The Determination of Exchange Rates

Exchange rates in the long run are determined primarily by the relative cost of goods and services. This is modeled using relative purchasing power parity.

Exchange rates in the short run are determined primarily by the movement of capital between countries. This is modeled using uncovered interest parity. To understand uncovered interest rate parity, we must first understand the determination of the forward exchange rate.

The Forward Exchange Rate

A forward contract is an agreement between two parties to exchange currencies at a specified date in the future at a specified exchange rate. The agreed upon exchange rate is known as the forward rate. The forward rate must be agreed upon at the time the forward contract is signed. It may differ from the spot rate at the time the contract is signed. The forward exchange rate is denoted as $F_{\$/€}$.

Example

A U.S. firm is planning to purchase machinery from a German firm on May 1, 2025. It will need to convert dollars to Euros on that date. It signs a forward contract to convert $1,000,000 into Euros at that date at a rate of $1.08 per Euro. This is slightly higher than the today's spot rate, which is $1.07 per Euro. Why not wait until May 1 to exchange currency? The dollar could depreciate against the Euro and the spot rate next May could be higher than $1.08. Signing a forward contract protects the firm from exchange rate fluctuations.

How is the Forward Rate Determined?

The forward rate is determined by interest rate differentials between the two countries.

Assume that U.S. investors can invest in either the U.S. or Europe. A savings account in the U.S. pays an interest rate of $i_{\$}$ while a savings account in Europe pays $i_{€}$, where i is expressed as a decimal.

1. Annual Return from Investing in the U.S.
   An investor with one dollar will have have $(1+i_{\$})$ dollars in one year.

2. Annual Return from Investing in Europe using the Forward Exchange Rate

An American investor will first have to change dollars into Euros. She does this using the current spot rate. She will have

$(1/ E_{\$/€})€$

The investor will next put the Euros in a savings account in Europe. In one year she will have

$[(1+ i_{€}) / E_{\$/€}]€$

She will then have to change her Euros back into dollars. Her rate of return will depend on what has happened to the exchange rate in the intervening year. She can protect herself from exchange rate fluctuations by using the forward rate. If uses the forward rate, she will have

$F_{\$/€} [(1+ i_{€}) / E_{\$/€}]\$$

$F_{\$/€}\,[(1+ i_{€}) / E_{\$/€}]\$$

Note that if F and E are equal, the investor will only have an interest return.

If F > E, then the future price of the Euro is greater than its current price. Therefore, there is a capital gain to holding Euros. The rate of return to holding Euros is greater than the interest return on Euros.

Arbitrage will force the two rates of return (investing in the U.S. and investing in Europe) to be equal. If the two rates of return are equal then

$$\underbrace{(1+i_{\$})}_{\text{U.S. return}} = \underbrace{(1+ i_{€})(F_{\$/€}/ E_{\$/€})}_{\text{European return}}$$

This relationship is known as covered interest parity. (CIP)

