model dat: $x_i = \mu + A \cdot i + w_i$, kde $w_i \sim N(0, \sigma_w^2)$, $i = 1, \dots, N$

log-věrohodnostní funkce:

$$\ell(\mu, A \mid \underline{x}) = -\sum_{i=1}^{N} \frac{(x_i - \mu - A i)^2}{2\sigma^2} - \frac{1}{2}\log(2\pi\sigma^2)$$

derivace $\ell$ pro odvození MLE:

$$\frac{1}{N}\frac{\partial \ell}{\partial \mu} = \frac{1}{N}\sum_{i=1}^{N} \frac{x_i - \hat{\mu} - \hat{A} i}{\sigma^2} = 0 \iff \hat{\mu} = \underbrace{\frac{1}{N}\sum_{i=1}^{N} x_i - \hat{A}\frac{1}{N}\sum_{i=1}^{N} i}$$

- závislé na $\hat{A}$
- nezávislé na $\sigma^2$

$$\frac{1}{N}\frac{\partial \ell}{\partial A} = \frac{1}{N}\sum_{i=1}^{N} i \cdot \frac{x_i - \hat{\mu} - \hat{A} i}{\sigma^2} = 0 \iff \frac{1}{N}\sum_{i=1}^{N} i x_i - \hat{\mu}\frac{1}{N}\sum_{i=1}^{N} i - \hat{A}\frac{1}{N}\sum_{i=1}^{N} i^2 = 0 \quad (*)$$

použijeme: 1) $\frac{1}{N}\sum_{i=1}^{N} i = \frac{N+1}{2}$ 2) $\frac{1}{N}\sum_{i=1}^{N} i^2 = \frac{1}{6}(2N+1)(N+1)$

(*) po dosazení 1), 2) a $\mu$

$$\frac{1}{N}\sum_{i=1}^{N} i x_i - \left(\frac{1}{N}\sum_{i=1}^{N} x_i - \hat{A}\frac{N+1}{2}\right)\frac{N+1}{2} - \hat{A}\frac{1}{6}(2N+1)(N+1) = 0$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(i x_i - \frac{N+1}{2}x_i\right) = \left(\frac{1}{6}(2N+1)(N+1) - (N+1)^2 \cdot \frac{1}{4}\right)\hat{A}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)x_i = \frac{1}{12}(N+1)\left(\underbrace{2(2N+1) - 3(N+1)}_{N-1}\right)\hat{A}$$

$$\boxed{\hat{A} = \frac{12}{N(N^2-1)}\sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)x_i}$$

po dosazení $\hat{A}$ do $\hat{\mu}$: $\boxed{\hat{\mu} = \frac{6}{N(N-1)}\sum_{i=1}^{N}\left(\frac{2N+1}{3} - i\right)x_i}$