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Analytical models of thermal aberrations in massive mirrors heated by high power laser beams

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Résumé. — Nous traitons le problème de l'échauffement des supermiroirs par des sources laser intenses. L'échauffement vient de l'absorption dans le substrat de silice ou de la dissipation dans les couches réfléchissantes. Dans chacun de ces cas, nous construisons des modèles analytiques originaux pour la température transitoire et les aberrations thermiques qui en résultent. Nous présentons des formules analytiques, basées sur des séries rapidement convergentes, et des résultats numériques concernant l'évolution de la température, et le développement des aberrations en séries de polynômes de Zernike.

Abstract. — We consider the problem of massive mirrors heated by intense laser sources. Heating occurs both by absorption in the silica substrate and by dissipation in the coating. In each case we derive original analytical models of the transient temperature and of the resulting thermal aberrations. Analytical formulas, based upon rapidly converging series, and numerical results are presented for the time dependent temperature and for the time dependent expansion of the aberrations in series of Zernike polynomials.

1. Introduction.

The heating of mirrors or other cylindrical solids like YAG rods has been yet studied in various experimental conditions and approximation schemes. But the resulting type of model depends strongly on the boundary conditions as well as on the adopted simplifications.

Some authors consider two or three-dimensional models but restrict themselves to steady state temperature distributions [1-8]. Others take time dependence into account but consider only one or two dimensions, building up simple models of plates, laser windows or slabs... [9-10]. Some rare authors finally consider three-dimensional mirrors and include the time dependences by using Green's integrals [11-13].

In this paper, we present for the first time an explicit analytical model including finite dimensions of the heated solid, the time dependence and radiation or convective losses.

2. Physical conditions.

The starting point of our work is the study of the thermoelastic behavior of the mirrors involved in interferometric gravitational antennas.

The details of these antennas planned for the foregoing years are now widely known. In the Italian and French VIRGO project as well as in the American LIGO project, a key subsystem is a Fabry-Pérot cavity with a long storage time, i.e. with a high surtension coefficient. In the required conditions for operation at an acceptable signal-to-noise ratio, hundreds of watts light power will cross the input mirror, and tens of kilowatts will be continuously reflected on its internal coated face.

Obviously, materials with very low intrinsic absorption, and very low losses coatings are to be used. Nevertheless, it is likely that at least several watts will be dissipated in the substrate and the coating, resulting in temperature gradients. These temperature gradients will in turn result in index gradients and geometrical alteration of the reflecting face, and finally, the optical properties of the mirror will be affected.

The requirements on optical surfaces and bulk index homogeneity being severe, an evaluation of the thermal behavior of the mirrors seems therefore necessary. Moreover, time-dependent solutions are especially useful in a dynamical model of a Fabry-Pérot cavity with a high surtension coefficient for example.

The mirrors that we consider are thick cylindrical blocks of pure silica, with a high reflectance coating on the inner face, and eventually an antireflecting coating on the outer face. Light power is thus expected to be dissipated by bulk absorption in the silica substrate, and by the coating losses. These mirrors are supposed to be suspended by thin wires in a vacuum vessel so that the heat losses are only due to thermal radiation (see Fig. 1).

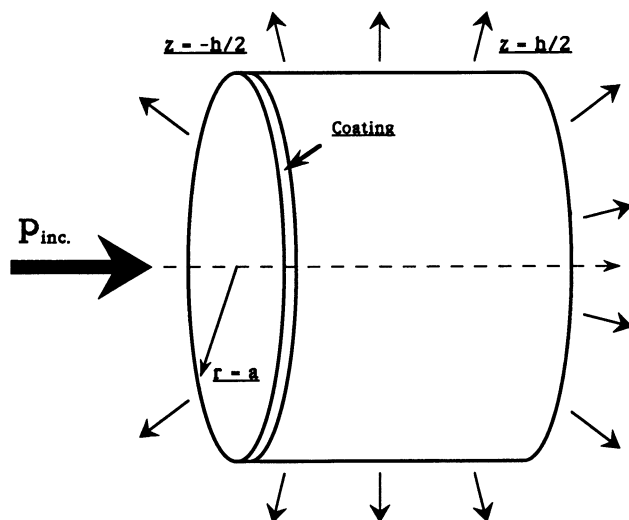


Fig. 1. — Sketch of the mirror. The incoming light can be absorbed either by the substrate (bulk absorption) or in the coating in the left face (coating absorption) ; the mirror losses heat only by thermal radiation.

We concern ourselves with the steady state and the transient temperature distribution in the mirrors with the special assumption of an axially symmetrical system (mirror-light beam), and within the approximation of low heating. Low heating is the restriction that the highest

temperature T_{\max} in the mirror is not much higher than the temperature T_{ext} (≈ 300 K) of the walls of the vacuum vessel :

$$\frac{T_{\max} - T_{\text{ext}}}{T_{\text{ext}}} \ll 1.$$

It is well known that the radiative heat losses of a surface element at temperature T are given by :

$$F = \sigma' [T^4 - T_{\text{ext}}^4]$$

where σ' is the Stefan-Boltzmann constant corrected for emissivity. With the above assumption, this becomes a linear expression :

$$F = 4 \sigma' T_{\text{ext}}^3 \delta T \quad (\delta T = T - T_{\text{ext}}).$$

The whole problem becomes thus linear, and we can treat separately the different contributions to heating : dissipation in the coating and bulk absorption. Note that convective heat losses can also be treated by the model, according to the above expression for F ; the coefficient of δT has simply to be modified.

3. Heating by absorption of power in the coatings.

3.1 STEADY STATE SOLUTION. — We consider a disk of radius a and thickness h . The radial coordinate is $0 \leq r \leq a$ and the axial coordinate $-h/2 \leq z \leq h/2$ (Fig. 1). The incoming light power is supposed to be dissipated in a thin layer on the $z = -h/2$ face, and we shall represent it by an incoming heat flux at $z = -h/2$. With no internal generation of heat, the Fourier equation is

$$\rho C \frac{\partial T}{\partial t} - K \Delta T = 0 \quad (1)$$

where $T(t, r, z)$ is the temperature distribution (K), we define in the following table the different coefficients and their numerical values in the case of silica :

ρ : density ($2\,202 \text{ kg m}^{-3}$),

C : specific heat ($745 \text{ J kg}^{-1} \text{ K}^{-1}$),

K : thermal conductivity ($1.38 \text{ W m}^{-1} \text{ K}^{-1}$).

