

1. REPORT SUBMISSION FORM



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PROJECTILE MOTION

By

TAN WEI LIANG

DECEMBER 2023

First Year Laboratory Report

PROJECTILE MOTION

3. ABSTRACT

The title of this experiment is Projectile Motion. This experiment explores projectile motion by analysing the trajectory of a steel ball launched at various velocities and angles relative to the horizontal using PHYWE's Projectile Motion experiment set. The study involves collecting data to establish correlations between the angle of projection and both the range and height of the projectile. The experimental results revealed a gravitational acceleration of $(9.71 \pm 0.01) \text{ ms}^{-2}$, exhibiting a 0.92% discrepancy from the theoretical value. Additionally, it was observed that the maximum range of the projectile is directly influenced by the initial speed of the launch. This investigation provides valuable insights into the fundamental principles of projectile motion and contributes to a better understanding of the relationship between launch parameters and the resulting trajectory. Further explore, the Time of flight can be obtained from gradient of graph of Maximum Range, S against Initial Speed, V_0 with angle of projection 45° is 1.66 s.

4. Acknowledgements

First and foremost, I express my deepest gratitude to Dr. Edmund Loh Wai Ming, our distinguished lecturer and examiner, for his invaluable guidance and unwavering support throughout our scientific exploration. His contributions as both an esteemed educator and the original creator of this laboratory manual have been instrumental to the success of our endeavours. I am truly thankful for his mentorship and the foundation he laid for our scientific understanding. I extend my sincere gratitude to my experiment partner, *Aina Imanina Binti Mohb Khozikin*. Her invaluable cooperation and dedication throughout both experiments were instrumental to the success of this project. I appreciate her commitment, expertise, and teamwork, which made these scientific endeavours both productive and enjoyable. A heartfelt acknowledgment is also extended to *Dr. John Soo Yue Han* for his dedicated efforts in revising and standardizing the manual in 2021, elevating its clarity and educational significance. This collective endeavor has significantly enhanced our scientific learning journey, and I extend genuine gratitude to everyone mentioned for their noteworthy contributions.

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8. INTRODUCTION

In this experiment, we will study the projectile motion of a steel ball using PHYWE's Projectile Motion experiment set. A steel ball is fired by a spring at different velocities and at different angles to the horizontal. Using the data collected, we will investigate the relationship between the range, height of projection, angle of projection and initial speed of the projectile.

9. THEORY

Projectile Motion

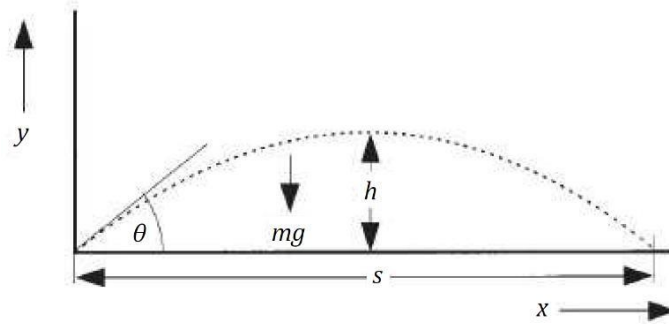


Figure 1: Movement of a mass point under the effect of gravitational force.

If a body of mass m moves in a constant gravitational field (gravitational force $m\vec{g}$), the motion lies in a plane (**Figure 1**). If the coordinate system is laid in this x - y plane, the *equation of motion* $\vec{r}(t)$ is

$$m \frac{d^2}{dt^2} \vec{r}(t) = m\vec{g}. \quad (1)$$

If $\vec{r} = (x, y)$ and $\vec{g} = (0, -g)$, and the initial position and velocity are $\vec{r}(0) = (0, 0)$ and $\vec{v}(0) = (v_0 \cos \phi, v_0 \sin \phi)$, we obtain the *coordinates* as a function of time t ,

$$\begin{aligned} x(t) &= v_0 t \cos \phi \\ y(t) &= v_0 t \sin \phi - \frac{1}{2} g t^2. \end{aligned} \quad (2)$$

From this, the *maximum height* of projection h is obtained as a function of the *angle of projection* ϕ ,

$$h = \frac{v_0^2}{2g} \sin^2 \phi, \quad (3)$$

and the *maximum range* s is

$$s = \frac{v_0^2}{g} \sin 2\phi. \quad (4)$$

Figure 2 shows the maximum height as a function of projection angle, and we see that the higher the initial velocity, the higher the maximum height. On the other hand, the maximum range is reached at the projection angle of 45° for every initial velocity, as shown in **Figure 3**. By choosing a logarithmic scale, a regression line can be applied to the measured data and used to determine the maximum range for arbitrary initial velocities (**Figure 4**).

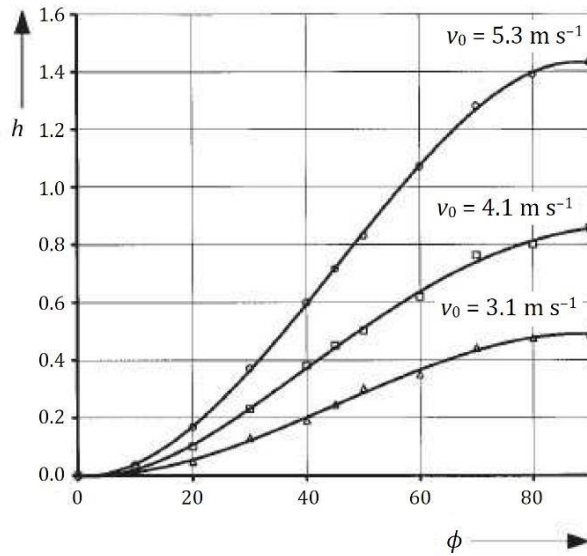


Figure 2: Maximum height of projection h as a function of angle of projection ϕ at different initial velocities v_0 .

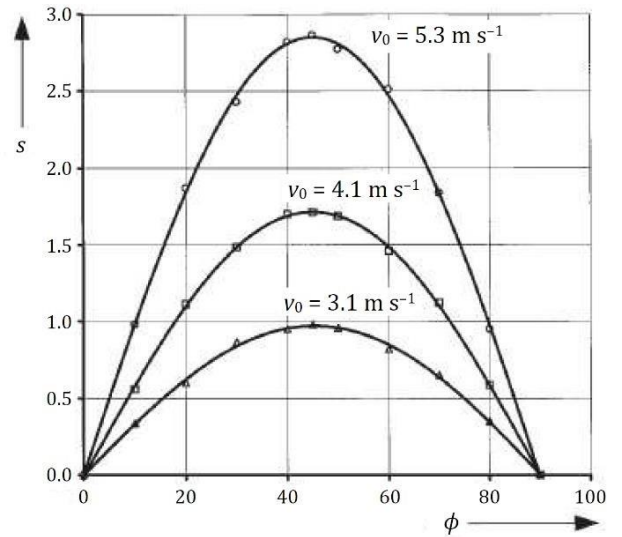


Figure 3: Maximum range s as a function of angle of projection ϕ at different initial velocities v_0 .

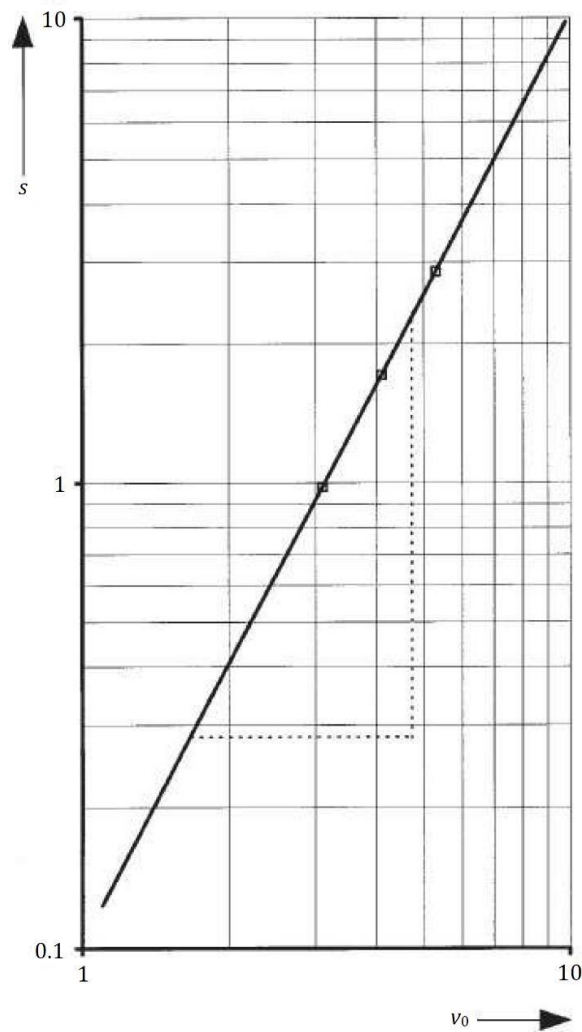


Figure 4: Maximum range s as a function of initial speed v_0 with a fixed angle of projection $\phi = 45^\circ$.

10.3. EXPERIMENTAL METHODOLOGY

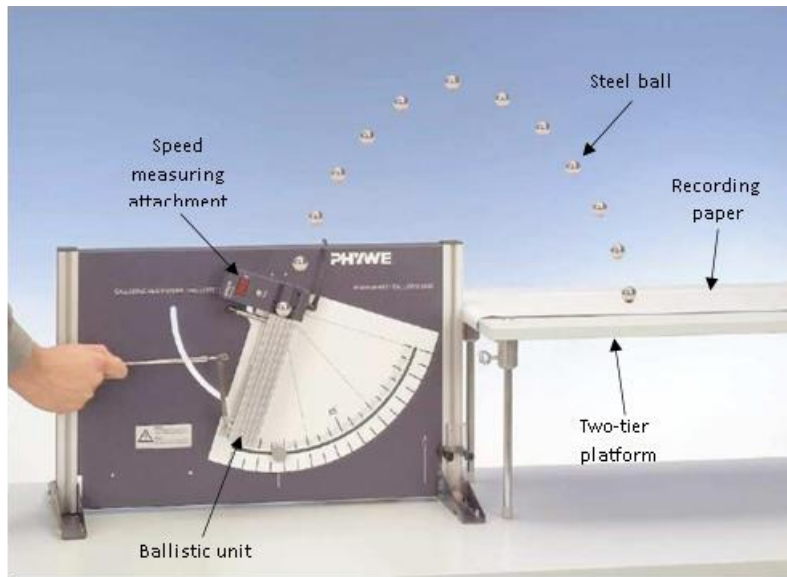


Figure 5: shows the experiment setup used in this study.

In part A, the apparatus, as illustrated in Figure 5, was set up with a recording strip securely affixed to the bench using adhesive tape. To measure the height of projection accurately, a meter scale was clamped in the barrel base, allowing for parallel movement along the projection plane. Impact points were marked on the recording strip, and for safety, an empty box was positioned behind the bench to catch the steel ball upon descent.

To initiate the experiment, the steel ball was placed on the striker within the ballistic unit, and the firing spring was pulled to the first tension stage, denoted as the initial speed $v_{0,1}$. The distance between the striker and the centre between the light barriers (d) was measured and recorded in Table 1. Subsequently, the ballistic unit was adjusted to set the angle of projection (ϕ) to 25° . The steel ball was then fired, and both the range (s) and the height of projection (h) were recorded in Table 1, along with the experimental initial speed (v_{exp}).

Experiments were repeated with varying angles of projection ($\phi = 35^\circ, 45^\circ, 55^\circ$, and 65°). Furthermore, the experiment was replicated for the second and third tension stages, corresponding to the second and third initial velocities ($v_{0,2}$ and $v_{0,3}$).

For the analysis phase, the actual initial speeds (v_0) were calculated using the formula $v_0 = \sqrt{v_{\text{exp}}^2 + gd \sin \phi}$, accounting for the time taken by the ball to cover the measuring distance. Graphs of maximum range (s) against angle of projection (ϕ) and maximum height of projection (h) against angle of projection (ϕ) were plotted for each initial velocity.

To determine the gravitational acceleration (g), two additional linear graphs s against $v_o^2 \sin 2\phi$ and h against $\frac{1}{2}(v_o \sin \phi)^2$ were plotted using the collected data for s against ϕ and h against ϕ . The value of gravitational acceleration (g) was then obtained from the slopes of these linear graphs. This systematic approach ensured precision in measuring and analysing the experimental data, facilitating a reliable determination of the gravitational acceleration of the Earth.

