$$Bmse(\hat{A}) = \iint p(A|x)\,p(x)\,(\hat{A}-A)^2\,dx\,dA =$$

$$= \int \Big[\underbrace{\int (A - E[A|x])^2\, p(A|x)\, dA}_{var(A|x)} \Big] p(x)\,dx$$

SLIDE 61: VOLBA APRIORNÍ PDF (2c)

$$= \int var(A|x)\,p(x)\,dx = var(A|x) = \frac{1}{\frac{N}{\sigma^2} + \frac{1}{\sigma_A^2}}$$

$$= \frac{\sigma^2}{N}\left(\frac{1}{1 + \frac{\sigma^2}{\sigma_A^2}\frac{1}{N}}\right) \leq \frac{\sigma^2}{N}$$

$p(x_i|\theta) = \theta e^{-\theta x_i}$

$p(\theta) = \lambda e^{-\lambda\theta}$

$$p(\theta|x) = \frac{p(x|\theta)\,p(\theta)}{c} = \frac{\theta^N \lambda\, e^{-\theta\sum x_i - \lambda\theta}}{c}$$

a) MAP: $l = \ln p(\theta|x) = \ln(p(x|\theta)) + \ln p(\theta) = N\log\theta - \theta\sum x_i + \log\lambda - \lambda\theta$ konst.

$$\frac{\partial l}{\partial\theta} = \frac{N}{\theta} - \sum x_i - \lambda \stackrel{!}{=} 0 \Rightarrow \hat{\theta} = \frac{N}{\lambda + \sum_{i=1}^{N} x_i}$$

b) MMSE: $$E[\theta|x] = \frac{\int_0^\infty \theta^{N+1} e^{-(\lambda+\sum x_i)\theta}\,d\theta}{\int_0^\infty \theta^N e^{-(\lambda+\sum x_i)\theta}\,d\theta} = \frac{N+1}{\lambda + \sum_{i=1}^{N} x_i}$$

SLIDE 62: METODY BAYES. ODHADŮ

$$\int_0^\infty \theta^{N+1} e^{-A\theta}\,d\theta = \left| \begin{matrix} u = \theta^{N+1} & u' = (N+1)\theta^N \\ v = -\frac{e^{-A\theta}}{A} & v' = e^{-A\theta} \end{matrix} \right| = \left[-\frac{\theta^{N+1} e^{-A\theta}}{A} \right]_0^\infty + \frac{N+1}{A}\int_0^\infty \theta^N e^{-A\theta}\,d\theta$$