Abstract

Going to make LIGO the best possible ever.

Adaptive Mode Matching in Advanced LIGO

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Preface

The era of gravitational waves astronomy was ushered in by the LIGO (Laser Interferometer Gravitational-Wave Observatory) collaboration with the detection of a binary black hole collision (Detection paper). The event that shook the foundation of space-time allowed mankind to view the cosmos in a way that had never been done previously.

Contents

Chapter 1

Introduction

1.1 Gravitational Waves

In 1915, Albert Einstein published his theory of general relativity.

The seminal equation in this theory is:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{1.1}$$

which is a set of 10 coupled second order differential equations that are nonlinear. In their complete form, equation [1] fully describes the interaction between space-time and mass-energy. To describe the physics in a highly curved space-time, one would have to fully solve the Einstein field equations numerically.

In the weak field approximation, the metric can be described as

$$g_{\mu\nu} \cong \eta_{\mu\nu} + h_{\mu\nu} \tag{1.2}$$

where $\eta_{\mu\nu}$ is the metric of flat space time and $|h_{\mu\nu}| \ll 1$ is the perturbation due to a gravitational field.

By plugging in equation 1.2 into equation 1.1 and using empty space we obtain the familiar wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)h_{\mu\nu} = 0 \tag{1.3}$$

which has a plane-wave solution of the form $h_{\mu\nu} = A_{\mu\nu}e^{ik_\nu x^\nu}$.

By using the gauge constraint $h^{\mu\nu}_{,\nu} = 0$, it follows that $A_{\mu\nu}k^\mu = 0$ which means that the gravitational wave amplitude is orthogonal to the propagation vector.

By further imposing transverse-traceless gauge, we can constrain the complex amplitude to two orthogonal

polarizations which have physical significance:

$$A_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{yx} & 0 \\ 0 & A_{xy} & -A_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{1.4}$$

Add Figure of GWs

It is natural to attempt to understand the physical interpretation of Eq.1.4 as an affect on the position of a free floating particle. Consider the four-velocity, U^α, in the transverse traceless gauge where the coordinate itself is attached the particles and incorporates any small wiggles that would shake the coordinates. Of course, any free particles will follow the geodesic equation

$$\nabla U^\alpha = \frac{\mathrm{d}}{\mathrm{d}\tau} U^\alpha + \Gamma^\alpha_{\mu\nu} U^\mu U^\nu = 0 \tag{1.5}$$

where $\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\gamma\alpha}(g_{\gamma\mu,\nu} + g_{\gamma\nu,\mu} - g_{\mu\nu,\gamma})$ are the famous Christoffel symbols. By evaluating the first term of the acceleration in Eq.1.5,

$$\left(\frac{\mathrm{d}U^\alpha}{\mathrm{d}\tau}\right)_0 = -\Gamma^\alpha_{00} = \frac{1}{2}\eta_{\mu\nu}(h_{\beta 0,0} + h_{0\beta,0} + h_{00,\beta}) \tag{1.6}$$

However, comparing Eq.1.4 and Eq.1.6, it is clear that if the particle is initially at rest, then a moment later it is still at rest! The term "at rest" is actually used liberally here since the coordinate system varies along with the gravitational wave.

Alternatively, one can ask if a gravitational wave passed by a pair of particles separated by length L, what would be the effect on the distance between two points? The proper distance is defined as

$$\delta l = \int g_{\mu\nu} dx^\mu dx^\nu = \int_0^L g_{xx} dx \approx |g_{xx}(x=0)|^{1/2} \approx [1 + \frac{1}{2}h_{xx}(x=0)]L \tag{1.7}$$

which shows us two very important points about the nature of gravitational waves. Firstly, the effect is very small since the length variation is, by definition, a small perturbation in flat space-time. Secondly, the effect is proportional to the initial separation between the particles. This means a detector which is large will have a better chance to measure these small effects, a point that drove the design of the Laser Interferometer Gravitational-Wave Interferometer (LIGO).

1.1.1 Measuring Gravitational Waves with Light

Even with the theoretical formulation of gravitational waves resolved by the 1970s, detection of GWs by ground-based detectors was still a controversial topic among scientists in the field. This was due to the incredible amount of accuracy needed to actually measure strain.

