DELFT UNIVERSITY OF TECHNOLOGY
FACULTY OF AEROSPACE ENGINEERING
Section Control & Simulation

AE4304P: Stochastic Aerospace Systems Practical

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Assignment Goal

The goal of this assignment is to gain practical hands-on experience with some of the concepts and methods covered in the AE4304 lecture series. In this assignment, you will familiarize yourself with the analysis of stochastic processes using time- and frequency-domain analysis techniques. Thereby, you will gain practical experience regarding the advantages, disadvantages, and limitations of these methods. Even though you will focus on the analysis of *simulated aircraft responses to atmospheric turbulence*, the applied methods and techniques are actually useful for the analysis of *any stochastic process or set of real, measured data!* Note that a very long measurement time or very high sampling frequency is feasible in simulation, but not for real test flights. It is good to play around with these values to investigate their effect, but also comment on the feasibility of the parameters that you choose.

Assignment

You are requested to perform calculations on the symmetrical aircraft responses for a rigid aircraft in symmetrical atmospheric turbulence conditions with an autopilot (pitch-damper) both **on** and **off**. Use the control law:

$$\delta_e = K_\theta \theta, \text{ with:}$$

- **Autopilot off**: $K_\theta = 0$,
- **Autopilot on**: $K_\theta \neq 0$.

Choose the value for K_θ such that the phugoid mode becomes better damped, i.e., find a value for K_θ such that $\zeta_{phugoid} \approx 0.5$.

The Power Spectral Density function of atmospheric turbulence is according to the model of Dryden. Perform calculations of aircraft responses to vertical turbulence, with:

$$L_g = 1500\, m \qquad \sigma_{w_g} = 2\, m/s.$$

Perform all calculations due to this turbulence component with the pitch-damper on and off for the aircraft configuration summarized in Table 2. Use MATLAB and carefully look at the examples in the lecture notes and the MATLAB files distributed on Brightspace.

Use the complete set of aircraft equations of motion, supplemented with a Dryden turbulence model, as given in the lecture notes **[1,2]**.

Perform the following calculations:

1. **Stability Analysis**

   Compare the stability of the aircraft with and without the above mentioned pitch-damper.

   In your report give:

   a) the state space system(s) you used in the simulations

   b) a stability analysis of the aircraft that evaluates the stability of all relevant eigenmodes (i.e., pole-zero map, eigenvalues, etc.), for both the aircraft with and without pitch damper

2. **Time-Domain Analysis**

   To generate simulated stochastic measurement data, you will perform time-domain simulations of your aircraft model(s) in the specified turbulence condition(s). You will simulate the responses of the aircraft state variables, as well as the load factor $n_z = -\frac{a_z}{g}$, with

   $$a_z = -\frac{d^2h}{dt^2} = -\frac{d}{dt}V \sin\gamma \approx -V\dot{\gamma}.$$

   Note: the load factor does not necessarily have to be extracted from the time signals of V and $\dot{\gamma}$, but can be directly calculated from the state-space system!

   In your report give:

   a) the time domain simulation results (time traces) of both the controlled and uncontrolled aircraft flying through turbulence. This includes all the aircraft states, including the load factor. It is not necessary to show the simulation results for the gust states.

3. **Spectral Analysis**

   Calculate the analytical and experimental Power Spectral Density functions of the above mentioned variables (so not the gust states). Calculate the:

   i) *analytical* Power Spectral Density functions using the state-space representation

   ii) *experimental* Power Spectral Density functions (periodograms) estimated using the MATLAB routine `fft.m`

   iii) *experimental* Power Spectral Density functions (periodograms) estimated using the MATLAB routine `pwelch.m` for the estimation of a smoothed periodogram. *Note:* for this routine you need to supply the sampling frequency of the data, as well as some settings!

   In your report give:

   a) the spectral analysis for the aircraft states and load factor only. Thus a spectral analysis regarding the gust states is not necessary

   b) a discussion on all the results and how you obtained them. Clearly **explain** the differences between the responses of the controlled and uncontrolled aircraft

   c) a discussion on any differences that were found between the calculation methods of the periodograms. This includes both a comparison between the methods and a possible explanation for any differences that were found

4. **Variances**

   Estimate the variances of all aircraft states and the load factor using:

i) the analytical power spectra,

ii) the experimental power spectra (both non-smoothed and smoothed periodograms),

iii) by using the MATLAB routine `var.m` on the simulated state variable time traces.

