Event-Triggered ℓ_2-Optimal Formation Control for Agents Modeled as LPV Systems

Hamideh Saadabadi and Herbert Werner

***Abstract*— This paper proposes a novel approach to event-triggered formation control for homogeneous, non-holonomic multi-agent systems with undirected interaction topology, where the non-holonomic vehicle dynamics are represented by polytopic linear-parameter-varying (LPV) models. The proposed event-triggered strategy is able to reduce the communication cost by transmitting information only when needed. To maintain a formation, each agent is equipped with an inner state-feedback loop that is time-triggered, while an outer position loop is closed by each agent individually through the communication network whenever a local trigger condition is satisfied. The control strategy can be implemented in a distributed manner; the trigger condition is based only on locally available information. The proposed method allows to simultaneously design a controller and a trigger level that guarantee stability and a bound on the overall ℓ_2 performance of the network. The synthesis problem is formulated as an LMI problem. Under the additional assumption that the agents are homogeneously scheduled, the synthesis problem can be decomposed to reduce its complexity to the size of a single agent, regardless of the number of agents, without degrading the performance. The effectiveness of the results is illustrated in a simulation scenario with non-holonomic agents modeled as dynamic unicycles.**

I. INTRODUCTION

Cooperative control of multi-agent systems has received considerable attention recently, because of its wide variety of applications including swarm robotics, sensor networks, and formation control of vehicles [1]- [2]. Most of the recent research reported on event-triggered control of networks is assuming LTI agent dynamics. However, mobile robots or autonomous vehicles are typically subject to non-holonomic constraints; they can be modeled as linear parameter-varying (LPV) systems [3], which facilitates an extension of existing results on formation control for LTI agents to non-holonomic agents.

In multi-agent systems, agents exchange information with neighbors; it is typically assumed that information about the state of neighbors is permanently available [4]. Such periodically sampled control schemes can however lead to excessive and unnecessary utilisation of communication resources [5]. To address this problem and to reduce the number of data transmits, distributed event-triggered strategies have been developed which can provide a more efficient use of network bandwidth, see e.g. [6]-[16] and also [20]-[26] In these papers the authors propose centralized and decentralized event triggered schemes for consensus or formation control of multi-agent system; they all are however restricted to agents modelled as LTI systems.

The formation control problem is a basic problem in cooperative control of multi-agent systems which has been studied widely [17]. A formation is achieved when each agent takes a pre-determined relative position with respect to its neighbors to attain a specified overall shape [18]-[19]. Typically autonomous vehicles e subject to non-holonomic constraints, so they cannot be modeled as LTI systems. Here we propose a distributed event-triggered control scheme for networks of agents that are modeled as polytopic LPV systems.

Until now a few studies have been reported on event-triggered control of LPV systems [27]-[32]. In [27]-[28] design of event-triggered state feedback and output feedback control for a discrete-time LPV system in a single loop is studied, with LMI conditions for bounds on the l_2- performance. In [29], event-triggered H_∞ state-feedback control is considered for LPV systems in a single loop, where sensors are however time-triggered and the trigger condition is not local; in [30] this approach is extended to event-triggered output feedback control. In [31]-[32], event triggered control for continuous-time switched single loop LPV systems is studied. In [33]-[34] event-triggered fault detection schemes for LPV systems are proposed. However, to the best of the authors' knowledge, so far no results on distributed event-triggered control of networks of LPV agents have been reported.

The contribution of this paper is a novel method for co-design of an event-triggered distributed LPV formation control strategy together with a trigger condition that depends only on locally available information, with guaranteed ℓ_2 performance. In this scheme only the output (position) is transmitted to neighbors; the synthesis problem is formulated as LMI problem. If in addition it is assumed that scheduling of the LPV agents is homogeneous (*i.e.* the scheduling variables of all agents at a given time take identical values), the synthesis complexity can be reduced via a decomposition approach to the size of a single agent, independent of the network size. For nonlinear/nonholonomic agents which are represented as quasi-LPV systems and move in formation, this assumption will be approximately satisfied. A simulation example suggests that the proposed scheme will still work well even when (as a result of a disturbance acting on an individual agent) this assumption is violated. The work reported here is building on earlier results on event-triggered control of a single LPV system presented in [27] and on

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networks of LTI systems in [16].

II. Preliminaries and Notation

We let L denote the (positive semi-definite) Laplacian of a graph G. For a connected graph, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is a vector of ones, denoted by $\mathbf{1}$. We then let $0 = \lambda_1(G) < \lambda_2(G) \leq ... \leq \lambda_N(G)$ denote the eigenvalues of L. Note that if G is undirected we have $L = L^T$ [35].

The Kronecker product of $A \in \mathbb{R}^{m\times n} := [a_{ij}]$ and $B \in \mathbb{R}^{p\times q}$ is denoted by $A \otimes B$ and is a $mp \times nq$ matrix defined by

$$A \otimes B := [a_{ij}B]$$

The Kronecker product facilitates the manipulation of matrices by the following expansion properties

- $(A\otimes B)(C\otimes D) = AC\otimes BD$
- $(A\otimes B)^T = A^T \otimes B^T$
- Let $A \in \mathbb{R}^{r\times s}$ and $B \in \mathbb{R}^{N\times N}$ then $(I_N \otimes A)(B\otimes I_s) = (B\otimes I_r)(I_N \otimes A) = B\otimes A$

For a given matrix M, we will use the notation

$$M_{(n)} = M \otimes I_n, \qquad \text{and} \qquad \check{M} = I_N \otimes M.$$

For $x(k) \in \mathbb{R}^n$, $\|x(k)\| = \sqrt{x^T(k)x(k)}$ is the Euclidean norm and $\| x \|_{\ell_2} = \sqrt{\sum_{k=0}^{\infty} \|x(k)\|^2}$ is the ℓ_2 signal norm. For a signal $x^i(k)$ we will also use the shorthand notation x^i_k. The induced l_2-norm of an LPV system $T(\theta_k)$ with input signal w_k, output signal z_k and scheduling parameter sequence $\theta_k \in \mathscr{F}_\Theta$ is defined as

$$\|T(\theta_k)\|_{\ell_2} = \sup_{\theta_k\in\mathscr{F}_\Theta} \sup_{0\neq w_k\in\ell_2} \frac{\|z_k\|_{\ell_2}}{\|w_k\|_{\ell_2}},$$

where we let $\mathscr{F}_\Theta$ denote the set of all admissible parameter trajectories.

