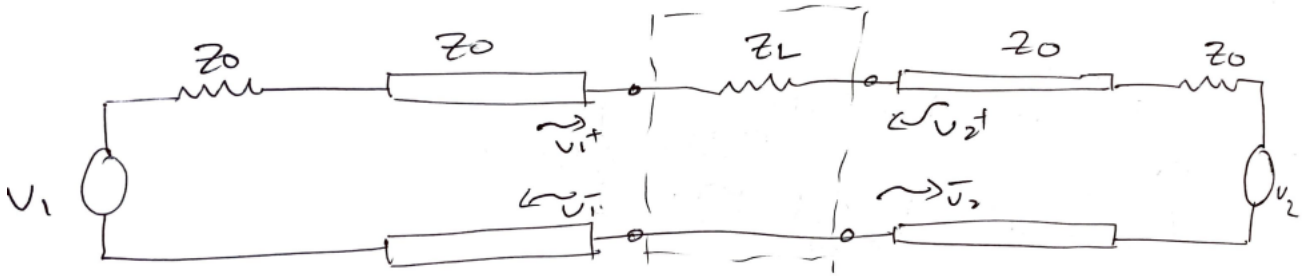


ASSIGNMENT 6

TADIPATRI UDAY KIRAN REDDY
EE19BTECH11038

April 16, 2022

Problem 1



$$V_2^+ = 0$$

$$\Gamma_{L1} = \frac{\frac{Z_L + Z_0}{Z_0} - 1}{\frac{Z_L + Z_0}{Z_0} + 1} = \frac{Z_L}{Z_L + 2Z_0} \quad (1)$$

$$\Rightarrow \frac{V_1^-}{V_1^+} = S_{11} = \frac{Z_L}{Z_L + 2Z_0} \quad (2)$$

Since source at Port 2 is matched network then,

$$V_2^- = (V_1^+ + V_1^-) \frac{Z_0}{Z_0 + Z_L} \quad (3)$$

$$\frac{V_2^-}{V_1^+} = S_{12} = (S_{11} + 1) \frac{Z_0}{Z_0 + Z_L} \quad (4)$$

$$\Rightarrow S_{12} = \frac{2Z_0}{Z_L + 2Z_0} \quad (5)$$

$$V_1^+ = 0$$

The above network is clearly symmetric which scattering matrix is symmetric.

$$\mathbf{S} = \begin{bmatrix} \frac{Z_L}{Z_L + 2Z_0} & \frac{2Z_0}{Z_L + 2Z_0} \\ \frac{2Z_0}{Z_L + 2Z_0} & \frac{Z_L}{Z_L + 2Z_0} \end{bmatrix} \quad (6)$$