1.1.2 Energy from a Gravitational Wave

1.1.3 Signal to Noise

1.1.4 Sources of Gravitational Waves

Compact Binary Inspirals

Continuous Waves

Bursts

Stochastic

Even with some of the most energetic events known to humanity such as the merger of neutron stars and black holes, the amount of strain expected is on the order of 10e-24.

[1]

1.1.5 Measuring Gravitational Waves with Light

Even with the theoretical formulation of gravitational waves resolved by the 1970s, detection of GWs by ground-based detectors was still a controversial topic among scientists in the field. This was due to the incredible amount of accuracy needed to actually measure strain.

1.2 The LIGO Instrument

In the simplest form, the LIGO instrument is an incredibly large Michaelson interferometer. If we imagine the output as a measure of the differential arm length, it becomes a natural way of detecting gravitational waves.

The LIGO instruments are considered dual-recycled Fabry-Perot interferometers.

1.2.1 Simple Michaelson

A simple interferometer consists of an input laser, a 50/50 beamsplitter, two mirrors and a photodetector at the output port to read the signal.

In Peter Saulson's book, there is a simple explaination of the light field exiting the anti-symmetric port.

However, the signal is proportional to the square of the strain, not good. [Black Paper on Signal extraction]

As show in section 1.1, the gravitational wave will modulate the proper length between two free floating points in space.

1.2.2 Fabry-Perot Cavities

A Fabry-Perot cavity is an optical system comprised of two mirrors and one laser input. The very simple condition that once the laser has made a round trip around the optics, it is the same shape at the same point. Conceptually, this may seem simple but in practice, controlling and sensing any optical cavity comes with many challenges.

Appendix

References:[Black, Rick's Paper]

1.2.3 Power-Recycled Interferometers

Power Recycling If the interferometer is operating such that the 4 km arms are exactly different in arms a pi over two times the wavelength, then the intensity of the light at antisymmetric port will be close to null. This means the power from the arms will reflect back towards the input laser. Ref[] shows the effect of adding a partially reflecting mirror to increase the optical gain of the Michaelson.

References: [Meers, Kiwamu]

1.2.4 Dual-Recycled Interferometers

Figure comparing the three cases of increased sensitivity.

References: [Kiwamu]

1.2.5 Fundamental Noise Sources

Ref: Evan Hall, GWINC

Noise budget:

Quantum Noise

Seismic Noise

Thermal Noise

Newtonian Noise

1.3   Squeezed States of Light

The Quantum Noise is a fundamental source that can be improved by modifying the quantum vacuum using correlated photons and injecting the states of light into the antisymmetric port of the interferometer. Within the LIGO community, this procedure of modifying quantum vacuum is called squeezing. Caves analytically derived the effects of quantum noise on the interferometer as well as the improvement due to i

References: [Caves, Dwyer, Kwee, Miao]

Chapter 2

The Fundamentals of Mode-Matching

Theory section of modematching. Gaussian beams, define relevant quantities, Gouy Phase!

In order to properly explain the effects of electromagnetic waves in the LIGO interferometers, it is useful to review the

References: [Kognelik and Li]

2.1 Gaussian Beam Optics

When dealing with length sensing degrees of freedoms such as section 1.2.2, the simple plane wave approximation is sufficient in describing the dynamics, however, when trying to understand the misalignment and mode-mismatch signals, it is necessary to incorporate Gaussian beam optics and associated their higher order modes.

Consider the famous Maxwell's equations in vacuum:

$$
\begin{aligned}
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{B} &= \mu \, \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \\
\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon}
\end{aligned}
\qquad ,
\tag{2.1}
$$

Concentrating on the electric field in vacuum, we arrive at the Helmholtz Equation

$$
(\nabla^2 + k^2)\mathbf{U}(\mathbf{r}, t) = 0
\tag{2.2}
$$

where $k = \frac{2\pi\nu}{c}$ is the wave number and $\mathbf{U}(\mathbf{r}, t)$ is the complex amplitude which can describe either the

electric or magnetic fields.

