**North Dakota**

**Mathematics**

**Content Standards**

**K-12**

**July 2023**

North Dakota Department of Public Instruction

Kirsten Baesler, State Superintendent

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**Document Revision Log**

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| --- | --- | --- |
| **Date Revised** | **Description of Revision** | **Page(s)** |
| 5/22/24 | Added Clarification:  **3.NO.NF.4** Within this standard, learners are going beyond 1. | 21 |
| 5/22/24 | Corrected Standard Description:  **4.DPS.D.2** Generate data and create line plots to display a data set of  fractions of a unit (½, ¼, ⅛). Solve problems involving addition and  subtraction of fractions by using information presented in line plots. | 27, 102, 103 |
| 5/22/24 | Corrected Standard Description:  **5.NO.NBT.1** Understand that in a multi-digit number, a digit in one place  represents ten times as much as it represents in the place to its right and  1/10 of what it represents in the place to its left. | 28, 76, 81 |
| 5/22/24 | Corrected Standard Description:  **5.DPS.D.1** Generate data and create line plots to display a data set of  fractions of a unit (½, ¼, ⅛). Use grade-level operations for fractions to  solve problems involving information presented in line plots. | 31, 102, 103 |
| 5/22/24 | Moved Clarification as it was in the incorrect place:  From 6.AR.RP.3 **-** To **6.AR.RP.5**. | 33 |
| 6/27/24 | Added clarification “Ordering numbers may be included for number  comparisons” to standards 1.NO.NBT.2, 2.NO.NBT.2, 3.NO.NBT.2,  3.NO.NF.5, 4.NO.NBT.2, 5.NO.NBT.2. | 14, 17, 20, 21,  24, 29 |
| 6/27/24 | Added clarification “Properties of two-dimensional shapes would include  identifying the number of sides and angles found in polygons and  triangles” to standard 4.GM.G.2. | 26 |
| 6/27/24 | Added clarification “Properties of two-dimensional shapes and the names  of specific shapes would include identifying the number of sides, angles,  and lines of symmetry found in polygons and triangles (equilateral,  isosceles, scalene)” to standard 5.GM.G.1. | 31 |

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**SUPERINTENDENT’S FOREWORD**

As the North Dakota State Superintendent, it is my honor to introduce the new North Dakota K-12  mathematics content standards. These standards represent a collective effort by North Dakota teachers and  content experts to provide our learners with the necessary mathematical knowledge and skills to tackle the  challenges of our ever-changing world.

Today, problem-solving and creative thinking are highly valued skills. Our students must be equipped with  algebraic and geometric reasoning abilities, allowing them to navigate novel situations and find innovative  solutions. However, these skills can only be fully developed on a strong foundation of numeracy. Therefore, it  is crucial that our learners possess robust numeracy skills, which serve as the bedrock for developing their  reasoning abilities.

The new mathematics content standards have been carefully crafted to embed the foundational skills  necessary for students to develop their geometric and algebraic reasoning. This progression of skills will  guide students throughout their K-12 academic journey, building the necessary background knowledge to  become skilled and innovative problem solvers.

While the North Dakota content standards serve as a statewide reference point for teaching mathematics, we  encourage local school districts to utilize them as a guide in developing their own customized curricula. We  recognize the importance of tailoring education to the unique needs and contexts of each community while  ensuring a solid alignment with the state standards.

The process of developing these standards involved collaboration between the North Dakota Department of  Public Instruction, North Dakota State University, and a team of dedicated North Dakota educators. Starting  in July 2022 and continuing through June 2023, this collective effort produced drafts that were made  available for public comment. We received invaluable feedback from teachers, administrators, parents, and  the community, which greatly contributed to the refinement of the standards.

Drawing upon previous North Dakota standards, standards from other states, and extensive research on mathematics content and skill development, the writing committee identified the foundational knowledge and skills that learners need to solve a variety of mathematical problems. Notably, in this 2023 version, the high  school standards are divided into two grade spans rather than broadly categorized. This change allows for more specific identification of assessed standards in grades 9-10 and provides districts with the opportunity  to align standards within their courses.

I would like to express my gratitude to the review committee, comprised of interested stakeholders from the  general public, who provided another layer of scrutiny and feedback. Their dedication and insights greatly  enhanced the quality and relevance of the standards. I am truly thankful to those who devoted their time and  talent to reviewing the draft standards and providing recommendations to the writing committee.

The primary architects of these standards are our North Dakota educators. Their expertise and commitment  to our students are unmatched. They have exemplified the best in North Dakota education by openly,  transparently, and collaboratively working on this document. Each member of the writing committee deserves  our heartfelt appreciation for their extensive research, rigorous analysis, and thoughtful deliberations. It is  through their hard work that these standards are now ready to be implemented in classrooms across our  great state.

I am confident that the adoption of these new mathematics content standards will empower our students,  equip them with the necessary mathematical skills, and foster their critical thinking abilities. Together, we can  prepare the next generation of problem solvers and innovators, ensuring a bright future for North Dakota and  beyond.

Sincerely,

Kirsten Baesler

North Dakota State Superintendent

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**INTRODUCTION**

Educational standards are statements designed to describe a clear path for students to gain the proficiency  required to learn increasingly complex material. The standards provide educational guidelines but do not  prescribe teaching practices, curricula, or assessment methods. The North Dakota Mathematics Content  Standards provide a rigorous and developmentally appropriate framework for instruction to increase student  achievement and provide students with a quality, equitable education. These standards help develop critical  and innovative thinking and problem-solving skills students will apply when meeting future postsecondary  and workforce demands.

The development of these new mathematics standards was a multi-phase process. State Superintendent of  Public Instruction Kirsten Baesler established a statewide committee through an application process that  included educators and higher education faculty. Over five two-day sessions, the committee reviewed the  existing standards, drafted new standards; and revised their work based upon input from two rounds of  public comments and two reviews by a content standards review committee representing business  interests, parents, and the public. The committee began its work in July 2022 and completed the  development of new standards in May 2023.

The 2023 Mathematics Content Standards identify math attributes that develop mathematical thought  processes woven within the content. The math attributes describe processes within mathematical concepts.  The math attributes summarize the mathematical practices found in the 2017 North Dakota Mathematics  Content Standards and are aligned with the 2022 North Dakota Learning Continuum, which identified essential knowledge, skills, and dispositions learners need to demonstrate throughout their lives.

The content standards identify essential skills and concepts across four categories which focus on  developing a conceptual understanding of math concepts as a learner progresses from learning  foundational arithmetic skills to applying those skills in algebra and geometry. The new standards require  procedural skills and fluency in using mathematical skills and concepts in various authentic problem  situations. The North Dakota Mathematics Content Standards identify skills in which proficiency is needed.

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**MATH ATTRIBUTES**

The math attributes contained in these standards summarize the mathematical practices found in the 2017  North Dakota Mathematics Content Standards and align with the 2022 North Dakota Learning Continuum.  These attributes will help learners solve authentic problems while connecting concepts, providing supporting  evidence, explaining the reasoning and efficiency of strategies used, and proving the accuracy of solutions.  The three attributes identified will be used by learners throughout their education and future careers. The  attributes are arranged by grade span.

A chart showing the progression of the Math Attributes is shown below:

|  |  |  |
| --- | --- | --- |
| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when solving novel situations.* | *Create connections within and  across concepts, using supporting  evidence to interpret how they  originate, extend, and relate to  other learning, ideas, and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and conclude  through exploration, generalization,  and validation.* |
| **K-2.MA.P** Learners can identify  and use strategies to problem solve situations and determine an appropriate solution. | **K-2.MA.C** Learners can make  connections and demonstrate  relationships using words,  pictures, or symbols. | **K-2.MA.R** Learners can use prior  knowledge and experiences to  explain their thinking. |
| **3-5.MA.P** Learners can develop  and carry out a logical plan to  problem-solve situations, reflect on  the reasonableness of solutions,  and explore alternate strategies with guidance. | **3-5.MA.C** Learners can make  connections and summarize  related ideas using supporting  evidence. | **3-5.MA.R** Learners can reason  logically based on experience and  knowledge, citing evidence to  support their reasoning and  conclusions. |
| **6-8.MA.P** Learners can analyze  information and formulate a  flexible, systematic plan to  problem-solve authentic situations  and reflect on the reasonableness  of the solution, making revisions when necessary. | **6-8.MA.C** Learners can create  connections within and across  concepts and provide examples of  how they relate to other learning  and ideas using supporting  evidence | **6-8.MA.R** Learners can reason  logically, citing evidence to  evaluate and explain what they  see, think, and conclude through  exploration and justification. |
| **9-12.MA.P** Learners can analyze,  execute, critique, and adapt  approaches and solutions when  problem-solving in novel situations. | **9-12.MA.C** Learners can create  connections within and across  concepts, using supporting  evidence to interpret how they  originate, extend, and relate to other learning, ideas, and life  experiences. | **9-12.MA.R** Learners can reason  logically, citing evidence to critique  and explain what they see, think,  and conclude through exploration,  generalization, and validation. |
| **Lifelong MA.P** Learners can  integrate their cumulative  knowledge and life experiences to  discern and prioritize information in  authentic situations, consider and  apply alternative methods of  resolution, and evaluate the  relevance, efficacy, and accuracy  of solutions. | **Lifelong MA.C** Learners can apply  connections and develop  generalizations within and across  concepts to execute effective  decision-making or generate new  ideas. | **Lifelong MA.R** Learners can  reason logically to discern the  validity of information and  synthesize it to formulate,  investigate, and critique claims and  evidence. |

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**HOW TO READ THIS DOCUMENT**

The content standards serve as a guide for districts to use as they develop curricula and select instructional  materials. These standards do not define how teachers teach.

The document is organized by category, sub-category, and standard and includes four categories defined  below:

|  |  |
| --- | --- |
| **Category** | **Definition** |
| **Number and Operations** | Learners will develop a foundational understanding of the number  system, operations, and computational fluency to create connections and solve problems within and across disciplines. |
| **Algebraic Reasoning** | Learners will look for, generate, and make sense of patterns,  relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel  situations. |
| **Geometry and Measurement** | Learners will use visualization, spatial reasoning, geometric  modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments. |
| **Data, Probability, and Statistics** | Learners will ask and answer questions by collecting, organizing,  and displaying relevant data, drawing inferences and conclusions,  making predictions, and understanding and applying basic concepts of probability. |

Each category progresses from kindergarten through grade 12, with the high school level divided into two  grade-span groups. Each category is split into sub-categories which are made up of the standards. The  elementary level focuses on building arithmetic skills and concepts, the middle level moves toward applying  those skills in pre-algebraic concepts, and the high school level refines and hones the skills needed to  develop the algebraic and geometric strategies to solve problems in the post-high school world.

**Kindergarten Grade**

**Level Category**

**Algebraic Reasoning (AR)**

*Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols  to represent mathematical models while adopting approaches and solutions in novel situations.***Operations and Algebraic Thinking (OA)**

*Learners will analyze patterns and relationships to generate and interpret numerical expressions. ***Standard Clarification**

**K.AR.OA.1** Automatically add and subtract within 5. **Standard**

Develop a flexible understanding of both vertical  and horizontal orientation. 

**Number Sub category**

**Coding: K.AR.OA.1**

**K – Grade Level or Grade Span**

**AR – Category**

**OA – Sub-category**

**1 – Standard Number**

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**NORTH DAKOTA  MATHEMATICS K-12 STANDARDS**

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**KINDERGARTEN**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring the skills necessary  for advancement through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and conclude  through exploration, generalization,  and validation.* |
| **K-2.MA.P** Learners can identify  and use strategies to problem solve situations and determine an appropriate solution. | **K-2.MA.C** Learners can make  connections and demonstrate relationships using words,  pictures, or symbols. | **K-2.MA.R** Learners can use prior  knowledge and experiences to  explain their thinking. |

(2022 North Dakota Learning Continuum)

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Counting and Cardinality (CC)**  *Learners will understand the relationship between numerical symbols, names, quantities, and counting  sequences.* | |
| **Standards** | **Clarification** |
| **K.NO.CC.1** Count verbally in sequential order by  ones and tens to 100, making accurate decuple  transitions (e.g., 89 to 90).  Count verbally forward from any given number within  100. |  |
| **K.NO.CC.2** Count backward from 20 by ones and  from a given number within 10. |  |
| **K.NO.CC.3** Identify and write any given numeral  within 20. | In a progression, students may identify and write  numerals sequentially prior to being able to identify and write any numeral within 20. |
| **K.NO.CC.4** Recognize and verbally label  arrangements, without counting, for briefly shown  collections up to 10 (e.g., “I saw 5.” How do you  know?” “I saw 3 and 2, that is 5.”). | Recognize without counting. Use scattered  arrangements for combinations up to 7. Structured  arrangements such as ten frames (utilizing 5+ and pair-wise patterns) can be utilized for combinations  up to 10. |
| **K.NO.CC.5** Count and tell how many objects up to  20 are in an arranged pattern or up to 10 objects in a  scattered configuration. Represent a quantity of up to 20 with a numeral. |  |
| **Base Ten (NBT)**  *Learners will understand the place value structure of the base-ten number system and represent, compare,  and perform operations with multi-digit whole numbers and decimals.* | |
| **Standards** | **Clarification** |
| **K.NO.NBT.1** Compose and decompose numbers  from 11 to 19 using a group of ten ones and some more ones using a model, drawing, or equation. |  |
| **K.NO.NBT.2** Compare two numbers between 1 and  20 using words greater than, less than, or equal to. | In a progression, students will use groups of objects for comparison prior to the end-of-year standard of  comparing numerals within 20. Ordering numbers  may be included for number clarifications. |

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| **Fractions (NF)**  *Learners will understand fractions and equivalency to represent, compare, and perform operations of  fractions and decimals.* | |
| *NOTE: Standards begin in first grade.* | |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols  to represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **Standards** | **Clarification** |
| **K.AR.OA.1** Automatically add and subtract within 5. | Develop a flexible understanding of both vertical and  horizontal orientation.  See Appendix B for recommended automatically. |
| **K.AR.OA.2** For any number from 1 to 9, find the  number that makes 10 when added to the given  number, sharing the answer with a model, drawing, or equation. |  |
| **K.AR.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way using verbal  explanations, objects, or drawings. |  |
| **K.AR.OA.4** Solve authentic word problems with  addition by putting together or adding to within 10. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **K.AR.OA.5** Solve authentic word problems with  subtraction by taking apart or taking from within 10. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **K.AR.OA.6** Recognize, duplicate, complete, and  extend repeating patterns in a variety of contexts  (e.g., shape, color, size, objects, sounds,  movements). | Use AB-ab, abc, aabb type patterns. |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Geometry (G)**  *Learners will compose and classify figures and shapes based on attributes and properties and represent and  solve problems using a coordinate plane.* | |
| **Standards** | **Clarification** |
| **K.GM.G.1** Name shapes and identify them as two dimensional (squares, circles, triangles, rectangles)  regardless of their orientations or overall sizes. |  |
| **K.GM.G.2** Name shapes and identify them as three dimensional (cubes and spheres) regardless of their  orientations or overall sizes. |  |
| **K.GM.G.3** Compare and classify two-dimensional  shapes to describe their similarities, differences, and attributes (squares, circles, triangles, rectangles). |  |
| **K.GM.G.4** Compose a geometric shape by  combining two or more simple shapes. |  |