We first consider the steady state solution of the form

$$T(r, z) = T_{\text{ext}} + T_{\infty}(r, z)$$

for which the boundary conditions are :

$$-K \frac{\partial T_{\infty}}{\partial r}(a, z) = 4 \sigma' T_{\text{ext}}^3 T_{\infty}(a, z) \quad (2)$$

$$-K \frac{\partial T_{\infty}}{\partial z}\left(r, -\frac{h}{2}\right) = \varepsilon I(r) - 4 \sigma' T_{\text{ext}}^3 T_{\infty}\left(r, -\frac{h}{2}\right) \quad (3)$$

$$-K \frac{\partial T_{\infty}}{\partial z}\left(r, \frac{h}{2}\right) = 4 \sigma' T_{\text{ext}}^3 T_{\infty}\left(r, \frac{h}{2}\right) \quad (4)$$

where $I(r)$ is the intensity distribution in the incoming light beam (W m^{-2}), and ε the efficiency of the conversion of light into heat power in the coating. In the steady state case, we have simply :

$$\Delta T_{\infty}(r, z) = 0. \quad (5)$$

A sufficiently general harmonic function solution of equation (5) may be expressed as :

$$T_{\infty}(r, z) = \sum_m [A_m e^{k_m z} + B_m e^{-k_m z}] J_0(k_m r)$$

where the constant k_m will be determined from condition (2) and constants A_m and B_m from conditions (3) and (4).

Equation (2) is satisfied if :

$$K k_m J_1(k_m a) = 4 \sigma' T_{\text{ext}}^3 J_0(k_m a) .$$

Let's call ζ_m the m -th solution of :

$$x J_1(x) - \tau J_0(x) = 0 \quad (6)$$

where we have introduced the reduced radiation constant :

$$\tau = 4 \sigma' T_{\text{ext}}^3 a / K .$$

The ζ_m may be obtained numerically by standard techniques, then

$$k_m = \zeta_m / a .$$

It is well known from the Sturm-Liouville problem that the functions

$$\left\{ J_0 \left(\zeta_m \frac{r}{a} \right), m \in \mathbb{N} \right\}$$

form an orthogonal basis for functions defined within the interval $[0, a]$ with normalization constants [14] :

$$\frac{a^2}{2 \zeta_m^2} [\tau^2 + \zeta_m^2] J_0(\zeta_m)^2 . \quad (7)$$

Then we can express $I(r)$ as a Dini series :

$$I(r) = \sum_m p_m J_0 \left(\zeta_m \frac{r}{a} \right)$$

so that equations (3) and (4) become :

$$\begin{cases} (\zeta_m - \tau) \gamma_m^2 A_m - (\zeta_m + \tau) B_m = - \frac{\varepsilon p_m a \gamma_m}{K} \\ (\zeta_m + \tau) A_m - (\zeta_m - \tau) \gamma_m^2 B_m = 0 \end{cases}$$

with

$$\gamma_m = \exp(-\zeta_m h / 2 a)$$

then we find

$$A_m = \frac{\varepsilon p_m a}{K} e^{-3 \zeta_m h / 2 a} \frac{\zeta_m - \tau}{(\zeta_m + \tau)^2 - (\zeta_m - \tau)^2 e^{-2 \zeta_m h / a}}$$

$$B_m = \frac{\zeta p_m a}{K} e^{-\zeta_m h / 2 a} \frac{\zeta_m + \tau}{(\zeta_m + \tau)^2 - (\zeta_m - \tau)^2 e^{-2 \zeta_m h / a}}$$

from which the solution is completely determined :

$$T_{\infty}(r, z) = \sum_m \frac{\varepsilon p_m a}{K} e^{-\zeta_m h/2a} \frac{(\zeta_m - \tau) e^{-\zeta_m(h-z)/a} + (\zeta_m + \tau) e^{-\zeta_m z/a}}{(\zeta_m + \tau)^2 - (\zeta_m - \tau)^2 e^{-2\zeta_m h/a}} J_0\left(\zeta_m \frac{r}{a}\right). \quad (8)$$

In the case of a Gaussian beam of waist w and power P :

$$I(r) = \frac{2P}{\pi w^2} e^{-2r^2/w^2} \quad (9)$$

in our numerical calculations $w = 2$ cm according to the Virgo specifications. It is easy to compute the coefficients p_m by the formula

$$p_m = \frac{2\zeta_m^2}{\zeta_m^2 + \tau^2} \frac{1}{J_0(\zeta_m)^2} \frac{1}{a^2} \int_0^a I(r) J_0\left(\zeta_m \frac{r}{a}\right) r dr. \quad (10)$$

If a is assumed much larger than the waist w , one finds :

$$p_m = \frac{P}{\pi a^2} \frac{\zeta_m^2}{(\zeta_m^2 + \tau^2) J_0(\zeta_m)^2} \exp\left[-\frac{1}{8} \zeta_m^2 \frac{w^2}{a^2}\right]. \quad (11)$$

In figure 2 we represent the stationary temperature distribution for $\varepsilon P = 1$ W absorbed power.

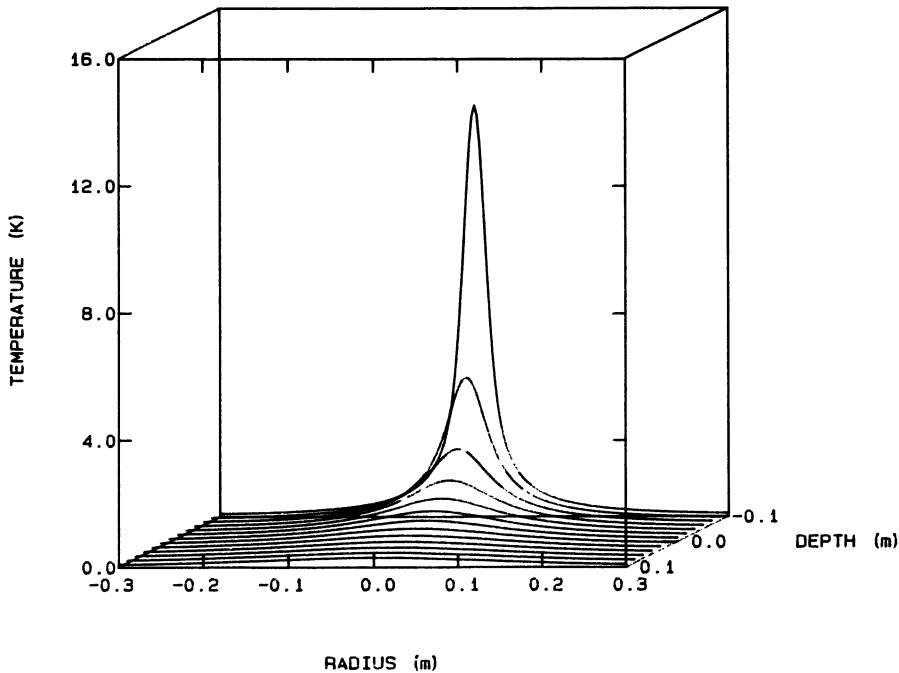


Fig. 2. — A map of the steady state temperature field for a coating absorption of 1 W. Note : for all the numerical results we have chosen $a = 0.3$ m and $h = 0.2$ m.

