SCHOOL OF PHYSICS
UNIVERSITI SAINS MALAYSIA

ZCT191/192 PHYSICS PRACTICAL I/II

ERROR ANALYSIS

Lab Manual

OBJECTIVES

1. *To understand how to correctly present a physical quantity with its error;*
2. *To estimate the resistivity of a copper wire and its error by measuring its length, resistance and diameter; and*
3. *To study the probability distribution function of the diameter of a copper wire.*

THEORY

Introduction

This experiment is the first experiment that you will conduct in your *first year undergraduate physics laboratory* (also known as *first year lab* or *lab 100*). Before you start, please read and understand the *Introduction to Error Analysis* guidebook given as it contains the fundamentals required to conduct all physics experiments from this point onwards.

This experiment is divided into two parts. **Part A** is about measuring the resistivity of copper, where the variables used to determine the resistivities are measured and their respective errors estimated. Subsequently, the resistivities are calculated and their related errors computed using the *propagation of errors*. In **Part B**, a statistical analysis on wire diameters will be performed to confirm that you understand basic error analysis, which has been covered in the guidebook mentioned. Just like all other experiments, two practical sessions are required to perform this experiment.

This experiment is a little different from the others, in the sense that all you need to do is to fill up the blanks and tables, cross out incorrect statements, analyse and comment on the experimental aspects in the worksheets given. No report submission is required for this experiment, unlike all other experiments in this lab. Remember to write your answers in the correct units and significant figures.

Resistivity

The resistance (R) for a conductor in cylindrical form is given by:

$$R = \rho \frac{l}{A} \tag{1}$$

where ρ, l, and A are the *resistivity*, length, and cross-sectional area of the conductor, respectively. Thus, the resistivity of a conductor can be calculated once l, R and A are known.

In this experiment, an SWG36 copper wire will be used as the conductor. Here, SWG stands for *standard wire gauge*, which is a unit for denoting wire size, while the number 36 is a code corresponding to a thickness of 0.1930 mm.

EQUIPMENT

1. SWG36 copper wire (length ~130 cm)
2. Micrometer
3. Metre rule
4. Multimeter
5. Sandpaper

PROCEDURE

Part A: Resistivity of A Copper Wire

In this experiment, you are required to make a few measurements on the length, resistance and diameter of a copper wire and calculate its resistivity.

A1: Length of Copper Wire

1. Measure the length (l) of the copper wire with the metre rule.
2. Repeat the measurement so that 8 readings are obtained, and record them in **Table 1**.
3. Calculate the average value of the length ($\bar{l}$) and its corresponding error.

A2: Resistance of Copper Wire

1. Remove the enamel coating at both ends of the copper wire to obtain better electrical conductivity.
2. Measure the resistance (R) of the copper wire with a multimeter. Ensure that you turn knob and read the scale carefully.
3. Repeat the measurement so that 8 readings are obtained, record them in **Table 2**.
4. Calculate the average value of the resistance ($\bar{R}$) and its corresponding error.

A3: Diameter of Copper Wire

1. Record the zero error of the micrometer up to 3 decimal places.
2. Remove a small part of the enamel coating until the bare copper wire is observed.
3. Measure the diameter of the copper wire with a micrometer.
4. Repeat **Step 3**, each time at a different area of the copper wire, until 8 readings are collected. Record your measurements in **Table 3**.
5. Calculate the average value of the diameter ($\bar{d}$) and its corresponding error.
6. Compare the measured value with the diameter d_0 given by the laboratory reference book, and find the percentage discrepancy between them.

Analysis

1. Compute the resistivity (ρ) of the copper wire using **Equation 1**, and find its corresponding error using the propagation of errors.
2. Measure the temperature of the laboratory.
3. By using the standard values of the resistivity of the SWG36 copper wire at temperatures $T = 0$ °C and 100 °C obtained from the laboratory reference book, plot a graph of ρ vs. T.
4. From the graph, obtain the standard resistivity at the laboratory temperature (ρ_0) and find its percentage discrepancy with the measured value.

Part B: Probability Distribution of Wire Diameter

In this part, you are required to perform statistical error analysis on a distribution of data.

Measurement

1. Obtain 8 readings of the diameter of the copper wire using the micrometer, each time at a different location.
2. Repeat **Step 1** twice, such that you have obtained 3 samples of diameter measurements (you can use your data obtained for **Part A3** as one sample). Record your data in **Table 4**. Note that samples $j = 4$ to 40 has been generated for you, thus you have a total sample size of $N = 40 \times 8$.

