Supplementary Document

A Distributed Linear Quadratic Discrete-Time Game Approach to Formation Control

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Algorithm for Problem 1

Algorithm 1 Nash Equilibrium via coupled Riccati difference equations

Input: each agent position and velocity or the state x^i at current time k
Output: each control inputs u_k^i and its agent position and velocity x_{k+1}^i

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Initialization P_T^i = Q_T^i,
for k = 1 : T − 1 do
    for j = T − 1 : −1 : 1 do
        for i = 1 : N do
            S^i = G^i R^{ii^{-1}} G^{i^T}
            Λ_k = I + Σ_{i=1}^N S^i P_{j+1}^i
            P_j^i = Q^i + F^T P_{j+1}^i Λ_k^{-1} F
        end for
    end for
    for i = 1 : N do
        u_k^i = −R^{ii^{-1}} G^{i^T} P_{k+1}^i Λ_k^{-1} F x_k
    end for
    x_{k+1} = F x_k + Σ_{i=1}^N G^i u_k^i
end for
```

Algorithm for Problem 2

In the same step as in the previous problem, we initialize $\tilde{P}_T = \tilde{Q}_T$. To implement the distributed framework in the open-loop solution, we provide the systematic way as in Algorithm 2. Since the solution is decoupled now, we can solve it globally and then retrieve its values locally.

Algorithm 2 Optimal control via a distributed framework

Input: relative state z at current time k
Output: relative control input a_k, relative states z_{k+1}, transformed control input $\hat{u}_k$, and transformed states $\hat{x}_{k+1}$

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Initialization P̃_T = Q̃_T, Φ^† = pinv(Φ)
for k = 1 : T − 1 do
    for j = T − 1 : −1 : 1 do
        P̃_j = Q̃ + F̃^T P̃_{j+1} F̃ + F̃^T P̃_{j+1} G̃ K̃_j
    end for
    K̃_k = −(R̃ + G̃ P̃_{k+1} G̃)^{-1} G̃^T P̃_{k+1} F̃
    a_k = K̃_k z_k
    z_{k+1} = F̃ z_k + G̃ a_k
    û_k = Φ^† a_k
    x̂_{k+1} = F x̂_k + G û_k
end for
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