CIV102-Matboard Bridge Project

Design Report

Group 606

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Introduction

Our group decided to make a beam bridge for this project after seriously considering the properties of the provided materials.

Firstly, we built a beam bridge after considering multiple models like truss bridges, suspension bridges, and arc bridges. Both truss and arch bridges require the structure to have a large flexural compressive strength, which is unsuitable for the provided matboard. Suspension bridges, on the other hand, are too difficult to build. In conclusion, our final choice is a beam bridge.

Secondly, we decided to place the glued parts near the bottom and top of the bridge. This is because the glue can withstand less shear stress than the matboard. According to the distribution graph of Q, the first moment of area, derived in the course note, is maximum at the centroid and minimum on the top and bottom sides. Then, according to Jourawski's equation, shear stress is proportional to Q, so the shear stress should be small on the top and bottom sides. Hence, we keep the glues away from the centroid to reduce shear stress.

Thirdly, the dimensions of the matboard are 813mm×1016mm×1.27mm. According to this, a matboard is not long enough to cover the horizontal distance between two supports, so we decided to use two parts, each 813mm long, overlapping to build a bridge that can cover this span.

Finally, we decided to reinforce the parts that need to withstand the greatest shear force and bending moment to maximize the bridge's capacity according to the shear envelope and bending moment envelope.

Iteration of Designs

Before analyzing the performance of this design, we have to clarify the testing environment and conditions, including shear force and bending moment. To carry a train in case 2 with a total weight of 446.7N, the maximum and minimum shear forces and bending moments are given by the following two graphs generated by codes. The codes that generated these two graphs are in Appendix B.

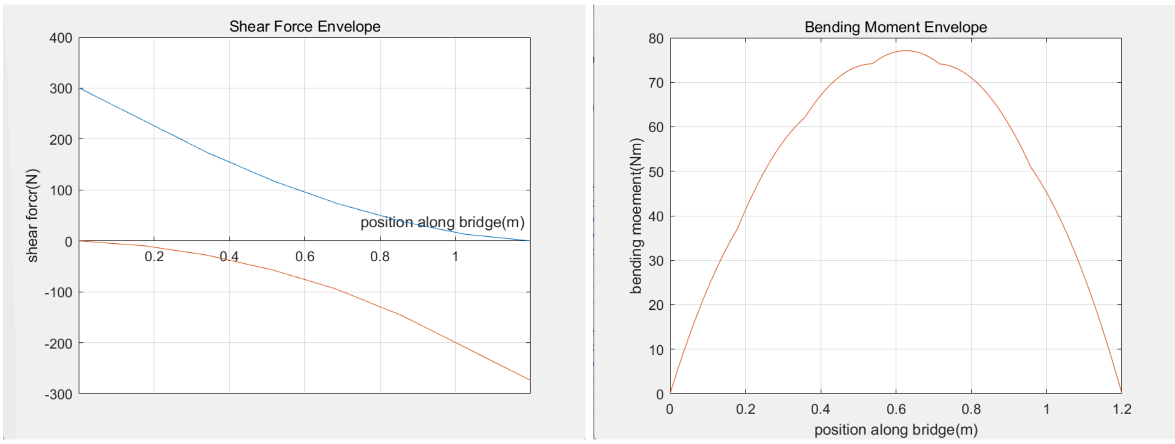


Figure 1. shear force envelope and bending moment envelope of load case 2 with a 446.7N weight train. The shear force and bending moment envelope of load case 1 are completely overshadowed by the above curves, so we will not pay any extra attention to them.