It is possible to express the solution to Eq. 2.2 as a plane wave with a modulated complex envelope

$$\mathbf{U}(\mathbf{r}) = \mathbf{A}(\mathbf{r})e^{-ikz} \tag{2.3}$$

By imposing the constraints which force the envelope to vary slowly with respect to the z-axis within the distance of one wavelength $\lambda = 2\pi/k$,

$$\left|\frac{\partial^2 \mathbf{A}}{\partial z^2}\right| \ll \left|k\frac{\partial \mathbf{A}}{\partial z}\right| \tag{2.4a}$$

$$\left|\frac{\partial^2 \mathbf{A}}{\partial z^2}\right| \ll \left|k\frac{\partial^2 \mathbf{A}}{\partial x^2}\right| \tag{2.4b}$$

$$\left|\frac{\partial^2 \mathbf{A}}{\partial z^2}\right| \ll \left|k\frac{\partial^2 \mathbf{A}}{\partial y^2}\right| \tag{2.4c}$$

the partial differential equation which arises is called the Paraxial Helmholtz Equation:

$$\nabla_T^2 A(r) - i2k\frac{\partial A(r)}{\partial z} = 0 \tag{2.5}$$

where $\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the transverse Laplacian. A simple solution for Eq.2.5 is the complex paraboloidal wave

$$A(\mathbf{r}) = \frac{A_0}{q(z)}e^{\frac{-ikr^2}{2q(z)}}, \quad q(z) = z + iz_0 \tag{2.6}$$

where z_0 is the Rayleigh range and is directly proportional to the square of the waist size. In order to separate the amplitude and phase portions of the wave, it is useful to rewrite $q(z)$ as

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi W^2(z)} \tag{2.7}$$

Plugging Eq.2.7 into 2.6 leads directly to the complex amplitude for a Gaussian Beam

$$U(r) = A_0\frac{W_0}{W(z)}e^{-\frac{r^2}{W^2(z)}}e^{-ikz-ik\frac{r^2}{2R(z)}+i\phi(z)} \tag{2.8}$$

where

$$W(z) = W_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2} \tag{2.9a}$$

$$R(z) = z\left[1 + \left(\frac{z}{z_0}\right)^2\right] \tag{2.9b}$$

$$\phi(z) = \tan^{-1}\left(\frac{z}{z_0}\right) \tag{2.9c}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}} \tag{2.9d}$$

- Gouy Phase + graph

- Intensity and Power

Hermite-Gauss Modes

The fundamental Gaussian beam is not the only solution which can be used to solve Eq.2.5. In fact, there exists a complete set of solutions that can solve the paraxial Helmholtz Equation, which are referred to as the higher order modes (HOMs)

$$U_{mn}(x,y,z) = A_{mn}\left[\frac{W_0}{W(z)}\right]\mathbb{G}_m\left(\frac{\sqrt{2}x}{W(z)}\right)\mathbb{G}_n\left(\frac{\sqrt{2}y}{W(z)}\right)$$
$$\times \exp\left\{-ikz - \frac{ik(x^2+y^2)}{2R(z)} + i(m+n+1)\phi(z)\right\} \tag{2.10}$$

where,

$$\mathbb{G}_{(}u) = \mathbb{H}(u)\exp(-u^2/2) \tag{2.11}$$

and $\mathbb{H}(u)$ are the well known Hermite polynomials. It is important to mention that the Gouy phase of the complex amplitude is different than the fundamental Gaussian beam by a factor of $(m+n+1)$. Also, the intensity distribution of these higher order modes are much different. Both of these facts will become extremely important in the following wavefront sensing discussion.

It is useful to normalize the Hermite-Gauss modes with respect to the overall power, which are derived in Appendix[].

Laguerre Modes

Another complete set of alternative solutions to Eq.2.5 exists which are called the Laguerre-Gauss modes

$$V_{\mu\nu}(\rho,\theta,z) = A_{\mu\nu}\left[\frac{W_0}{W(z)}\right]\mathbb{L}_\nu^\mu\left(\frac{\sqrt{2}x}{W(z)}\right)$$
$$\times \exp\left\{-ikz - \frac{ik\rho^2}{2R(z)} + i(\mu+2\nu+1)\phi(z)\right\} \tag{2.12}$$

where $\mathbb{L}_\nu^\mu\left(\frac{\sqrt{2}x}{W(z)}\right)$ is the Laguerre polynomial function. Both equations 2.10 and 2.12 are able to fully describe any complex electromagnetic amplitude; and because they both form complete sets, there is a

rotation which can map from one basis to the other [ref Bond and Biejergasern]

$$U_{\mu\nu}^{LG}(x,y,z) = \sum_{k}^{N} i^k b(n,m,k) U_{N-k,k}^{HG}(x,y,z) \tag{2.13}$$

where

$$b(n,m,k) = \sqrt{\left(\frac{(N-k)!k!}{2^N n!m!}\right)} \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}t^k}[(1-t)^m(1+t)^m]|_{t=0} \tag{2.14}$$

