

ASSIGNMENT 8

TADIPATRI UDAY KIRAN REDDY
EE19BTECH11038

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Problem 1

$$V_{in} = AV_{in}^3$$

$$20\log_{10}Aa^3 = -0.5 \implies a = \sqrt[3]{\frac{0.944}{|A|}}$$

Problem 2

Given $V_o = A_1V_{in} + A_2V_{in}^2 + A_3V_{in}^3 + A_4V_{in}^4 + A_5V_{in}^5$. Even powers have even harmonics and odd powers have odd harmonics. Thus we consider only odd power.

$$\cos^3(\omega t) = \frac{3}{4}\cos(\omega t) + \frac{1}{4}\cos(3\omega t)$$

$$\cos^5(\omega t) = \frac{5}{8}\cos(\omega t) + \frac{5}{16}\cos(3\omega t) + \frac{1}{16}\cos(5\omega t)$$

If $V_{in} = a\cos(\omega t)$,

$$V_o = A_1a + A_3\frac{3}{4}a^3 + A_5\frac{5}{8}a^5 \implies G = A_1 + A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^4$$

$$20\log_{10}\left(A_1 + A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^4\right) = 20\log_{10}(A_1) - 1$$

$$\implies 1 + \frac{A_3}{A_1}\frac{3}{4}a^2 + \frac{A_5}{A_1}\frac{5}{8}a^4 = 0.891$$

$$\implies 0.108 + \frac{A_3}{A_1}\frac{3}{4}a^2 + \frac{A_5}{A_1}\frac{5}{8}a^4 = 0 \implies a = \sqrt{\frac{-6A_3 \pm \sqrt{36A_3^2 - 17.4A_1A_5}}{10A_5}}$$

For IIP3,

$$IM3 = \frac{A_1}{A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^5}$$

$$\implies -A_1 + A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^4 = 0 \implies IIP3 = \sqrt{\frac{-6A_3 \pm \sqrt{36A_3^2 + 2.5A_1A_5}}{10A_5}}$$

Problem 3

Given, $I_{DS} = \frac{k_n}{2}(V_{DC} + V_{in} - I_{DS}R_S - V_T)^2$. We would like to find coefficients of the equation,

$$i_{ds} = A_1 V_{in} + A_2 V_{in}^2 + A_3 V_{in}^3$$

Where, $A_1 = \frac{\partial i_{ds}}{\partial V_{in}}$, $A_2 = \frac{1}{2} \frac{\partial^2 i_{ds}}{\partial V_{in}^2}$ and $A_3 = \frac{1}{6} \frac{\partial^3 i_{ds}}{\partial V_{in}^3}$

$$\frac{\partial i_{ds}}{\partial V_{in}} = k_n(V_{GS} - V_T) \left(1 - \frac{\partial i_{ds}}{\partial V_{in}} R_s\right)$$

Let $k_n(V_{DC} + V_{in} - I_{DS}R_S - V_T) = \alpha$; $\Rightarrow \frac{\partial \alpha}{\partial V_{in}} = k_n$, then,

$$\begin{aligned} \frac{\partial i_{ds}}{\partial V_{in}} &= \frac{\alpha}{1 + \alpha} \\ \frac{\partial^2 i_{ds}}{\partial V_{in}^2} &= \frac{\frac{\partial \alpha}{\partial V_{in}}}{(1 + \alpha)^2} \Rightarrow \frac{\partial^2 i_{ds}}{\partial V_{in}^2} = \frac{k_n}{(1 + \alpha)^3} \\ \frac{\partial^3 i_{ds}}{\partial V_{in}^3} &= \frac{-3k_n \frac{\partial \alpha}{\partial V_{in}}}{(1 + \alpha)^4} \Rightarrow \frac{\partial^3 i_{ds}}{\partial V_{in}^3} = \frac{-3k_n^2}{(1 + \alpha)^5} \\ \Rightarrow i_{ds} &= \frac{\alpha}{1 + \alpha} V_{in} + \frac{k_n}{2(1 + \alpha)^3} V_{in}^2 - \frac{k_n^2}{2(1 + \alpha)^5} V_{in}^3 \end{aligned}$$

$$\Rightarrow P_{1dB} = \sqrt{0.29} \frac{(1 + \alpha)^2}{k_n}$$

$$\Rightarrow IIP3 = \sqrt{\frac{8}{3}} \frac{(1 + \alpha)^2}{k_n}$$

Problem 4

Here, $V_o = (I_- - I_+)R_L$, now applying same square law here,

$$\begin{aligned} V_o &= \frac{k_n}{2} R_s \left((V_{cm} - V_T + \frac{V_{in}}{2})^2 - (V_{cm} - V_T - \frac{V_{in}}{2})^2 \right) \\ \Rightarrow V_o &= \frac{1}{2} k_n R_s (V_{cm} - V_T) V_{in} \end{aligned}$$

Since $(I_- + I_+)|_{V_{in}=0} = I_B \Rightarrow (V_{cm} - V_T) = \frac{I_B}{k_n}$.

$$V_o = \sqrt{k_n I_B} R_s V_{in}$$

Clearly there is no 3rd order component thus $P_{1dB} = \infty dB$ and $IIP3 = \infty dBm$.

Problem 5

$$\begin{aligned} V_x &= V_s + R_s I_x = V_s - g_{m1} V_x R_s \\ \Rightarrow V_x &= \frac{V_s}{1 + g_{m1} R_s} \\ I_x &= -g_{m1} V_x \end{aligned}$$

Input impedance here is $-V_x/I_x = 1/g_{m1}$. So if $R_s = 1/g_{m1}$ then the input is matched.

$$\begin{aligned} V_o &= (g_{m1}R_1 + g_{m2}R_2)V_x \\ \Rightarrow V_o &= \frac{g_{m1}R_1 + g_{m2}R_2}{1 + g_{m1}R_s}V_s \end{aligned}$$

Problem 6