1. **Design zero**

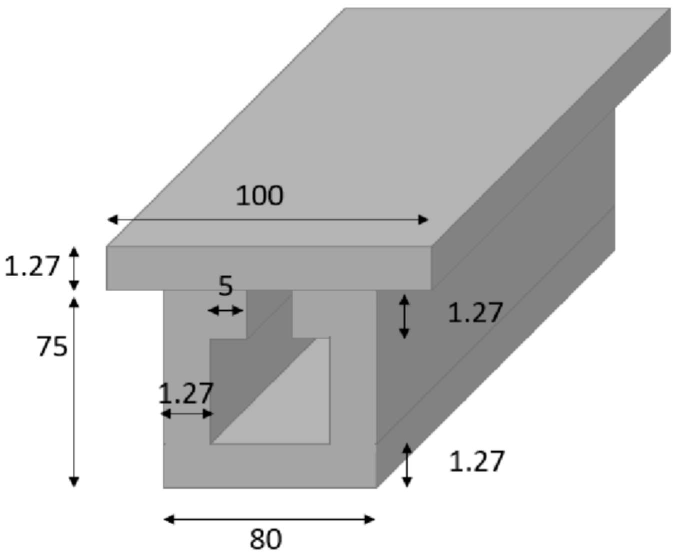


Figure 2 cross-section of design zero

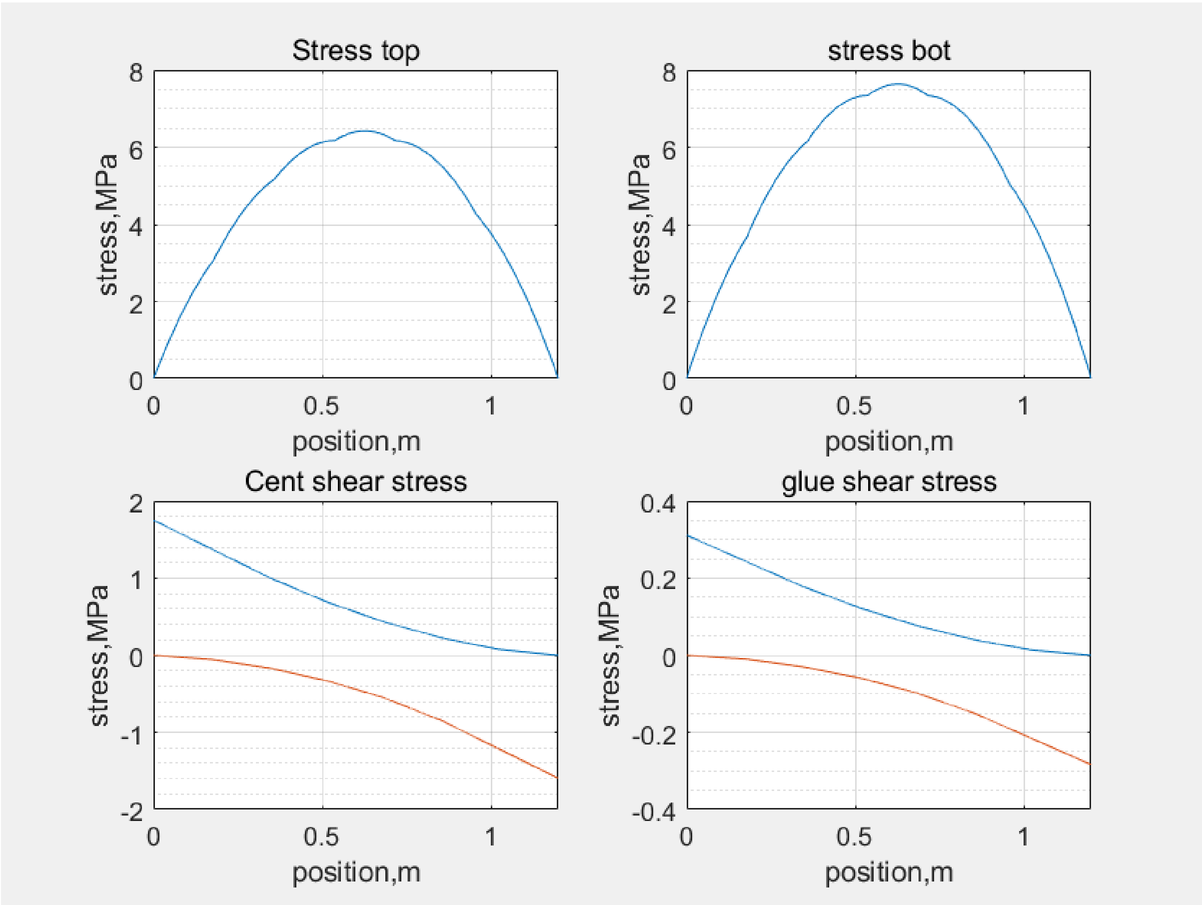
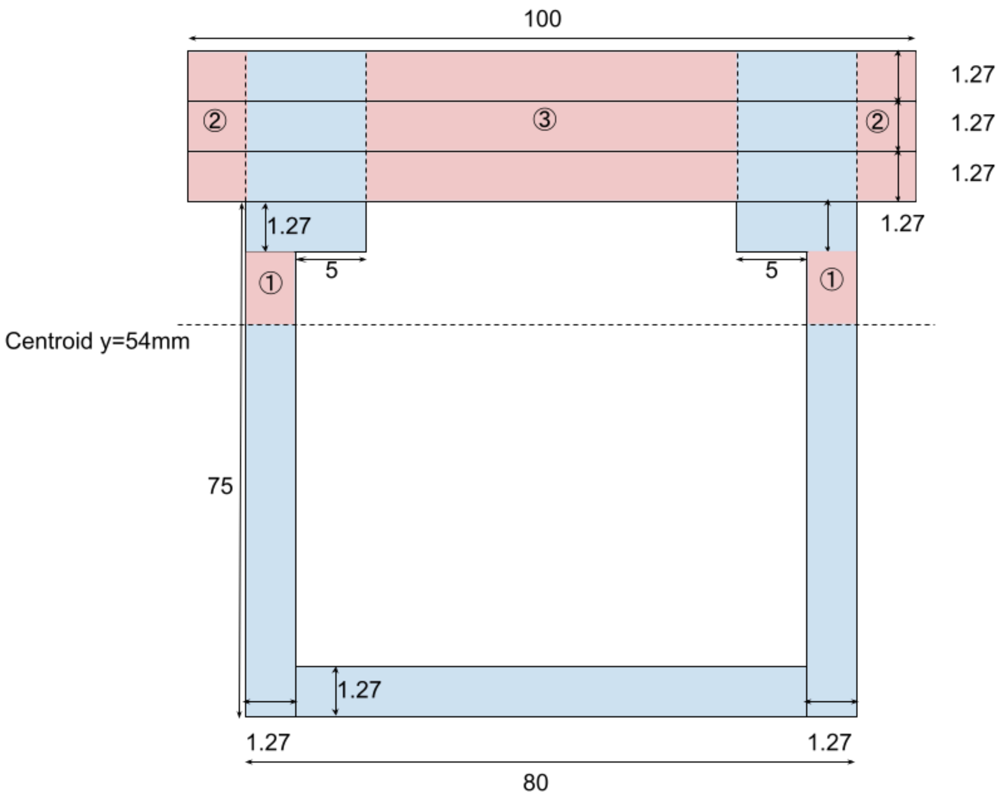


Figure 3, graphs about design zero under load case 2 with 446.7N train

Since there are only two supports available for the bridge, it will always experience flexural compression on the top and tension at the bottom. According to the data of the matboard, it can withstand tensile forces 5 times better than compression. However, the graph shows that the bridge experiences nearly the same force on top and bottom. Thus, it can be optimized a lot by moving the centroid of the cross-section upwards.