2.1.1 Misalignment and Mode Mismatch

Morrison and Anderson derived a simplistic way of how small misalignments and mismodematched cavities can couple the fundamental Gaussian beam into various higher order modes. This is done by taking a linear cavity and using its perfectly matched Gaussian beam as a reference, and then varying the input electric field with small perturbations and expanding in terms of the cavity modes. As long as the mismatches are small, it is possible to consider only the first few terms of the expansion which have gained power from the fundamental mode.

Consider the first three modes of Eq.2.10 in one dimension and normalized to set the total optical power to unity (derived in Appendix[]):

$$
\begin{aligned}
U_0(r) &= \left(\frac{2}{\pi w^2(z)}\right)^{1/4} e^{-r^2/w^2(z)} \\
U_1(r) &= \left(\frac{2}{\pi w^2(z)}\right)^{1/4} \frac{2r}{w(z)} e^{-r^2/w^2(z)} \\
U_2(r) &= \left(\frac{2}{\pi w^2(z)}\right)^{1/4} \left(\frac{r^2}{w^2(z)} - \frac{1}{4}\right) e^{-r^2/w^2(z)}
\end{aligned}
\tag{2.15}
$$

Beam Axis Tilted

If the input beam into an optical cavity is tilted by an angle α with respect to the nominal cavity axis as shown in Figure [], the wave front of the input beam will have an extra phase propagation relative to the cavity that is approximately proportional to $e^{ik\alpha r}$. By implementing the small angle approximation, which is valid if the misalignment is much smaller than the divergence angle of the fundamental mode $k\alpha r << 1$, the resultant input beam is

$$\Psi \approx U_0(r)e^{ik\alpha r} \approx U_0(r)(1 + ik\alpha r) = U_0(r) + \frac{ik\alpha w(z)}{\sqrt{2\pi}} U_1(r) \tag{2.16}$$

Here the factor associated with the first higher order mode is complex, indicating there is a 90 degree phase difference between the fundamental and off axis mode.

Beam Axis Displaced

If the input beam is displaced in the transverse direction by a quantity Δr, the resultant waveform will be

$$
\begin{aligned}
\Psi &= U_0(r + \Delta r) \\
&= \left(\frac{2}{\pi w^2(z)}\right)^{1/4} e^{-(r+\Delta r)^2/w^2(z)} \\
&= \left(\frac{2}{\pi w^2(z)}\right)^{1/4} e^{-(r^2+2r\Delta r+\Delta r^2)/w^2(z)} \\
&\approx \left(\frac{2}{\pi w^2(z)}\right)^{1/4} e^{-r^2/w^2(z)} e^{-2r\Delta r/w^2(z)} \\
&\approx \left(\frac{2}{\pi w^2(z)}\right)^{1/4} e^{-r^2/w^2(z)} \left(1 - \frac{2r\Delta r}{w^2(z)}\right) \\
&= \left(U_0(r) - \sqrt{\frac{2}{\pi}}\frac{\Delta r}{w(z)} U_1(r)\right)
\end{aligned}
\tag{2.17}
$$

Similarly to a tilted input beam axis, the displaced beam axis couples power to the first higher order mode, however, the latter does not have a 90 degree phase difference previously seen in the former. This point is of extreme importance when trying to discern between the two effects as shown in Section []. Although comparing the two cases in Figure, one can already seen the difference between the wavefronts in the near field, $z \ll z_R$, and the far field, $z \gg z_R$.

In the near field, there is no phase difference due to a displaced beam, but there is one for a tilted beam. Conversely, in the far field, there is no phase difference due to a tilted beam, but there is one from a displaced beam. In order to implement a closed loop feedback system, the wavefront sensors discussed in Section[] will use this precise logic to extract an error signal.