III. Problem Formulation

A. Agent Model

We consider a group of N mobile agents with (identical) nonlinear dynamics, each modeled as quasi-LPV system $P(\theta^i_k)$ with state space realization

$$\begin{aligned} x^i_{k+1} &= A(\theta^i_k)\, x^i_k + B_d(\theta^i_k)\, d^i_k + B_u(\theta^i_k)\, u^i_k \\ y^i_k &= C_y(\theta^i_k)\, x^i_k \qquad i = 1,\ldots,N, \end{aligned} \tag{1}$$

where $x^i(k) \in \mathbb{R}^{n_x}$, $u^i(k) \in \mathbb{R}^{n_u}$, $y^i(k) \in \mathbb{R}^{n_y}$, $z^i(k) \in \mathbb{R}^{n_z}$ and $d^i(k) \in \mathbb{R}^{n_d}$ denote the state, input, transmitted output, performance output and disturbance input, respectively, of agent i at time k. The model matrices $A(\theta^i_k)$, $B_u(\theta^i_k)$, $B_d(\theta^i_k)$, $C_y(\theta^i_k)$, $C_z(\theta^i_k)$ and $D_z(\theta^i_k)$ depend affinely on the time-varying vector of scheduling variables $\theta^i_k \in \Theta \subset \mathbb{R}^{n_\theta}$ of agent i, which is restricted to a compact set Θ at all times.

We assume that the parameter set Θ is represented as a polytope in terms of vertex vectors $\theta_v \in \mathbb{R}^{n_\theta}$

$$\Theta = \left\{ \theta \in \mathbb{R}^{n_\theta} \mid \theta = \sum_{l=1}^{s} \alpha_l \theta_{v.l}, \sum_{l=1}^{s} \alpha_l = 1, \alpha_l \geq 0, l = 1,\ldots,s \right\}.$$

Defining vertex model matrices $A^l = A(\theta_{v.l})$, $B^l_u = B_u(\theta_{v.l})$ etc, the LPV model (1) can be expressed in terms of the convex coordinates $\alpha_l(\theta^i_k)$ as

$$\begin{bmatrix} A(\theta^i_k) & B_u(\theta^i_k) & B_d(\theta^i_k) \\ C_y(\theta^i_k) & 0 & 0 \\ C_z(\theta^i_k) & 0 & D_z(\theta^i_k) \end{bmatrix} = \sum_{l=1}^{s} \alpha_l(\theta^i_k) \begin{bmatrix} A^l & B^l_u & B^l_d \\ C^l_y & 0 & 0 \\ C^l_z & 0 & D^l_z \end{bmatrix}. \tag{2}$$

B. Event-Triggered Formation Control

We assume that a group of N agents (1) is exchanging information through a communication network that is represented by an undirected, connected graph G; here the graph will reflect the desired geometric shape of a formation. Note that in this case we have $L = L^T$. All agents are equipped with (identical) local event-triggered state feedback controllers in PI structure, as shown in Fig. 1. Agent i will transmit its output y^i_k to its neighbors (in $\mathscr{N}_i$) only when a local trigger condition is violated, and each agent will update its control input whenever it receives new information. Let $\{\ldots, k^i_{m_i-1}, k^i_{m_i}, k^i_{m_i+1}, \ldots\}$ denote the time instants at which agent i updates its control input, and note that these trigger instants are not synchronized between agents. In Fig. 1, "ET" represents a block that transmits signal y^i_k when the local trigger condition is violated. The recipients maintain the most recently transmitted value of this signal as an estimate, denoted by $\hat{y}^i_k = y_{k^i_{m_i}}$, $k^i_{m_i} \leq k < k^i_{m_i+1}$, that is held constant until an update is received.

The local, event-triggered and gain-scheduled state feedback law for agent i is

$$u^i_k = F_\zeta(\theta^i_k)\, \zeta^i_k + F_x(\theta^i_k)\, x^i_k, \tag{3}$$

where

$$\zeta^i_{k+1} = \zeta^i_k + \hat{\eta}^i_k \tag{4}$$

integrates the estimated formation error

$$\hat{\eta}^i_k = \sum_{j\in\mathscr{N}_i} (\hat{y}^j_k - \hat{y}^i_k + f^{ij}_k), \qquad k^i_{m_i} \leq k < k^i_{m_i+1} \tag{5}$$

that is based on the latest data transmitted by neighbors; here f^{ij}_k denotes a desired distance vector between agents i and j. As discussed below, the distance vectors are represented by a formation reference vector f_k that is injected into the formation feedback loop. Note that the internal state feedback loop through F_x is updated at each sampling instant, whereas feedback through F_ζ that involves information from neighbors is updated only when such information is received. As the agent models are affine in θ^i_k, we impose affine parameter dependence also on the controllers, i.e.

$$F_x(\theta^i_l) = \sum_{l=1}^{s} \alpha_l(\theta^i_k) F^l_x, \quad F_\zeta(\theta^i_l) = \sum_{l=1}^{s} \alpha_l(\theta^i_k) F^l_\zeta,$$

where F^l_x and F^l_ζ denote the vertex controllers at vertex l.

To formulate the trigger conditions, we introduce the local update error

$$e^i_k = \hat{y}^i_k - y^i_k. \tag{6}$$

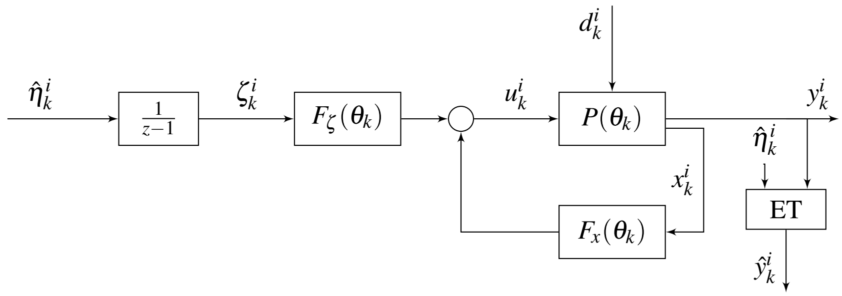

Fig. 1. Single agent with local state feedback through $F_x(\theta_k^i)$ and information from neighbors $\hat{\eta}_k^i$

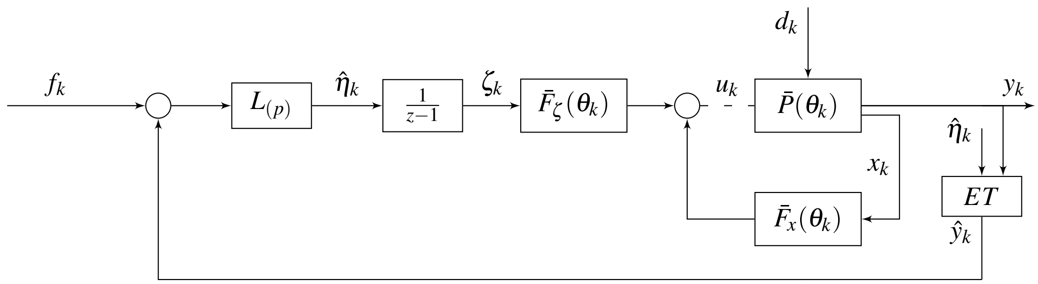

Fig. 2. Formation control loop

Note that (5) can then be written as

$$\hat{\eta}_k^i = \eta_k^i + \sum_{j \in \mathcal{N}_i} (e_k^j - e_k^i), \qquad k_{m_i}^i \le k < k_{m_i+1}^i \tag{7}$$

where $\eta_k^i = \sum_{j \in \mathcal{N}_i} (y_k^j - y_k^i + f_k^{ij})$ is the true (but unknown) formation error at time k.