Analysis

1. Using an interval of $d = 2 \times 10^{-3}$ mm, divide each reading into bins of data, and record the number of readings (n) for each bin in **Table 5**. Choose a suitable starting point and bin interval ranges such that all the readings could be included.
   [*Hint: you can sort the data and produce the graphs required using Microsoft Excel.*]
2. Plot a histogram of n/N vs. d.
3. Estimate the mean diameter of the copper wire, $(\bar{d})$, its standard deviation (s), and its standard error (s_{m}).
4. Sketch a smooth Gaussian curves that could represent the data distribution given by the histogram. This can be done by equating the maximum value of the histogram $(n/N)_{\max}$ with the peak of the Gaussian function (at $x = \mu$), such that

$$\left(\frac{n}{N}\right)_{\max}\frac{1}{\Delta x} = \frac{1}{\sigma\sqrt{2\pi}}, \tag{2}$$

   where Δx is the bin interval, required to adjust the area of the Gaussian curve. Sketch the Gaussian curve $f(\mu, \sigma)$ using $\mu = d_{\max}$, the centre point of the bin with $(n/N)_{\max}$ and the value of σ obtained from **Equation 2**.
5. Calculate the resistivity $\bar{\rho}$ of the copper wire by using the values of R and l obtained from **Parts A1** and **A2**, but d obtained from **Part B**.
6. Estimate the error $\delta\rho$ using propagation of errors.
7. Compare the $\bar{\rho}$ obtained from this experiment (ρ_{B}) with the standard value ρ_0 and the value of $\bar{\rho}$ obtained from **Part A** (ρ_{A}).

REFERENCES

1. School of Physics, USM (2021). *Introduction to Error Analysis*. Universiti Sains Malaysia.
2. Taylor, J. R. (1997). *An introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (2nd Ed.)*. University Science Books.

ACKNOWLEDGEMENT

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ZCT191/192 PHYSICS PRACTICAL I/II
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Worksheets

OBJECTIVES

1. *To understand how to correctly present a physical quantity with its error;*
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3. *To study the probability distribution function of the diameter of a copper wire.*

ERROR ANALYSIS WORKSHEET 1

Instructions: please complete **Worksheet 1** by the end of the first session of your experiment.

Name : TAN WEI LIANG Date : 23-10-2023
Partner's Name : AINA IMANINA BINTI MOHD KHOZIKIN Group : M5B

Part A1: Length of a Copper Wire

Table 1: The length of the copper wire.

Measurement, i	**Length of wire, $l \pm 0.1$ cm**	$l_i - \bar{l}$ **(m)**	$(l_i - \bar{l})^2$ **(m^2)**
1	129.5	-1.875$\times 10^{-3}$	3.51563$\times 10^{-6}$
2	130.0	3.125$\times 10^{-3}$	9.76563$\times 10^{-6}$
3	129.8	1.125$\times 10^{-3}$	1.26563$\times 10^{-6}$
4	129.7	0.125$\times 10^{-3}$	1.56250$\times 10^{-8}$
5	129.7	0.125$\times 10^{-3}$	1.56250$\times 10^{-8}$
6	129.4	-2.875$\times 10^{-3}$	8.26562$\times 10^{-6}$
7	129.7	0.125$\times 10^{-3}$	1.56250$\times 10^{-8}$
8	129.7	0.125$\times 10^{-3}$	1.56250$\times 10^{-8}$

1. Sample size, N : 8
2. Average length, $\bar{l} = \frac{1}{N}\sum_{i=1}^{N} l_i$: 1.296875 m
3. Variance, $s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(l_i - \bar{l})^2$: 0.032678571 m^2
4. Standard deviation, s : 1.807722×10^{-3} m
5. Standard error, $s_m = \frac{s}{\sqrt{N}}$: 6.39126×10^{-4} m
6. Length of copper wire, $\bar{l} \pm \delta l$: 1.2969 ± 0.0006 m

Part A2: Resistance of Copper Wire

Table 2: The resistance of the copper wire.

Measurement, i	**Resistance of wire, $R \pm 0.1\ \Omega$**	$R_i - \bar{R}$ **(Ω)**	$(R_i - \bar{R})^2$ **(Ω^2)**
1	0.8	-0.05	0.0025
2	0.8	-0.05	0.0025
3	0.9	0.05	0.0025
4	0.8	-0.05	0.0025
5	0.9	0.05	0.0025
6	0.9	0.05	0.0025
7	0.8	-0.05	0.0025
8	0.9	0.05	0.0025

1. Sample size, N : 8
2. Average resistance, $\bar{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$: 0.850 Ω
3. Variance, $s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(R_i - \bar{R})^2$: 0.002857143 Ω^2
4. Standard deviation, s : 0.053452248 Ω
5. Standard error, $s_{\mathrm{m}} = \frac{s}{\sqrt{N}}$: 0.018898224 Ω
6. Resistance of copper wire, $\bar{R} \pm \delta R$: 0.85 ± 0.02 Ω

Part A3: Diameter of Copper Wire

Table 3: The diameter of the copper wire.

