Python Example Optimization

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1 Python example for Optimization

1.1 PQMF Design

Take the frequency response of the baseband prototype or window function:

$$H(\omega) = DTFT(h(n))$$

Then we need to find (optimize) a function h(n), such that it fulfills: 1. **Attenuation**: The **stopband attenuation** should be high after the neighboring band to **minimize aliasing**:

$$|H(\omega)| \approx 0 \quad for \quad 1.5 \cdot \frac{\pi}{N} < |\omega| < \pi$$

2. **Unity condition**: Sum of magnitude squared frequency responses of 2 **neighboring bands** should be close to to a **constant**, here $2N^2$, to achieve **near perfect reconstruction**:

$$|H(\omega)|^2 + |H(\omega + \pi/N)|^2 \approx 2 \cdot N^2 \quad for \quad 0 \leqslant \omega < \frac{\pi}{N}$$

To fulfill these requirements, we now have an **optimization problem**. Python has powerful optimization libraries to find a solution. Take a very simple example: **find the minimum** of the function of 2 variables,

$$f(x_1, x_2) = sin(x_1) + cos(x_2)$$

- In Python we write it as a function in file **functionexamp.py:**

1.1.1 functionexamp

In [1]:
```
#functionăexampleăwithăseveralăunknownsă(variables)ăforăoptimization
#GeraldăSchuller,ăNov.ă2016
import numpy as np

def functionexamp(x):
    #x:ăarrayăwithă2ăvariables

    y=np.sin(x[0])+np.cos(x[1])
    return y
```

- Next we use the library scipy.optimize to find a minimum, and use its function "minimize". We save it for instance as **optimizationExample.py**

1.1.2 optimizationExample

```
In [2]: #Optimizationăexample,ăseeăalso:
        #https://docs.scipy.org/doc/scipy-0.18.1/reference/optimize.html
        #GeraldăSchuller,ăNov.ă2016
        #runăităwithă"pythonăoptimizationExample.py"ăinăaăterminaăshell
        #orătypeă"ipython"ăinăaăterminaăshellăandăcopyălinesăbelow:

        import numpy as np
        import scipy.optimize as optimize
        from functionexamp import functionexamp

        #Exampleăforă2ăunknowns,ăargs:ăfunction-name,ăstartingăpoint,ămethod:
        from functionexamp import *
        xmin = optimize.minimize(functionexamp, [-1.0, -3.0], method='CG')
        print xmin
```

```
     fun: -1.0
     jac: array([  7.45058060e-09,   7.45058060e-09])
 message: 'Optimization terminated successfully.'
    nfev: 32
     nit: 3
    njev: 8
  status: 0
 success: True
       x: array([ 0.21460184, -1.78539816])
```

And call it with ** *Python optimizationExample.py* **
Observe: We indeed obtain the minimum at $x_1 = /2, x_2 = $.

1.1.3 PQMF Optimization, Python Example, Optimization Function

1.1.4 optimfuncQMF

```
In [3]: import numpy as np
        import scipy as sp
        import scipy.signal as sig
        import matplotlib.pyplot as plt
```

```
%matplotlib inline

def optimfuncQMF(x):
    """Optimization function for a PQMF Filterbank
    x: coefficients to optimize (first half of prototype h because of symmetry)
    err: resulting total error
    """
    N = 4 #4 subbands
    h = np.append(x, np.flipud(x))
    f, H_im = sig.freqz(h)
    H = np.abs(H_im) #only keeping the real part

    posfreq = np.square(H[0:512/N])

    #Negative frequencies are symmetric around 0:
    negfreq = np.flipud(np.square(H[0:512/N]))

    #Sum of magnitude squared frequency responses should be closed to unity (or N)
    unitycond = np.sum(np.abs(posfreq + negfreq - 2*(N*N)*np.ones(512/N)))/512

    #plt.plot(posfreq+negfreq)

    #High attenuation after the next subband:
    att = np.sum(np.abs(H[1.5*512/N:]))/512

    #Total (weighted) error:
    err = unitycond + 100*att
    return err
```