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| **Measurement (M)**  *Learners will represent and calculate measurement data, including time, money, and geometric  measurement, and convert like measurement units within a given system.* | |
| **Standards** | **Clarification** |
| **K.GM.M.1** Compare and order two objects with a  common measurable attribute. | In a progression, students will describe and  understand common measurable attributes (e.g., length and weight) for ordering and comparisons. |
| **K.GM.M.2** Tell time related to daily life (today,  yesterday, tomorrow, morning, afternoon, night). | This concept is foundational learning for time. Days of the week and concepts of a.m. and p.m. are  included. |
| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.* | |
| **Data (D)**  *Learners will represent and interpret data.* | |
| **Standard** | **Clarification** |
| **K.DPS.D.1** Sort and classify objects (up to 10)  based on attributes and explain the reasoning used. |  |

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**FIRST GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using supporting  evidence to interpret how they  originate, extend, and relate to other learning, ideas, and life  experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization, and validation.* |
| **K-2.MA.P** Learners can identify  and use strategies to problem solve situations and determine an appropriate solution. | **K-2.MA.C** Learners can make  connections and demonstrate  relationships using words, pictures,  or symbols. | **K-2.MA.R** Learners can use prior  knowledge and experiences to  explain their thinking. |

(2022 North Dakota Learning Continuum)

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Counting and Cardinality (CC)**  *Learners will understand the relationship between numerical symbols, names, quantities, and counting  sequences.* | |
| **Standards** | **Clarification** |
| **1.NO.CC.1** Count forward by ones and tens from  any given point within 120. | Students practice their understanding of numbers in  the range of 120 by reading, writing, and verbally  counting. |
| **1.NO.CC.2** Count backward by ones and tens from  a given number within 120. | Students practice their understanding of numbers in  the range of 120 by reading, writing, and verbally counting. |
| **1.NO.CC.3** Represent several objects with a written  numeral up to 120. | In a progression, students may write their numeral patterns sequentially prior to being able to represent  any numeral or quantity in the range of 120. |
| **1.NO.CC.4** Recognize and verbally label  arrangements, without counting, for briefly shown  collections up to 20 (e.g., “I saw 16.” How do you know?” “I saw 10 and 6, that is 16.”). | Recognize without counting. Structured  arrangements such as twenty frames (utilizing 10+  and pair wise patterns) can be utilized for  combinations up to 20. |
| **1.NO.CC.5** Skip count forward and backward by 5s  and 10s from multiples and recognize the patterns of  up to 10 skip counts. | **Assessment Boundary**: Start from any multiple and  move forward or backward by 5 or 10 (e.g., 15, 10, 5, etc.). Range of 5-50 and 10-100. |
| **Base Ten (NBT)**  *Learners will understand the place value structure of the base-ten number system and represent, compare,  and perform operations with multi-digit whole numbers and decimals.* | |
| **Standards** | **Clarification** |
| **1.NO.NBT.1** Demonstrate that the two digits of a two-digit number represent a composition of some  tens and some ones. | Students may use concrete models, drawings, or written numerals to show a place value  understanding of tens and ones. |
| **1.NO.NBT.2** Compare two two-digit numbers using  symbols >, <, and =. Justify comparisons based on the value of tens and ones. | Ordering numbers may be included for number  comparisons. |

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| **1.NO.NBT.3** Add within 100 using a two-digit  number and a one-digit number. Use concrete models, drawings, and strategies that reflect an  understanding of place value. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **1.NO.NBT.4** Subtract multiples of 10 within 100  using concrete models, drawings, and strategies that reflect an understanding of place value. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **1.NO.NBT.5** Mentally add or subtract 10 to or from a given two-digit number and explain the reasoning  used. |  |
| **Fractions (NF)**  *Learners will understand fractions and equivalency to represent, compare, and perform operations of  fractions and decimals.* | |
| **Standard** | **Clarification** |
| **1.NO.NF.1** Partition circles and rectangles into two and four equal shares using the language halves  and fourths. |  |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **Standards** | **Clarification** |
| **1.AR.OA.1** Automatically add and subtract within 10. | Develop a flexible understanding of both vertical and  horizontal orientation.  See Appendix B for recommended automaticity. |
| **1.AR.OA.2** For any number from 1 to 19, find the  number that makes 20 when added to the given  number, sharing the answer with a model, drawing, or equation. | Students use composition and decomposition  strategies for combinations of 20 (e.g., “I have 17  and I need 3 to make 20” or “Tell me two numbers that go together to make 20.”). |
| **1.AR.OA.3** Decompose numbers less than or equal  to 20 into pairs in more than one way. |  |
| **1.AR.OA.4** Solve authentic word problems with  addition, including three numbers and unknowns,  within 20. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **1.AR.OA.5** Solve authentic word problems with  subtraction, including unknowns, within 20. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **1.AR.OA.6** Distinguish and use the +, -, and =  symbols accurately in an equation. | In a progression, students learn the meaning of an  equal sign, including if equations are true and false, solving on both sides if needed. |
| **1.AR.OA.7** Identify, create, complete, and extend patterns that are repeating, increasing, and  decreasing in a variety of contexts. | Example: shape, color, size, objects, and/or  numerical patterns. |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Geometry (G)**  *Learners will compose and classify figures and shapes based on attributes and properties and represent and  solve problems using a coordinate plane.* | |
| **Standards** | **Clarification** |
| **1.GM.G.1** Name shapes and identify them as two dimensional (trapezoids, rhombuses, pentagons,  hexagons, octagons). | **Assessment Boundary**: Includes shapes from  K.GM.G.1. |

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| **1.GM.G.2** Name and identify solids as three dimensional (cylinders, cones, triangular prisms, and  rectangular prisms). | **Assessment Boundary**: Includes shapes from  K.GM.G.2. |
| **1.GM.G.3** Determine geometric attributes of two dimensional and three-dimensional shapes  (squares, circles, triangles, rectangles, trapezoids,  rhombuses, pentagons, hexagons, octagons, cubes,  spheres, cylinders, cones, triangular prisms, and rectangular prisms). | **Assessment Boundary**: Includes shapes from  K.GM.G.3. |
| **1.GM.G.4** Compose a geometric shape or solid by  combining multiple two-dimensional shapes and/or  three-dimensional solids (squares, circles, triangles,  rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders,  cones, triangular prisms, and rectangular prisms). |  |
| **Measurement (M)**  *Learners will represent and calculate measurement data, including time, money, and geometric  measurement, and convert like measurement units within a given system.* | |
| **Standards** | **Clarification** |
| **1.GM.M.1** Measure the length of an object as a whole number of same-size, non-standard units from  end to end. | Non-standard units may include paperclips, cubes,  popsicle sticks, etc. |
| **1.GM.M.2** Compare the lengths of three objects  using a common measurable attribute. |  |
| **1.GM.M.3** Tell and write time to the hour and half hour (including o-clock and half past) using analog  and digital clocks. |  |
| **1.GM.M.4** Identify and tell the value of a dollar bill,  quarter, dime, nickel, and penny. |  |
| **1.GM.M.5** Count collections of coins (pennies,  nickels, and dimes) relating to counting patterns by  1s, 5s, and 10s up to one dollar. | This standard includes a mixture of coins (pennies,  nickels, and dimes) up to one dollar. Students may  start by counting one coin up to one dollar but are  expected to apply their counting patterns of 1s, 5s, and 10s. |
| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data (D)**  *Learners will represent and interpret data.* | |
| **Standards** | **Clarification** |
| **1.DPS.D.1** Collect, organize and represent data with  up to three categories using picture and bar graphs. |  |
| **1.DPS.D.2** Analyze data by answering descriptive  questions. | Ask and answer questions about the total number of  data points, how many in each category, and how  many more or less are in one category than in another. |

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**SECOND GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using supporting  evidence to interpret how they  originate, extend, and relate to other learning, ideas, and life  experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization, and validation.* |
| **K-2.MA.P** Learners can identify  and use strategies to problem solve situations and determine an  appropriate solution. | **K-2.MA.C** Learners can make  connections and demonstrate  relationships using words, pictures,  or symbols. | **K-2.MA.R** Learners can use prior  knowledge and experiences to  explain their thinking. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Counting and Cardinality (CC)**  *Learners will understand the relationship between numerical symbols, names, quantities, and counting  sequences.* | |
| **Standards** | **Clarification** |
| **2.NO.CC.1** Count forward from any given number  within 1000. |  |
| **2.NO.CC.2** Count backward from any given number  within 1000. |  |
| **2.NO.CC.3** Read and write numbers up to 1000  using standard, word, and expanded forms. | Spelling is not assessed. |
| **2.NO.CC.4** Skip count forward and backward by 2s  and 100s and recognize the patterns of skip counts. | **Assessment Boundary**: Start from any multiple and  move forward or backward by 2s or 100 (e.g., 20, 18, 16, etc.). Range 2-20 and 100-1,000. |
| **Base Ten (NBT)**  *Learners will understand the place value structure of the base-ten number system and represent, compare,  and perform operations with multi-digit whole numbers and decimals.* | |
| **Standards** | **Clarification** |
| **2.NO.NBT.1** Understand that the three digits of a three-digit number represent a composition of some  hundreds, some tens, and some ones. |  |
| **2.NO.NBT.2** Compare two three-digit numbers using  symbols >, <, and =. Justify comparisons based on the value of hundreds, tens, and ones. | Ordering numbers may be included for number  comparisons. |
| **2.NO.NBT.3** Add within 100 using place value  strategies and/or the relationship between addition  and subtraction. | Develop a flexible understanding of both vertical and  horizontal orientation. The representation of whole number sums within 100 on a number line diagram  may be included. |
| **2.NO.NBT.4** Subtract within 100 using place value  strategies and/or the relationship between addition  and subtraction. | Develop a flexible understanding of both vertical and  horizontal orientation. The representation of whole number differences within 100 on a number line diagram may be included. |
| **2.NO.NBT.5** Mentally add or subtract 10 or 100 to or  from a given number between 100 and 900. |  |

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| **Fractions (NF)**  *Learners will understand fractions and equivalency to represent, compare, and perform operations of  fractions and decimals.* | |
| **Standards** | **Clarification** |
| **2.NO.NF.1** Partition circles and rectangles into two,  three, or four equal shares. Describe the shares  using the language of halves, thirds, fourths, half of, a third of, and a fourth of. |  |
| **2.NO.NF.2** Recognize that identical wholes can be  equally divided in different ways. |  |
| **2.NO.NF.3** Recognize that partitioning shapes into  more equal shares creates smaller shares. |  |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **Standards** | **Clarification** |
| **2.AR.OA.1** Automatically add and subtract within 20. | See Appendix B for recommended automaticity. |
| **2.AR.OA.2** Apply the properties of operations to  solve addition and subtraction equations within 100  and justify thinking. | Properties of Operations – See Appendix A, Table 1. |
| **2.AR.OA.3** Solve one- and two-step authentic word  problems with addition within 100, including the use of unknowns. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **2.AR.OA.4** Solve one- and two-step authentic word problems with subtraction within 100, including the  use of unknowns. | Develop a flexible understanding of both vertical and  horizontal orientation. |
| **2.AR.OA.5** Use repeated addition to find the total  number of objects arranged in a rectangular array. | **Assessment Boundary**: Proficiency is limited up to  a 5 x 5 rectangular array. |
| **2.AR.OA.6** Identify a group of objects from 0 to 20  as even or odd by showing even numbers as a sum of two equal parts. |  |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Geometry (G)**  *Learners will compose and classify figures and shapes based on attributes and properties and represent and  solve problems using a coordinate plane.* | |
| **Standards** | **Clarification** |
| **2.GM.G.1** Identify two-dimensional shapes  (parallelograms and quadrilaterals). | **Assessment Boundary**: Include shapes from K.GM.G.1 and 1.GM.G.1 while adding  parallelograms and quadrilaterals. |
| **2.GM.G.2** Identify two-dimensional shapes found  within three-dimensional shapes. |  |
| **2.GM.G.3** Compose geometric shapes having  specified geometric attributes, such as a given  number of edges, angles, faces, vertices, and/or sides. | Composition includes drawing, building, or creating. |

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| **Measurement (M)**  *Learners will represent and calculate measurement data, including time, money, and geometric  measurement, and convert like measurement units within a given system.* | |
| **Standards** | **Clarification** |
| **2.GM.M.1** Measure the length of an object using two  different standard units of measurement. Describe  how the two measurements relate to the size of the units chosen. | **Assessment Boundary**: Different standard units of  measurement may include inches, feet, centimeters,  and meters. |
| **2.GM.M.2** Estimate and measure to determine how  much longer one object is than another, expressing  the difference with a standard unit of measurement. |  |
| **2.GM.M.3** Tell and write time to the nearest five  minutes (including quarter after and quarter to) with a.m. and p.m. using analog and digital clocks. |  |
| **2.GM.M.4** Count collections of money (quarters, dimes, nickels, and pennies) relating to counting  patterns by 1s, 5s, and 10s up to one dollar. |  |
| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.* | |
| **Data (D)**  *Learners will represent and interpret data.* | |
| **Standards** | **Clarification** |
| **2.DPS.D.1** Formulate questions and collect,  organize, and represent data with up to four  categories using single-unit scaled picture and bar graphs. |  |
| **2.DPS.D.2** Generate data and create line plots  marked in whole-number units. |  |
| **2.DPS.D.3** Analyze data and interpret the results to solve one-step comparison problems using  information from the graphs. |  |

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**THIRD GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and conclude  through exploration,*  *generalization, and validation.* |
| **3-5.MA.P** Learners can develop  and carry out a logical plan to  problem-solve situations, reflect  on the reasonableness of  solutions, and explore alternate  strategies with guidance. | **3-5.MA.C** Learners can make  connections and summarize  related ideas using supporting  evidence. | **3-5.MA.R** Learners can reason  logically based on experience and  knowledge, citing evidence to  support their reasoning and  conclusions. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Counting and Cardinality (CC)**  *Learners will understand the relationship between numerical symbols, names, quantities, and counting  sequences.* | |
| **Standard** | **Clarification** |
| **3.NO.CC.1** Read and write numbers up to 10,000 using objects or visual representations, including  standard, word, and expanded forms. | Spelling is not assessed. |
| **Base Ten (NBT)**  *Learners will understand the place value structure of the base-ten number system and represent,  compare, and perform operations with multi-digit whole numbers and decimals.* | |
| **Standards** | **Clarification** |
| **3.NO.NBT.1** Compare two four-digit numbers using  symbols >, <, and =. Justify comparisons based on  the value of thousands, hundreds, tens, and ones. |  |
| **3.NO.NBT.2** Apply place value understanding to  round whole numbers to the nearest 10 or 100. | Ordering numbers may be included for number  comparisons. |
| **3.NO.NBT.3** Add and subtract within 1000 using  place value strategies, algorithms, and/or the relationship between addition and subtraction. | Apply a flexible understanding of both vertical and  horizontal orientation. |
| **3.NO.NBT.4** Multiply one-digit whole numbers by  multiples of 10 within 100. | Apply a flexible understanding of both vertical and  horizontal orientation. |
| **Fractions (NF)**  *Learners will understand fractions and equivalency to represent, compare, and perform operations of  fractions and decimals.* | |
| **Standards** | **Clarification** |
| **3.NO.NF.1** Partition two-dimensional figures into  equal areas and express the area of each part as a  unit fraction of the whole. Describe using the  language of sixths, eighths, a sixth of, and an eighth of. | Two-dimensional figures are partitioned into halves,  fourths, and thirds in prior grades (see 1.NO.NF.1  and 2.NO.NF.1). |

