CONVEX OPTIMISATION
TUTORIAL 12

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(a)

Given $f(x) = sup_{c \in \mathcal{C}} \overline{c}^T \overline{x}$. Clearly $\overline{c}^T \overline{x}$ is an affine function and it is convex. Operation *sup* will preserve the convexity of the function. Thus f is convex.

(b)

Given primal (P),

$$\begin{array}{rl} \max & \overline{c}^T \overline{x} \\ \text{subject to} & \mathbf{F}\overline{c} \leq \overline{g} \end{array} \equiv \begin{array}{rl} \min & -\overline{c}^T \overline{x} \\ \text{subject to} & \mathbf{F}\overline{c} \leq \overline{g} \end{array} \equiv f(\overline{x})$$

Now we find the dual of this problem by taking infimum of Lagrange function over $\overline{x}$.

$$\mathcal{L}(\overline{x}, \overline{\lambda}) = -\overline{c}^T \overline{x} + \lambda^T (\mathbf{F}\overline{c} - \overline{g})$$
$$g(\overline{\lambda}) = inf_{\overline{x}} \mathcal{L}(\overline{x}, \overline{\lambda})$$

Therefore the dual problem (D) is,

$$\begin{array}{rl} \min & \overline{\lambda}^T \overline{g} \\ \text{subject to} & \mathbf{F}^T \overline{\lambda} = \overline{x} \\ & \lambda \geq 0 \end{array}$$

$f(\overline{x})$ is the optimal of (P) and dual gap for LPs is zero. Thus $f(\overline{x})$ is the optimal for (D) also.

(c)

We can replace the objective in the *Robust LP* with the dual problem.

$$\begin{array}{rl} \min\limits_{\overline{x}} \min\limits_{\overline{\lambda}} & \overline{\lambda}^T \overline{g} \\ \text{subject to} & \mathbf{A}\overline{x} \geq \overline{b} \\ & \mathbf{F}^T \overline{\lambda} = \overline{x} \\ & \lambda \geq 0 \end{array}$$

Clearly both $\overline{x}$ and $\overline{\lambda}$ thus the above problem can might as well written as,

$$\begin{aligned} \min_{\overline{x},\overline{\lambda}} \quad & \overline{\lambda}^T \overline{g} \\ \text{subject to} \quad & \mathbf{A}\overline{x} \geq \overline{b} \\ & \mathbf{F}^T \overline{\lambda} = \overline{x} \\ & \lambda \geq 0 \end{aligned}$$

Initial problem was not convex because both $\overline{x}$ and $\overline{c}$ are variable and $\overline{c}^T \overline{x}$ is not convex. But once we tranform the subproblem to dual we get a LP.