ASSIGNMENT 3

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Problem 1

From previous assignment we found out that, Characterstic impedance is $\sqrt{\frac{R+j\omega L}{G+j\omega C}}$. Here it is given that $\frac{L}{R} = \frac{C}{G}$ which means $\mathbf{z_0} = \sqrt{\frac{\mathbf{R}}{\mathbf{G}}}\Omega$.
Propagation constant is $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{RG} + j\omega\sqrt{LC} = \alpha + j\beta$
Phase velocity is $V_p = \frac{\omega}{\beta} = 1/\sqrt{LC}$
Observations

- The characterstic impedance of the line is purely resistive.
- z_0 does not depend on the frequency of operation.
- α is frequency independent.
- β is linear function of ω.
- Phase velocity is same at all frequency which means the line is **Distortion less**.

Problem 2

Given lossless transmission line with a load Z_L

$$Z_{in}(x) = z_0 \frac{1 + \Gamma_L e^{-2\gamma x}}{1 - \Gamma_L e^{-2\gamma x}}$$

$$\Gamma_L = \frac{Z_L - z_0}{Z_L + z_0}$$

$$Z_{in}(-l/4) = z_0 \frac{1 + \Gamma_L e^{2j\frac{2\pi}{\lambda}\frac{\lambda}{4}}}{1 - \Gamma_L e^{2j\frac{2\pi}{\lambda}\frac{\lambda}{4}}}$$

$$\implies Z_{in} = z_0 \frac{1 - \frac{Z_L - z_0}{Z_L + z_0}}{1 + \frac{Z_L - z_0}{Z_L + z_0}}$$

$$\implies \boxed{Z_{in} = \frac{z_0^2}{Z_L}}$$

Problem 3

First we calculate Z_{in}.

$$Z_{in}(-l) = z_0 \frac{1 + \Gamma_L e^{2\gamma l}}{1 - \Gamma_L e^{2\gamma l}}$$

$$\implies \boxed{V_{in} = V_s \frac{Z_{in}}{z_0 + Z_{in}} = V_s \frac{1+\Gamma_L e^{2\gamma l}}{2}} \tag{1}$$

$$V_o = V(0) \tag{2}$$

$$V_{in} = V(-l) \tag{3}$$

$$\implies \frac{V_o}{V_{in}} = \frac{V_o^+(1+\Gamma_L)}{V_o^+ e^{-\gamma l}(1+\Gamma_L e^{2\gamma l})} = \frac{1+\Gamma_L}{e^{-\gamma l} + e^{\gamma l}\Gamma_L} \tag{4}$$

$$\implies \boxed{V_o = V_s \frac{1+\Gamma_L}{2} e^{\gamma l}} \tag{5}$$

For maximising V_o,

$$\frac{\partial V_o}{\partial Z_L} = 0 \tag{6}$$

$$\frac{\partial \left(V_s \frac{Z_L}{Z_L + z_o} e^{\gamma l} \right)}{\partial Z_L} = 0 \tag{7}$$

$$\implies \frac{1}{(Z_L + z_o)^2} = 0 \tag{8}$$

$$\implies \boxed{Z_L \to \infty} \tag{9}$$

$$\frac{\partial^2 V_o}{\partial Z_L^2} @ Z_L \to \infty < 0 \implies \text{It is local maxima.} \tag{10}$$

Thus the keeping a open circuit near load will maximise the $\boxed{V_o = V_s e^{\gamma l}}$ and corresponding $\boxed{V_{in} = V_s \frac{1+e^{2\gamma l}}{2}}$