ASSIGNMENT 8

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Problem 1

$$V_{in} = AV_{in}^3$$

$$20log_{10}Aa^3 = -0.5 \implies a = \boxed{P_{0.5dB} = \sqrt[3]{\frac{0.944}{|A|}}}$$

Problem 2

Given $V_o = A_1V_{in} + A_2V_{in}^2 + A_3V_{in}^3 + A_4V_{in}^4 + A_5V_{in}^5$. Even powers have even harmonics and odd powers have odd harmonics. Thus we consider only odd power.

$$cos^3(\omega t) = \frac{3}{4}cos(\omega t) + \frac{1}{4}cos(3\omega t)$$
$$cos^5(\omega t) = \frac{5}{8}cos(\omega t) + \frac{5}{16}cos(3\omega t) + \frac{1}{16}cos(5\omega t)$$

If $V_{in} = acos(\omega t)$,

$$V_o = A_1a + A_3\frac{3}{4}a^3 + A_5\frac{5}{8}a^5 \implies G = A_1 + A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^4$$
$$20log_{10}\left(A_1 + A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^4\right) = 20log_{10}(A_1) - 1$$
$$\implies 1 + \frac{A_3}{A_1}\frac{3}{4}a^2 + \frac{A_5}{A_1}\frac{5}{8}a^4 = 0.891$$

$$\implies 0.108 + \frac{A_3}{A_1}\frac{3}{4}a^2 + \frac{A_5}{A_1}\frac{5}{8}a^4 = 0 \implies a = \boxed{P_{1db} = \sqrt{\frac{-6A_3 \pm \sqrt{36A_3^2 - 17.4A_1A_5}}{10A_5}}}$$

For IIP3,

$$IM3 = \frac{A_1}{A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^5}$$

$$\implies -A_1 + A_3\frac{3}{4}a^2 + A_5\frac{5}{8}a^4 = 0 \implies \boxed{IIP3 = \sqrt{\frac{-6A_3 \pm \sqrt{36A_3^2 + 2.5A_1A_5}}{10A_5}}}$$

Problem 3

Given, $I_{DS} = \frac{k_n}{2}(V_{DC} + V_{in} - I_{DS}R_S - V_T)^2$. We would like to find coefficients of the equation,

$$i_{ds} = A_1 V_{in} + A_2 V_{in}^2 + A_3 V_{in}^3$$

Where, $A_1 = \frac{\partial i_{ds}}{\partial V_{in}}$, $A_2 = \frac{1}{2}\frac{\partial^2 i_{ds}}{\partial V_{in}^2}$ and $A_3 = \frac{1}{6}\frac{\partial^3 i_{ds}}{\partial V_{in}^3}$

$$\frac{\partial i_{ds}}{\partial V_{in}} = k_n(V_{GS} - V_T)\left(1 - \frac{\partial i_{ds}}{\partial V_{in}} R_s\right)$$

Let $k_n(V_{DC} + V_{in} - I_{DS}R_S - V_T) = \alpha$; $\implies \frac{\partial \alpha}{\partial V_{in}} = k_n$, then,

$$\frac{\partial i_{ds}}{\partial V_{in}} = \frac{\alpha}{1+\alpha}$$

$$\frac{\partial^2 i_{ds}}{\partial V_{in}^2} = \frac{\frac{\partial \alpha}{\partial V_{in}}}{(1+\alpha)^2} \implies \frac{\partial^2 i_{ds}}{\partial V_{in}^2} = \frac{k_n}{(1+\alpha)^3}$$

$$\frac{\partial^3 i_{ds}}{\partial V_{in}^3} = \frac{-3k_n \frac{\partial \alpha}{\partial V_{in}}}{(1+\alpha)^4} \implies \frac{\partial^3 i_{ds}}{\partial V_{in}^3} = \frac{-3k_n^2}{(1+\alpha)^5}$$

$$\implies i_{ds} = \frac{\alpha}{1+\alpha} V_{in} + \frac{k_n}{2(1+\alpha)^3} V_{in}^2 - \frac{k_n^2}{2(1+\alpha)^5} V_{in}^3$$

$$\implies \boxed{P_{1dB} = \sqrt{0.29}\frac{(1+\alpha)^2}{k_n}}$$

$$\implies \boxed{IIP3 = \sqrt{\frac{8}{3}}\frac{(1+\alpha)^2}{k_n}}$$

Problem 4

Here, $V_o = (I_- - I_+)R_L$, now applying same square law here,

$$V_o = \frac{k_n}{2} R_s((V_{cm} - V_T + \frac{V_{in}}{2})^2 - (V_{cm} - V_T - \frac{V_{in}}{2})^2)$$