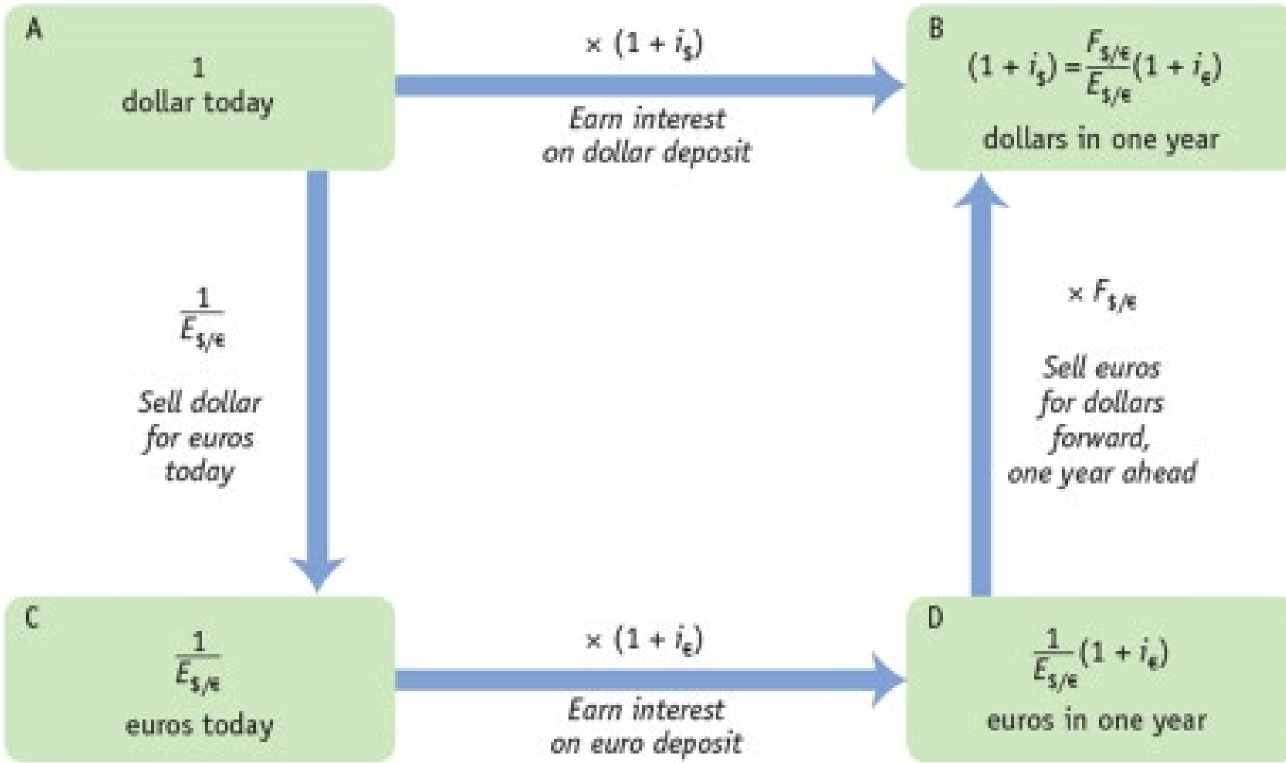

Figure 13.8
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Calculating the Forward Exchange Rate

Covered interest parity can be used to solve for the forward rate:

$F_{\$/€} = E_{\$/€} * (1+i_{\$}) / (1+ i_{€})$

If the home country interest rate is greater than the foreign country interest rate, the forward rate will be greater than the spot rate. Put differently, the country with the higher interest rate will have a depreciating currency.

Numerical Example

$i_\$ = .03$
$i_€ = .01$
$E_{\$/€} = 1$

$F_{\$/€} = E_{\$/€} * (1+i_\$) / (1+ i_€)$
$F_{\$/€} = 1*1.03/1.01 = 1.0198$

This means that the future price of a Euro is 1.0198, or 1.98% higher than today’s price.

CIP Approximation

$$i_{\$} = i_{€} + \underbrace{(F_{\$/€} - E_{\$/€})/ E_{\$/€}}_{\text{Forward Premium}}$$

or

$$F_{\$/€} = E_{\$/€}(1 + i_{\$} - i_{€})$$

Arbitrage using the Forward Rate

Let $E^e_{\$/€}$ = the expected spot rate. If the expected spot rate is different from the forward rate, investors can make money through arbitrage.

If $F_{\$/€} < E^e_{\$/€}$ Buy euros at the forward rate and sell euros at the expected exchange rate

If $F_{\$/€} > E^e_{\$/€}$ Sell euros at the forward rate and buy euros at the expected exchange rate

Example

Let $F_{\$/€} = 1$ and $E^e_{\$/€} = .98$.
Sign a forward contract to exchange 1,000,000 € into $1,000,000 (sell euros for $ in the future). If your expectation is correct, one year from the spot rate, $E_{\$/€}$ will equal .98. You can buy 1,000,000 € for $980,000. Then sell the 1,000,000 € for $1,000,000 using the forward contract that you previously signed. This gives you a profit of $20,000.