Agent i will transmit the current value of its output y_k^i to its neighbors whenever the condition

$$e_k^{i\,T} e_k^i \le \sigma \cdot \hat{\eta}_k^{i\,T} \hat{\eta}_k^i \tag{8}$$

is violated, where $\sigma > 0$ is a trigger level to be determined. Note that each agent is able to evaluate this condition using locally available information only.

Define stacked signal vectors

$$x_k = \begin{bmatrix} x_k^1 \\ \vdots \\ x_k^N \end{bmatrix}, \; y_k = \begin{bmatrix} y_k^1 \\ \vdots \\ y_k^N \end{bmatrix}, \; \theta_k = \begin{bmatrix} \theta_k^1 \\ \vdots \\ \theta_k^N \end{bmatrix}$$

and similarly for all other signals, where $\theta_k \in \bar{\Theta} = \oplus_{i+1}^N \Theta$. The updated stacked formation error vector is

$$\hat{\eta}_k = \begin{bmatrix} \hat{\eta}_k^1 \\ \vdots \\ \hat{\eta}_k^N \end{bmatrix} = \begin{bmatrix} \eta_{k_{m_1}^1}^1 \\ \vdots \\ \eta_{k_{m_N}^N}^N \end{bmatrix}, \quad \underline{k} \le k < \bar{k}$$

where

$$\underline{k} = \max_{j,m_j} (k_{m_j}^j \le k), \quad \bar{k} = \min_{j,m_j} (k < k_{m_j}^j).$$

Note that satisfying the local trigger condition (8) for all agents $i = 1, \ldots, N$ is equivalent to

$$e_k^T e_k \le \sigma \cdot \hat{\eta}_k^T \hat{\eta}_k. \tag{9}$$

We further define

$$\bar{A}(\theta_k) = \mathrm{diag}\big(\mathrm{A}(\theta_k^1), \ldots, \mathrm{A}(\theta_k^N)\big)$$

as a block diagonal matrix with the scheduled A matrices of all agents along the diagonal, and similarly $\bar{B}_u(\theta_k)$, $\bar{B}_d(\theta_k)$, $\bar{C}_y(\theta_k)$, $\bar{C}_z(\theta_k)$ and $\bar{D}_z(\theta_k)$, to obtain a qLPV state space realization $\bar{P}(\theta_k)$ of the overall network.

The open-loop network can then be represented as

$$\begin{aligned} x_{k+1} &= \bar{A}(\theta_k)\, x_k + \bar{B}_d(\theta_k)\, d_k + \bar{B}_u(\theta_k)\, u_k \\ \zeta_{k+1} &= \zeta_k + \hat{\eta}_k \\ y_k &= \bar{C}_y(\theta_k)\, x_k, \end{aligned} \tag{10}$$

where ζ_k is the integrated estimate of the formation error. With

$$\begin{aligned} \bar{F}_x(\theta_k) &= \mathrm{diag}\big(\mathrm{F_x}(\theta_k^1), \ldots, \mathrm{F_x}(\theta_k^N)\big) \\ \bar{F}_\zeta(\theta_k) &= \mathrm{diag}\big(\mathrm{F}_\zeta(\theta_k^1), \ldots, \mathrm{F}_\zeta(\theta_k^N)\big), \end{aligned}$$

the control law (3) becomes

$$u_k = [\bar{F}_x(\theta_k) \;\; \bar{F}_\zeta(\theta_k)] \begin{bmatrix} x_k \\ \zeta_k \end{bmatrix} = \bar{F}(\theta_k) \psi_k \tag{11}$$

where in the second equation we introduced the augmented state vector ψ_k and the augmented state feedback gain $\bar{F}(\theta_k)$.

C. Problem Formulation

The problem addressed in this paper is the following: For a given agent model (1), communication graph G and positive constant γ, find scheduled state feedback gain matrices $F_x(\theta_k^i)$ and $F_\zeta(\theta_k^i)$ in (3) and a trigger level σ in (8) such that the group of agents (10) under the distributed feedback law (11) is stable, and moreover, such that the closed-loop system $T_{zd}(\theta_k)$ from disturbance input d to the performance output z (defined in the next section) satisfies

$$\|T_{zd}\|_{\ell_2} \leq \gamma,$$

IV. Controller Synthesis

From (5) and (6) we have

$$\hat{\eta}_k = L_{(n_y)}(y_k + f_k) + L_{(n_y)}e_k. \tag{12}$$

Using this and (10) we can represent the configuration shown in Fig. 2 with the loop cut open at the agent input u_k (marked by a dashed line) as

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ \zeta_{k+1} \end{bmatrix} &= \begin{bmatrix} \bar{A} & 0 \\ L_{(n_y)}\bar{C}_y & I \end{bmatrix} \begin{bmatrix} x_k \\ \zeta_k \end{bmatrix} + \begin{bmatrix} \bar{B}_u \\ 0 \end{bmatrix} u_k \\ &+ \begin{bmatrix} \bar{B}_d \\ 0 \end{bmatrix} d_k + \begin{bmatrix} 0 \\ L_{(n_y)} \end{bmatrix} f_k + \begin{bmatrix} 0 \\ L_{(n_y)} \end{bmatrix} e_k, \\ y_k &= \begin{bmatrix} \bar{C}_y & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \zeta_k \end{bmatrix}. \end{aligned} \tag{13}$$

