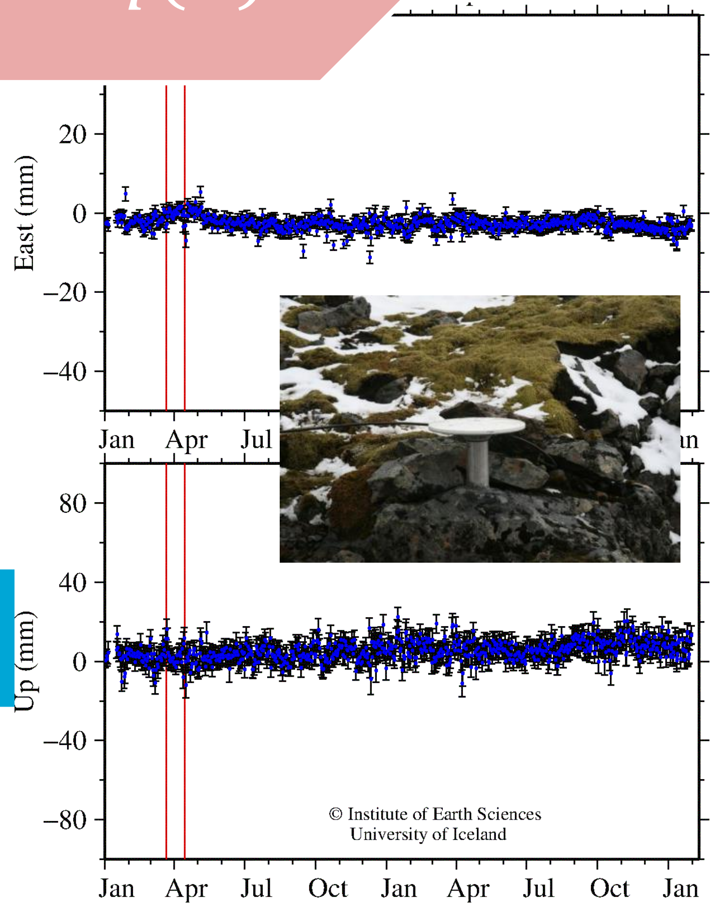
Ingredients

Input data Y → estimate parameters of interest x → Output data $\hat{X} = q(Y)$

You wil need …

- … a model to describe relation between Y and x
- … to select and apply an appropriate estimation method
- … to apply uncertainty propagation to assess the precision of $\hat{X}$
- … to apply tests to assess validity of our model
  - to account for errors in data
  - to choose best model from different candidates

example:
change detection



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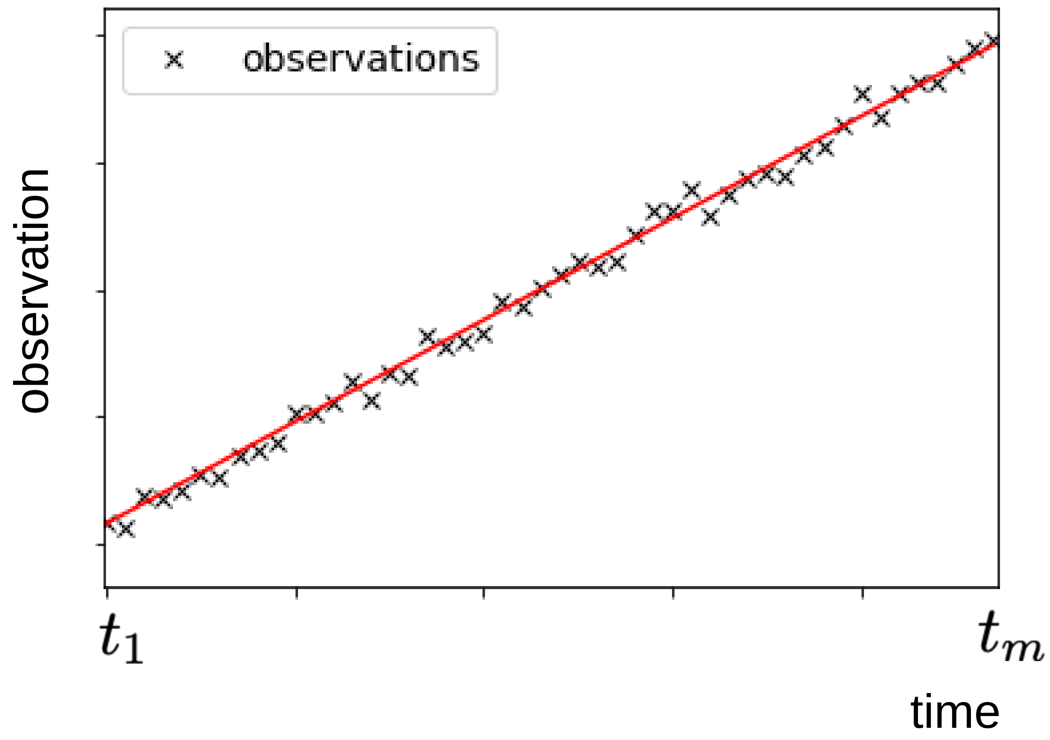
Examples