If $E^e_{\$/€} > F_{\$/€}$, you would sign a forward contract to trade $ for € in the future (buy € in the future).

If investors are risk neutral, then arbitrage will lead to
$E^e_{\$/€} = F_{\$/€}$
The forward rate will reflect investor expectations about the future spot rate.

Uncovered Interest Rate Parity (UIP)

If the forward rate reflects expectations of the exchange rate, then $E^e_{\$/€} = F_{\$/€}$ and the zero profit arbitrage condition becomes

$$E^e_{\$/€} = E_{\$/€} * (1+i_\$) / (1+ i_€)$$

Rewriting

$$(1+i_\$) = (1+ i_€)(E^e_{\$/€} / E_{\$/€})$$

The domestic return equals the expected foreign return. This is known as uncovered interest parity (UIP).

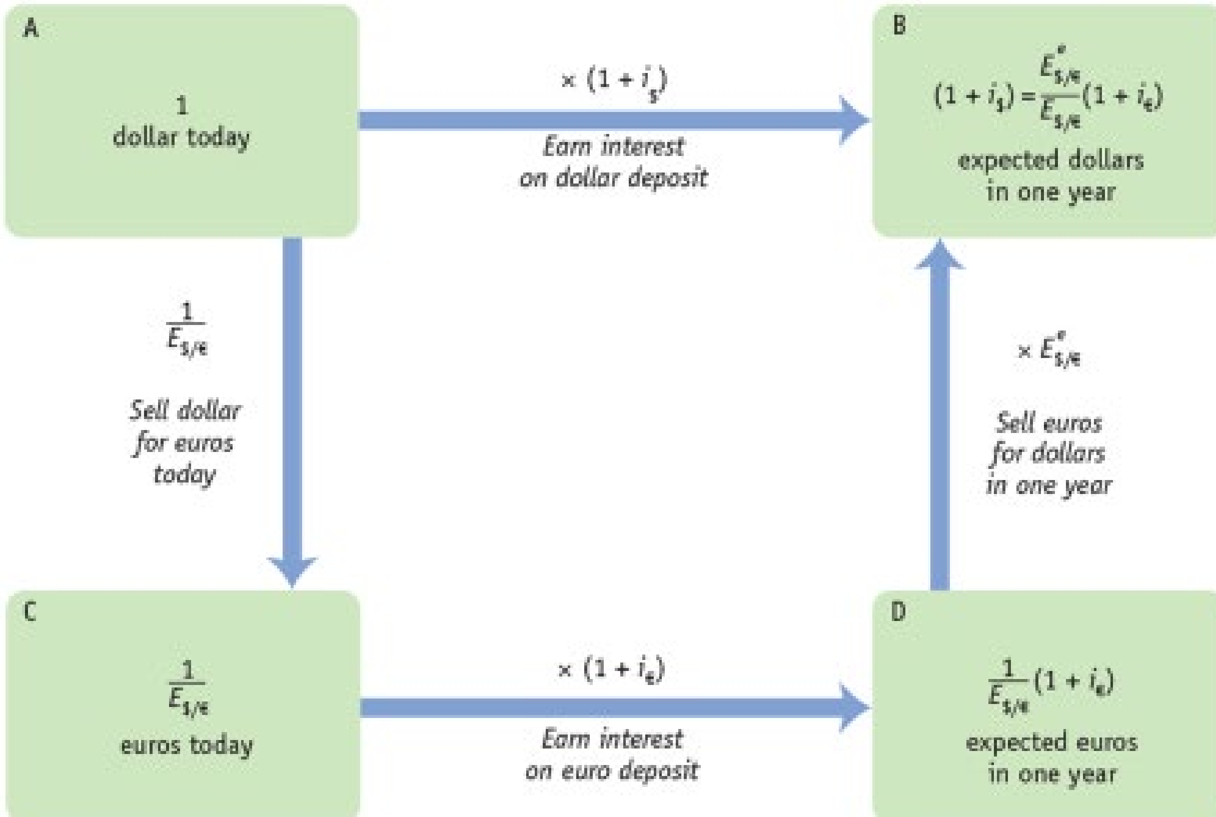

Figure 13.10
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UIP Approximation