In Part B of the experiment, the objective is to investigate the relationship between the maximum range (s) and the initial speed (v_0) of the steel ball. The procedure for this part is outlined as follows.

The experimental setup from Part A was replicated for continuity. The steel ball was placed on the striker in the ballistic unit, and the firing spring was pulled to the first tension stage, representing the first initial speed ($v_{0,1}$). The distance between the striker and the centre between the light barriers (d) was measured and recorded in Table 2.

Subsequently, the ballistic unit was adjusted to set the angle of projection (ϕ) to 45° . The steel ball was then fired, and both the range (s) and the initial firing speed (v_{exp}) were recorded. Steps were repeated for the second and third tension stages, corresponding to the second and third initial velocities ($v_{0,2}$ and $v_{0,3}$).

For the analysis phase, the actual initial speeds (v_0) were calculated using the formula $v_o = \sqrt{v_{\text{exp}}^2 + gd \sin \phi}$. Subsequently, a graph of maximum range (s) against initial speed (v_0) was plotted. This graph aims to reveal the nature of the relationship between the maximum range achieved by the steel ball and its initial speed.

This investigation provides valuable insights into the projectile motion of the steel ball, allowing for a comprehensive understanding of how the initial speed influences the maximum range attained. The plotted graph serves as a visual representation of this relationship, aiding in the interpretation of the experimental results.

4. DATA ANALYSIS

PART A

Angle of Projection, ϕ	Range, s (± 0.01 m)	Height of Projection, h (± 0.01 m)	Experimental Initial Speed, v_{exp} (± 0.01 m s $^{-1}$)	Actual Initial Speed, v_0 (m s $^{-1}$)
First initial speed, $v_{0,1}$ (m s $^{-1}$) [Distance, $d_1 = 0.050 \pm 0.01$ m]				
25°	0.485	0.035	2.32	2.36
35°	0.570	0.039	2.29	2.35
45°	0.590	0.170	2.30	2.37
55°	0.495	0.210	2.27	2.36
65°	0.460	0.250	2.30	2.39
Second initial speed, $v_{0,2}$ (m s $^{-1}$) [Distance, $d_2 = 0.065 \pm 0.01$ m]				
25°	1.095	0.080	3.33	3.37
35°	1.220	0.190	3.27	3.33
45°	1.230	0.290	3.28	3.35
55°	1.170	0.410	3.29	3.37
65°	0.785	0.450	3.12	3.21
Third initial speed, $v_{0,3}$ (m s $^{-1}$) [Distance, $d_3 = 0.075 \pm 0.01$ m]				
25°	1.685	0.160	4.27	4.31
35°	1.890	0.330	4.24	4.29
45°	2.105	0.510	4.27	4.33
55°	1.945	0.680	4.26	4.33
65°	1.415	0.720	3.99	4.07

Table 1: Experimental Data set for Part A experiment.

[Calculated by Excel: Appendices 1]

Actual Initial Speed, v_0 (m s $^{-1}$)	Uncertainty of Actual Initial speed, Δv_0 (m s $^{-1}$)
First initial speed, $v_{0,1}$ (m s $^{-1}$) [Distance, $d_1 = 0.050 \pm 0.01$ m]	
2.36	0.05
2.35	0.07
2.37	0.08
2.36	0.09
2.39	0.10
Second initial speed, $v_{0,2}$ (m s $^{-1}$) [Distance, $d_2 = 0.065 \pm 0.01$ m]	
3.37	0.05
3.33	0.06
3.35	0.08
3.37	0.09
3.21	0.10
Third initial speed, $v_{0,3}$ (m s $^{-1}$) [Distance, $d_3 = 0.075 \pm 0.01$ m]	
4.31	0.05
4.29	0.06
4.33	0.07
4.33	0.08
4.07	0.09

Table 2: Actual Initial Speed, v_0 (m s $^{-1}$) and uncertainty

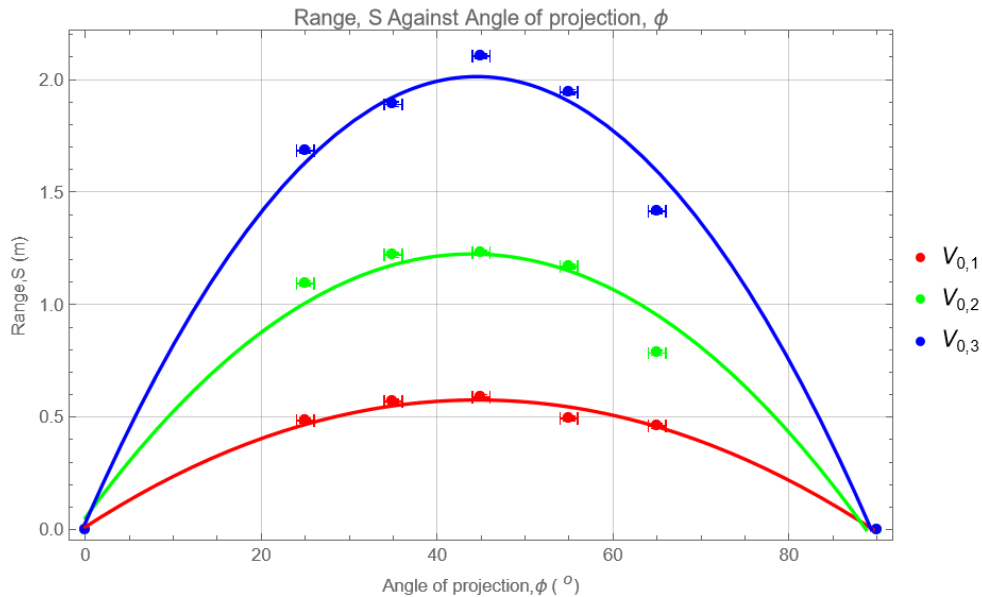
[Calculated by Excel: Appendices 3]

Angle of Projection, ϕ	Range, s (± 0.01 m)	Theoretical Range, (m)	Percentage of discrepancy	Height of Projection, h (± 0.01 m)	Theoretical Height of Projection (m)	Percentage of discrepancy
First initial speed, $v_{0,1}$ (m s^{-1}) [Distance, $d_1 = 0.050 \pm 0.01$ m]						
25°	0.485	0.435	11.51	0.035	0.051	30.97
35°	0.570	0.529	7.75	0.039	0.093	57.88
45°	0.590	0.573	3.04	0.170	0.143	18.76
55°	0.495	0.534	7.22	0.210	0.190	10.25
65°	0.460	0.446	3.13	0.250	0.239	4.54
Second initial speed, $v_{0,2}$ (m s^{-1}) [Distance, $d_2 = 0.065 \pm 0.01$ m]						
25°	1.095	0.887	23.47	0.080	0.103	22.62
35°	1.220	1.062	14.86	0.190	0.186	2.18
45°	1.230	1.144	7.52	0.290	0.286	1.40
55°	1.170	1.088	7.55	0.410	0.388	5.56
65°	0.785	0.805	2.44	0.450	0.431	4.32
Third initial speed, $v_{0,3}$ (m s^{-1}) [Distance, $d_3 = 0.075 \pm 0.01$ m]						
25°	1.685	1.451	16.16	0.160	0.169	5.38
35°	1.890	1.763	7.21	0.330	0.309	6.93
45°	2.105	1.911	10.14	0.510	0.478	6.74
55°	1.945	1.796	8.30	0.680	0.641	6.05
65°	1.415	1.294	9.39	0.720	0.693	3.82

Table 3: Range and Height of projection with the Theoretical values and Percentage of discrepancy

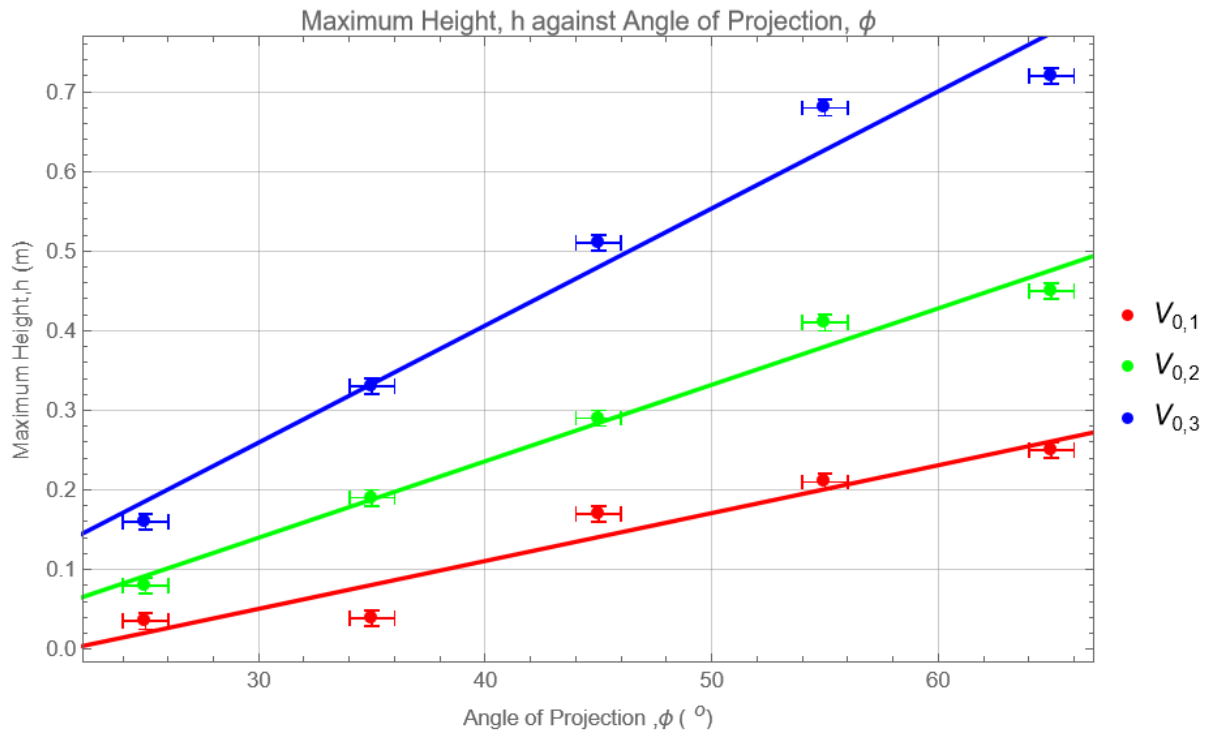
[Calculated by Excel: Appendices 2]

Based on the **Table 3**, increase in the initial speed of projection results in a decrease in the percentage of discrepancy of both range and height. Conversely, in the case of the angle of projection, an increase leads to a decrease in the percentage of discrepancy both range and height.