2. Design Based On Design Zero



For design 2, we made the top part of the cross-section thicker. However, the ratio between tensile and compressive stress is still far from the ideal 5:1. At this point, we have used most of the matboard, so it is not probable to further thicken the top flange.

Sections that are susceptible to large local buckling are colored in red. We will focus on calculating those parts later on.

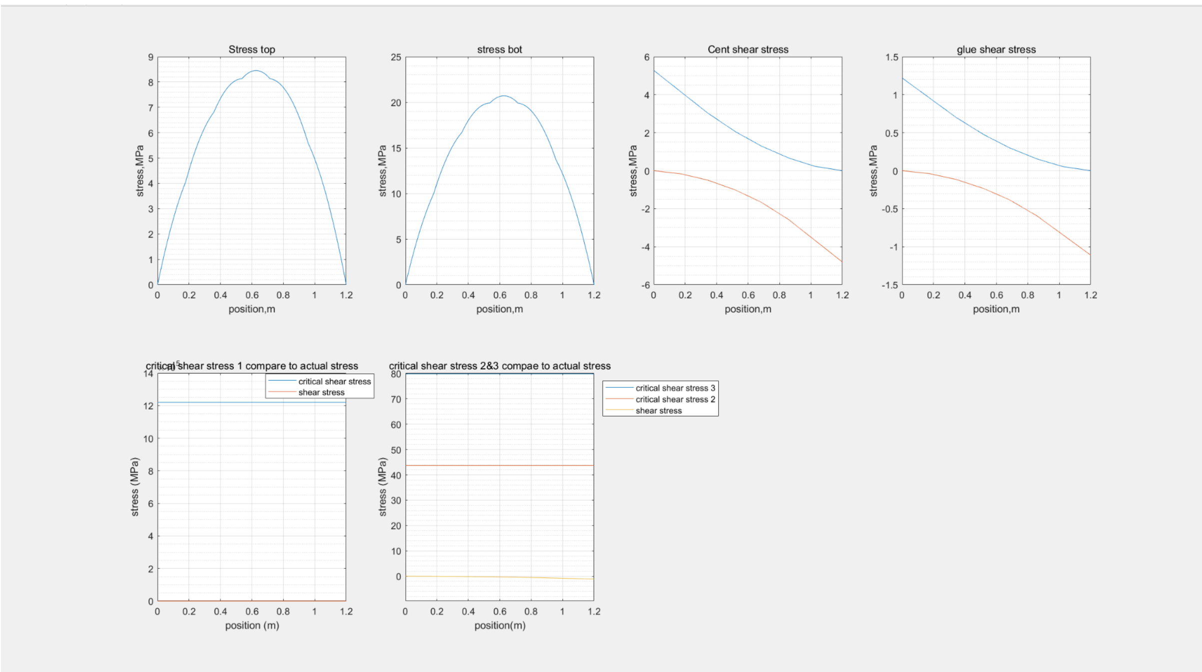


Figure 5 diagrams for design 2 under load case 2 with a 1340N train

To demonstrate the performance of this design better, we calculated the scenario of a train weighing 1340N to cross the bridge. According to the diagram, the bridge failed for both flexural compressive stress and shear stress to a similar extent.

