ASSIGNMENT 4

TADIPATRI UDAY KIRAN REDDY
EE19BTECH11038

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Problem 1

First we find out the V^+ using below procedure and here $\Gamma_L = \frac{R_L - z_0}{R_L + z_0}$

$$V(x) = V^+ e^{-\gamma x} \left(1 + \Gamma_L e^{2\gamma x}\right) \tag{1}$$

$$I(x) = \frac{V^+}{z_0} e^{-\gamma x} \left(1 - \Gamma_L e^{2\gamma x}\right) \tag{2}$$

$$\implies V_{in} = V_s - I(-l) r_s = V(-l) \tag{3}$$

$$\implies V_s - V^+ \frac{r_s}{z_0} e^{\gamma l} \left(1 - \Gamma_L e^{-2\gamma l}\right) = V^+ e^{\gamma l} \left(1 + \Gamma_L e^{-2\gamma l}\right) \tag{4}$$

$$\implies V^+ = V_s \frac{e^{-\gamma l}}{\frac{r_s}{z_0}(1 - \Gamma_L e^{-2\gamma l}) + (1 + \Gamma_L e^{-2\gamma l})} \tag{5}$$

We use this V^+ to compute V_0,

$$V_0 = V(0) = V^+ (1 + \Gamma_L) \implies V_0 = V_s \frac{(1 + \Gamma_L) e^{-\gamma l}}{\frac{r_s}{z_0}(1 - \Gamma_L e^{-2\gamma l}) + (1 + \Gamma_L e^{-2\gamma l})} \tag{6}$$

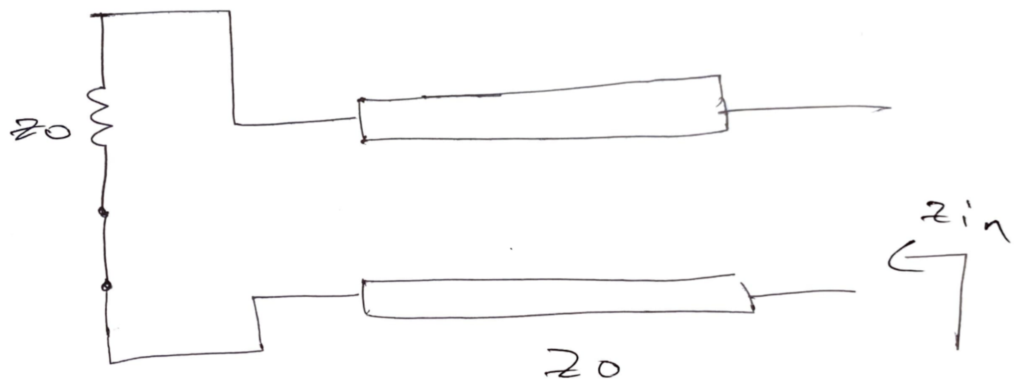
$$\implies \boxed{V_0 = V_s \frac{2 z_0 R_L}{(z_0^2 + R_L r_s)(e^{\gamma l} - e^{-\gamma l}) + (z_0(R_L + r_s))(e^{\gamma l} + e^{-\gamma l})}} \tag{7}$$

Sanity check
As $l = 0$,

$$V_0 = V_s \frac{2 z_o R_L}{z_0 (R_L + r_s) 2} = V_s \frac{R_L}{r_s + R_L} \tag{8}$$

Problem 2

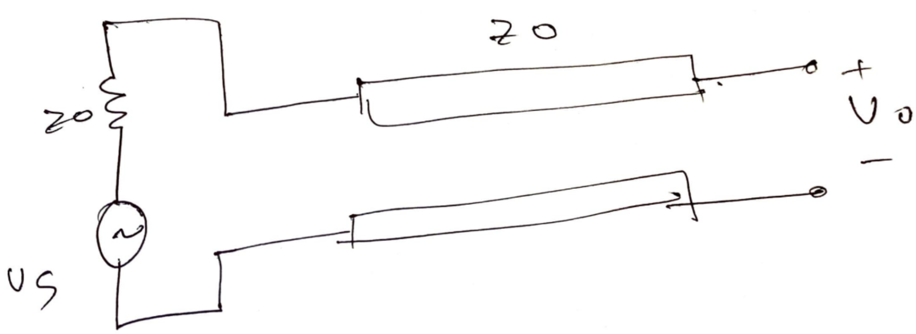
Thevinin resistance
It is just input impedance but from load side, thus input resistance is load which means $\Gamma_L = 0$ since it is matching network.



$$Z_{th} = Z_{in}(l) = z_0 \frac{1 + \Gamma_L e^{-2\Gamma_L l}}{1 - \Gamma_L e^{-2\Gamma_L l}} \tag{9}$$

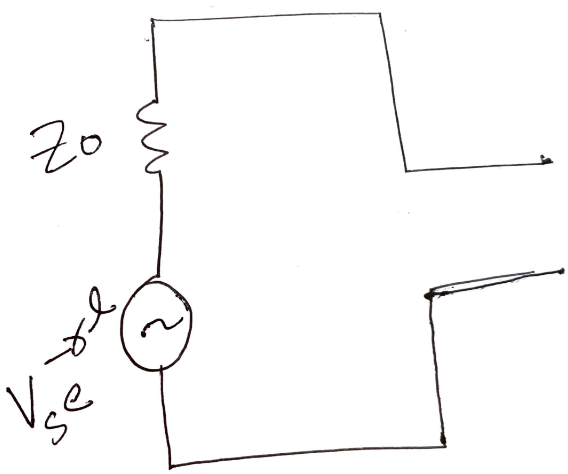
$$\boxed{Z_{th} = z_0} \tag{10}$$

Thevinin voltage
As obtained from previous problem here load is open circuited.



$$V_{th} = V_0|_{R_L \to \infty} = V_s \frac{2z_0}{(\frac{z_0^2}{R_L} + z_0)(e^{\gamma l} - e^{-\gamma l}) + (z_0(1 + \frac{z_0}{R_L}))(e^{\gamma l} + e^{-\gamma l})}|_{R_L \to \infty} \tag{11}$$

$$\boxed{V_{th} = V_s e^{-\gamma l}} \tag{12}$$



Problem 3

$$\frac{V_2}{V_1} = \frac{V(l)}{V(0)} = \frac{e^{j\beta l} + \Gamma_L e^{-j\beta l}}{1 + \Gamma_L} \tag{13}$$

Since it is open circuit, $\Gamma_L = 1$

$$V_2 = V_1(e^{j\beta l} + e^{-j\beta l})/2 \tag{14}$$

Delay can be computes using phase velocity $V_p = 1/\beta = l/t_d$

This means that delay in that transmission line is $\boxed{t_d = \beta l = \sqrt{L_0 C_0} l = z_0 C_0 l = z_0 C_T}$

Since, $z_0 = \sqrt{\frac{L_0}{C_0}}$ and $C_T = C_0 l$