**Figure 6:** Graph of Range, S against Angle of Projection, ϕ

[Display by Mathematica: Appendices 5]

Based on Figure 6, it can be observed that a higher initial speed of projection leads to a greater range, assuming the angle of projection remains constant.



$$V_{0,1}: y = 0.00601x - 0.12965$$

$$V_{0,2}: y = 0.0096x - 0.148$$

$$V_{0,3}: y = 0.0147x - 0.1815$$

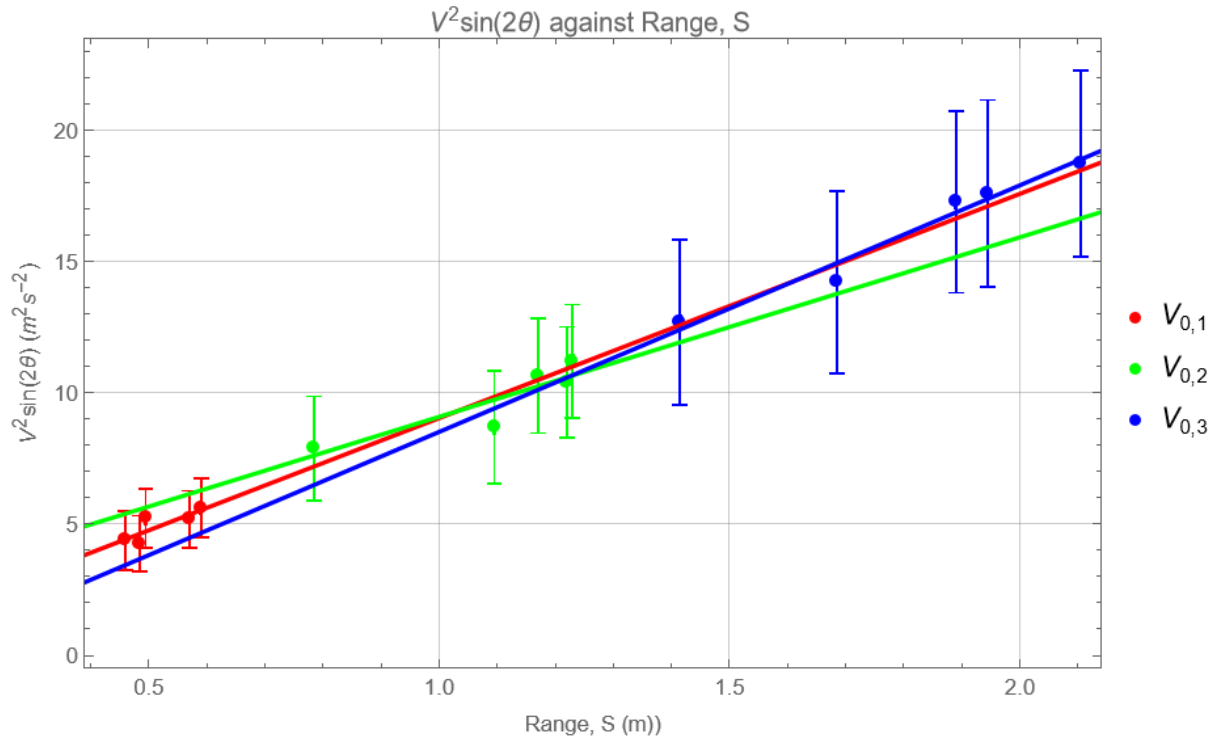
Figure 7: Graph of Maximum Height, h against Angle of Projection, ϕ

[Display by Mathematica: Appendices 6]

In theory, the graph of the maximum height, h against the angle of projection, ϕ is expected to exhibit a slightly logistic increase, particularly becoming more pronounced at higher initial speeds. However, Figure 7 suggests modelling this relationship as a linear graph. This linear approximation is suitable when the initial speed is low, simplifying the representation of the projectile motion. Notably, it becomes evident that a higher initial speed of projection corresponds to a greater maximum height, assuming the angle of projection remains constant.

Angle of Projection, ϕ	Range, s (± 0.01 m)	Height of Projection, h (± 0.01 m)	$v_o^2 \sin 2\phi$ ($m^2 s^{-2}$)	Uncertainty of $v_o^2 \sin 2\phi$ ($m^2 s^{-2}$)	$v_o^2 \sin 2\theta$ ($m^2 s^{-2}$)	Uncertainty of $v_o^2 \sin 2\theta$ ($m^2 s^{-2}$)
First initial speed, $v_{0,1}$ ($m s^{-1}$) [Distance, $d_1 = 0.050 \pm 0.01$ m]						
25°	0.485	0.035	4.27	1.06	0.50	0.03
35°	0.570	0.039	5.19	1.08	0.91	0.06
45°	0.590	0.170	5.62	1.12	1.40	0.10
55°	0.495	0.210	5.23	1.12	1.87	0.16
65°	0.460	0.250	4.38	1.13	2.35	0.21
Second initial speed, $v_{0,2}$ ($m s^{-1}$) [Distance, $d_2 = 0.065 \pm 0.01$ m]						
25°	1.095	0.080	8.70	2.14	1.01	0.05
35°	1.220	0.190	10.42	2.11	1.82	0.09
45°	1.230	0.290	11.22	2.16	2.81	0.15
55°	1.170	0.410	10.67	2.19	3.81	0.21
65°	0.785	0.450	7.89	1.99	4.23	0.27
Third initial speed, $v_{0,3}$ ($m s^{-1}$) [Distance, $d_3 = 0.075 \pm 0.01$ m]						
25°	1.685	0.160	14.23	3.48	1.66	0.08
35°	1.890	0.330	17.29	3.47	3.03	0.12
45°	2.105	0.510	18.75	3.55	4.69	0.19
55°	1.945	0.680	17.62	3.56	6.29	0.27
65°	1.415	0.720	12.69	3.15	6.80	0.33

Table 4: Values of $v_o^2 \sin 2\phi$ and $v_o^2 \sin 2\theta$ with uncertainties
[Calculated by Excel: Appendices 3]



$$V_{0,1}: y = 8.55253x + 0.490685$$

$$V_{0,2}: y = 6.84422x + 2.25135$$

$$V_{0,3}: y = 9.40063x - 0.880336$$

Figure 8: Graph of $v_o^2 \sin 2\theta$ against Range, S

[Display by Mathematica: Appendices 7]

Since,

$$R = v_o t \cos \theta = (v_o \cos \theta) \left(\frac{2v_o \sin \theta}{g} \right) = \frac{2v_o^2 \sin \theta \cos \theta}{g} = \frac{v_o^2 \sin 2\theta}{g}$$

$$v_o^2 \sin 2\theta = gR, v_o^2 \sin 2\theta \propto R, \text{ which } g \text{ is a constant}$$

Thus, gravitational acceleration, g is the gradient of graph of $v_o^2 \sin 2\theta$ against Range, S

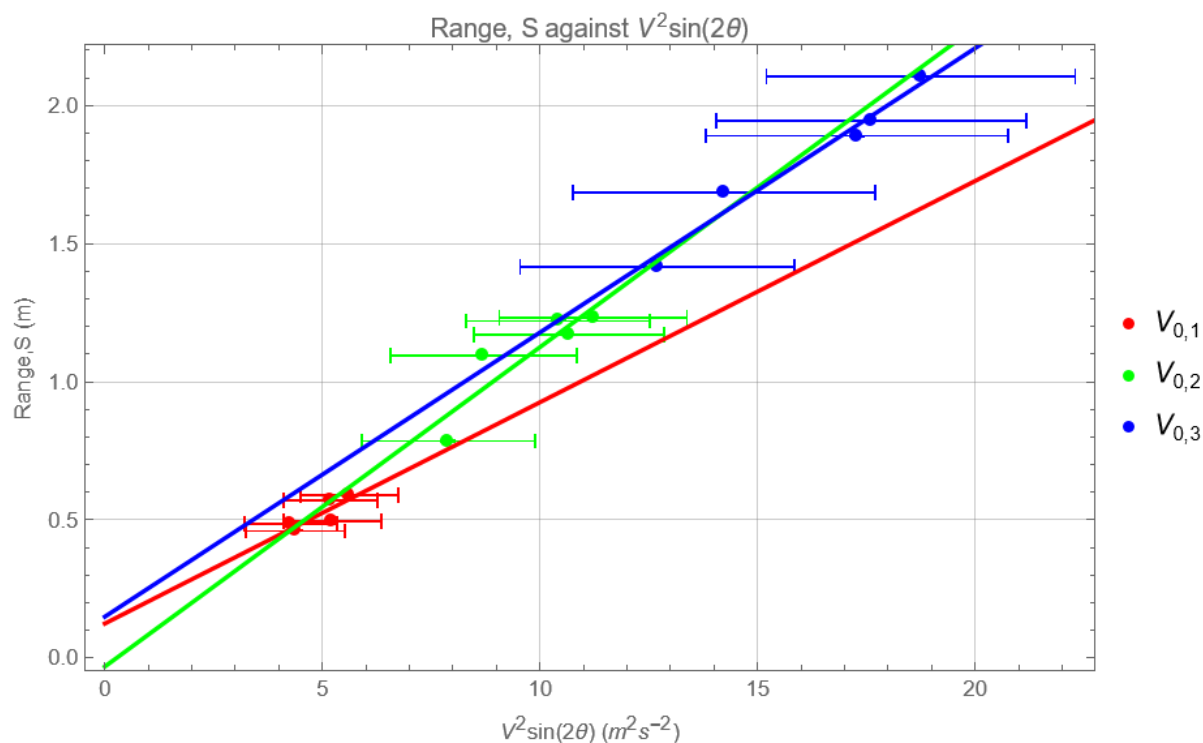
Initial velocity of projection, V_o	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_o = 9.81 ms^{-2}$
$V_{0,1}$	8.55	12.84
$V_{0,2}$	6.84	30.28
$V_{0,3}$	9.40	4.18

Table 5: The gravitational acceleration, g obtained from the graph of $v_o^2 \sin 2\theta$ against Range, S

[Calculated by Mathematica: Appendices 14]

Calculate the percentage of discrepancy,

$$\frac{|g - g_o|}{g_o} \times 100\%, \text{ where } g_o \text{ is } 9.81 ms^{-2}$$



$$V_{0,1}: y = 0.0801324 x + 0.124306$$

$$V_{0,2}: y = 0.115682 x - 0.031367$$

$$V_{0,3}: y = 0.102927 x + 0.149223$$

Figure 9: Graph of Range, S against $v_o^2 \sin 2\theta$

[Display by Mathematica: Appendices 8]

Since,

$$R = v_o t \cos \theta = (v_o \cos \theta) \left(\frac{2v_o \sin \theta}{g} \right) = \frac{2v_o^2 \sin \theta \cos \theta}{g} = \frac{v_o^2 \sin 2\theta}{g}$$

$$R = \frac{1}{g} v_o^2 \sin 2\theta, R \propto v_o^2 \sin 2\theta, \text{ which is } \frac{1}{g} \text{ a constant}$$

Thus, gravitational acceleration, g is reciprocal of the gradient of graph of $v_o^2 \sin 2\theta$ against Range, S

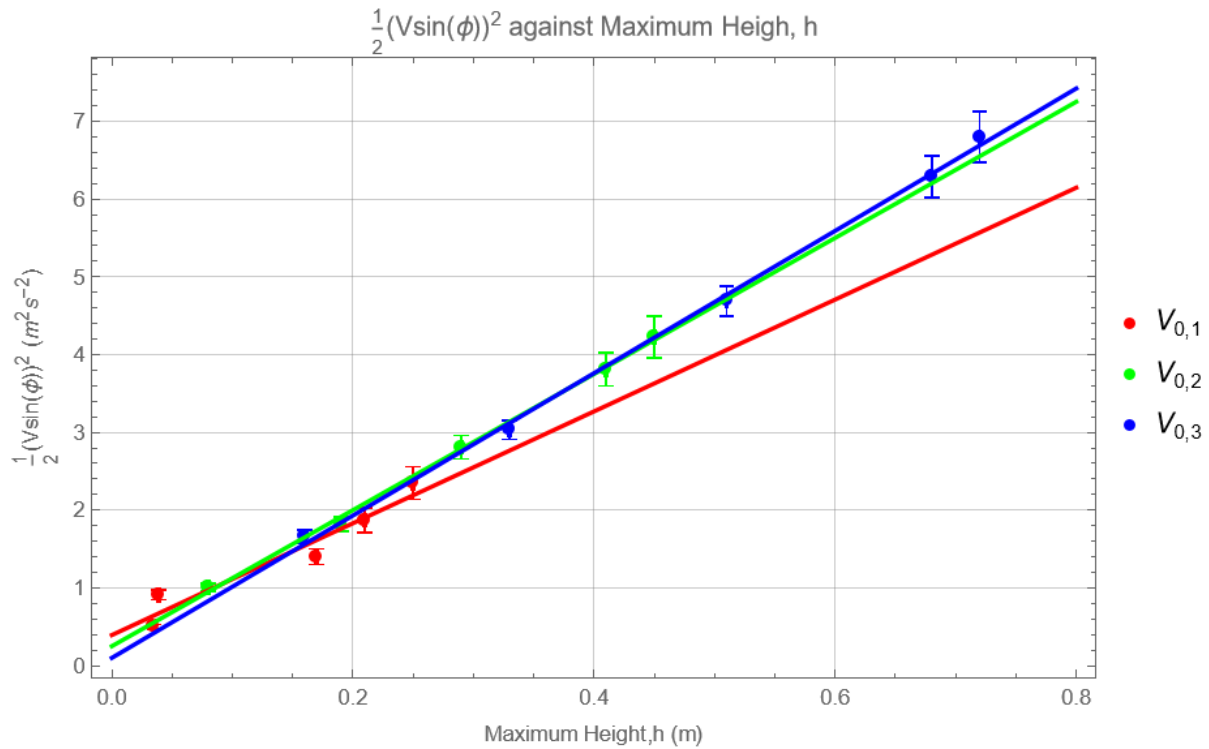
Initial velocity of projection, V_o	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_o = 9.81 ms^{-2}$
$V_{0,1}$	12.48	27.22
$V_{0,2}$	8.64	11.93
$V_{0,3}$	9.72	0.92

Table 6: The gravitational acceleration, g obtained from the graph of Range, S against $v_o^2 \sin 2\theta$

[Calculated by Mathematica: Appendices 14]

Calculate the percentage of discrepancy,

$$\frac{|g - g_o|}{g_o} \times 100\%, \text{ where } g_o \text{ is } 9.81 ms^{-2}$$



$$V_{0,1}: y = 7.19187x + 0.393385$$

$$V_{0,2}: y = 8.75192x + 0.250456$$

$$V_{0,3}: y = 9.15398x + 0.100088$$

Figure 10: Graph of $\frac{1}{2}(v \sin \phi)^2$ against Maximum Height, h

[Display by Mathematica: Appendices 9]

Since,

$$v^2 = u^2 + 2as$$

$$0^2 = (v_0 \sin \theta)^2 + 2(-g)h$$

$$0 = (v_0 \sin \theta)^2 - 2gh$$

$$h = \frac{(v_0 \sin \theta)^2}{2g}$$

$$\frac{1}{2}(v_0 \sin \theta)^2 = gh, \frac{1}{2}(v_0 \sin \theta)^2 \propto h \text{ which } g \text{ is a constant}$$

Thus, gravitational acceleration, g is the gradient of graph of $\frac{1}{2}(v_0 \sin \theta)^2$ against Maximum Height, h .

