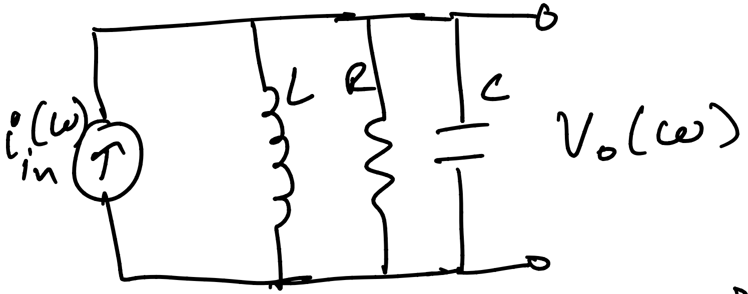
EE5192
Assignment - I

Q1)

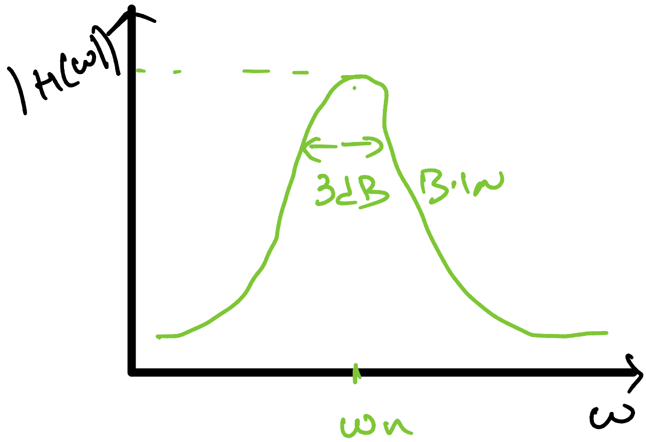


Admittance (Y_{eff}) $= \dfrac{1}{SL} + \dfrac{1}{R} + SC \Rightarrow \dfrac{S^2LC + \frac{SL}{R} + 1}{SL}$

$$H(s) = \frac{V_o(s)}{i_{in}(s)} = \frac{1}{Y_{eff}(s)}$$

$$\Rightarrow \boxed{H(s) = \frac{s/C}{s^2 + \frac{s}{RC} + \frac{1}{LC}}}$$

$\omega_n = \dfrac{1}{\sqrt{LC}}$; $\dfrac{\omega_n}{Q} = \dfrac{1}{RC} \Rightarrow \boxed{Q = R\sqrt{\dfrac{C}{L}}}$



Let 3dB - frequencies be

$\omega_1 = \omega_n + \Delta\omega; \; \omega_2 = \omega_n - \Delta\omega$

Assuming $\Delta\omega << \omega_n$

$$|Y(\omega_n + \Delta\omega)| = \frac{1}{\sqrt{2}R}$$

$$Y(\omega_n + \Delta\omega) = \frac{1+j}{R}$$

as $\Delta\omega << \omega_n$

$$\Rightarrow Y(\omega_n + \Delta\omega) = Y'(\omega_n)\,\Delta\omega + Y(\omega_n)$$

$$= \frac{\partial}{\partial\omega}\left(j\left(\omega C - \frac{1}{\omega L}\right)\right)\Bigg|_{\omega_n} \Delta\omega + \frac{1}{R}$$

$$\Rightarrow Y(\omega_n + \Delta\omega) = \frac{1}{R} + 2\Delta\omega C j$$

$$\Rightarrow \boxed{\Delta\omega = \frac{1}{2RC}}$$

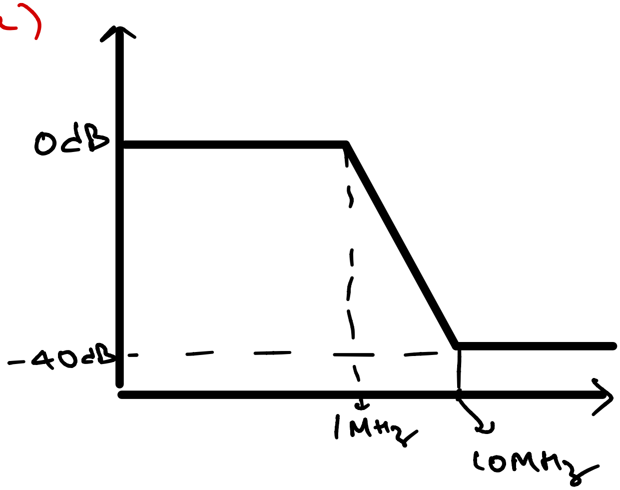
$$\omega'_{3dB} = \omega_n + \Delta\omega = \frac{1}{\sqrt{LC}} + \frac{1}{2RC}$$