We now introduce a performance output

$$z_k = \begin{bmatrix} L_{(n_y)}\bar{C}_y & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \zeta_k \end{bmatrix} + \begin{bmatrix} \bar{D}_z \\ 0 \end{bmatrix} d_k + \begin{bmatrix} 0 \\ L_{(n_y)} \end{bmatrix} f_k \tag{14}$$

that will later be used for tuning the formation control loop. Note that when shaping filters are used in a loop shaping design, the filter dynamics need to be included in the model matrices $\bar{A}$, $\bar{B}_d$. For ease of notation the dependence of the model matrices on θ_k is suppressed. The above open-loop model can then be written in a more compact form as

$$\begin{aligned} \psi_{k+1} &= \breve{A}\psi_k + \breve{B}_u u_k + \breve{B}_d d_k + \breve{f}_k + v_k \\ z_k &= \breve{C}_z \psi_k + \breve{D}_z d_k + \breve{f}_k \\ y_k &= \breve{C}_y \psi_k \end{aligned} \tag{15}$$

where

$$\psi_k = \begin{bmatrix} x_k \\ \zeta_k \end{bmatrix}, \quad v_k = \begin{bmatrix} 0 \\ L_{(n_y)} \end{bmatrix} e_k, \quad \breve{f}_k = \begin{bmatrix} 0 \\ L_{(n_y)} \end{bmatrix} f_k.$$

Under the feedback law (3), (11) the augmented closed-loop system becomes

$$\begin{aligned} \psi_{k+1} &= \breve{A}_c \psi_k + \breve{B}_d d_k + \breve{f}_k + v_k \\ z_k &= \breve{C}_z \psi_k + \breve{D}_z d_k + \breve{f}_k \\ y_k &= \breve{C}_y \psi_k \end{aligned} \tag{16}$$

where $\breve{A}_c = \breve{A} + \breve{B}_u \bar{F}$.

We can remove the formation reference $\breve{f}_k$ in (16) by assuming w.l.o.g. that in the state space model in (1) the transmitted outputs $y_k^i \in \mathbb{R}^{n_y}$ are identical to the top n_y entries of the state vector, *i.e.*

$$C_y = [I_{n_y}\ 0], \quad x_k^i = \begin{bmatrix} y_k^i \\ \xi_k^i \end{bmatrix}$$

At network level we define

$$h_k = y_k + f_k, \quad x_k^{h.i} = \begin{bmatrix} h_k^i \\ \xi_k^i \end{bmatrix}, \quad \Psi_k^h = \begin{bmatrix} x_k^h \\ \zeta_k \end{bmatrix}.$$

The closed-loop system (16) can then be written as

$$\begin{aligned} \psi_{k+1}^h &= \breve{A}_c \psi_k^h + \breve{B}_d d_k + v_k \\ z_k^h &= \breve{C}_z \psi_k^h + \breve{D}_z d_k \\ h_k &= \breve{C}_y \psi_k^h \end{aligned} \tag{17}$$

Note that $\psi^h = 0$ is an unforced equilibrium of (17).

The configuration resulting from closing all loops in the network via the feedback law (3) (the dashed line) is shown in Fig. 2.

Next we present a result that provides a way of constructing a distributed state feedback control scheme from the solution to an LMI problem.

Theorem 1: The closed-loop system (17) with distributed event-triggered controller (11) and local trigger condition (9) is stable and satisfies

$$\|T_{zd}\|_{\ell_2} \leq \gamma,$$

if there exist symmetric positive definite matrices $S^l \in R^{n_x \times n_x}$, matrices $G^l \in R^{n_x \times n_x}$, $K^l \in R^{n_u \times n_x}$, and a scalar $\sigma_x > 0$, such that the repeated block diagonal matrices $\breve{S}^l \in R^{Nn_x \times Nn_x}$, matrices $\breve{G}^l \in R^{Nn_x \times Nn_x}$, $\breve{K}^l \in R^{Nn_u \times Nn_x}$ satisfy (18) (see next page) for $l = 1, \ldots, n$.

If (18) is feasible, the vertex controllers for each agent can be constructed from the solution as $F^l = K^l (G^{l^{-1}})^T$, and the trigger condition from $\sigma = \sigma_x^{-1}$.

Proof: See Appendix.

The size of the LMI condition (18) depends on the number of agents and is potentially huge. If we make the following assumption, the complexity of the synthesis problem can be reduced to the size of a single agent.

Assumption 1: Scheduling in the group of agents (10) is homogeneous, i.e. $\theta_k^i = \theta_k^j\ \forall\ 1 \leq i, j \leq N$ and $\forall k \geq 0$.

We then have the following result.

Theorem 2: The closed-loop system (17) with distributed event-triggered controller (11) and local trigger condition (9) is stable and satisfies

$$\|T_{zd}\|_{\ell_2} \leq \gamma,$$

if there exist symmetric positive definite matrices $S^l \in R^{n_x \times n_x}$, matrices $G^l \in R^{n_x \times n_x}$, $K^l \in R^{n_u \times n_x}$, and a scalar $\sigma_x > 0$, such that (19) is satisfied for $l = 1, \ldots, n$.

If (19) is feasible, the vertex controllers for each agent can be constructed from the solution as $F^l = K^l (G^{l^{-1}})^T$, and the trigger condition from $\sigma = \sigma_x^{-1}$.

Proof: See Appendix.

Note that whereas Assumption 1 is approximately satisfied when agents are moving in a formation, it may be violated when a disturbance acts on an individual agent. Simulation results shown below suggest that even then the proposed scheme performs well.

V. LPV Representation of a Non-Holonomic Vehicle

To motivate the use of LPV models for agents in formation control problems, we will introduce a model of an non-holonomic system that can be used as a simplified model of

$$
\begin{bmatrix}
\breve{G}^l + \breve{G}^{l^T} - \breve{S}^l & 0 & 0 & \breve{G}^l \breve{A}^{l^T} + \breve{K}^{l^T} \breve{B}_u^{l^T} & \breve{G}^l \breve{C}_y^{l^T} & \breve{G}^l \breve{C}_z l^T \\
0 & I & 0 & I & \breve{C}_y^{l^T} & 0 \\
0 & 0 & \gamma^2 I & \breve{B}_d^{l^T} & 0 & \breve{D}_z^{l^T} \\
\breve{A}^l \breve{G}^{l^T} + \breve{B}_u^l \breve{K}^l & I & \breve{B}_d^l & \breve{S}^l & 0 & 0 \\
\breve{C}_y^l \breve{G}^{l^T} & \breve{C}_y^l & 0 & 0 & \sigma_x I & 0 \\
\breve{C}_z^l \breve{G}^{l^T} & 0 & \breve{D}_z^l & 0 & 0 & I
\end{bmatrix} > 0 \tag{18}
$$

