Teaching Dimension

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Definition

$\mathrm{TD}(C) = \max_{c \in C} \left(\min_{\tau \in T(c)} |\tau|\right)$

Problem 2

By problem setting, $C = \{c_{ab} : 1 \le a \le b \le n\}$, we could calculate the cardinality $|C| = \sum_{a=1}^{n-1} \sum_{b=a}^{n} 1 = n^2 - n$.

For an teaching instance c_{ab}, we can see we need at least $T(c_{ab}) = \{a, b\}$ to specify it. By this specification, with the setting $x_i = i$ for all $i \in [1, n] \cup \mathbb{Z}$, we can interpret a teaching set $T(c_{ab})$ as the rule $\mathrm{sgn}(x_i) = \begin{cases} + & \text{if } i \in [a, b] \\ - & \text{otherwise} \end{cases}$. The max cardinality $\max |T(c_{ab})| = 2$ for all a, b. Therefore, $\mathrm{TD}(C) = 2$. $\square$

In case that $C = \{c_{ab}\}$ with a, b be constants, as there is only one teaching instance, we do not need to distinguish it from other instances, so $T(c_{ab}) = 0$ and $\mathrm{TD}(C) = 0$.

The $T(c_{ab})$ above is the maximum situation for the original C, because we could use only $a = n - 1$ to specify $c_{n-1,n}$ and only $b = 2$ to specify $c_{1,2}$, in which case $T(c_{ab}) = 1$.

Problem 3

By problem setting, $C = \{c_{ab}, \bar{c}_{ab} : 1 \le a \le b \le n\}$. For an teaching instance c_{ab}, we can see we need at least $T(c_{ab}) = \{a, b, p\}$ where $p = +$ to specify it, because we need to distinct the complement case: for $\bar{c}_{ab}$, $T(\bar{c}_{ab}) = \{a, b, p\}$ where $p = -$. By this specification, with the setting $x_i = i$ for all $i \in [1, n] \cup \mathbb{Z}$ and the notation c'_{abp} defined by $c'_{ab+} = c_{ab}$ and $c'_{ab-} = \bar{c}_{ab}$, we can interpret a teaching set $T(c'_{abp})$ as the rule $\mathrm{sgn}(x_i) = \begin{cases} + \cdot p & \text{if } i \in [a, b] \\ - \cdot p & \text{if } i \in [a, b] \end{cases}$. The max cardinality $\max |T(c_{ab})| = 3$ for all a, b. Therefore, $\mathrm{TD}(C) = 3$. $\square$

If we remove one case (either c_{ab} or $\bar{c}_{ab}$ from C), we do not need p to distinguish instances, so we reduce this setting to Problem 2.