Example 1 Sampling

January 25, 2017

0.1 Make a sine wave which at 44100 Hz sampling rate has a frequency of 400 Hz at 1 second duration. Hence we need 44100 samples, and 400 periods of our sinusoid in this second. Hence we can write our signal in Python as:

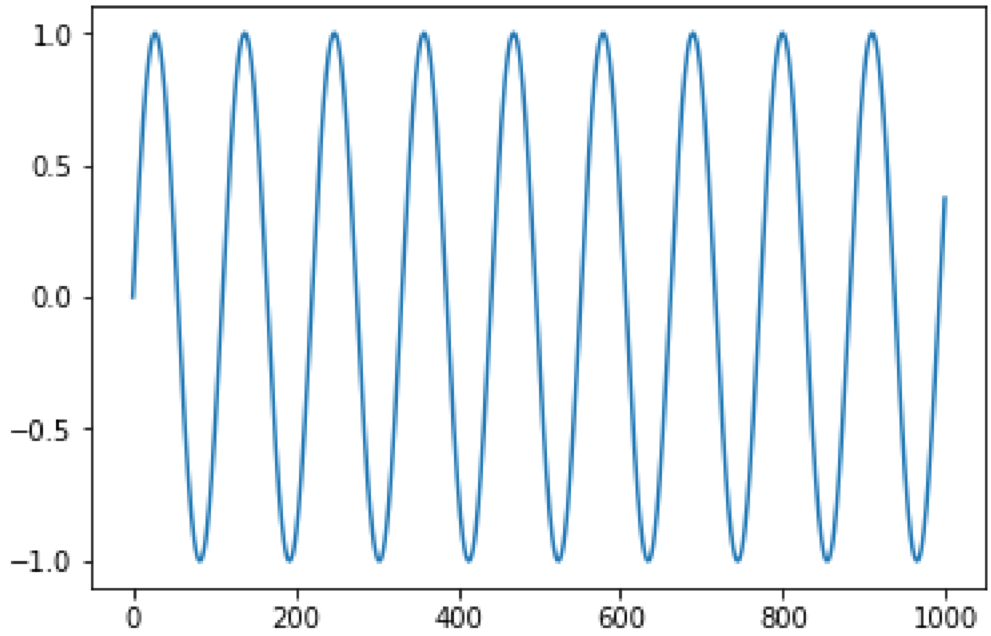
```
In [1]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt
        fs = 44100
        f = 400.0
        s = np.sin(2*np.pi*f*np.arange(0, 1, 1.0/fs))
```

0.1.1 To listen to it, we use our sound library "sound.py“, which you can find on Moodle Webpage:

```
In [2]: from sound import sound

In [3]: sound((2**15)*s,fs)

* done
```

0.1.2 Now plot the first 1000 samples:

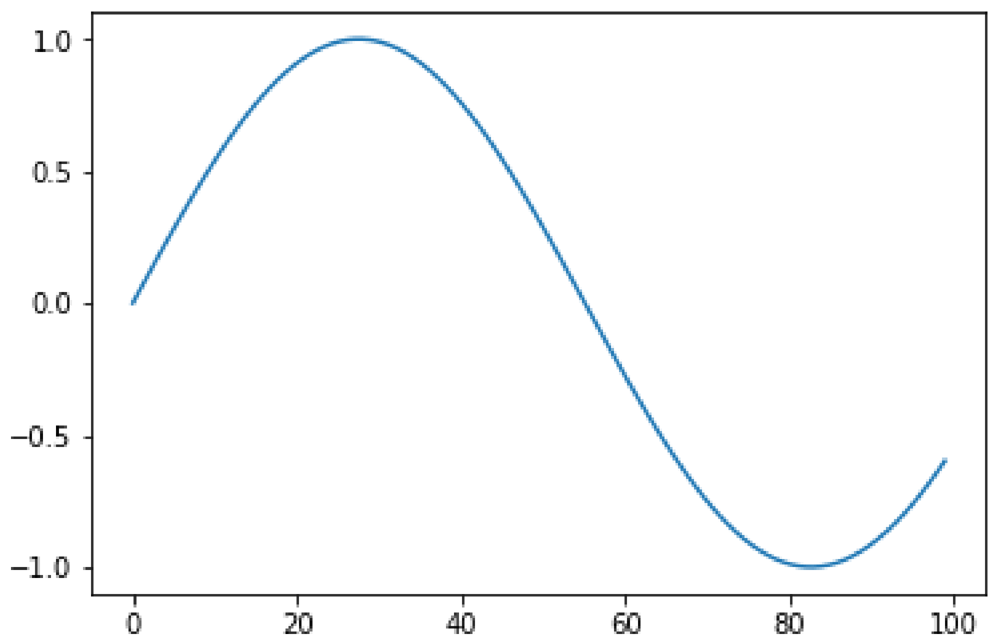
```
In [4]: plt.plot(s[0:1000])

Out[4]: [<matplotlib.lines.Line2D at 0x8400bb0>]
```