$$
\begin{bmatrix}
G^l + G^{l^T} - S^l & 0 & 0 & G^l \tilde{A}_i^{l^T} + K^{l^T} \tilde{B}_u^{l^T} & G^l \tilde{C}_y^{l^T} & G^l \tilde{C}_{z.i} l^T \\
0 & I & 0 & I & \tilde{C}_y^{l^T} & 0 \\
0 & 0 & \gamma^2 I & \tilde{B}_d^{l^T} & 0 & \tilde{D}_z^{l^T} \\
\tilde{A}_i^l G^{l^T} + \tilde{B}_u^l K^l & I & \tilde{B}_d^l & S^l & 0 & 0 \\
\tilde{C}_y^l G^{l^T} & \tilde{C}_y^l & 0 & 0 & \sigma_x I & 0 \\
\tilde{C}_{z.i}^l G^{l^T} & 0 & \tilde{D}_z^l & 0 & 0 & I
\end{bmatrix} > 0. \tag{19}
$$

with modal subsystems

$$
\tilde{A}_i^l = \begin{bmatrix} A^l & 0 \\ \lambda_i C_y^l & I \end{bmatrix}, \quad \tilde{B}_d^l = \begin{bmatrix} B_d^l \\ 0 \end{bmatrix}, \quad \tilde{B}_u^l = \begin{bmatrix} B_u^l \\ 0 \end{bmatrix}, \quad \tilde{C}_y^l = \begin{bmatrix} C_y^l & 0 \end{bmatrix}, \quad \tilde{C}_{z.i}^l = \begin{bmatrix} \lambda_i C_z^l & 0 \end{bmatrix},
$$

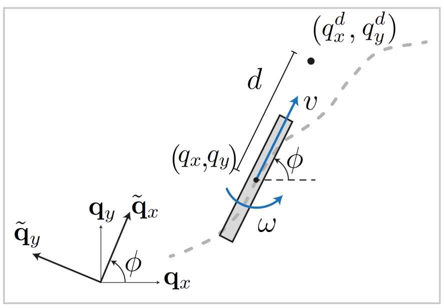

Fig. 3. Mobile robot with handle

wheeled mobile robots. The position of the robot in a plane is given by the coordinates $x(t)$ and $y(t)$, and its orientation by the angle $\phi(t)$.

The constraint on the mobile robot shown in Fig. 3 is typical for non-holonomic constraints on wheeled vehicles or robots: even though they have three total degrees of freedom (position in two axes and orientation), the two controllable degrees of freedom are acceleration (or braking) and steering.

A. Non-holonomic system with handle

In order to represent the non-honomic dynamics in the form of a quasi-LPV system, we introduce a rotation and a "handle point", as shown in Fig. 3, with position (q_x^d, q_y^d), based on the coordinate system $(\mathbf{q_x}, \mathbf{q_y})$. The dynamic equations of the handle are

$$
\begin{aligned}
\dot{q}_x^d &= v\cos\phi - \omega\ d\sin\phi \\
\dot{q}_y^d &= v\sin\phi + \omega\ d\cos\phi \\
\dot{\phi} &= \omega \\
\dot{v} &= \frac{u_1}{m} \\
\dot{\omega} &= \frac{u_2}{J}
\end{aligned} \tag{20}
$$

where u_1 and u_2 are the two control inputs, a force and a torque, respectively [38].

A time-varying transformation

$$
T\big(\phi(t)\big) = \begin{bmatrix}
\cos\phi(t) & \sin\phi(t) & 0 & 0 & 0 \\
-\sin\phi(t) & \cos\phi(t) & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{21}
$$

is used to align the body frame of the mobile robot with the direction of motion. The transformed equations for the handle point (q_x^d, q_y^d), based on the coordinate system $(\tilde{\mathbf{q}}_\mathbf{x}, \tilde{\mathbf{q}}_\mathbf{y})$ are given by

$$
\begin{aligned}
\dot{\tilde{q}}_x^d &= v + \omega\tilde{q}_y^d \\
\dot{\tilde{q}}_y^d &= -\omega\tilde{q}_x^d + \omega d \\
\dot{\phi} &= \omega \\
\dot{v} &= \frac{u_1}{m} \\
\dot{\omega} &= \frac{u_2}{J}
\end{aligned} \tag{22}
$$

An LPV model is then obtained by writing the state space model as

$$
\begin{bmatrix} \dot{\tilde{q}}_x^d \\ \dot{\tilde{q}}_y^d \\ \dot{\phi} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix}
0 & \omega & 0 & 1 & 0 \\
-\omega & 0 & 0 & 0 & d \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} \tilde{q}_x^d \\ \tilde{q}_y^d \\ \phi \\ v \\ \omega \end{bmatrix}
+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/m & 0 \\ 0 & 1/J \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{23}
$$

which has the form

$$
\dot{x}(t) = A\big(\omega(t)\big)x(t) + Bu(t) \tag{24}
$$

Note that this model is affine in the scheduling parameter ω and has a constant input matrix B. This model is a dynamic extension of the kinematic LPV model used in [3]

We use Euler discretization to discretize the model with sampling time T_s and obtain the discrete-time model

$$x_{k+1} = (I + T_s \cdot A(\omega_k))x_k + (T_s \cdot B)u_k \tag{25}$$

VI. SIMULATION RESULTS

o illustrate the effectiveness of the proposed method, a simulation example is given. Consider a network of three agents with complete graph communication, which also indicates the geometric shape of the formation, specified by a constant formation reference vector

$$f_k^T = [1, 0, 0, 0.5, 0.5, -0.5]$$

The Laplacian of this graph is

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

Each agent is modelled as (25). Agents 1 to 3 are required to attain the desired formation under the proposed the event-triggered control scheme, starting from random initial positions. At time $t = 70$ a disturbance is applied to the X-Y position of agent 1. In Fig. 4-7, the X-Y position of agent 1 and the trigger times and formation error for agents 1 to 3 are shown for two different trigger levels, one with $\sigma = 0.015$ and the other with $\sigma = 0.000125$. As we can see in table 1 and the figures show that for the smaller value of σ, data is more frequently transmitted . When a disturbance is added, transmission of more data is needed to attenuate it. In the figure 5 and 7 as you can see, the effect of disturbance is more sever in figure 5 and we have more frequent trigger happens, the reason is that the trigger threshold is larger and it allows larger error to be detected, but when we have smaller trigger threshold it react when the error is smaller and to reject the disturbance less frequent trigger is necessary.