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| **3.NO.NF.2** Represent and understand a fraction as  a number on a number line. |  |
| **3.NO.NF.3** Represent equivalent fractions using  visual representations and number lines. |  |
| **3.NO.NF.4** Recognize whole numbers as fractions and express fractions that are equivalent to whole  numbers. | Within this standard, learners will go beyond 1. |
| **3.NO.NF.5** Compare fractions of the same whole  having the same numerators or denominators, using  symbols >, <, and = by reasoning about their size  (fractions should be limited to denominators of 2, 3, 4, 6, and 8 and should not exceed the whole). | Ordering numbers may be included for number  comparisons. |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **Standards** | **Clarification** |
| **3.AR.OA.1** Using mental strategies, multiply and  divide basic facts within 100. Automatically multiply  and divide up to 5 x 5 and 10s facts. | Develop a flexible understanding of both vertical  and horizontal orientation. Students will continue to  learn multiplication and division within the range of  basic facts to 100, but automaticity is expected  within the range of 5 x 5 and 10s facts. Continued  automaticity of facts continues in 4.AR.OA.1.  See Appendix B for recommended automaticity. |
| **3.AR.OA.2** Apply the properties of operations to  solve multiplication and division equations and  justify thinking. | Apply a flexible understanding of both vertical and  horizontal orientation.  Properties of Operations – See Appendix A,  Table 1.  **Assessment Boundary**: Learners utilize  commutative, associative, and distributive properties without formal language. |
| **3.AR.OA.3** Solve two-step authentic word problems using addition and subtraction within 1000, including  equations with a letter as an unknown. |  |
| **3.AR.OA.4** Use strategies and visual models to  solve authentic word problems with multiplication  within 100, including unknowns, using grouping models and equations. |  |
| **3.AR.OA.5** Use strategies and visual models to  solve authentic word problems with division within 100, including unknowns, using grouping models  and equations. |  |
| **3.AR.OA.6** Identify arithmetic patterns and explain  them using the properties of operations. | Properties of Operations – See Appendix A,  Table 1.  Example: Observe that 4 times a number is always  even and explain why 4 times a number can be  decomposed into two equal addends.  **Assessment Boundary**: Learners utilize  commutative, associative, and distributive  properties without formal language. |

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| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Geometry (G)**  *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.* | |
| **Standards** | **Clarification** |
| **3.GM.G.1** In two-dimensional shapes, identify lines, angles (right, acute, obtuse), and perpendicular and  parallel lines. | Two-dimensional shapes include quadrilaterals and  right triangles. |
| **3.GM.G.2** Sort quadrilaterals into categories based  on attributes. | Quadrilaterals may share attributes, and attributes  can define a larger category. (See Appendix D) |
| **3.GM.G.3** Identify lines of symmetry in  quadrilaterals. |  |
| **Measurement (M)**  *Learners will represent and calculate measurement data, including time, money, and geometric  measurement, and convert like measurement units within a given system.* | |
| **Standards** | **Clarification** |
| **3.GM.M.1** Measure lengths using rulers marked  with halves and fourths of an inch. |  |
| **3.GM.M.2** Measure and estimate liquid volumes and  masses of objects using standard units. Solve one step authentic word problems involving masses or  volume given in the same units. | Standard units: grams (g), kilograms (kg), and  liters (l). |
| **3.GM.M.3** Tell and write time to the nearest minute  and measure time intervals in minutes. |  |
| **3.GM.M.4** Solve elapsed time authentic word  problems on the hour and the half-hour, using a  variety of strategies. |  |
| **3.GM.M.5** Solve authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies  using the $ and ¢ symbols appropriately. | **Assessment Boundary**: Word problems do not  include the use of decimals. |
| **3.GM.M.6** Solve problems involving the perimeters  of rectangles given the side lengths or when given  the perimeter and unknown side length(s). | Use rectangles with the same perimeter and  different areas or with the same area and different  perimeters. |
| **3.GM.M.7** Recognize area as an attribute of plane  figures and understand concepts of area  measurement. | A square with a side length of 1 unit, called “a unit  square,” is said to have “one square unit” of area  and can be used to measure area. A plan figure,  which can be covered without gaps or overlaps by n  unit squares, is said to have an area of n square units. |
| **3.GM.M.8** Find the area of a rectangle with whole number side lengths by modeling with unit squares;  show that area can be additive and is the same as would be found by multiplying the side lengths. |  |

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| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data (D)**  *Learners will represent and interpret data.* | |
| **Standards** | **Clarification** |
| **3.DPS.D.1** Formulate questions to collect, organize, and represent data with more than four categories  using scaled picture and bar graphs. | This includes collecting observations, surveys, or experiments to collect data to best-fit hypotheses or  questions. |
| **3.DPS.D.2** Generate data and create line plots  marked in whole numbers, halves, and fourths of a  unit. |  |
| **3.DPS.D.3** Analyze data and make simple  statements to solve one- and two-step problems using information from the graphs. |  |

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**FOURTH GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring necessary skills for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using supporting  evidence to interpret how they  originate, extend, and relate to other learning, ideas, and life  experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization, and validation.* |
| **3-5.MA.P** Learners can develop  and carry out a logical plan to  problem-solve situations, reflect  on the reasonableness of  solutions, and explore alternate strategies with guidance. | **3-5.MA.C** Learners can make  connections and summarize  related ideas using supporting  evidence. | **3-5.MA.R** Learners can reason  logically based on experience  and knowledge, citing evidence  to support their reasoning and  conclusions. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Counting and Cardinality (CC)**  *Learners will understand the relationship between numerical symbols, names, quantities, and counting  sequences.* | |
| **Standard** | **Clarification** |
| **4.NO.CC.1** Read numbers to the millions place,  including word, standard, and expanded form.  Write numbers to the millions place, including standard and expanded form. | Students are not expected to write word form to the  millions. Spelling is not assessed. |
| **Base Ten (NBT)**  *Learners will understand the place value structure of the base-ten number system and represent,  compare, and perform operations with multi-digit whole numbers and decimals.* | |
| **Standards** | **Clarification** |
| **4.NO.NBT.1** Understand that in a multi-digit whole  number, a digit in one place represents ten times what it represents in the place to its right. |  |
| **4.NO.NBT.2** Compare two numbers to the millions  place and decimals to the hundredths place, using  symbols >, <, and =. Justify comparisons based on the value of the digits. | Students compare two numbers to the millions  place and decimals to the hundredth place. In a  progression, students may practice reading and writing numbers and decimals prior to comparing.  Ordering numbers may be included for number  comparisons. |
| **4.NO.NBT.3** Apply place value understanding to  round multi-digit whole numbers to any place. |  |
| **4.NO.NBT.4** Add and subtract multi-digit whole numbers to the one million place using strategies,  including the algorithm. | Apply a flexible understanding of both vertical and  horizontal orientation. |
| **4.NO.NBT.5** Multiply a whole number of up to four  digits by a one-digit whole number and multiply two  two-digit numbers. Show and justify the calculation using equations, rectangular arrays, and models. | Apply a flexible understanding of both vertical and  horizontal orientation. |

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| **4.NO.NBT.6** Find whole-number quotients and  remainders with up to four-digit dividends and one digit divisors using place value strategies. Show and justify the calculation using equations, rectangular  arrays, and models. |  |

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| **Fractions (NF)**  *Learners will understand fractions and equivalency to represent, compare, and perform operations of  fractions and decimals.* | |
| **Standards** | **Clarification** |
| **4.NO.NF.1** Express equivalent fractions with a  denominator of 10 and a denominator of 100 to generate a decimal notation. |  |
| **4.NO.NF.2** Explain and demonstrate how a mixed  number is equivalent to a fraction greater than one  and how a fraction greater than one is equal to a  mixed number using visual fraction models and  reasoning strategies (proper and improper fractions  limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). | Example: 1 1 = 4 and 4 =1 13 3 3 3 |
| **4.NO.NF.3** Generate equivalent fractions using  numerical representations, visual representations,  and number lines (proper and improper fractions  limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). |  |
| **4.NO.NF.4** Demonstrate how equivalent fractions  are generated by multiplying a fraction equivalent to  1 or the properties of multiplication (proper and  improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). |  |
| **4.NO.NF.5** Compare and order fractions having,  unlike numerators or denominators. Record  comparisons using the symbols >, <, and =. Justify  using a visual fraction model (proper and improper  fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). |  |
| **4.NO.NF.6** Solve authentic word problems by  adding and subtracting fractions and mixed numbers with like denominators (proper and  improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). | In a progression, students would learn how to add  and subtract fractions and mixed numbers to apply  the understanding to word problems. |
| **4.NO.NF.7** Solve problems by multiplying fractions  and whole numbers using visual fraction models  (proper and improper fractions limited to  denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). | **Assessment Boundary**: Model with visuals how  fractions are multiplied, rather than using a standard  algorithm for multiplication with fractions. |

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| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **Standards** | **Clarification** |
| **4.AR.OA.1** Automatically multiply and divide  through 10 x 10. | Apply a flexible understanding of both vertical and  horizontal orientation. Automaticity in the range of  5 x 5 and 10s facts is in standard 3.AR.OA.1. Continued automaticity of facts continues in  standard 5.AR.OA.1.  See Appendix B for recommended automaticity. |

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| **4.AR.OA.2** Identify and apply the properties of  operations for addition, subtraction, multiplication,  and division and justify thinking. | Apply a flexible understanding of both vertical and  horizontal orientation.  Properties of Operations – See Appendix A,  Table 1.  **Assessment Boundary**: Learners utilize  commutative, associative, identity, and distributive  properties. |
| **4.AR.OA.3** Solve multi-step authentic word  problems using the four operations, including  problems with interpreted remainders. Represent  problems using equations, including a symbol as an unknown. | **Assessment Boundary**: Use drawings and  equations with symbols for the unknown number  (variable) to represent a problem. |
| **4.AR.OA.4** Find factor pairs and multiples within the range of 1-36 while classifying numbers as prime or  composite. |  |
| **4.AR.OA.5** Interpret multiplication equations as a  comparison. Represent multiplicative comparisons  as multiplication equations. | Example: Interpret 35 = 5 x 7 as a statement that  35 is 5 times as many as 7 and 7 times as many as  5.  Example: Kari has 3 marbles; Greg has 7 times as  many. How many marbles does Greg have? 3 x 7 = 21 or 7 x 3 = 21. |
| **4.AR.OA.6** Generate a number or shape pattern  that follows a given rule while identifying apparent  features of the pattern that were not explicit in the rule itself. | (e.g., Given a rule “add 3” and the starting number  of 1, generate terms in the resulting sequence and  observe that the terms appear to alternate between odd and even numbers.) |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate  the characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Geometry (G)**  *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.* | |
| **Standards** | **Clarification** |
| **4.GM.G.1** Identify, label, and draw points, lines, line  segments, rays, and angles (right, acute, obtuse). |  |

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| **4.GM.G.2** Classify two-dimensional figures based  on the presence or absence of parallel or  perpendicular lines, or the presence or absence of angles of specified size. | See Appendix D for guidance.  Properties of two-dimensional shapes would include  identifying the sides and angles found in polygons and  triangles.  **Assessment Boundary**: Shapes are classified by  their attributes and not their formal name. |
| **4.GM.G.3** Draw lines of symmetry in two  dimensional figures. |  |
| **Measurement (M)**  *Learners will represent and calculate measurement data, including time, money, and geometric  measurement, and convert like measurement units within a given system.* | |
| **Standards** | **Clarification** |
| **4.GM.M.1** Know the relative sizes of measurement  units within one system of units including km, m,  cm; kg, g; lb., oz.; l, ml; hr., min., sec. Record measurement equivalents in a two-column table. |  |
| **4.GM.M.2** Generate simple conversions from a  larger unit to a smaller unit to solve authentic  problems within a single system of measurement, both customary and metric systems. |  |

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| **4.GM.M.3** Identify and use the appropriate tools,  operations, and units of measurement, both customary and metric, to solve problems involving  time, length, weight, mass, and capacity. |  |
| **4.GM.M.4** Solve authentic word problems involving  dollar bills, quarters, dimes, nickels, and pennies  using $ and ¢ symbols and decimal notation appropriately. |  |
| **4.GM.M.5** Apply the area and perimeter formulas for  rectangles, including connected rectangular figures,  in problems. | Example:  A house owner wants to buy sod for his backyard.  The sod is sold in square meters. Determine how  many square meters of sod are needed to cover the  backyard pictured below. |
| **4.GM.M.6** Measure angles in whole-number  degrees using a protractor. Using a protractor and ruler, draw angles of a specified measure. |  |
| **4.GM.M.7** Recognize angle measures as additive  and solve addition and subtraction problems to find  unknown angles on a diagram. | Example:  If angle BAD is 58° and angle BAC measures 32°,  what is the measure of angle CAD?  A diagram of a circle with arrows and a circle with a circle and a circle with a circle with a circle with a circle with a circle with a circle with a circle with a circle with a  Description automatically generated |

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| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data (D)**  *Learners will represent and interpret data.* | |
| **Standards** | **Clarification** |
| **4.DPS.D.1** Formulate questions to collect, organize,  and represent data to reason with math and across  disciplines. | Choose the visual representation that best displays  the data collected (e.g., pictograph, bar graph, and  tallies). |
| **4.DPS.D.2** Generate data and create line plots to  display a data set of fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction  of fractions by using information presented in line plots. |  |
| **4.DPS.D.3** Utilize graphs and diagrams to represent  and solve authentic word problems using the four  operations involving whole numbers, benchmark fractions, and decimals. | **Assessment Boundary**: This includes distances,  intervals of time, liquid volumes, masses of objects,  and money. |

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**FIFTH GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using supporting  evidence to interpret how they  originate, extend, and relate to other learning, ideas, and life  experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization, and validation.* |
| **3-5.MA.P** Learners can develop  and carry out a logical plan to  problem-solve situations, reflect  on the reasonableness of  solutions, and explore alternate strategies with guidance. | **3-5.MA.C** Learners can make  connections and summarize  related ideas using supporting  evidence. | **3-5.MA.R** Learners can reason  logically based on experience  and knowledge, citing evidence  to support their reasoning and  conclusions. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Counting and Cardinality (CC)**  *Learners will understand the relationship between numerical symbols, names, quantities, and counting  sequences.* | |
| **Standard** | **Clarification** |
| **5.NO.CC.1** Read and write decimals to the thousandths, including standard, word, and  expanded forms. | Spelling is not assessed. |
| **Base Ten (NBT)**  *Learners will understand the place value structure of the base-ten number system and represent,  compare, and perform operations with multi-digit whole numbers and decimals* | |
| **Standards** | **Clarification** |
| **5.NO.NBT.1** Understand that in a multi-digit number, a digit in one place represents ten times as much as it represents in the place to its right and  1/10 of what it represents in the place to its left. |  |
| **5.NO.NBT.2** Compare two decimals to the  thousandth place using symbols >, <, and =.  Justify comparisons based on the value of the digits. | Ordering numbers may be included for number  comparisons. |
| **5.NO.NBT.3** Apply place value understanding to  round decimals to any place. |  |
| **5.NO.NBT.4** Multiply multi-digit whole numbers  using strategies flexibly, including the algorithm. | Apply a flexible understanding of both vertical and  horizontal orientation. Mastery of the multiplication  algorithm is expected. |
| **5.NO.NBT.5** Use concrete models, drawings, place  value strategies, properties of operations and/or  relationships to add, subtract, and multiply decimals  to hundredths. | Properties of Operations – See Appendix A,  Table 1.  Division of decimals is found within the sixth-grade  standards. |