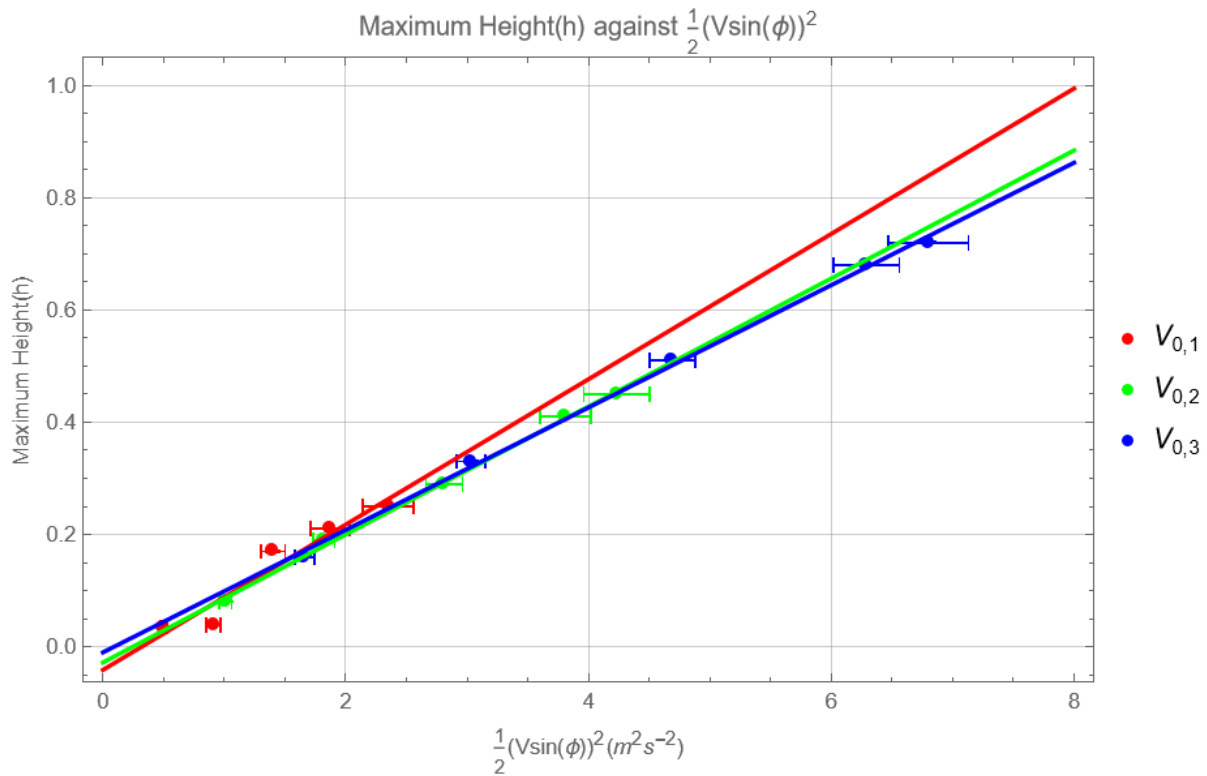
Initial velocity of projection, V_0	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_0 = 9.81 \text{ ms}^{-2}$
$V_{0,1}$	7.19	26.71
$V_{0,2}$	8.75	10.81
$V_{0,3}$	9.15	6.73

Table 7: The gravitational acceleration, g obtained from the graph of $\frac{1}{2}(v_0 \sin \theta)^2$ against Maximum Height, h

[Calculated by Mathematica: Appendices 14]

Calculate the percentage of discrepancy,

$$\frac{|g - g_0|}{g_0} \times 100\%, \text{ where } g_0 \text{ is } 9.81 \text{ ms}^{-2}$$



$$V_{0,1}: y = 0.129464x - 0.0412259$$

$$V_{0,2}: y = 0.114019x - 0.0279573$$

$$V_{0,3}: y = 0.109029x - 0.00997522$$

Figure 11: Graph of Maximum Height, h against $\frac{1}{2}(v_o \sin \phi)^2$

[Display by Mathematica: Appendices 10]

Since,

$$v^2 = u^2 + 2as$$

$$0^2 = (v_o \sin \theta)^2 + 2(-g)h$$

$$0 = (v_o \sin \theta)^2 - 2gh$$

$$h = \frac{(v_o \sin \theta)^2}{2g}$$

$$h = \frac{1}{2g}(v_o \sin \phi)^2, h \propto \frac{1}{2g}(v_o \sin \phi)^2 \text{ which is } \frac{1}{g} \text{ a constant}$$

Thus, gravitational acceleration, g is reciprocal of the gradient of graph of $\frac{1}{2}(v_o \sin \phi)^2$ against Maximum Height, h .

Initial velocity of projection, V_o	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_o = 9.81 ms^{-2}$
$V_{0,1}$	7.72	21.30
$V_{0,2}$	8.77	10.60
$V_{0,3}$	9.17	6.52

Table 8: The gravitational acceleration, g obtained from the graph of Maximum Height, h against $\frac{1}{2}(v_o \sin \phi)^2$

[Calculated by Mathematica: Appendices 14]

Calculate the percentage of discrepancy,

$$\frac{|g - g_o|}{g_o} \times 100\%, \text{ where } g_o \text{ is } 9.81 ms^{-2}$$

Type of Graph	Initial velocity of projection, V_0	Gravitational acceleration, $g \text{ (ms}^{-2}\text{)}$	Percentage of discrepancy from $g_0=9.81 \text{ ms}^{-2}$
Graph of $v_0^2 \sin 2\theta$ against Range, S	$V_{0,1}$	8.55	12.84
	$V_{0,2}$	6.84	30.28
	$V_{0,3}$	9.40	4.18
Graph of Range, S against $v_0^2 \sin 2\theta$	$V_{0,1}$	12.48	27.22
	$V_{0,2}$	8.64	11.93
	$V_{0,3}$	9.72	0.92
Graph of $\frac{1}{2}(v_0 \sin \theta)^2$ against Maximum Height, h	$V_{0,1}$	7.19	26.71
	$V_{0,2}$	8.75	10.81
	$V_{0,3}$	9.15	6.73
Graph of Maximum Height, h against $\frac{1}{2}(v_0 \sin \theta)^2$	$V_{0,1}$	7.72	21.30
	$V_{0,2}$	8.77	10.60
	$V_{0,3}$	9.17	6.52

Table 9: Results of gravitational acceleration, g, percentage of discrepancy with various initial velocity of projection and type of graph

According to the data presented in the table, the incremental adjustments in the initial velocity of projection result in the gravitational acceleration values converging towards the actual gravitational acceleration of Earth. Notably, the graph's gradient derived from $\frac{1}{g}$ yields a more precise estimation of the gravitational acceleration compared to the gradient obtained using g.

Analysing the graphs generated from the parameters of Range, S and $v_0^2 \sin 2\theta$, it is evident that they exhibit a smaller percentage of discrepancy at the highest initial velocity of projection, in contrast to the graphs derived from parameters such as Maximum Height, h and $\frac{1}{2}(v_0 \sin \theta)^2$. Consequently, the gravitational acceleration of Earth is determined by selecting the value with the smallest percentage of discrepancy, amounting to 0.92%, resulting in $(9.72 \pm 0.01) \text{ ms}^{-2}$, in close proximity to the accepted value of $g_0=9.81 \text{ ms}^{-2}$. This refined selection enhances the accuracy of the gravitational acceleration determination.

PART B

Experimental Initial Speed, v_{exp} (m s^{-1})	Distance, d ($\pm 0.01 \text{ m}$)	Range, s ($\pm 0.01 \text{ m}$)	Actual Initial Speed, v_0 (m s^{-1})
2.31	0.050	0.595	2.38
2.95	0.065	1.215	3.03
4.08	0.075	2.100	4.14

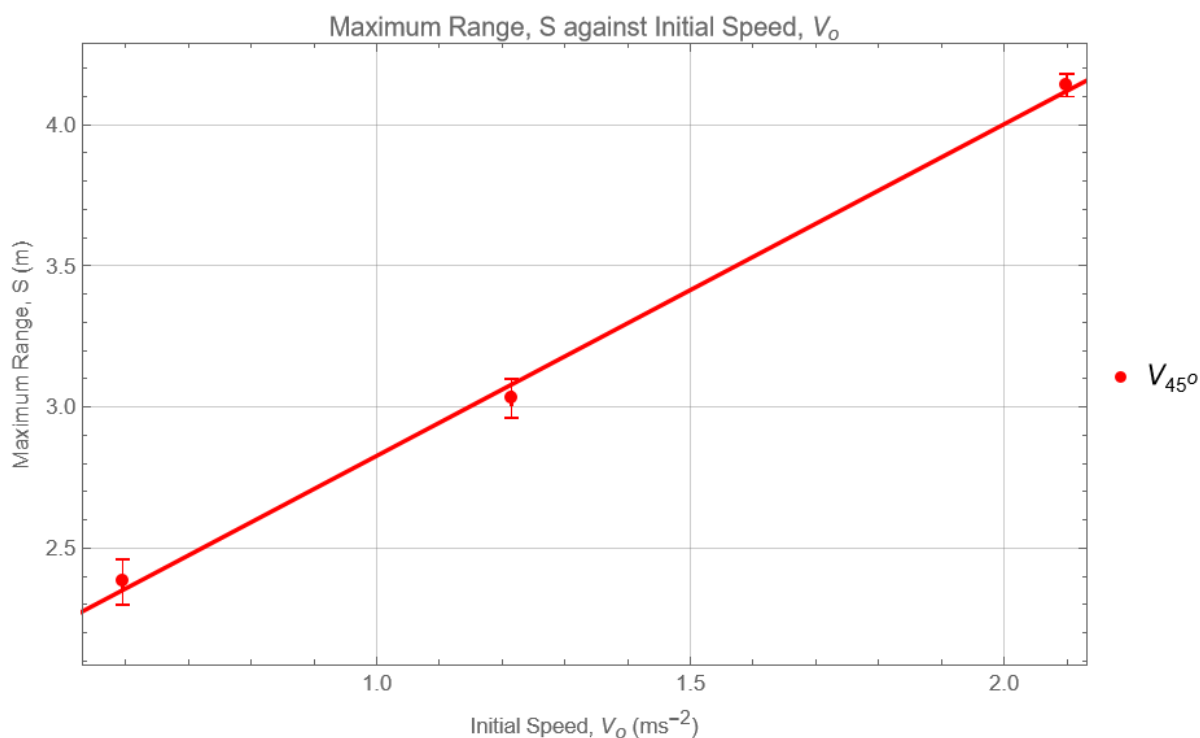
Table 10: Data for **Part B**.

[Calculated by Excel: Appendices 4]

Experimental Initial Speed, v_{exp} (m s^{-1})	Distance, d ($\pm 0.01 \text{ m}$)	Range, s ($\pm 0.01 \text{ m}$)	Uncertainty of Range, ∇s ($\pm 0.01 \text{ m}$)	Actual Initial Speed, v_0 (m s^{-1})	Uncertainty of Actual Initial Speed, ∇v_0 (m s^{-1})
2.31	0.050	0.595	0.001	2.38	0.08
2.95	0.065	1.215	0.001	3.03	0.08
4.08	0.075	2.100	0.001	4.14	0.07

Table 11: Range and Actual Initial speed with projection angle 45° with the uncertainties

[Calculated by Excel: Appendices 4]



$$V_{45^\circ}: y = 1.17523x + 1.65162$$

Figure 12: Graph of Maximum Range, S against Initial Speed, V_o

[Display by Mathematica: Appendices 11]

Based on Figure 10, it can be observed that a higher initial speed of projection leads to a greater range, when the angle of projection remains 45° .