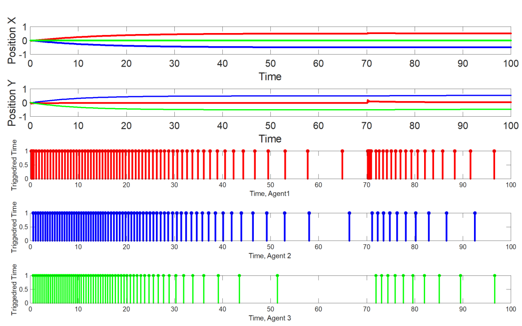

Fig. 4. X-Y position of agent 1 and trigger times of agent2 1,2 and 3 for $\sigma = 0.015$.

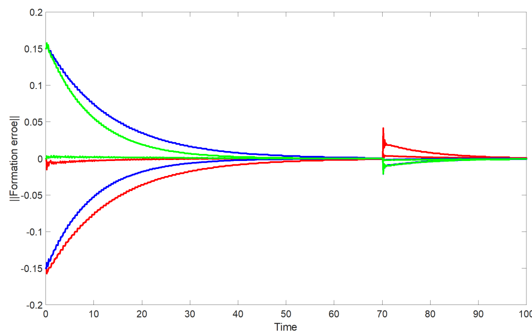

Fig. 5. Formation error of agents 1,2,3 for $\sigma = 0.015$.

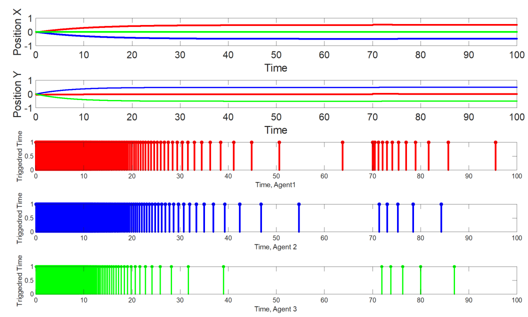

Fig. 6. X-Y position of agent 1 and trigger times of agents 1,2 and 3 for $\sigma = 0.000125$.

TABLE I

DATA TRANSMISSION PERCENTAGE FOR DIFFERENT VALUES OF σ

σ	0.015	0.000125
Data transmission percentage	7%	20%

VII. CONCLUSIONS

In this paper, we propose a distributed event-triggered formation control scheme for non-holonomic agents modeled as LPV systems, that reduces the communication load of the shared network while guaranteeing stability and a level of control performance for the networked. The synthesis

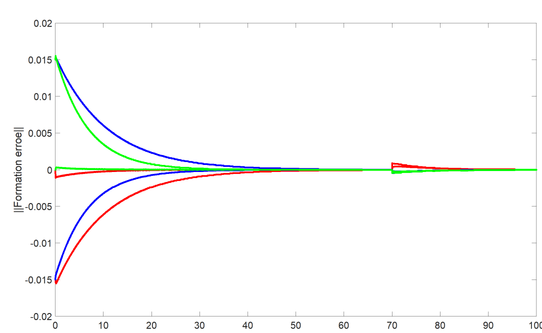

Fig. 7. Formation error of agents 1,2,3 for $\sigma = 0.000125$.

problem is formulated as LMI problem; its size can be reduced to that of a single agent when assuming homogeneous scheduling. A simulation example illustrates the practicality of the proposed method.

REFERENCES

[1] R. Olfati-Saber, J. Alex Fax, M. M. Richard, (2007). Consensus and cooperation in networked multi-agent systems. *Proc. IEEE*, , vol. 97, no. 1, pp. 215-233.

[2] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," in IEEE Transactions on Automatic Control, vol. 49, no. 9, pp. 1465-1476, Sept. 2004, doi: 10.1109/TAC.2004.834433.

[3] A. Mendez Gonzalez, C. Hoffmann, C. Radisch, H. Werner; LPV Formation Control for a Class of Non-Holonomic Agents with Directed and Switching Communication Topologies, 54th IEEE Conference on Decision and Control, 2015, Osaka, JP.

[4] E. Garcia, Y. Cao, H. Yu, P. Antsaklis, and D. Casbeer, "Decentralised event-triggered cooperative control with limited communication," Int. J.Control, vol. 86, no. 9, pp. 1479–1488, 2013.

[5] K. Zhu, D. Ma, and J. Zhao, "Event-triggered control for a switched LPV system with applications to aircraft engines," in IET Control Theory and Applications, vol. 12, no. 10, pp. 1505-1514, 3 7 2018.

[6] D. V. Dimarogonas, K.H. Johansson, "Event-triggered cooperative control", Proc. of the 2009 European Control Conference, pp. 3015-3020, 2009.

[7] D. V. Dimarogonas, E. Frazzoli, and K. J. Johansson, "Distributed event-triggered control for multi-agent systems,"IEEE Trans. Autom.Control, vol. 57, no. 5, pp. 1291–1297, May 2012.

[8] SG. S. eyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus," Automatica, vol. 49,no. 1, pp. 245–252, Jan. 2013.

[9] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", IEEE Trans. Autom. Control, vol. 52, no. 9, pp. 1680-1685, Sep. 2007.

[10] M. Mazo, P. Tabuada, "On event-triggered and self-triggered control over sensor/actuator networks", Decision and Control 47th IEEE Conference on. IEEE, pp. 435-440, 2008.

[11] E. Garcia, Y. Cao, D. W. Casbeer, "Decentralized event-triggered consensus with general linear dynamics", Automatica, vol. 50, no. 10, pp. 2633-2640, Oct. 2014.

[12] T. H. Cheng, Z. Kan, R.J. Klotz, J. M. Shea, and W.E. Dixon, (2017). "Event-triggered control of multi-agent systems for fixed and time-varying network topologies," IEEE Trans. Autom. Control, vol. 62, no. 10, pp. 5365–5371,Oct.

[13] Z. Liu, Z. Chen, and Z. Yuan, "Event-triggered average-consensus of multi-agent systems with weighted and direct topology," J. Syst. Sci.Complex., vol. 25, no. 5, pp. 845–855, 2012.

[14] X. Chen, F. Hao, "Event-triggered average consensus control for discrete-time multi-agent systems", IET Control Theory Appl., vol. 6, no. 16, pp. 2493-2498, Nov. 2012.

[15] T. Yang, Z. Meng, D.V. Dimarogonas, K. H. Johansson, "Global consensus for discrete-time multi-agent systems with input saturation constraints", Automatica, vol. 50, no. 2, pp. 499-506, Feb. 2014.

[16] A. Amini, Z. Zeinaly, A. Mohammadi and A. Asif, "Performance Constrained Distributed Event-triggered Consensus in Multi-agent Systems," 2019 American Control Conference (ACC), Philadelphia, PA, USA, 2019, pp. 1830-1835.

[17] K. K. Oh, M. C. Park and H. S. Ahn, "A survey of multi-agent formation control", Automatica, vol. 53, pp. 424-440, 2015.