3. New designs based on the information after studying the former designs

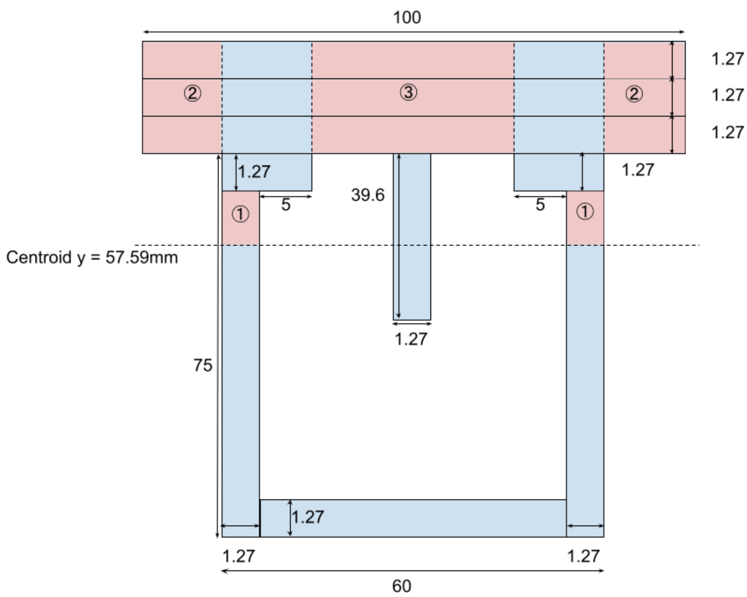


Figure 6 cross-section of design 3

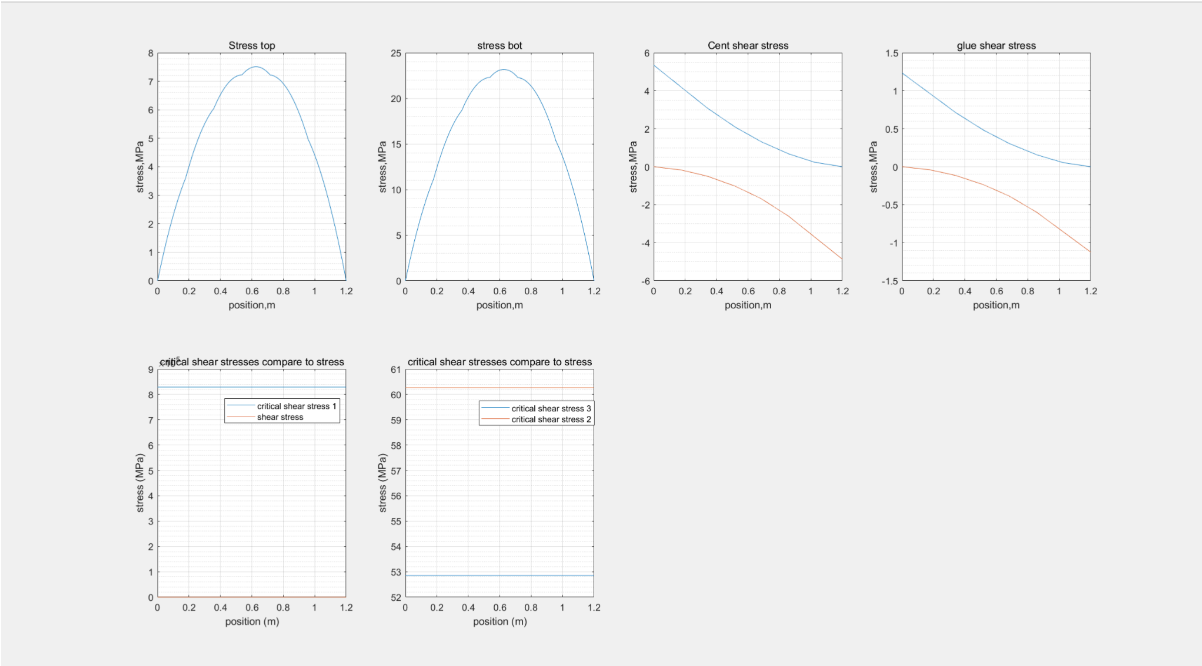


Figure 8 diagrams of design 3

According to this, the y-bar calculated is 57.6mm, I is 0.575×10^{6}mm.

The motivation behind adding a long protrusion at the top section of the beam was to raise the overall centroidal axis to resist more flexural compressive stress. The protrusion here will in theory increase the second moment of area and raise the centroidal axis. It had better performance when compared to design 2, although it still failed when doing the same 1340 N train test.

Other than the bending moment, this design also cannot resist high shear stress on the centroid, especially on the endpoints of the bridge, around 0m and 1.2m, where the shear stress is the largest. In the 1340N train test, it failed to carry shear force at 0m - 0.2m and 1 - 1.2m.

4. Final design (Theoretically)

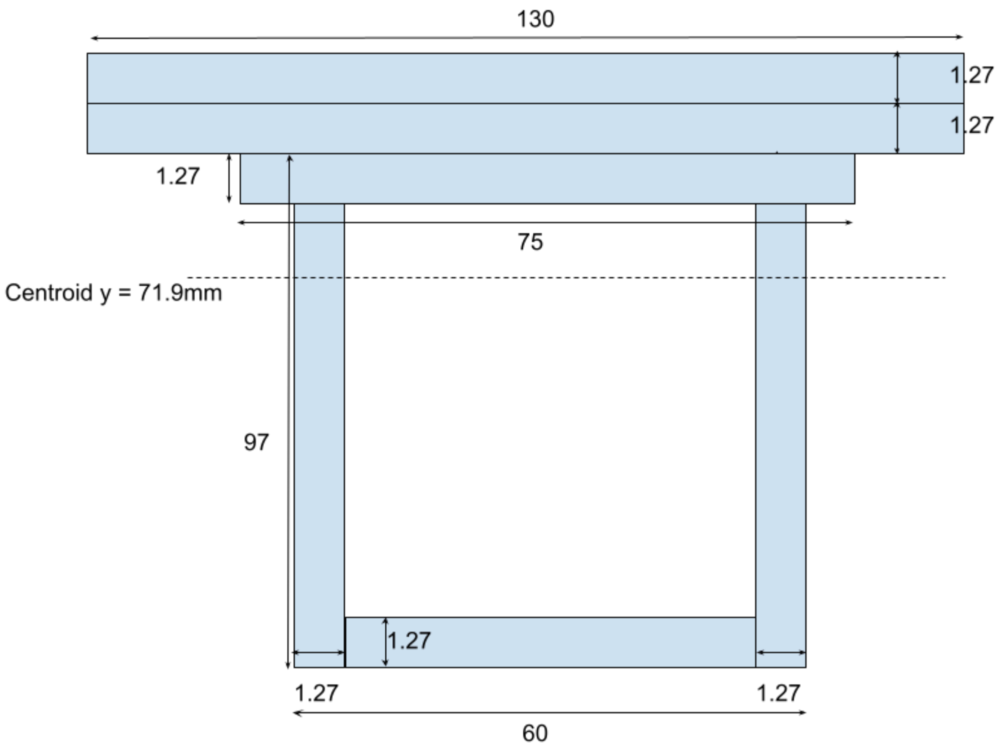
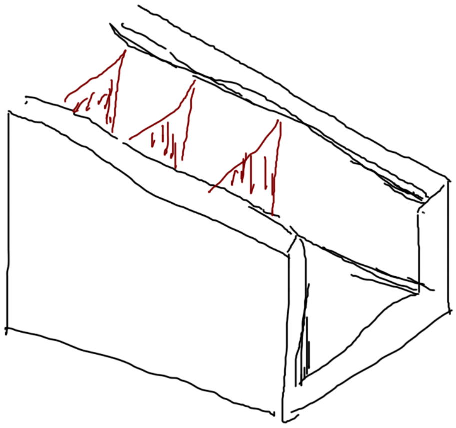


Figure 9

From design 3, we found that the bridge was still weak against compressive stress and from 0.350m to 0.890m, the compressive stress went over the limit in which the material can handle. From 0.000mm to 0.002mm the maximum shear stress was above the limit. Therefore based on this, our final design will focus on improving the bridge at those positions focusing on increasing the second moment of area I, raise the centroid higher, and to reduce the likelihood for thin plate buckling.