$$\omega''_{3dB} = \omega_n - \Delta\omega = \frac{1}{\sqrt{LC}} - \frac{1}{2RC}$$

$$\boxed{\omega_{3dB\text{-}B.W} = \frac{1}{RC}}$$

Q2)

(a)

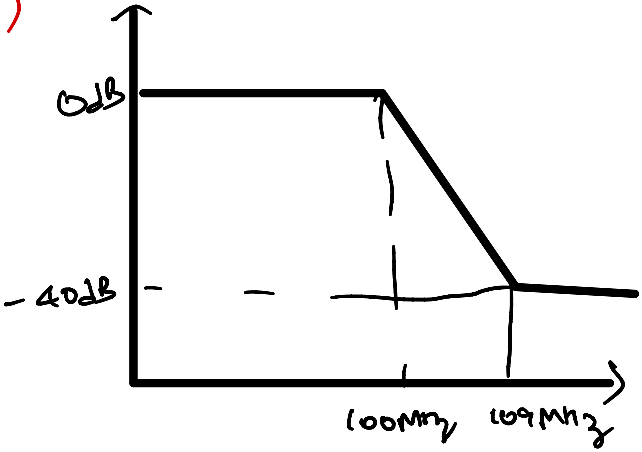


$$\text{Slope} = \frac{40\,dB}{\log_{10}\frac{10}{1}}$$

$$\Rightarrow \text{Slope} = 40\,dB/Dec$$

$\therefore$ It should be 2^{nd} order filter

(b)

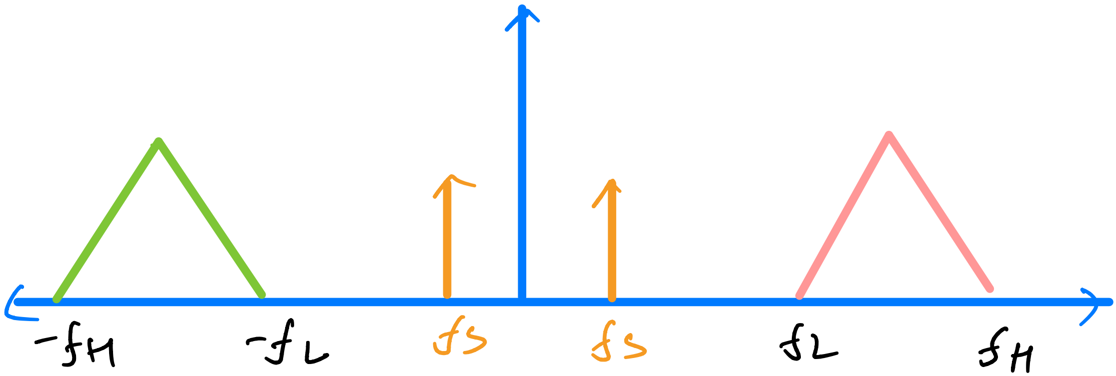


$$\text{Slope} = \frac{40\,dB}{\log_{10} \frac{109}{100}} \Rightarrow \text{Slope} = 1068.76\,dB/Dec$$

