SCHOOL OF PHYSICS
UNIVERSITI SAINS MALAYSIA

ZCT191/192 PHYSICS PRACTICAL I/II

1EM5 ALTERNATING CURRENT RESONANCE

Lab Manual

OBJECTIVES

1. *To determine the resonant frequency of a series circuit;*
2. *To determine the Q factor and resistance of a series circuit;*
3. *To study the phase difference between the current and applied voltage for a series circuit; and*
4. *To determine the resonant frequency of a parallel resonant circuit.*

THEORY

Introduction

The purpose of this experiment is to investigate the characteristics of resonant circuits containing the three basic components in electronics, which are the resistor (R), inductor (L), and capacitor (C). Whether simple or complicated, most circuits consist of these three components.

The resistance of a pure resistor does not vary with frequency; however inductors and capacitors possess characteristics that depend on frequency, which results in *phase shifts* between the applied voltage and current. When *alternating current* (AC) flows through a resistor, the applied voltage and current are in phase. However, the applied voltage leads the current by ¼ of a cycle (90°) for an inductor, while the current leads the applied voltage by ¼ of a cycle for a capacitor. The presence of R, L and C introduces an *impedance* within a circuit.

RLC Circuits

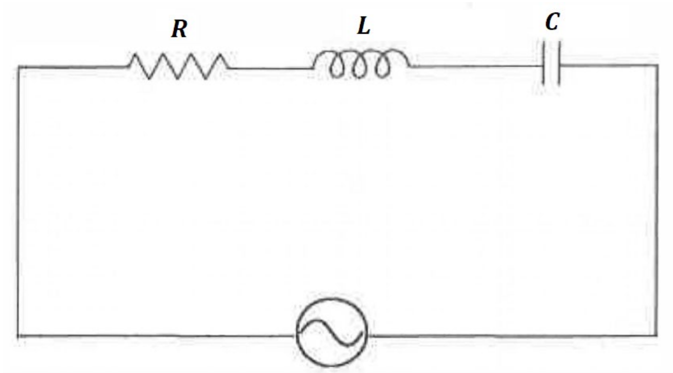

Figure 1: An oscillator connected to an RLC circuit in series.

Consider the RLC series circuit in **Figure 1**. When the circuit is supplied with an AC sinusoidal voltage source, the resultant current is an applied voltage as a function of frequency. Using complex notation (see **APPENDIX**), the *impedance* within a circuit is given by

$$Z = R + i(X_L - X_C), \tag{1}$$

where R is the *resistance*, $X_L = 2\pi L$ the *inductive reactance*, and $X_C = 1/2\pi C$ the *capacitive reactance*. Thus, the value of *effective current* (I) is

$$I = \frac{V}{Z} = \frac{V}{R + i(X_L - X_C)}. \tag{2}$$

The absolute value of I can be written as

$$|I| = \frac{|V|}{\sqrt{R^2 + (X_L - X_C)^2}}, \tag{3}$$

or in frequency terms,

$$|I| = \frac{|V|}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}. \tag{4}$$

Equation (4) shows that when f approaches zero, $|I|$ also approaches zero; when f approaches infinity, $|I|$ approaches zero as well. It is thus clear that $|I|$ possess a maximum value at a frequency f_0, and this maximum value occurs when

$$2\pi f_0 L - \frac{1}{2\pi f_0 C} = 0, \tag{5}$$

in which we get the *critical frequency* f_0 as

$$f_0 = \frac{1}{2\pi\sqrt{LC}}. \tag{6}$$

At this frequency, the current I will possess a maximum value of

$$|I|_0 = \frac{|V|}{R}, \tag{7}$$

whereas any frequency f other than f_0 will result in a current $|I| < |V|/R$. Qualitatively, the frequency response in such a series circuit is shown in **Figure 2**.

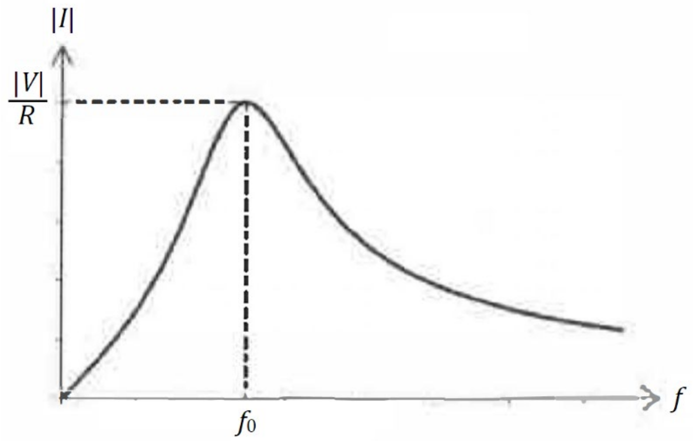

Figure 2: Current versus frequency in a series circuit.