Since,

$$S_{max} = V_{o,x} \times T$$

where $V_{o,x}$ is the horizontal component of the initial velocity. For a launch angle of 45° ,

$$V_{o,x} = \frac{V_o}{\sqrt{2}}$$

Thus,

$$S_{max} = \frac{T}{\sqrt{2}} \times V_o$$

$$S_{max} \propto V_o, \text{ which is } \frac{T}{\sqrt{2}} \text{ a constant}$$

Thus, $\frac{T}{\sqrt{2}}$ is the gradient of graph of Maximum Range, S against Initial Speed, V_o

$$\text{Gradient} = \frac{T}{\sqrt{2}} = 1.17523 \text{ s}$$

$$\text{Time of Flight, } T = 1.17523\sqrt{2} \text{ s} = 1.66 \text{ s}$$

Angle of Projection, ϕ	First initial speed, v_0 (m s^{-1})	Range, s ($\pm 0.01 \text{ m}$)	$v_0^2 \sin 2\phi$ ($\text{m}^2 \text{s}^{-2}$)	Uncertainty of $v_0^2 \sin 2\phi$ ($\text{m}^2 \text{s}^{-2}$)
45°	2.38	0.595	5.62	1.12
45°	3.03	1.215	5.23	1.12
45°	4.1	2.100	4.38	1.13

Table 12: Values of Uncertainty of $v_0^2 \sin 2\phi$ in projection angle 45° with uncertainties

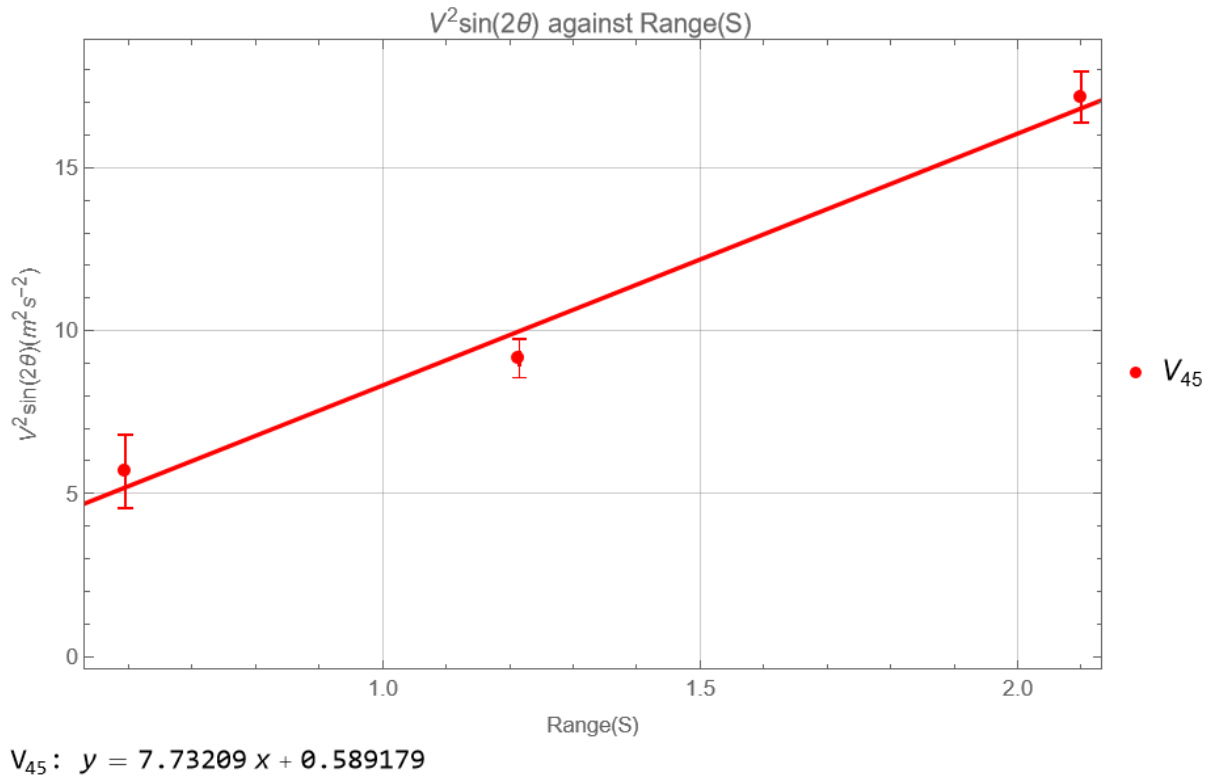


Figure 13: Graph of $v_o^2 \sin 2\theta$ against Range, S in angle of projection 45°

[Display by Mathematica: Appendices 12]

Since,

$$R = v_o t \cos \theta = (v_o \cos \theta) \left(\frac{2v_o \sin \theta}{g} \right) = \frac{2v_o^2 \sin \theta \cos \theta}{g} = \frac{v_o^2 \sin 2\theta}{g}$$

$$v_o^2 \sin 2\theta = gR, v_o^2 \sin 2\theta \propto R, \text{ which } g \text{ is a constant}$$

Thus, gravitational acceleration, g is the gradient of graph of $v_o^2 \sin 2\theta$ against Range, S

Initial velocity of projection, V_o	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_o = 9.81 ms^{-2}$
V_{45}	7.73	21.20

Table 13: The gravitational acceleration, g obtained from the graph of $v_o^2 \sin 2\theta$ against Range, S

[Calculated by Mathematica: Appendices 14]

Calculate the percentage of discrepancy,

$$\frac{|g - g_o|}{g_o} \times 100\%, \text{ where } g_o \text{ is } 9.81 ms^{-2}$$

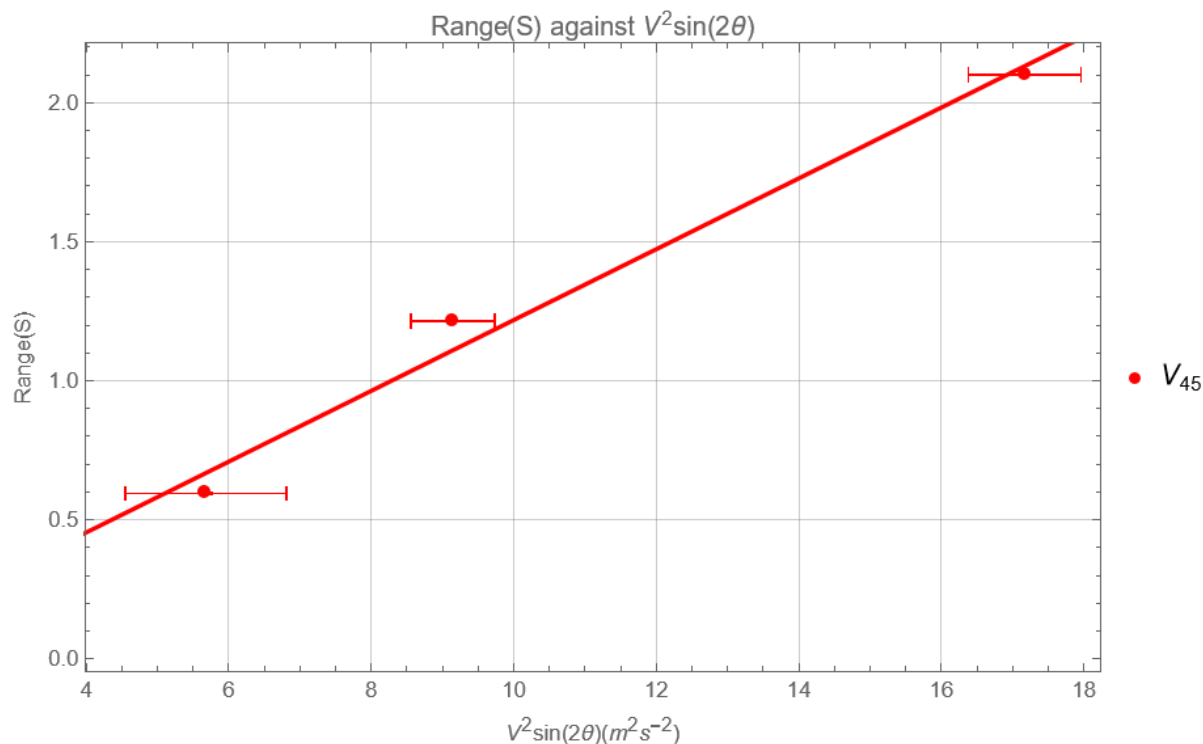


Figure 14: Graph of Range, S against $v_o^2 \sin 2\theta$ in angle of projection 45°

[Display by Mathematica: Appendices 13]

Since,

$$R = v_o t \cos \theta = (v_o \cos \theta) \left(\frac{2v_o \sin \theta}{g} \right) = \frac{2v_o^2 \sin \theta \cos \theta}{g} = \frac{v_o^2 \sin 2\theta}{g}$$

$$R = \frac{1}{g} v_o^2 \sin 2\theta, R \propto v_o^2 \sin 2\theta, \text{ which is } \frac{1}{g} \text{ a constant}$$

Thus, gravitational acceleration, g is reciprocal of the gradient of graph of $v_o^2 \sin 2\theta$ against Range, S

Initial velocity of projection, V_o	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_o = 9.81 ms^{-2}$
V_{45}	7.85	19.98

Table 14: The gravitational acceleration, g obtained from the graph of Range, S against $v_o^2 \sin 2\theta$

[Calculated by Mathematica: Appendices 14]

Calculate the percentage of discrepancy,

$$\frac{|g - g_o|}{g_o} \times 100\%, \text{ where } g_o \text{ is } 9.81 ms^{-2}$$

Type of graph	Initial velocity of projection, V_o	Gravitational acceleration, g (ms^{-2})	Percentage of discrepancy from $g_o=9.81 ms^{-2}$
Graph of $v_o^2 \sin 2\theta$ against Range, S in angle of projection 45°	V_{45}	7.73	21.20
Graph of Range, S against $v_o^2 \sin 2\theta$ in angle of projection 45°	V_{45}	7.85	19.98

Table 15: Comparison results obtained from Graph of $v_o^2 \sin 2\theta$ against Range, S with Graph of Range, S against $v_o^2 \sin 2\theta$ in angle of projection 45°

Based on the table, affirmed that Graph of $v_o^2 \sin 2\theta$ against Range, Compared to Graph of Range, S against $v_o^2 \sin 2\theta$ having more accuracy value of gravitational acceleration with lower percentage of discrepancy from actual value.

11.5. DISCUSSION AND CONCLUSION

Discussion

In Part A of the experiment, a comprehensive examination of projectile motion unveils intriguing dynamics. From the calculation of raw data, higher initial speeds resulted in decreased discrepancies, aligning with fundamental principles of projectile motion. Additionally, an increase in the angle of projection demonstrated a similar effect, suggesting certain launch angles contribute to more accurate predictions. Initially, the range experiences a swift ascent, followed by a gradual increase as the angle of projection escalates. Remarkably, the range attains its zenith at a 45° projection angle, in alignment with theoretical expectations. After this optimal angle, the range undergoes a precipitous decline with increasing projection angles beyond 45° .

Simultaneously, the height of the projectile exhibits a proportional augmentation in response to the varying projection angles. This phenomenon is rooted in the sustained increase in the vertical component, persisting even beyond the 45° angle. Consequently, the height demonstrates a consistent increase as the projection angle varies.

Moreover, the influence of initial speed on the projectile's range is discernible, with an escalation in the initial speed correlating with a notable increase in the overall range. Noteworthy is the resilience of the observed pattern of range variation with respect to projection angles across different tension stages, corresponding to diverse initial velocities. This consistency underscores the robust nature of the observed patterns, providing valuable insights into the intricate interplay between launch parameters and resultant projectile motion. The stepwise increments in the initial velocity of projection lead to gravitational acceleration values that converge towards the actual gravitational acceleration of Earth. Significantly, the

graph's slope derived from $\frac{1}{g}$ provides a more accurate estimation of gravitational acceleration compared to the slope obtained using g .