Linear trend model:

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_m \end{bmatrix}$$

$$= \mathrm{Ax} + \epsilon$$



Unknowns:

x_1 initial value at $t = 0$

x_2 slope

Model formulation

- Observable Y : stochastic quantity (due to random errors)
  → an observable ("to be observed quantity") has a certain probability distribution
- Observation vector y : realization of Y
  → the measured value(s)
- Parameter vector x : deterministic, but unknown
- Random errors ϵ : stochastic with $\epsilon \sim N(0, \Sigma_\epsilon)$
- Functional model (linear case) : $\mathbb{E}(Y) = \underset{m \times n}{\mathrm{A}} \cdot \mathrm{x}$ or $Y = \mathrm{A} \cdot \mathrm{x} + \epsilon$
- Design matrix A : describes functional relationship between Y and x

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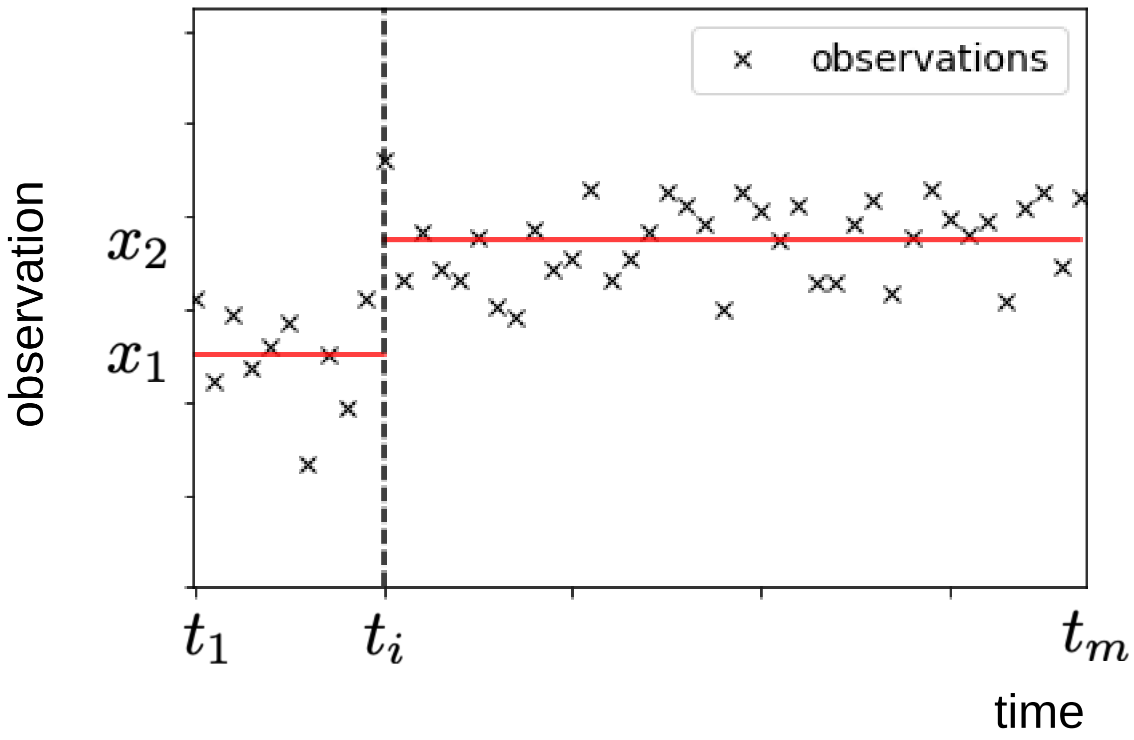
Examples