$$i_{\$} = i_{€} + \underbrace{(E^e_{\$/€} - E_{\$/€})/ E_{\$/€}}_{\text{Expected appreciation of Euro}}$$

If the interest rates are expressed as percentages, this can also be written as

$$i_{\$} = i_{€} + \underbrace{\%\Delta E^e_{\$/€}}_{\text{expected \% appreciation of Euro}}$$

where $\%\Delta E^e_{\$/€} = 100*(E^e_{\$/€} - E_{\$/€})/ E_{\$/€}$

The right hand side of the equation is the foreign return. This has two parts: an interest return and an expected capital gain or loss on holding foreign currency.

Arbitrage states that the domestic return must equal the foreign return. If the foreign interest rate is lower than the domestic interest rate, it must make up that difference with an exchange rate appreciation in order to equalize the two rates of return.

$$\%\Delta E^e_{\$/€} = i_{\$} - i_{€}$$

If
$i_{\$} > i_{€}$, € expected to appreciate
$i_{\$} < i_{€}$, € expected to depreciate

The country with the lower interest rate is expected to have an appreciating currency.

Numerical Examples of UIP

$i_{\$}$	$i_{€}$	$\%\Delta E^e_{\$/€}$
3	1	?
?	2	2
4	?	-1

Answers

$i_{\$}$	$i_{€}$	$\%\Delta E^e_{\$/€}$
3	1	**2**
4	2	2
4	**5**	-1

Calculating the Spot and Expected Exchange Rates

Once the expected growth rate of the exchange rate is known, the expected exchange rate or the spot rate can be calculated.

The expected exchange rate is just equal to the current exchange rate times one plus the expected growth rate in the exchange rate:

$$E^e_{\$/€} = E_{\$/€} * (1 + \%\Delta E^e_{\$/€}/100)$$

Under interest rate parity, when interest rates are expressed as decimals, this equals

$$E^e_{\$/€} = E_{\$/€} * (1 + i_\$ - i_€)$$

This states that the expected future exchange rate equals the spot rate times one plus the interest rate differential. In words,

Future E = Current E * (1 + interest differential)

This equation of course could also be used to solve for the spot rate:

$$\underbrace{E_{\$/€}}_{\text{Spot}} = \underbrace{E^e_{\$/€}}_{\text{Future}} / \underbrace{(1 + i_\$ - i_€)}_{\text{(1 + interest diffential)}}$$

UIP and Relative PPP Compared

Relative PPP makes a prediction about the actual exchange rate in the long run.
UIP makes a prediction about the expected exchange rate in the short run.
The relative PPP prediction is based on inflation differentials.
The UIP prediction is based on interest rate differentials.

PPP

$\%\Delta\ E_{\$/FGN} = \pi_{US} - \pi_{FGN}$

UIP

$\%\Delta E^{e}_{\$/€} = i_{\$} - i_{€}$

Numerical Examples of UIP

$i_{\$}$	$i_{€}$	$\Delta E^e_{\$/€}/ E_{\$/€}$	$E^e_{\$/€}$	$E_{\$/€}$
4	1	?	?	2
3	2	?	3	?

Solutions

$i_{\$}$	$i_{€}$	$\Delta E^e_{\$/€}/ E_{\$/€}$	$E^e_{\$/€}$	$E_{\$/€}$
4	1	**3**	?	2

The exchange rate is expected to grow by 3%. Thus $E^e_{\$/€}$ is 3% higher than E:
$E^e_{\$/€} = (1 + .03)*2 = 2.06$

$i_{\$}$	$i_{€}$	$\Delta E^e_{\$/€}/ E_{\$/€}$	$E^e_{\$/€}$	$E_{\$/€}$
3	2	**1**	3	?