Upon scrutinizing the graphs generated from the parameters of Range, S and $v_o^2 \sin 2\phi$, it becomes evident that they exhibit a smaller percentage of discrepancy at the highest initial velocity of projection, in contrast to the graphs derived from parameters such as maximum Height, h and $\frac{1}{2}(v_o \sin \phi)^2$. Consequently, the determination of Earth's gravitational acceleration involves selecting the value with the smallest percentage of discrepancy, which amounts to 0.92%. This selection results in a refined value of gravitational acceleration of earth is $(9.72 \pm 0.01) \text{ ms}^{-2}$. This precision enhances the reliability of our discussion in the physics report's analysis and interpretation section.

For Part B, higher initial speed of projection leads to a greater range, when the angle of projection remains 45° prove that the maximum projectile range as a function of initial speed. Based on the graph, Time of Flight, $T = 1.66 \text{ s}$ can be obtained from the graph of Maximum Range, S against Initial Speed, V_o . Under the projection angle 45° proving again that Graph of $v_o^2 \sin 2\phi$ against Range, S Compared to Graph of Range, S against $v_o^2 \sin 2\phi$ having more accuracy value of gravitational acceleration with lower percentage of discrepancy from actual value.

In the projectile motion experiment, sources of error were identified to understand potential discrepancies in the results. Human factors, including reaction time during projectile release and visual judgment in tracking motion, were recognized as sources of uncertainty. Variability among individuals and the potential impact of fatigue further contribute to potential errors associated with human factors. Another source of error is imperfect launch conditions, characterized by variations in initial velocity, launch angle, and direction, posed

challenges in achieving consistent and precise launch parameters. This variability introduces complexities that can lead to observable discrepancies in the observed projectile motion.

The suggestion of improving the experiment is utilization of high-speed photography technology significantly improves the accuracy of measurements during projectile motion, specifically in capturing the trajectory and velocity of the projectile during its flight. This technological integration minimizes errors associated with rapid changes in motion. The concurrent use of real-time analysis software further enhances the experiment's efficiency by allowing prompt processing of captured images, reducing data processing delays. Incorporating multi-angle photography provides a more comprehensive view of the projectile's motion, aiding in the identification and correction of potential parallax errors. The strategic placement of calibration markers contributes to precise measurements of height and trajectory when employing high-speed photography.

Conclusion

From this experiment, concluded that the range, S and height, h of projection are functions of the angle of projection. The experimental gravitational acceleration is $(9.71 \pm 0.01)ms^{-2}$ exhibits a small percentage of discrepancy of 0.92% when compared to the theoretical value $9.81 ms^{-2}$. There is insignificant deviation between the theoretical and experimental gravitational accelerations. Also conclude that the maximum projectile range as a function of initial speed, that a higher initial speed of projection leads to a greater range. Further explore, the Time of flight obtained from gradient of graph of Maximum Range, S against Initial Speed, V_0 with angle of projection 45° is 1.66 s.

12. 6. REFERENCES

1. PHYWE (2019). Student's Sheet for *Projectile Motion (P2131100)*.
2. PhysChem EMU (2020). *EMU Physics Department: "Projectile Motion" Experiment*. Retrieved 5 Aug 2021 from [youtube.com](https://www.youtube.com).

13. APPENDICES

Appendices 1: Calculation of data in PART A by EXCEL

1st Initial Velocity								
Distance, d1 = m								
Angle, °		Range, m	Height, m	Experimental Velocity, ms-1	Actual Velocity,	$(V_o)^2 \sin(2\theta)$	$0.5(V_o \sin(\theta))^2$	
25	0.436332313	0.485	0.035	2.32	2.36	4.27	0.50	
35	0.610865238	0.570	0.039	2.29	2.35	5.19	0.91	
45	0.785398163	0.590	0.170	2.30	2.37	5.62	1.40	
55	0.959931089	0.495	0.210	2.27	2.36	5.23	1.87	
65	1.134464014	0.460	0.250	2.30	2.39	4.38	2.35	
2nd Initial Velocity								
Distance, d2 = m								
Angle, °		Range, m	Height, m	Experimental Velocity, ms-1	Actual Velocity,	$(V_o)^2 \sin(2\theta)$	$0.5(V_o \sin(\theta))^2$	
25	0.436332313	1.095	0.080	3.33	3.37	8.70	1.01	
35	0.610865238	1.220	0.190	3.27	3.33	10.42	1.82	
45	0.785398163	1.230	0.290	3.28	3.35	11.22	2.81	
55	0.959931089	1.170	0.410	3.29	3.37	10.67	3.81	
65	1.134464014	0.785	0.450	3.12	3.21	7.89	4.23	
3rd Initial Velocity								
Distance, d3 = m								
Angle, °		Range, m	Height, m	Experimental Velocity, ms-1	Actual Velocity,	$(V_o)^2 \sin(2\theta)$	$0.5(V_o \sin(\theta))^2$	
25	0.436332313	1.685	0.160	4.27	4.31	14.23	1.66	
35	0.610865238	1.890	0.330	4.24	4.29	17.29	3.03	
45	0.785398163	2.105	0.510	4.27	4.33	18.75	4.69	
55	0.959931089	1.945	0.680	4.26	4.33	17.62	6.29	
65	1.134464014	1.415	0.720	3.99	4.07	12.69	6.80	

Appendices 2: Calculation of theoretical range and height with percentage discrepancy by Excel

1st Initial Velocity											
Distance, d1 = 0.050m											
Angle, °	Angle, rad.	Range, m	Theoretical Range	Percentage of discrepancy	Height, m	Theoretical Height	Percentage of discrepancy	Experimental Velocity, ms-1	Actual Velocity,V	Distance	
25	0.436332313	0.485	0.435	11.51	0.035	0.051	30.97	2.32	2.36	0.05	
35	0.610865238	0.570	0.529	7.75	0.039	0.093	57.88	2.29	2.35		
45	0.785398163	0.590	0.573	3.04	0.170	0.143	18.76	2.30	2.37		
55	0.959931089	0.495	0.534	7.22	0.210	0.190	10.25	2.27	2.36		
65	1.134464014	0.460	0.446	3.13	0.250	0.239	4.54	2.30	2.39		
2nd Initial Velocity											
Distance, d2 =0.065 m											
Angle, °	Angle, rad.	Range, m	Theoretical Range	Percentage of discrepancy	Height, m	Theoretical Height	Percentage of discrepancy	Experimental Velocity, ms-1	Actual Velocity,V	Distance	
25	0.436332313	1.095	0.887	23.47	0.080	0.103	22.62	3.33	3.37	0.065	
35	0.610865238	1.220	1.062	14.86	0.190	0.186	2.18	3.27	3.33		
45	0.785398163	1.230	1.144	7.52	0.290	0.286	1.40	3.28	3.35		
55	0.959931089	1.170	1.088	7.55	0.410	0.388	5.56	3.29	3.37		
65	1.134464014	0.785	0.805	2.44	0.450	0.431	4.32	3.12	3.21		
3rd Initial Velocity											
Distance, d3 =0.075 m											
Angle, °	Angle, rad.	Range, m	Theoretical Range	Percentage of discrepancy	Height, m	Theoretical Height	Percentage of discrepancy	Experimental Velocity, ms-1	Actual Velocity,V	Distance	
25	0.436332313	1.685	1.451	16.16	0.160	0.169	5.38	4.27	4.31	0.075	
35	0.610865238	1.890	1.763	7.21	0.330	0.309	6.93	4.24	4.29		
45	0.785398163	2.105	1.911	10.14	0.510	0.478	6.74	4.27	4.33		
55	0.959931089	1.945	1.796	8.30	0.680	0.641	6.05	4.26	4.33		
65	1.134464014	1.415	1.294	9.39	0.720	0.693	3.82	3.99	4.07		

Appendices 3: Uncertainty of calculation in PART A by EXCEL

1st Initial Velocity												
Distance, d1 = 0.050m												
Angle, °	Angle, rad.	Angle of 1 ° in rad.	Range, m	d range	Height, m	d height	Experimental Velocity, ms-1	d Vexp	d Vexp^2	Actual Velocity,V		
25	0.436332313	0.017453293	0.485	0.001	0.035	0.001	2.32	0.01	0.05	2.36		
35	0.610865238	0.017453293	0.570	0.001	0.039	0.001	2.29	0.01	0.05	2.35		
45	0.785398163	0.017453293	0.590	0.001	0.170	0.001	2.30	0.01	0.05	2.37		
55	0.959931089	0.017453293	0.495	0.001	0.210	0.001	2.27	0.01	0.05	2.36		
65	1.134464014	0.017453293	0.460	0.001	0.250	0.001	2.30	0.01	0.05	2.39		
2nd Initial Velocity												
Distance, d2 =0.065 m												
Angle, °	Angle, rad.	Angle of 1 ° in rad.	Range, m	d range	Height, m	d height	Experimental Velocity, ms-1	d Vexp	d Vexp^2	Actual Velocity,V		
25	0.436332313	0.017453293	1.095	0.001	0.080	0.001	3.33	0.01	0.07	3.37		
35	0.610865238	0.017453293	1.220	0.001	0.190	0.001	3.27	0.01	0.07	3.33		
45	0.785398163	0.017453293	1.230	0.001	0.290	0.001	3.28	0.01	0.07	3.35		
55	0.959931089	0.017453293	1.170	0.001	0.410	0.001	3.29	0.01	0.07	3.37		
65	1.134464014	0.017453293	0.785	0.001	0.450	0.001	3.12	0.01	0.06	3.21		
3rd Initial Velocity												
Distance, d3 =0.075 m												
Angle, °	Angle, rad.	Angle of 1 ° in rad.	Range, m	d range	Height, m	d height	Experimental Velocity, ms-1	d Vexp	d Vexp^2	Actual Velocity,V		
25	0.436332313	0.017453293	1.685	0.001	0.160	0.001	4.27	0.01	0.09	4.31		
35	0.610865238	0.017453293	1.890	0.001	0.330	0.001	4.24	0.01	0.08	4.29		
45	0.785398163	0.017453293	2.105	0.001	0.510	0.001	4.27	0.01	0.09	4.33		
55	0.959931089	0.017453293	1.945	0.001	0.680	0.001	4.26	0.01	0.09	4.33		
65	1.134464014	0.017453293	1.415	0.001	0.720	0.001	3.99	0.01	0.08	4.07		

d sin(theta)	d d sin(theta)	Vexp ² - g d sin(theta)	d Vexp ² - g d sin(theta)	d Vo	(V _o) ² SIN(2*theta)	d (V _o) ² SIN(2*theta)	0.5*(V _o SIN(theta)) ²	d 0.5*(V _o SIN(theta)) ²	Distance	
0.02	0.79	5.59	0.25	0.05	4.27	1.06	0.50	0.03	0.05	
0.03	1.24	5.53	0.33	0.07	5.19	1.08	0.91	0.06		
0.04	1.70	5.64	0.39	0.08	5.62	1.12	1.40	0.10		
0.04	2.12	5.55	0.45	0.09	5.23	1.12	1.87	0.16		
0.05	2.47	5.73	0.49	0.10	4.38	1.13	2.35	0.21		
d sin(theta)	d d sin(theta)	Vexp ² - g d sin(theta)	d Vexp ² - g d sin(theta)	d Vo	(V _o) ² SIN(2*theta)	d (V _o) ² SIN(2*theta)	0.5*(V _o SIN(theta)) ²	d 0.5*(V _o SIN(theta)) ²	Distance	
0.03	1.02	11.36	0.34	0.05	8.70	2.14	1.01	0.05	0.065	
0.04	1.62	11.06	0.43	0.06	10.42	2.11	1.82	0.09		
0.05	2.21	11.21	0.52	0.08	11.22	2.16	2.81	0.15		
0.05	2.76	11.35	0.59	0.09	10.67	2.19	3.81	0.21		
0.06	3.21	10.31	0.64	0.10	7.89	1.99	4.23	0.27		
d sin(theta)	d d sin(theta)	Vexp ² - g d sin(theta)	d Vexp ² - g d sin(theta)	d Vo	(V _o) ² SIN(2*theta)	d (V _o) ² SIN(2*theta)	0.5*(V _o SIN(theta)) ²	d 0.5*(V _o SIN(theta)) ²	Distance	
0.03	1.18	18.54	0.40	0.05	14.23	3.48	1.66	0.08	0.075	
0.04	1.87	18.40	0.51	0.06	17.29	3.47	3.03	0.12		
0.05	2.56	18.75	0.61	0.07	18.75	3.55	4.69	0.19		
0.06	3.19	18.75	0.69	0.08	17.62	3.56	6.29	0.27		
0.07	3.71	16.59	0.75	0.09	12.69	3.15	6.80	0.33		