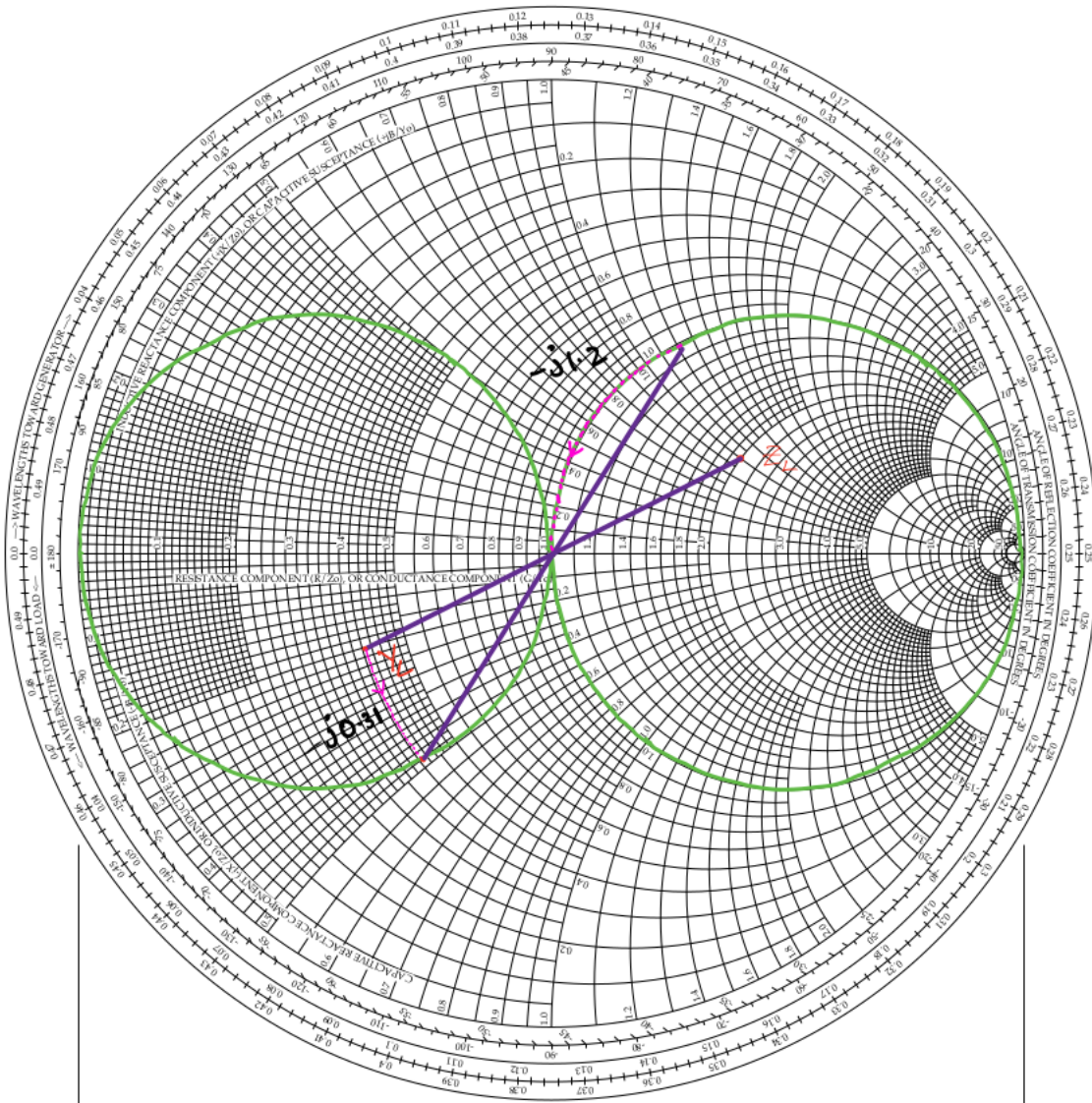
In this case we get,

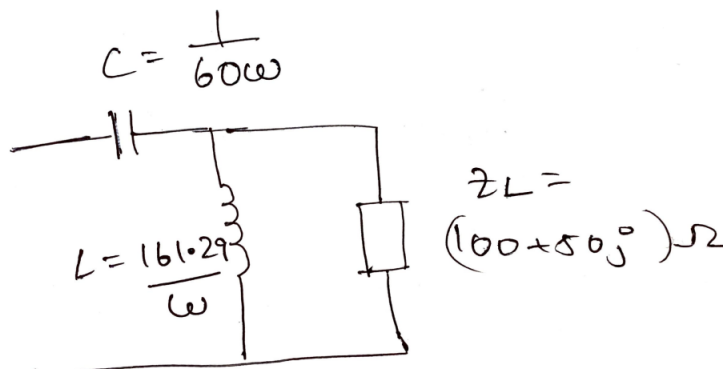
$$\mathbf{S} = \begin{bmatrix} 0.0476 & 0.9524 \\ 0.9524 & 0.0476 \end{bmatrix}$$

(7)