Equation (2) can be used to obtain the *phase angle* between the applied voltage and the current. Using the properties of complex conjugates, we get

$$\begin{aligned} I &= \frac{V}{R + i(X_L - X_C)} \frac{R - i(X_L - X_C)}{R - i(X_L - X_C)} \\ &= \frac{VR - iV(X_L - X_C)}{1 + \left(\frac{X_L - X_C}{R}\right)^2}. \end{aligned} \tag{8}$$

From this expression, we can see that the phase angle θ is

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right). \tag{9}$$

From equations (6) and (9), when $f = f_0$ and $X_L = X_C$, we get $\theta = \tan^{-1}(0)$ or $\theta = 0$, i.e. the applied voltage and resultant current are in the same phase at frequency f_0. This known as the *resonance* condition, where the net impedance is purely resistive. It is found that at resonance, the current is maximum and the impedance is minimum.

The Q Factor

At resonance, $X_L = X_C$. Using equation (6), we get

$$X_L = 2\pi f_0 L = 2\pi\left(\frac{1}{2\pi\sqrt{LC}}\right)L = \sqrt{\frac{L}{C}} = X_0. \tag{10}$$

Thus, it can be shown that at resonance,

$$2\pi L = \frac{X_0}{f_0}, \qquad \frac{1}{2\pi C} = f_0 X_0. \tag{11}$$

Inserting equation (11) into equation (4), we get

$$|I| = \frac{|V|}{R}\frac{1}{\sqrt{1+\left(\frac{X_0}{R}\right)^2\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2}} = \frac{|V|}{R}\frac{1}{\sqrt{1+Q^2\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2}}, \tag{12}$$

where $Q = X_0/R$ is known as the *Q factor* of the circuit. For practical purposes, we can rewrite equation (12) as

$$\frac{|I|}{|I|_0} = \frac{1}{\sqrt{1+Q^2\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2}}. \tag{13}$$

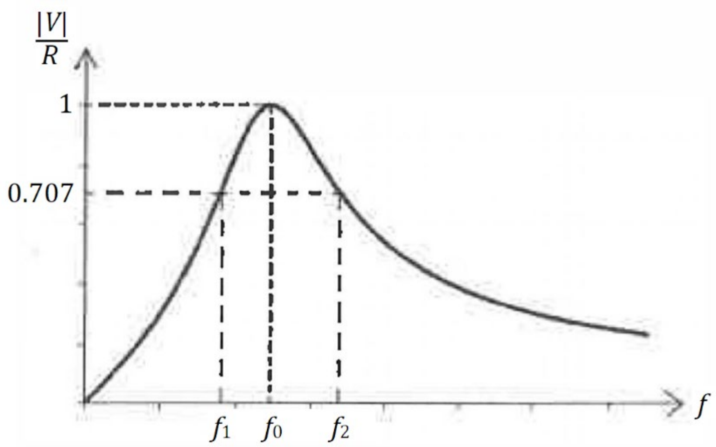

Figure 3: Graph of $|I|/|I_0|$ versus frequency f.

The Q factor can be obtained through graphical methods, as shown in **Figure 3**. From the experiment, the Q factor can be determined from the relationship

$$Q = \frac{f_0}{f_2 - f_1}, \tag{14}$$

where f_1 and f_2 are known as *half-power frequencies*, when the power $P = P_0/2$, or when $|I| = |I_0|/\sqrt{2} = 0.707|I_0|$.

Parallel Resonant Circuits

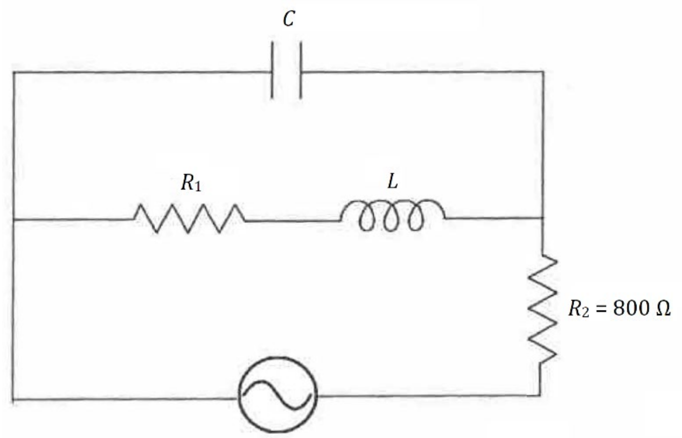

Figure 4: A parallel resonant circuit.