Again the exchange rate is expected to grow by 1% so that $E^e_{\$/€}$ is 1% higher than E, or equivalently, E is 1% lower than $E^e_{\$/€}$:
$E = E^e_{\$/€} / 1.01 = 3/1.01 = 2.97$

How does UIP Affect the Spot Rate?

UIP states that the domestic return equals the expected foreign return.

$i_{\$} = i_{€} + \%\Delta E^{e}_{\$/€}$

But which of these variables adjust so that UIP holds?
Interest rates are determined by central banks and expectations are formed by market participants.
The spot exchange rate is therefore the variable that adjusts so that uncovered interest parity holds.

Graphing the UIP Relationship

Hold constant $i_\$$, $i_€$ and $E^e_{\$/€}$
Then the foreign return will be inversely related to the spot rate ($E_{\$/€}$). A lower spot rate means a low purchase price of the foreign currency, which will result in a positive capital gain when the foreign currency is sold in the future.

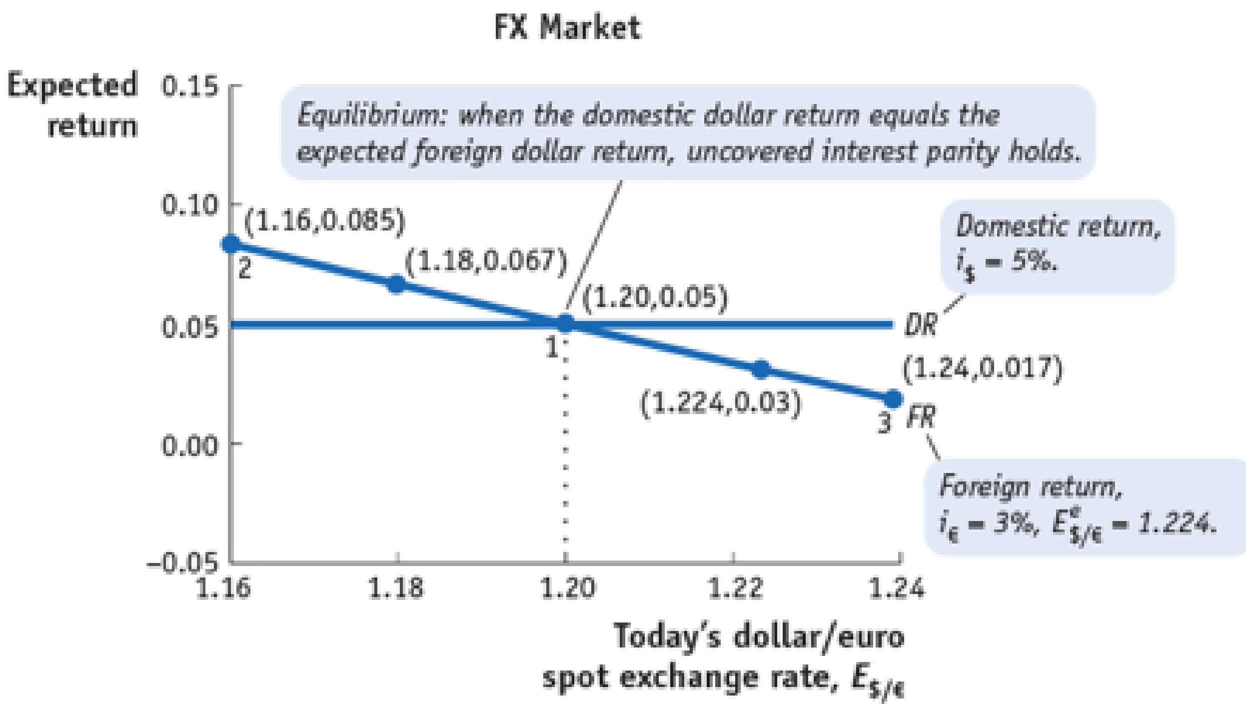

Figure 15.2
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1 Exchange Rates and Interest Rates in the Short Run: UIP and FX Market Equilibrium

