CVIČENÍ 8: PŘ 3

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

$$l = x \log \lambda - \log x! - \lambda$$

$$\frac{\partial l}{\partial \lambda} = \frac{\sum x_i}{\lambda} - 1 \stackrel{!}{=} 0 \Rightarrow \lambda = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\frac{\partial^2 l}{\partial \lambda^2} = -\frac{\sum x_i}{\lambda^2} \Rightarrow var(\hat{\lambda}) \geq \frac{1}{E\left[\frac{\sum x_i}{\lambda^2}\right]} = \frac{\lambda}{N}$$

$$f(x) = \theta e^{-\theta x}$$

$$l = \log \theta - \theta x$$

$$\frac{\partial l}{\partial \theta} = \frac{1}{\theta} - \sum x_i \stackrel{!}{=} 0 \Rightarrow \frac{1}{\theta} = \sum x_i$$

Pareto: $F(x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha} \Rightarrow f(x) = \begin{cases} \frac{\alpha \beta^{\alpha}}{x^{\alpha+1}} & x \geq \beta \\ 0 & x < \beta \end{cases}$

$$l(\alpha) = \log \alpha + \alpha \log \beta - (\alpha+1) \log x$$

$$\frac{\partial l}{\partial \alpha} = N\frac{1}{\alpha} + N \log \beta - \log \sum x_i = 0 \Leftrightarrow \hat{\alpha} = \frac{N}{\sum \log \frac{x_i}{\beta}}$$