Step model

$$\mathbb{E}\left(\begin{bmatrix} Y_1 \\ \vdots \\ Y_{i-1} \\ Y_i \\ \vdots \\ Y_m \end{bmatrix}\right) = \underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}}_{\text{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{x}}$$



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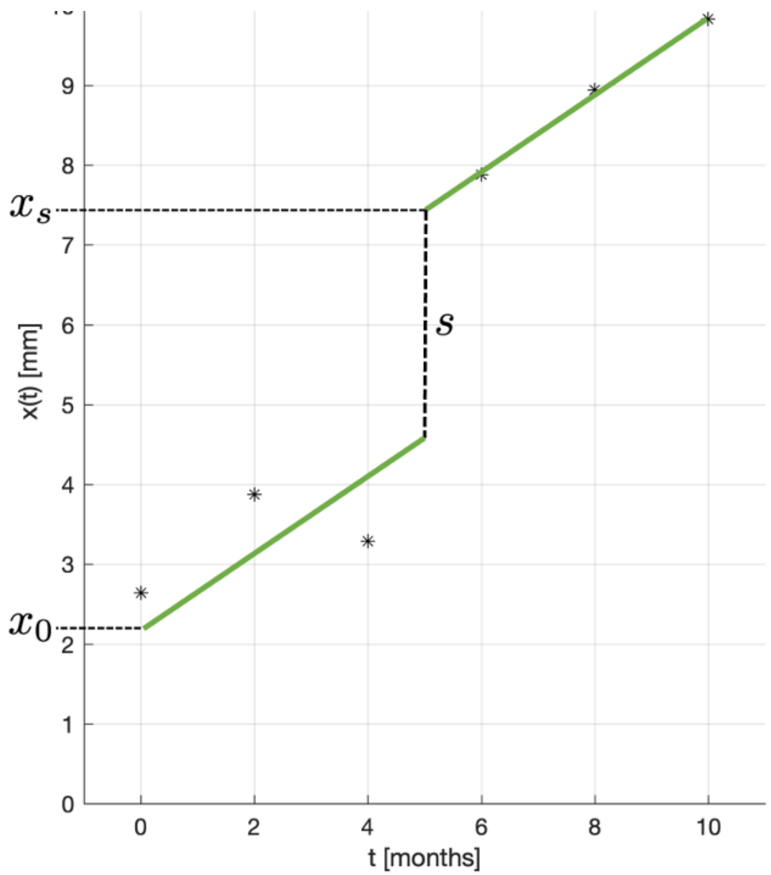
Part 7: Sensing and Observation

The distance x between a fixed benchmark and a moving benchmark on a landslide is measured at times $t = 0, 2, 4, 6, 8, 10$ months. The observations are shown in the figure.

It is assumed that normally the distance is changing at a constant rate. It is known, however, that at $t = 5$ months there was a sudden slip of the landslide, causing an additional change in distance at that time.



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https://mude.citg.tudelft.nl/2024/files/Exams/22_Q1_resit.html

Observations y collected,

we have a functional model A,

how to estimate x?

Observations y collected, we know A, how to estimate x?

for now we ignore the random errors

A linear system $\mathrm{y} = \underset{m \times n}{\mathrm{A}} \cdot \mathrm{x}$

We will consider overdetermined systems with $rank(\mathrm{A}) = n < m$

Hence we have more observations than unknowns

Redundancy = $m - n$

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Example of overdetermined system with $rank(A) = n$

$$\underbrace{\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}}_{\mathrm{y}} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{\mathrm{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathrm{x}}$$

→ no solution

$$\underbrace{\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}}_{\mathrm{y}} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{\mathrm{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathrm{x}}$$

