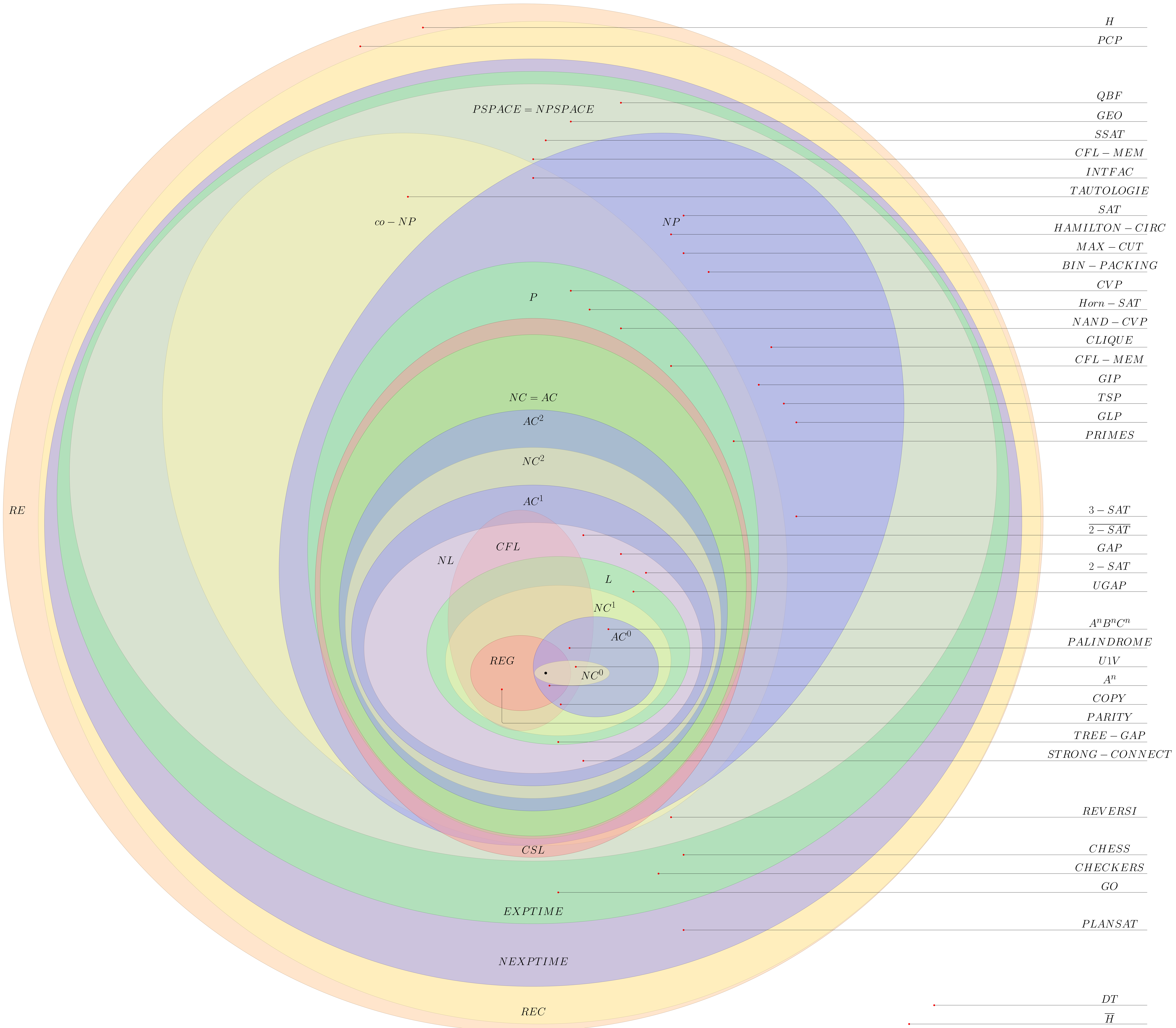

COMPLEXITY DIAGRAM

Definitions

The Comlexity Classes

- $NONE = \emptyset$
- $NC^0 =$ Functions computable by constant-depth, bounded-fanin circuits.
- $AC^0 =$ Class of decision problems solvable by a nonuniform family of Boolean circuits, with polynomial size, depth $O(log^i(n))$, and unbounded fanin.
- $REG = DSPACE(O(1))$: The decision problems that can be solved in constant space
- $NC^1 =$ Class of decision problems solvable by a nonuniform family of Boolean circuits, with polynomial size, depth $O(log(n))$, and fan-in 2.
- $CFL =$ Context-Free Languages
- $AC^1 =$ Class of decision problems solvable by a nonuniform family of Boolean circuits, with polynomial size, depth $O(log(n))$
- $L =$ Class of decision problems solvable by a Turing machine restricted to use an amount of memory logarithmic in the size of the input.
- $NL = coNL$ Class of decision problems solvable by a nondeterministic Turing machine restricted to use an amount of memory logarithmic in the size of the input.
- $NC = AC$: Decidable in polylogarithmic time on a parallel computer with a polynomial number of processors.
- $CSL = NLINSPACE$ The class of languages generated by context-sensitive grammars.
- $P = Polysize(uniform) =$ The class of decision problems solvable in polynomial time by a Turing machine.
- NP = Nondeterministic Polynomial-Time
- $coNP$= Complement of NP
- $PSPACE = NPSPACE = AP = APP = IP$
- $EXPTIME = \bigcup_{k\in\mathbb{N}} DTIME(2^{n^k})$
- $NEXPTIME = \bigcup_{k\in\mathbb{N}} NTIME(2^{n^k})$
- REC= Class of decidable Problems
- RE is the class of decision problems for which a „yes“ answer can be verified by a Turing machine in a finite amount of time. (If the answer is „no,“ on the other hand, the machine might never halt.)