Measurement, i	**Diameter of wire, $d \pm 0.01$ mm**	**$d_i - \bar{d}$ (m)**	**$(d_i - \bar{d})^2$ (m^2)**
1	0.21	2.5×10^{-6}	6.25×10^{-12}
2	0.23	2.25×10^{-5}	5.0625×10^{-10}
3	0.20	-7.5×10^{-6}	5.625×10^{-11}
4	0.19	-1.75×10^{-5}	3.0625×10^{-10}
5	0.17	-3.75×10^{-5}	1.40625×10^{-9}
6	0.23	2.25×10^{-5}	5.0625×10^{-10}
7	0.20	-7.5×10^{-6}	5.625×10^{-11}
8	0.23	2.25×10^{-5}	5.0625×10^{-10}

1. Zero error : 0 m
2. Sample size, N : 8
3. Average diameter, $\bar{d} = \frac{1}{N}\sum_{i=1}^{N} d_i$: 2.075×10^{-4} m
4. Variance, $s^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2$: 4.78571×10^{-10} m^2
5. Standard deviation, s : 2.18763×10^{-5} m
6. Standard error, $s_{\mathrm{m}} = \frac{s}{\sqrt{N}}$: 7.73443×10^{-6} m
7. Diameter of copper wire, $\bar{d} \pm \delta d$: $(2.075 \pm 0.077) \times 10^{-4}$ m
8. Diameter of copper wire (lab reference book), d_0 : 1.93×10^{-4} m
9. Percentage discrepancy, $\frac{|\bar{d} - d_0|}{d_0} \times 100\%$: 7.51 %

Analysis

Fill in the blanks with the correct answers, uncertainties and units.

1. The measured values of the length (l), resistance (R) and diameter (d) of the SWG36 copper wire are
   a. $l = \bar{l} \pm \delta l$, $l = 1.2969 \pm 0.0006$ m
   b. $R = \bar{R} \pm \delta R$, $R = 0.85 \pm 0.02$ Ω
   c. $d = \bar{d} \pm \delta d$, $d = (2.075 \pm 0.077) \times 10^{-4}$ m

2. The best estimate of the resistivity of the copper wire is $\rho = 2.40 \times 10^{-8}$ Ωm.
   [Show your calculations in the space provided below. Use the correct units in your final answer.]

$$\bar{\rho} = \frac{\bar{R}A}{\bar{l}}$$
$$= \frac{\bar{R} \times \pi(\frac{\bar{d}}{2})^2}{\bar{l}}$$
$$= \frac{\pi \bar{R} \bar{d}^2}{4\bar{l}}$$
$$= \frac{\pi \times 0.85 \times (2.075 \times 10^{-4})^2}{4 \times 1.2969}$$
$$= 2.216350892 \times 10^{-8} \ \Omega\text{m}$$

3. The uncertainty of the resistivity of the copper wire is $\delta\rho = 1.95 \times 10^{-9}$ Ωm.
   [Show your calculations in the space below. Hint: use the variance formula from the guidebook.]

$$\frac{\delta\rho}{\bar{\rho}} = \sqrt{(\frac{\delta R}{\bar{R}})^2 + (2 \times \frac{\delta d}{\bar{d}})^2 + (\frac{\delta l}{\bar{l}})^2}$$
$$\delta\rho = \bar{\rho}\sqrt{(\frac{\delta R}{\bar{R}})^2 + (2 \times \frac{\delta d}{\bar{d}})^2 + (\frac{\delta l}{\bar{l}})^2}$$
$$= 2.216350892 \times 10^{-8}\sqrt{(\frac{0.02}{0.85})^2 + (2 \times \frac{0.077 \times 10^{-4}}{2.075 \times 10^{-4}})^2 + (\frac{0.0006}{1.2969})^2}$$
$$= 1.725623915 \times 10^{-9} \ \Omega\text{m}$$

4. The final estimated resistivity of the SWG36 copper wire is therefore
   $\rho = \bar{\rho} \pm \delta\rho = (\ 2.22 \pm 0.17\ ) \times 10^{-8}$ Ωm.

5. The temperature in the first-year lab is $T = 26.0$ °C.

6. The standard values of the resistivity for the SWG36 copper wire SWG 36 at different temperatures are
$\rho_{0\,°\mathrm{C}} \quad = 1.55 \times\ 10^{-8}\ \ \Omega\mathrm{m}$
$\rho_{100\,°\mathrm{C}} = 2.24 \times\ 10^{-8}\ \ \Omega\mathrm{m}$

