$$\hat{\sigma}^2 = \frac{1}{n}\sum(X_i - \bar{X}_n)^2 \Rightarrow E[\hat{\sigma}^2] = E\left[\frac{1}{n}\sum_i (X_i - \bar{X}_n)^2\right]$$

CVIČENÍ 3
Př. 2c

$$= \frac{1}{n} E\left[\sum_i \left((X_i - \mu) - (\bar{X}_n - \mu)\right)^2\right] = \frac{1}{n} E\left[\sum_i \left((X_i - \mu)^2 + (\bar{X}_n - \mu)^2 - 2(X_i - \mu)(\bar{X}_n - \mu)\right)\right]$$

a) $E[\Sigma(X_i - \mu)^2] = \sum E[(X_i - \mu)^2] = n\sigma^2$

b) $E[\sum_i (\bar{X}_n - \mu)^2] = n E[(\bar{X}_n - \mu)^2] = n \cdot \frac{\sigma^2}{n} = \sigma^2$

c) $E[\sum_i (X_i\bar{X}_n - \mu\bar{X}_n - \mu X_i + \mu^2)] = E[n\bar{X}_n^2 - 2n\mu\bar{X}_n + \mu^2 \cdot n]$

$= n E[\bar{X}_n^2 - 2\mu\bar{X}_n + \mu^2] = n E[(\bar{X}_n - \mu)^2] = \sigma^2$

$\Rightarrow E[\hat{\sigma}^2] = \frac{1}{n}(n\sigma^2 + \sigma^2 - 2\sigma^2) = \frac{n-1}{n}\sigma^2$ X

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(3) $\mathrm{Var}(\hat{A}) = \mathrm{Var}\left(\frac{1}{n}\Sigma X_i\right) = \frac{1}{n^2}\sum \mathrm{var}(X_i) = \frac{\sigma^2}{n}$

CVIČENÍ 4
PŘ. 2

$\mathrm{Var}(\check{A}) = \mathrm{Var}\left(\frac{1}{n+2}(2X_1 + 2X_n + \sum_{i=2}^{n-1} X_i)\right) = \frac{1}{(n+2)^2}\sum$

$= \frac{1}{(n+2)^2}\left[4\sigma^2 + 4\sigma^2 + (n-2)\sigma^2\right]$ $\left(2\mathrm{var}X_1 + 2\mathrm{var}X_n + \sum_{i=2}^{n-1}\mathrm{var}X_i\right)$

$= \frac{n+2+4}{(n+2)^2}\sigma^2 = \frac{n+6}{(n+2)^2}\sigma^2$

PRO $n > 2$

$\frac{n+6}{(n+2)^2} > \frac{1}{n} \Leftarrow n^2 + 6n > n^2 + 4n + 4$