Scale-freeness and Growth Stability of Realistic Network Models

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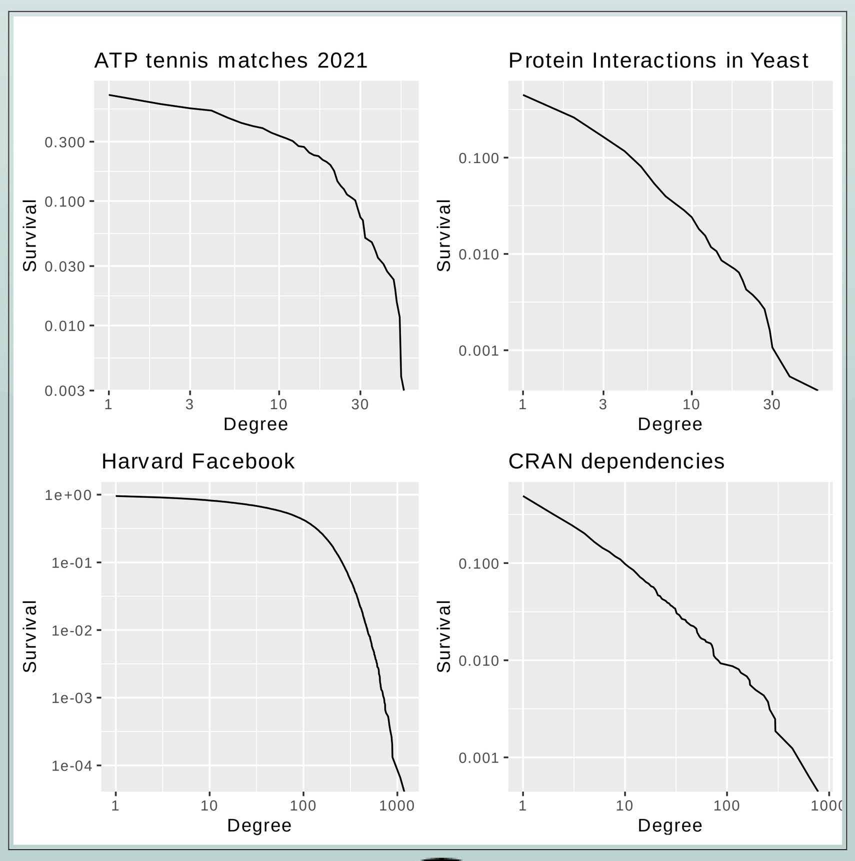
Barabási-Albert Model

1. **Growth:** At each time step, add a node to the network that will connect to $m \leq m_0$ nodes (already in the network) with m edges.

2. **Preferential Attachment:** The probability that an edge from the new node connects to node i is proportional to its current degree k_i i.e $k_i / \Sigma_j k_j$.

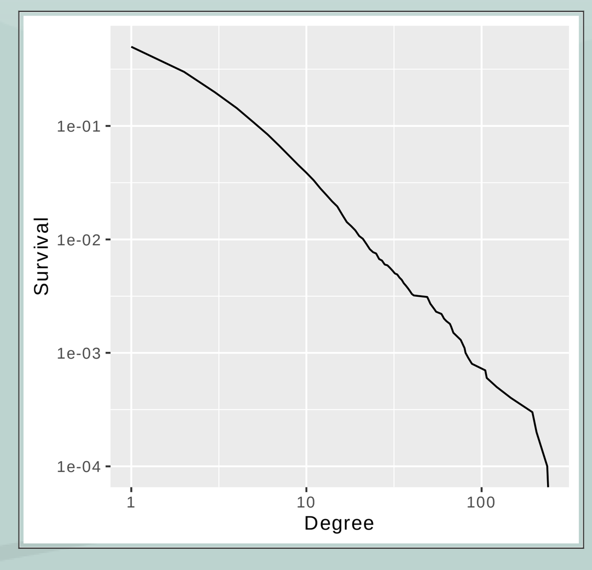
Survival Functions of Real Networks



Barabási-Albert Realistic?