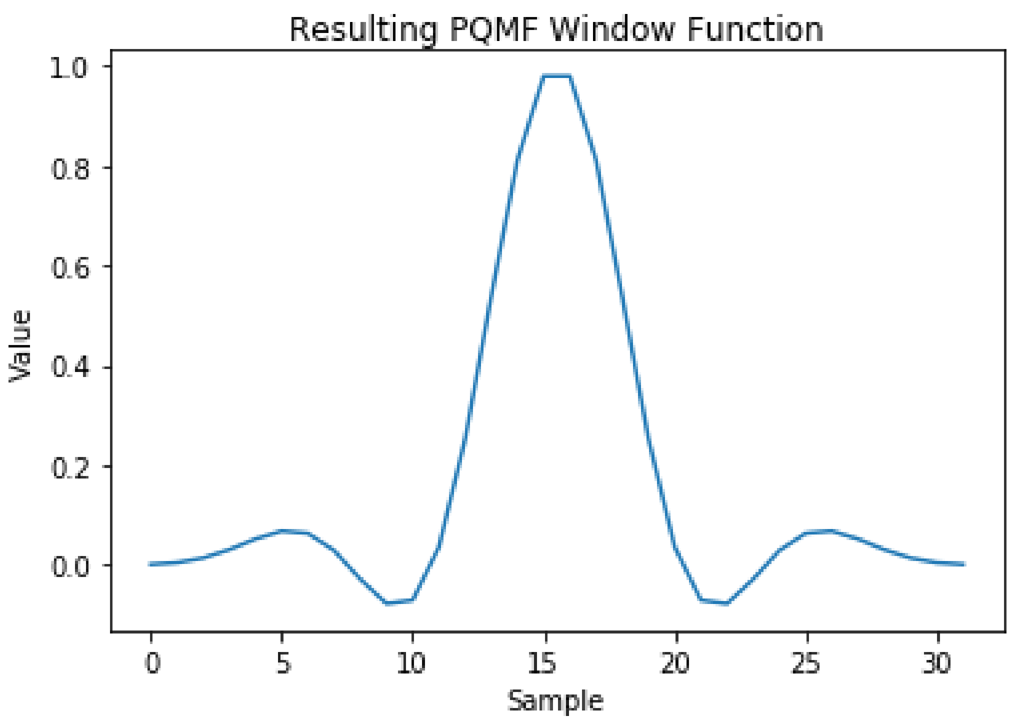
1.1.5 PQMF Optimization, Python Example, Optimizer

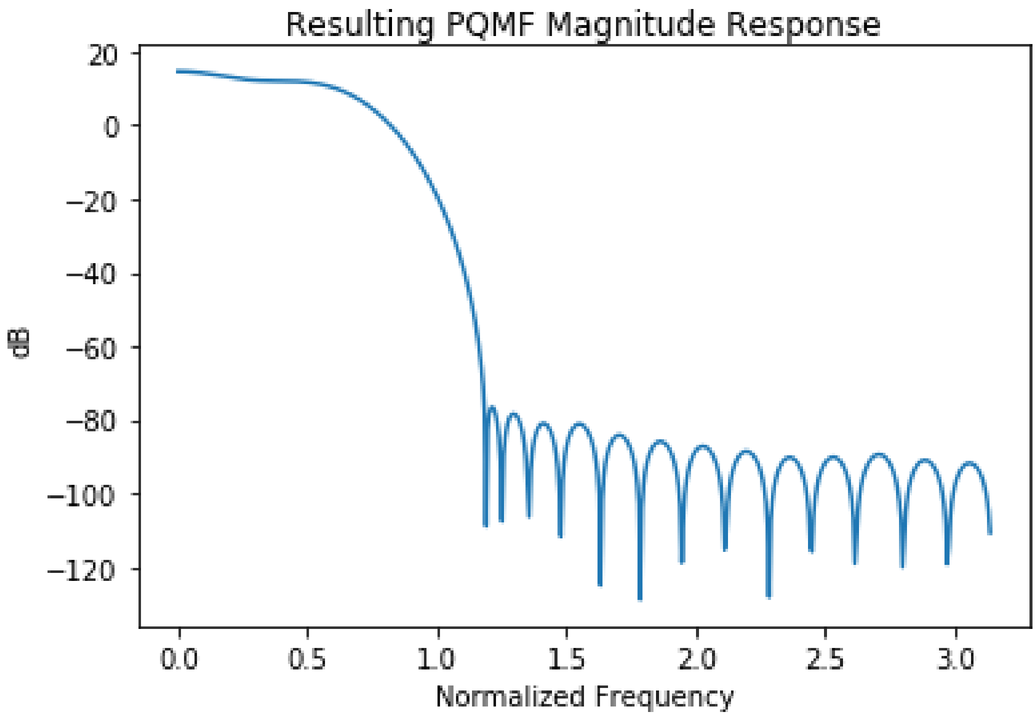
Now we have a function to minimize, and we can use Pythons powerful optimization library to minimize this function:

```
In [4]: import numpy as np
        import matplotlib.pyplot as plt
        import scipy.optimize as opt
        import scipy.signal as sig
        from optimfuncQMF import optimfuncQMF

        #optimize for 16 filter coefficients:
        xmin = opt.minimize(optimfuncQMF, 16*np.ones(16), method='SLSQP')
        xmin = xmin["x"]

        #Restore symmetric upper half of window:
```

```
h = np.concatenate((xmin, np.flipud(xmin)))
plt.plot(h)
plt.title('Resulting PQMF Window Function')
plt.xlabel('Sample')
plt.ylabel('Value')
plt.show()

f, H = sig.freqz(h)
plt.plot(f, 20*np.log10(np.abs(H)))
plt.title('Resulting PQMF Magnitude Response')
plt.xlabel('Normalized Frequency')
plt.ylabel('dB')
plt.show()
```

```
optimfuncQMF.py:26: VisibleDeprecationWarning: using a non-integer number instead of an integer
  att = np.sum(np.abs(H[1.5*512/N:]))/512
```