0.1.3 Next plot the first 100 samples:

```
In [5]: plt.plot(s[0:100])
```

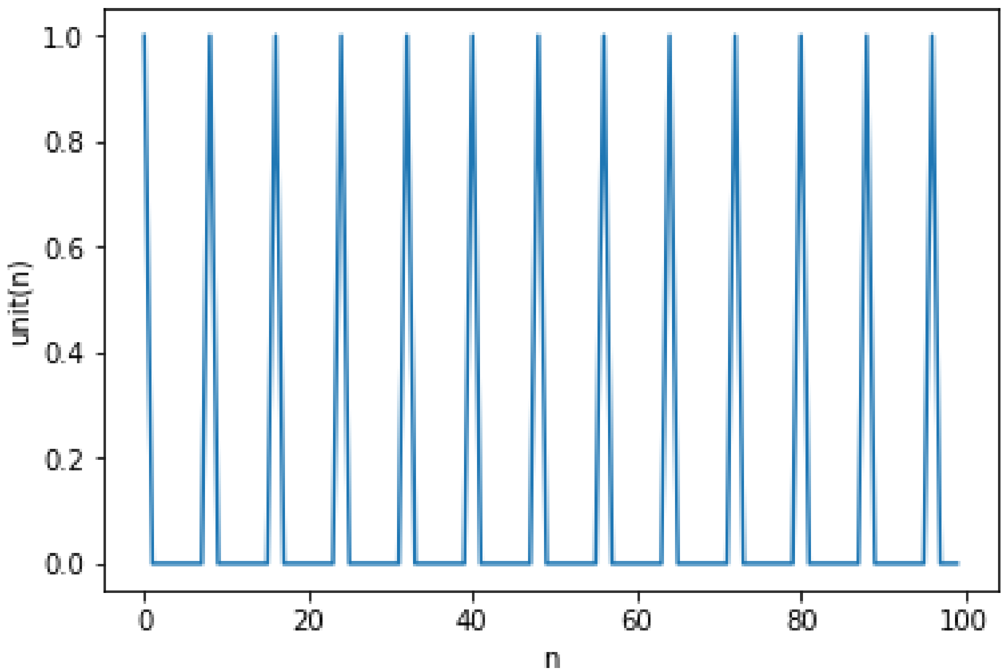
```
Out[5]: [<matplotlib.lines.Line2D at 0x8647750>]
```



Now we can multiply this sine tone signal with a unit pulse train, with N=8 i.e., downsampling while keeping the zeros. ### We generate the unit impulse train,

```
In [6]: unit = np.zeros(44100)
        unit[0::8] = 1
        plt.plot(unit[0:100])
        plt.xlabel('n')
        plt.ylabel('unit(n)')
```

```
Out[6]: <matplotlib.text.Text at 0x86786b0>
```



0.1.4 Listen to it, with scaling to the value range for 16 bit/sample:

```
In [7]: sound(unit*2.0**15,44100)
```

```
* done
```