3.2 TRANSIENT SOLUTION.

3.2.1 *General principles.* — A time dependent solution of equation (1) may be written as :

$$T(t, r, z) = T_{\text{ext}} + T_{\infty}(r, z) + T_{\text{tr}}(t, r, z) \quad (12)$$

where $T_{\infty}(r, z)$ is the steady state solution found above, satisfying the conditions (2), (3) and (4). $T_{\text{tr}}(t, r, z)$ is solution of equation (1), and satisfies the homogeneous boundary conditions :

$$-K \frac{\partial T_{\text{tr}}}{\partial r}(a, z) = 4 \sigma' T_{\text{ext}}^3 T_{\text{tr}}(a, z) \quad (13)$$

$$-K \frac{\partial T_{\text{tr}}}{\partial z}(r, \pm h/2) = \pm 4 \sigma' T_{\text{ext}}^3 T_{\text{tr}}(r, \pm h/2). \quad (14)$$

But we add the Cauchy data :

$$T(0, r, z) = T_{\text{ext}} + T_0(r, z) \quad (15)$$

where $T_0(r, z)$ is an arbitrary function. The heat equation is now satisfied by a series of the form :

$$\begin{aligned} T_{\text{tr}}(t, r, z) = & \sum_{p, m} e^{-\alpha_{pm} t} A_{pm} \cos(u_p z/a) J_0(\xi_m r/a) \\ & + \sum_{p, m} e^{-\beta_{pm} t} B_{pm} \sin(v_p z/a) J_0(\xi_m r/a) \end{aligned} \quad (16)$$

with the time constants

$$\alpha_{pm} = \frac{K}{\rho C a^2} [u_p^2 + \xi_m^2], \quad \beta_{pm} = \frac{K}{\rho C a^2} [v_p^2 + \xi_m^2] \quad (17)$$

in the following we shall use the characteristic time t_c :

$$t_c = \frac{\rho C a^2}{K}.$$

The ξ_m having the same definition as above, the condition (13) is fulfilled. Now the conditions (14) give the dispersion equations for the even and odd modes respectively :

$$u = \tau \cot \left[\frac{uh}{2a} \right] \quad (18)$$

$$v = -\tau \tan \left[\frac{vh}{2a} \right]. \quad (19)$$

The constant u_p (resp. v_p) is the p -th solution of equation (18) (resp. (19)), the $\{u_p, v_p\}$ may be numerically computed with standard methods.

The functions

$$\left\{ \cos \left(u_p \frac{z}{a} \right), \sin \left(v_p \frac{z}{a} \right), p = 1, 2, \dots \right\}$$

form an orthogonal basis for functions defined within the interval $[-h/2, h/2]$.

We need the normalization constants s_p and c_p , defined by

$$\begin{aligned} \int_{-h/2}^{h/2} \sin^2 (v_p z/a) dz &= \frac{h}{2} \left[1 - \frac{a}{v_p h} \sin (v_p h/a) \right] = \frac{h}{2} s_p \\ \int_{-h/2}^{h/2} \cos^2 (u_p z/a) dz &= \frac{h}{2} \left[1 + \frac{a}{u_p h} \sin (u_p h/a) \right] = \frac{h}{2} c_p. \end{aligned} \quad (20)$$

The initial condition (15) is satisfied if

$$\sum_{p,m} [A_{pm} \cos (u_p z/a) + B_{pm} \sin (v_p z/a)] J_0(\xi_m r/a) = T_0(r, z) - T_{\text{ext}} - T_{\infty}(r, z).$$

Assuming that the right hand side of the preceding equation admits a Dini expansion :

$$\sum_m \theta_m(z) J_0(\xi_m r/a)$$

we may determine the constants A_{pm} , B_{pm} by

$$\begin{aligned} A_{pm} &= \frac{2}{hc_p} \int_{-h/2}^{h/2} \cos (u_p z/a) \theta_m(z) dz \\ B_{pm} &= \frac{2}{hs_p} \int_{-h/2}^{h/2} \sin (v_p z/a) \theta_m(z) dz. \end{aligned}$$

3.2.2 Heating from zero. — When the initial state is thermal equilibrium with uniform temperature equal to the surrounding vessel's (before switching on the laser), we have the special case $T_0(r, z) = T_{\text{ext}}$, and we obtain thus from equation (8) :

$$\theta_m(z) = - \frac{\varepsilon p_m a}{K} e^{-\xi_m h/2a} \frac{(\xi_m - \tau) e^{-\xi_m(h-z)/a} + (\xi_m + \tau) e^{-\xi_m z}}{(\xi_m + \tau)^2 - (\xi_m - \tau)^2 e^{-2\xi_m h/a}}.$$

A direct calculation using the dispersion equations gives

$$\begin{aligned} e^{\pm \xi_m z/a} &= \frac{2a}{h} [e^{\xi_m h/2a} (\xi_m + \tau) - e^{-\xi_m h/2a} (\xi_m - \tau)] \sum_{p>0} \frac{\cos (u_p h/2a)}{(\xi_m^2 + u_p^2) c_p} \cos (u_p z/a) \\ &\pm \frac{2a}{h} [e^{\xi_m h/2a} (\xi_m + \tau) + e^{-\xi_m h/2a} (\xi_m - \tau)] \sum_{p>0} \frac{\sin (v_p h/2a)}{(\xi_m^2 + v_p^2) s_p} \sin (v_p z/a) \end{aligned}$$

for

$$-\frac{h}{2} < z < \frac{h}{2}$$

and we find

$$A_{pm} = -2 \frac{\varepsilon p_m a^2 \cos (u_p h/2a)}{Kh (\xi_m^2 + u_p^2) c_p}, \quad B_{pm} = 2 \frac{\varepsilon p_m a^2 \sin (v_p h/2a)}{Kh (\xi_m^2 + v_p^2) s_p}$$

so that the time dependent temperature field may be expressed as

$$\begin{aligned} T(t, r, z) &= T_{\text{ext}} + 2 \frac{\varepsilon a^2}{Kh} \sum_{p,m} p_m [1 - e^{-\alpha_{pm} t}] \frac{\cos (u_p h/2a)}{(\xi_m^2 + u_p^2) c_p} \cos (u_p z/a) J_0(\xi_m r/a) \\ &- 2 \frac{\varepsilon a^2}{Kh} \sum_{p,m} p_m [1 - e^{-\beta_{pm} t}] \frac{\sin (v_p h/2a)}{(\xi_m^2 + v_p^2) s_p} \sin (v_p z/a) J_0(\xi_m r/a). \end{aligned} \quad (21)$$

In figure 3, we represent the temperature evolution of the warmest point ($z = -h/2$, $r = 0$) for 1 W absorbed power. The characteristic time is about 30 h. In figures 4 and 5 we show transversal and axial cuts at different times.