→ $\hat{\mathrm{x}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

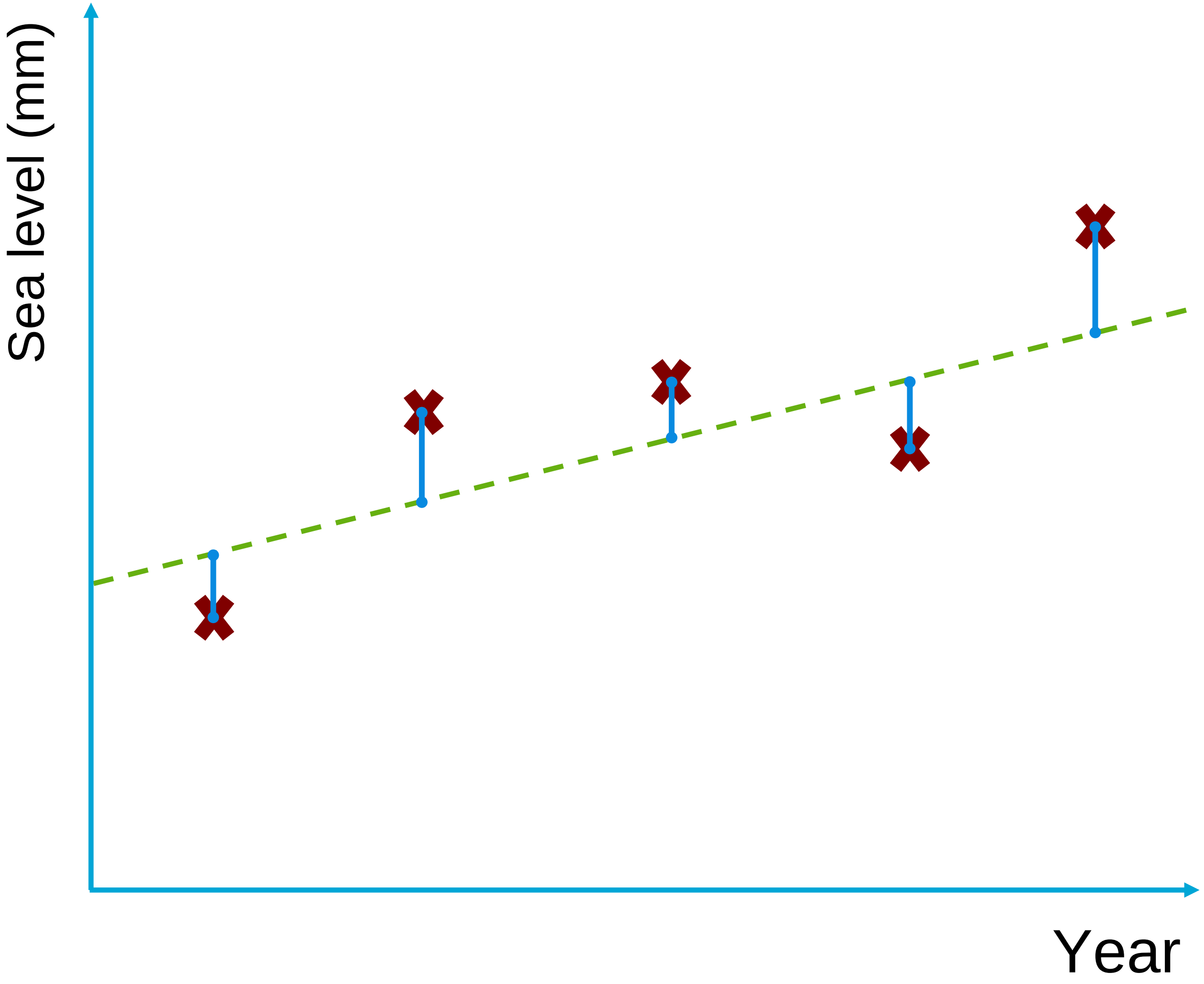
→ in case of perfect measurements, i.e., errors equal to 0

Overdetermined system

Account for random errors,
otherwise generally no solution

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \\ 1 & t_4 \\ 1 & t_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \end{bmatrix}$$

unknowns : 2 parameters + 5 errors
but only 5 observations…
many possible solutions



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Overdetermined system

Account for random errors,
otherwise generally no solution

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \\ 1 & t_4 \\ 1 & t_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \end{bmatrix}$$