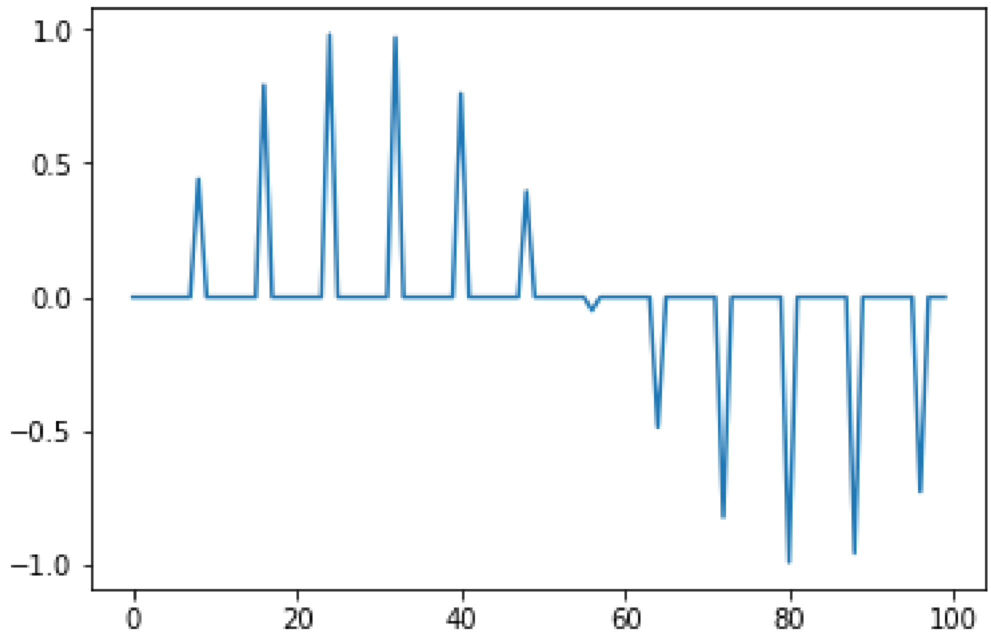
0.1.5 The multiplication with the unit impulse train:

```
In [8]: sdu=s*unit
```

(This multiplication is also called frequency mixing“). ### Now plot the result, the first 100 samples:

```
In [9]: plt.plot(sdu[0:100])
```

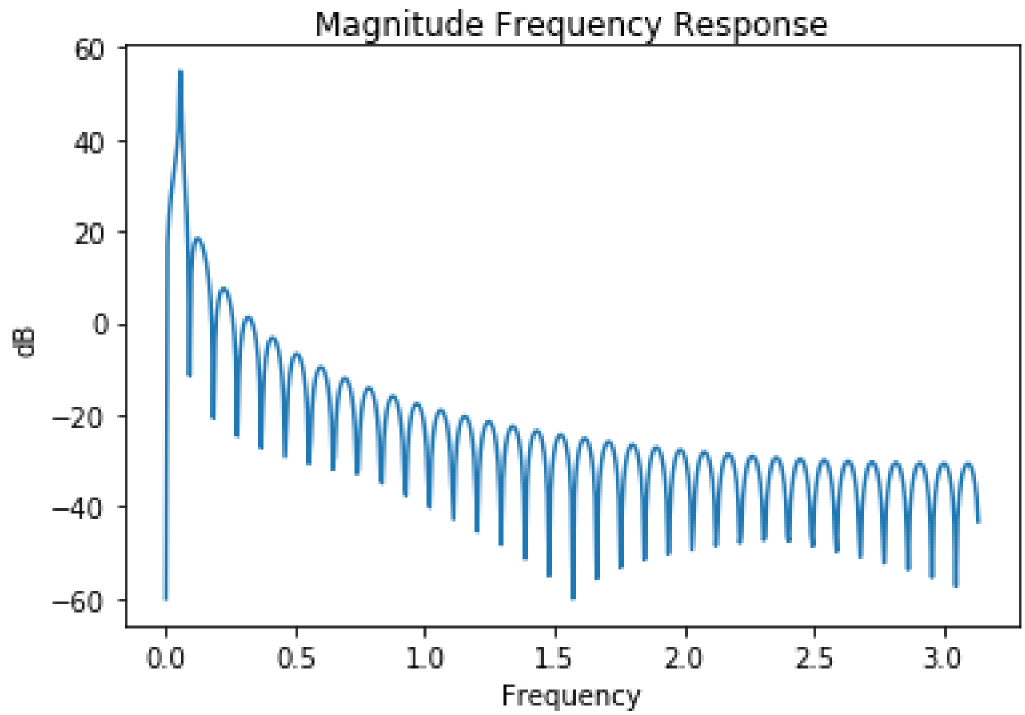
```
Out[9]: [<matplotlib.lines.Line2D at 0x8938f50>]
```



This is our signal still with the zeros in it. Now take a look at the magnitude spectrum (in dB) of the original signal s:

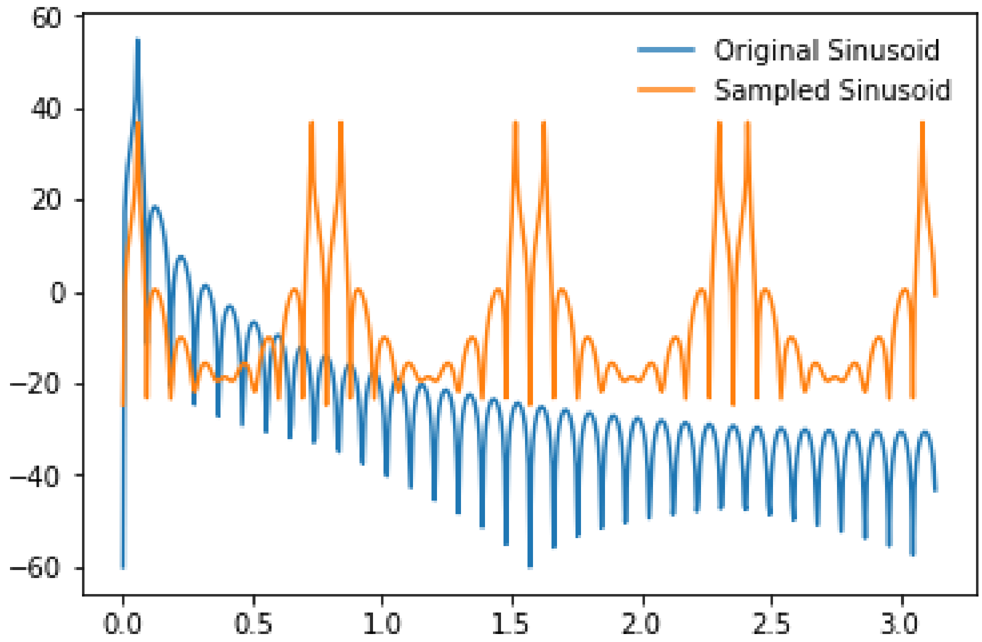
```
In [10]: from scipy.signal import freqz
         w, h = freqz(s)
         plt.plot(w, 20*np.log10(np.abs(h) + 1e-3)) #Adding 1e-3 to avoid log(0)
         plt.xlabel('Frequency')
         plt.ylabel('dB')
         plt.title('Magnitude Frequency Response')
```

```
Out[10]: <matplotlib.text.Text at 0x122a11f0>
```



The plot shows the magnitude of the frequency spectrum of our signal. Observe that the frequency axis (horizontal) is a normalized frequency, normalized to the Nyquist frequency as , in our case 22050 Hz. Hence our sinusoid should appear as a peak at normalized frequency 400.0/22050∗pi=0.05699, which we indeed see. ### Now we can compare this to our signal with the zeros, sdu:

```
In [11]: ws, hs = freqz(sdu) #ws, hs are normalized frequency and magnitude for sampled signal
         plt.plot(w, 20*np.log10(np.abs(h) + 1e-3))
         plt.plot(ws, 20*np.log10(np.abs(hs) + 1e-3))
         plt.legend(('Original Sinusoid', 'Sampled Sinusoid'))

Out[11]: <matplotlib.legend.Legend at 0x12532610>
```



Here we can see the original line of our 400 Hz tone, and now also the 7 new aliasing components. Observe that always 2 aliasing components are close together. This is because the original 400 Hz tone also has a spectral peak at the negative frequencies, at -400 Hz, or at normalized frequency -0.05699.