Waist Size Shifted

$$
\begin{aligned}
\Psi &= U_0\big(r, w(z) = w_0/(1+\epsilon)\big) \\
&= \left(\frac{2}{\pi w_0^2}\right)^{1/4} \sqrt{1+\epsilon}\ \ e^{-r^2(1+\epsilon)^2/w_0^2} \\
&\approx \left(\frac{2}{\pi w_0^2}\right)^{1/4} (1+\epsilon/2)\ \ e^{-r^2/w_0^2}\ \ e^{-2r^2\epsilon/w_0^2} \\
&\approx \left(\frac{2}{\pi w_0^2}\right)^{1/4} (1+\epsilon/2)\ \ e^{-r^2/w_0^2}\ \ (1 - 2r^2\epsilon/w_0^2) \\
&\approx \left(\frac{2}{\pi w_0^2}\right)^{1/4} \left(1 + 2\epsilon\left(\frac{1}{4} - \frac{r^2}{w_0^2}\right)\right)\ \ e^{-r^2/w_0^2} \\
&= U_0(r) + 2\epsilon U_2(r)
\end{aligned}
\tag{2.18}
$$

Waist Position Shifted

Figure: Modematching References: [Own Calculations]

2.2 Wavefront Sensing

Heterodyne detection using a modal decomposition of the full electric field allows the use of wavefront sensors to extract an error signal from the optical system. Hefetz et.al Ref[Sigg and Nergis] created a formalism to describe the use of wavefront sensors by creating frequency sidebands which accumulate a different Gouy phase than the electric field at the carrier frequency when passed through the optical system.

Fundamentally, the purpose of wavefront sensing is to detect the content of higher order modes due to physical disturbances of the optical cavity (ie. mode mismatch or misalignment). In other words, it is examining the difference of basis sets between the incoming eigenmodes and the cavity eigenmodes.

Consider a general equation for an electric field which is a linear combination of all higher order modes of the complex amplitude

$$E(x,y,z) = \sum_{m,n}^{\infty} a_{mn} U_{mn}(x,y,z) \tag{2.19}$$

where $U_{mn}(x,y,z)$ are the eigenmodes of the described in Eq[] and a_{mn} is the complex amplitude. It is also convenient in the following analysis to use vectors when describing the composition of the electric fields.

$$|E(x,y,z)\rangle = \begin{pmatrix} E_{00} \\ E_{01} \\ E_{10} \\ \vdots \end{pmatrix} \tag{2.20}$$

When creating a theory that involves laser beams, it is useful to define operators that are important in describing physical situations. For example, laser beams propagate through space and pick up phase according to Eq.2.10 which can be represented by the spatial propagation operator,

$$\hat{P}_{mn,kl} = \delta_{mn}\delta_{kl} \quad \exp[-ik(z_2 - z_1)]\exp[i(m+n+1)\phi(z)] \tag{2.21}$$

However, it is useful to compare how the fundamental Gaussian mode propagates compared to the higher order modes,

$$\hat{\eta}_{\mu\nu} = \begin{pmatrix} e^{i\phi} & 0 & 0 & 0 & 0 \\ 0 & e^{2i\phi} & 0 & 0 & 0 \\ 0 & 0 & e^{2i\phi} & 0 & 0 \\ 0 & 0 & 0 & e^{3i\phi} & 0 \\ 0 & 0 & 0 & 0 & e^{3i\phi} \end{pmatrix} \tag{2.22}$$

From the aabove diagonal elements it is clear that the higher order modes have an extra phase compared to the fundamental 00 mode, this effect will be extremely important on how an error signal can be derived from the optical system.

Fundamentally, the purpose of wavefront sensing is to detect the content of higher order modes due to physical disturbances of the optical cavity (ie. mode mismatch or misalignment). In other words, it is examining the difference of basis sets between the incoming eigenmodes and the cavity eigenmodes; this can be described by [ref Hefetz]

$$|E(x, y, z_2)\rangle = \hat{M}(x, y, z_1, z_2) |E(x, y, z_1)\rangle \tag{2.23}$$

where $\hat{M}(x, y, z_1, z_2)$ is the misalignment operator. Since we are using the paraxial approximation, the z-components of the misalignment operator are small so we can approximate $\hat{M}(x, y, z_1, z_2) \approx \hat{M}(x, y)$ and the expectation value is

