

Ex1

March 15, 2017

0.1 Example:

Design a **warped low pass filter** with cuto frequency of $0.05 * \pi$ (π is the Nyquist frequency). Observe: here this frequency is the end of passband, with frequency warping close to the Bark scale of human hearing.

First as a comparison: design an **unwarped filter** with 4 coefficients/taps with these specifications:

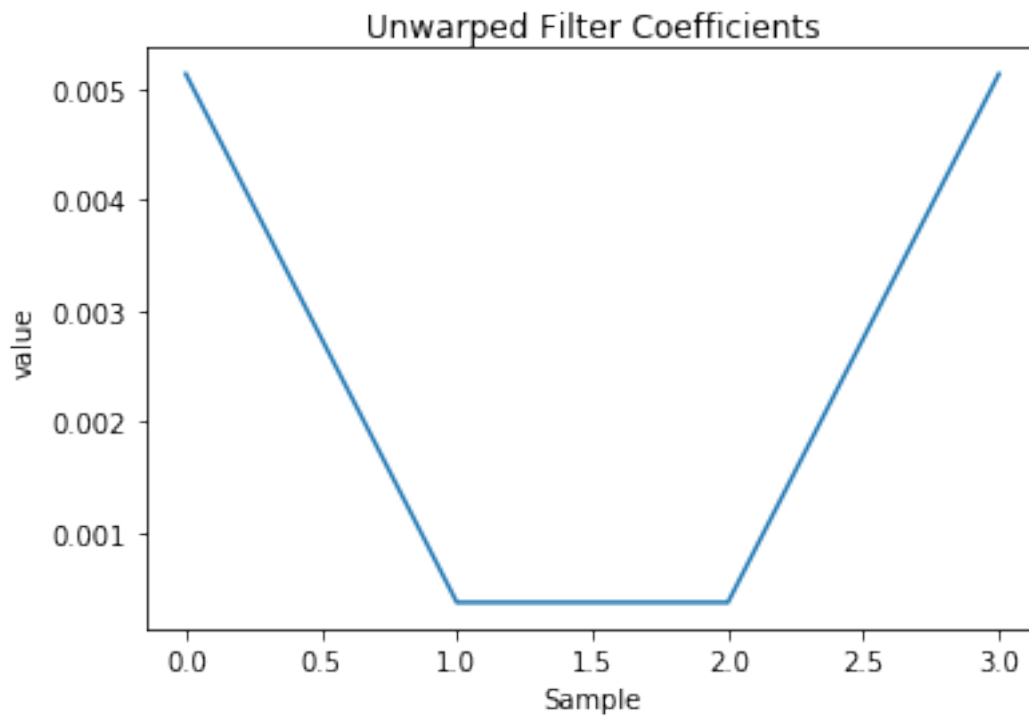
In Python:

```
In [1]: import scipy.signal as sp
        cunw = sp.remez(4, [0, 0.025, 0.025+0.025, 0.5], [1,0], [1, 100])
        print cunw
```

