ASSIGNMENT 6

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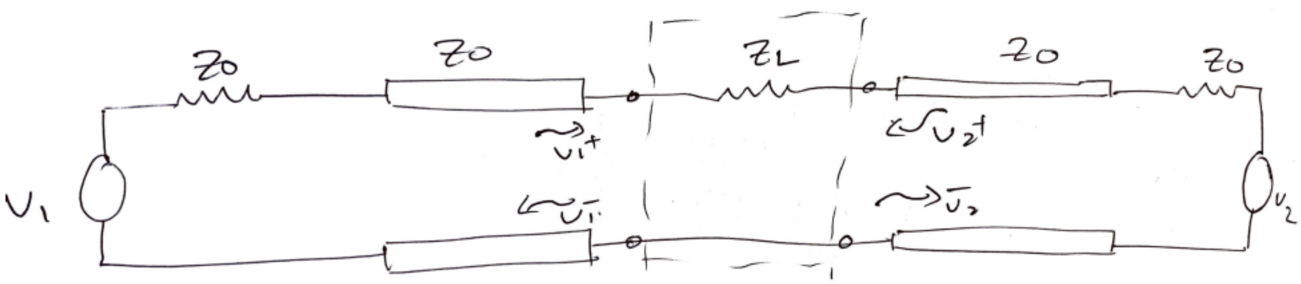
Problem 1



$V_2^+ = 0$

$$\Gamma_{L1} = \frac{\frac{Z_L+Z_0}{Z_0} - 1}{\frac{Z_L+Z_0}{Z_0} + 1} = \frac{Z_L}{Z_L + 2Z_0} \tag{1}$$

$$\implies \frac{V_1^-}{V_1^+} = \boxed{S_{11} = \frac{Z_L}{Z_L + 2Z_0}} \tag{2}$$

Since source at Port 2 is matched network then,

$$V_2^- = (V_1^+ + V_1^-)\frac{Z_0}{Z_0 + Z_L} \tag{3}$$

$$\frac{V_2^-}{V_1^+} = S_{12} = (S_{11} + 1)\frac{Z_0}{Z_0 + Z_L} \tag{4}$$

$$\implies S_{12} = \boxed{\frac{2Z_0}{Z_L + 2Z_0}} \tag{5}$$

$V_1^+ = 0$

The above network is clearly symetric which scattering matrix is symetric.

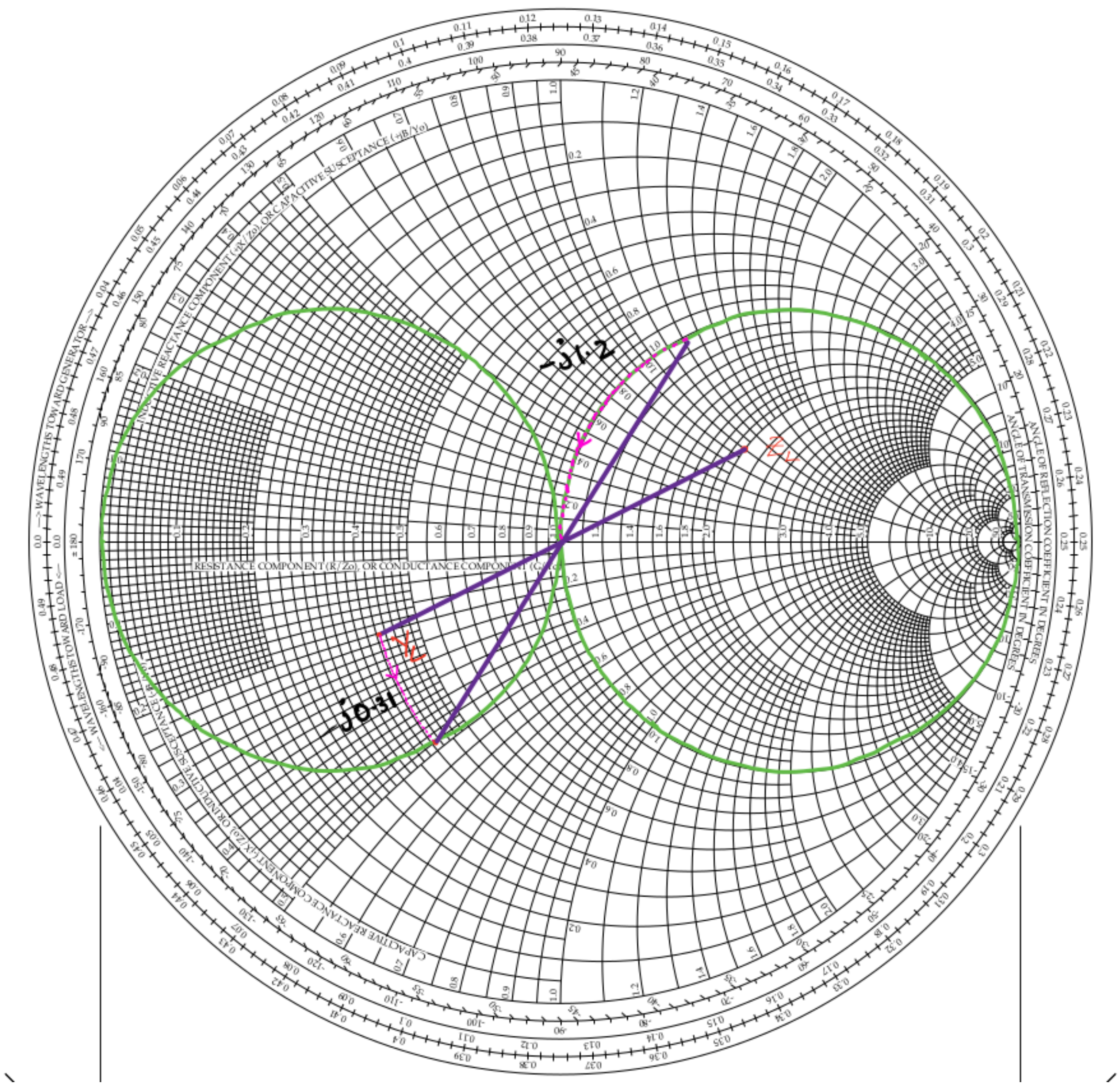
$$\boxed{\mathbf{S} = \begin{bmatrix} \frac{Z_L}{Z_L+2Z_0} & \frac{2Z_0}{Z_L+2Z_0} \\ \frac{2Z_0}{Z_L+2Z_0} & \frac{Z_L}{Z_L+2Z_0} \end{bmatrix}} \tag{6}$$