+ Simple
+ Interpretable
+ Power law degree distribution
- Not accurate to a lot real networks

Survival of simulated network



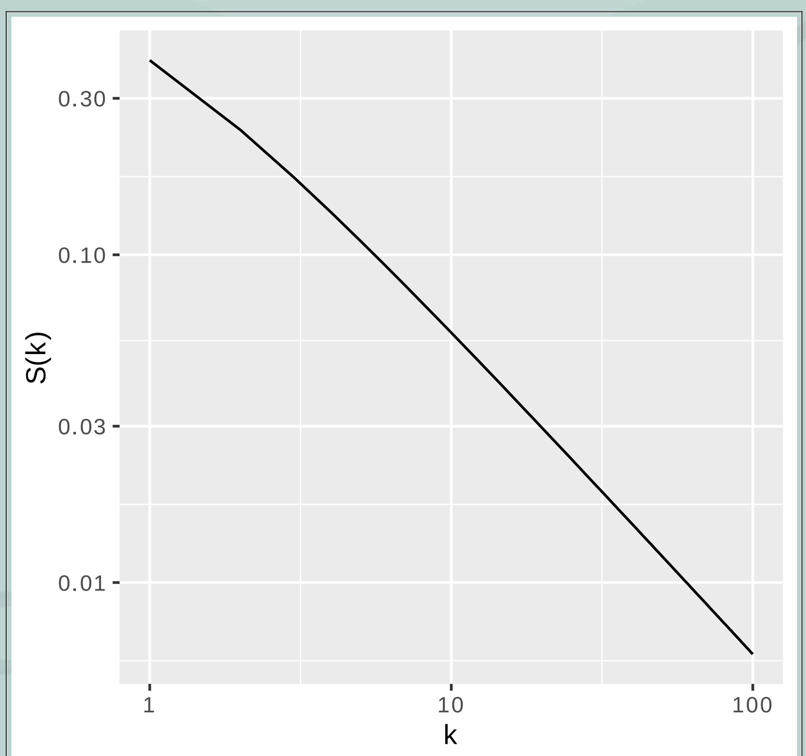
Power Law

$$f(k) = \zeta(\alpha+1)^{-1} k^{-(\alpha+1)}$$

For some $\alpha \in \mathbb{R}^+$.

A **very simple** model that is only characterised by only one parameter α.

The survival function of this model for $\alpha = 1$ is below:



What are we doing?

Networks are common in many fields of research such as:

- **Sports Modelling**
- **Social Networks**
- **Biological Systems**

We are interested in modelling how these networks may have formed.

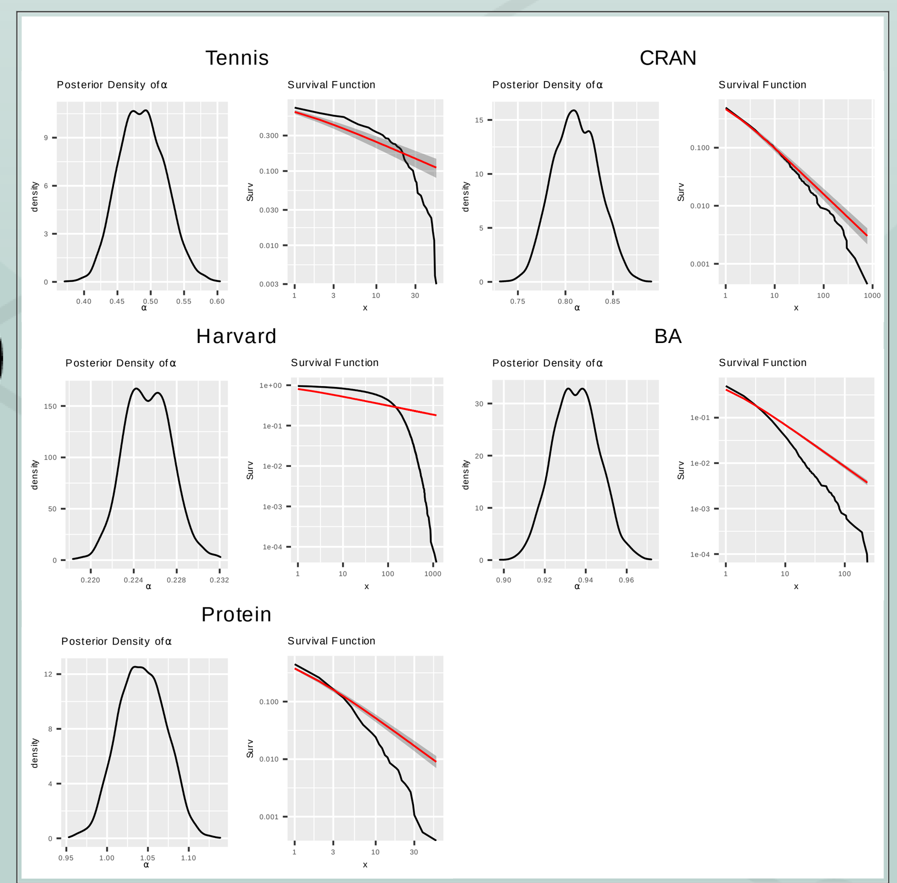
We want to find a model that improves on the Barabási-Albert model that is:

- Fairly **simple**
- Represents real networks growth accurately

The first step of this is to find a suitable model for the degree distribution of real networks.

How good is the power law?

Fitting the power law to data from real networks alongside the simulated network.



Clearly **not adequate** for the full set of data real or simulated.

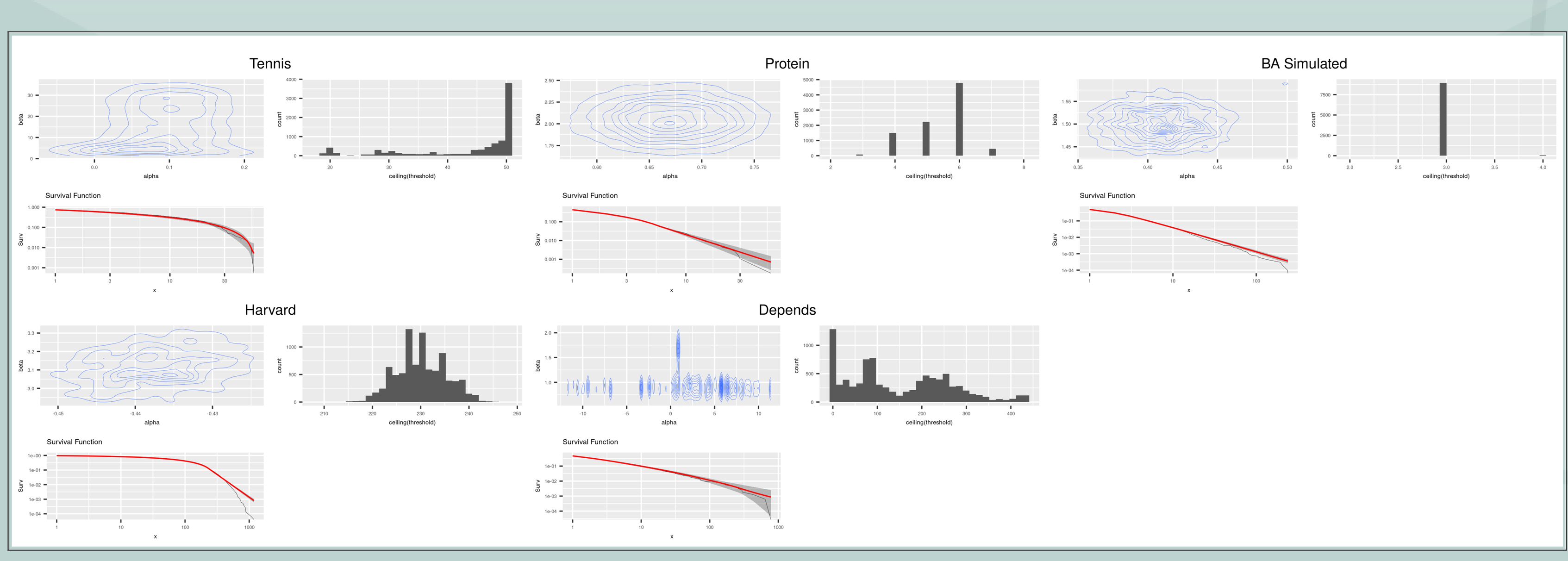
Power Law Mixture

$$f(k) = (1-\phi)\mathbb{I}_{\leq v}(k)\, k^{-(\alpha+1)} [\zeta(\alpha+1) - \zeta(\alpha+1, v+1)]^{-1} + \phi\, \mathbb{I}_{>v}(k)\, k^{-(\beta+1)} \zeta(\beta+1, v+1)$$

For some $\alpha \in \mathbb{R}^+$, $\beta \in \mathbb{R}$, and $\phi \in (0,1)$.