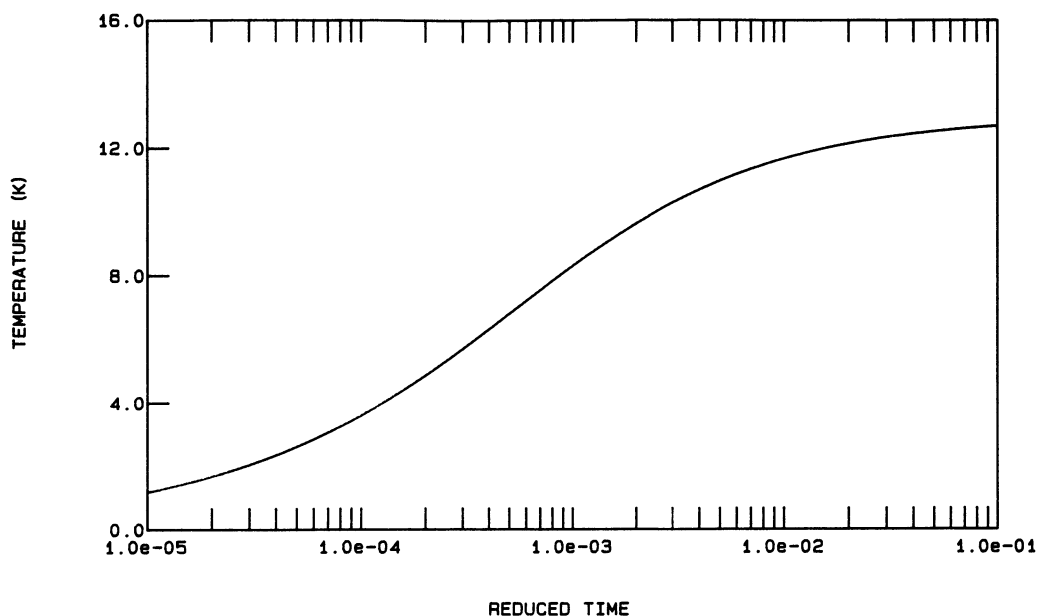


Fig. 3. — Evolution of the temperature of the « warm point » at the centre of the coating. The reduced time is t/t_c and the absorbed power is 1 W in the coating.

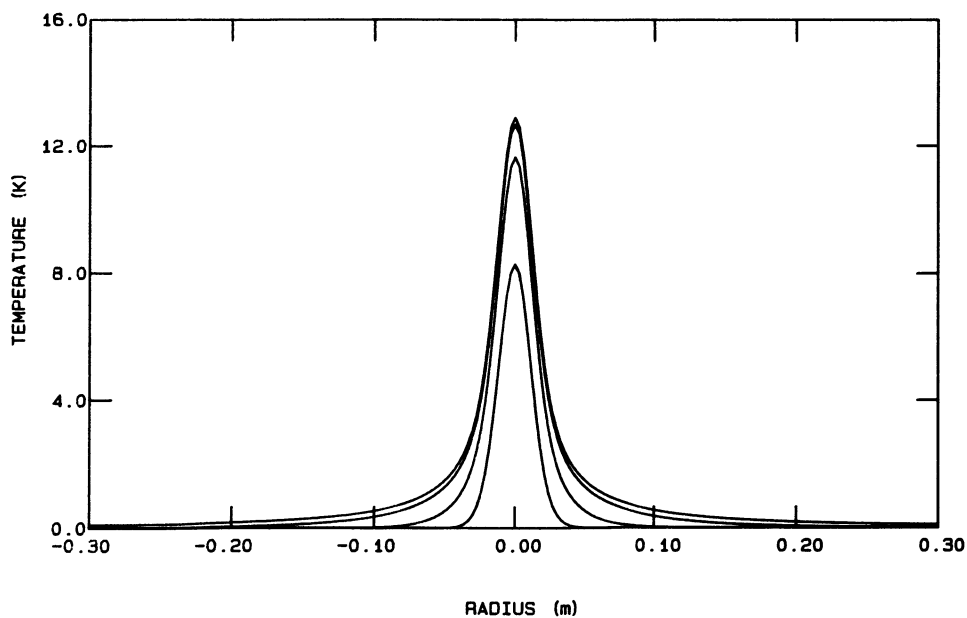


Fig. 4. — Successive transversal cuts of the temperature field in the coating plane at the times $t_c/1000$, $t_c/100$, $t_c/10$ and t_c for 1 W absorbed in the coating.

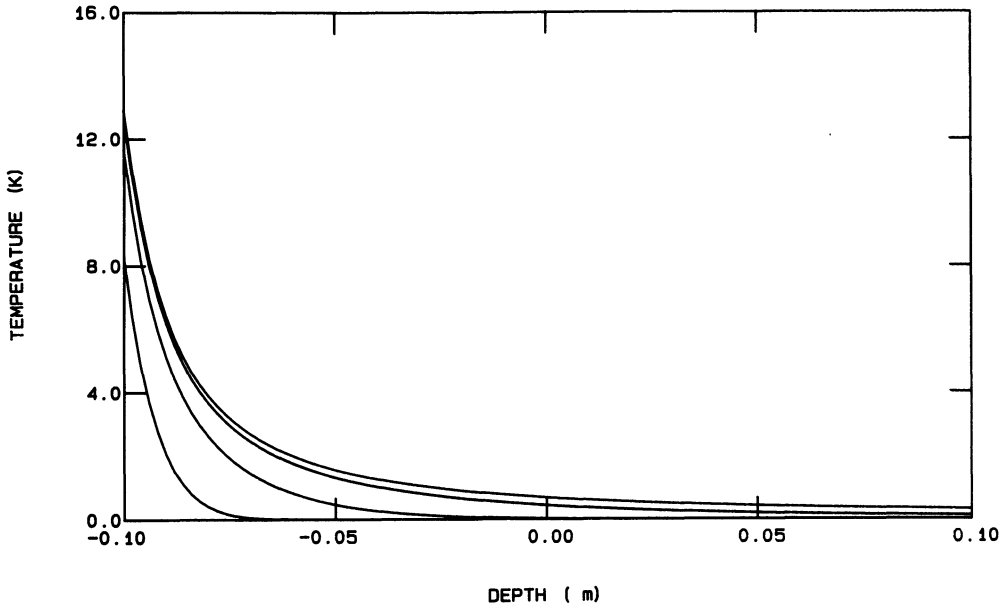


Fig. 5. — Successive axial cuts of the temperature field at the times, $t_c/1000$, $t_c/100$, $t_c/10$ and t_c for 1 W absorbed in the coating.