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| **5.NO.NBT.6** Find whole-number quotients and  remainders with up to four-digit dividends and two digit divisors using place value strategies. Show and  justify the calculation by using equations,  rectangular arrays, and/or area models. | Division procedures, including the algorithm, are  included in sixth grade. |
| **5.NO.NBT.7** Explain patterns in the number of  zeros of the product when multiplying a number by  powers of 10.  Explain patterns in the placement of the decimal  point when a decimal is multiplied or divided by a  power of 10.  Use whole-number exponents to denote powers  of 10. |  |
| **Fractions (NF)**  *Learners will understand fractions and equivalency to represent, compare, and perform operations of  fractions and decimals.* | |
| **Standards** | **Clarification** |
| **5.NO.NF.1** Generate equivalent forms of commonly used fractions and decimals (e.g., halves, fourths,  fifths, tenths). | This standard includes writing fractions in the lowest  terms. |
| **5.NO.NF.2** Explain why multiplying a given number  by a fraction greater than one results in a product  greater than the given number and explain why  multiplying a given number by a fraction less than  one results in a product smaller than the given number. |  |
| **5.NO.NF.3** Solve authentic word problems by  adding and subtracting fractions and mixed numbers with unlike denominators using visual  fraction models and equations. | In a progression, students may practice adding and  subtracting fractions and mixed numbers with unlike denominators prior to using the understanding in  word problems. |
| **5.NO.NF.4** Solve authentic word problems by  multiplying fractions and mixed numbers using visual fraction models and equations. | **Assessment Boundary**: Model with visuals how  fractions are multiplied, rather than using the standard algorithm for multiplication with fractions. |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **Standards** | **Clarification** |
| **5.AR.OA.1** Automatically multiply and divide  through 12 x 12. | Apply a flexible understanding of both vertical and  horizontal orientation. Automaticity of facts is also in  standards 3.AR.OA.1 and 4.AR.OA.1.  See Appendix B for recommended automaticity. |
| **5.AR.OA.2** Analyze problems using the order of operations to solve and evaluate expressions while  justifying thinking. | Apply a flexible understanding of both vertical and  horizontal orientation. |

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| **Operations and Algebraic Thinking (OA)**  *Learners will analyze patterns and relationships to generate and interpret numerical expressions.* | |
| **5.AR.OA.3** Write simple expressions that record  calculations with numbers. Interpret numerical  expressions without evaluating them. | Example: Express the calculation “add 8 and 7, then  multiply by 2 as 2x(8 + 7).  Recognize that 3x(18,932 + 921) is three times as  large as 18,932 + 921 without having to calculate  the indicated sum or product. |
| **5.AR.OA.4** Find factor pairs and multiples within the  range of 1-100 while classifying numbers as prime or composite. |  |
| **5.AR.OA.5** Generate two numerical patterns using  two given rules and form ordered pairs consisting of  corresponding terms from the two patterns.  (Graphing on a coordinate plane). | Example: Given the rule “add 3” and the starting  number of 0 and given the rule “add 6” and the  starting number of 0, generate terms in the resulting  sequences and, in this case, observe that the terms  in one sequence are twice the corresponding terms  in the other sequence. Explain informally why this is  so. |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Geometry (G)**  *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.* | |
| **Standards** | **Clarification** |
| **5.GM.G.1** Classify two-dimensional figures in a  hierarchy based on properties. | See Appendix D.  Properties of two-dimensional shapes and the names of  the specific shapes would include identifying the  number of sides, angles, and lines of symmetry found in  polygons and triangles (equilateral, isosceles, scalene). |
| **5.GM.G.2** Identify the x-coordinate and y-coordinate to graph and name points in the first quadrant of the  coordinate plane. | In a progression, students may begin by learning about the origin in direction with the axis and how the  coordinates correspond. |
| **5.GM.G.3** Form ordered pairs and graph points in  the first quadrant on the coordinate plane to solve  authentic word problems. |  |
| **Measurement (M)**  *Learners will represent and calculate measurement data, including time, money, and geometric  measurement, and convert like measurement units within a given system.* | |
| **Standards** | **Clarification** |
| **5.GM.M.1** Generate conversions among different sized standard measurement units within a given  measurement system, both customary and metric.  Use these conversions to solve multi-step,  authentic word problems. |  |
| **5.GM.M.2** Find the area and perimeter of a  rectangle, including connected rectangular figures,  with fractional side lengths. | Example: |

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| **5.GM.M.3** Recognize volume as an attribute of  rectangular prisms and measure volume by counting unit cubes. | In a progression, students may begin by recognizing  that volume is additive when measuring volume by counting unit cubes. |

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| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data (D)**  *Learners will represent and interpret data.* | |
| **Standards** | **Clarification** |
| **5.DPS.D.1** Generate data and create line plots to  display a data set of fractions of a unit (½, ¼, ⅛). Use grade-level operations for fractions to solve  problems involving information presented in line plots. |  |
| **5.DPS.D.2** Utilize graphs and diagrams to  represent, analyze, and solve authentic word  problems using information presented in one or more tables or line plots including whole numbers,  fractions, and decimals. | The DPS.D category in K-4 describes graphs and  tables that students are expected to learn. 5.DPS.D.2 encompasses all graphs, and the problem now dictates which visual representation  students should use. |

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**SIXTH GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization, and validation.* |
| **6-8.MA.P** Learners can analyze  information and formulate a  flexible, systematic plan to  problem-solve authentic  situations and reflect on the  reasonableness of the solution,  making revisions when  necessary. | **6-8.MA.C** Learners can create  connections within and across  concepts and provide examples  of how they relate to other  learning and ideas using  supporting evidence. | **6-8.MA.R** Learners can reason  logically, citing evidence to  evaluate and explain what they  see, think, and conclude through  exploration and justification. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Number Systems (NS)**  *Learners will expand their knowledge of the number system to create connections and solve problems  within and across concepts.* | |
| **Standards** | **Clarification** |
| **6.NO.NS.1** Explain and show the relationship  between non-zero rational numbers and their  opposites using horizontal and vertical number  lines, including authentic problems.  Use rational numbers to represent quantities in authentic contexts and explain the meaning of 0 in  certain situations. | This is the concept of absolute value, but formal  notation is not required at this level. |
| **6.NO.NS.2** Write, interpret, and explain statements of order for rational numbers on a number line and  in authentic contexts. | A statement of order could be a list of numbers, a  statement of inequality, or a description. |
| **Operations (O)**  *Learners will expand their computational fluency to create connections and solve problems within and  across concepts.* | |
| **Standards** | **Clarification** |
| **6.NO.O.1** Divide multi-digit whole numbers up to  four-digit dividends and two-digit divisors using  strategies or procedures. | Learners should be able to reason using number  relationships and logic to choose an efficient  strategy to solve each problem. Procedures may include the standard algorithm. |
| **6.NO.O.2** Add and subtract fractions and decimals up to the hundredth place, including authentic  problems. | Fractions include mixed numbers and improper  fractions. |

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| **6.NO.O.3** Apply multiplication and division of  fractions and decimals to solve and interpret  problems using visual models, including authentic  problems. | Fractions include mixed numbers and improper  fractions.  **Assessment Boundary**: Decimal division is limited  to problems where either the dividend or divisor is a  whole number, and the other is a decimal up to the hundredth place. |
| **6.NO.O.4** Determine the greatest common factor of  two whole numbers less than or equal to 100 and  the least common multiple of two whole numbers  less than or equal to 12. | This leads to algebraic topics, including factoring  expressions and the distributive property with  variables. The focus should not be on simplifying  fractions or finding the least common denominators.  See Appendix B for recommended automaticity. |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Ratios and Proportional Relationships (RP)**  *Learners will use ratios, rates, and proportions to model relationships and solve problems.* | |
| **Standards** | **Clarification** |
| **6.AR.RP.1** Describe the concept of a ratio  relationship between two quantities using ratio  language and visual models. | Visual models may include tables of equivalent  ratios, tape diagrams, double number line diagrams,  etc.  This includes part-to-part and part-to-whole ratios. |
| **6.AR.RP.2** Describe and calculate a unit rate when  given a ratio relationship between two quantities  using rate language and visual models. | Visual models may include tables of equivalent  ratios, tape diagrams, double number line diagrams,  etc.  The focus should be ratios and rates but use  previous fraction knowledge to support the work. |
| **6.AR.RP.3** Make and use tables of equivalent ratios, tape diagrams, double number line diagrams,  and equations to solve problems involving ratios, rates, and unit rates, including authentic problems. |  |
| **6.AR.RP.4** Calculate a percent of a quantity as a  rate per 100. Solve problems using ratio reasoning  involving finding the whole when given a part and the percent. |  |
| **6.AR.RP.5** Convert measurement units within and  between measurement systems using ratio  reasoning given conversion factors. | This is the introduction to conversions between  measurement units. |
| **Expressions and Equations (EE)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adapting approaches in novel situations.* | |
| **Standards** | **Clarification** |
| **6.AR.EE.1** Read, write, and evaluate numerical expressions, including expressions with whole  number exponents and grouping symbols. | This standard includes evaluating expressions using  the order of operations, including parentheses. |
| **6.AR.EE.2** Read and evaluate algebraic  expressions, including expressions with whole  number exponents and grouping symbols.  Write algebraic expressions to represent simple and  authentic situations. |  |

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| **6.AR.EE.3** Identify when two expressions are  equivalent.  Apply the properties of operations to generate  equivalent expressions. | Properties of Operations – See Appendix A,  Table 1.  Both numeric and algebraic expressions are  included.  Two expressions are equivalent when they represent the same number regardless of which  value is substituted into them. |
| **6.AR.EE.4** Describe the concept of a solution of an  equation and an inequality.  Determine whether a given number is a solution to  an equation or an inequality. |  |
| **6.AR.EE.5** Write and solve equations of the form x + p = q and px = q for cases in which p and q are  non-negative whole numbers or decimals, including  authentic problems. |  |
| **6.AR.EE.6** Write a statement of inequality of the  form x > c or the form x < c to represent a constraint  or condition.  Recognize that inequalities of the form x > c or the  form x < c have infinitely many solutions; represent  solutions of such inequalities on number line diagrams. | Inequalities are represented by the following <, >, ≤, ≥, ≠.  **Assessment Boundary**: This does not include  compound inequalities at this level. |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Area and Volume (AV)**  *Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.* | |
| **Standards** | **Clarification** |
| **6.GM.AV.1** Derive the relationship of the areas of  triangles using the area of rectangles.  Calculate the areas of triangles and quadrilaterals  by composing and/or decomposing them into  rectangles and triangles, including authentic  problems. | Learners should develop a fluent way of finding the  area of a triangle.  Using the shape composition and decomposition  skills acquired in earlier grades, Learners learn to  develop area formulas for parallelograms and then  triangles. They learn how to address three different  cases for triangles:  • a height that is a side of a right triangle, • a height that “lies over the base,”  • and a height that is outside the triangle. |
| **6.GM.AV.2** Describe the concept of volume of a  right rectangular prism.  Apply given formulas to calculate the volume of right rectangular prisms, including fractional edge  lengths, including authentic problems. | In fifth grade, there is a similar standard with whole  numbers only. This understanding is extended to  fractional sizes. |

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| **Geometric Figures (GF)**  *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics  of figures, perform transformations, and construct logical arguments.* | |
| **Standards** | **Clarification** |
| **6.GM.GF.1** Identify and position ordered pairs of  rational numbers in all four quadrants of a coordinate plane. |  |
| **6.GM.GF.2** Draw polygons in the coordinate plane  given coordinates for the vertices.  Determine the length of a side joining points with the same first or second coordinate, including  authentic problems. | The focus is not on integer operations.  The sides of polygons should not be diagonal. |
| **6.GM.GF.3** Represent three-dimensional figures  using nets made up of rectangles and triangles  (right prisms and pyramids whose bases are  triangles and rectangles).  Calculate the surface area of prisms with  rectangular and triangular bases using nets, including authentic problems. | **Assessment Boundary**: This standard does not  include knowing and applying surface area  formulas; the focus is on individual areas of the net. |
| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data Analysis (D)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing  inferences, conclusions, and making predictions.* | |
| **Standards** | **Clarification** |
| **6.DPS.D.1** Write a statistical question that can be  answered using measures of center or variability of  a data set. | **Assessment Boundary**: Measures of center: mean  and median. Measures of variability: range and  mean absolute deviation. |
| **6.DPS.D.2** Calculate measures of center (median  and mean) and variability (range and mean absolute  deviation) to answer a statistical question.  Identify mode(s) if they exist. |  |
| **6.DPS.D.3** Identify outliers by observation and  describe their effect on measures of center and  variability.  Justify which measures would be appropriate to  answer a statistical question. |  |
| **6.DPS.D.4** Display numerical data in plots on a  number line, including dot plots and histograms.  Describe any overall patterns in data, such as gaps,  clusters, and skews. | Overall shape in this context refers to the shape of a  graphical representation of data including uniform,  skewed, symmetric, and normal (bell-shaped). |
| **Probability (P)**  *Learners will understand and apply basic concepts of probability.* | |
| *NOTE: Standards begin in seventh grade.* | |

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**SEVENTH GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and conclude  through exploration,*  *generalization, and validation.* |
| **6-8.MA.P** Learners can analyze  information and formulate a  flexible, systematic plan to  problem-solve authentic  situations and reflect on the  reasonableness of the solution,  making revisions when  necessary. | **6-8.MA.C** Learners can create  connections within and across  concepts and provide examples  of how they relate to other  learning and ideas using  supporting evidence. | **6-8.MA.R** Learners can reason  logically, citing evidence to  evaluate and explain what they  see, think, and conclude through  exploration and justification. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Number Systems (NS)**  *Learners will expand their knowledge of the number system to create connections and solve problems  within and across concepts.* | |
| **Standards** | **Clarification** |
| **7.NO.NS.1** Describe the absolute value of a number  as its distance from zero on a number line. | Learners should be introduced to the notation of  absolute value at this level. |
| **7.NO.NS.2** Recognize common fractions and  decimal equivalencies up to a denominator of 10.  Convert a rational number to a decimal using  technology. | See Appendix B for recommended automaticity. |
| **Operations (O)**  *Learners will expand their computational fluency to create connections and solve problems within and  across concepts.* | |
| **Standards** | **Clarification** |
| **7.NO.O.1** Add, subtract, multiply, and divide  integers using visual models and properties of  operations in multi-step problems, including  authentic problems. | Properties of Operations – See Appendix A,  Table 1.  Visual models may include algebra tiles, colored  chips, number lines, etc. |
| **7.NO.O.2** Add, subtract, multiply, and divide non negative fractions in multi-step problems, including  authentic problems. |  |

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| **7.NO.O.3** Add, subtract, multiply, and divide non negative decimals to the hundredth place in multi step problems using strategies or procedures,  including authentic problems. | Learners should be able to reason using number  relationships and logic to choose an efficient  strategy to solve each problem.  Procedures can include the standard algorithm.  **Assessment Boundary**: Division is limited to  problems where either the dividend or divisor is a  whole number, and the other is a decimal up to the hundredth place. |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Ratios and Proportional Relationships (RP)**  *Learners will use ratios, rates, and proportions to model relationships and solve problems.* | |
| **Standards** | **Clarification** |
| **7.AR.RP.1** Calculate unit rates associated with  ratios of rational numbers, including ratios of  lengths, areas, and other quantities measured in like or different units. | Unit rates may be represented as fractions,  decimals, and/or percents. |
| **7.AR.RP.2** Analyze the relationship between the  dependent and independent variables of a  proportional relationship using graphs and tables.  Explain what a point (x, y) on the graph of a  proportional relationship means in terms of the  situation, with special attention to the points (0, 0) and (1, k) where k is the unit rate. |  |
| **7.AR.RP.3** Identify the constant of proportionality in  tables, graphs, equations, diagrams, and  descriptions of proportional relationships.  Represent proportional relationships by an equation  of the form y = kx, where k is the constant of  proportionality, and describe the meaning of each variable (y, k, x) in the context of the situation. |  |
| **7.AR.RP.4** Use proportional relationships to solve  multi-step problems involving ratios, percents, and  scale drawings of geometric figures, including  authentic problems. | The focus should be on the conceptual  understanding of a proportional relationship, not the  procedural methods of solving these problems.  Conceptual methods can include using ratio tables,  tape diagrams, double number lines, etc. |
| **Expressions and Equations (EE)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adapting approaches in novel situations.* | |
| **Standards** | **Clarification** |
| **7.AR.EE.1** Apply properties of operations as  strategies to add, subtract, factor, and expand linear  expressions involving variables, integers, and/or  nonnegative fractions and decimals with an emphasis on writing equivalent expressions. | Properties of Operations – See Appendix A,  Table 1. |