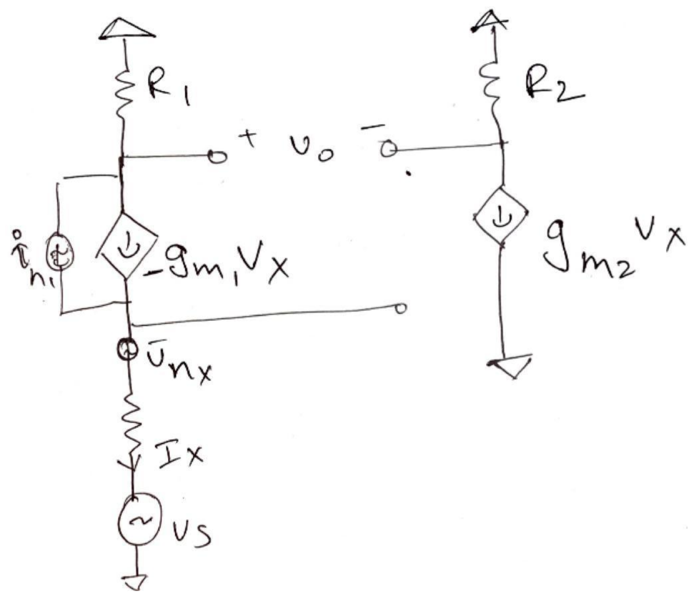
$$\implies V_o = \frac{1}{2} k_n R_S (V_{cm} - V_T) V_{in}$$

Since $(I_- + I_+)|_{V_{in}=0} = I_B \implies (V_{cm} - V_T) = \frac{I_B}{k_n}$.

$$V_o = \sqrt{k_n I_B} R_s V_{in}$$

Clearly there is no 3rd order component thus $P_{1dB} = \infty dB$ and $IIP3 = \infty dBm$.

Problem 5



$$V_x = V_s + R_s I_x = V_s - g_{m1} V_x R_s 1$$
$$\implies V_x = \frac{V_s}{1 + g_{m1} R_s}$$
$$I_x = -g_{m1} V_x$$

Input impedance here is $-V_x/I_x = 1/g_{m1}$. So if $\boxed{R_s = 1/g_{m1}}$ then the input is matched.

$$V_o = (g_{m1}R_1 + g_{m2}R_2)V_x$$
$$\implies V_o = \frac{g_{m1}R_1 + g_{m2}R_2}{1 + g_{m1}R_s} V_s$$
$$\boxed{Gain = \frac{g_{m1}R_1 + g_{m2}R_2}{1 + g_{m1}R_s}}$$

Now we find gains due to some noise causing elements,

$$\frac{\partial V_o}{\partial V_{n,x}} = g_{m1}R_1 + g_{m2}R_2$$
$$\frac{\partial V_o}{\partial V_{n,R_1}} = 1$$
$$\frac{\partial V_o}{\partial V_{n,R_2}} = -1$$
$$\frac{\partial V_o}{\partial i_{n2}} = -R_2$$

To find the gain through M1, this changes V_x and in turns V_{o-} changes along with V_{o+}.

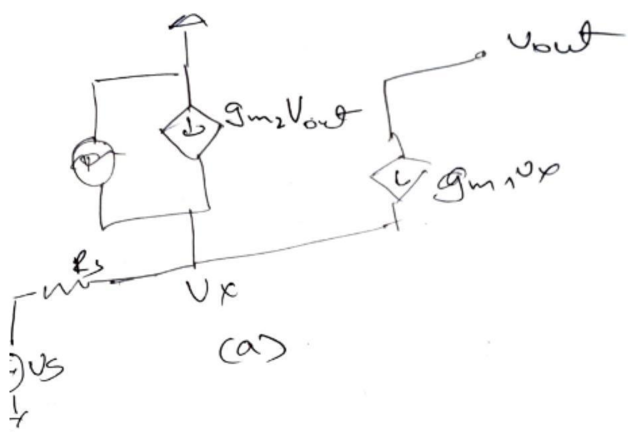
$$\Delta V_{o+} = i_{n1} R_1$$
$$\Delta V_{o-} = g_{m2} i_{n1} R_s R_2$$
$$\implies \frac{\partial V_o}{\partial i_{n1}} = R_1 - g_{m2} R_2 R_s$$