4. Heating by bulk absorption.

4.1 STEADY STATE SOLUTION. — We consider now the case where the light beam crosses the transparent disk with a weak distributed attenuation characterized by the constant α (m^{-1}) and no losses on the faces, so that there is only internal heat generation and zero incoming heat fluxes at the boundaries. The Fourier equation is now :

$$\rho C \frac{\partial T}{\partial t} - K \Delta T = \alpha I(r) \quad (22)$$

where the z dependence of the light intensity due to attenuation was dropped out as giving second order terms in α . $I(r)$ is supposed to have the Dini expansion given by equations (9) and (10), so that we can write the steady state solution under the form :

$$T_\infty(r, z) = \sum_m \left[A_m \cosh \left(\xi_m \frac{z}{a} \right) + \frac{\alpha p_m a^2}{K \xi_m^2} \right] J_0 \left(\xi_m \frac{r}{a} \right) \quad (23)$$

expecting it to be even with respect to z . The boundary conditions are homogeneous :

$$-K \frac{\partial T}{\partial r}(a, z) = 4 \sigma' T_{\text{ext}}^3 T_\infty(a, z) \quad (24)$$

$$-K \frac{\partial T}{\partial r}(r, \pm h/2) = \pm 4 \sigma' T_{\text{ext}}^3 T_\infty(r, \pm h/2). \quad (25)$$

Condition (24) is fulfilled if k_m has the definition (6), and condition (25) gives the value of A_m :

$$A_m = - \frac{\tau \alpha p_m a^2}{K \xi_m^2} \frac{2}{(\xi_m + \tau) \exp(\xi_m h/2a) - (\xi_m - \tau) \exp(-\xi_m h/2a)}.$$

We get the final expression :

$$T_{\infty}(r, z) = \frac{\alpha a^2}{K} \sum_m \frac{p_m}{\zeta_m^2} \left[1 - \frac{2 \tau \cosh(\zeta_m z/a)}{(\zeta_m + \tau) e^{\zeta_m h/2a} - (\zeta_m - \tau) e^{-\zeta_m h/2a}} \right] J_0(\zeta_m r/a). \quad (26)$$

This stationary temperature distribution is shown in figure 6.

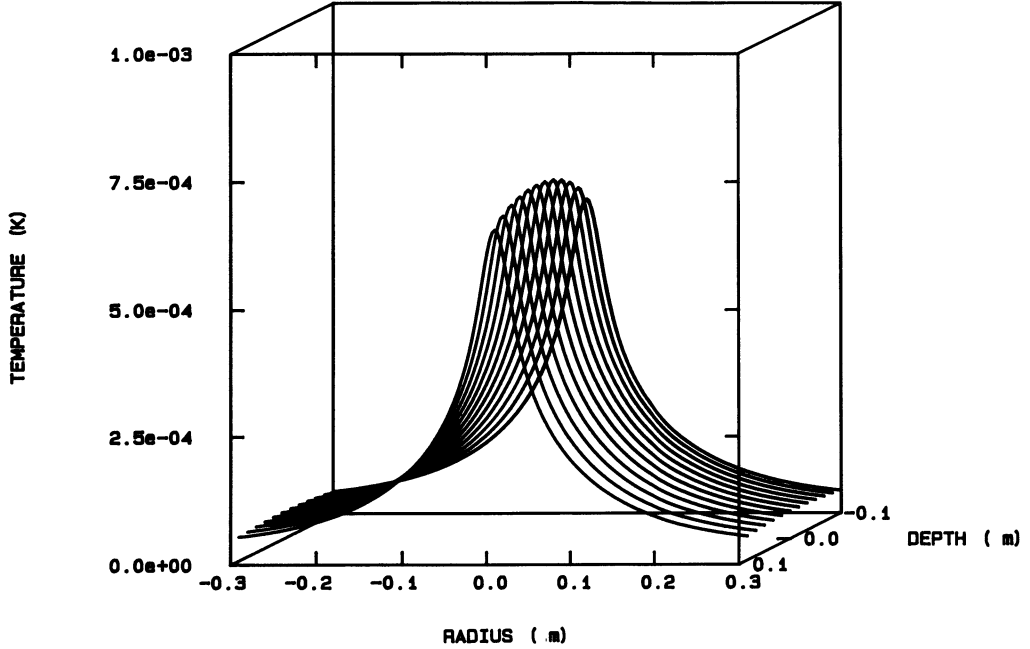


Fig. 6. — An example of map of the steady state temperature field in the case of a bulk absorption ; the absorbed power is 2 mW.m^{-1} .

4.2 TRANSIENT SOLUTION. — As in the previous section, the time dependent solution of equation (22) will be taken of the form

$$T(t, r, z) = T_{\text{ext}} + T_{\infty}(r, z) + T_{\text{tr}}(t, r, z)$$

where T_{tr} is a solution of the homogeneous heat equation, and owing to the required even dependence on z , may be expressed as :

$$T_{\text{tr}}(t, r, z) = \sum_{p, m} C_{pm} e^{-\alpha_{pm} t} \cos(u_p z/a) J_0(\zeta_m r/a)$$

where

$$\alpha_{pm} = \frac{K}{\rho C a^2} (\zeta_m^2 + u_p^2)$$

ζ_m and u_p having the same values as in the precedent section, it is easy to verify that the boundary conditions (24) and (25) are satisfied. We can now take into account the Cauchy data

$$T(0, r, z) = T_{\text{ext}} + T_0(r, z).$$

Within the interval $-h/2 < z < h/2$, we can expand the z dependence of T_∞ in a Fourier series ; T_∞ having the Dini expansion

$$T_\infty(r, z) = \sum_m \theta_m(z) J_0(\zeta_m r/a)$$

with

$$\theta_m(z) = \frac{\alpha a^2 p_m}{K \zeta_m^2} \left[1 - \frac{2 \tau \cosh(\zeta_m z/a)}{(\zeta_m + \tau) \exp(\zeta_m h/2 a) - (\zeta_m - \tau) \exp(-\zeta_m h/2 a)} \right]$$

we find

$$\theta_m(z) = \sum_p C'_{pm} \cos(u_p z/a)$$

where the coefficients C'_{pm} are given by :

$$C'_{pm} = \frac{4 \alpha a^3 p_m}{Kh \zeta_m^2} \left[\frac{\sin(u_p h/2 a)}{u_p} - \frac{\tau \cos(u_p h/2 a)}{\zeta_m^2 + u_p^2} \right] \frac{1}{c_p}$$

by choosing $C_{pm} = -C'_{pm}$ we obtain the time dependent solution satisfying the initial condition :

$$T(t, r, z) = T_{\text{ext}} + \sum_{p, m} C'_{pm} [1 - e^{-\alpha_{pm} t}] \cos(u_p z/a) J_0(\zeta_m r/a). \quad (27)$$

Figure 7 shows the evolution of the warmest point ($r = z = 0$). Figures 8 and 9 show transversal and axial cuts of the temperature field for different times. We took a $10^{-5}/\text{m}$ absorption rate which corresponds to optimistic estimates for very pure silica and 200 W incident light power.