After a lot of iterations, at least 12 of them, which we have no more vigor to record, we come up with a theoretical cross section which is shown above. During those iterations, we found out that making the deck wider is the best way to improve I and the position of the centroid, so we moved more material to the deck, making it wider. We also increased the total height of the bridge from 80mm to 100mm to further increase the second moment of area. At first we thought about tremendously increasing the width of the deck, but later on we realized that it will fail due to local buckling. After considering all the factors the above cross section is our final answer, on paper.

The above design is enough to let a 1200N train pass through, but it does leave us with a large area of spare matboard, so we decided to add multiple diaphragms perpendicular to the deck in the middle part of the bridge, where the bending moment is the largest, and on the ends of the bridge, where the shear force is the largest:



Diaphragms colored in brown

Figure 10
We do not know how to calculate the new critical load, but from research and studying similar real life bridges we can conclude that it will help a lot.

The length of the matboard also does not allow for a complete bridge without splices, so we decided to make 2 half parts of the bridge, and stick them together using the spare matboards.

5. Final Design To Construct

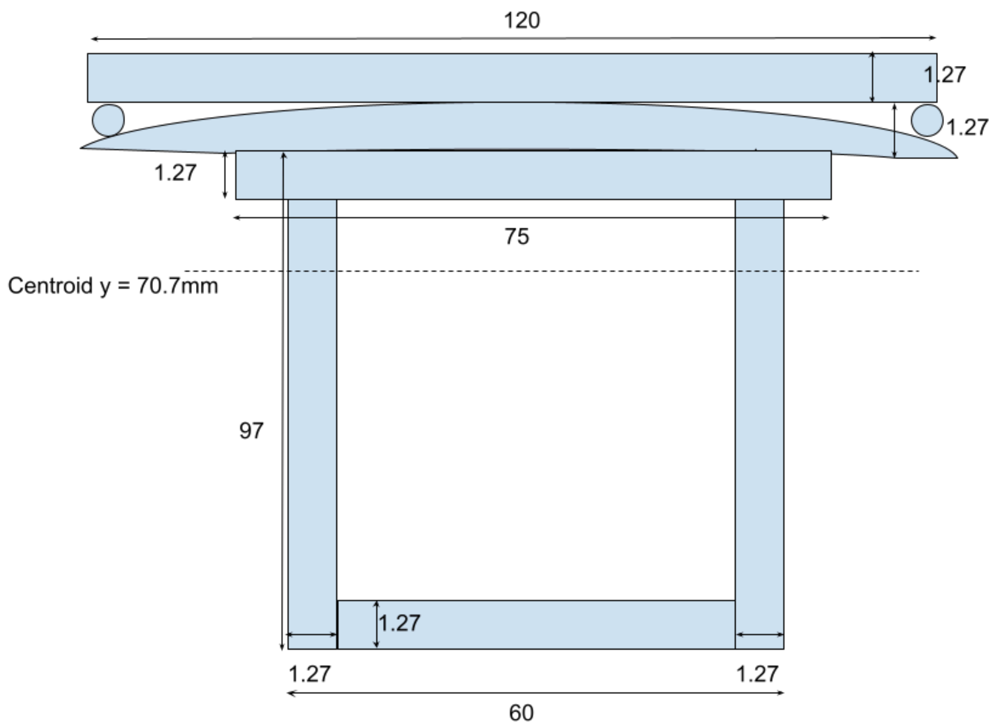


Figure 11

During the process of scheduling the usage of the matboard, we found out that although our last design has theoretically enough area of the matboard, realistically we cannot place the sections on the matboard for it to work. After discussion we decided to reduce the width of the deck to 120mm.

After doing some research we also discovered a technique called pre-stressing. If we bend the lower half of the deck upwards and stick to the top part, which is flat, using some leftover matboard (indicated by the 2 circles in the graph), as shown in the graph above, we can cancel out a portion of the weight of the train. We also do not know how to calculate this effect, so we ignored it in the calculation, knowing that it can only make the bridge stronger.

In the process of construction we had to place the splice point of the bridge in the middle, where the bending moment is the largest. We were afraid that this is the weakest point of the bridge, and reinforced this position.