[18] X. Yu and L. Liu, "Distributed Formation Control of Non-holonomic Vehicles Subject to Velocity Constraints," in IEEE Transactions on Industrial Electronics, vol. 63, no. 2, pp. 1289-1298, Feb. 2016.

[19] T. Liu and Z. P. Jiang, "Distributed formation control of non-holonomic mobile robots without global position measurements,"Automatica,vol. 49, no. 2, pp. 592–600, 2013.

[20] X. Ge and Q.-L. Han, "Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism", IEEE Trans. Ind. Electron., vol. 64, no. 10, pp. 8118-8127, Oct. 2017.

[21] W. Zhu, W. Cao and Z. Jiang, "Distributed Event-Triggered Formation Control of Multi-agent Systems via Complex-Valued Laplacian," in IEEE Transactions on Cybernetics.

[22] C. Viel, S. Bertrand, M. Kieffer, and H. Piet-Lahanier, "Distributed event-triggered control for multi-agent formation stabilization," IFAC-PapersOnLine, vol. 50, no. 1, pp. 8025–8030, 2017.

[23] A. Amini, A. Asif and A. Mohammadi, "Dynamic Event-triggered Formation Control for Multi-agent Systems: A Co-design Optimization Approach," 2020 American Control Conference (ACC), Denver, CO, USA, 2020, pp. 707-712.

[24] Li. X, Dong. X, Li. Q, and Ren. Z, "Event-triggered time-varying formation control for general linear multi-agent systems,"J. Frankl. Inst.,Feb. 2018.

[25] R. Toyota, T. Namerikawa, "Event-Triggered Formation Control of a Generalized Multi-Agent System", 2018 57th Annual Conference of the Society of Instrument and Control Engineers of Japan (SICE).

[26] A. Eqtami. A, Dimarogonas. D. V, Kyriakopoulos.K . J, "Event-triggered control for discrete-time systems", Proc. Amer. Control Conf., pp. 4719-4724, Jun. 2010.

[27] A. Golabi, N. Meskin, R. Tóth, J. Mohammadpour, and T. Donkers, "Event-triggered control for discrete-time linear parameter varying systems," 2016 American Control Conference (ACC), Boston, MA, 2016, pp. 3680-3685.

[28] H. Saadabadi, H. Werner, "Event-Triggered Dynamic Output Feedback Control For Discrete-Time Polytopic Linear Parameter-Varying Systems," 2020 IEEE Conference on Decision and Control (CDC).

[29] S. Li, D. Sauter, B. Xu, "Co-design of event-triggered H_∞ control for discrete-time linear parameter varying systems with network-induced delays," Journal of the Franklin Institute, Volume 352, Issue 5, Pages 1867-1892, 2015.

[30] J. J. Huang, X. Z. Pan, X. Z. Hao, "Event-triggered and self-triggered H∞ output tracking control for discrete-time linear parameter-varying systems with network-induced delays", International Journal of Systems Science, 2020.

[31] J. Liu, D. Chang and D. Ma, "Event-Triggered H_∞ Tracking Control for Switched LPV Systems," 2019 Chinese Control Conference (CCC), Guangzhou, China, 2019, pp. 1235-1240.

[32] K. Zhu, D. Ma, and J. Zhao, "Event-triggered control for a switched LPV system with applications to aircraft engines," in IET Control Theory and Applications, vol. 12, no. 10, pp. 1505-1514, 3 7 2018.

[33] A. Golabi, M. Davoodi, N. Meskin, J. Mohammadpour and R. Tóth, "Event-triggered fault detection for discrete-time LPV systems," 2016 Second International Conference on Event-based Control, Communication, and Signal Processing (EBCCSP), Krakow, 2016, pp. 1-8.

[34] S. Li, S. Jiang, and F. Pan, "Event-Triggered Fault Detection for Networked LPV Systems," Circuits Syst Signal Process 38, 2992–3019 (2019)

[35] M. Mesbahi and M. Egerstedt, Graph-Theoretic Methods in Multiagent Networks, NJ, Princeton:Princeton Univ. Press, 2010.

[36] P. Massioni, M. Verhaegen, "Distributed control for identical dynamically coupled systems: A decomposition approach", IEEE Transactions on Automatic Control, vol. 54, pp. 124-135, 2009.

[37] A. Eichler, H. Werner, "Performance Bounds on Decomposable Systems", American Control Conference, 2013.

[38] J. R. T. Lawton, R. W. Beard and B. J. Young, "A decentralized approach to formation maneuvers," in IEEE Transactions on Robotics and Automation, vol. 19, no. 6, pp. 933-941, Dec. 2003.

Proof of Theorem 1

Consider the Lyapunov function candidate

$$V(\psi_k^h,\theta_k)=\left(\psi_k^h\right)^T\check{P}(\theta_k)\psi_k^h,\quad P(\theta_k)>0\ \forall\theta_k\in\bar{\Theta}$$

where $\check{P}=I_N\otimes P(\theta_k)$. Stability and ℓ_2 performance γ are guaranteed with trigger condition (9) if we have along all admissible trajectories

$$\begin{aligned}&\psi^{hT}_{k+1}\check{P}(\theta_{k+1})\psi^h_{k+1}-\psi^{hT}_k\check{P}(\theta_k)\psi^h_k\\&\leq\ -z_k{}^Tz_k+\gamma^2d_k{}^Td_k-\sigma\hat{\eta}_k^T\hat{\eta}_k+e_k^Te_k.\end{aligned}\tag{26}$$

Note that the formation error estimate can be written as $\hat{\eta}_{k+1}=L_{(n_y)}\bar{C}_y(\theta_k)x_k+L_{(n_y)}e_k$. Substituting this in (26) yields after some rearrangement

$$\begin{aligned}&\big(\breve{A}_c\psi_k^h+\breve{B}_d d_k+Le_k\big)^T\breve{P}\big(\breve{A}_c\psi_k^h+\breve{B}_d d_k+Le_k\big)-\psi_k^{hT}\breve{P}\psi_k^h \;\le\\ &\quad\psi_k^{hT}\breve{C}_z^T\breve{C}_z\psi_k^h+\psi_k^{hT}\breve{C}_z^T D_z d_k+d_k^T\breve{D}_z^T\breve{C}_z\psi_k^h+d_k^T\breve{D}_z^T\breve{D}_z d_k\\ &\quad+\gamma^2 d_k^T d_k-\sigma(\psi_k^h+e_k)^T\breve{C}_y^T L^T L\breve{C}_y(\psi_k^h+e_K)+e^T(k)e(k)\end{aligned} \tag{27}$$