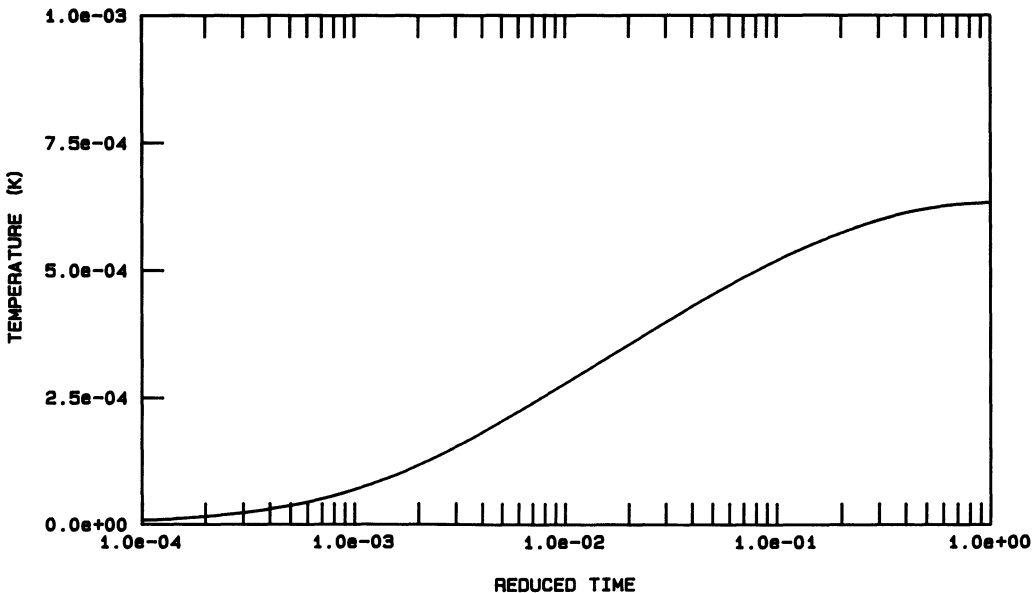


Fig. 7. — Evolution of the temperature of the « warm point » : the point at the centre of the silica cylinder ; the absorbed power in the bulk is $2 \text{ mW} \cdot \text{m}^{-1}$.

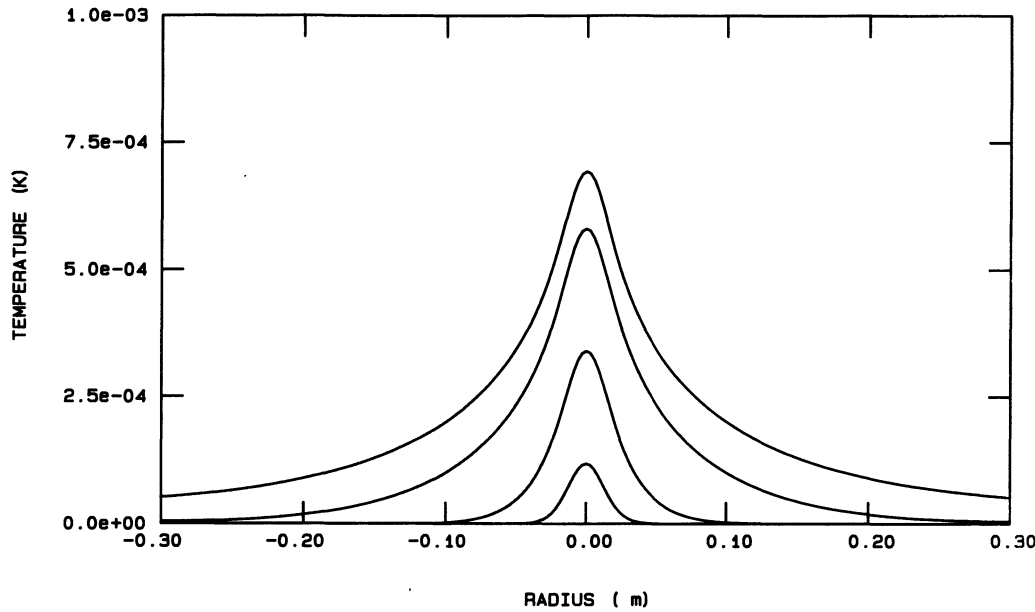


Fig. 8. — Successive transversal cuts of the temperature field in the central plane ($z = 0$) at the times $t_c/1\,000$, $t_c/100$, $t_c/10$ and t_c when the absorbed power is 2 mW.m^{-1} in the bulk.

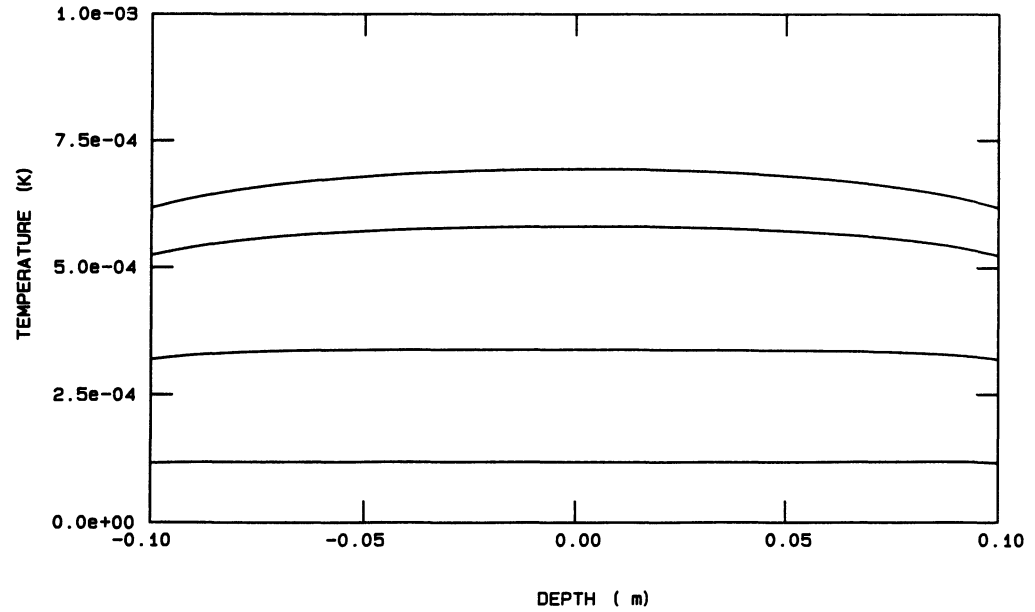


Fig. 9. — Successive axial cuts of the temperature field at the times $t_c/1\,000$, $t_c/100$, $t_c/10$ and t_c for on 2 mW.m^{-1} absorbed in the bulk.

5. Thermal aberrations due to index gradients.

Transparent dielectric materials such as silica have in general temperature dependent refraction indices. The local index $n(x, y, z)$ may be related to the local temperature $T(x, y, z)$ by

$$n(x, y, z) = n_0 + \frac{dn}{dT} [T(x, y, z) - T_0]$$

where n_0 is the reference index corresponding to the temperature T_0 . For silica, we have $dn/dT = -0.87 \times 10^{-5} \text{ K}^{-1}$.

