Dokažte, že μ je střední hodnota Normálního rozložení!

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X] = \int_{-\infty}^{+\infty} x \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} x\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \left\{ \begin{array}{l} \text{substituce} \\ y = x-\mu \\ dy = dx \end{array} \right\} =$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} (y+\mu) e^{-\frac{y^2}{2\sigma^2}} dy = \frac{1}{\sqrt{2\pi\sigma^2}} \left(\int_{-\infty}^{+\infty} y\, e^{-\frac{y^2}{2\sigma^2}} dy + \mu \int_{-\infty}^{+\infty} e^{-\frac{y^2}{2\sigma^2}} dy \right) =$$

$$= \underbrace{\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} \underbrace{y}_{\text{lichá fce}}\, \underbrace{e^{\frac{-y^2}{2\sigma^2}}}_{\text{sudá fce}} dy}_{\text{lichá fce} \Rightarrow = 0} + \mu \underbrace{\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} e^{-\frac{y^2}{2\sigma^2}} dy}_{\int_{-\infty}^{+\infty} f(x)dx = 1} = 0 + \mu \cdot 1 = \mu$$