# RESEARCH ARTICLE

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# An off-axis Hartmann sensor for the measurement of absorption-induced wavefront distortion in advanced gravitational wave interferometers

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**Abstract** We describe a novel off-axis Hartmann wavefront sensor, developed for the measurement of wavefront distortions induced in the mirrors (test masses) of advanced gravitational wave interferometers by residual absorption of the circulating laser power.

 $\textbf{Keywords} \ \ \text{Hartmann sensor} \ \cdot \ \ \text{Wavefront distortion} \ \cdot \ \ \text{Gravitational wave}$  interferometers

### 1 Introduction

Advanced gravitational wave interferometers will have circulating optical powers of order a few kW in the power-recycling cavity and about 500 kW in the arm cavities [1]. Even extremely small absorption by the substrates and coatings of the optical components in these cavities will result in thermal gradient within the optics, which will lead to significant distortion of the optical wavefront via the thermo-optic and elasto-optic effects and thermo-elastic deformation [2].

Absorption in the substrates of the input test masses (ITM), which are inside the power-recycling cavity, and in the coatings of the ITM and end test masses (ETM), which reflect the power in the arm cavities, are two particularly important sources of wavefront distortion [1]. These distortions could significantly reduce the sensitivity of an advanced interferometer by reducing the sideband power build-up in the power-recycling cavity and increasing the carrier power leaking out the dark port of the interferometer [3]. Thus, their effects must be compensated [3], which requires that the distortion in each mirror be measured independently, without interfering with the circulating power in the interferometer.

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**Fig. 1** A schematic of a prototypical Hartmann sensor. The Hartmann plate contains an array of apertures that produce a set of Hartmann rays. The transverse aberration of the Hartmann rays at the CCD is used to calculate the change in local slope of the wavefront due to the optic.

The High Power Test Facility (HPTF), a collaboration between the Australian Consortium for Gravitational Astronomy (ACIGA) and the LIGO project, located at Gingin in Western Australia, will be used to investigate the effect of the absorption in substrates and coatings in optical cavities that have high circulating power. The optical configuration of this facility has been designed to yield wavefront distortions with parameters that are directly relevant to advanced interferometers. Initially, we shall measure the effect of absorption in the substrate of the ITM, and investigate its compensation [4].

We are developing a novel off-axis Hartmann sensor that can be used to measure wavefront distortion without interfering with the circulating power. Hartmann sensors [5, 6], one variant of which is shown in Fig. 1, measure wavefront distortion by recording the positions of the "Hartmann rays" at the CCD camera before and after the distortion is introduced. Since the measured transverse displacement or aberration of each ray is proportional to the change in slope of the wavefront at the point at which the ray passes through the optic [7], the wavefront distortion can be determined by integration.

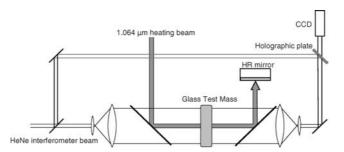
We have chosen to use a Hartmann sensor rather than a Shack–Hartmann sensor [8] to monitor the wavefront distortion. It does not require a micro-lens array, which could introduce interference between adjacent rays, as the light source is coherent, and it can have larger beam sizes at the CCD thereby reducing the effect of irregularities in pixel responsivity. Further, the effective optical 'lever arm' in a Hartmann sensor could be larger than for a Shack–Hartman sensor if a large area CCD were used, thereby further improving the sensitivity. The poorer spatial resolution of the Hartmann sensor is unimportant in this application as the wavefront distortion occurs over an area typically 2 cm diameter in the HPTF test and 10–20 cm diameter in an advanced interferometer, and is not expected to contain high spatial frequencies.

In this paper, we outline the development of the Hartmann sensor, including validation of the sensor, and preliminary measurements.

### 2 Hartmann sensor validation

The Hartmann sensor is validated by comparing the wavefront distortion measured using a stored-beam holographic interferometer [9], shown in Fig. 2, with that measured using an on-axis Hartmann sensor, shown in Fig. 3, and the off-axis Hartmann sensor, shown in Fig. 4.

In a stored-beam interferometer, the holographic beam-splitter is recorded while the heating beam is blocked, thereby recording and thus removing the effect



**Fig. 2** A schematic of the stored-beam holographic interferometer. The holographic beam-splitter is recorded while the heating beam is blocked. The output surface of the glass test mass is imaged onto the CCD. Note that the glass used for the test mass is chosen to have a low absorption and the heating beam is retro-reflected through the test mass to provide a power absorption that is almost independent of the axial position. The glass was chosen to produce the distortion encountered in an advanced interferometer.

of the non-common static aberrations in the arms of the interferometer. Replaying the hologram while the heating beam is blocked results in a zero interference fringe as the beams transmitted through the hologram are identical to the holographically reconstructed beams. Unblocking the heating beam produces additional wavefront distortion as the interferometer beam passes through the glass test mass and thus the transmitted and reconstructed beams differ. The resultant interference pattern can be measured accurately by tilting one of the beams incident on the hologram prior to unblocking the heating beam, thereby introducing straight interference fringes. The added distortion is then determined by measuring the displacement of the fringes.