0.1.6 Now also listen to the signal with the zeros:

```
In [12]: sound(sdu*2.0**15,44100)
```

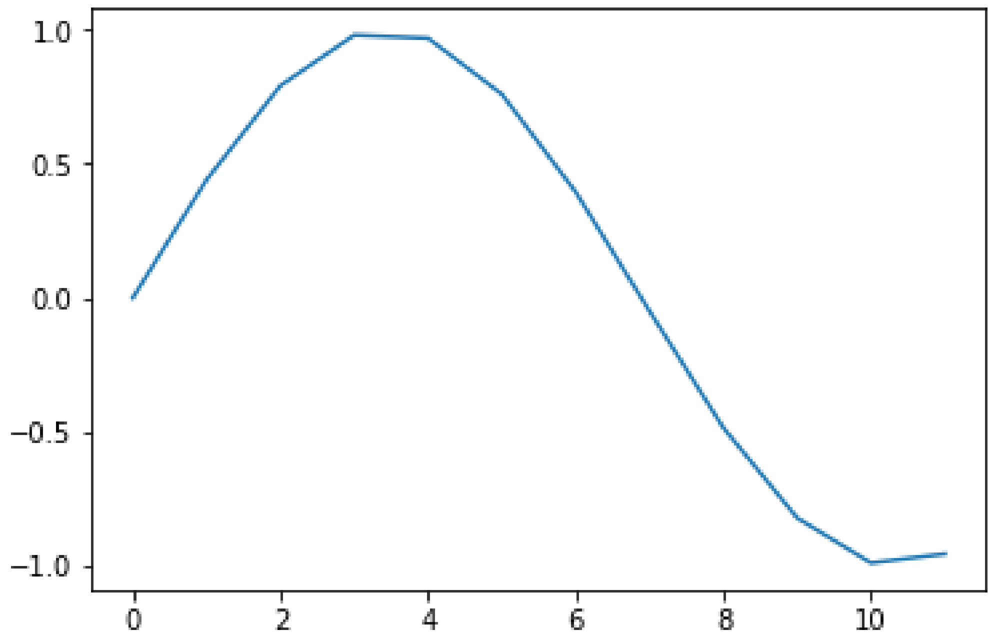
```
* done
```

Here you can hear that it sounds quite different from the original, because of the strong aliasing components!

0.1.7 Removing the zeros

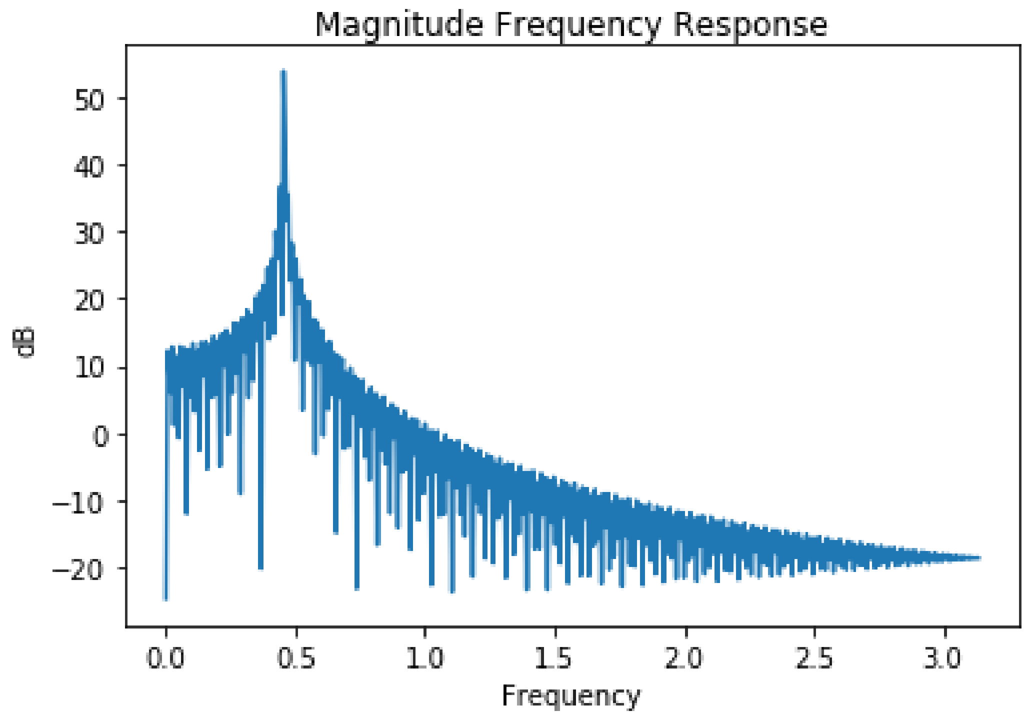
```
In [13]: #Taking every 8th sample which in our case are the only non-zero values
         sd = sdu[0:44100:8]
         plt.plot(sd[0:100/8])
```

```
Out[13]: [<matplotlib.lines.Line2D at 0x126622f0>]
```



0.1.8 We can now take a look at the spectrum with

```
In [14]: wd, hd = freqz(sd)
         plt.plot(wd, 20*np.log10(np.abs(hd) + 1e-3))
         plt.xlabel('Frequency')
         plt.ylabel('dB')
         plt.title('Magnitude Frequency Response')

Out[14]: <matplotlib.text.Text at 0x12745f30>
```



Observe that the sine signal now appear at normalized frequency of 0.455, a factor of 8 higher than before, with the zeros in it, because we reduced the sampling rate by 8. This is because we now have a new Nyquist frequency of 22050/8 now, hence our normalized frequency becomes 4003.14/2205080.455 . This means removing the zeros scales or stretches our frequency axis.

Observe that here we only have 100/812 samples left.