Data & Graphs for Capacity of our Bridge

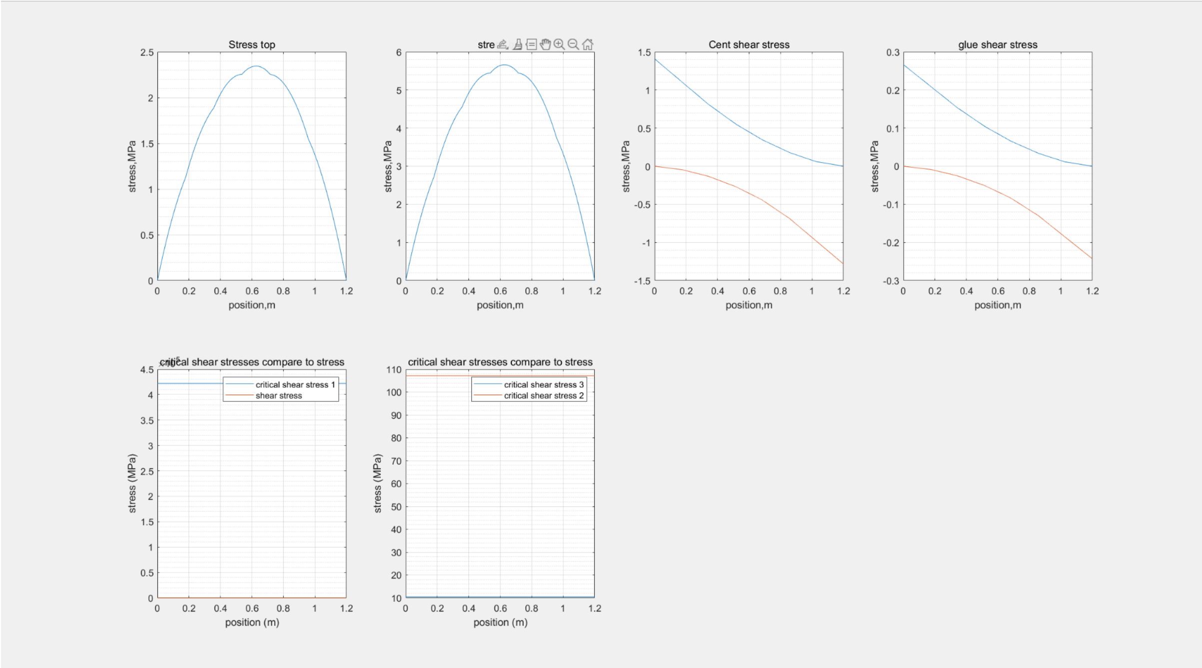


Figure 12

Performance and Factor of Safety:

Max Flexural Compressive Stress: 2.35Mpa

Allowable: 6.00Mpa

FOS = $\frac{6.00Mpa}{2.35Mpa} = 2.55$

Max Flexural Tensile Stress: 5.66Mpa

Allowable: 30.0Mpa

FOS = $\frac{30.0Mpa}{5.66Mpa} = 5.30$

Max Shear at Centroid: 1.403Mpa

Allowable: 4.00Mpa

FOS = $\frac{4.00Mpa}{1.403Mpa} = 2.85$

Max Shear at Glue: 0.266 Mpa

Allowable: 2.00Mpa

FOS = $\frac{2.00Mpa}{0.266Mpa} = 7.52$

Max Shear Stress Capacity On Fledge: 0.266Mpa

Allowable: 10.44Mpa

FOS = $\frac{10.44Mpa}{0.266Mpa} = 39.2$

Max Shear Stress Capacity On Web: 1.400 Mpa

Allowable: 422×10^3

FOS = $\frac{422 \times 10^3 Mpa}{1.400Mpa} = 301 \times 10^3$

Pictures of our Workspace

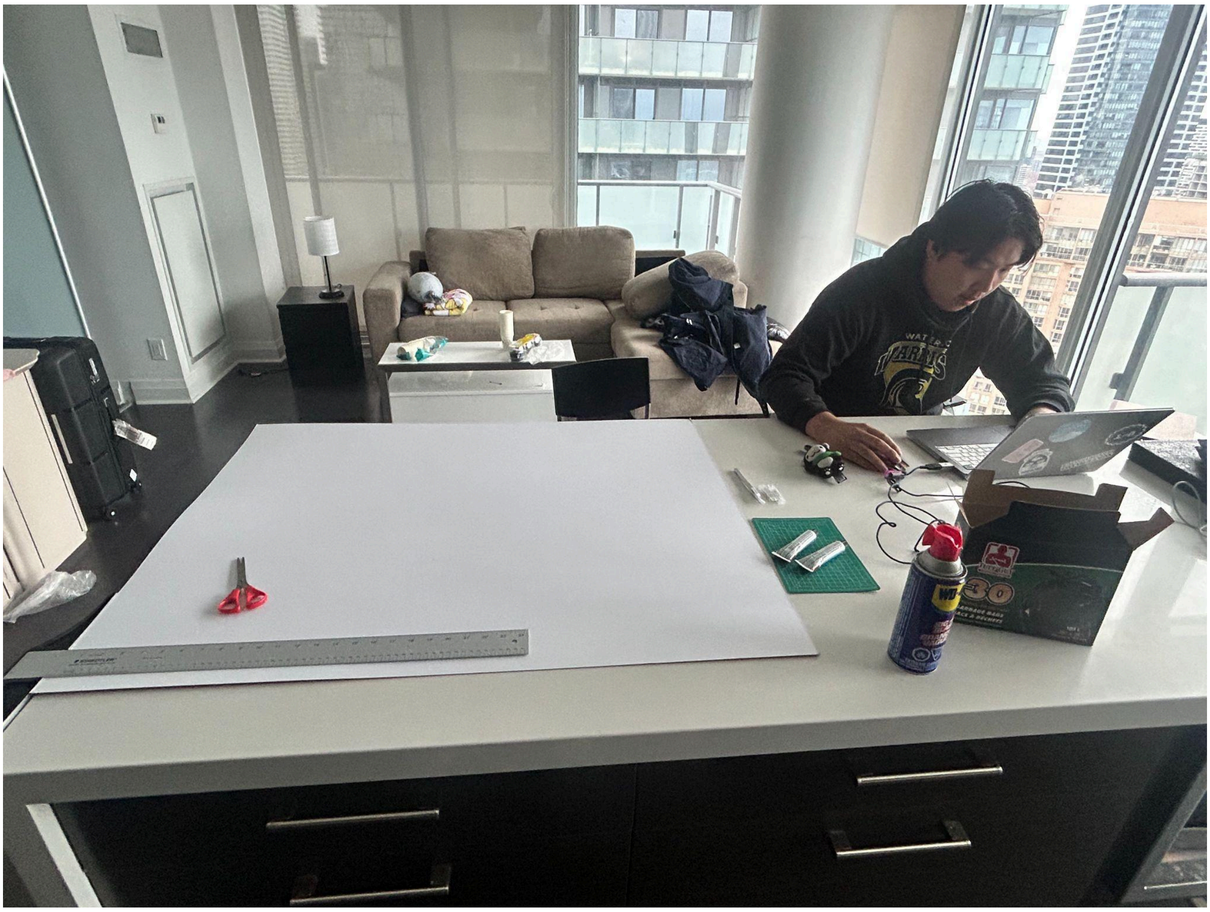


Figure 13 workspace before building

Figure 14 workspace after building

Appendix A(resources and formulas)

1 Q distribution

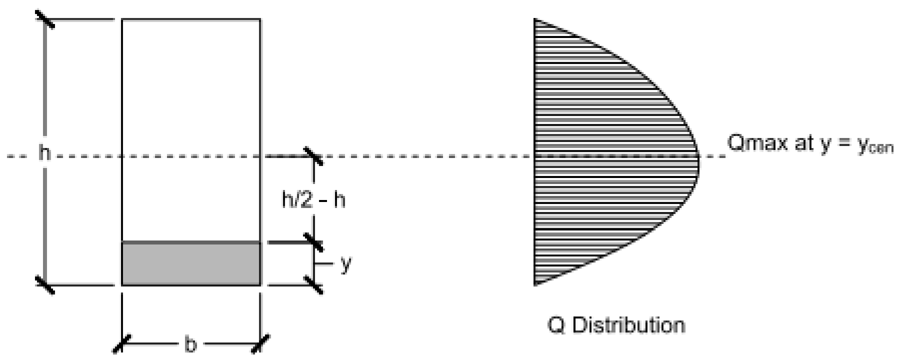


Fig. 25.8 – Derivation of Q for a rectangular cross section and resulting distribution over the member height.

2 Jourawski's equation on the course note

$$\tau = \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} \frac{Q}{Ib} = \frac{VQ}{Ib} \tag{25.8}$$

<u>AppendixB(codes)</u>

Matlab code used to get the graphs

```
clear
clc
maximum_shear = [];
minimum_shear = [];

x_plot = [];

for point = 0:0.001:1.2; % This loop will now iterate in 1mm increments
    shear = [];
    moment=[];
    for x = -0.52:0.001:2.056;
        position = [-0.856, -0.680, -0.516, -0.340, -0.176, 0] + x;
        position_on_bridge = [];
        current_shear=0;
        total_moment = 0;
        for i = 1:length(position);
            if position(i) >= 0 && position(i) <= 1.2;
                position_on_bridge = [position_on_bridge, position(i)];
                if i==1|i==2;
                    total_moment=total_moment+position(i)*66.7*1.35;
                else;
                    total_moment = total_moment + position(i) * 66.7; % Moment contribution from each point
                end;
            end;
        end;

        reactionA = total_moment / 1.2;
        if length(position_on_bridge)<=2;
            reactionB = (1.35*66.7* length(position_on_bridge)) - reactionA;
        else;
            reactionB = (66.7*(length(position_on_bridge)-2))+2*66.7*1.35-reactionA;
        end;

        shearforces = [reactionB];

        for i = 1:length(position_on_bridge); %get shear forces
            if i==1 | i==2
                shearforces=[shearforces,reactionB-i*66.7*1.35];
            else;
                shearforces = [shearforces, shearforces(i) - 66.7];
            end;

        end;
        position_index = find(position_on_bridge < point, 1, 'last');
        if isempty(position_index) % If no positions are before the point
            current_shear = shearforces(1);% Find the shear force just before the current point
        else;
            current_shear = shearforces(position_index+1);
        end;
        shear = [shear, current_shear];
    end
    maximum_shear = [maximum_shear, max(shear)];
    minimum_shear = [minimum_shear, min(shear)];
    x_plot = [x_plot, point];
end

maxmoment=[0.0, 0.6121074999999999, 1.22255, 1.8313275, 2.43844, 3.0438875, 3.6476699999999997, 4.2497875, 4.850239999999999, 5.4490275, 6.046149999999999, 6.641607499999997, 7.2353999999999985, 7.827527500000003, 8.41799, 9.006787500000002, 9.593919999999999, 10.179387500000002, 10.763190000000002, 11.345327499999998,
w=446.7;
for i =1:1201
    maxmoment(i)=maxmoment(i)*w/1000;
end
```

```
figure(1)
plot(x_plot, maximum_shear);
grid on
hold on;
plot(x_plot, minimum_shear);
set(gca,'XAxisLocation','origin');
ylabel('shear force(N)')
xlabel("position along bridge(m)")
title("Shear Force Envelope")
set(gca,'YTickLabel',num2str(get(gca,'YTick')','%.2f'))

figure(2)
plot(x_plot,maxmoment);
hold on
set(gca,'XAxisLocation','origin');
ylabel('bending moment(Nm)')
xlabel('position along bridge(m)')
title("Bending Moment Envelope")
grid on


E=4000%MPa;
mu=0.2;

S_top=[];
S_bot=[];
T_cent=[];
t_cent=[];
tglue=[];
Tglue=[];

flux3=[];
flux2=[];
flux1=[];
for i = 1:1201;
    y= 70.712;
    I= 962936.96232;
    Q= [8539.78, 11464.05];
    Qcent=Q(2);
    Qglue=Q(1);
    fluxcase3=6*((95-1.27-y)/(1.27*6))^2*(E*pi^2)/(12*(1-mu^2));
    fluxcase1=4*(1.27*4/(60-2*1.27))^2*(E*pi^2)/(12*(1-mu^2));
    fluxcase2=0.425*(1.27*2/30)^2*(E*pi^2)/(12*(1-mu^2));
  S_top=[S_top,maxmoment(i)*(100-y)/I*1000];
  S_bot=[S_bot,maxmoment(i)*y/I*1000];
  T_cent=[T_cent, (maximum_shear(i)*Qcent)/(I*1.27*2)];%MPa
  Tglue=[Tglue, (maximum_shear(i)*Qglue)/(I*5*2)];
  t_cent=[t_cent, (minimum_shear(i)*Qcent)/(I*1.27*2)];
  tglue=[tglue, (minimum_shear(i)*Qglue)/(I*5*2)];
  flux1=[flux1,fluxcase1];
  flux2=[flux2,fluxcase2];
  flux3=[flux3,fluxcase3];

end
```

```
figure;%plot shear stresses

subplot(2,4,1);
plot(x_plot,S_top)
ylabel('stress,MPa')
xlabel('position,m')
title('Stress top')
grid on
set (gca,"YMinorGrid","on")
subplot(2,4,2)
plot(x_plot,S_bot)
ylabel('stress,MPa')
xlabel('position,m')
title("stress bot")
grid on
set (gca,"YMinorGrid","on")
subplot(2,4,3)
plot(x_plot,T_cent)
hold on
plot(x_plot,t_cent)
ylabel('stress,MPa')
xlabel('position,m')
title("Cent shear stress")
grid on
set (gca,"YMinorGrid","on")
subplot(2,4,4)
plot(x_plot,Tglue)
hold on
plot(x_plot,tglue)
ylabel('stress,MPa')
xlabel('position,m')
title('glue shear stress')
grid on
set (gca,"YMinorGrid","on")
subplot(2,4,5)
plot(x_plot,flux3,DisplayName="critical shear stress 1");
hold on
plot(x_plot,T_cent,DisplayName="shear stress");
ylabel('stress (MPa)');
legend()
title ('critical shear stresses compare to stress')
xlabel('position (m)')
grid on
set (gca,"YMinorGrid","on")
subplot(2,4,6)

plot(x_plot,flux2,DisplayName="critical shear stress 3");
hold on
plot(x_plot,flux1,DisplayName="critical shear stress 2");
ylabel('stress (MPa)');
legend()
title ('critical shear stresses compare to stress')
xlabel('position (m)')
grid on
set (gca,"YMinorGrid","on")
```