7. Plot of ρ vs. T:

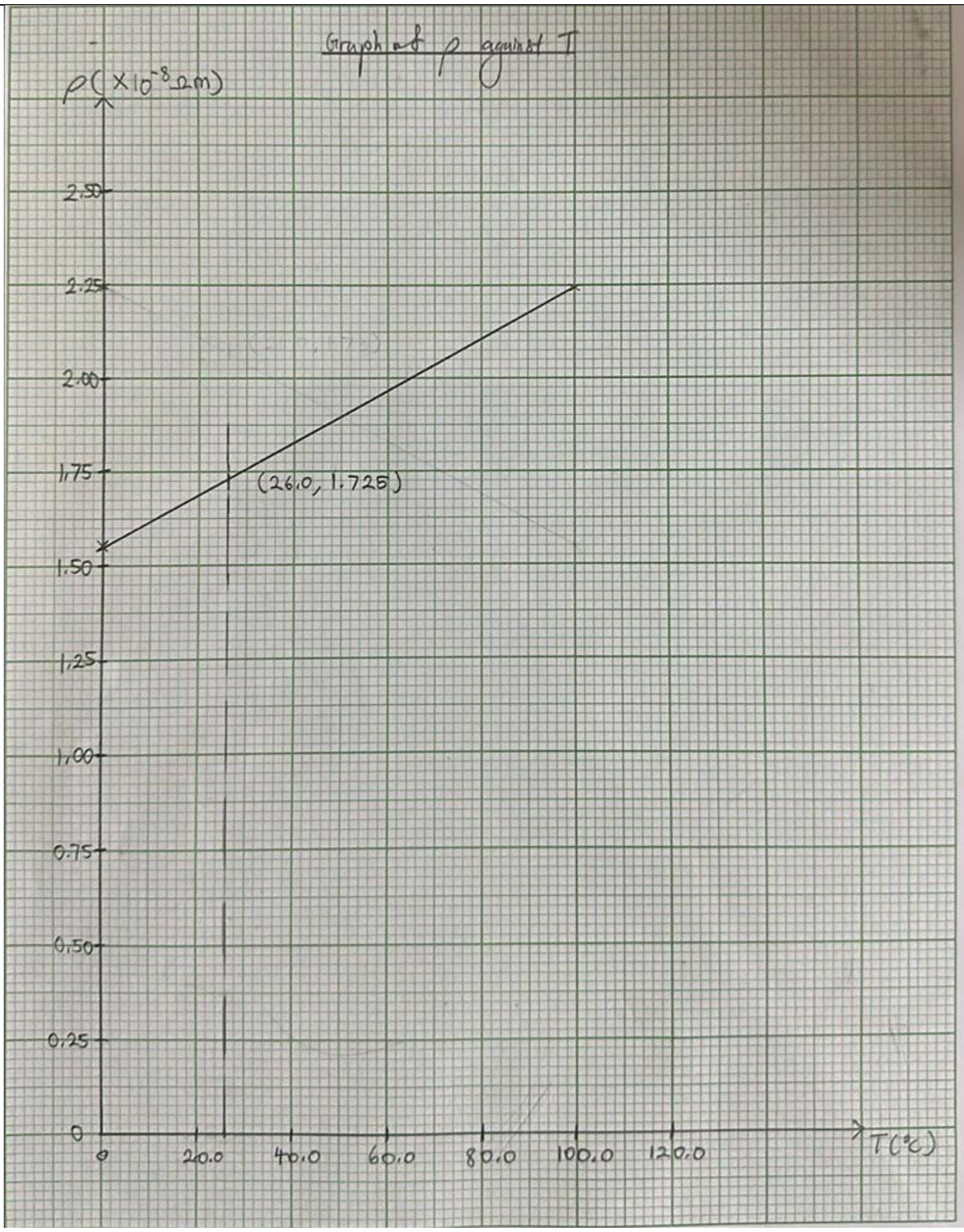

Figure 1: Plot of ρ vs. T.

8. From the graph, the estimated standard resistivity of a copper wire at laboratory temperature is $\rho_0 = 1.73 \times\ 10^{-8}\ \Omega\mathrm{m}$.

9. The percentage discrepancy between the standard resistivity and the experimental resistivity of the copper wire is 28.31 %.
   [Show your calculations in the space below, and comment if your result is accurate.]

percentage discrepancy, $\frac{|\bar{\rho}-\rho_o|}{\rho_o} \times 100\% = \frac{|2.22\times 10^{-8}-1.73\times 10^{-8}|}{1.73\times 10^{-8}} \times 100\%$
$= 28.31\%$

Since the percentage of discrepancy is large, the standard resistivity and the experimental resistivity of the copper wire are not consistent and indicated the presence of systematic error.

10. The sources of systematic errors observed in this experiment are:
    [Comment if they are significant.]

The length of the copper wire is influenced by changes in surrounding temperature, causing linear expansion. This statement suggests that temperature can affect the length of the copper wire. This is a systematic error because it consistently influences measurements in a specific way.

The multimeter lacks the necessary sensitivity to detect small changes in resistance and offers limited resolution. This is a systematic error because it consistently affects the measurements by limiting the precision and sensitivity of the equipment.

Inconsistencies in enamel coating removal, if the process of removing enamel coating from the copper wire is not done consistently across all measurements or if it inadvertently damages the wire. This is a systematic error because it consistently affects the measurements by the copper wire is not done consistently across all measurements.

11. Other sources of errors in this experiment include:
[List down at least three sources, and explain briefly how these errors affected your results.]

When data falls between the smallest division on the micrometre scale, it can be challenging to accurately determine the nearest value, especially for individuals with poor eyesight. This is not a systematic error. It's more related to the experimenter's limitations and doesn't consistently affect measurements in the same way. It's more of a random error.

During length measurements, the copper wire may not be perfectly aligned or straight, which can introduce errors in the data. This is not a systematic error. It's more related to the setup and handling of the experiment, leading to random errors.

The resistivity-temperature graph is not necessarily linear in theoretical models, and the provided standard values for SWG36 copper resistivity may lack the necessary information to represent real-world results for data analysis. This is not a systematic error. It's related to the theoretical understanding and the accuracy of the provided resistivity data, but it doesn't consistently affect measurements in the same way.