When resonance occurs in a *parallel resonant circuit*, the impedance is maximum but the current in the circuit becomes minimum. The resonant frequency (f_0) for a parallel circuit as shown in **Figure 4** is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}. \tag{15}$$

PROCEDURE

Part A: Resonant Frequency of a Series Circuit

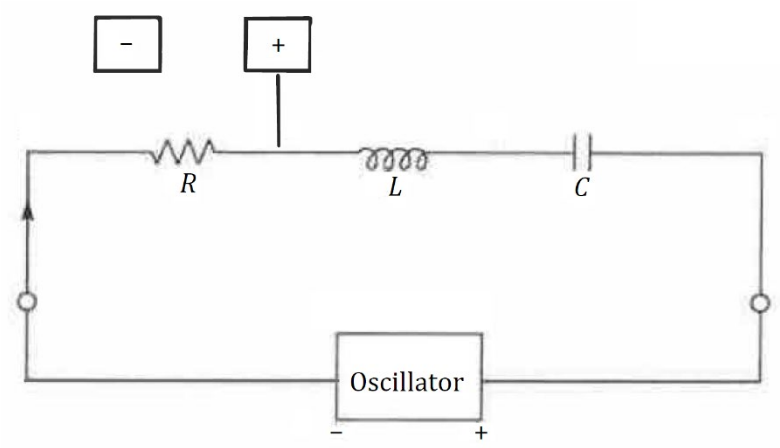

Figure 5: Experimental setup for Parts A and B.

Measurement

1. Connect the circuit as shown in **Figure 5**, using resistors, inductors and capacitors with $R = 10$ kΩ, $L = 0.5$ H and $C = 2100$ pF, respectively.
2. Connect the cathode ray oscilloscope (CRO) across R.
3. Plot the values of $V_R/V_{R,\max}$ against the frequency f of the oscillator, where $V_{R,\max}$ is the maximum voltage value across R.
4. Determine the resonant frequency f_0 from the plot.
5. Compare this value of f_0 obtained with the value obtained from theory.

Part B: The Q Factor

Measurement

1. Connect the circuit as shown in **Figure 5** using the same values of L and C as in **Part A**, but replace R with values of 12 kΩ and 8 kΩ.
2. Plot $V_R/V_{R,\max}$ against the corresponding frequencies f and determine the values of Q for the two values of R used.
3. Compare the values of Q you obtained in the experiment with the values obtained from theory.
4. Show how Q varies with R graphically.

Part C: The Phase Difference between Current and Applied Voltage

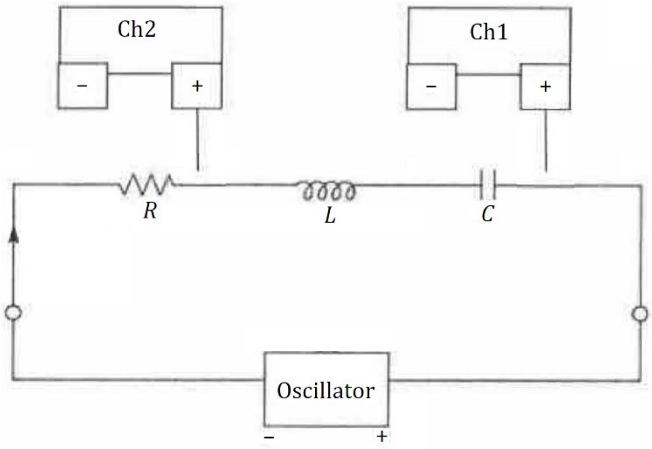

Figure 6: Experimental setup for Part C.

Measurement

1. Connect the circuit as shown in **Figure 6** using the same values of L and C as in **Part A**, but replace R with a value of 800 Ω.
2. Using the CRO with an oscillator frequency f given by your lecturer-in-charge, measure the voltage across R, L and C using the X-plate.
3. Measure the voltage across R using the Y-plate.
4. Draw the Lissajous curve obtained, and determine the phase angle for the circuit.
5. Compare the phase angle obtained from the experiment with the value obtained from theory.
6. Starting with $f = 0$ Hz, increase f until it reaches maximum (∞), and discuss what happens there.