Observe: We get almost 100 dB Stopband attenuation, much more than with the MDCT!

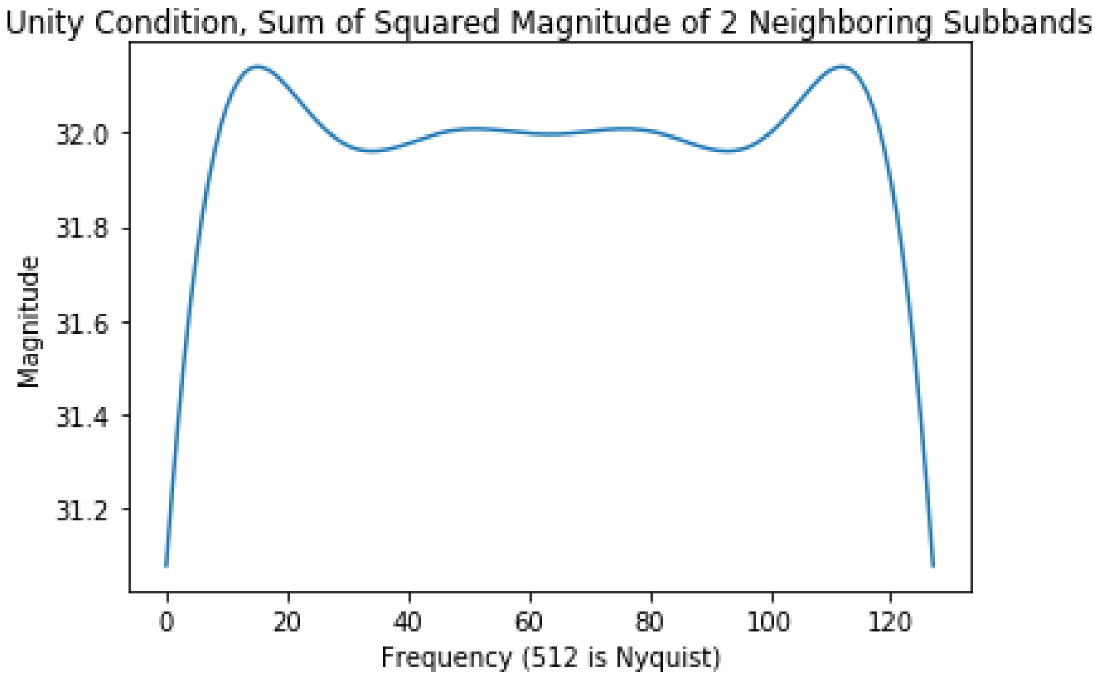
1.1.6 PQMF Optimization, Python Example, Unity Condition

We can test the PQMF Unity condition (slide 19)

$$|H(\omega)^2| + |H(\omega + \pi/N)^2| \approx 2 \cdot N^2 \quad for \quad 0 \leq \omega < \frac{\pi}{N}$$

($2N^2 = 32$), with the following Python code

```
In [5]: N = 4
        f, H_im = sig.freqz(h)
        posfreq = np.square(H[0:512/N])
        negfreq = np.flipud(np.square(H[0:512/N]))
        plt.plot((np.abs(posfreq) + np.abs(negfreq)))
        plt.xlabel('Frequency (512 is Nyquist)')
        plt.ylabel('Magnitude')
        plt.title('Unity Condition, Sum of Squared Magnitude of 2 Neighboring Subbands')
        plt.show()
```



Observe: * We obtain a 4-band filter bank with filter length of 32 taps, hence 8 times overlap. * The stopband attenuation reaches almost 100 dB, almost right after the passband, much more than with the MDCT!