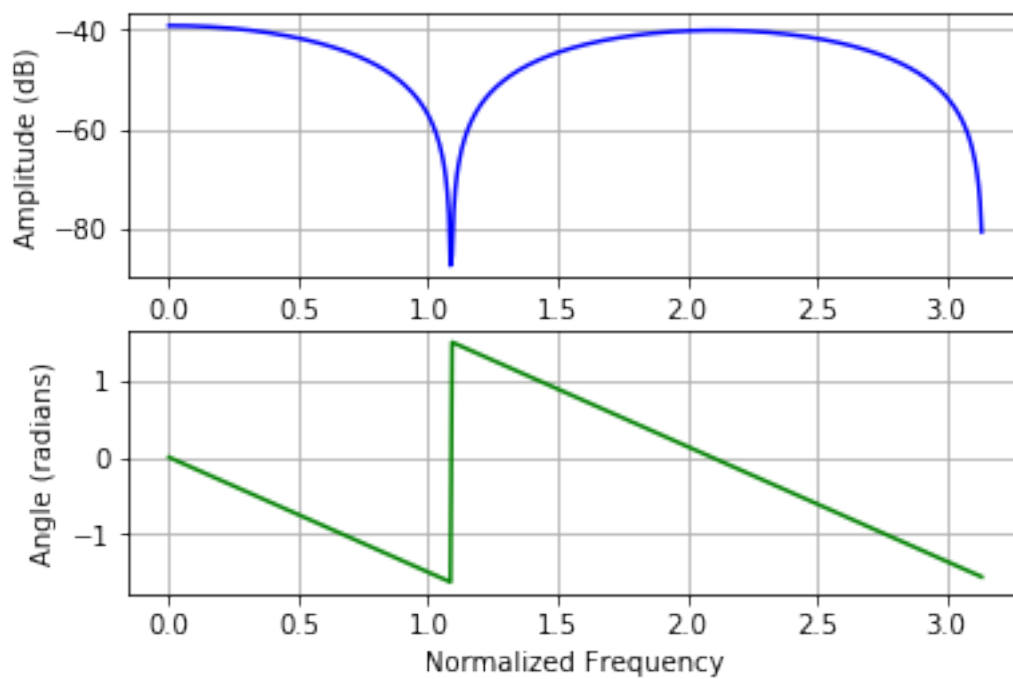
```
[ 0.00513655  0.00037423  0.00037423  0.00513655]
```

```
In [2]: #impulse response:
        %matplotlib inline
        import matplotlib.pyplot as plt
        plt.plot(cunw)
        plt.xlabel('Sample')
        plt.ylabel('value')
        plt.title('Unwarped Filter Coefficients')
```

```
Out[2]: <matplotlib.text.Text at 0x12222d50>
```



```
In [3]: #frequency response:
from freqz import freqz
freqz(cunw, 1)
```



```
Out[3]: array([ 1.10215610e-02 +0.00000000e+00j,
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3.39607038e-06 -1.84470302e-04j ,    8.49090316e-07 -9.22508663e-05j] )

```

Here we can see that this is not a good filter. The passband is too wide (up to about 0.15), and there is almost no stopband attenuation (in the range of 0.5 to 0.9). So this filter is probably **useless** for our application.

Now design the FIR low pass filter (4th order), which we then want to frequency warp in the next step, with a warped cut frequency. First we have to compute the allpass coefficient a for our allpass filter which results in an approximate Bark warping, according to Julius O. Smith and Jonathan S. Abel, "Bark and ERB Bilinear Transforms," IEEE Transactions on Speech and Audio Processing, vol. 7, no. 6, pp. 697 – 708, November 1999: ###

$$a = 1.0674 \cdot \left(\frac{2}{\pi} \cdot \arctan(0.6583 \cdot f_s) \right)^{0.5} - 0.1916$$

with f_s as the sampling frequency in kHz. Our warped design is then,

```
In [4]: from warpingphase import *
import numpy as np
#warping allpass coefficient:
a = 1.0674 * (2 / np.pi * np.arctan(0.6583 * 32)) ** 0.5 - 0.1916
print a
```

0.859559698991

```
In [5]: # with f_s=32 in kHz. from [1]
# The warped cutoff frequency then is:
fcw = -warpingphase(0.05 * np.pi, 0.85956)
print fcw
```