The length scale of the spatial variations of T being much larger than the wavelength of the light, we may use the geometric optics approximation for evaluating the phase distortion of a wave passing through the thermal lens. The optical path obeys the eikonal equation

$$[\nabla \Phi]^2 = n^2. \quad (28)$$

At first order, we may consider ϕ as the sum of the reference optical path plus a perturbation $\psi(x, y, z)$:

$$\phi(r, z) = n_0 z + \psi(r, z)$$

so that, by neglecting the squares of the derivatives of ψ , equation (28) becomes:

$$\frac{\partial \psi}{\partial r} = \frac{dn}{dT} [T(r, z) - T_0].$$

The total optical path distortion due to thermal lensing in the substrate is thus after a passing through the silica:

$$\psi(r) = \frac{dn}{dT} \int_{-h/2}^{h/2} [T(r, z) - T_0] dz. \quad (29)$$

As dn/dT is negative the thermal lens is a diverging lens: the optical path is bigger at the borders than at the center of the mirror.

The axisymmetric aberration may be expanded as usually on a basis of Zernike polynomials $R_{2q}^{(0)}(r/b)$ within the disk of radius $b < a$ corresponding to the optical zone of the silica block. (The block is much wider than the light beam, and possible aberrations outside the beam are meaningless: in the case of a Gaussian beam, b could be 3 times the waist w). We shall thus write:

$$\psi(r) = \sum_q c_{2q} R_{2q}^{(0)}(r/b) \quad (30)$$

where the coefficients c_{2q} are obtained from

$$c_{2q} = \frac{4q+2}{b^2} \int_0^b \psi(r) R_{2q}^{(0)}(r/b) r dr. \quad (31)$$

The expression found for the temperature distributions in the preceding sections are series of Bessel functions J_0 and evaluation of (31) will use the Nijboer relation [15]

$$\int_0^1 J_0(sp) R_{2q}^{(0)}(\rho) \rho d\rho = (-1)^q \frac{J_{2q+1}(s)}{s}. \quad (32)$$

5.1 CASE OF HEATING BY COATING ABSORPTION. — When the temperature distribution is given by equation (21), the preceding method gives the time dependent thermal aberration :

$$\psi_{\text{coat}}(t, r) = \frac{dn}{dT} \frac{2 \varepsilon a^3}{Kh} \sum_{m,p} p_m [1 - e^{-\alpha_{pm} t}] \frac{\sin(u_p h/a)}{[\zeta_m^2 + u_p^2] u_p c_p} J_0(\zeta_m r/a). \quad (33)$$

An expansion in a series of Zernike polynomials such as (30) gives in the case of a Gaussian beam of radius w , and $\varepsilon P = P_{\text{abs}}$ (absorbed power), transient coefficients

$$c_{2q}(t) = (-1)^q (2q+1) \frac{4 P_{\text{abs}} a^2 dn/dT}{\pi K h b} \times \sum_{p,m} \frac{[1 - e^{-\alpha_{pm} t}] \zeta_m e^{-\zeta_m^2 w^2/8 a^2} \sin(u_p h/a) J_{2q+1}(\zeta_m b/a)}{[\zeta_m^2 + \tau^2][\zeta_m^2 + u_p^2] J_0(\zeta_m)^2 u_p c_p}. \quad (34)$$

We have plotted in figure 10 the time dependence of the first c coefficients. The coefficient c_0 was omitted because it represents only an uniform additional phase.

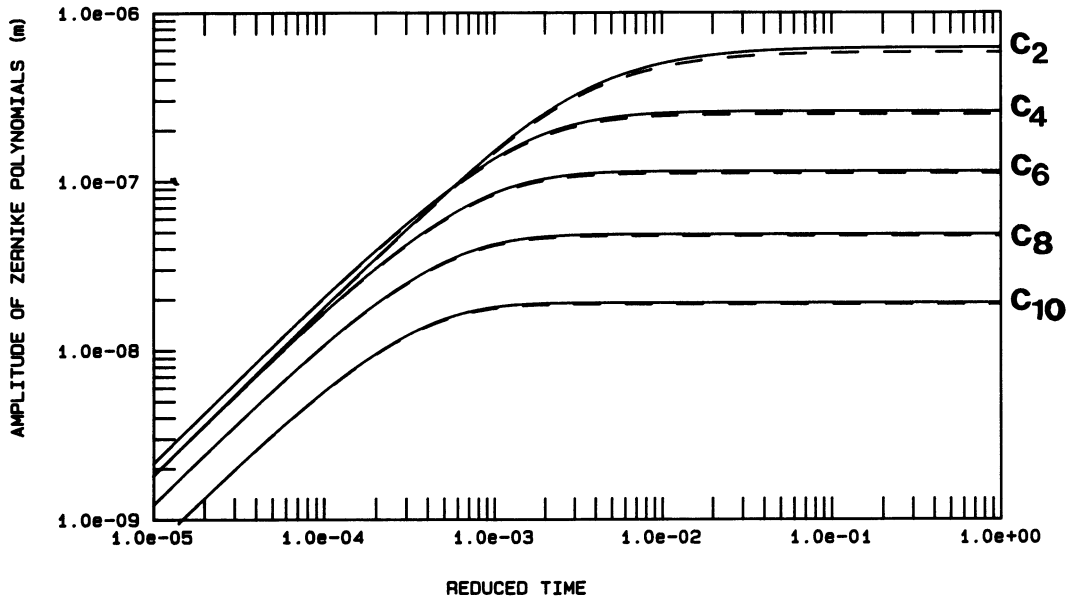


Fig. 10. — Evolution of the five first coefficients of the Zernike polynomials except c_0 . The bulk absorption datas are plotted in plain lines and the coating absorption ones in dashed lines. In both cases the total absorbed power is 1 W.

5.2 CASE OF HEATING BY BULK ABSORPTION. — When the temperature distribution is given by equation (29), the time dependent aberration is

$$\psi_{\text{bulk}}(t, r) = \frac{dn}{dT} \frac{4 \alpha a^4}{Kh} \sum_{p,m} \frac{p_m}{\zeta_m^2} [1 - e^{-\alpha_{pm} t}] \left[\frac{1 - \cos(u_p h/a)}{u_p} - \frac{\tau \sin(u_p h/a)}{\zeta_m^2 + u_p^2} \right] \frac{J_0(\zeta_m r/a)}{c_p u_p} \quad (35)$$

and the expansion in Zernike polynomials, with $P_{\text{abs}} = h\alpha P$, gives the coefficients :

$$c_{2q}(t) = (-1)^q (2q+1) \frac{8 P_{\text{abs}} a^3}{\pi K h^2 b} \frac{dn}{dT} \times \\ \times \sum_{p,m} \frac{[1 - e^{-\alpha_{pm} t}] e^{-\zeta_m^2 w^2 / 8 a^2}}{\zeta_m [\zeta_m^2 + \tau^2] J_0(\zeta_m)^2} \left[\frac{1 - \cos(u_p h/a)}{u_p} - \frac{\tau \sin(u_p h/a)}{[\zeta_m^2 + u_p^2]} \right] \frac{J_{2q+1}(\zeta_m b/a)}{c_p u_p} \quad (36)$$

see figure 10.