When the copper wire is sanded, it generates heat, raising the temperature and subsequently increasing resistance. Immediate resistance measurements using a multimeter in this scenario can yield inaccurate data. This is not a systematic error. It's more related to the timing and procedure of measurements and introduces a random error due to the delay between sanding and measurement.

12. The fractional uncertainty of the measured resistivity of the SWG36 copper wire is 7.66 %.
[Show your calculations in the space below, and comment if your result is precise.]

Fractional uncertainty, $\frac{\Delta\rho}{\bar{\rho}} \times 100\% = \frac{0.17 \times 10^{-8}}{2.22 \times 10^{-8}} \times 100\%$

$$= \frac{17}{222} \times 100\%$$
$$= 7.66\ \%$$

Since the fractional uncertainty is small, the result of measurement is precise.

13. The precision of the resistivity measurement can improve if
[List down at least three suggestions.]

The temperature of labs is set as constant temperature to avoid the linear expansion of copper wire. The measuring apparatus in smaller scale division or more sensitivity is suggested to use in measurement. When using a multimeter for measurement, ensure that the probes are clamped properly and tightly onto the copper wire. During sanding the copper wire, must be sanding gently and stop immediately when observed the shiny surface is formed to avoid over sanding. After sanding the surface layer of wire, it should be wait until the friction-generated temperature drops to lab temperature before measurement. After sanding, the copper must always keep in straight to avoid the deformation of the cross-sectional area when it was bent before the measurement of diameter. The copper wire should always be kept straight when taking measurements to obtains accurate length. The more larger sample size is suggested to take in experiment for the more accuracy average results and smaller standard error.

Conclusion

1. The resistivity of the copper wire at temperature26.0 °C is $(2.22 \pm 0.17) \times 10^{-8}$ Ωm.
2. The agreement between experimental and standard values is ~~good~~ / not good*. The systematic errors are significant / ~~insignificant~~*.
*[*Strike out whichever is not applicable.]*

ERROR ANALYSIS WORKSHEET 2

Instructions: please complete **Worksheet 2** by the end of the second session of your experiment.

Name : TAN WEI LIANG Date : 30-10-2023
Partner's Name : AINA IMANINA BINTI MOHD KHOZIKIN Group : M5B

Data

Table 4: Diameter d (10^{-3} mm) measurements of the copper wire for samples $j = 1$ to 40. Students are required to obtain three samples of readings only, as the remaining 37 samples have been provided for you.

j	i							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	220	190	190	200	200	190	200	200
2	190	190	190	190	190	210	190	190
3	190	190	190	190	190	190	190	190
4	196	206	191	193	192	190	195	193
5	197	193	193	198	188	191	191	195
6	203	201	196	195	196	197	193	189
7	201	205	203	197	196	189	197	200
8	200	195	193	196	201	188	196	195
9	195	201	206	198	196	194	189	198
10	194	198	199	198	196	206	196	198
11	201	199	193	191	190	189	190	188
12	192	190	191	197	190	189	193	188
13	197	198	200	199	206	198	195	200
14	191	190	198	196	196	201	196	196
15	194	205	202	203	197	191	193	199
16	199	195	198	199	205	206	208	192
17	204	196	199	199	192	193	196	197
18	193	200	191	191	192	199	203	198
19	196	193	193	203	205	194	194	199
20	199	193	195	188	199	199	197	190
21	197	194	191	195	195	195	203	206
22	197	196	200	197	198	195	194	195
23	190	197	189	195	193	195	197	189
24	195	197	196	199	205	195	198	196
25	191	194	197	198	199	200	199	194
26	201	193	196	197	192	196	198	198
27	191	197	195	196	187	195	197	202
28	197	197	193	201	195	193	193	193
29	196	193	199	199	191	195	191	195
30	203	206	195	195	189	195	197	195
31	193	203	201	201	195	205	196	199
32	196	192	196	201	195	196	197	196
33	196	195	197	198	195	197	199	195
34	200	196	206	193	195	192	198	192
35	196	201	196	199	193	201	197	209
36	196	195	195	195	196	198	197	195
37	197	195	194	196	196	200	198	198
38	193	194	190	193	194	190	196	195
39	195	197	195	199	193	196	193	213
40	192	196	193	192	196	199	198	194

1.	Sample size, N	:	320	
2.	Average diameter, $\bar{d} = \frac{1}{N}\sum_{i=1,j=1}^{N} d_{ij}$	:	1.96025×10^{-4}	m
3.	Variance, $s^2 = \frac{1}{N-1}\sum_{i=1,j=1}^{N}\left(d_{ij}-\bar{d}\right)^2$	:	2.1084×10^{-11}	m^2
4.	Standard deviation, s	:	$4.591731699 \times 10^{-6}$	m
5.	Standard error, $s_{\mathrm{m}} = \frac{s}{\sqrt{N}}$	:	$2.566856053 \times 10^{-7}$	m
6.	Diameter of copper wire, $\bar{d} \pm \delta d$	:	$(1.960 \pm 0.003) \times 10^{-4}$	m

Table 5: The frequency distribution for the diameter readings of the copper wire.