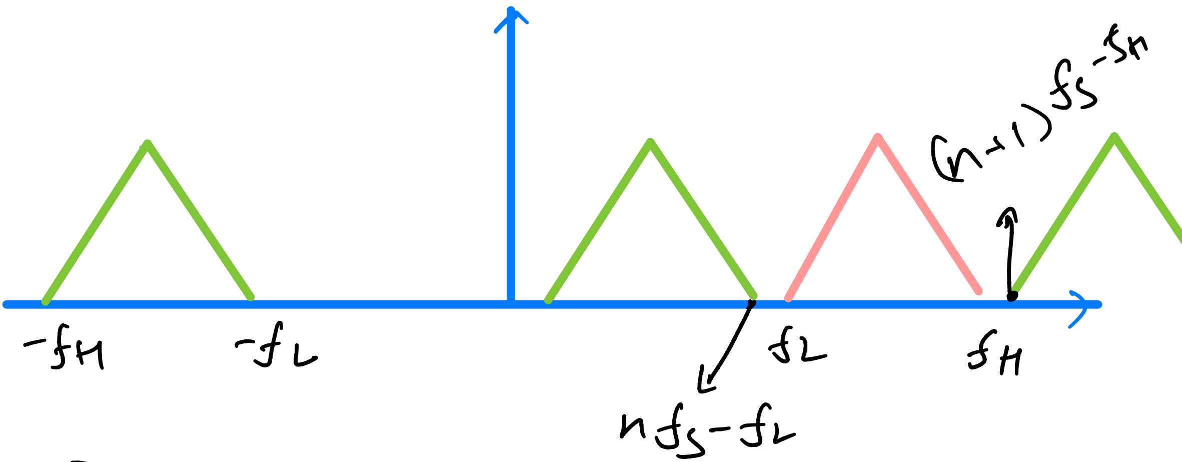
$\therefore$ It should be atleast 54^{th} order

Q3)

Bandpass Sampling



Let us analyse corner cases.



$$\Rightarrow \quad nf_s - f_L \le f_L \; ; \; (n+1)f_s - f_H \ge f_H$$

$$\Rightarrow \quad f_s \in \left[\frac{2f_H}{n+1}, \frac{2f_L}{n}\right]$$

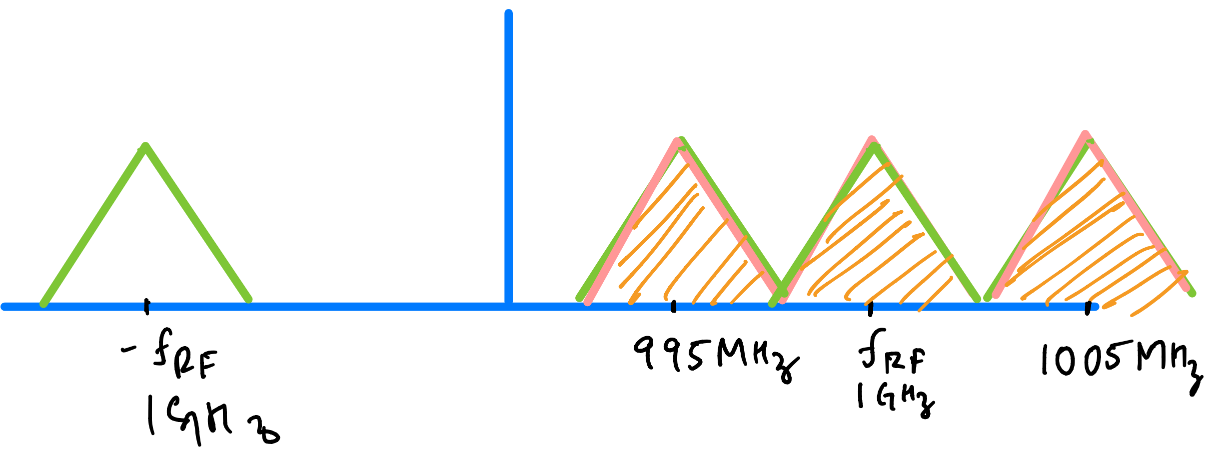
Here 'n' represents ; # of spectrums which fits with given bandwidth

$$\Rightarrow \quad n+1 = \left\lfloor \frac{f_H}{f_H - f_L} \right\rfloor$$

$$\Rightarrow \quad f_s \in \left[\frac{2 f_H}{\left\lfloor \frac{f_H}{f_H - f_L} \right\rfloor}, \frac{2 f_L}{\left\lfloor \frac{f_H}{f_H - f_L} \right\rfloor - 1} \right]$$

a) CASE I

$f_s = 5\,MHz$; $f_{RF} = 1\,GHz$



$$f_s \in \left[\frac{2\times(1002.5)}{\left\lfloor \frac{1002.5}{5} \right\rfloor}, \frac{2\times(997.5)}{\left\lfloor \frac{1002.5}{5} \right\rfloor - 1} \right]$$