To nullify the gain we need $\boxed{R_2 = \frac{R_1}{g_{m2} R_s} = \frac{1}{g_{m1} g_{m2} R_s}}$.

$$NF = \frac{4KTR_s + 4KTR_1 + 4KTR_2 + 4KTr_2 g_{m2} R_2^2 + 4KTr_1 g_{m1}(R_1 - g_{m2} - R_2 R_s)^2}{4KTR_s}$$
$$\implies \boxed{NF = 1 + \frac{R_1}{R_s} + \frac{\gamma g_{m2} R_2^2}{R_s} + \frac{\gamma g_{m1}(R_1 - g_{m2} R_2 R_s)^2}{R_s}}$$

Problem 6

(a)



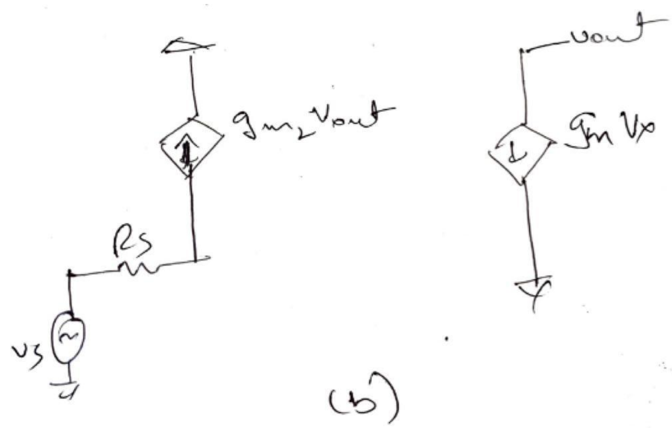
Clearly here noise generated by M2 must be compensated with small signal,

$$g_{m2} V_o = i_{n2} \implies V_o / i_{n2} = 1/g_{m2}$$

Since there is no gain by M1,

$$NF = 1 + \gamma g_{m2} R_s$$

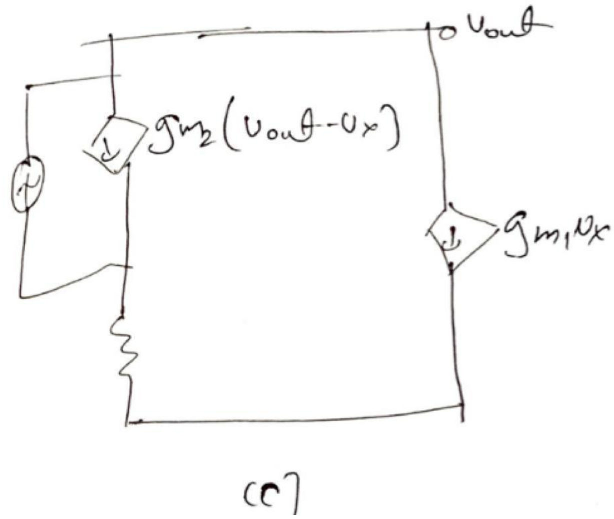
(b)



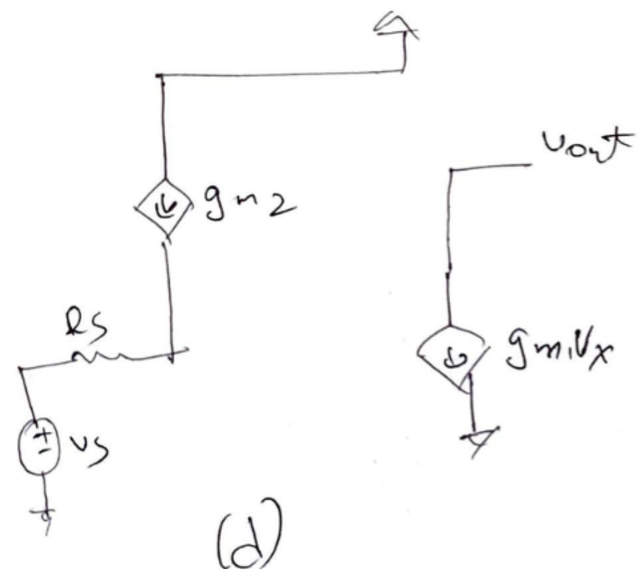
Here also it is the same case as before,

$$NF = 1 + \gamma g_{m2} R_s$$

(c)



(d)



Here apart from M2 even M1 comes into picture because of drain and source connection,

$$NF = 1 + \gamma g_{m2} R_s + \frac{\gamma}{g_{m1} R_s}$$