$$M_{mn,kl} = \langle U_{mn}(x, y, z_1)| M(x, y) |U_{kl}(x, y, z_2)\rangle \tag{2.24}$$

where the product is an integral over the transverse space $\iint_{D(x,y)} \mathrm{d}x\mathrm{d}y$

$$\hat{\Theta}_{\mu\nu} = \begin{pmatrix} 1 & 2i\theta_x & 2i\theta_y & 0 & 0 \\ 2i\theta_x & 1 & 0 & 0 & 0 \\ 2i\theta_y & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.25}$$

$$\hat{\mathbb{D}}_{\mu\nu} = \begin{pmatrix} 1 & \alpha_x/\omega_0 & \alpha_y/\omega_0 & 0 & 0 \\ \alpha_x/\omega_0 & 1 & 0 & 0 & 0 \\ \alpha_y/\omega_0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.26}$$

$$\hat{\mathbb{Z}}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & \Delta z_x & \Delta z_y \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \Delta z_x & 0 & 0 & 1 & 0 \\ \Delta z_y & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.27}$$

where $\Delta z_{(x,y)} = \frac{i}{\sqrt{2}} \frac{\lambda b}{2\pi\omega_0}$

$$\hat{\mathbb{Z}}_{0,\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & \Delta z_{0,x} & \Delta z_{0,y} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \Delta z_{0,x} & 0 & 0 & 1 & 0 \\ \Delta z_{0,y} & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.28}$$

where $\Delta z_{0,(x,y)} = \frac{1}{\sqrt{2}} \frac{\omega' - \omega_0}{\omega_0}$

Figure: Beat frequency between two modes.

Example: Fabry Perot Cavity

Example: Simple Michaelson

2.3   Effects of Mode-matching on Squeezing

Still not clear to me. In the most simple sense, a loss is akin to coupling quantum vacuum into a squeezed state. References: [Miao, Sheon, Kimble, Dwyer]

e

Chapter 3

Simulating Mode-Matching with Finesse

3.1 How it works

Summary of how Finesse works (input output matrix), how it handles HOMs

3.2 Finesse Simulations

3.2.1 ALIGO Design with FC and Squeezer

3.2.2 Looking at just Modal Change

3.2.3 QM Limited Sensitivity

3.3 Results

* Signal recycling cavity mismatches

    * Mismatches before the OMC

    * Mismatch contour graph: Comparing all of ALIGO cavities

    * Optical Spring pops up at 7.4 Hz in the Signal-to-Darm TF, re-run with varying SRM Trans which should.

Chapter 4

Experimental Mode Matching Cavities at Syracuse

In conjunction with Sandoval et al, the adaptive modematching table top experiment was able to show the feasibility of a fully dynamical system.

4.1 Adaptive Mode Matching

Real time digital system and model.

Signal chain.

4.2 Actuators

4.2.1 Thermal Lenses

Fabian's work and UFL paper.

4.2.2 Translation Stages

4.3 Sensors

4.3.1 Bullseye Photodiodes

As stated in section [], we saw that the error signal for a mode-mismatched cavity has cylindrical symmetry due to the beating between the LG-01 and the LG-00 mode. This means that the quadrant photodiodes would not have a way to detect the modal content of a cavity. One way to solve this is to introduce a type of detector that can sense the power outside versus the power inside (ugh re write this).

In [Rana's IO final design document]

Why do we need bullseye photodiodes, meantion the geometry of the error signal.

Derivations in the appendix.

Picture of BPD

Pitch and Yaw sensing matrix

Explain why we need $\omega_0 = \sqrt{2} r_0$

The ratio of the out over in will give:

$$\text{Power Ratio} = \frac{\text{P}_2 + \text{P}_3 + \text{P}_4}{\text{P}_1} = \frac{e^{-2r_0^2/\omega_0^2}}{1 - e^{-2r_0^2/\omega_0^2}} \approx 0.582 \tag{4.1}$$

4.3.2 Mode Converters

Chapter 5

Mode Matching Cavities at LIGO Hanford

5.1   Active Wavefront Control System

5.2   SRC

The importance of mode-matching actually goes beyond reducing the amount of losses in coupled cavities. It also is important for cavity stability. If we look at the g-factor of a cavity, it is required through ABCD transformations that the values lay between 0 and 1. For the signal recycling cavity, if the round trip gouy phase is off by a few millimeters, the stability of the cavity can be compromised.

