2. 1. $X \sim N(\mu, \sigma^2) \Rightarrow f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\boxed{\text{CVIČENÍ 2}}$

$h(x) = Y = \frac{X-\mu}{\sigma}$

$h^{-1}(y) = X = \sigma Y + \mu$

$$f_Y(y) = f_X(\sigma Y+\mu)\cdot\left|\frac{d(\sigma Y+\mu)}{dy}\right| =$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}}\cdot|\sigma| = \frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}$$

3. 2 $\hat\sigma^2 = \frac{1}{n}\sum(x_i-\mu)^2$

$\boxed{\text{CVIČENÍ 3}}$

$$E[\hat\sigma^2] = \frac1n\sum E[(x_i-\mu)^2] = \cancel{\frac1n \sum E[(x_i^2 - 2x_i\mu}$$

$$= \frac1n\sum \underbrace{E[(X_i - E[X_i])^2]}_{\sigma^2} = \sigma^2 \quad \checkmark$$

$$\hat\sigma^2 = \frac1n\sum(X_i-\bar X_n)^2$$

$$E[\hat\sigma^2] = \frac1n\sum_i E\left[\left(X_i - \frac1n\sum_j X_j\right)^2\right] = \frac1n\sum_i E\left[\left(\frac{nX_i - \sum X_j}{n}\right)^2\right]$$

$$\left(= \frac{1}{n^3}\sum_i E\left[\left(\sum_j (X_i - X_j)\right)^2\right]\right)$$

$$= \frac{1}{n^3}\sum_i E\left[n^2X_i^2 + \left(\sum X_j\right)^2 - 2nX_i\sum X_j\right]$$

$$= \frac{1}{n^3}\sum_i n^2\sigma^2 + n^2\sigma^2 - 2n\sigma^2$$

$$= \frac{n-1}{n}\sigma^2$$

SLOŽITÉ

$$E\left[\frac1n\sum(X_i-\bar X_n)^2\right] =$$

$$= E\left[\frac1n\sum[(X_i-\mu)-(\bar X_n-\mu)]^2\right] =$$

$$= E\left[\frac1n\sum (x_i-\mu)^2 + (\bar X_n-\mu)^2 - \underbrace{2(x_i-\mu)(\bar X_n-\mu)}\right]$$

$$= \frac1n\left(Var(X_i)\cdot n + n\cdot Var(\bar X_n)\right) \qquad E[x_i\bar x_n + \mu^2 - x_i\mu - \bar x_n\mu] = 0$$

$$\hookrightarrow \frac1n(\sigma^2 n +$$