ASSIGNMENT 5

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Problem 1

Effective $Q_{eff} = Q_1 + Q2$, where $Q_i = \sqrt{\frac{R_{out}}{R_{in}} - 1}$. Since this is 2 stage low pass matching network let us take a intermediate jump of resistance R_i.

$$Q_1 = \sqrt{\frac{R_{HI}}{R_i} - 1} = \frac{\omega L_1}{R_i + R_{rs_1}} = \omega C_1 R_{HI} \tag{1}$$

$$Q_2 = \sqrt{\frac{R_i}{R_{LO}} - 1} = \frac{\omega L_2}{R_{LO} + R_{rs_2}} = \omega C_2 R_i \tag{2}$$

The loss in the network is because of lossy inductors which have a Q of 20.

$$\eta = \frac{P_L}{P_L + P_{rs_1} + P_{rs_2}} = \frac{P_L}{P_L + \frac{E_{L_1} + E_{L_2}}{Q}} \tag{3}$$

$$\eta = \frac{P_L}{P_L + \frac{Q_1 P_I + Q_2 P_L}{Q}} = \frac{1}{1 + \frac{Q_1 \frac{P_I}{P_L} + Q_2}{Q}} \tag{4}$$

Here $\frac{P_I}{P_2}$ is inverse of efficiency of only 2nd stage which is $1 + \frac{Q_2}{Q}$. Therefore,

$$\eta = \frac{1}{1 + \frac{Q_1 + Q_2 + Q_1 Q_2 / Q}{Q}} \tag{5}$$

$$max\{\eta\} \iff min\{Q_1 + Q_2 + Q_1 Q_2 / Q\} \tag{6}$$

$$\implies \frac{\partial}{\partial R_i}(Q_1 + Q_2 + Q_1 Q_2 / Q) = 0 \tag{7}$$

$$Q(\frac{\partial Q_1}{\partial R_i} + \frac{\partial Q_2}{\partial R_i}) + Q_1 \frac{\partial Q_2}{\partial R_i} + Q_2 \frac{\partial Q_1}{\partial R_i} = Q\left[-\frac{R_{HI}}{2R_i^2 Q_1} + \frac{1}{2R_{LO} Q_2}\right] + Q_1\left[\frac{1}{2R_{LO} Q_2}\right] - Q_2\left[\frac{R_{HI}}{2R_i^2 Q_1}\right] = 0 \tag{8}$$

$$\tag{9}$$

Simplifying the above equation we end up getting,

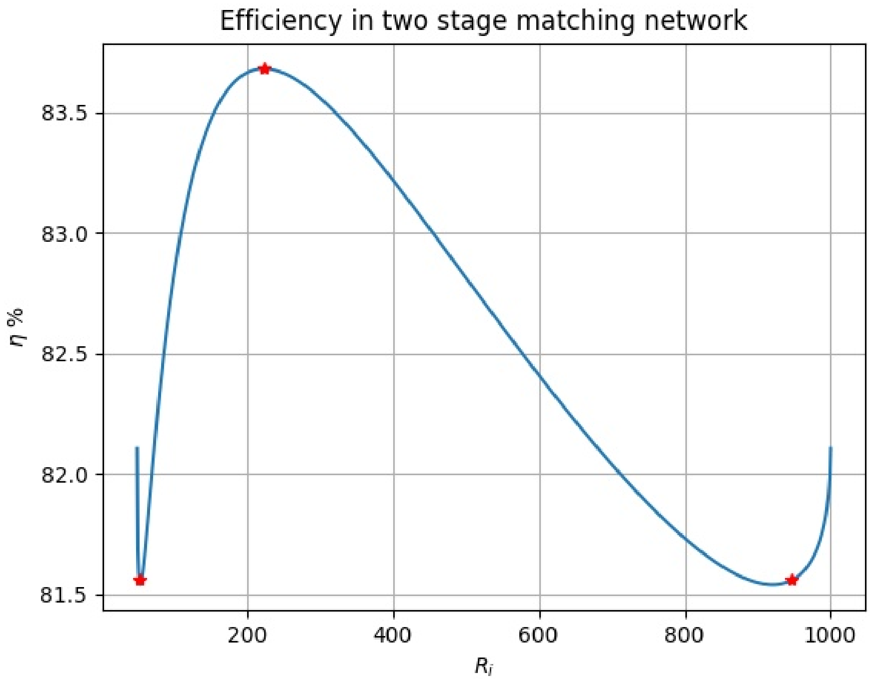
$$R_i^2 Q_1 (Q_1 + Q) = R_{HI} R_{LO} Q_2 (Q_2 + Q) \tag{10}$$