Bin Ranges (10^{-3} mm) [E.g. 185.5 – 187.5]	**Number of Readings, n**	**Ratio, n/N (Total Sample Size, N =320_)**
184.5-186.5	0	0
186.5-188.5	6	0.01875
188.5-190.5	36	0.1125
190.5-192.5	25	0.078125
192.5-194.5	42	0.13125
194.5-196.5	81	0.253125
196.5-198.5	51	0.159375
198.5-200.5	36	0.1125
200.5-202.5	15	0.046875
202.5-204.5	9	0.028125
204.5-206.5	14	0.04375
206.5-208.5	1	0.003125
208.5-210.5	2	0.00625
210.5-212.5	0	0
212.5-214.5	1	0.003125
214.5-216.5	0	0
216. 5-218.5	0	0
218.5-220.5	1	0.003125

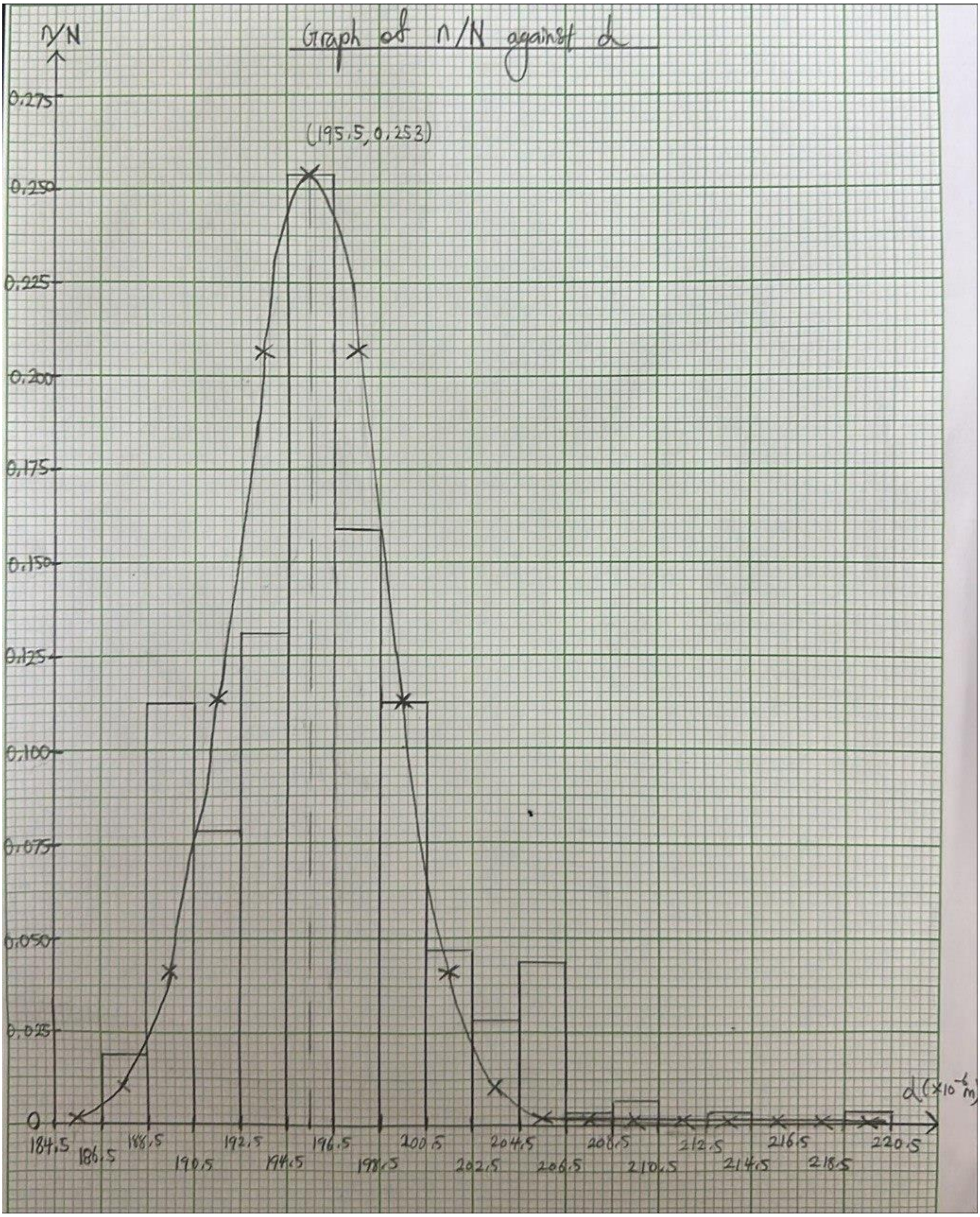

Figure 2: Histogram of n/N vs. d.

Analysis

1. The maximum point of the histogram is located at $d_{\max} = 1.955\times\ 10^{-4}\ m$, with a value of $(n/N)_{\max} = 0.253$.