1.61200904751

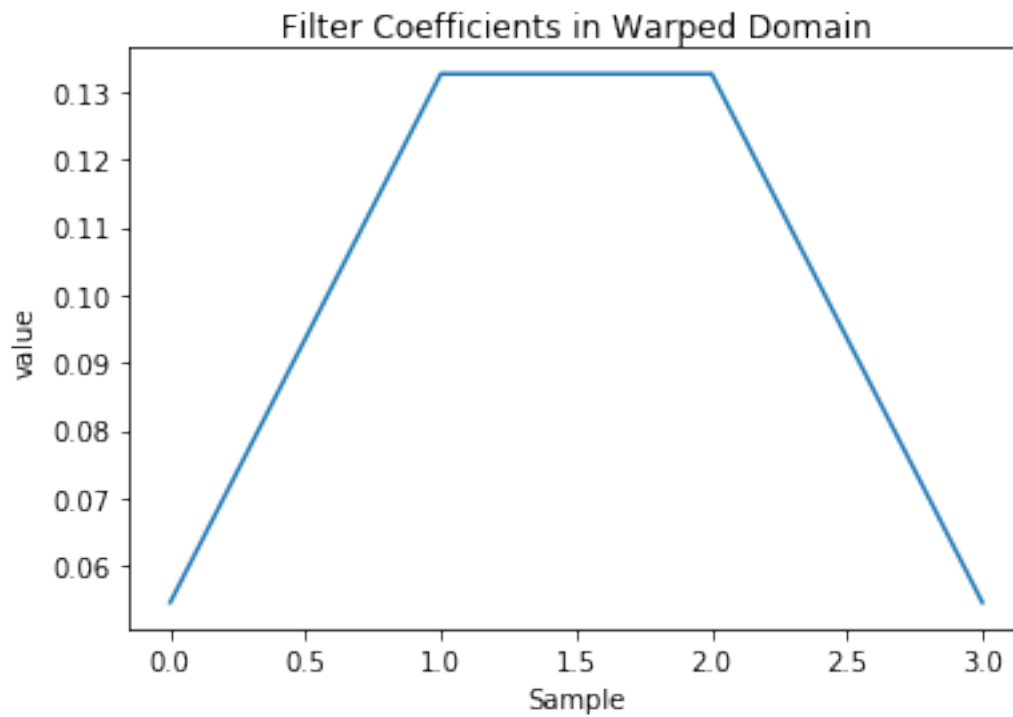
```
In [6]: # fcw = 1.6120; %in radians
# filter design:
# cutoff frequency normalized to nyquist:
fcny=fcw/np.pi
print fcny
```

0.513118416439

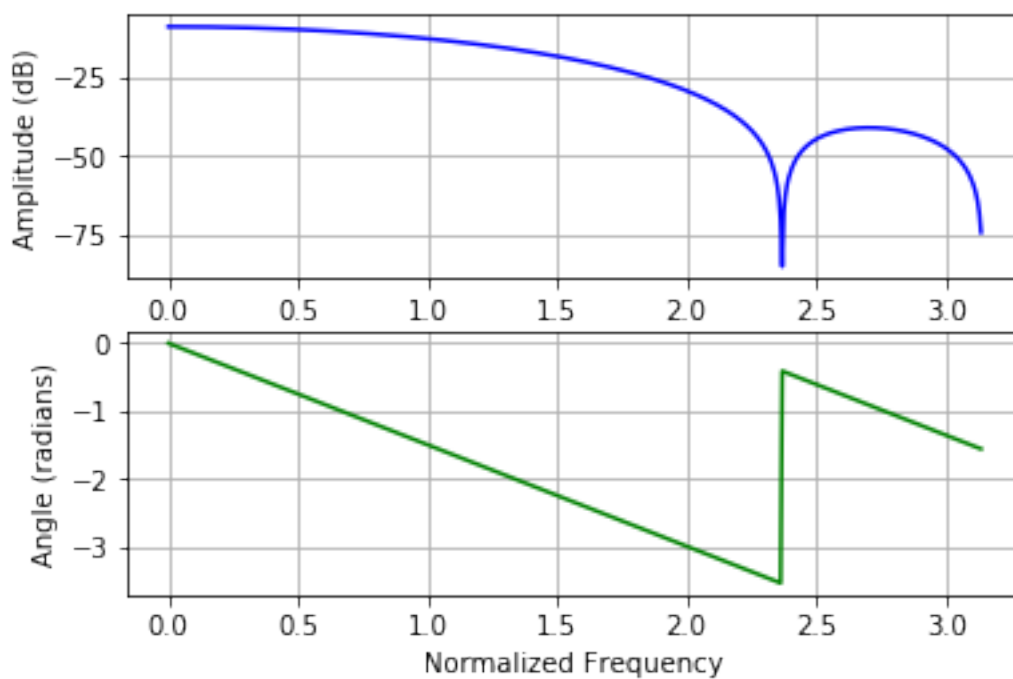
```
In [7]: c = sp.remez(4, [0, fcny/2.0, fcny/2.0+0.1, 0.5], [1, 0],[1, 100])
```

```
In [8]: #The resulting Impulse Response:
plt.plot(c);
plt.xlabel('Sample')
plt.ylabel('value')
plt.title('Filter Coefficients in Warped Domain')
```

Out[8]: <matplotlib.text.Text at 0x124e5850>



In [9]: *#The resulting Frequency response:*
`freqz(c,1)`



```
Out[9]: array([ 3.74442549e-01 +0.00000000e+00j,  
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