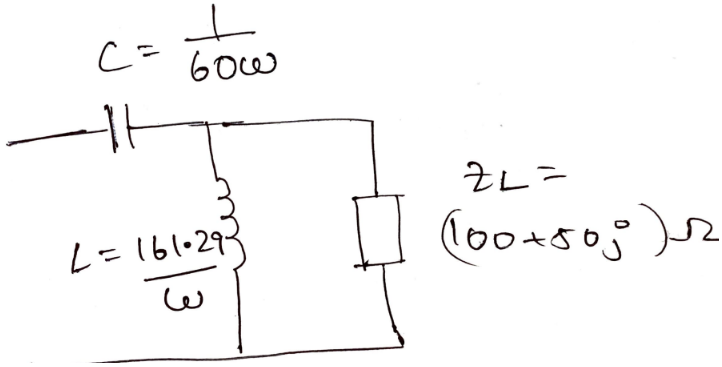
In this case we get,

$$\boxed{\mathbf{S} = \begin{bmatrix} 0.0476 & 0.9524 \\ 0.9524 & 0.0476 \end{bmatrix}} \tag{7}$$

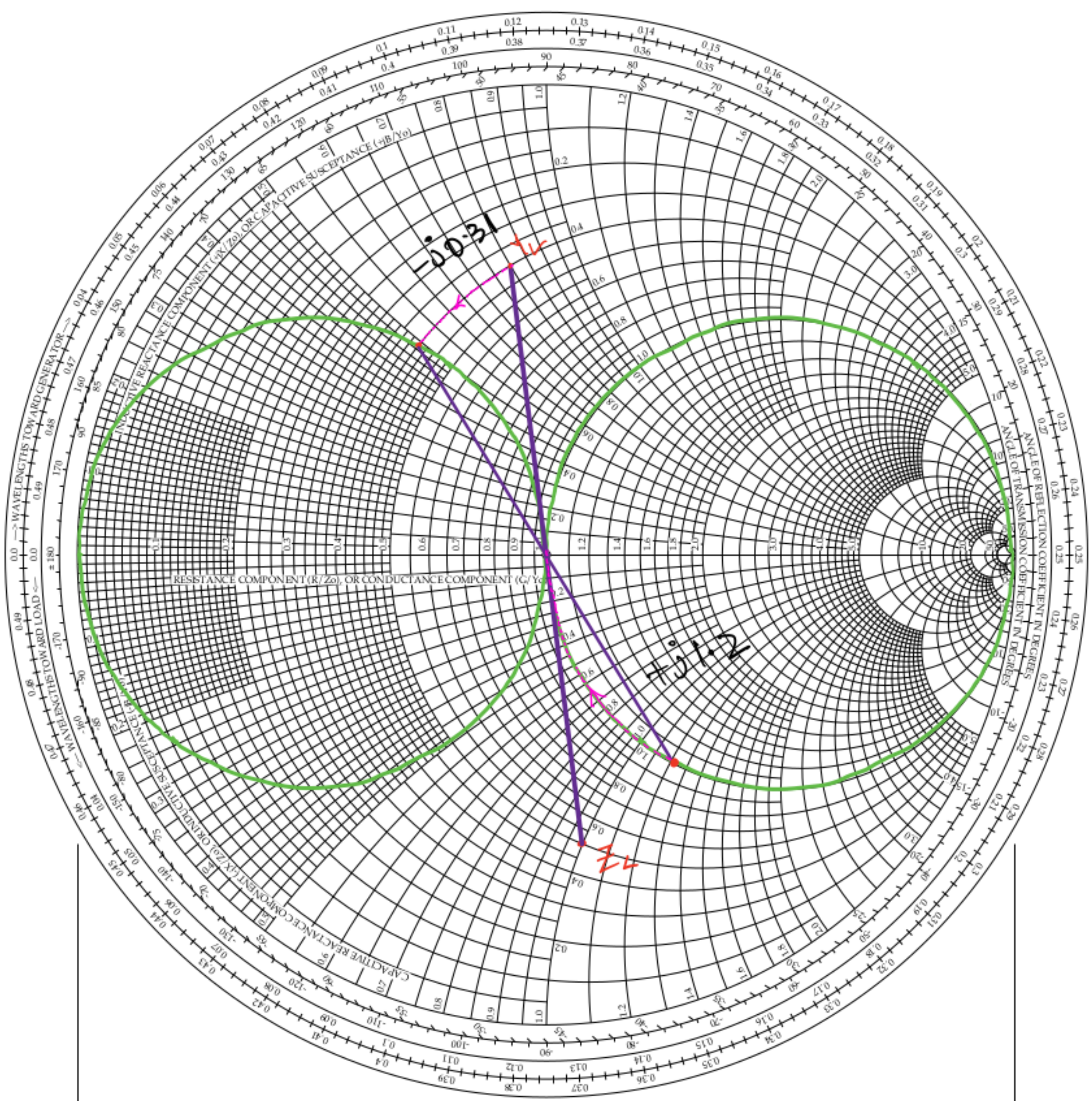
Problem 2

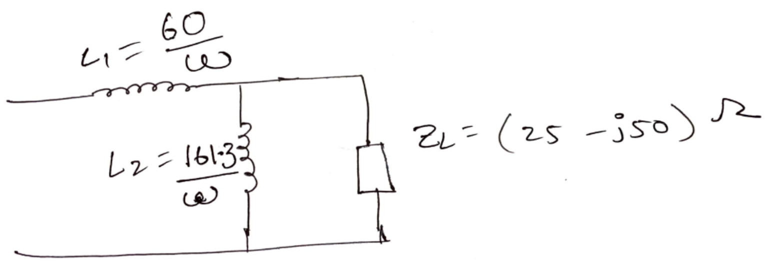
(a)





(b)





Problem 3

At DC we see that $Z_{in} = Z_L = 50\Omega$ which means the line is lossless $\implies \alpha = 0$.
At 1GHz when lossless,

$$Z_{in}(x) = Z_0 \left[\frac{Z_L + Z_0 jtan(\beta_{1GHz}l)}{Z_0 - jZ_L tan(\beta_{1GHz}l)}\right] \tag{8}$$

$$4 = z_0 \left[\frac{1 + z_0 jtan(\beta_{1GHz}l)}{z_0 - jtan(\beta_{1GHz}l)}\right] \tag{9}$$

$$\implies 3z_0 - j\left(tan(\beta_{1GHz}l)[z_0^2 + 4]\right) = 0 \tag{10}$$

The above equation is valid iff $z_0 = 0$ and $\beta_{1GHz}l = n\pi$. This means that,

$$z_0 = \frac{1}{50}\sqrt{\frac{L_0}{C_0}} = 0 \tag{11}$$

$$2\pi.10^9.\sqrt{L_0 C_0}l = n\pi \tag{12}$$

The above two equation implies that $\boxed{L_0 = 0}$.

$$\boxed{Z_0 = 0\Omega; t_{PD} = 0s} \tag{13}$$