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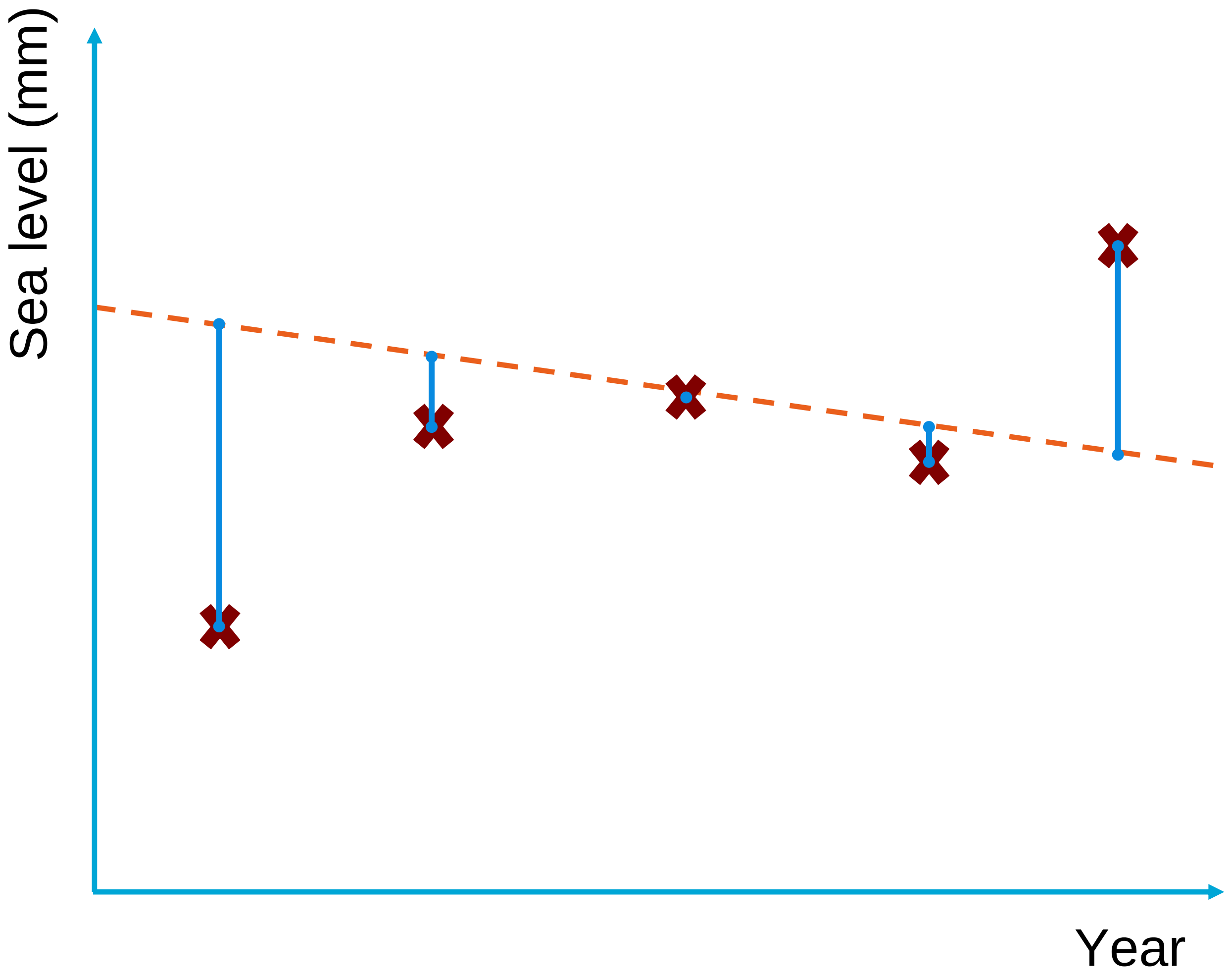

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Least squares criterion?

What is the least-squares criterion?

minimize the mean of the errors	0%
minimize the mean of the absolute errors	0%
minimize the sum of the squared errors	0%
minimize the sum of the absolute errors	0%

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What is the least-squares criterion?

minimize the mean of the errors ##.##%

minimize the mean of the absolute errors ##.##%

minimize the sum of the squared errors ##.##%

minimize the sum of the absolute errors ##.##%

RESULTS SLIDE

Quiz: what is the least-squares criterion?

- minimize the mean of the errors
- minimize the mean of the absolute errors
- minimize the sum of the absolute errors
- minimize the sum of the squared errors

Least-squares principle

- Linear model: $$\mathrm{y} = \mathrm{A}\mathrm{x} + \epsilon$$
- Objective: $$\min_{\mathrm{x}}(\epsilon^T \epsilon) = \min_{\mathrm{x}}(\mathrm{y} - \mathrm{A}\mathrm{x})^T(\mathrm{y} - \mathrm{A}\mathrm{x})$$

Minimize the sum of squared errors (i.e., optimization problem)

- Gradient (first-order partial derivatives) = 0
- Hessian (matrix with second-order partial derivatives) > 0