Python codes are used to calculate bending moment envelope and calculate geometry properties (I,Q, y-bar)

```python
def get_all(cross,m,glue_y):
    y_bar = 0
    height,y_bar,i = autoy(cross)
    top = m * (height - y_bar) / i
    bottom = m * y_bar / i
    print("Stress Top: ", top)
    print("Stress Bottom: ", bottom)
    #autoy([[77.46,t,t/2],[2*t,75-t,(75-t)/2],[2*(5+t),t,75+t/2],[100,3t,75+2.5*t]],[75+t,75+2*t,75+3*t])

def autoy(cross,glue_y):
    '''glue_y = [y1,y2,y3,...]'''
    '''cross = [[b,h,y],[...],[...],[...],...]'''
    '''m = Moment'''
    area_sum = find_area_sum(cross)
    height = find_height(cross)
    fi = 0
    y_bar = 0
    i = 0
    for element in range(len(cross)):
        fi = fi + cross[element][0] * cross[element][1] * cross[element][2]
    y_bar = fi / area_sum
    y_bar *= 1000
    y_bar = y_bar // 1
    y_bar = y_bar / 1000
    print("Y Bar (Bottom Up): ", y_bar)
    i = find_I(cross,y_bar)
    q = find_Q(cross,y_bar,y_bar)
    q_q = q[0]
    print("Q(Centroid): ", q_q)
    for i in range(len(glue_y)):
        q_here = find_Q(cross,y_bar,glue_y[i])
        print("Q(Glue) at",q_here[1],"is",q_here[0])
    return (height,y_bar,i)

def find_Q(cross, y_bar,interest):
    res = 0
    for element in range(len(cross)):
        b = cross[element][0]
        h = cross[element][1]
        y = cross[element][2]
        top = y + h/2
        down = y - h/2
        if down >= interest:
            continue
        elif top <= interest:
            res = res + b*h*(y_bar - y)
        else:
            reduce = top - interest
            h = h - reduce
            y = y - (reduce/2)
            res = res + b*h*(y_bar - y)
    return (res,interest)


def find_area_sum(cross):
    res = 0
    for element in range(len(cross)):
        res = res + cross[element][0] * cross[element][1]
    print("Cross Section Area: ", res)
    return res

def find_height(cross):
    res = 0
    for element in range(len(cross)):
        res = res + cross[element][1]
    print("Total Height: ", res)
    return res

def find_I(cross,y_bar):
    i = 0
    for element in range(len(cross)):
        i = i + cross[element][0] * cross[element][1] * cross[element][1] * cross[element][1] / 12
        i = i + cross[element][0] * cross[element][1] * (abs(cross[element][2] - y_bar))**2
    print("Second I:", i)
    return i
```