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| **7.AR.EE.2** Write and solve equations of the form  px + q = r and p(x + q) = r , including authentic  problems. | Properties of Equality – See Appendix A, Table 2.  **Assessment Boundary**: q and r are integers, and p  is an integer or a positive fraction/decimal. |
| **7.AR.EE.3** Write and solve one- or two-step  inequalities where coefficients and solutions are  integers and/or non-negative fractions and  decimals, including authentic problems.  Graph the solution set of the inequality and interpret  it in the context of the problem. | Properties of Inequality – See Appendix A, Table 3.  **Assessment Boundary**: At this level, compound  inequalities are not included. |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Area and Volume (AV)**  *Learners will use visualization and spatial reasoning to solve authentic and mathematical problems  involving area, surface area, and volume of geometric figures.* | |
| **Standards** | **Clarification** |
| **7.GM.AV.1** Describe the relationship between the  circumference and diameter of a circle (pi).  Apply given formulas to calculate the area and  circumference of a circle, including authentic problems. | The focus of the first part is to develop an  understanding of the concept of pi. |
| **7.GM.AV.2** Calculate areas of polygons by  composing and/or decomposing them into  rectangles and triangles, including authentic  problems.  Solve problems involving the surface area of prisms and right pyramids using nets, including authentic  problems. | **Assessment Boundary**: The standard does not  include knowing and applying surface area  formulas; the focus is on the individual areas of the  net. |
| **7.GM.AV.3** Solve problems involving the volume of  prisms and composite solids, including authentic  problems. | Any problem can be used, provided the base can be  decomposed into triangles and/or rectangles. |
| **Geometric Figures (GF)**  *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics  of figures, perform transformations, and construct logical arguments.* | |
| **Standards** | **Clarification** |
| **7.GM.GF.1** Draw triangles from given conditions  using appropriate tools.  Defend whether a unique triangle, multiple triangles,  or no triangle can be constructed when given three measures of angles or sides. | Appropriate tools could include protractors, rulers,  compasses, and/or technology.  Ensure learners understand the triangle  classifications and vocabulary. |
| **7.GM.GF.2** Describe the following angle-pair  relationships: supplementary angles,  complementary angles, vertical angles, and  adjacent angles.  Solve for an unknown angle in a figure by applying  facts about these angles. | **Assessment Boundary**: Solving for unknown  angles does not include algebraic expressions  involving operations. The focus is finding unknown  angle measures, not solving equations. |

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| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data Analysis (D)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing  inferences and conclusions, and making predictions.* | |
| **Standards** | **Clarification** |
| **7.DPS.D.1** Identify the strengths and weaknesses of  a population sample, including bias in the process of  the data collection. |  |
| **7.DPS.D.2** Analyze and draw inferences about a  population using single and multiple random samples by using given measures of center and  variability for the numerical data set. |  |
| **Probability (P)**  *Learners will understand and apply basic concepts of probability.* | |
| **Standards** | **Clarification** |
| **7.DPS.P.1** Develop a probability model to find  probabilities of theoretical events and contrast  probabilities from an experimental model. | This is the first time learners have been exposed to  the concept of probability. The basic concepts of probability and likelihood will need to be developed  before fully addressing this standard. |
| **7.DPS.P.2** Develop a probability model to find  theoretical probabilities of independent compound  events. | Examples of probability models can include  organized lists, tree diagrams, area models, and  simulations.  **Assessment Boundary**: Learners are not expected  to use formulas (formal procedures). |

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**EIGHTH GRADE**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement  through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and conclude  through exploration,*  *generalization, and validation.* |
| **6-8.MA.P** Learners can analyze  information and formulate a  flexible, systematic plan to  problem-solve authentic  situations and reflect on the  reasonableness of the solution,  making revisions when  necessary. | **6-8.MA.C** Learners can create  connections within and across  concepts and provide examples  of how they relate to other  learning and ideas using  supporting evidence. | **6-8.MA.R** Learners can reason  logically, citing evidence to  evaluate and explain what they  see, think, and conclude through  exploration and justification. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Number Systems (NS)**  *Learners will expand their knowledge of the number system to create connections and solve problems  within and across concepts.* | |
| **Standards** | **Clarification** |
| **8.NO.NS.1** Compare and classify real numbers  within the real number system. |  |
| **8.NO.NS.2** Use rational approximations of irrational  numbers to compare the size of irrational numbers,  locate them on a number line diagram, and estimate  the value of irrational expressions involving one operation. | Expressions can include examples such as  2π or √2 + 11. |
| **8.NO.NS.3** Use scientific notation to represent very  large or very small quantities.  Interpret scientific notation generated by  technology.  Compare and order numbers in both scientific and  standard notation. |  |

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| **Operations (O)**  *Learners will expand their computational fluency to create connections and solve problems within and  across concepts.* | |
| **Standards** | **Clarification** |
| **8.NO.O.1** Evaluate mentally the square roots of  perfect squares up to 225 and cube roots of perfect  cubes up to 1000. | This standard supports standard 8.NO.NS.2.  This is the first learners are introduced to the idea of  radicals. Connections should be made to the area  of a square and the volume of a cube.  See Appendix B for recommended automaticity. |
| **8.NO.O.2** Add, subtract, multiply, and divide rational  numbers using strategies or procedures. | Learners should be able to reason using number  relationships and logic to choose an efficient  strategy to solve each problem.  Procedures can include the standard algorithm. |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Expressions and Equations (EE)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adapting approaches in novel situations.* | |
| **Standards** | **Clarification** |
| **8.AR.EE.1** Explain the relationship between  repeated multiplication and the properties of integer  exponents.  Apply a single exponent property to generate  equivalent numeric and algebraic expressions that  include numerical coefficients. |  |
| **8.AR.EE.2** Use square root and cube root symbols  to represent solutions to equations of the form x² = p and x³ = p, where p is a non-negative rational  number. | Attention should be drawn to the conceptual  understanding of the number of solutions to these  equations. |
| **8.AR.EE.3** Explain the characteristics of a linear  relationship, including identifying the slope and y intercept in tables, graphs, equations, and descriptions. |  |
| **8.AR.EE.4** Represent linear relationships using  tables, graphs, equations, and descriptions when  given a relationship in one of these forms. | **Assessment Boundary**: Equations must be of the  form y = mx + b. |
| **8.AR.EE.5** Solve linear equations with rational  number coefficients and variables on both sides,  including equations that require using the  distributive property and/or combining and  collecting like terms.  Interpret the number of solutions.  Give examples of linear equations in one variable  with one solution, infinitely many solutions, or no  solutions. |  |

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| **8.AR.EE.6** Read, write, and evaluate numerical and  algebraic expressions including expressions  involving absolute value.  Solve and graph equations of the form |x|=r where r  is a nonnegative rational number. | Attention should be drawn to the conceptual  understanding about the number of solutions to  these equations and why r cannot be a negative  value.  Graphs should be done on a number line with attention being drawn to the symmetry of the  solutions. |
| **8.AR.EE.7** Solve and graph inequalities in one  variable with rational number coefficients and  variables on both sides, including inequalities that  require using the distributive property and/or combining like terms. | **Assessment Boundary**: This level does not  include compound inequalities. |
| **8.AR.EE.8** Graph linear inequalities in two  variables on a coordinate plane. Interpret the  possible solutions in the context of authentic problems. | **Assessment Boundary**: This level does not  include compound inequalities. Inequalities must be  given in the slope-intercept form. |
| **Functions (F)**  *Learners will develop a foundational knowledge of functions and use them to model relationships between  quantities.* | |
| **Standards** | **Clarification** |
| **8.AR.F.1** Defend whether a relation is a function  from various representations using appropriate function language. | **Assessment Boundary**: Function language does  not include function notation at this level. |
| **8.AR.F.2** Compare and contrast properties of two  linear functions, each represented in a different way  (algebraically, graphically, numerically in tables, and/or by descriptions). |  |
| **8.AR.F.3** Compare and contrast linear and non linear functions represented in different ways  (algebraically, graphically, numerically in tables,  and/or by descriptions). | Non-linear is a general term that refers to any  function that does not change at a constant rate.  This standard is not requiring any specific type of  non-linear function, such as quadratic or  exponential. |
| **8.AR.F.4** Model a linear function between two  quantities by creating a table, graph, and equation.  Interpret the rate of change and initial value of a  linear function in terms of the situation it models. | Modeling is applying mathematics learners know to  solve problems arising in everyday life, society, and  the workplace. See Appendix C for the modeling  process. |
| **8.AR.F.5** Describe qualitatively the functional  relationship between two quantities by analyzing a  graph including where the function is constant,  increasing, or decreasing; linear or nonlinear; and  discrete or continuous.  Create a graph that exhibits the qualitative features  of a function described. |  |

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| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate  the characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Area and Volume (AV)**  *Learners will use visualization and spatial reasoning to solve problems involving area, surface area, and  volume of geometric figures.* | |
| **Standard** | **Clarification** |
| **8.GM.AV.1** Apply given formulas to solve problems involving the volume of cones, cylinders, and  spheres, including authentic problems. |  |
| **Geometric Figures (GF)**  *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the  characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Standards** | **Clarification** |
| **8.GM.GF.1** Perform single transformations to a  figure on the coordinate plane and determine  whether the figures are congruent or similar. | **Assessment Boundary**: Reflections on the  coordinate plane are limited to over the x- or y-axis.  Rotations are limited to multiples of 90° rotations  about the origin.  Centers for dilation on the coordinate plane are  limited to the origin.  Formal (coordinate) notations are not expected at  this level. |
| **8.GM.GF.2** Describe the characteristics of  transformations on the coordinate plane using  transformation language. | **Assessment Boundary**: For translations, use  distance and direction.  For reflection, use an axis as a line of reflection.  For rotations about the origin, use direction  (clockwise and counterclockwise) and degree (90, 180, 270, 360). |
| **8.GM.GF.3** Name the type of transformation(s) needed to map a pre-image to its image. | **Assessment Boundary**: Sequences should be  limited to two transformations. |
| **8.GM.GF.4** Describe the following angle-pair  relationships: interior and exterior angles of  triangles and angles formed when a transversal  cuts parallel lines or intersecting lines.  Solve for an unknown angle in a figure by applying  facts about these angles. |  |
| **8.GM.GF.5** Describe the relationship between the  leg lengths and the hypotenuse length of a right  triangle.  Determine whether a triangle is a right triangle  using this relationship. |  |
| **8.GM.GF.6** Apply the Pythagorean Theorem to  determine unknown side lengths in right triangles in  two and three dimensions on and off a coordinate plane, including authentic problems. | This does not include the distance formula. |

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| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Data Analysis (D)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing  inferences and conclusions, and making predictions.* | |
| **Standards** | **Clarification** |
| **8.DPS.D.1** Interpret scatter plots for bivariate  measurement data to investigate patterns such as  clustering, outliers, positive or negative association, linear association, and nonlinear association. |  |
| **8.DPS.D.2** Draw an informal trend line on a given  scatter plot with a linear association and justify its fit  by describing the closeness of the data points to the line. |  |
| **8.DPS.D.3** Solve authentic problems in the context  of bivariate measurement data by interpreting the slope and intercept(s) and making predictions using  a linear model. |  |
| **8.DPS.D.4** Construct and interpret a two-way table  summarizing bivariate categorical data collected from the same subjects. | Interpretations can include calculating joint and  marginal relative frequencies. |
| **Probability (P)**  *Learners will understand and apply basic concepts of probability.* | |
| *NOTE: There are no probability standards at this level. Probability concepts are further developed in ninth  and tenth grade.* | |

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**NINTH AND TENTH GRADES**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for  advancement through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization and validation.* |
| **9-12.MA.P** Learners can analyze,  execute, critique, and adapt  approaches and solutions when  problem-solving in novel  situations. | **9-12.MA.C** Learners can create  connections within and across  concepts, using supporting  evidence to interpret how they  originate, extend, and relate to  other learning, ideas, and life experiences. | **9-12.MA.R** Learners can reason  logically, citing evidence to  critique and explain what they  see, think, and conclude through  exploration, generalization, and  validation. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Standards** | **Clarifications** |
| **9-10.NO.1** Explain how the definition of rational  exponents follows from extending the properties of  integer exponents; rewrite simple expressions  involving radicals and rational exponents using the  properties of exponents. | Example: �x3=x32  Example: �43= �412�3=23=8 |
| **9-10.NO.2** Perform basic operations on simple  radical expressions to write a simplified equivalent  expression. | Basic operations include addition, subtraction,  multiplication, and division (e.g., rationalizing the  denominator (no conjugation)).  Example:  Simplify: 2√9 - 3√9 + √4 √7  Example:  Rationalize: 1√2 |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **46** | P a g e

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| **9-10.NO.3** Choose and interpret the scale and the  units in graphs and data displays. | Example:  Are there any false impressions in this graph? If so,  how could you change the scale to alleviate false  impressions?  Example: Gary sold candy bars to raise money for  the German Club. He raised a total of $1000 for selling 400 candy bars. Graph the relationship  between candy bars sold and the total raised. |
| **9-10.NO.4\*** Define appropriate quantities and units  for the purpose of descriptive modeling. | Example:  When carpeting a room, learners may consider  whether it is best to use square feet or square  yards. When considering a remodeling project, they  may choose such units as cost per room, cost per  month of the project, or cost per contractor.  Example:  It takes Jeb 4.5 hours to run 50 kilometers. What is  Jeb’s rate in minutes per mile?  Solution:  4.5hr 1.609 km 60min  50km ⋅ 1 mi ⋅1 hr min/ m ≈ 8.689 min/mi |
| **9-10.NO.5** Choose a level of accuracy or precision  appropriate to limitations on measurement when  reporting quantities. | This standard applies across all high school grade  levels.  Example:  When using a ruler, learners report their  measurements based on the ruler's precision (e.g.,  to the nearest 1/16 or the nearest 1/32). They are  able to measure accurately.  Example:  If you play soccer and you always hit the left  goalpost instead of scoring, then you are not  accurate; you are precise.  Example:  When using a ruler, learners are able to measure  accurately. |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **47** | P a g e

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|  | Example:  When calculating the cost of a road trip, learners  are given the cost of gasoline to the thousandth  place. When reporting the trip cost, learners  determine what level of precision (to the  hundredths place or thousandths place) is appropriate and why. |
| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Standards** | **Clarification** |
| **9-10.AR.1** Use the structure of an expression (i.e.,  quadratic and exponential) to identify ways to  rewrite it. | This standard includes rewriting expressions by  factoring, combining like terms, using factoring  techniques, applying distributive property, applying  operations with polynomials, and recognizing  patterns and structures in expressions.  Example:  See 9a2- 4b2 as (3a)2- (2b)2 and recognize it as a  difference of squares that can factor as  (3a-2b)(3a+2b).  Example:  See x4- y4 as ( ) ( ) 2 2 2 2 x y − , thus recognizing it as a  difference of squares that can be factored as  ( )( ) 2222 xyxy − + and further to  ( )( )( ) 2 2 xyxyx y −+ + . |
| **9-10.AR.2** Rearrange formulas to isolate a quantity  or variable(s) of interest using the same reasoning  as in solving equations. | This standard applies across all high school grade  levels.  Example: Rearrange V=IR to solve for the  resistance R in Ohm’s Law. |
| **9-10.AR.3\*** Create equations and inequalities in  one variable and use them to solve problems. Include equations arising from linear, quadratic,  and exponential functions. | Limit inequalities to linear and quadratic. |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **48** | P a g e

