1 Forward and inverse kinematics

1.1 UR5

Universal Robots (UR1,-3,-5,-10) are multifunctional, flexible robots that can be used for various tasks. The number stands for the working load and therefore also the size of the robot. These robots are place saving and can be easily reprogrammed for new requirements.

ref1

The URs consist out of six rotational joints and seven fixed links. Therefore there are eight different constellations of joint angles to reach a determined end-effector position. That means that for given joint parameters we can calculate one unique position, but for a given position there are eight different sets of angles. These eight constellations can be described as lefty/righty, up/down and flip/noflip. With lefty/righty we can distinguish between the second joint being on the right or left side of the imaginary plane going through the origin of the coordinate system of the first joint. This distinction makes sence if the coordinate system of the second joints has an offset according to the coordinate system of the first joint. Up/down describes the relation between the coordinate systems of the third and the fourth joint. If the fourth joint remains in the same position, the third joint can be below or above it. Therefore we distinguish between up and down. Finally flip/noflip describes the wrist (5th joint) being flipped or not, meaning rotated by ϕ or $\phi + 180°$.

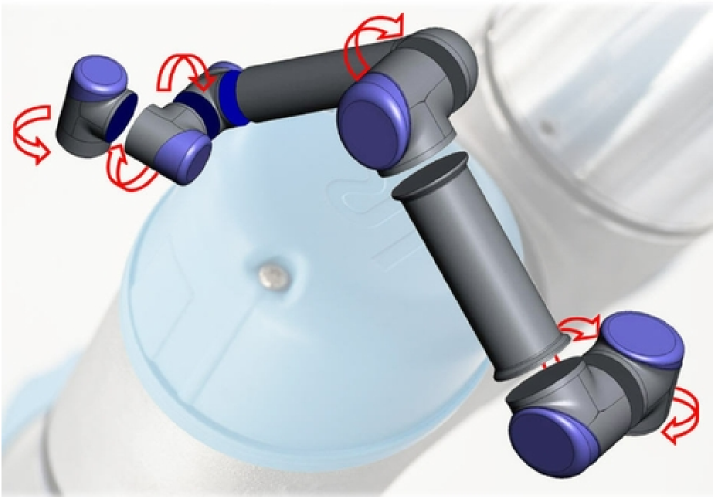

Abbildung 1.1: Joints of the UR, ref2

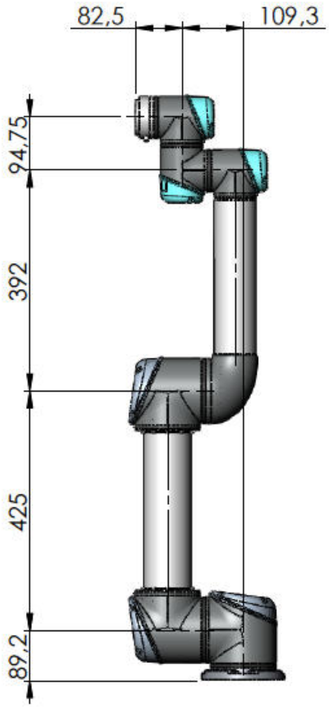

Abbildung 1.2: Dimensions of the UR, ref3

1.2 Forward kinematics

The forward kinematics of this kind of robot can be calculated easily with the Denavit-Hartenberg-procedure if the parameters are known. Denavit-Hartenberg-parameters describe the relation between two coordinate systems with a maximum of four different parameters: two rotation angles and two linear translations. These can be fixed or variable. In our case with six joints we have seven coordinate systems and six transfomations. Coordinate system zero is not attached to any of the joints and is used as the reference frame.

1.3 Backward kinematics

The backward kinematics has been realised with the algorithm of paper ref4

Its starting point is the desired pose that we want to move our robot to. With this pose and the given Denavit-Hartenberg-parameters we calculate intermediate poses from which the six angles can be found step by step. Previously calculated angles can furthermore be used to calculate the next ones. Since there are different angles possible to reach the same pose, it is important to name the current angle that is being used to calculate the next one. Lefty/righty, up/down and flip/noflip are current names. In case that one angle has two solutions the current solution exist with its particular name so that the distinction between different possibilities is made.