This condition can be written as

$$\begin{bmatrix}\psi_k^{hT} & e_\psi^T & d_k{}^T\end{bmatrix} M(\theta_k)\begin{bmatrix}\psi_k^h\\ e_k\\ d_k\end{bmatrix}<0 \tag{28}$$

where

$$M=\begin{pmatrix}\breve{P}-\breve{A}_c^T\breve{P}\breve{A}_c-\breve{C}_z^T\breve{C}_z+\sigma\breve{C}_y^T L^T L\breve{C}_y & -\breve{A}_c^T\breve{P}L-\sigma\breve{C}_y^T L^T L\breve{C}_y & -\breve{A}_c^T\breve{P}\breve{B}_d-\breve{C}_z^T\breve{D}_z\\ -L^T\breve{P}\breve{A}_c-\sigma\breve{C}_y^T L^T L\breve{C}_y & -L^T\breve{P}L-\sigma\breve{C}_y^T L^T L\breve{C}_y+I & -L^T\breve{P}\breve{B}_d\\ -\breve{B}_d^T\breve{P}\breve{A}_c+\breve{D}_z^T\breve{C}_z & -\breve{B}_d^T\breve{P}L & \gamma^2 I-\breve{B}_d^T\breve{P}\breve{B}_d+\breve{D}_z^T\breve{D}_z\end{pmatrix} \tag{29}$$

Thus, a sufficient condition for closed-loop stability and a guaranteed performance level γ is that $M(\theta_k)>0\ \forall\theta_k\in\bar{\Theta}$. Rewriting this as

$$M=\begin{bmatrix}\breve{P} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & \gamma^2 I\end{bmatrix}-\begin{bmatrix}\breve{A}_c^T\breve{P} & \sigma\breve{C}_y^T L^T & \breve{C}_z^T\\ L^T\breve{P} & \sigma\breve{C}_y^T L^T & 0\\ \breve{B}_d^T\breve{P} & 0 & \breve{D}_z^T\end{bmatrix}\begin{bmatrix}\breve{P} & 0 & 0\\ 0 & \sigma I & 0\\ 0 & 0 & I\end{bmatrix}^{-1}\begin{bmatrix}\breve{P}\breve{A}_c & \breve{P}L & \breve{P}\breve{B}_d\\ \sigma L\breve{C}_y & \sigma L\breve{C}_y & 0\\ \breve{C}_z & 0 & \breve{D}_z\end{bmatrix}>0 \tag{30}$$

and using a Schur complement argument, it follows that (26) is equivalent to $\breve{P}(\theta_k)>0$ and

$$\begin{bmatrix}\breve{P} & 0 & 0 & \breve{A}_c^T\breve{P} & \sigma\breve{C}_y^T L^T & \breve{C}_z^T\\ 0 & I & 0 & L^T\breve{P} & \sigma\breve{C}_y^T L^T & 0\\ 0 & 0 & \gamma^2 I & \breve{B}_d^T\breve{P} & 0 & \breve{D}_z^T\\ * & * & * & \breve{P} & 0 & 0\\ * & * & * & * & \sigma I & 0\\ * & * & * & * & * & I\end{bmatrix}>0 \tag{31}$$

being true for all $\theta_k\in\bar{\Theta}$. Multiplying (31) from left and right with

$$\operatorname{diag}\big(\breve{G}(\theta_k),I,I,\breve{P}^{-1}(\theta_k),\sigma^{-1}I,I\big)$$

where

$$G(\theta_k)=\sum_{l=1}^{n}\alpha_l(k)G_l,\qquad \breve{G}(\theta_k)=I_N\otimes G(\theta_k),$$

and defining $\breve{S}(\theta_k)=\breve{P}^{-1}(\theta_k)$ and $K^l=\bar{F}(G^l)^{-1}$, we obtain (18) (see page 5), where the (1,1) block of the resulting block matrix has been replaced by the right hand side of $\breve{G}_l\breve{S}_l^{-1}\breve{G}_l^T\ge\breve{G}_l+\breve{G}_l^T-\breve{S}_l$. Note that this also ensures that G_l is invertible.

Proof of Theorem 2

Because the Laplacian L is symmetric, there exists a matrix Z such that

$$\det Z\neq 0\quad\text{and}\quad Z^T LZ=\operatorname{diag}(\lambda_1,\dots,\lambda_N).$$

Since the underlying graph is connected, we have that $\lambda_1=0$ and $\lambda_2,\dots,\lambda_N>0$, and w.l.o.g. we assume that the columns of Z have been arranged such that

$$Z^T LZ=\begin{bmatrix}0 & 0_{1\times(N-1)}\\ 0_{(N-1)\times 1} & \Lambda\end{bmatrix} \tag{32}$$

where $\Lambda=\operatorname{diag}(\lambda_2,\dots,\lambda_N)>0$. Moreover, we assume w.l.o.g. that Z has been scaled such that $Z^TZ=I$. Let z_i denote the i^{th} column of Z, and define $\bar{Z}=[z_2,\dots,z_N]$, then we have

$$\Lambda=\bar{Z}^T L\bar{Z}\in\mathbb{R}^{(N-1)\times(N-1)}.$$

A congruence transformation can now be employed to decompose the "large" synthesis condition (18) into a set of $N-1$ "small" conditions, each parameterized by one of the non-zero eigenvalues of L. For this purpose define

$$T=\operatorname{diag}\big(\bar{Z}_{(n_x)},\bar{Z}_{(n_y)},\bar{Z}_{(n_x+n_y)},\bar{Z}_{(n_x)},\bar{Z}_{(n_y)},\bar{Z}_{(n_y)}\big)$$

and note that we have $T^{-1}=T^T$. We first apply a permutation to (18) that rearranges the individual agent model matrices into blocks of the form of the modal subsystems in (19). Left and right multiplying the permuted version of (18) with T^T and T, respectively, block-diagonalizes - as a result of the mixed-product rule for the Kronecker product (under homogeneous scheduling) - the left hand side and thus decouples (18) into the $N-1$ equivalent small conditions (19), parameterized by the non-zero eigenvalues of L appearing as factors in $\tilde{A}_i^l$ and $\tilde{C}_{z.i}^l$. Since the resulting block matrices are affine in λ_i, condition (18) is then equivalent to (19) being true for λ_2 and λ_N.

The above established (19) to be a sufficient condition for stability of the network under trigger condition (9). As for performance, note that the block-diagonalizing congruence transformation implies also a transformation of input and output signals. However, from $Z^TZ=I$, the condition number of the transformation matrix T is 1 and the performance bound holds on the original network; see [36]- [37].

Observing that the local trigger condition (8) implies (9) completes the proof.