5.3   Beam Jitter

Current measurements of mode-matching.

Chapter 6

Solutions for Detector Upgrades

* SR3 Heater

 * SRM Heater

 * Bullseye photodetectors

 * Operation: range (in terms of watts and

 * Translation stages

 * Mechanical description (Solidworks designs)

 * Constraints (range, vacuum, alignment, integration)

 * Electronics

 * Software

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Appendix A

Resonator Formulas

Ray Trace: Round trip phase, cavity stability

TF: analytical derivation

Important quantities: Finesse, cavity pole, free spectral range

Effect of higher order modes into the cavity, mode scanning. All comes from round trip Gouy phase.

Appendix B

Hermite Gauss Normalization

According to Eq.2.10, the higher order modes in the Hermite Gauss basis has the intensity profile,

$$I_{mn}(x,y,z) = |A_{mn}|^2 \left[\frac{W_0}{W(z)} \right]^2 \mathbb{G}_n^2 \left(\frac{\sqrt{2}x}{W(z)} \right) \mathbb{G}_n^2 \left(\frac{\sqrt{2}y}{W(z)} \right) \tag{B.1}$$

It is useful to normalize the first few lowest order modes with respect to the total optical power since the Gaussian beam will couple to them the most due either misalignment or mode mismatch as seen in section [].

In one dimension, the total optical power for the first 3 modes are

$$P_0(x,y,z) = \int_{-\infty}^{\infty} |A_0|^2 \left[\frac{W_0}{W(z)} \right] e^{-2x^2/w^2(z)} dx$$

$$P_1(x,y,z) = \int_{-\infty}^{\infty} |A_1|^2 \left[\frac{W_0}{W(z)} \right] \frac{8x^2}{w^2(z)} e^{-2x^2/w^2(z)} dx \tag{B.2}$$

$$P_2(x,y,z) = \int_{-\infty}^{\infty} |A_2|^2 \left[\frac{W_0}{W(z)} \right] \left(\frac{8x^2}{w^2(z)} - 2 \right)^2 e^{-2x^2/w^2(z)} dx$$

In two dimensions, the total optical power for the first 3 modes are

$$P_{00}(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A_{00}|^2 \left[\frac{W_0}{W(z)} \right]^2 e^{-2x^2/w^2(z)} e^{-2y^2/w^2(z)} dx dy$$

$$P_{10}(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A_{10}|^2 \left[\frac{W_0}{W(z)} \right]^2 \frac{8x}{w^2(z)} e^{-2x^2/w^2(z)} e^{-2y^2/w^2(z)} dx dy \tag{B.3}$$

$$P_{20}(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A_{20}|^2 \left[\frac{W_0}{W(z)} \right]^2 \left(\frac{8x^2}{w^2(z)} - 2 \right)^2 e^{-2x^2/w^2(z)} e^{-2y^2/w^2(z)} dx dy$$

By setting the equations above to unity, the normalization factors become

$$A_0 = \left(\frac{2}{\pi w_0^2} \right)^{1/4}$$

$$A_1 = \left(\frac{2}{\pi w_0^2} \right)^{1/4} \frac{1}{\sqrt{2}} \tag{B.4}$$

$$A_2 = \left(\frac{2}{\pi w_0^2} \right)^{1/4} \frac{1}{8}$$

$$A_{00} = \sqrt{\frac{2}{\pi w_0^2}}$$

$$A_{10} = \sqrt{\frac{1}{\pi w_0^2}} \tag{B.5}$$

$$A_{20} = \sqrt{\frac{1}{4\pi w_0^2}}$$

Therefore the normalized modes are

Appendix C

Bullseye Photodiode Characterization

C.1 DC

$$\text{Power} = \int_A^B |A_{00}|^2 e^{\frac{-2r^2}{\omega_0^2}} 2\pi r dr$$
$$= -|A_{00}|^2 \frac{\pi\omega_0^2}{2} e^{\frac{-2r^2}{\omega_0^2}} \Big|_A^B \tag{C.1}$$

