$$\left[\begin{array}{l} Z = X+Y = h(X), \\ X = Z - Y = h^{-1}(Z) \Rightarrow \frac{dh^{-1}(z)}{dz} = 1 \end{array}\right] \quad f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$$

SLIDE 23: DALŠÍ PŘÍKLADY TRANSFORMACE HUSTOT

$$f_{XY}(x,y) = f_{XY}(z-y,y) \Rightarrow f_Z(z) = \int_{\mathbb{R}} f_{XY}(z-y,y)\,dy =$$

$$= \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-y)^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\,dy =$$

$$= \frac{1}{2\pi} \int_{\mathbb{R}} e^{-\frac{z^2+y^2-2zy+y^2}{2}}\,dy = \frac{1}{2\pi} \int_{\mathbb{R}} e^{-\frac{2y^2-2yz+\frac{1}{2}z^2+\frac{1}{2}z^2}{2}}\,dy =$$

$$= \frac{1}{2\pi} \int_{\mathbb{R}} e^{-\frac{\left(\sqrt{2}y - \frac{1}{\sqrt{2}}z\right)^2}{2}} e^{-\frac{z^2}{4}}\,dy = \frac{1}{2\pi} e^{-\frac{z^2}{4}} \frac{1}{\sqrt{2}} \int_{\mathbb{R}} e^{-\frac{x^2}{2}} =$$

$$\left| \begin{array}{l} \sqrt{2}y - \frac{1}{\sqrt{2}}z = x \\ \sqrt{2}\,dy = dx \end{array} \right| \qquad = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{2}} e^{-\frac{z^2}{4}}$$

OBRÁCENĚ: $f(x,y) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2}} e^{-\frac{(x+y)^2}{2}}$ NENÍ JOINT DIST!

$f(x) = \int_{\mathbb{R}} e^{-\frac{(x+y)^2}{2}}\,dy = \sqrt{2\pi}$ $\qquad f(x,y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$ JE JOINT PDF