

$$① \quad f(x) = \theta e^{-x\theta}$$

SLIDE 47: MLE VLASTNOSTI

$$l(\theta|x) = \sum_{i=1}^N \log \theta - x_i \theta$$

$$\frac{\partial l}{\partial \theta} = N \frac{1}{\theta} - \sum_{i=1}^N x_i \stackrel{!}{=} 0 \Rightarrow \frac{1}{\theta} = \frac{1}{N} \sum_{i=1}^N x_i \Rightarrow \theta = \frac{N}{\sum_{i=1}^N x_i}$$

$$② \quad f(x) = \frac{\beta}{2\alpha \Gamma(\frac{1}{\beta})} e^{-\left(\frac{x-\mu}{\alpha}\right)^\beta}$$

SLIDE 47: MLE VLASTNOSTI

$$l(\mu|x) = \log \beta - \log 2\alpha \Gamma(\frac{1}{\beta}) - \sum_{i=1}^N \left(\frac{x_i - \mu}{\alpha}\right)^\beta$$

$$\frac{\partial l}{\partial \mu} = 0 - 0 + \sum_{i=1}^N \frac{\beta}{\alpha} \left(\frac{x_i - \mu}{\alpha}\right)^{\beta-1} \stackrel{!}{=} 0 \Leftrightarrow \sum_{i=1}^N (x_i - \mu)^{\beta-1} = 0$$

$$③ \quad f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

CVIČENÍ 7: PŘ 1

$$l(\mu|x) = -\frac{1}{2} \log 2\pi - \frac{(x-\mu)^2}{2\sigma^2}$$

$$\frac{\partial l}{\partial \mu} = \sum_{i=1}^N \frac{x_i - \mu}{\sigma^2} \stackrel{!}{=} 0 \Leftrightarrow \frac{1}{N} \sum_{i=1}^N x_i = \mu$$

$$\frac{\partial l}{\partial \sigma^2} = -\frac{1}{2\sigma^2} + \frac{(x-\mu)^2}{2(\sigma^2)^2} \stackrel{!}{=} 0 \Leftrightarrow \frac{1}{2(\sigma^2)^2} \sigma^2 = (x-\mu)^2 \Rightarrow \frac{1}{\sigma^2} = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$