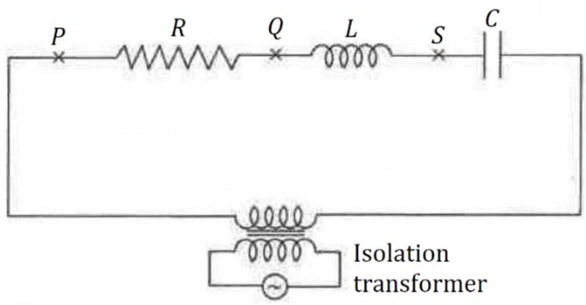

Figure 7: A circuit with an isolation transformer.

7. Connect the isolation transformer (**Figure 7**) to the signal source in order to isolate the ground of the supply with the ground of the oscilloscope.
8. Set the signal frequency to 3.8 kHz, and use the value of $R = 10$ kΩ.
9. Connect one end of the oscilloscope earth to point Q.
10. Connect the oscilloscope Y (**Channel 2**) to P and the oscilloscope X (**Channel 1**) to point S (between L and C).
11. Obtain the trace and sketch it in your report.
12. Set the display to XY, and obtain the phase between the two signals in the Lissajous curve.
13. Connect the scope ground to P, **Channel 2** to Q and **Channel 1** to S.

14. From the Lissajous curve shown, obtain the phase angle, and compare this to the value obtained from theory.
15. Now interchange L and C, then repeat **Steps 9** to **14**.
16. From the phase angles obtained in **Steps 13** and **15**, determine which phase leads and which phase lags the signal.

Part D: Resonant Frequency of a Parallel Circuit

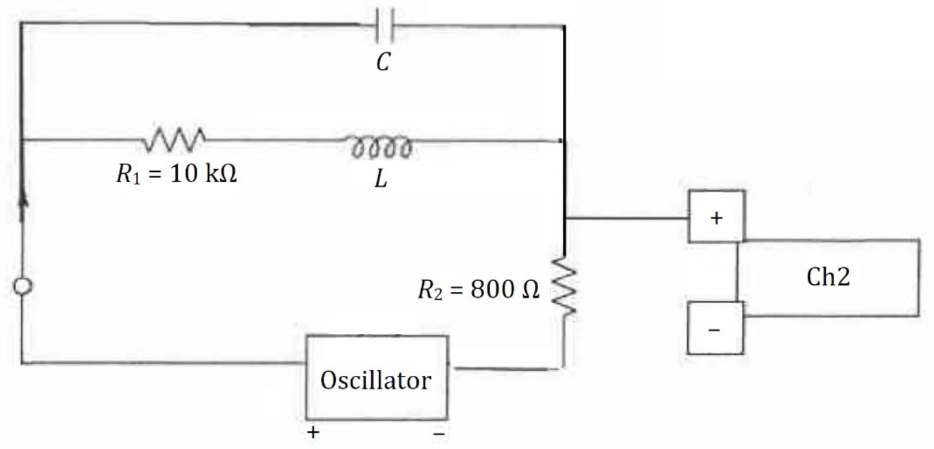

Figure 8: Experimental setup for Part D.

Measurement

1. Connect the circuit as shown in **Figure 8**.
2. Connect the Y-plate of the CRT across R_2, and measure its voltage.
3. Plot V_{R_2} versus the oscillator frequency, and determine the resonant frequency of the circuit.
4. Compare the value of f_0 obtained from experiment with the value obtained from theory.

REFERENCES

1. Halliday, D., Resnick, R. & Walker, J. (2020) *Principles of Physics (11th ed.)*. Wiley.
2. Korneff, T. (1966). *Introduction to Electronics (1st ed.)*. Academic Press.
3. Brophy, J. J. (1990). *Basic Electronics for Scientists*. McGraw-Hill.
4. Mitchell, F. H. (1969). *Essentials of Electronics*. Addison-Wesley.

ACKNOWLEDGEMENT

The original creator of this lab manual is unknown. This manual was revised and standardised by *Dr. John Soo Yue Han* in 2021.

APPENDIX

The Complex Plane

The *complex plane* is widely used in studying AC circuits, where vectors are pictured in components along a *real axis* and an *imaginary axis*. Components along the imaginary axis is multiplied with the complex number i where $i^2 = -1$. For example, a current $I = 4 + 3i$ can be visualised in **Figure 9** below.

