Supplementary Document

A Distributed Linear Quadratic Discrete-Time Game Approach to Multi-Agent Consensus

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Algorithm for Problem 1: Nash Strategy

Algorithm 1 Nash Equilibrium via coupled Riccati difference equations

Input: each agent position and velocity or the state x^i at current time k
Output: each control inputs u^i_k and its agent position and velocity x^i_{k+1}

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Initialization P^i_T = Q^i_T,
for k = 1 : T − 1 do
    for j = T − 1 : −1 : 1 do
        for i = 1 : N do
            S^i = G^i R^{ii^{-1}} G^{i^T}
            Λ_k = I + Σ_{i=1}^N S^i P^i_{j+1}
            P^i_j = Q^i + F^T P^i_{j+1} Λ_k^{-1} F
        end for
    end for
    for i = 1 : N do
        u^i_k = −R^{ii^{-1}} G^{i^T} P^i_{k+1} Λ_k^{-1} F x_k
    end for
    x_{k+1} = F x_k + Σ_{i=1}^N G^i u^i_k
end for
```

Initialization $P^i_T = Q^i_T$,
for $k = 1 : T-1$ **do**
  for $j = T-1 : -1 : 1$ **do**
    for $i = 1 : N$ **do**
      $S^i = G^i {R^{ii}}^{-1} {G^i}^T$
      $\Lambda_k = I + \sum_{i=1}^{N} S^i P^i_{j+1}$
      $P^i_j = Q^i + F^T P^i_{j+1} \Lambda_k^{-1} F$
    end for
  end for
  for $i = 1 : N$ **do**
    $u^i_k = -{R^{ii}}^{-1} {G^i}^T P^i_{k+1} \Lambda_k^{-1} F x_k$
  end for
  $x_{k+1} = F x_k + \sum_{i=1}^{N} G^i u^i_k$
end for

Algorithm for Problem 2: Distributed LQDTG via Receding Horizon

Algorithm 2 Receding horizon for the distributed framework

Input: agent's position and velocity or the state x_k at current time k, $\Phi^\dagger = \text{pinv}(\Phi)$, where $\Phi = (I_n \otimes (-D^T))$
Output: control inputs $\hat{u}^i_k$ and state $\hat{x}_k$

Initialization $x(0) = x_k$, prediction horizon N_p
while consensus has not been achieved **do**
  form the relative dynamics z_k
  obtain $\tilde{P}^i_0$ (19) to calculate $a^{i\star}_{k+\delta}$ in (22)
  form $a^\star_{k+\delta} = [{a^{i\star}_{k+\delta}}^T, ..., {a^{M\star}_{k+\delta}}^T]^T$
  calculate the values $\hat{u}^\star_{k+\delta} = \Phi^\dagger a^\star_{k+\delta}$
  retrieve the values of $\hat{u}^{i\star}_{k+\delta}$ from $\hat{u}^\star_{k+\delta}$ and substitute the resulting value to the true state (5) $\to \hat{x}_k$
  $k = k + \delta$
end while