2. The percentage difference between $\bar{d}$ and $d_{\max}$ is 0.26 %.
   [Show your calculations in the space below, and comment if your results are consistent with each other.]

percentage difference, $\frac{|\bar{d}-d_{\max}|}{\frac{|\bar{d}+d_{\max}|}{2}} \times 100\% = \frac{|1.960\times 10^{-4}-1.955\times 10^{-4}|}{\frac{|1.960\times 10^{-4}+1.955\times 10^{-4}|}{2}} \times 100\%$

$= 0.26\ \%$

Since, the percentage difference is small, both result of diameter is consistent. The percentage difference is smaller than 1.0% indicated the results are very closer.

3. The area under the Gaussian curve is $2.00\times\ 10^{-6}$ m.
   [Show your workings below.]

$$\begin{aligned} Area\ under\ the\ Gaussian\ Curve &= number\ of\ (5\times 5) block \times width \times height \\ &= 80 \times 2 \times 0.0125 \times 10^{-6} \\ &= 2.00 \times 10^{-6} m \end{aligned}$$

4. The standard deviation (σ) of the sketched Gaussian curve is $3.154 \times 10^{-6} m$.
   [Show all the steps that you have used to obtain the answer.]

$$(\frac{n}{N})_{max} \frac{1}{\Delta x} = \frac{1}{\sigma\sqrt{2\pi}}$$

$$\begin{aligned} \sigma &= \frac{\Delta x}{\left(\frac{n}{N}\right)_{max} \sqrt{2\pi}} \\ &= \frac{2}{0.253\sqrt{2\pi}} \\ &= 3.153693916 \times 10^{-3} mm \\ &= 3.154 \times 10^{-6} m \end{aligned}$$

5. The percentage difference between the standard deviation obtained using statistical analysis (s) and the standard deviation obtained using a best-fit Gaussian curve (σ) is 37.13 % .
[Show your calculations in the space below, and comment if your results are consistent with each other. Determine which is a more appropriate standard deviation for the diameter of the copper wire, and if the sample distribution approximates a Gaussian distribution well.]

percentage discrepancy, $\frac{|s-\sigma|}{\frac{|s+\sigma|}{2}} \times 100\% = \frac{|4.592\times 10^{-6} - 3.154\times 10^{-6}|}{\frac{|4.592\times 10^{-6}+3.154\times 10^{-6}|}{2}} \times 100\%$

$$= 37.13\ \%$$

Since the percentage difference is large, both results are not consistent. For achieving accuracy, the method in step 4 is the preferred method in comparison to that of step 3. The calculation of area under the curve through grid-based measurements on graph paper is susceptible to imperfections and inaccuracies, leading to a greater error. in contrast, the method relying on equations to compute standard deviation is favoured for its reduced reliance on approximation variables, thereby enhancing its accuracy.

In a normal experiment, you will not have sufficient time to collect 40 sets of data with 8 readings for each set to make the above estimation of mean and standard deviation. Usually, only one set of data is obtained, and all the errors will be estimated from this set of data.

6. Taking only data from sample $j = 1$ from **Table 4**, The best estimate of the diameter of the copper wire is $d_1 = \bar{d}_1 \pm \delta d_1 = (1.9875\ \pm\ 0.0365)\ \times\ 10^{-4}$, which is within 1.39% percentage difference from $\bar{d}$ calculated using all 40 samples.
[Show your workings below, and comment if both results are consistent with one another.]

percentage difference, $\frac{|\bar{d}-\bar{d}_1|}{\frac{|\bar{d}+\bar{d}_1|}{2}} \times 100\% = \frac{|1.960\times 10^{-4} - 1.9875\times 10^{-4}|}{\frac{|1.960\times 10^{-4}+1.9875\times 10^{-4}|}{2}} \times 100\%$

$$= 1.39\ \%$$

Since the percentage of difference is 1.39%, both results are consistent.

7. The fractional uncertainties of $\bar{d}_1$ and $\bar{d}$ are 1.84 % and 0.15 %, respectively.
[Show your workings below, and determine which measurement is more precise.]

Fractional uncertainty, $\frac{\delta d_1}{\bar{d}_1} \times 100\% = \frac{0.0365 \times 10^{-4}}{1.9875\times 10^{-4}} \times 100\%$

$$= 1.84\%$$

Fractional uncertainty, $\frac{\delta d}{\bar{d}} \times 100\% = \frac{0.003\times 10^{-4}}{1.960 \times 10^{-4}} \times 100\%$

$$= 0.15\ \%$$

the fractional uncertainties of $\bar{d}$ is smaller than $\bar{d}_1$ indicate the measurements conducted in Part B yielded more precise results.