Finally figure 11 shows an example of the thermal lens profile in the steady state for the two cases (coating and bulk absorption) and for 1 W absorbed power. The profile is limited to an optical zone of 5 cm radius according to the VIRGO specifications.

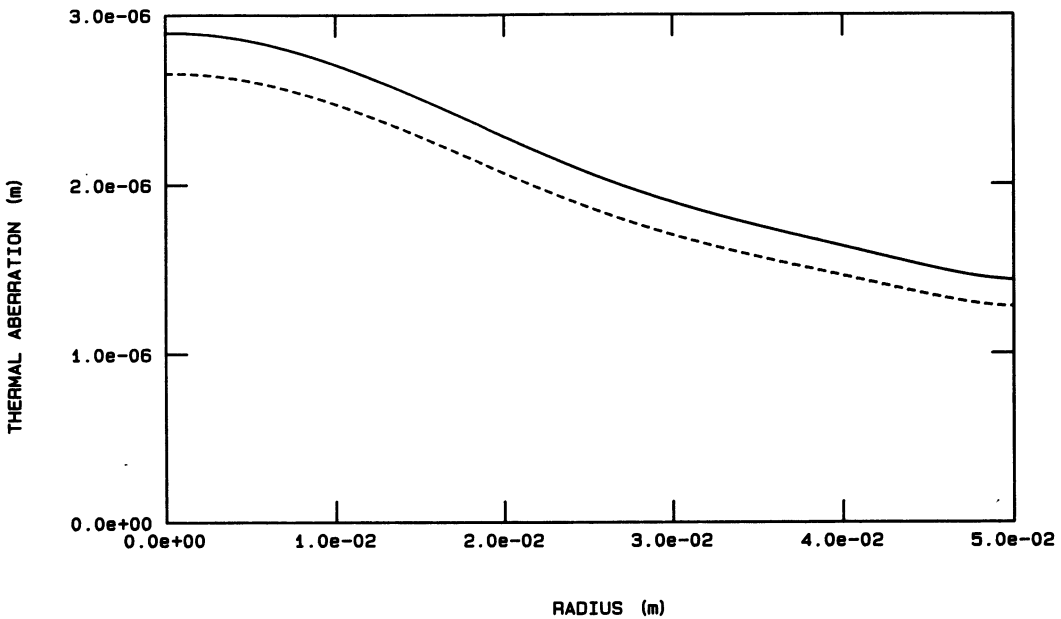


Fig. 11. — The shape of the thermal lens when the steady state is established. We give the absolute value of the thermal aberration, dn/dT being negative. The plain line refers to the bulk absorption, and the dashed one to the coating absorption. In both cases the total absorbed power is 1 W.

6. Conclusion.

A time dependent modelization of the temperature field in transparent mirrors caused by absorption of light either in the coatings or in the bulk material has been derived and numerically exploited. It shows that the steady state temperature is about 13 K/W in the first case (coating dissipation) and about 2 K/W in the second one (bulk absorption) ; nevertheless the two mechanisms of light power dissipation give roughly equal thermal leasing (about $1.5 \mu\text{m/W}$ at the center) for equal dissipated powers. When the substrate is made of pure silica, the dissipation rate by bulk absorption is however expected to be much smaller than the losses in the coatings, and we can therefore conclude that the thermal lensing is mainly due to the heating by faces. In a foregoing paper we will complete this work by studying the

thermoelastic deformations of the mirror in the cases of absorption either in the bulk or in the coating.

Clearly, dissipation of 1 W on the coatings would have a huge perturbing effect on the Virgo cavities and we have either to find very low dissipation materials, or to correct for the induced aberration. But when such dissipative elements are included in a resonant cavity, a non-linear coupling occurs between the thermal system which determines the optical tuning, and the optical system which determines the rate of heating. We are therefore developing a numerical simulator of a dissipative cavity, including the preceding results.

References

- [1] FOSTER J. D. and OSTERINK L. M., Thermal effects in a Nd : YAG Laser, *J. Appl. Phys.* **41** (1970) 3656-3663.
- [2] OGRODNIK R. F., An analysis of optical effects caused by thermally induced mirror deformations, *Appl. Opt.* **9** (1970) 2028-2034.
- [3] CUTOLO A., GAY P., SOLIMENO S., Mirror deformations and wavefront aberrations caused by c.w. high power laser beams, *Opt. Acta* **27** (1980) 1105-1116.
- [4] EGGLESTON J. M., KANE T. J., KUHN K., UNTERNAHRER J. and BYER R. L., The slab geometry laser-Part I : Theory, *IEEE J. Quantum Electron.* **QE-20** (1984) 289-300.
- [5] EGGLESTON J. M., KANE T. J., KUHN K., UNTERNAHRER J. and BYER R. L., The slab geometry laser-Part II : Thermal effects in a finite slab, *IEEE J. Quantum Electron.* **QE-21** (1985) 1195-1209.
- [6] TANKOVSKY N. S. and IVANOV D. V., Photodisplacement imaging — A Static regime, *Appl. Phys. Lett.* **46** (1985) 639-641.
- [7] TANKOVSKY N. S., IVANOV D. V. and YORDANOV B., Evaluation of the local heat losses in solids through an optical method, *Revue Phys. Appl.* **21** (1986) 339-341.
- [8] BLOISI F., VICARI L., CAVALIERE P., MARTELLUCCI S., QUARTIERI J., MORMILE P. and PIERATTINI G., Laser induced thermal profiles in thermally and optically thin films, *Appl. Phys. B* **47** (1988) 67-69.
- [9] GRENINGER C. E., Thermally induced wave-front distortions in laser windows, *Appl. Opt.* **25** (1986) 2474-2475.
- [10] BUROV J. I. and IVANOV D. V., Thermodisplacement imaging of silicon membranes, *J. Phys. France* **47** (1986) 549-552.
- [11] JACKSON W. B., AMER N. M., BOCCARA A. C. and FOURNIER D., Photothermal deflection spectroscopy and deflection, *Appl. Opt.* **20** (1981) 1333.
- [12] OLMSTEAD M. A., AMER N. M., KOHN S., FOURNIER D. and BOCCARA A. C., Photothermal displacement spectroscopy : an optical probe for solids and surfaces, *Appl. Phys. A* **32** (1983) 141-154.
- [13] MYERS C. J. and ALLEN R. C., Development of an analytical mirror model addressing the problem of thermoelastic deformation, *Appl. Opt.* **24** (1985) 1933-1939.
- [14] ABRAMOWITZ M. and STEGUN I. A., Handbook of mathematical functions, National Bureau of Standards, *Appl. Math. Ser.* **55** (1970).
- [15] BORN M. and WOLF E., Principles of Optics (Oxford : Pergamon Press) 1975.