- Solution $$\hat{\mathrm{x}} = \left(\mathrm{A}^T\mathrm{A}\right)^{-1}\mathrm{A}^T \cdot \mathrm{y}$$

Least-squares solution

Functional model:

$$\mathrm{y} = \mathrm{A}\mathrm{x} + \epsilon$$

Least squares solution

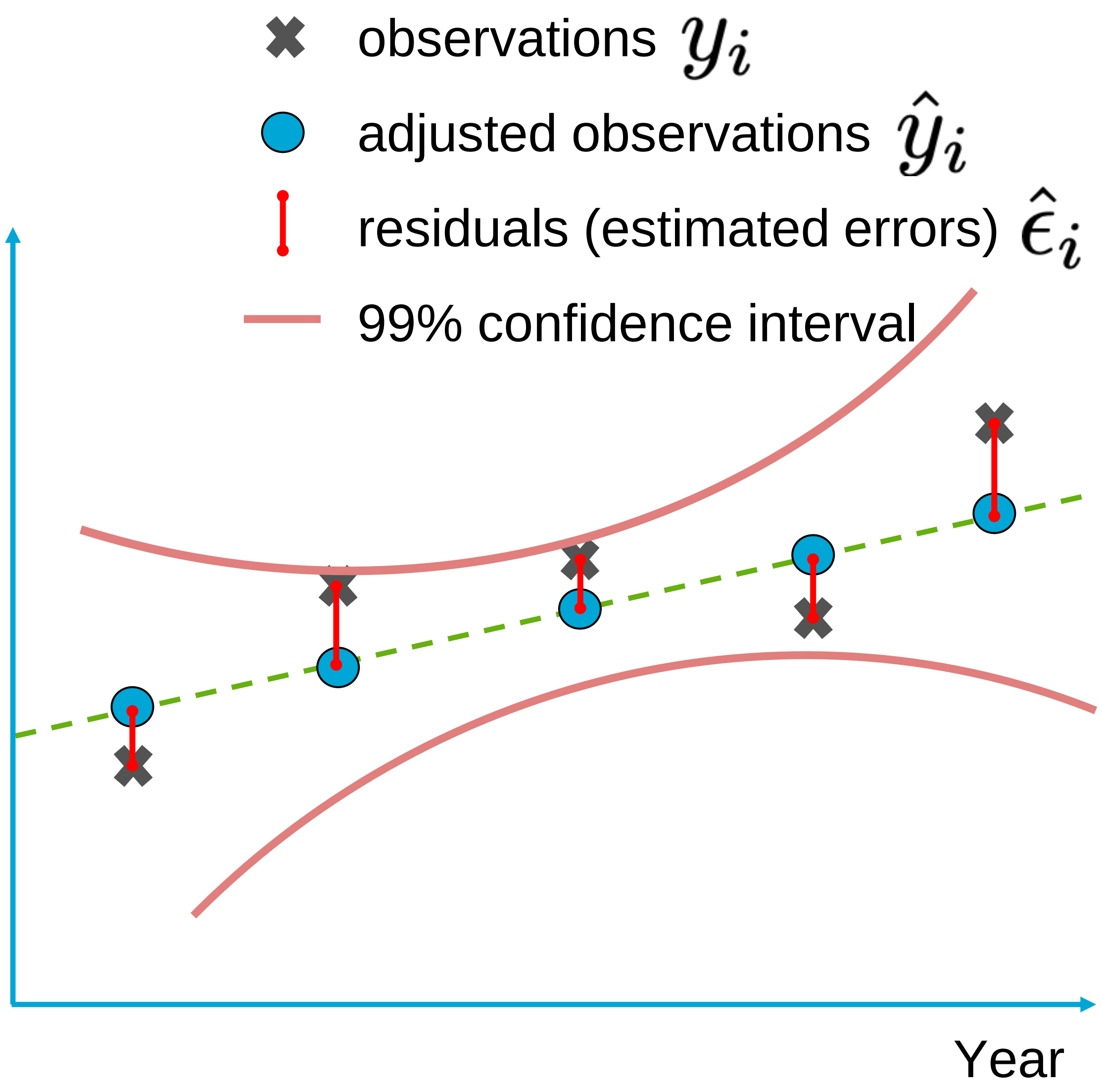
$$\hat{\mathrm{x}} = \left(\mathrm{A}^T\mathrm{A}\right)^{-1}\mathrm{A}^T \cdot \mathrm{y}$$

Adjusted (predicted) observations:

$$\hat{\mathrm{y}} = \mathrm{A}\hat{\mathrm{x}}$$

Residuals (estimated errors):

$$\hat{\epsilon} = \mathrm{y} - \hat{\mathrm{y}}$$



Open questions

- is least-squares the best way to estimate the parameters (fit model)?