The implemented algorithm works as follows:

- Calculate eight max possible angle configurations that lead to the desired pose
- Check if the pose is possible with the robots dimensions and/or the given maximal joint changes
- Calculate the $\pm$ 360°-versions of the found angles since the robot might be able to move in the range of $\pm$ 360° and get eight new possibilities
- Choose one angle set of the 16 sets with the minimal angle sum with respect to the initial angles

2 Handy-Eye-calibration

The hand-eye-calibration is a current procedure to determine unknown rigid transformations, e.g. transformations between the end-effector and a tool attached to it or a transformation between the robot and the tracking system, by taking measurements with the tracking device in different poses of the robot. For good precision and accuracy you need many different poses and they need to be linearly independant from each other.

In this project the transformation between the robots end-effector and the coil had to be determined as well as the transformation between the robots base and the coordinate system of the tracking device. The coil needed to be placed in a certain translation and rotation with respect to the head of the imaginary patient therefore it was not enough to know only the end-effector coordinates that can be calculated by the direct kinematics. The hand-eye-calibration uses the closed loop of four rigid transformations. That means that we can express one transformation or a combination of two by means of the remained ones. Two of these transformations are fixed and the other two can vary. Figure 2.1 shows the installation.

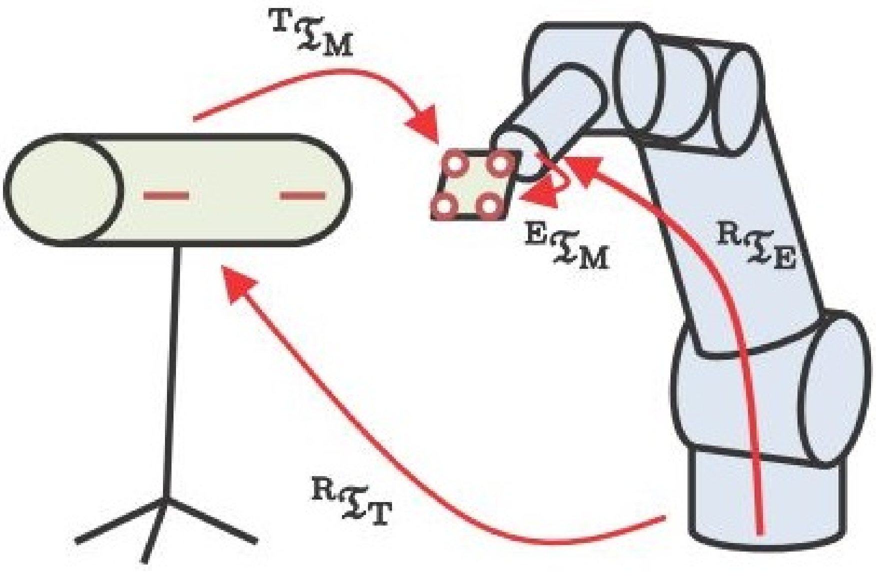

Abbildung 2.1: Hand-Eye-calibration setup

$$M =^R T_E \tag{2.1}$$

$$X =^E T_M \tag{2.2}$$

$$N =^T T_M \tag{2.3}$$

$$Y =^R T_T \tag{2.4}$$

We are interested in the fixed ones ${}^E\mathrm{T}_M$ and ${}^R\mathrm{T}_T$ whereas the variable ones can be calculated or measured. We can write

$$M \cdot X = Y \cdot N \tag{2.5}$$

or

$$M_i \cdot X = N_i \cdot Y \tag{2.6}$$

where index i denotes one set of measurements.

These equations can be rearranged and written as

$$A \cdot w = b \tag{2.7}$$

where A and b contain manipulated values of M_i and N_i so that we can fill these matrices with actual measurement values to calculate w. After the calculation the solution vector w contains the 24 not trivial values of X and Y. The last step of the calibration is to orthonormalize X and Y since the calculation of w provides independant values for the rotation and translation. If the translation can be taken as given, the vectors of the rotation matrix need to be orthonormal.

ref5