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| **9-10.AR.4\*** Create linear and exponential equations  in two or more variables to represent relationships  between quantities.  Graph equations on coordinate axes with  appropriate labels and scales. | Limit to situations requiring evaluation of  exponential functions at integer inputs.  Example:  The cost to rent a car is $50 plus $0.25 per mile driven. Write and graph an equation to represent  the situation. |
| **9-10.AR.5** Justify each step in solving a linear  equation that may or may not have a solution. | Use justifiable comments such as “combine like  terms,” and “distributive property,” within the  explanation.  Example:  2(3x - 5) + 3x - 2(6 + x) = 5x -3 + 6x +17  Solution:  2(3x-5)+3x-2(6-x)=11x+14 Combine like terms  6x-10+3x-12+2x=11x+14 Distributive Prop. 11x-22=11x+14 Combine like terms -22=0x+14 Additive Inverse -22≠14 |
| **9-10.AR.6** Solve linear equations and inequalities  (to include compound inequalities) in one variable. | Examples:  Solve:  -4(x - 3) + 8 < -10 + 2x  Solve:  - 3 y + 7 > 10  4  Solve:  6 + 7d < 6d - 5 or 3d - 7 < 5 + 6d  Solve:  4 (4x - 3) = 1 (5x - 5)  3 7 |
| **9-10.AR.7\*** Solve a system of linear equations  graphically and algebraically.  Create and solve a system of linear equations in  context. | Example:  Solve:  { 21 3y 12x  − =− −  2 x 4y  =− −  Example:  Sarah had $12,100 to invest. She decided to invest  her money in bonds and mutual funds. She  invested a portion of the money in bonds paying  8% interest per year and the remainder in a mutual  fund paying 9% per year. After one year, the total  income she had earned from the investments was $1,043. How much had she invested in each rate? |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **49** | P a g e

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| **9-10.AR.8** Graph the solution set to a two-variable  system of linear inequalities.  Create and graph the solution set to a two-variable  system of linear inequalities in context. | Example:   ≥− +  ≥ −  y x1  Solve by graphing:  y x1  Example:  The girls’ swim team is hosting a fundraiser. They  would like to raise at least $500. They are selling  candles for $5 and flower arrangements for $6. The  girls estimate that, at most, they will sell 200 items.  • Write a system of inequalities to represent  this situation.  • Graph each inequality on a grid. |
| **9-10.AR.9** Solve absolute value equations and  inequalities in one or two variables. | Example:  Solve: 3|x + 2| -6 = 6  Solution: x = 2 or x = -6  Example:  Solve y = |x + 3| - 2 graphically.  Example:  Solve y < |x + 3| - 2 graphically. |
| **9-10.AR.10** Solve quadratic equations in one  variable by inspection (e.g., for x² = 49) taking  square roots, the quadratic formula, and factoring,  as appropriate to the initial form of the equation. |  |
| **9-10.AR.11** Add, subtract, and multiply  polynomials. | Focus on polynomial expressions that simplify to  forms that are linear or quadratic. |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **50** | P a g e

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| **Functions (F)**  *Learners will develop a foundational knowledge of functions and use them to model relationships between  quantities.* | |
| **Standards** | **Clarification** |
| **9-10.AR.F.1** Determine whether a relationship is a  function given a table, graph, or words, identifying x  as an element of the domain and f(x) as an element  in the range.  Determine the domain and range of a function in  context. | Example:  State the domain and range of the function graphed  at the right.  Solution:  Domain: -6 ≤ x ≤ 6 or [-6,6]  Range: 0 ≤ y ≤ 6 or [0,6] |
| **9-10.AR.F.2\*** Use function notation, evaluate  functions for inputs in their domains and interpret  statements that use function notation in context. | Example:  Suppose f(d) = 0.5d + 50 where f(d) represents the  cost of renting a car driven d miles. Evaluate f(200)  and interpret the result.  Example:  Given P(s)=4s, where P(s) represents the perimeter  of a square whose side length is s, P is a function of s. |
| **9-10.AR.F.3\*** Sketch the key features (to include  intercepts, maximums, minimums, and lines of  symmetry, where applicable) of linear, exponential,  and quadratic functions modeling the relationship  between two quantities using tables, graphs, written  descriptions, and equations. | Example:  Given f(x) = x2- 4*.* Graph the function and identify  the intercepts, maximums, minimums, and any  symmetry.  Solution:  Intercepts: (-2,0), (2,0),  (0,-4)  Relative Minimum: (0,-4)  Symmetric to the y-axis |
| **9-10.AR.F.4\*** Relate the domain of a linear,  quadratic, or exponential function to its graph and,  where applicable, to the quantitative relationship it  describes. | Example:  A vehicle depreciates roughly 20% per year.  Suppose the situation is modeled by the equation  f(x) = 39,900 (0.8)x. Graph the function and justify  the domain. |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **51** | P a g e

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| **9-10.AR.F.5\*** Calculate and interpret the rate of  change of linear, quadratic, or exponential functions (presented algebraically or as a table) over  specified intervals.  Estimate the rate of change from a graph. | Focus on linear, quadratic, and exponential  functions whose domains are a subset of the  integers.  Example:  Jamie went on a bike trip and stopped regularly at  half-hour intervals. At each break, he recorded his  total distance since leaving home.  Stops Time (h) Distance (km)  1st 0.5 7  2nd 1 15  3rd 1.5 21  4th 2 24  5th 2.5 28  6th 3 36  What was Jamie’s average speed, in km/h, during the first half-hour? During the last half-hour? Justify  why the speeds are different. |
| **9-10.AR.F.6\*** Write a function defined by an  expression in different but equivalent forms to  reveal and explain the different properties of the  function.  a. Use appropriate forms of linear, quadratic,  and exponential functions to show zeros,  extreme values, and symmetry (where  applicable) and interpret them in context.  b. Use the properties of an exponential  function to classify it as growth or decay. | Example:  Identify the percent rate of change in functions such t  as y =(1.02)t, y =(0.97)t, y =(1.01)12t, y =(1.2)10and classify them as representing exponential  growth or decay.  Example:  Given a quadratic function, explain the meaning of  the zeros of the function. That is if  f(x) = x2- 7x + 12 =(x – 4) (x – 3), then  f(4) = 0 and f(3) = 0.  Example:  A toy rocket is launched at 128 ft/sec from a height  of 5 feet. What is the maximum height of the rocket,  and when does the rocket reach that height?  Example: 8t = 23t  Example: The expression t 1.15 can be rewritten as 12t 1~~12~~ 12t 1.15 1.012      ≈  to reveal the approximate     equivalent monthly interest rate if the annual rate is  15%. |
| **9-10.AR.F.7\*** Compare key features of two linear,  exponential, or quadratic functions, each  represented in a different way (algebraically,  graphically, numerically in tables, or by verbal  descriptions). | Example:  Given a graph of one quadratic function and an  algebraic representation for another function, say  which has the larger maximum.  Example:  Compare the intercepts of two functions, one  represented graphically, and the other is  represented symbolically. |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **52** | P a g e

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| **9-10.AR.F.8\*** Identify situations that can be  modeled with linear, quadratic, and exponential  functions.  Justify the most appropriate model for a situation  based on the rate of change over equal intervals.  Include situations in which a quantity grows or  decays. | Example:  A person earning $10 per hour experiences a  constant rate of change in salary given the number  of hours worked.  Example:  The number of bacteria on a dish doubles every  hour and will have equal factors over equal intervals. |
| **9-10.AR.F.9\*** Identify the effect of transformations on  the graph of a linear, absolute value, or quadratic  function by replacing f(x) with af(x), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive  and negative).  Find the value of a, h, and k given the graph of the  function. |  |
| **9-10.AR.F.10\*** Find the inverse of a linear function  and describe the relationship between the domain,  range, and graph of the function and its inverse in  context. | Example:  An internet provider charges an initial equipment  fee of $500 and an additional $100 per month for  using their satellite internet service. The following  function represents this linear relationship: C(x) = 100x + 500 where x is the number of months  and C(x) is the combined cost.  Find C-1(x) and describe this inverse relationship. |
| **9-10.AR.F.11\*** Interpret the parameters in a linear,  quadratic, or exponential function in context. | Parameters for a linear: values of m and b  Parameters of a quadratic: values of a and c  Parameters for an exponential: values of a and b  Example:  In the equation y = mx + b *,* m and b are  parameters that specify the particular line  represented by the equation.  Example:  A cell phone plan costs $40 a month plus 2 cents  per minute of usage. Write a function that shows  the monthly cost of using your cell phone and  interpret the parameters.  Answer: C = 0.02m + 40  The monthly cost is 40 dollars plus two cents times  the number of minutes used. The minimum cost per  month is $40 (at m = 0).  Example:  Interpret ½h (b₁ + b₂) as the product of the height of a trapezoid and the average of its base lengths. |
| **9-10.AR.F.12** Identify, using graphs or tables, the  solution(s) to linear and exponential functions f(x) = g(x) as x-value(s) that result in equivalent  y-values. | Example:  Use a graphing calculator to find and justify the  approximate solution(s) to the system below. x f(x) 2   =  = −  g(x) 4x 4  Solution(s): x = 2 therefore f(2) = 4, g(2) = 4 |

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| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate  the characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Standards** | **Clarification** |
| **9-10.GM.1** Know precise definitions and notations  of angle, circle, perpendicular line, parallel line, and  line segment based on the undefined notions of point, line, and plane. | Example:  An angle is composed of two rays that share a  common initial point. |
| **9-10.GM.2** Represent transformations in the plane.  Describe transformations as functions taking points  in the plane as inputs and giving other points as  outputs.  Compare transformations that preserve distance  and angle to those that do not (i.e., rigid versus  non-rigid motion). |  |
| **9-10.GM.3** Describe the rotations and reflections of  a triangle, rectangle, parallelogram, trapezoid, or  regular polygon that map each figure onto itself or another figure. |  |
| **9-10.GM.4** Develop or verify the characteristics of  rotations, reflections, and translations in angles,  circles, perpendicular lines, parallel lines, and line  segments. | Example:  Using patty paper or geometry software,  develop/verify that the reflection line is the perpendicular bisector of the segment that  connects the pre-image to its image. |
| **9-10.GM.5** Draw the image of a figure that has  undergone a series of transformations [rotation(s),  reflection(s), or translation(s)] of a geometric figure  using a variety of methods (e.g., graph paper, tracing paper, or geometry software). | Learners must be able to perform and draw a  series of transformations as well as describe said  transformations to successfully produce the  resulting image. |
| **9-10.GM.6** Predict the effect of a specified rigid  motion on a given figure using geometric  descriptions of rigid motions.  Determine whether two figures are congruent using  the definition of congruence in terms of rigid motions. | Learners must be able to predict and recognize  rigid motions and use them to justify congruence. |
| **9-10.GM.7** Use the definition of congruence, based  on rigid motions, to show two triangles are congruent if and only if their corresponding sides  and corresponding angles are congruent. |  |
| **9-10.GM.8** Prove two triangles are congruent using  the congruence theorems. | “Proof” may take on various forms (flow, paragraph,  2-column, informal). |
| **9-10.GM.9** Prove and apply theorems about lines  and angles. | “Proof” may take on a variety of forms (flow,  paragraph, 2-column, informal).  Theorems include but are not limited to vertical  angles are congruent; when a transversal crosses  parallel lines, alternate interior angles are  congruent and corresponding angles are  congruent; points on a perpendicular bisector of a  line segment are exactly those equidistant from the segment’s endpoints. |

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| **9-10.GM.10** Prove and apply theorems about  triangles. | “Proof” may take on a variety of forms (flow,  paragraph, 2-column, informal).  Theorems include but are not limited to measures  of interior angles of a triangle sum to 180°; base  angles of isosceles triangles are congruent; the  segment joining midpoints of two sides of a triangle  is parallel to the third side and half the length; the medians of a triangle meet at a point. |
| **9-10.GM.11** Prove and apply theorems about  parallelograms. | “Proof” may take on a variety of forms (flow,  paragraph, 2-column, informal).  Theorems include but are not limited to opposite  sides are congruent, opposite angles are  congruent, the diagonals of a parallelogram bisect  each other, and rectangles are parallelograms with congruent diagonals. |
| **9-10.GM.12** Make basic geometric constructions  (e.g., segment, angle, bisectors, parallel and  perpendicular lines) with a variety of tools and methods. | Tools may include a compass and straightedge,  string, reflective devices, paper folding, or dynamic  geometric software. |
| **(+) 9-10.GM.13** Apply basic constructions to create  polygons such as equilateral triangles, squares,  and regular hexagons inscribed in circles. | Learners can use technology or compass and  straightedge to accomplish the construction. |
| **9-10.GM.14** Verify experimentally and justify the  properties of dilations given by a center and a scale factor. |  |
| **9-10.GM.15** Use transformations to decide if two  given figures are similar.  Apply the meaning of similarity for triangles as the  equality of all corresponding pairs of angles and the  proportionality of all corresponding pairs of sides. |  |
| **9-10.GM.16** Prove similarity theorems about  triangles. | “Proof” may take on a variety of forms (flow,  paragraph, 2-column, informal). |
| **9-10.GM.17** Apply knowledge of congruence and  similarity criteria for triangles to solve problems and to prove relationships in various geometric figures. |  |
| **9-10.GM.18** Recognize how the properties of  similar right triangles allow the trigonometric ratios  to be defined and determine the sine, cosine, and  tangent of an acute angle in a right triangle. | Example:  Verify experimentally that the side ratios in similar  right triangles depend upon the measure of an  acute angle in the triangle due to the preservation  of angle measure in similarity. Use this discovery to  develop definitions of the trigonometric ratios for acute angles. |
| **(+) 9-10.GM.19** Explain and use the relationship between the sine and cosine of complementary  angles. |  |
| **9-10.GM.20\*** Solve applied problems involving right  triangles using trigonometric ratios, the  Pythagorean Theorem, and special right triangles  (30°-60°-90° and 45°-45°-90°). |  |

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| **(+) 9-10.GM.21\*** Solve unknown sides and angles of non-right triangles using the Laws of Sines and  Cosines. |  |
| **9-10.GM.22** Apply theorems about relationships  between line segments and circles or angles and  circles formed by radii, diameter, secants, tangents,  and chords to find unknown lengths or angles. | Example: solve for x: Example: Solve for x: Solution: x = 20 Solution: x = 25ο |
| **(+) 9-10.GM.23** Construct the incenter and  circumcenter of a triangle.  Relate the incenter and circumcenter to the  inscribed and circumscribed circles. | Learners may use technology to perform the  constructions. |
| **(+) 9-10.GM.24** Construct a tangent line from a  point outside a given circle to the circle. |  |
| **9-10.GM.25** Explain and use the formulas for arc  length and area of sectors of circles. |  |
| **9-10.GM.26** Recognize that the radian measure of an  angle is the ratio of the length of the arc to the length  of the radius of a circle. |  |
| **9-10.GM.27** Develop and verify the slope criteria for  parallel and perpendicular lines.  Apply the slope criteria for parallel and  perpendicular lines to solve problems. | Example:  Find the equation of a line parallel or perpendicular  to a given line that passes through a given point. |
| **9-10.GM.28** Verify simple geometric theorems  algebraically using coordinates.  Verify algebraically, using coordinates, that a given  set of points produces a particular type of triangle  or quadrilateral. | This standard allows for coordinate proof.  Example: Given a rhombus with vertices at (2, 0),  (-2, 0), (0, 3) and (0, -3), verify that the diagonals  are perpendicular.  Example:  Verify algebraically whether a figure defined by four  given points in the coordinate plane is a rectangle. |
| **9-10.GM.29** Determine the midpoint or endpoint of  a line segment using coordinates.  (+) Find the point on a directed line segment  between two given points that partitions the segment in a given ratio. | (+) Example:  Find the coordinate pair that is 2/3 the distance  from the point (2, 3) to (-4, 7). |
| **9-10.GM.30\*** Compute perimeters of polygons and areas of triangles, parallelograms, trapezoids, and  kites using coordinates. |  |