Natural extension of vanilla power law model, remains somewhat simple.

Fitting this model to the same data sets:



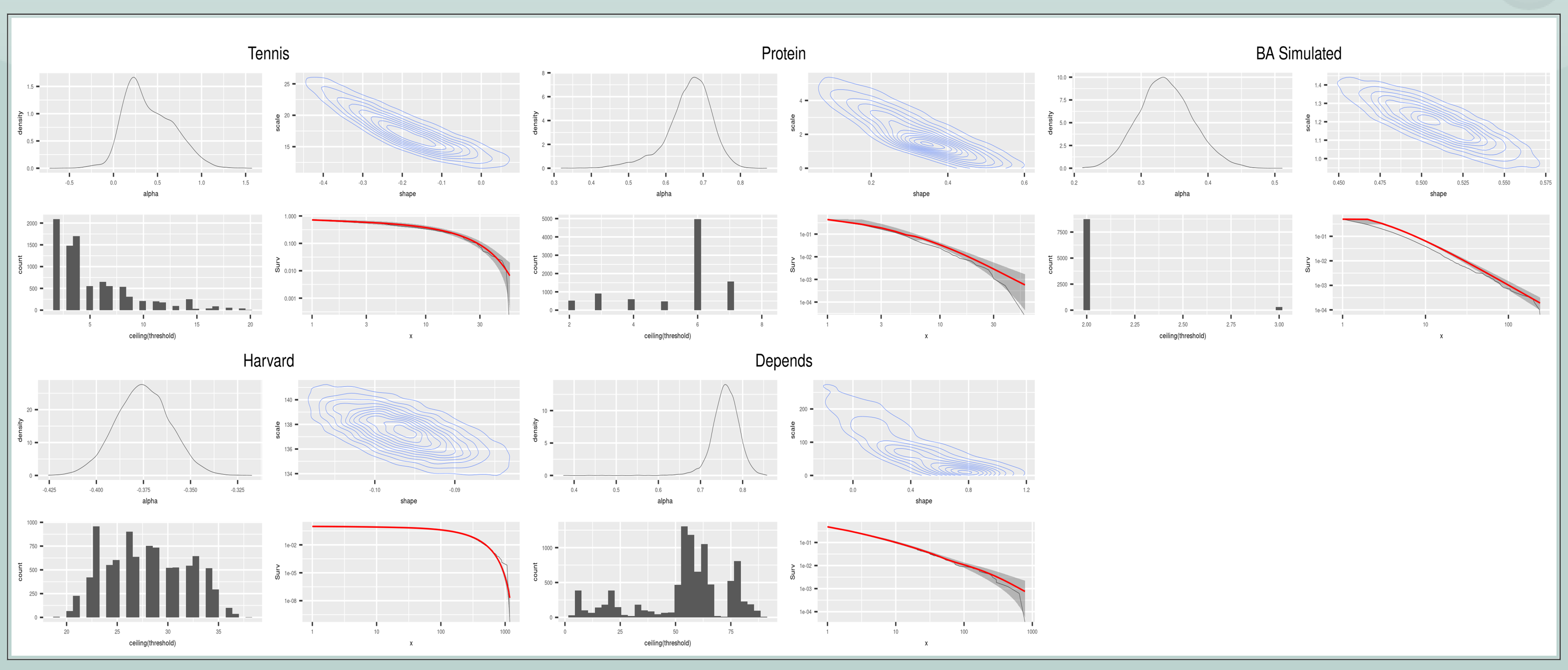
Works **better than the power law**, but could still be improved.

Power Law – Integer Generalised Pareto Mixture

$$f(k) = (1-\phi)\mathbb{I}_{\leq v}(k)\, k^{-(\alpha+1)} [\zeta(\alpha+1) - \zeta(\alpha+1, v+1)]^{-1} + \phi\, \mathbb{I}_{>v}(k)\, G(k; v, \sigma_0, \xi)$$

For some $\alpha \in \mathbb{R}^+$, $\sigma_0 \in \mathbb{R}^+$, $\xi \in \mathbb{R}$, and $\phi \in (0,1)$

More **complex**, that requires discretised version of GPD.



Conclusions and The Future

PL-IGP is the best model we have found so far, could now measure changes in parameters over time. This will then inform modification of the Barabási-Albert model.