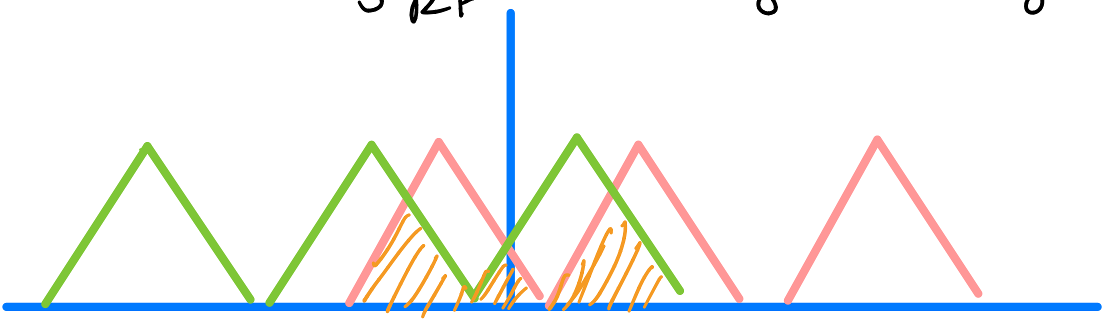
$$\Rightarrow f_{s_{min}} = 10.025\, MHz$$

$$f_{s_{max}} = 10.0251256\, MHz$$

CASE II

$$f_s = 5\, MHz;\ f_B = 5\, MHz$$

$$f_{RF} = 1\, GHz + 3\, MHz$$



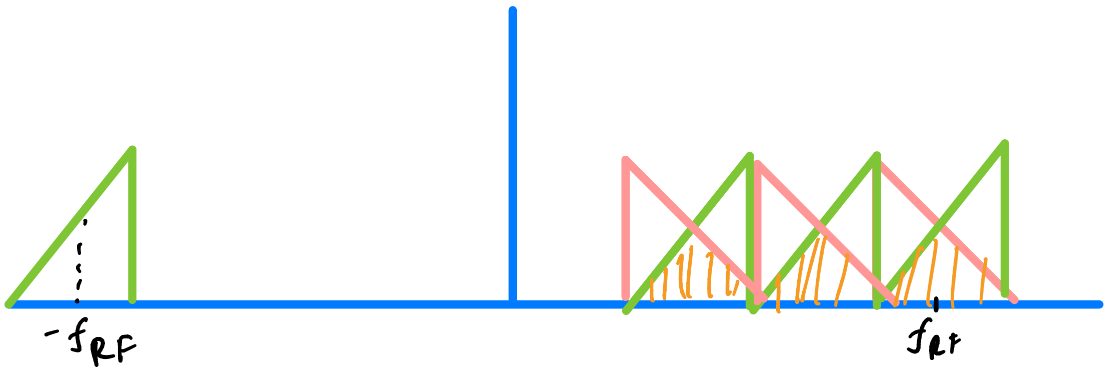
$$f_s \in \left[\frac{2\times(1005.5)}{\left\lfloor \frac{1005.5}{5} \right\rfloor}, \frac{2\times(1000.5)}{\left\lfloor \frac{1005.5}{5} \right\rfloor - 1} \right]$$

$\Rightarrow f_{s_{min}} = 10.0049751 \text{ MHz}$

$f_{s_{max}} = 10.005 \text{ MHz}$

b) CASE I

$f_s = 5 \text{ MHz}$; $f_B = 5 \text{ MHz}$; $f_{RF} = 1 \text{ GHz}$



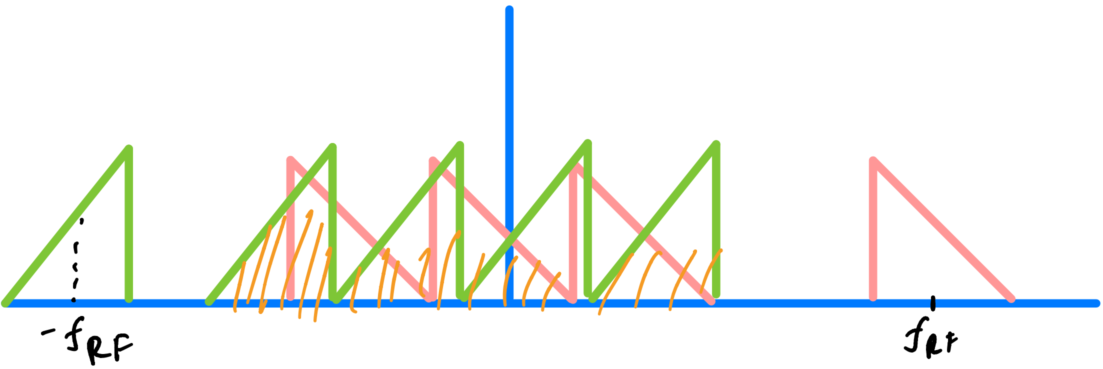
Since the specs remain same compared to a) CASE I