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| **9-10.GM.31** Explain derivations of the formulas for  the circumference of a circle, area of a circle, and  volume of a cylinder, pyramid, and cone. | May use dissection arguments, Cavalieri’s  Principle, or informal limit arguments.  Example:  The area of a circle can be reduced by rearranging  the sectors of two semi-circles to form a rough  rectangle.  Area :  =r⋅1⋅Circumference  2  =r⋅1⋅2πr  2  =πr2 |
| **9-10.GM.32** Calculate the surface area for prisms,  cylinders, pyramids, cones, and spheres to solve problems. |  |
| **9-10.GM.33** Know and apply volume formulas for  prisms, cylinders, pyramids, cones, and spheres to  solve problems. | Example:  Find the volume of the composite figure below: |
| **9-10.GM.34** Identify the shapes of two-dimensional  cross-sections of three-dimensional objects and  identify three-dimensional objects generated by rotations of two-dimensional objects. |  |
| **9-10.GM.35\*** Apply concepts of density based on  area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). |  |
| **9-10.GM.36\*** Apply geometric methods to solve  design problems (e.g., designing an object or  structure to satisfy physical constraints or minimize  cost; scaling a model). | Example:  Learners design a soft drink package that  minimizes surface area and cost.  Example:  Design an art sculpture composed of at least 4  solids. Calculate the amount of material used to  build it. |

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| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying  basic concepts of probability.* | |
| **Standards** | **Clarification** |
| **9-10.DPS.1\*** Represent data with plots on the real  number line (dot plots, histograms, and box plots). |  |
| **9-10.DPS.2\*** Compare the center (median, mean)  and spread (interquartile range, standard deviation) of two or more different data sets using statistics  appropriate to the shape of the data distribution. | Learners may use technology to find the standard  deviation. |
| **9-10.DPS.3\*** Represent data on two quantitative  variables on a scatter plot and describe how the  variables are related.  a. Fit a linear function to the data (with or  without technology) if appropriate.  b. Compute (using technology) and interpret  the correlation coefficient of a linear fit.  c. Interpret the meaning of the slope and y intercept of the linear model in context.  d. Interpolate and extrapolate the linear model  to predict values. |  |
| **9-10.DPS.4\*** Distinguish between correlation and  causation. | Example:  It is noted that there is a high correlation between  people who eat ice cream daily and their annual job  salary. Does eating ice cream predict salary or vice-versa? |
| **9-10.DPS.5\*** Describe events as subsets of a  sample space (the set of outcomes) using  characteristics (or categories) of the outcomes or  as unions, intersections, or complements of other events (“or,” “and,” “not”). | Example:  Given a classroom of 30 learners, list the subset of  learners in the room who are blonde and have blue  eyes. |
| **9-10.DPS.6\*** Recognize that event A is  independent of event B if the probability of event A  does not change in response to the occurrence of  event B.  Apply the formula P(A and B) = P(A)·P(B) given  that events A and B are independent. | Understand that two events, A and B, are  independent if the probability of A and B occurring  together is the product of their chances and use  this characterization to determine if they are  independent. |
| **9-10.DPS.7\*** Recognize that the conditional  probability of an event A given B is the probability  that event A will occur given the knowledge that  event B has already occurred.  Calculate the conditional probability of A given B  and interpret the answer in context. | Example: A math teacher gave her class two tests.  25% of the class passed both tests and 42% of the  class passed the first test. What percent of those  who passed the first test also passed the second  test?  Solution:  P(A and B) = P(passed both) = 0.25  P(A) = P(passed the first test) = 0.42  Find P(B) given that P(A) is true:  = P(A and B)/P(A) = 0.25/0.42 = 0.6 = 60%  Therefore, 60% of those students who passed the  first test also passed the second test. |

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| **9-10.DPS.8\*** Apply the formula  P(A or B) = P(A) + P(B) – P(A and B) and interpret  the answer in context. |  |
| **9-10.DPS.9\*** Determine the number of outcomes  using permutations and combinations in context. |  |
| **9-10.DPS.10\*** Construct and interpret two-way  frequency tables of data for two categorical  variables.  Use the two-way table as a sample space to decide  if events are independent and to approximate  conditional probabilities. | Example:  Collect data from a random sample of learners in  your school on their favorite subject among  mathematics, science, and English. Estimate the  probability that a randomly selected student from  your school will favor science, given that the  student is in 10th grade. Do the same for other  subjects and compare the results.  Example:  Compare the chance of having lung cancer if you  are a smoker with the possibility of being a smoker  if you have lung cancer. |

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**ELEVENTH AND TWELFTH GRADES**

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| **Math Attributes (MA)**  *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for  advancement through participation in various relevant learning experiences.* | | |
| **Problem-Solving (P)** | **Connections (C)** | **Reasoning and Proof (R)** |
| *Analyze, execute, evaluate, and  adapt approaches and solutions  when problem-solving in novel  situations.* | *Create connections within and  across concepts, using*  *supporting evidence to interpret  how they originate, extend, and relate to other learning, ideas,  and life experiences.* | *Reason logically, citing relevant  evidence to explain and critique  what they see, think, and*  *conclude through exploration,  generalization and validation.* |
| **9-12.MA.P** Learners can analyze,  execute, critique, and adapt  approaches and solutions when  problem-solving in novel  situations. | **9-12.MA.C** Learners can create  connections within and across  concepts, using supporting  evidence to interpret how they  originate, extend, and relate to  other learning, ideas, and life experiences. | **9-12.MA.R** Learners can reason  logically, citing evidence to  critique and explain what they  see, think, and conclude through  exploration, generalization, and  validation. |

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* | |
| **Standards** | **Clarification** |
| **11-12.NO.1**.Rewrite complex expressions involving  radicals and rational exponents using the properties  of exponents. | This standard is an extension of 9-10.NO.1. Example:  3 2  4xy  Simplify:  3 4 4  32x y  1  =  3 3 2  8x y  1  3  1 1y.  = =  32 2 1  33 3 3  2x y 2xy y  3  y  =  2xy |

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| **11-12.NO.2** Perform operations on complex radical  expressions and simplify radicals to write  equivalent expressions. | This standard is an extension of 9-10.NO.2.  Operations include addition, subtraction,  multiplication, and division (e.g., rationalizing the  denominator and performing conjugation).  Example: �63x4+2�28x3- 5x2√7  Solution:  4 32  = +−  9 7 x 2 28x 5x 7  2 2  =+−  3x 7 4x 7x 5x 7  2  =− +  2x 7 4x 7x |
| **11-12.NO.3** Demonstrate that the sum or product of  two rational numbers is rational; that the sum of a  rational number and an irrational number is  irrational, and that the product of a nonzero rational  number and an irrational number is irrational. | Example:  Evaluate √2 ∙ √4 and identify which subset of the  real number system the solution is in.  Solution:  √8 = 2√2, which is irrational. |
| **11-12.NO.4\*** Use units to understand problems and  to guide the solution of multi-step problems (e.g.,  unit analysis).  Choose and interpret units consistently in formulas.  Choose and interpret the scale and the units in  graphs and data displays. | This standard is an extension of 9-10.NO.3 and  9-10.NO.4.  Example:  Blood sugar level is measured in milligrams of  glucose per deciliter of blood volume. If a person’s  blood sugar level measures 128 mg/dL, what is this in grams per liter? |
| **11-12.NO.5\*** Choose a level of accuracy or  precision appropriate to limitations on  measurement when reporting quantities. | This standard applies to all high school  mathematics.  Example:  When using a ruler, learners choose to report their  measurements based on the precision of the ruler  (e.g., to the nearest 1/16 or the nearest 1/32).  Example:  If you are playing soccer and you always hit the left  goal post instead of scoring, then you are not  accurate; you are precise.  Example:  When using a ruler, learners are able to measure  accurately.  Example:  When calculating the cost of a road trip, learners  are given the cost of gasoline to the thousandth  place. When reporting the cost of the trip, learners  determine what level of precision (to the  hundredths place or to the thousandth place) is  appropriate and why. |

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| **11-12.NO.6** Know there is a complex number i such  that i² = -1, and every complex number has the form a + bi with a and b real.  Understand the hierarchal relationships among  subsets of the complex number system. | Knowledge of complex numbers extends and  reinforces student knowledge of the real number  system.  Example: √8 is a complex number because it can  be written in the form √8 + 0i.  √8 is also a real number since its imaginary  coefficient is 0.  √8 is also an irrational number because it cannot  be written as a ratio of two integers. |
| **11-12.NO.7** Use the definition i² = -1 and the  commutative, associative, and distributive  properties to add, subtract, and multiply complex  numbers. | Knowledge of complex numbers extends and  reinforces student knowledge of basic operations  and properties of the real number system.  Example:  (2 + 3i) + (4 - 5i) = 6 - 2i  (2+ 3i) - (4 - 5i ) = -2 + 8i  (2 + 3i)(4 - 5i) = 8 -10i + 12i -15i2  = 8 + 2i + 15  = 23 + 2i |
| **11-12.NO.8** Use conjugates to find quotients of  complex numbers. |  |
| **11-12.NO.9** Apply the Fundamental Theorem of  Algebra to determine the number of zeros for  polynomial functions.  Find all solutions to a polynomial equation. | This standard applies to multiple high school  mathematics levels. |
| **(+) 11-12.NO.10** Represent complex numbers on  the complex plane in rectangular, trigonometric,  and polar forms.  Find the modulus (absolute value) of a complex  number.  Explain why the rectangular, trigonometric, and  polar forms of a given complex number represent the same number. |  |
| **(+) 11-12.NO.11** Represent addition, subtraction,  multiplication, conjugation, powers, and roots of  complex numbers geometrically on the complex  and/or polar plane; use properties of this  representation for computation. | Example:  �1 - i√3�3= -8 because ( ) 1i3 − written in polar form  is   2,300  ο . Applying de Moivre’s Theorem yields 3 3      2,300 2 ,300 3 8,180 = ⋅=      οο ο  a = 8 cos 1 80∘= -8  b = 8 sin 1 80∘= 0  a + bi = -8 + 0i |

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| **(+) 11-12.NO.12** Extend polynomial identities to the  complex numbers. | Example:  Rewrite x2+ 4 as (x + 2i)(x - 2i).  Polynomial identities include but are not limited to: (a +b)2 = a2 + 2ab + b2  (a + b)(c + d) = ac + ad + bc + bd  a2- b2 = (a + b)(a - b)  a3 + b3 = (a + b)(a2- ab + b2)  a3- b3= (a - b) (a2+ ab + b2)  x2+ (a + b)x + ab = (x + a)(x + b) |
| **(+) 11-12.NO.13** Apply the Fundamental Theorem  of Algebra to find all roots of a polynomial equation  and determine the nature (i.e., integer, rational, irrational, real, complex) of the roots. | This standard applies to multiple high school  mathematics levels. |
| **(+) 11-12.NO.14** Recognize vector quantities as  having both magnitude and direction, writing them  in polar form. |  |
| **(+) 11-12.NO.15** Find the components of a vector  by subtracting the coordinates of an initial point from the coordinates of a terminal point. |  |
| **(+) 11-12.NO.16** Solve problems involving magnitude and direction that can be represented by  vectors. |  |
| **(+) 11-12.NO.17** Add and subtract vectors. a. Add vectors end-to-end, component-wise,  and by the parallelogram rule. Know that the  magnitude of a sum of two vectors is  typically not the sum of the magnitudes.  b. Given two vectors in magnitude and  direction form, determine the magnitude and  direction of their sum.  c. Understand that vector subtraction **v**-**w** is  defined as **v**+(-**w**), where -**w** is the additive  inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction.  Represent vector subtraction graphically by  connecting the tips in the appropriate order and  using the components to perform vector  subtraction. |  |
| **(+) 11-12.NO.18** Multiply a vector by a scalar. a. Represent scalar multiplication graphically  by scaling vectors and possibly reversing  their direction. Use the components to  perform scalar multiplication  (e.g., as c(vx, vy) = (cvx , cvy)).  b. Compute the magnitude of a scalar multiple  c**v** using ||c**v**|| = |c|**v**.  c. Compute the direction of cv knowing that when |c|**v** ≠ 0, the direction of c**v** is either  along **v** (for c > 0) or against **v** (for c < 0). |  |

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| **(+) 11-12.NO.19** Represent data in a matrix.  Perform operations (i.e., addition, subtraction,  multiplication) on matrices of appropriate  dimensions to solve problems and in context.  Know that matrix multiplication is not commutative. |  |

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| **Algebraic Reasoning (AR)**  *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to  represent mathematical models while adopting approaches and solutions in novel situations.* | |
| **Standards** | **Clarification** |
| **11-12.AR.1\*** Rearrange multi-variable formulas to  highlight a quantity of interest. | In grades 11-12, the linear, exponential, or  quadratic types of problems should draw from more  complex situations than those addressed in grades  9-10. This standard is an extension of 9-10.AR.2.  Example: Solve A=P(1 + r)t for r. |
| **11-12.AR.2** Use the structure of an expression (to  extend to polynomial and rational expressions) to  identify ways to rewrite it. | Learners in grades 11-12 extend their focus to  polynomial and rational expressions. This standard  is an extension of 9-10.AR.1  Example:  See 9a2- 4b2 as (3a)2- (2b)2 and recognize it as a  difference of squares that can be factored as (3a - 2b)(3a + 2b).  Example:  See x4 - y4 as (x²)² - (y²)², thus recognizing it as a  difference of squares that can be factored as (x² – y²)(x² + y²), and further to (x - y)(x + y)(x2 + y2). |
| **11-12.AR.3\*** Interpret expressions that represent a  quantity in context.  a. Interpret parts of an expression, such as  terms, factors, and coefficients.  b. Interpret complicated expressions by  viewing one or more of their parts as a  single entity. | Learners in grades 11-12 extend their focus to  polynomial and rational expressions.  Example: Interpret 12 h(b1+b2) as the product of the height of a trapezoid and the average of its  base lengths. |
| **11-12.AR.4\*** Choose and produce an equivalent  form of an expression to reveal and explain  properties of the quantity represented by the  expression.  a. Factor a quadratic expression to reveal the  zeros of the function it defines.  b. Use the properties of exponents to  transform exponential expressions.  c. Complete the square in a quadratic  expression to produce an equivalent  expression. | This standard is an extension of 9-10.AR.4.  Example:  Given a quadratic function explain the meaning of  the zeros of the function. That is if  f(x) = 6x2- 11x - 10 , then f(x) can be factored into  f(x) = (2x - 5)(3x + 2).  Therefore, f �52� =0 and f �-23� =0.  Example:  Find the center and radius of the circle whose equation is x2+ y2- 6x + 2y - 6=0. |

