$x[n] = s[n] + w[n]$ $\quad w[n] \sim N(0, \sigma_n^2)$ $\quad \sigma_n^2 = \left(\frac{1}{2}\right)^n$

$s[n] = \frac{1}{2} s[n-1] + u[n]$ $\quad u[n] \sim N(0, \sigma_u^2)$, $\sigma_u^2 = 2$

$s[-1] \sim N(0,1)$

$a = \frac{1}{2}$

PRED: $\hat{s}[n|n-1] = \frac{1}{2}\hat{s}[n-1|n-1] \Rightarrow \hat{s}[0|-1] = \frac{1}{2}\hat{s}[-1|-1] = E[s[-1]] = 0$

min PRED MSE: $\Pi[n|n-1] = \frac{1}{4}\Pi[n-1|n-1] + \sigma_u^2 \Rightarrow M[0|-1] = \frac{1}{4}\Pi[-1|-1] + 2 =$

$= \frac{1}{4} E[(s[-1] - \hat{s}[-1])^2] + 2 =$

$= \frac{1}{4}\,\mathrm{var}[s[-1]] + 2 = \frac{1}{4} + 2 = \frac{9}{4}$

Gain: $K[n] = \frac{\Pi[n|n-1]}{\sigma_n^2 + \Pi[n|n-1]} \Rightarrow K[0] = \frac{\frac{9}{4}}{\left(\frac{1}{2}\right)^0 + \frac{9}{4}} = \frac{9}{4} \cdot \frac{4}{13} = \frac{9}{13}$

KOREKCE: $\hat{s}[n|n] = \hat{s}[n|n-1] + K[n]\left(x[n] - \hat{s}[n|n-1]\right) \Rightarrow$

$\hat{s}[0|0] = 0 + \frac{9}{13}\left(x[0] - 0\right) = \frac{9}{13} x[0]$

min MSE: $\Pi[n|n] = (1 - K[n])\,\Pi[n|n-1] = \frac{4}{13} \cdot \frac{9}{4} = \frac{9}{13}$

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