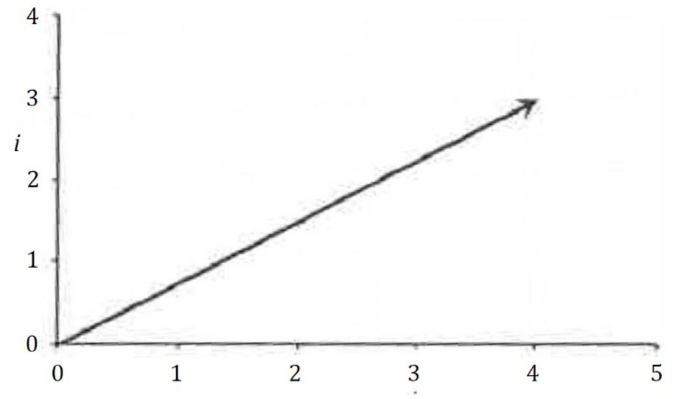

Figure 9: The vector $I = 4 + 3i$ in the complex plane.

A *complex conjugate* of a complex quantity must be defined to obtain its *absolute value*. The complex conjugate of $I = 4 + 3i$ is $I^* = 4 - 3i$, and the absolute value of I is therefore

$$\begin{aligned} |I| &= \sqrt{II^*} \\ &= \sqrt{(4+3j)(4-3j)} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5. \end{aligned} \tag{16}$$

The angle θ between I and the real axis is defined as $\tan\theta = \mathrm{Im}(I)/\mathrm{Re}(I)$, where Im and Re refers to the imaginary and real components of I. Thus, the angle θ for the same number $4 + 3i$ above is

$$\tan\theta = \frac{\mathrm{Im}(I)}{\mathrm{Re}(I)} = \frac{3}{4} \tag{17}$$

Complex Impedance

In the complex plane, resistance is pictured on the real axis, inductive reactance is pictured in the $+i$ direction on the imaginary axis, while the capacitive reactance is pictured in the $-i$ direction on the imaginary axis. For example, we consider the current I in the phasor circuit shown in **Figure 10** below.

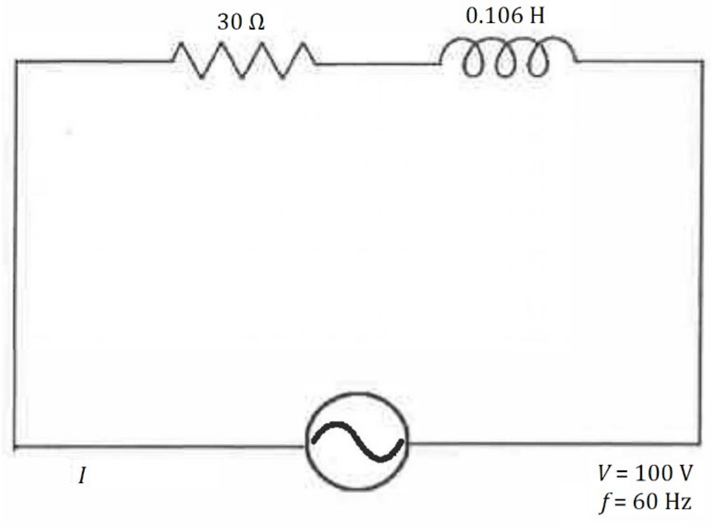

Figure 10: Example of a phasor circuit.

The total impedance of the circuit above is $Z = R + iX_L = 30 + i2\pi fL = 30 + 39.96i$. The complex current is therefore

$$I = \frac{V}{Z} = \frac{100}{30 + 39.96i} = \frac{100}{30 + 39.96i}\frac{30 - 39.96i}{30 - 39.96i} = 1.2 - 1.6i, \tag{18}$$

and the absolute value of the current is therefore $|I| = \sqrt{II^*} = 2$ A.

To the draw the phasor diagram, we need to calculate the values of V_R and V_L,

$$V_\mathrm{R} = IR = 36 - 48i, \tag{19}$$
$$V_\mathrm{L} = I(iX_\mathrm{L}) = (1.2 - 1.6i)(40i) = 64 + 48i. \tag{20}$$

The values of V_R and V_L are visualised in the phasor diagram as shown in **Figure 11**.

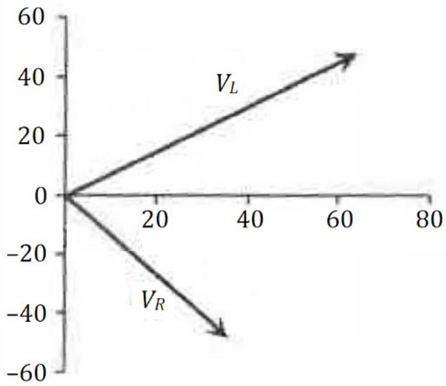

Figure 11: Representation of V_R and V_L on the complex plane.

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