Some Missing Classes

- $EEXPTIME$
- $NEEXPTIME$
- $EXPSPACE$
- BPP
- BQP

The Languages

- $A^n = \{x|x \in \{a\}^*\} \in REG$
- $A^nB^n = \{a^nb^n|n \geq 1\} \in CFL$
- $A^nB^nC^n = \{a^nb^nc^n|n \geq 1\} \in CSL$
- $BIN - PACKING$ =Objects of different volumes must be packed into a finite number of bins of capacity V in a way that minimizes the number of bins used. $\in NP - c$
- $DT := \{i|\Phi_i^1 \text{is total}\} \notin RE$
- $CHECKERS =$ The generalized version of the game $\in EXPTIME - c$
- $CHESS =$ The generalized version of the game chess $\in EXPTIME - c$
- $CFL - MEM =$ Context Free Grammar Membership
- $CLIQUE =$ If a graph contains a clique of at least a given size k. $\in NP - c$
- $COPY = \{uv|u = v\}$
- $CVP =$ Circuit Value Problem $\in P - c$
  - $NAND - CVP = CVP$ only using NAND-components $\in P$
- $GAP = PATH = \{\langle D, s, t\rangle \,|\text{Dir. Graph } D \text{ has path from } s \text{to } t\} \in NL - c$
  - $UGAP = GAP$ in an underícted graph $\in L - c$
  - $TREE - GAP = GAP$ in a tree (DAG) $\in L - c$
- $GEO \in PSCACE - c$
- $GIP =$ Graph isomorphism problem: Determining whether two graphs are isomorphic $\in NP$
- $GLP = \{(A, b) \in \mathbb{Z}^{m,n} \times \mathbb{Z}^m | \exists x \in \mathbb{Z}^n : Ax \leq b\} \in NP - c$
- $GO =$ The generalized version of the japanese game Go $\in EXPTIME - c$
- $H := \{\langle i, x\rangle \,| x \in dom(\Phi_i^1)\} \in RE - c$
  - $\overline{H} = \{\langle i, x\rangle \,| x \notin H\} \notin RE$
- $HAMILTON - CIRC =$ If A graph contains a hamilton circle. $\in NP - c$
- $INTFAC =$ integer factorization is the process of breaking down a composite number into smaller non-trivial divisors, which when multiplied together equal the original integer. $\in NP \cap co - NP$
- $MAX - CUT =$ The Maximum Cut of a Graph. $\in NP - c$
- $PALINDROME =$ if a word is a palindrome. $\in CFL$
- $PARITY = \{a \in \{0, 1\}^* | \#1 \text{ is odd}\} \in REG$
- $PCP =$ Post correspondence problem $\in NP - c$
- $PLANSAT =$ set off all strings $s \subseteq L^*$ such that s is the statement of a solvable problem. $\in NEXPTIME - c$ (Without certain restrictions the problem is $\in EXPSPACE - c$.)
- $PRIMES =$ Determining whether an input number is prime. $\in P$
- $QBF =$ Satisfiability problem for quantified boolean formula $\in PSPACE - c$
- $REVERSI =$ Generalized Version of the board game. $\in PSPACE - c$
- $SAT =$ Boolean satisfiability problem $\in NP - c$
  - $2 - SAT =$ SAT with CNF with 2 variables per clause $\in NL - c$
  - $\overline{2 - SAT} = \{x|x \notin 2 - SAT\} \in NL - c$
  - $3 - SAT =$ SAT with CNF with 3 variables per clause $\in NP - c$
  - $Horn - SAT =$ SAT with a conjunction of Horn clauses. $\in P - c$
  - $SSAT =$ Probabilistic satisfiability problem $\in PSPACE - c$
- $STRONG - CONNECT \in NL$
- $TAUTOLOGY =$ If a given boolean formula is a tautology. $\in co - NPc$
- $TSP =$ Traveling sales person $\in NP - c$
- $U1V = \{u1v||u| = |v|\}$

Relations

Unsolved Problems

- $NL \subseteq EXPTIME$
  This includes the questions of „$NL \neq P$?“, „$P \neq NP$?“, „$PSAPCE \neq P$?“ and „$EXPTIME \neq PSAPCE$?“
- Is NC propper? $\Leftrightarrow$ $NC^i \neq NC^{i+1}$? (and is AC propper?)
- Is EXPTIME=NEXPTIME? (A direct result of P=NP would be EXPTIME=NEXPTIME)

Known Subsets

- Chomsky hierarchy:
  $REG = TYPE3 \subset DCFL \subset CFL = TYPE2 \subset CSL = TYPE1 \subset RE = TYPE0$
- $NC^0 \subset AC^0 \subset NC^1 \subset L \subseteq NL \subseteq NC = AC \subseteq P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE = NEXPSAPCE \subseteq REC \subseteq RE$
- $NL \subsetneq PSPACE \subsetneq EXPSPACE$
- $NC \subsetneq PSPACE$ and $NC \subseteq CSL$

Savich's Theorem

For any function $f(n) \geq log(n)$: $DSPACE(f(n)) \subseteq NSPACE(f(n))$
$\Rightarrow$ $L^2 = NL$, $PSPACE = NPSPACE$, $EXPSPACE = NEXPSPACE$, ...