Appendices 4: Calculation and uncertainties of data in PART B by EXCEL

1st Initial Velocity										
Angle, °	Angle, rad.	Angle of 1 ° in rad.	Range, m	d range	Experimental Velocity, ms-1	d Vexp	d Vexp^2	Actual Velocity,V	dsin(theta)	
45	0.785398163	0.017453293	0.595	0.001	2.31	0.01	0.05	2.38	0.04	
45	0.785398163	0.017453293	1.215	0.001	2.95	0.01	0.06	3.03	0.05	
45	0.785398163	0.017453293	2.100	0.001	4.08	0.01	0.08	4.14	0.05	

d dsin(theta)	Vexp^2+gdsin(theta)	d Vexp^2+gdsin(theta)	d Vo	(V_o)^2* SIN(2*theta)	delta (V_o)^2* SIN(2*theta)	0.5*(V_o SIN(theta))^2	delta 0.5*(V_o SIN(theta))^2	Distance	
1.70	5.68	0.39	0.08	5.68	1.13	0.50	0.04	0.05	
2.21	9.15	0.51	0.08	3.03	0.59	0.91	0.06	0.065	
2.56	17.17	0.60	0.07	4.14	0.79	1.40	0.06	0.075	

Appendices 5: Code in Mathematica for Graph of Range, S against Angle of Projection, ϕ

```

V1={{0,0},{25,0.485},{35,0.570},{45,0.590},{55,0.495},{65,0.460},{90,0}};
V2={{0,0},{25,1.095},{35,1.220},{45,1.230},{55,1.170},{65,0.785},{90,0}};
V3={{0,0},{25,1.685},{35,1.890},{45,2.105},{55,1.945},{65,1.415},{90,0}};

V1error={
{Around[25,1],Around[0.485,0.01]},{Around[35,1],Around[0.570,0.01]},{Around[45
,1],Around[0.590,0.01]},{Around[55,1],Around[0.495,0.01]},{Around[65,1],Around
[0.460,0.01]}};
V2error={
{Around[25,1],Around[1.095,0.01]},{Around[35,1],Around[1.220,0.01]},{Around[45
,1],Around[1.230,0.01]},{Around[55,1],Around[1.170,0.01]},{Around[65,1],Around
[0.785,0.01]}};
V3error={
{Around[25,1],Around[1.685,0.01]},{Around[35,1],Around[1.890,0.01]},{Around[45
,1],Around[2.105,0.01]},{Around[55,1],Around[1.945,0.01]},{Around[65,1],Around
[1.415,0.01]}};

(*Extracting x and y values for each group*)
x1=V1[[All,1]];
y1=V1[[All,2]];
x2=V2[[All,1]];
y2=V2[[All,2]];
x3=V3[[All,1]];
y3=V3[[All,2]];

(*Fitting curves for each group*)
fit1=Fit[Transpose[{x1,y1}],{1,x,x^2},x];
fit2=Fit[Transpose[{x2,y2}],{1,x,x^2},x];
fit3=Fit[Transpose[{x3,y3}],{1,x,x^2},x];

(*Plotting the data points and best-fit curves with extended range*)
Show[
ListPlot[{V1,V2,V3},PlotStyle->{Red,Green,Blue}],
ListPlot[{V1error,V2error,V3error},PlotStyle->{Red,Green,Blue},PlotLegends-
>{"!\(\(*SubscriptBox[\(V\), \(\theta, 1\)\]\)", "!\(\(*SubscriptBox[\(V\), \(\theta,
2\)\]\)", "!\(\(*SubscriptBox[\(V\), \(\theta,
3\)\]\)"}],Plot[{fit1,fit2,fit3},{x,0,100},PlotStyle-
>{Red,Green,Blue},PlotRange->{{0,100},{0,Max[y3]}},Frame->True,FrameLabel-
>{"Angle of projection, $\phi$  ( °)","Range,S (m)",GridLines->Automatic,PlotLabel-
>"Range, S Against Angle of projection,  $\phi$ ",ImageSize->500]

```

Appendices 6: Code in Mathematica for Graph of Maximum Height, h against Angle of Projection, ϕ

```
(*Define the datasets*)V1={{25,0.035},{35,0.039},{45,0.170},{55,0.210},{65,0.250}};
V2={{25,0.080},{35,0.190},{45,0.290},{55,0.410},{65,0.450}};
V3={{25,0.160},{35,0.330},{45,0.510},{55,0.680},{65,0.720}};

V1error={
{Around[25,1],Around[0.035,0.01]},{Around[35,1],Around[0.039,0.01]},{Around[45,1],Around[0.170,0.01]},{Around[55,1],Around[0.210,0.01]},{Around[65,1],Around[0.250,0.01]}};
V2error={
{Around[25,1],Around[0.080,0.01]},{Around[35,1],Around[0.190,0.01]},{Around[45,1],Around[0.290,0.01]},{Around[55,1],Around[0.410,0.01]},{Around[65,1],Around[0.450,0.01]}};
V3error={
{Around[25,1],Around[0.160,0.01]},{Around[35,1],Around[0.330,0.01]},{Around[45,1],Around[0.510,0.01]},{Around[55,1],Around[0.680,0.01]},{Around[65,1],Around[0.720,0.01]}};

(*Fit linear models to the data*)
fitV1=LinearModelFit[V1,x,x];
fitV2=LinearModelFit[V2,x,x];
fitV3=LinearModelFit[V3,x,x];

(*Extract equations as strings*)
eq1=ToString[TraditionalForm[y==fitV1["BestFit"]]];
eq2=ToString[TraditionalForm[y==fitV2["BestFit"]]];
eq3=ToString[TraditionalForm[y==fitV3["BestFit"]]];

(*Plot the data and the best-fit lines*)
Column[{combinedPlot=Show[ListPlot[{V1error,V2error,V3error},PlotStyle->{Red,Green,Blue},PlotLegends->{"!"\(\*SubscriptBox[\(V\), \((0, 1\))]\)" ,!"\(\*SubscriptBox[\(V\), \((0, 2\))]\)" ,!"\(\*SubscriptBox[\(V\), \((0, 3\))]\)"},Plot[{fitV1[x],fitV2[x],fitV3[x]},{x,0,90},PlotStyle->{Red,Green,Blue},PlotRange->{{0,90},Automatic}],Frame->True,FrameLabel->{"Angle of Projection ,\[\Phi\] (^o)","Maximum Height,h (m)"},GridLines->Automatic,PlotLabel->"Maximum Height, h against Angle of Projection, \[\Phi\]",ImageSize->500],Column[{Row[{"!"\(\*SubscriptBox[\(V\), \((0, 1\))]\)": ",eq1}],Row[{"!"\(\*SubscriptBox[\(V\), \((0, 2\))]\)": ",eq2}],Row[{"!"\(\*SubscriptBox[\(V\), \((0, 3\))]\)": ",eq3"}]}]}
```

Appendices 7: Code in Mathematica for Graph of $v_o^2 \sin 2\theta$ against Range, S

```
(*Define the data*)

V1={ {0.485,4.27},{0.570,5.19},{0.590,5.62},{0.495,5.23},{0.460,4.38}};
V2={ {1.095,8.70},{1.220,10.42},{1.230,11.22},{1.170,10.67},{0.785,7.89}};
V3={ {1.685,14.23},{1.890,17.29},{2.105,18.75},{1.945,17.62},{1.415,12.69}};

(*error bar*)

V1error={
{Around[0.485,0.001],Around[4.27,1.06]}, {Around[0.570,0.001],Around[5.19,1.08]}, {Around[0.590,0.001],Around[5.62,1.12]}, {Around[0.495,0.001],Around[5.23,1.12]}, {Around[0.460,0.001],Around[4.38,1.13]}
};

V2error={
{Around[1.095,0.001],Around[8.70,2.14]}, {Around[1.220,0.001],Around[10.42,2.11]}, {Around[1.230,0.001],Around[11.22,2.16]}, {Around[1.170,0.001],Around[10.67,2.19]}, {Around[0.785,0.001],Around[7.89,1.99]}
};

V3error={
{Around[1.685,0.001],Around[14.23,3.48]}, {Around[1.890,0.001],Around[17.29,3.47]}, {Around[2.105,0.001],Around[18.75,3.55]}, {Around[1.945,0.001],Around[17.62,3.56]}, {Around[1.415,0.001],Around[12.69,3.15]}
};

(*Extract x and y values for each set of data*)

x1=V1[[All,1]];
y1=V1[[All,2]];
x2=V2[[All,1]];
y2=V2[[All,2]];
x3=V3[[All,1]];
y3=V3[[All,2]];

(*Perform linear regression to find the best-fit lines*)

fit1=LinearModelFit[V1,x,x];
fit2=LinearModelFit[V2,x,x];
fit3=LinearModelFit[V3,x,x];

(*Extract equations as strings*)

eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];
eq2=ToString[TraditionalForm[y==fit2["BestFit"]]];
eq3=ToString[TraditionalForm[y==fit3["BestFit"]]];

(*Display the equations outside the graph with extended line plot*)

Column[{Show[ListPlot[{V1error,V2error,V3error},PlotStyle->{Red,Green,Blue},PlotLegends->{"!"(*SubscriptBox[V, \0, 1]),!"(*SubscriptBox[V, \0, 2]),!"(*SubscriptBox[V, \0, 3])}],Plot[{fit1[x],fit2[x],fit3[x]}, {x,0,2.5},PlotStyle->{Red,Green,Blue},PlotRange->{{0,2.5},Automatic}],Frame->True,FrameLabel->{"Range, S (m)",!"(*SuperscriptBox[V, \2])\sin(2\Theta) (m^2s^-2)",GridLines->Automatic,PlotLabel->!"(*SuperscriptBox[V, \2])\sin(2\Theta) against Range, S ",ImageSize->500],Column[{Row[{!"(*SubscriptBox[V, \0, 1]),": ",eq1}],Row[{!"(*SubscriptBox[V, \0, 2]),": ",eq2}],Row[{!"(*SubscriptBox[V, \0, 3]),": ",eq3}]}]}
```

Appendices 8: Code in Mathematica for Graph of Range, S against $v_o^2 \sin 2\theta$

```
{Around[14.23,3.48],Around[1.685,0.001]}, {Around[17.29,3.47],Around[1.890,0.001]}, {Around[18.75,3.55],Around[2.105,0.001]}, {Around[17.62,3.56],Around[1.945,0.001]}, {Around[12.69,3.15],Around[1.415,0.001]}};

(*Extract x and y values for each set of data(*Define the data*)V1={4.27,0.485},{5.19,0.570},{5.62,0.590},{5.23,0.495},{4.38,0.460}};

V2={{8.70,1.095},{10.42,1.220},{11.22,1.230},{10.67,1.170},{7.89,0.785}};

V3={{14.23,1.685},{17.29,1.890},{18.75,2.105},{17.62,1.945},{12.69,1.415}};

(*error bar*)

V1error={

{Around[4.27,1.06],Around[0.485,0.001]}, {Around[5.19,1.08],Around[0.570,0.001]}, {Around[5.62,1.12],Around[0.590,0.001]}, {Around[5.23,1.12],Around[0.495,0.001]}, {Around[4.38,1.13],Around[0.460,0.001]}};

V2error={

{Around[8.70,2.14],Around[1.095,0.001]}, {Around[10.42,2.11],Around[1.220,0.001]}, {Around[11.22,2.16],Around[1.230,0.001]}, {Around[10.67,2.19],Around[1.170,0.001]}, {Around[7.89,1.99],Around[0.785,0.001]}};

V3error={

*)

x1=V1[[All,1]];

y1=V1[[All,2]];

x2=V2[[All,1]];

y2=V2[[All,2]];

x3=V3[[All,1]];

y3=V3[[All,2]];

(*Perform linear regression to find the best-fit lines*)

fit1=LinearModelFit[V1,x,x];

fit2=LinearModelFit[V2,x,x];

fit3=LinearModelFit[V3,x,x];

(*Extract equations as strings*)

eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];

eq2=ToString[TraditionalForm[y==fit2["BestFit"]]];

eq3=ToString[TraditionalForm[y==fit3["BestFit"]]];

(*Display the equations outside the graph with extended line plot*)

Column[{Show[ListPlot[{V1error,V2error,V3error},PlotStyle->{Red,Green,Blue},PlotLegends->{"V1", "V2", "V3"},Plot[fit1[fit2[fit3[x]],{x,0,25},PlotStyle->{Red,Green,Blue},PlotRange->{0,25},Automatic],Frame->True,FrameLabel->{"V", "sin(2\Theta) sin(2\Theta)} (m^2s^-2)", "Range, S (m)",GridLines->Automatic,PlotLabel->"Range, S against V^2 sin(2\Theta)",ImageSize->500],Column[{Row[{"V", "eq1"},Row[{"V", "eq2"},Row[{"V", "eq3"}]}]]]
```