$$\text{P}_{in} = \text{Power}\Big|_0^{r_0} = |A_{00}|^2 \frac{\pi\omega_0^2}{2}[1 - e^{\frac{-2r_0^2}{\omega_0^2}}] \tag{C.2}$$

$$\text{P}_{out} = \text{Power}\Big|_{r_0}^{\infty} = |A_{00}|^2 \frac{\pi\omega_0^2}{2}[e^{\frac{-2r_0^2}{\omega_0^2}}] \tag{C.3}$$

$$\text{P}_{total} = \text{P}_{in} + \text{P}_{out} \tag{C.4}$$

$$\omega = \sqrt{\frac{\text{P}_{total}}{|A_{00}|^2 \pi/2}} \tag{C.5}$$

$$\text{DC Power Ratio} = \frac{P_{out}}{P_{in}} = \frac{e^{-2r_0^2/\omega_0^2}}{1 - e^{-2r_0^2/\omega_0^2}} \approx 0.582 \tag{C.6}$$

C.2 RF

$$\text{P}_{RF} = \int_A^B |A_{01}|^2 \big(1 - \frac{2r^2}{\omega_0^2}\big) e^{\frac{-2r^2}{\omega_0^2}} 2\pi r dr$$
$$= -|A_{00}|^2 \frac{\pi}{2}\omega_0^2 e^{\frac{-2r^2}{\omega_0^2}} \left(1 + \frac{4r_4}{\omega_0^4}\right)\Big|_A^B \tag{C.7}$$

$$\mathrm{P}_{in} = \mathrm{P}_{RF}\Big|_{0}^{r_0} = -|A_{01}|^2 \frac{\pi}{2}\omega_0^2 \left(e^{\frac{-2r^2}{\omega_0^2}}\left(1 + \frac{4r_4}{\omega_0^4}\right) - 1 \right) \tag{C.8}$$

$$\mathrm{P}_{out} = \mathrm{P}_{RF}\Big|_{r_0}^{\infty} = -|A_{01}|^2 \frac{\pi}{2}\omega_0^2 e^{\frac{-2r^2}{\omega_0^2}}\left(1 + \frac{4r_4}{\omega_0^4}\right) \tag{C.9}$$

$$\text{RF Power Ratio} = \frac{P_{out}}{P_{in}} = \frac{e^{-2r_0^2/\omega_0^2}}{1 - e^{-2r_0^2/\omega_0^2}} \approx 2.7844 \tag{C.10}$$

Appendix D

Overlap of Gaussian Beams

Referenced in section

The full Gaussian beam overlap is important in quantitatively defining the amount of power loss obtained when a cavity is mismatched a incoming laser field.

First we define an arbitrary Gaussian beam in cylindrical coordinates:

$$
\begin{aligned}
|A(r)\rangle &= \frac{A_0}{q(z)} e^{\frac{-ikr^2}{2q(z)}} \\
&= \frac{A_0}{q(z)} e^{\frac{-ikr^2(z-iz_0)}{2|q(z)|^2}}
\end{aligned}
\tag{D.1}
$$

where A_0 is a real amplitude, $q(z) = z + iz_0$ is the complex beam parameter, k is the wave number, and r is the radial variable in the transverse direction.

First we normalize the overlap integral to unity:

$$
\langle A(r)|A(r)\rangle = \frac{|A_0|^2}{z^2 + z_0^2} \int_0^\infty e^{\frac{-kr^2 z_0}{|q(z)|^2}} 2\pi r dr = 1
\tag{D.2}
$$

Normalization factor is this:

$$
A_0 = \sqrt{\frac{kz_0}{\pi}}
\tag{D.3}
$$

For two Gaussian beams with arbitrary q-parameters

$$
|A_i\rangle = \frac{A_{0,i}}{q_i} e^{\frac{-ikr^2(z-iz_0)}{2|q_i|^2}}
\tag{D.4}
$$

where $z_{0,i}$ is the waist size of one particular beam.

The power overlap is:

$$\text{Overlap} = \langle A_1 | A_2 \rangle = 4 \frac{\sqrt{z_{0,1} z_{0,2}}}{|q_1 - q_2^*|^2} \tag{D.5}$$

Bibliography

[1] Peter Saulson. *Fundamentals of Gravitational Wave Detectors*. World Scientific, 2016. 6