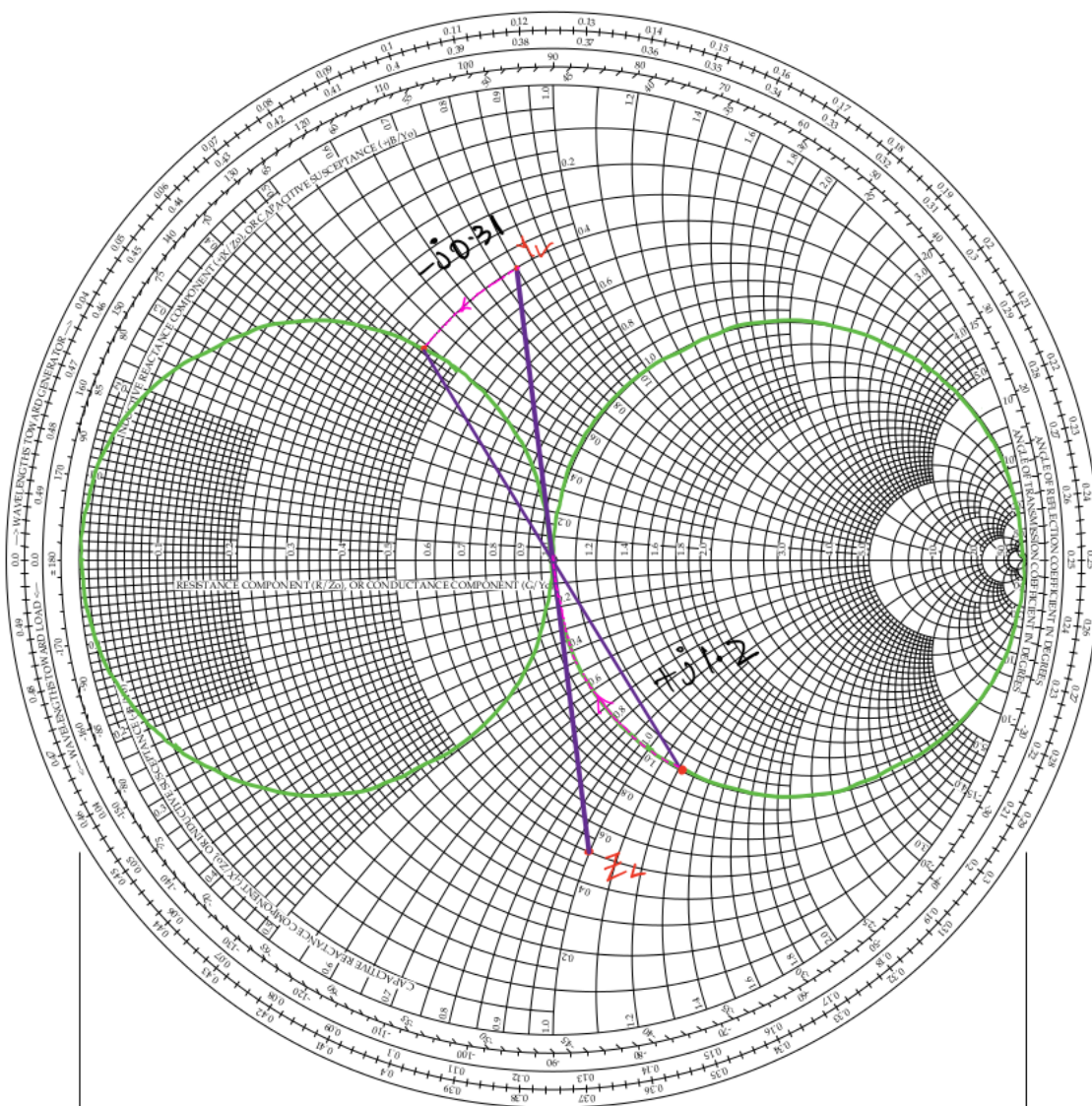
Problem 2

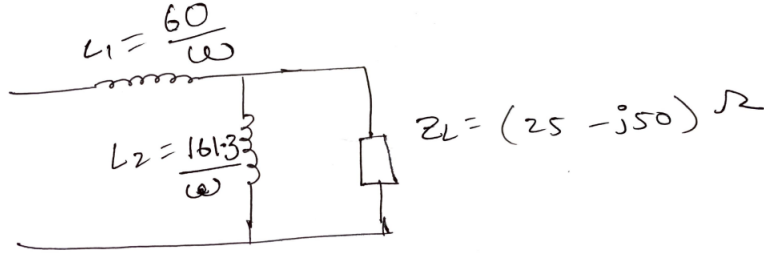
(a)





(b)





Problem 3

At DC we see that $Z_{in} = Z_L = 50\Omega$ which means the line is lossless $\implies \alpha = 0$.

At 1GHz when lossless,

$$Z_{in}(x) = Z_0 \left[\frac{Z_L + Z_0 j \tan(\beta_{1GHz} l)}{Z_0 - j Z_L \tan(\beta_{1GHz} l)} \right] \quad (8)$$

$$4 = z_0 \left[\frac{1 + z_0 j \tan(\beta_{1GHz} l)}{z_0 - j \tan(\beta_{1GHz} l)} \right] \quad (9)$$

$$\implies 3z_0 - j (\tan(\beta_{1GHz} l) [z_0^2 + 4]) = 0 \quad (10)$$

The above equation is valid iff $z_0 = 0$ and $\beta_{1GHz} l = n\pi$. This means that,

$$z_0 = \frac{1}{50} \sqrt{\frac{L_0}{C_0}} = 0 \quad (11)$$

$$2\pi \cdot 10^9 \cdot \sqrt{L_0 C_0} l = n\pi \quad (12)$$

The above two equation implies that

$$L_0 = 0$$

$$Z_0 = 0\Omega; t_{PD} = 0s \quad (13)$$