Assignment 1

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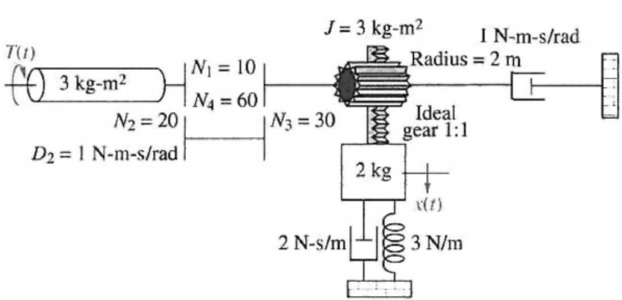
Question 40

For the combined transitional and rotational system shown below, find the transfer function , $G(s) = X(s)/T(s)$.



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Solution

We can solve this problem in two ways,

–By writing Free body equations.

–Using analogy of electrical mesh equations with mechanical systems.

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Analogy of Mesh equations with Mechanical Systems

In laplcaian Domain,Free body equations will be,

$$F(s) = Ms^2X(s); T(s) = Js^2\Theta(s)$$

F(s) = K X(s); T(s) = K $\Theta(s)$
F(s) = $f_v sX(s)$; $T(s) = f_v s\Theta(s)$

Since the Mechanical Model is Linear-Time-Invariant Systems, we can add the impedences and in case of
system having N Degrees of Freedom, We can add its Forces also into the final equation,

Contd.

For Eg, A system having 2 DOF,
Sum of Applied Forces on $X_1 = X_1(s)$(Sum of impedences at X_1) - $X_2(s)$(Sum of impedences across X_1 and X_2);
Sum of Applied Forces on $X_2 = -X_1(s)$(Sum of impedences across X_1 and X_2) $+ X_2(s)$(Sum of impedences at X_2);

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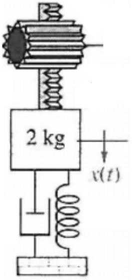
Solution

One thing we can observe is that there is only one-degree of freedom.
Seeing everything from the Gear's perspective,
All the impedances get scaled up due presence of the gears.



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In Laplacian Domain,
For rotational systems:
$T_g =$ (Sum of all Impedences about the gear)$\Theta_g(s)$+Torque applied by the transnational motion,
Here T_g is torque on gear and $\Theta_g(s)$ is angular displacement of the gear.

$$J_e = 3(\frac{N_4 N_2}{N_1 N_3}^2) + 3; J_e = 51,$$

$$\mathrm{D}_e = D_2(\frac{N_2}{N_1}^2) + 1; D_e = 5,$$
$$\implies T_g(s) = (51s^2 + 5s)\Theta_g(s) + T_{trans}(s)$$

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Contd.

For transitional systems:
$F(s) = (Ms^2 + K + f_v s)X(s)$
Since, Gear Ratio is 1:1,
Torque applied by transnational and rotational is the same,

$$\implies T_{ongear}(s) = F(s)r,$$

$$\mathrm{r}\Theta_g(s) = X(s) \implies \Theta_g = 0.5X(s),$$
$$\implies T_g(s) = 0.5(51s^2 + 5s)X(s) + r(Ms^2 + K + f_v s)X(s),$$
$$\mathrm{T}_g(s) = \frac{59s^2+13s+12}{2}X(s),$$

And we know that Torque $\propto$ Number of teeth,
$\implies T(s) = (\frac{N_4 N_2}{N_1 N_3})T_g(s).$
$\implies G(s) = X(s)/T(s) = \frac{X(s)}{4T_g(s)} = \frac{8}{59s^2+13s+12}$

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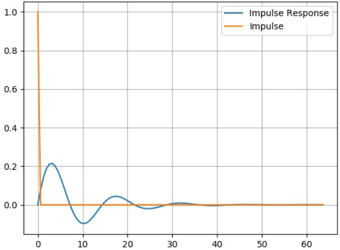
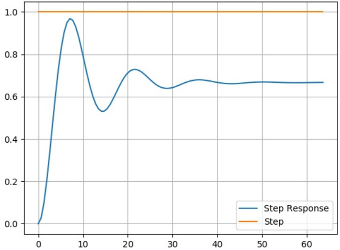
Plots





Find the codes in,
`https://github.com/TUdayKiranReddy/Control_Systems/blob/master/Assignment_1/C_2_Q_40.py`