Equilibrium in the FX Market: An Example

TABLE 12-1

Interest Rates, Exchange Rates, Expected Returns, and FX Market Equilibrium: A Numerical Example The foreign exchange (FX) market is in equilibrium when the domestic and foreign returns are equal. In this example, the dollar interest rate is 5%, the euro interest rate is 3%, and the expected future exchange rate (one year ahead) is = 1.224 \$/€. The equilibrium is highlighted in bold type.

	(1)	(2)	(3)	(4)	(5)	(6) = (2) + (5)
	Interest Rate on Dollar Deposits (annual)	Interest Rate on Euro Deposits (annual)	Spot Exchange Rate (today)	Expected Future Exchange Rate (in 1 year)	Expected Euro Appreciation against Dollar (in 1 year)	Expected Dollar Return on Euro Deposits (annual)
	Domestic Return (\$)					Foreign Expected Return (\$)
	$i_\$$	$i_€$	$E_{\$/€}$	$E^e_{\$/€}$	$\frac{E^e_{\$/€} - E_{\$/€}}{E_{\$/€}}$	$i_€ + \frac{E^e_{\$/€} - E_{\$/€}}{E_{\$/€}}$
	0.05	0.03	1.16	1.224	0.0552	0.0852
	0.05	0.03	1.18	1.224	0.0373	0.0673
Market equilibrium	0.05	0.03	1.20	1.224	0.02	0.05
	0.05	0.03	1.22	1.224	0.0033	0.0333
	0.05	0.03	1.24	1.224	−0.0129	0.0171

Shifts in the Foreign Exchange Market

1. An increase in the domestic interest rate

The DR line shifts upwards by the increase in the interest rate. At the initial spot rate, the domestic return is now greater than the foreign return. There will be a greater demand for the domestic currency, pushing the price of $ up and Euros down ($E_{\$/€}$ fall). However a reduction in $E_{\$/€}$ will increase the foreign return. The exchange rate will fall until the domestic and foreign returns are equal.

2. An increase in the foreign interest rate

The FR line shifts upwards by the increase in the interest rate. At the initial spot rate, the foreign return is now greater than the domestic return. There will be a greater demand for the foreign currency, pushing the price of Euros up ($E_{\$/€}$ increase). The increase in $E_{\$/€}$ will decrease the foreign return. The exchange rate will rise until the domestic and foreign returns are equal.

3. An increase in the expected exchange rate

The FR line shifts rightwards (horizontally) by the increase in expected exchange rate. At the initial spot rate, the foreign return is now greater than the domestic return. There will be a greater demand for the foreign currency, pushing the price of Euros up ($E_{\$/€}$ increase). The increase in $E_{\$/€}$ will decrease the foreign return. The exchange rate will rise until the domestic and foreign returns are equal.