$\Rightarrow f_{s_{min}} = 10.025 \text{ MHz}$

$f_{s_{max}} = 10.0251256 \text{ MHz}$

CASE -II

$f_s = 5\,MHz$; $f_B = 5\,MHz$; $f_{RF} = 1\,GHz + 3\,MHz$

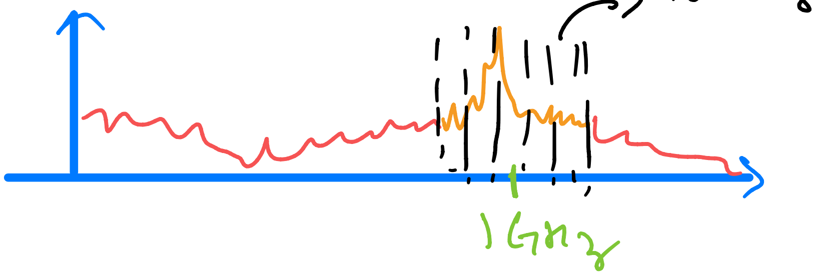


Specs match @ CASE II

$\Rightarrow f_{s_{min}} = 10.0049751\,MHz$

$f_{s_{max}} = 10.005\,MHz$

Q 4) Given



Specs of Channel Select filter

CF = 300 MHz ; B.W = 10 MHz

First & last channels center frequency would be 1 GHz ± 45 MHz

Now, we should be able to translate these signals to 300 MHz by an local Oscillator whose frequency is can be controlled

$\Rightarrow$ Range of f_{LO} should be

$$f_{LO} - f_{in} = 300\,\text{MHz}$$

$$\Rightarrow f_{LO} = f_{in} + 300\,\text{MHz}$$

(a)

$\therefore \; f_{LO} \in [1.255, 1.345]\,GHz$

(b)

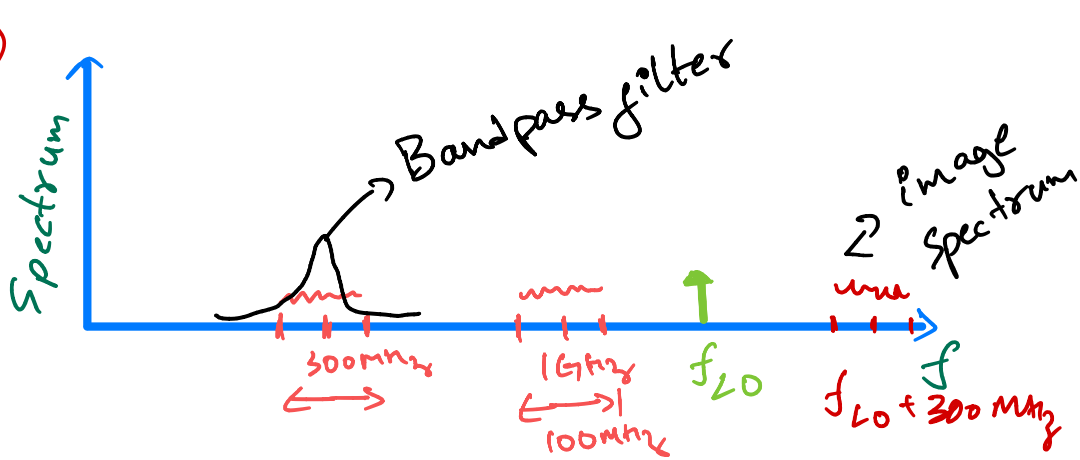


Image spetrum will be of frequency
$f_{LO} + 300MHz$
which will be,

$$f_{image} = f_{LO} + 300MHz$$

$\therefore \; f_{image} \in [1.555, 1.645]\,GHz$

$\Rightarrow$ Center frequency is 1.6GHz

Span of image is 90MHz