In your report give:

a) a table with the variances calculated using the different methods

b) an explanation for differences in variance between the two aircraft

c) an explanation for the differences between the different calculation methods

Report Deadline & Assessment

The deadline for submitting the report is **April 7, 2024, 17:00**. If you miss the deadline, your next chance for handing in an AE4304P report will be next year! Please note that you get separate grades for the AE4304P practical (report) and for the AE4304 course (exam). Both grades must be ≥ 6. The scoring distribution for this AE4304P assignment is given in Table 1.

Table 1: The score distribution for AE4304P. The final grade is 1 + your score.

Assignment (1 pt)	pt
Correct implementation of turbulence condition and/or model reduction	0.5
Correct aircraft stability analysis	0.5
Spectral Analysis (3 pt)	
Spectra calculated for all required variables and methods	0.5
Spectra calculated correctly	1
Spectra plotted clearly (plot size, x/y-axis labels, units)	0.5
Discussion of results for different conditions/models	0.5
Discussion of results for different methods	0.5
Variances (3 pt)	
Variances calculated for required variables and methods	0.5
Variances calculated correctly	1
Variances presented clearly (table, significant digits, units)	0.5
Discussion of results for different conditions/models	0.5
Discussion of results for different methods	0.5
Report (2 pt)	
Quality of report Introduction	0.5
Quality of report Conclusion	0.5
Overall report quality	1.0

Getting help

For questions regarding your assignment, please send an e-mail to dr.ir. E. J. J. Smeur (ae4304-project@tudelft.nl). Always first make an appointment by email before coming to my office.

References

1. Mulder, J. A. & Van Staveren, W.H.J.J. & Van der Vaart, J.C. & De Weerdt, E., *Flight Dynamics*. Lecture Notes AE3302.

2. Mulder, J. A. & Van der Vaart, J. C. & Van Staveren, W.H.J.J. & Chu, Q.P. & Mulder, M., *Aircraft Responses to Atmospheric Turbulence*. Lecture Notes AE4304.

Table 2: Stability and control derivatives for the Cessna Ce500 Citation I, landing (1).

Aircraft : Cessna Ce500 Citation I

Configuration : landing

$x_{c.g.}$	=	0.30	$\bar{\mathrm{c}}$							
W	=	44675	N	V	=	51.4	m/sec	μ_b	=	11
m	=	4556	kg	h	=	0	m	$\mathrm{K_X^2}$	=	0.012
S	=	24.2	m^2	ρ	=	1.225	$\mathrm{kg/m^3}$	$\mathrm{K_Z^2}$	=	0.037
$\bar{c}$	=	2.022	m	μ_c	=	76		$\mathrm{K_{XZ}}$	=	0.002
b	=	13.36	m	l_h	=	5.5	m	$\mathrm{K_Y^2}$	=	0.980

C_{X_0}	=	0	C_{Z_0}	=	-1.1360			
C_{X_u}	=	-0.2173	C_{Z_u}	=	-2.2720	C_{m_u}	=	0
C_{X_α}	=	0.4692	C_{Z_α}	=	-5.1300	C_{m_α}	=	-0.4000
			$C_{Z_{\dot{\alpha}}}$	=	-1.4050	$C_{m_{\dot{\alpha}}}$	=	-3.6150
C_{X_q}	=	0	C_{Z_q}	=	-3.8400	C_{m_q}	=	-7.3500
C_{X_δ}	=	0	C_{Z_δ}	=	-0.6238	C_{m_δ}	=	-1.5530

C_{Y_β}	=	-0.9896	C_{ℓ_β}	=	-0.0772	C_{n_β}	=	0.1628
C_{Y_p}	=	-0.0870	C_{ℓ_p}	=	-0.3415	C_{n_p}	=	-0.0108
C_{Y_r}	=	0.4300	C_{ℓ_r}	=	0.2830	C_{n_r}	=	-0.1930
$C_{Y_{\delta_a}}$	=	0	$C_{\ell_{\delta_a}}$	=	-0.2349	$C_{n_{\delta_a}}$	=	0.0286
$C_{Y_{\delta_r}}$	=	0.3037	$C_{\ell_{\delta_r}}$	=	0.0286	$C_{n_{\delta_r}}$	=	-0.1261