CVIČENÍ 5:
PŘ 1 (VEKTOROVĚ)

$X \sim N(\mu, \sigma^2) \Rightarrow f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\ell = \log f(x) = -\frac{1}{2}\log 2\pi\sigma^2 - \frac{(x-\mu)^2}{2\sigma^2}$

$\frac{\partial \ell}{\partial \mu} = \frac{x-\mu}{\sigma^2}$ ① $\frac{\partial^2 \ell}{\partial \mu^2} = -\frac{1}{\sigma^2}$ ② $\frac{\partial^2 \ell}{\partial \mu \partial \sigma^2} = -\frac{x-\mu}{(\sigma^2)^2}$

$\frac{\partial \ell}{\partial \sigma^2} = -\frac{1}{2}\frac{1}{\sigma^2} + \frac{(x-\mu)^2}{2(\sigma^2)^2}$ ③ $\frac{\partial^2 \ell}{\partial \sigma^2 \partial \mu} = -\frac{x-\mu}{(\sigma^2)^2}$ ④ $\frac{\partial^2 \ell}{\partial (\sigma^2)^2} = \frac{1}{2}\frac{1}{(\sigma^2)^2} - \frac{(x-\mu)^2}{(\sigma^2)^3}$

$\Rightarrow FIM = \begin{pmatrix} E\left[\frac{1}{\sigma^2}\right] & E\left[\frac{x-\mu}{(\sigma^2)^2}\right] \\ E\left[\frac{x-\mu}{(\sigma^2)^2}\right] & E\left[\frac{1}{2}\frac{1}{(\sigma^2)^2} + \frac{(x-\mu)^2}{(\sigma^2)^3}\right] \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2}\frac{1}{(\sigma^2)^2} \end{pmatrix}$

$X, Y \sim N(0,1)$

$f_{X,Y}(x,y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$

$z_1 = X - Y = h_1(x,y)$

$z_2 = X + Y = h_2(x,y)$

$\oplus \; X = \frac{z_1+z_2}{2} = h_1^{-1}(z_1,z_2)$

$\ominus \; Y = \frac{z_2-z_1}{2} = h_2^{-1}(z_1,z_2)$

$J = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$f(z_1,z_2) = \frac{1}{2\pi} e^{-\frac{\left(\frac{z_1+z_2}{2}\right)^2 + \left(\frac{z_2-z_1}{2}\right)^2}{2}} \cdot \frac{1}{2} = \frac{1}{4\pi} e^{-\frac{z_1^2+z_2^2+2z_1z_2+z_2^2+z_1^2-2z_1z_2}{8}}$

$= \frac{1}{4\pi} e^{-\frac{z_1^2+z_2^2}{4}} \Rightarrow f_{z_1}(z_1) = \frac{1}{\sqrt{4\pi}} e^{-\frac{z_1^2}{4}}$

SLIDE 36: TRANSFORMACE HUSTOTY (VEKTOROVĚ)