Assuming $Q_2, Q_1 << Q$,

$$2R_i^2 Q_1 Q = R_{HI} R_{LO} Q_2 Q \tag{11}$$

$$\implies 4R_i^4 - R_{HI} R_i^3 + R_{HI}^2 R_{LO} R_i - R_{HI}^2 R_{LO^2} = 0 \tag{12}$$

$$\implies R_i = 223.6, 52.7, 947.213 \tag{13}$$



Plugging in these values in the equation (3) we get highest value for R=223.6

$$\implies \boxed{R_i = 223.6} \tag{14}$$

$$\implies \boxed{Q_1 = 1.863, Q_2 = 1.863} \tag{15}$$

From equation (5) we get,

$$\boxed{\eta = 83.69\%} \tag{16}$$

From equation (1) and (2) we get,

$$L_1 = Q_1 \frac{R_i + \omega L_1/Q}{\omega} \tag{17}$$

$$\implies L_1 = \frac{R_i}{\omega(1/Q_1 - 1/Q)} \tag{18}$$

$$\boxed{L_1 = 73.1nH} \tag{19}$$

$$L_2 = \frac{R_{LO}}{\omega(1/Q_2 - 1/Q)} \tag{20}$$

$$\boxed{L_2 = 16.35nH} \tag{21}$$

$$C_1 = \frac{Q_1}{\omega R_{HI}} \tag{22}$$

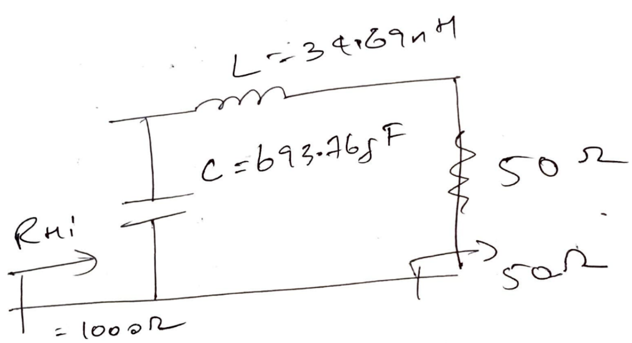
$$\boxed{C_1 = 0.29fF} \tag{23}$$

$$C_2 = \frac{Q_2}{\omega R_i} \tag{24}$$

$$\boxed{C_2 = 1.3fF} \tag{25}$$

Problem 2

(a)



$R_{HI} = 1000\Omega, R_{LO} = 50\Omega; \omega = 2\pi 10^9$

$$Q = \sqrt{\frac{1000}{50} - 1} \implies \boxed{Q = 4.359} \tag{26}$$

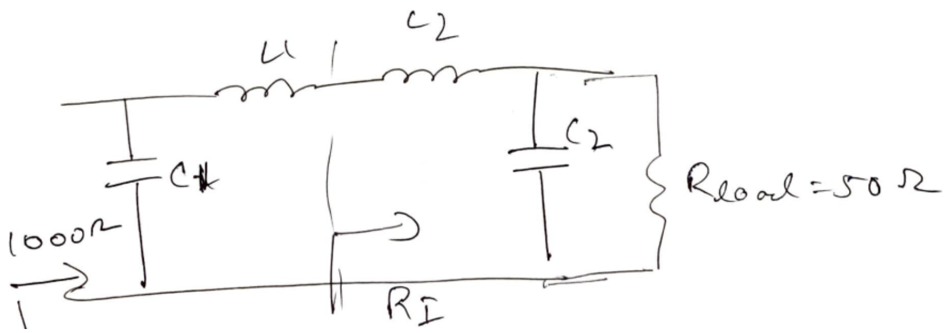
$$Q = \frac{\omega L}{R_{LO}} \implies \boxed{L = 34.69nH} \tag{27}$$

$$Q = \omega C R_{Hi} \implies \boxed{C = 693.76fF} \tag{28}$$

(b)

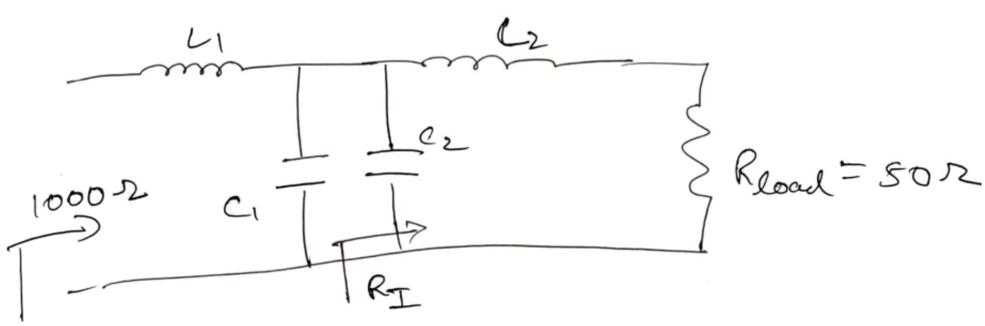
Given $Q_{loaded} = 5$ which means Q of network is 10.

(i) Low-pass Π



$$Q_1 = \sqrt{\frac{1000}{R_I} - 1} \tag{29}$$
$$Q_2 = \sqrt{\frac{50}{R_I} - 1} \tag{30}$$
$$Q = \sqrt{\frac{1000}{R_I} - 1} + \sqrt{\frac{50}{R_I} - 1} = 10 \tag{31}$$
$$\implies 104R_I^2 - 2100R_I + 9025 = 0 \implies \boxed{R_I = 13.99\Omega} \tag{32}$$

(i) Low-pass T



$$Q_1 = \sqrt{\frac{R_I}{R_{HI}} - 1} \tag{33}$$
$$Q_2 = \sqrt{\frac{R_I}{R_{LO}} - 1} \tag{34}$$
$$Q = \sqrt{\frac{R_I}{1000} - 1} + \sqrt{\frac{R_I}{50} - 1} = 10 \tag{35}$$
$$\implies R_I^2(3.61e-4) - 4.2R_I + 10400 = 0 \tag{36}$$
$$\boxed{R_I = 3574.26\Omega} \tag{37}$$