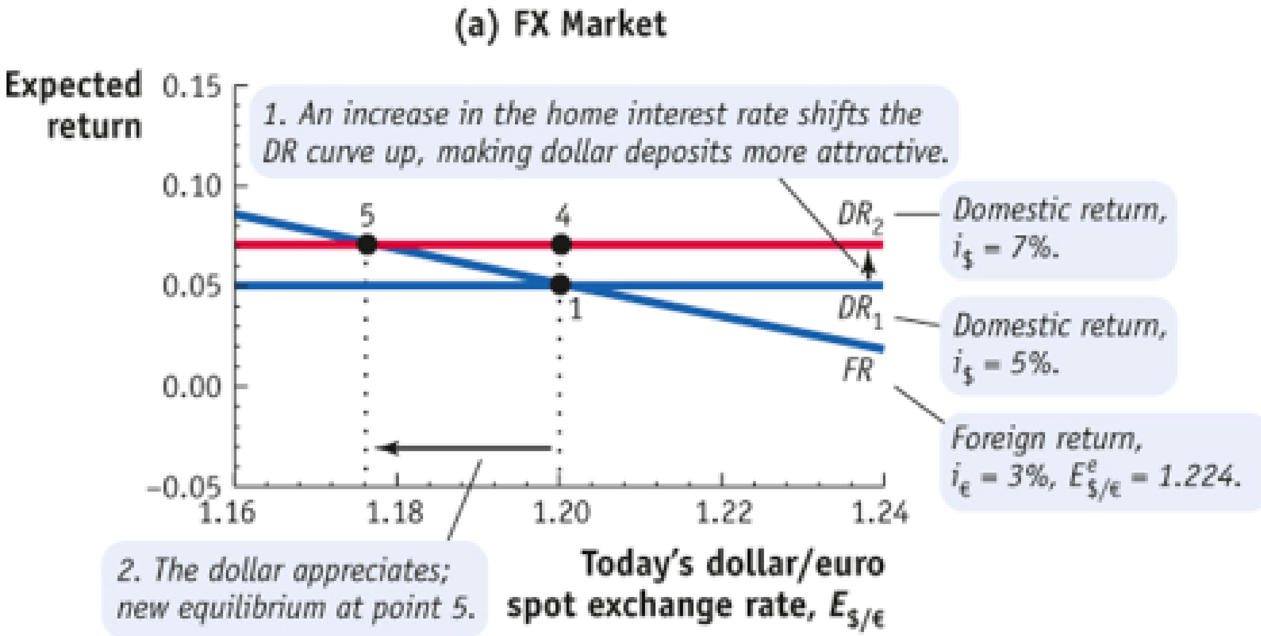

Figure 15.3a

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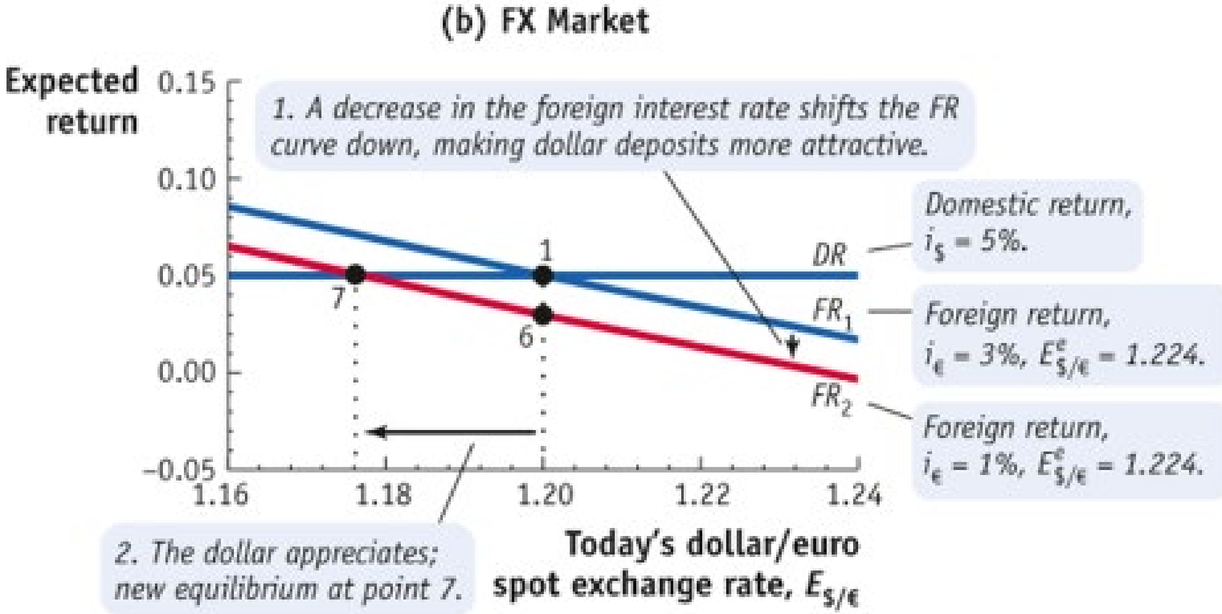