Appendices 9: Code in Mathematica for Graph of $\frac{1}{2}(v_o \sin \phi)^2$ against Maximum Height, h

```

V1={ {0.035,0.50},{0.039,0.91},{0.170,1.40},{0.210,1.87},{0.250,2.35}};
V2={ {0.080,1.01},{0.190,1.82},{0.290,2.81},{0.410,3.81},{0.450,4.23}};
V3={ {0.160,1.66},{0.330,3.03},{0.510,4.69},{0.680,6.29},{0.720,6.80}};

(*error bar*)
V1error={ {Around[0.035,0.001],Around[0.50,0.03]},{Around[0.039,0.001],Around[0.91,0.06]},{Around[0.170,0.001],Around[1.40,0.10]},{Around[0.210,0.001],Around[1.87,0.16]},{Around[0.250,0.001],Around[2.35,0.21]}};

V2error={ {Around[0.080,0.001],Around[1.01,0.05]},{Around[0.190,0.001],Around[1.82,0.09]},{Around[0.290,0.001],Around[2.81,0.15]},{Around[0.410,0.001],Around[3.81,0.21]},{Around[0.450,0.001],Around[4.23,0.27]}};

V3error={ {Around[0.160,0.001],Around[1.66,0.08]},{Around[0.330,0.001],Around[3.03,0.12]},{Around[0.510,0.001],Around[4.69,0.19]},{Around[0.680,0.001],Around[6.29,0.27]},{Around[0.720,0.001],Around[6.80,0.33]}};

(*Extract x and y values for each set of data*)
x1=V1[[All,1]];
y1=V1[[All,2]];
x2=V2[[All,1]];
y2=V2[[All,2]];
x3=V3[[All,1]];
y3=V3[[All,2]];

(*Perform linear regression to find the best-fit lines*)
fit1=LinearModelFit[V1,x,x];
fit2=LinearModelFit[V2,x,x];
fit3=LinearModelFit[V3,x,x];

(*Extract equations as strings*)
eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];
eq2=ToString[TraditionalForm[y==fit2["BestFit"]]];
eq3=ToString[TraditionalForm[y==fit3["BestFit"]]];

(*Display the equations outside the graph with extended line plot*)
Column[{Show[ListPlot[{V1error,V2error,V3error},PlotStyle->{Red,Green,Blue},PlotLegends->{"\["SubscriptBox["V"], \{0, 1\}]\)", "\["SubscriptBox["V"], \{0, 2\}]\)", "\["SubscriptBox["V"], \{0, 3\}]\)"},Plot[{fit1[x],fit2[x],fit3[x]},{x,0,0.8},PlotStyle->{Red,Green,Blue}],Frame->True,FrameLabel->{"Maximum Height,h (m)", "\["FractionBox["V"], \{2\}]\)(Vsin[\Phi])\["SuperscriptBox["", \{2\}]\) (m^2s^-2)"},GridLines->Automatic,PlotLabel->"\["FractionBox["V"], \{2\}]\)(Vsin[\Phi])\["SuperscriptBox["", \{2\}]\) against Maximum Height, h",ImageSize->500,PlotRange->{{0,0.8},Automatic}],Column[{Row[{"\["SubscriptBox["V"], \{0, 1\}]\): ",eq1}],Row[{"\["SubscriptBox["V"], \{0, 2\}]\): ",eq2}],Row[{"\["SubscriptBox["V"], \{0, 3\}]\): ",eq3}]}]}]
```

Appendices 10: Code in Mathematica for Graph of *Maximum Height, h against $\frac{1}{2}(v_o \sin \phi)^2$*

```

V1={ {0.50,0.035}, {0.91,0.039}, {1.40,0.170}, {1.87,0.210}, {2.35,0.250} };
V2={ {1.01,0.080}, {1.82,0.190}, {2.81,0.290}, {3.81,0.410}, {4.23,0.450} };
V3={ {1.66,0.160}, {3.03,0.330}, {4.69,0.510}, {6.29,0.680}, {6.80,0.720} };

(*error bar*)
V1error={
{Around[0.50,0.03],Around[0.035,0.001]}, {Around[0.91,0.06],Around[0.039,0.001]},
{Around[1.40,0.10],Around[0.170,0.001]},
{Around[1.87,0.16],Around[0.210,0.001]},
{Around[2.35,0.21],Around[0.250,0.001]}};
V2error={
{Around[1.01,0.05],Around[0.080,0.001]},
{Around[1.82,0.09],Around[0.190,0.001]},
{Around[2.81,0.15],Around[0.290,0.001]},
{Around[3.81,0.21],Around[0.410,0.001]},
{Around[4.23,0.27],Around[0.450,0.001]}};
V3error={
{Around[1.66,0.08],Around[0.160,0.001]},
{Around[3.03,0.12],Around[0.330,0.001]},
{Around[4.69,0.19],Around[0.510,0.001]},
{Around[6.29,0.27],Around[0.680,0.001]},
{Around[6.80,0.33],Around[0.720,0.001]}};

(*Extract x and y values for each set of data*)
x1=V1[[All,1]];
y1=V1[[All,2]];
x2=V2[[All,1]];
y2=V2[[All,2]];
x3=V3[[All,1]];
y3=V3[[All,2]];

(*Perform linear regression to find the best-fit lines*)
fit1=LinearModelFit[V1,x,x];
fit2=LinearModelFit[V2,x,x];
fit3=LinearModelFit[V3,x,x];

(*Extract equations as strings*)
eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];
eq2=ToString[TraditionalForm[y==fit2["BestFit"]]];
eq3=ToString[TraditionalForm[y==fit3["BestFit"]]];

(*Display the equations outside the graph with extended line plot*)

Column[{Show[ListPlot[{V1error,V2error,V3error},PlotStyle->{Red,Green,Blue},PlotLegends->{"\!\(\*SubscriptBox[\(V\), \!(0, 1)\])", "\!\(\*SubscriptBox[\(V\), \!(0, 2)\])", "\!\(\*SubscriptBox[\(V\), \!(0, 3)\])"}],Plot[{fit1[x],fit2[x],fit3[x]}, {x,0,8},PlotStyle->{Red,Green,Blue}],Frame->True,FrameLabel->{"\!\(\*FractionBox[\!(1\), \!(2)\])/(V sin(\[Phi]))!\(\*SuperscriptBox[\!(0\), \!(2)\])/(m^2 s^2)", "Maximum Height(h)",GridLines->Automatic,PlotLabel->"Maximum Height(h) against \!\(\*FractionBox[\!(1\), \!(2)\])/(V sin(\[Phi]))!\(\*SuperscriptBox[\!(0\), \!(2)\])"}],ImageSize->500,PlotRange->{{0,8},Automatic}],Column[{Row[{"\!\(\*SubscriptBox[\(V\), \!(0, 1)\])": "eq1"},Row[{"\!\(\*SubscriptBox[\(V\), \!(0, 2)\])": "eq2"},Row[{"\!\(\*SubscriptBox[\(V\), \!(0, 3)\])": "eq3"}]}]}]}

```

Appendices 11: Code in Mathematica for Graph of Maximum Range, S against Initial Speed, V_0

```

(*Define the data*)
V={{0.595,2.38},{1.215,3.03},{2.100,4.14}};

(*error bar*)
Verror={{Around[0.595,0.001],Around[2.38,0.08]},
{Around[1.215,0.001],Around[3.03,0.07]},
{Around[2.100,0.001],Around[4.14,0.04]}};

(*Extract x and y values for each set of data*)
x1=V[[All,1]];
y1=V[[All,2]];

(*Perform linear regression to find the best-fit lines*)
fit1=LinearModelFit[V,x,x];

(*Extract equations as strings*)
eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];

(*Display the equations outside the graph*)
Column[{Show[ListPlot[{Verror},PlotStyle->{Red},PlotLegends->{"Subscript[V,
45^o]"}],Plot[{fit1[x]},{x,0,3},PlotStyle->{Red}],Frame->True,FrameLabel->{"Initial Speed, Subscript[V,
o] (ms^-2)","Maximum Range, S (m)"},GridLines->Automatic,PlotLabel->"Maximum Range, S against
Initial Speed, Subscript[V, o]",ImageSize->500],Column[{Row[{"Subscript[V, 45^o]: ",eq1}]}]}]

```

Appendices 12: Code in Mathematica for Graph of $v_o^2 \sin 2\theta$ against Range, S in angle of projection 45°

```
(*Define the data*)V45={{0.595,5.68},{1.215,9.15},{2.1,17.17}};

(*error bar*)
V45error={{Around[0.595,0.001],Around[5.68,1.13]},{Around[1.215,0.001],Around[9.15,0.59]},{Around[2.1,0.001],Around[17.17,0.79]}};

(*Extract x and y values for each set of data*)
x1=V45[[All,1]];
y1=V45[[All,2]];

(*Perform linear regression to find the best-fit lines*)
fit1=LinearModelFit[V45,x,x];

(*Extract equations as strings*)
eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];

(*Display the equations outside the graph with extended line plot*)
Column[{Show[ListPlot[V45error,PlotStyle->{Red},PlotLegends->{"!"\(*SubscriptBox[V, \
(45\)]\)}],Plot[{fit1[x]},{x,0,2.5},PlotStyle->{Red},PlotRange->{{0,2.5},Automatic}],Frame-
>True,FrameLabel->{"Range(S)",!"\(*SuperscriptBox[V, \ (2\)]\)\sin(2\Theta)(m^2s^-
2)"},GridLines->Automatic,PlotLabel->!"\(*SuperscriptBox[V, \ (2\)]\)\sin(2\Theta) against
Range(S) ",ImageSize->500],Column[{Row[{"!"\(*SubscriptBox[V, \ (45\)]\): ",eq1}]]}]}
```

Appendices 13: Code in Mathematica Graph of Range, S against $v_o^2 \sin 2\theta$ in angle of projection 45°

```
(*Define the data*)V45={{5.68,0.595},{9.15,1.215},{17.17,2.1}};

(*error bar*)
V45error={{Around[5.68,1.13],Around[0.595,0.001]},{Around[9.15,0.59],Around[1.215,0.001]},{Around[17.17,0.79],Around[2.1,0.001]}};

(*Extract x and y values for each set of data*)
x1=V45[[All,1]];
y1=V45[[All,2]];

(*Perform linear regression to find the best-fit lines*)
fit1=LinearModelFit[V45,x,x];

(*Extract equations as strings*)
eq1=ToString[TraditionalForm[y==fit1["BestFit"]]];

(*Display the equations outside the graph with extended line plot*)
Column[{Show[ListPlot[{V45error},PlotStyle->{Red},PlotLegends->{"!(\(*SubscriptBox[(V\), \ (45\)]\)")"},Plot[{fit1[x]},{x,0,25},PlotStyle->{Red},PlotRange->{{0,25},Automatic}],Frame->True,FrameLabel->{"!(\(*SuperscriptBox[(V\), \ (2\)]\)sin(2\ [Theta]) (m^2s^-2)", "Range(S)"},GridLines->Automatic,PlotLabel->"Range(S) against !( \(*SuperscriptBox[(V\), \ (2\)]\)sin(2\ [Theta])",ImageSize->500],Column[{Row[{!(\(*SubscriptBox[(V\), \ (45\)]\)": "eq1}]]}]}
```

Appendices 14:

Code in Mathematica for the percentage of discrepancy of gravitational acceleration on Earth

```
(*Percentage of discrepancy*)
gteory=9.81;
g1={8.55,6.84,9.40}
Pd=Abs[(g1-gteory)/gteory*100 ]
g2={12.48,8.64,9.72}
Pd=Abs[(g2-gteory)/gteory*100 ]
g3={7.19,8.75,9.15}
Pd=Abs[(g3-gteory)/gteory*100 ]
g4={7.72,8.77,9.17}
Pd=Abs[(g4-gteory)/gteory*100 ]
g5={7.73}
Pd=Abs[(g5-gteory)/gteory*100 ]
g6={7.85}
```