Annual Progress Review

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1 Research Progress

This research has been focused on networks and modelling how they grow. The main focus has so far been the degree distribution of both real world networks as well as networks simulated from various models.

1.1 Degree Distribution of Real Networks

The research began by looking at the degree distributions of real networks, with the aim of gaining some insight to how they may have grown and will continue to. Looking at Figure 1.1 it is clear that the degree distribution of networks can have various different shapes on a log-log scale. Some look almost linear, but others look more concave.

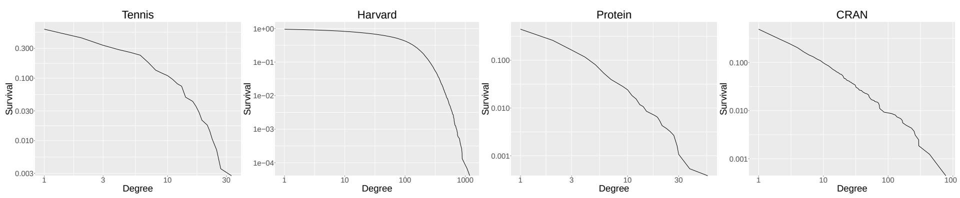

Figure 1.1: Survival function of the degrees.

1.2 Generative Network Models

Network generation is a complex subject and so finding a fairly simple way of modelling how real networks might have grown is very attractive and the one of the reasons for the popularity of the Barabási-Albert model.

1.2.1 Barabási-Albert Model

First introduced by Barabási and Albert (1999), this is a simple network growth model whereby, when a vertex is added to the network it connects to those already in the network with probability proportional to their degree. More formally, starting with m_0 nodes , the edges between them chosen arbitrarily such that each node has at least one link. The network then develops using two steps:

1. Growth

   At each time step, add a node to the network that will connect to $m \leq m_0$ nodes (already in the network) with m edges.

2. Preferential Attachment

   The probability that an edge from the new node connects to node i is proportional to its current degree k_i i.e $k_i / \sum_j k_j$.

This model has been proven through various means to produce a network that follows a power law with exponent 3. The method used in Barabási, Albert, and Jeong (1999) is unclear and it is not stated where some of the equations used come from. So, as part of reviewing the literature, we set out to make the proof of this more thorough with the idea of applying the method to more complex models.

1.2.2 Degree Distribution of the Barabási-Albert Model

We first consider the degree of a particular vertex in the network k_i, and notice that at any given time step the degree of vertex i can gain up to m edges with each edge connecting with probability $p_i(t) = k_i / \sum_j k_j$. This bears some resemblance to a self exciting poisson process.

Consider the Hawkes' process with random intensity at time t given by $R_i(t) \sim \mathrm{Bin}(m, p_i(t))$, where $p_i(t)$

References

Barabási, Albert-László, and Réka Albert. 1999. "Emergence of Scaling in Random Networks." *Science* 286 (5439): 509–12. https://doi.org/10.1126/science.286.5439.509.

Barabási, Albert-László, Réka Albert, and Hawoong Jeong. 1999. "Mean-Field Theory for Scale-Free Random Networks." *Physica A: Statistical Mechanics and Its Applications* 272 (1): 173–87. https://doi.org/https://doi.org/10.1016/S0378-4371(99)00291-5.