8. By using the value of $\bar{d}$ calculated using all 40 samples, and the values of $\bar{l}$ and $\bar{R}$ calculated in **Part A**, the best estimate of the resistivity of the copper wire is $\bar{\rho} = 1.977 \times 10^{-8}\ \Omega\text{m}$.
[Show all the steps that you have used to obtain the answer.]

$$\bar{\rho} = \frac{\bar{R}A}{\bar{l}}$$
$$= \frac{\bar{R} \times \pi(\frac{\bar{d}}{2})^2}{\bar{l}}$$
$$= \frac{\pi\bar{R}\bar{d}^2}{4\bar{l}}$$
$$= \frac{\pi \times 0.85 \times (1.960 \times 10^{-4})^2}{4 \times 1.2969}$$
$$= 1.977490745 \times 10^{-8}\ \Omega\text{m}$$

9. By using δl and δR obtained in **Part A** as well as δd obtained in **Part B**, the uncertainty of the resistivity of the copper wire is $\delta\rho = 4.693 \times 10^{-10}\ \Omega\text{m}$.
[Show all the steps that you have used to obtain the answer.]

$$\frac{\delta\rho}{\bar{\rho}} = \sqrt{(\frac{\delta R}{\bar{R}})^2 + (2 \times \frac{\delta d}{\bar{d}})^2 + (\frac{\delta l}{\bar{l}})^2}$$
$$\delta\rho = \bar{\rho}\sqrt{(\frac{\delta R}{\bar{R}})^2 + (2 \times \frac{\delta d}{\bar{d}})^2 + (\frac{\delta l}{\bar{l}})^2}$$
$$= 1.977490745 \times 10^{-8}\sqrt{(\frac{0.02}{0.85})^2 + (2 \times \frac{0.003 \times 10^{-4}}{1.960 \times 10^{-4}})^2 + (\frac{0.0006}{1.2969})^2}$$
$$= 4.693024893 \times 10^{-10}\ \Omega\text{m}$$

10. The final estimated resistivity of the SWG36 copper wire at temperature 26.0 °C is $\rho = \bar{\rho} \pm \delta\rho = (1.98 \pm 0.05) \times 10^{-8}\ \Omega\text{m}$.

11. The percentage difference between the resistivity obtained in **Part A** and **Part B** is11.43 %.
[Show your workings below, and comment if the results are consistent with each other.]

percentage difference, $\frac{|\bar{\rho}_A - \bar{\rho}_B|}{\frac{|\bar{\rho}_A + \bar{\rho}_B|}{2}} \times 100\% = \frac{|2.22\times 10^{-8} - 1.98\times 10^{-8}|}{\frac{|2.22\times 10^{-8} + 1.98\times 10^{-8}|}{2}} \times 100\%$

$= 11.43\ \%$

The presence of large percentage difference suggests two results are not consistent.

Given that the sample size in Part B of 320 is larger than that in Part A of 8, the standard resistivity value obtained in Part B is considered more accurate in comparison to that in Part A. Consequently, a significant and large discrepancy is observed.

12. From **Figure 1**, the standard resistivity value for the SWG36 copper wire at the same temperature is $\rho_0 = 1.73 \times 10^{-8}\ \Omega\text{m}$, which is within 14.45 % percentage discrepancy with the results obtained in **Part B**.
[Show your workings below, and comment if your result is accurate.]

percentage discrepancy, $\frac{|\bar{\rho}_B - \rho_0|}{\rho_0} \times 100\% = \frac{|1.98\times 10^{-8} - 1.73\times 10^{-8}|}{1.73\times 10^{-8}} \times 100\%$

$= 14.45\ \%$

The large percentage discrepancy, exceeding 10%, indicates a significant inconsistency between the two results.

Conclusion

1. The resistivity of the copper wire at temperature 26.0 °C is $(1.98 \pm 0.05) \times 10^{-8}\ \Omega\text{m}$.
2. The percentage discrepancy between the standard and measured values of ρ is significant / ~~insignificant~~*.
*[*Strike out whichever is not applicable.]*

ABSTRACT

The title of this experiment is Error Analysis. This physics experiment delves into the comprehensive study of error analysis during the measurement of the resistivity of copper wire. Employing error propagation techniques and statistical analysis of wire diameters, the objective is to gain a profound understanding of correctly present a physical quantity along with its associated error, and estimating the resistivity of a copper wire, along with its error, through the measurement of length, resistance, and diameter. Additionally, the experiment explores the probability distribution function of the diameter of a copper wire. The experiment is divided into two parts: Part A and Part B. In Part A, the measurement of length, resistance, and diameter of SWG 36 copper wire is utilized to calculate the resistivity, resulting in a value of $(2.22 \pm 0.17) \times 10^{-8}$ Ωm. From the graph, the estimated standard resistivity of a copper wire at laboratory temperature is $\rho_0 = 1.73 \times 10^{-8}$ Ωm at temperature 26.0 °C.The experiment reveals a 28.31% percentage discrepancy between the standard and experimental resistivity, indicating significant systematic errors. The agreement between experimental and standard values is not good. The systematic errors are significant in part A. In Part B, the diameter of SWG 36 copper wire is determined from a sample size of 320, and errors are estimated using statistical analysis. The experiment demonstrates that the method for obtaining standard deviation through a best-fit Gaussian curve is more precise than traditional statistical analysis. The resistivity of the copper wire is determined as $(1.98 \pm 0.05) \times 10^{-8}$ Ωm , with a 14.45% percentage discrepancy between the standard and measured values. The percentage discrepancy between the standard and measured values of ρ is also significant in part B. In conclusion, this experiment highlights the inevitability of uncertainties and errors in measurements, as well as the importance of precise analysis methods, and emphasizes the need for meticulous error analysis in physical experiments.