  → e.g., by taking into account the distribution of ϵ → Weighted Least-Squares estimation

  Best Linear Unbiased estimation

  Maximum Likelihood estimation

- what if forward model is not linear? → Non-linear Least Squares estimation
- quality assessment? → Quality and testing

Keep the application in mind!

Decisions to be made based on monitoring and sensing:

- can we safely continue with gas extraction / water injection/extraction / CO_2 sequestration?
- do we need to build higher dikes based on sea level rise predictions / observed deformations?
- do we need to evacuate a region due to risk of landslide, volcano eruptions, tsunami, …?
- is railway maintenance needed?
- is a safe underkeel clearance of ships approaching Rotterdam guaranteed?
- … (etcetera etcetera etcetera)

Need proper data processing and quality assessment of the results

In this part: focus on sensing and monitoring applications

Estimation principles also needed for model verification and validation, regression analysis, machine learning

Weighted Least-Squares estimation

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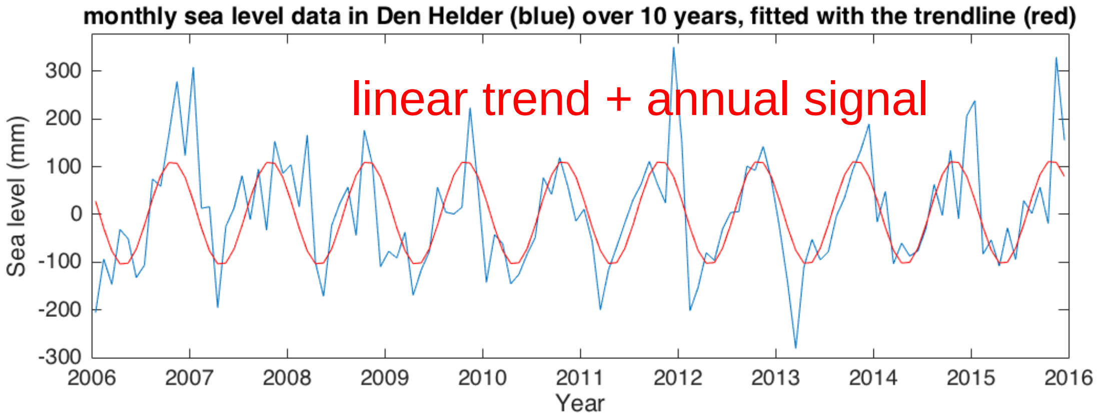
Leas-squares…



- Linear model: $$\mathrm{y} = \mathrm{Ax} + \epsilon$$
- Objective $$\min_{\mathrm{x}}(\epsilon^T \epsilon) = \min_{\mathrm{x}}(\mathrm{y} - \mathrm{Ax})^T(\mathrm{y} - \mathrm{Ax})$$
- Solution: $$\hat{\mathrm{x}} = \left(\mathrm{A}^T \mathrm{A}\right)^{-1} \mathrm{A}^T \cdot \mathrm{y}$$

- … treats all observations equally

- But what if observations are collected with different sensors, with different measurement precision?
  - ➢ only use the observations from the best one? NO
  - ➢ give different weights to the observations? YES

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Least-squares…

- Linear model: $\mathrm{y} = \mathrm{Ax} + \epsilon$

- Introduce a weight matrix W

- Objective: $\min\limits_{\mathrm{x}}(\epsilon^T W \epsilon)$

- For example with a diagonal weight matrix:

$$\epsilon^T W \epsilon = \begin{bmatrix} \epsilon_1 & \epsilon_2 & \cdots & \epsilon_m \end{bmatrix} \begin{bmatrix} W_{11} & & & O \\ & W_{22} & & \\ & & \ddots & \\ O & & & W_{mm} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_m \end{bmatrix} = \sum_{i=1}^{m} W_{ii} \cdot \epsilon_i^2$$

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An observation with a larger weight is supposed to have a smaller error; this is considered in this minimization problem