Figure 15.3b
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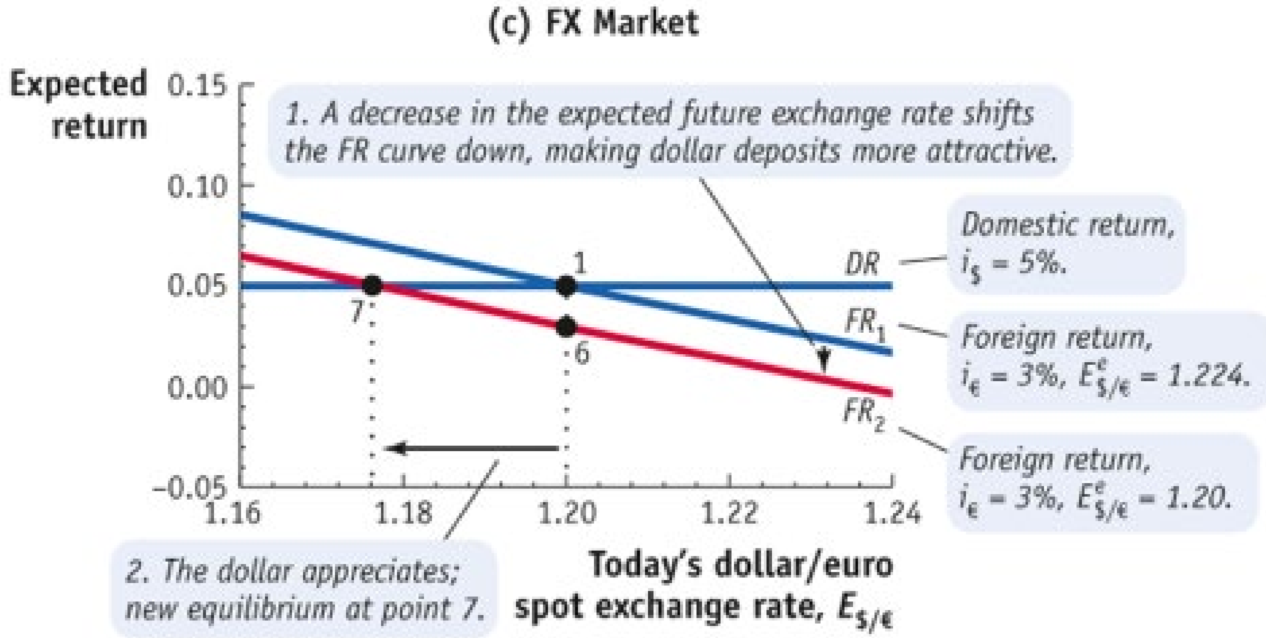

Figure 15.3c

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Calculating the Spot Rate

The UIP relationship can be used to calculate the spot rate:

The spot rate just equals its future expected rate divided by its expected growth rate:

$E_{\$/€} = E^e_{\$/€} / (1 + \Delta E^e_{\$/€} / E_{\$/€})$

But UIP states that
$\Delta E^e_{\$/€} / E_{\$/€} = i_\$ - i_€$

Therefore
$E_{\$/€} = E^e_{\$/€} / (1 + i_\$ - i_€)$

Example
$i_\$ = .07$
$i_€ = .03$
$E^e_{\$/€} = 1.224$

$E_{\$/€} = 1.224 / (1 + .07 - .03) = 1.224/1.04 = 1.177$