As indicated above, the reference spot positions for the Hartmann sensor are recorded with the heating beam blocked. It is then unblocked and the aberrated spot positions are recorded. An example of the calculated transverse aberrations is shown in Fig. 5. The ratio of the transverse aberration to the distance between the test mass and the CCD camera gives the change in the local slope of the wavefront, and is equal to the gradient of the wavefront distortion [7].

Since the Hartmann rays in the off-axis sensor propagate at an angle to the cylindrical axis of the test mass, the transverse aberration of each ray will not simply reflect the wavefront distortion that would be experienced by the eigenmode

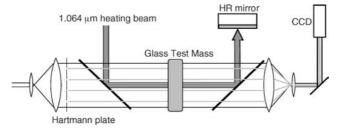


Fig. 3 A schematic of the on-axis Hartmann sensor. The reference spot positions are recorded with the heating beam blocked. It is then unblocked and the aberrated spot positions are then recorded.

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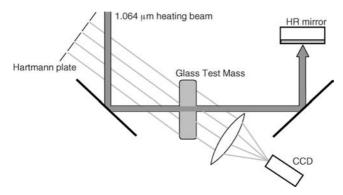


Fig. 4 A schematic of the off-axis Hartmann sensor.

of the interferometer, which propagates axially though the test mass. The distortion can be estimated, however, if it is assumed that the temperature increase due to the absorption is independent of the axial position in the cylindrical test mass, as occurs for substrate absorption in the ITM of advanced interferometers and in our tests. Then, the optical path difference (OPD) acquired by the Hartmann ray shown in Fig. 6 can be written, for small distortions:

$$OPD = \frac{\mathrm{d}n}{\mathrm{d}T} \int_{z=-h/2}^{z=h/2} \Delta T \left( x \left( z \right), y \left( z \right) \right) \sqrt{\tan^2 \theta + 1} \, \mathrm{d}z \tag{1}$$

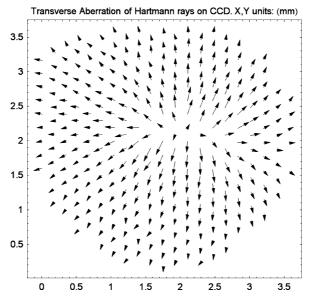
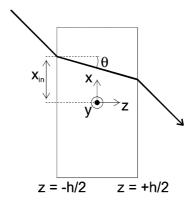


Fig. 5 An example of the measured transverse aberrations, shown as a vector field, recorded by the off-axis Hartmann sensor.



**Fig. 6** The geometry used to determine the expected OPD of a particular Hartmann ray. The ray enters the test mass at  $(x_{in}, y_{in}, z = -h/2)$ . Transverse aberrations are measured in the coordinate frame of the CCD camera and transformed via a suitable rotation into the coordinate frame of the test mass.

where the temperature increase is given by

$$\Delta T(x, y) = \sum_{i=0}^{N} A_i (x^2 + y^2)^i,$$
 (2)

and it is assumed that the distortion does not change the path within the test mass and thus  $x(z) = z \tan \theta + x_{in}$  and  $y(z) = y_{in}$ .

Since the transverse aberration is proportional to the gradient of the OPD, we can use the measured transverse aberrations to determine the best estimate of the polynomial coefficients by defining

$$\chi^{2} = \left[ \left( \frac{\partial OPD}{\partial x} - TA_{x} \right)^{2} + \left( \frac{\partial OPD}{\partial y} - TA_{y} \right)^{2} \right]$$
 (3)

where  $TA_x$  and  $TA_y$  are the x and y components of the measured transverse aberration. The usual 'least squares' minimization of  $\chi^2$  is then used to determine the best estimate of the coefficients  $A_i$ .

A preliminary comparison of the wavefront distortion that would be experienced by a laser beam propagating along the axis of the test mass, as determined using the off-axis Hartmann sensor and the stored-beam interferometer is shown in Fig. 7. The discrepancy between the two curves in this result is believed due to the very noisy preliminary Hartmann data, caused by the poor dynamic range of the CCD camera used, which significantly limited the precision with which the centroids of the spots could be calculated.

## 3 Summary

We have described an off-axis Hartmann sensor for measuring wavefront distortion induced by absorption in the substrate of the input test masses of an advanced gravitational wave interferometer. We have shown how the wavefront distortion

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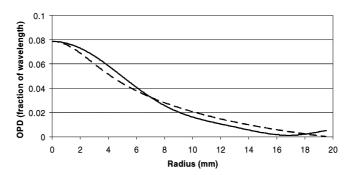


Fig. 7 A comparison of the wavefront distortion that would be experienced by a laser beam propagating along the axis of the test mass, as determined using the stored-beam interferometer (dashed line) and the off-axis Hartmann sensor (solid line). A HeNe laser,  $\lambda = 632.8$  nm, was used for the measurement.

that would be experience by a laser beam propagating along the axis of the test mass can be extracted from the transverse aberrations measured using an off-axis Hartmann sensor. A preliminary measurements showing that the wavefront distortion measured using an off-axis Hartmann sensor is consistent with that measured using an interferometer has also been presented. We are currently replacing the CCD camera used for the Hartmann sensor.

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