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| **11-12.AR.5** Add, subtract, multiply, and divide  rational expressions.  Understand that rational expressions form a system  analogous to rational numbers, closed under  addition, subtraction, multiplication, and division by  a nonzero rational expression. | Learners in grades 11-12 extend their  understanding beyond the quadratic expressions. |
| **11-12.AR.6** Rewrite simple rational expressions in  different forms. Write a(x)/b(x) in the form of q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are  polynomials with the degree of r(x) less than the  degree of b(x), using inspection, long division, or  technology for the more complicated examples. | Example: Use long division to rewrite:  3x3- 2x2+ 4x - 3  x2 + 3x + 3  in the form:28x + 30  (3x -11)+x2 + 3x + 3 |
| **11-12.AR.7\*** Create equations and inequalities and  use them to solve problems. Include equations  arising from linear and quadratic equations and  simple rational and exponential equations. | Learners in grades 11-12 use all available types of  functions to create such equations, including root  functions, but constrain to simple cases. This  standard is an extension of 9-10.AR.3. |
| **11-12.AR.8\*** Create equations in two or more  variables to represent relationships between  quantities.  Graph equations on coordinate axes with proper  labels and scales. | In grades 11-12, the linear, exponential, or  quadratic types of problems should draw from more  complex situations than those addressed in grades  9-10. This standard is an extension of 9-10.AR.4.  Example:  Every time Pinocchio lies, his nose grows about  20% of its size. Originally his nose is 2 inches long.  Write an equation that models the situation and  graph the model.  How many lies would he have to tell before his  nose is longer than 3 feet? |
| **11-12.AR.9\*** Represent constraints by equations or  inequalities and by systems of equations and/or  inequalities and interpret solutions as viable or non viable options in a modeling context. | In grades 11-12, the linear, exponential, or  quadratic types of problems should draw from more  complex situations than those addressed in grades  9-10. This is an extension of 9-10.AR.8  Example:  Willy Wonka’s Chocolate Factory makes *Wonka  Bars* and *The Everlasting Gobstopper*, among other  amazing treats. Oompa Loompas and Fuzzy Fizzies work on each item. The Oompa Loompas  spend 6 minutes making a *Wonka Bar* and 4  minutes mixing the ingredients for an *Everlasting  Gobstopper.* There are enough Oompa Loompas  for up to 6000 worker minutes per day. The Fuzzy  Fizzies spend about 1 minute wrapping each  *Wonka Bar* and 2 minutes wrapping each  *Everlasting Gobstopper*. There are enough Fuzzy  Fizzies for a maximum of 1200 worker minutes per  day.  Given the above constraints, find the feasible region for the number of *Wonka Bars* and  *Everlasting Gobstoppers* that can be made per day. |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **65** | P a g e

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|  | Solution:  Oompa Loompas:  (6 min/bar)(x bars) + (4 min/y gob) ≤ 6000 min.  Fuzzy Fizzies:  (1 min/bar)(x bars) + (2 min/gob)(y gob) ≤ 1200 min. using substitution, y = 150, x = 900 if the  maximum number of hours are worked. Therefore,  the feasible region for the number of *Wonka Bars* made in a day is 0 ≤ x ≤ 900, and the feasible  region for the number of *Everlasting Gobstoppers* is 0 ≤ y ≤ 150. |
| **(+) 11-12.AR.10** Derive the quadratic formula from the  form 0 = ax² + bx + c. |  |
| **11-12.AR.11** Solve quadratic equations with real  coefficients that have solutions of the form a + bi  and  a - bi. | This standard is an extension of 9-10.AR.10.  Example:  x2 + 2x + 5 = 0  -2 ±�(2)2 - 4(1) (5)   x =  2(1)  x = -2 ± √-16  2  x = -2 ± 4i  2  x = -1 ± 2i |
| **11-12.AR.12** Solve simple rational and radical  equations in one variable and identify extraneous  solutions. | Example:  Solve:  6 − x = x  ( 6 − x ~~)~~2 = x2  6 − x = x2  x2 + x − 6 = 0  (x + 3)(x − 2) = 0  x = 2 or x = −3  Check 2 and -3 in the original equation:  6 − 2 = 4 = 2  6 −(−3) = 9 = 3  Because -3 does not satisfy the original equation, -3 is an extraneous solution. 2 is the only solution  to the equation. |
| **11-12.AR.13** Add, subtract, and multiply  polynomials beyond quadratics.  Understand that polynomials form a system  comparable to integers, namely, they are closed  under the operations of addition, subtraction, and  multiplication. | Learners in grades 11-12 extend their  understanding beyond quadratic polynomials. This  standard is an extension of 9-10.AR.11. |
| **11-12.AR.14** Identify zeros of polynomial equations  when suitable factorizations are available.  Use the zeros to construct a rough graph of the  function defined by the polynomial. |  |

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| **11-12.AR.15** Apply the Factor and Remainder  Theorems to determine efficiently whether a linear  expression is a factor of a polynomial equation.  Apply the Remainder Theorem in context. | Pre-requisite knowledge for this standard includes  an understanding of polynomial division and  factoring.  Example:  The total production of eggs in billions in the United  States can be modeled by the function  f(x) = 0.007x3- 0.149x2 + 1.534x + 84.755, where x is the number of years since 2000. Predict the total  production of eggs in 2025.  Solution: f(25) = 139.355 billion eggs |
| **11-12.AR.16** Using graphs, technology, tables, or  successive approximations, show that the  solution(s) to the equation f(x) = g(x) are the x value(s) that result in the y-values of f(x) and g(x)  being the same. | Grades 11-12 will include combinations of linear,  polynomial, rational, radical, absolute value,  exponential and logarithmic functions. This  standard is an extension of 9-10.AR.F.12.  Example: Use a graphing calculator to find and  justify the approximate solution(s) to the system  below.  f(x) = x + 4   g(x) = 4 − x2  Solutions*:* x = 0 and x = -1.  f(0) = 4, g (0) = 4; f(-1) = 3, g(-1) = 3 |
| **11-12.AR.17** Solve a simple system consisting of a  linear equation and a quadratic equation in two  variables algebraically and graphically. | This standard is an extension of 9-10.AR.8  Example:  2 y x 2x 3   =−−  = −  Solve:  y 2x 3  Solutions: (0,-3), (4,5) |
| **(+) 11-12.AR.18** Find the inverse of a matrix if it  exists and use it to solve systems of linear  equations (using technology for matrices of  dimension 3 × 3 or greater). | Example: Solve using technology:  a b c  1 0 0  1 2 0  d e f  � = �  �∙ �  �  �  0 1 0  3 4 -1  g h i  0 0 1  5 7 2 |
| **(+) 11-12.AR.19** Solve a system of equations in  three or more variables with matrices (using technology). |  |
| **(+) 11-12.AR.20** Apply the Binomial Theorem for  the expansion of (ax + by)ⁿ in powers of x and y for  a positive integer n and integers a and b. |  |

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| **Functions (F)**  *Learners will develop a foundational knowledge of functions and use them to model relationships between  quantities.* | |
| **Standards** | **Clarification** |
| **11-12.AR.F.1\*** Write a function that describes a  relationship between two quantities.  a. Combine standard function types using  arithmetic operations.  b. Compose functions. | Example: Build a function that models the  temperature of a cooling body by adding a constant  function to a decaying exponential and relate these  functions to the model.  Example: If T(y) is the temperature in the  atmosphere as a function of height, and h(t) is the  height of a weather balloon as a function of time,  then T(h(t)) is the temperature at the location of the weather balloon as a function of time. |
| **11-12.AR.F.2\*** Calculate and interpret the average  rate of change of a function (presented symbolically  or as a table) over a specified interval. Estimate the  rate of change from a graph. | This standard is an extension of 9-10.AR.F.5  Example:  Find the rate of change over the interval 0 ≤ x ≤ 3  given the graph below.  Solution:  The average rate of change of a function y = f(x)  over an interval a ≤ x ≤ b is ∆y= f(b) - f(a). ∆x b - a  Therefore, the estimated average rate of change for  the function graphed above is 7 - 1  0 - 3 = -2. |
| **11-12.AR.F.3\*** Write a function defined by an  expression in different but equivalent forms to  reveal and explain the different properties of the  function.  a. Use the process of factoring and completing  the square in a quadratic function to show  zeros, minimum/maximum, and symmetry of  the graph, and interpret these in terms of  context.  b. Use the properties of exponents to interpret  expressions for exponential functions. | This standard is an extension of 9-10.AR.F.6.  Example:  Identify the percent rate of change in functions such  as  y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10,  and classify them as representing exponential  growth or decay*.* |

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| **11-12.AR.F.4\*** Identify the effect of transformations  on the graph of a function by replacing f(x) with  af(x), f(bx), f(x - h), and f(x) + k, for specific values  of a, h, and k (both positive and negative).  Find the value of a, b, h, and k given the graph of  the function.  Recognize even and odd functions from their  graphs and equations. | Technology may be used to experiment with the  effects of transformations on a graph. This standard  is an extension of 9-10.AR.F.9.  Learners in grades 11-12 will use transformations  of functions to find models as learners consider  increasingly more complex situations; note the  effect of multiple transformations on a single graph  and the common effect of each transformation  across function types. |
| **11-12.AR.F.5\*** Find inverse functions.  a. Verify by composition that one function is  the inverse of another.  b. Recognize that the graph of a function and  its inverse are reflection images over the  line y = x*.*  c. Produce an invertible function from a non invertible function by restricting the domain. | Learners in grades 11-12 will extend to simple  rational, simple radical, and simple exponential  functions. This standard is an extension of 9- 10.AR.F.10.  Example:  Find the inverse for each function:  f(x) = 2x3 or f(x) = x + 1 for x ≠ 1. x - 1 |
| **11-12.AR.F.6\*** Apply the inverse relationship  between exponents and logarithms to solve problems. |  |
| **11-12.AR.F.7\*** Compare key features of two  functions each represented in a different way  (algebraically, graphically, numerically, in tables, or  by verbal descriptions). | Grades 11-12 focus on using key features to guide  the selection of the proper type of model function.  This standard is an extension of 9-10.AR.F.7.  Example:  Given a graph of one quadratic function and an  algebraic expression for another, say which has the  larger maximum.  Example:  Compare the intercepts of two functions, one  represented graphically and the other symbolically. |
| **11-12.AR.F.8\*** Use tables, graphs, verbal  descriptions, and equations to interpret and sketch  the key features of a function modeling the  relationship between two quantities. | Key features include domain, range, intercepts;  intervals where the function is increasing, decreasing,  relative maximums and minimums; symmetries; end  behavior; and periodicity. This standard is an  extension of 9-10.AR.F.3.  Example:  Given f(x)=x2- 4. Graph the function and identify the  intercepts, intervals, where the function is increasing,  decreasing, positive, negative, the relative maximum  and minimum, and any symmetry.  Solution:  Intercepts: (-2, 0), (2, 0),  (0,-4)  Relative Minimum: (0, -4)  Increasing: x > 0  Decreasing: x < 0  Positive: x < -2, x > 2  Negative: -2 < x < 2  Symmetric to the y-axis |

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| **11-12.AR.F.9\*** Relate the domain of a function to its  graph and, where applicable, to the quantitative  relationship it describes. | Grades 11-12 emphasize the selection of a model  function based on behavior of data and content. This standard is an extension of 9-10.AR.F.4.  Example:  If the function h(n) gives the number of person hours it takes to assemble n engines in a factory,  then the positive integers would be an appropriate  domain for the function. |
| **11-12.AR.F.10\*** Graph functions expressed  symbolically and show key features of the graph, by  hand in simple cases and using technology for more complicated cases.  a. Graph square root, cube root, piecewise defined, step, and absolute value functions. b. Graph polynomial functions, identifying  zeros when suitable factorizations are  available and showing end behavior.  c. Graph exponential and logarithmic  functions, showing intercepts and end  behavior.  d. Graph f(x) = sin x and f(x) = cos x as  representations of periodic phenomena. | This standard addresses portions of 9-10.AR.F.3.  Example:  Solve the annual compound interest formula A = P(1 + r)t for t and draw a graph of time vs.  amount for a given rate and principal amount, showing intercepts and end behavior. Compare this  graph to the graph of amount vs. time. |
| **(+) 11-12.AR.F.11\*** Analyze and graph functions  expressed symbolically (by hand in simple cases  and using technology for more complicated cases),  identifying key features of the graph.  a. (+) Graph rational functions, identifying  domain, range, asymptote(s), removable  and non-removable discontinuities,  intercepts, behavior at the asymptote(s),  and end behavior.  b. (+) Graph trigonometric functions, showing  period, midline, phase shift, and amplitude. | This standard is an extension of 9-10.AR.F.3. |
| **11-12.AR.F.12\*** Compare the end behavior of  linear, quadratic, and exponential functions using  graphs and/or tables to show that a quantity  increasing exponentially eventually exceeds a  quantity increasing as a linear or quadratic function. |  |
| **11-12.AR.F.13\*** Determine whether a linear,  quadratic, polynomial, exponential, logarithmic, or  trigonometric model fits a situation.  Determine an appropriate mathematical model in  context (with or without technology). | This standard is an extension of 9-10.AR.F.7. |

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| **11-12.AR.F.14\*** Write arithmetic and geometric  sequences both recursively and with an explicit  formula and convert between the two forms.  Use sequences to model situations. | Example:  Allen is training for a biking race and begins his  workout regimen by biking 10 miles on day one and  increasing his mileage by 2 miles per day for the  next 15 days. Express the situation with a recursive  and explicit formula.  Solution:  Explicit Formula: an = 2n + 8  Recursive Formula:  �a₁ = 10  aₙ₋₁ +2, n>1, n∈Z |
| **11-12.AR.F.15\*** Use properties of logarithms to  express the solution to abᶜᵗ = d where a, c, and d  are real numbers and b is a positive real number.  Evaluate the logarithm using technology when  appropriate. | Example:  2t  =  3e 317  317 lne ln3         2t  =  317 2t ln3         =  t 2.330  ≈  Using a calculator and rounding t to the nearest  thousandth: t ≈ 2.330 |
| **11-12.AR.F.16** Extend right triangle trigonometry  and apply knowledge of the unit circle to determine  values of sine, cosine, and tangent for multiples of π/3, π/4, and π/6*.* | This standard is an extension of 9-10.GM.18 and  9-10.GM.20. |
| **11-12.AR.F.17** Use the Pythagorean Identity sin² (θ) + cos² (θ) = 1 to find sin (θ), cos (θ), or tan  (θ) given sin (θ), cos (θ), or tan (θ) and the  quadrant of the angle. | Example:  Given θ is a Quadrant II angle and sin θ = 4/5, find cos θ using the Pythagorean Identity. |
| **(+) 11-12.AR.F.18** Explain how the unit circle in the  coordinate plane enables the extension of  trigonometric functions to all real numbers,  interpreted as radian measures of angles traversed counterclockwise around the unit circle. | Example:  Find sin 7π6 |
| **(+) 11-12.AR.F.19** Use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x,  and 2π - x in terms of their values for x, where x is  any real number. |  |
| **(+) 11-12.AR.F.20** Use the unit circle to explain  symmetry (odd and even) and periodicity of trigonometric functions. |  |
| **(+) 11-12. AR.F.21** Create a trigonometric function  to model periodic phenomena. |  |
| **(+) 11-12. AR.F.22** Restrict the domain of a  trigonometric function to construct its inverse. |  |

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| **(+) 11-12. AR.F.23\*** Use inverse functions to solve  trigonometric equations that arise in modeling  contexts; evaluate the solutions and interpret them  in context. | Example:  A surveyor marks off points D, E, and F and  records that the measure of angle D is 40.2  degrees, d = 100 m, and f = 500 m. Explain why  there is a problem with the surveyor’s  measurements.  Solution: Using the Law of Sines,  sin∠ F ≈ 3.227 , which is not possible. The  surveyor has a measurement error.  The surveyor has a measurement error. |
| **(+) 11-12. AR.F.24** Know and apply the addition and subtraction formulas for sine, cosine, and  tangent to solve problems. |  |
| **Geometry and Measurement (GM)**  *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate  the characteristics of figures, perform transformations, and construct logical arguments.* | |
| **Standards** | **Clarification** |
| **11-12.GM.1** Write the equation of a conic section  given its special features.  Convert between the standard form and general  form equations of conic sections. | Conic sections include the circle, ellipse, parabola,  and hyperbola.  Key features include:  • Circle – center, radius  • Parabola – vertex, focus, directrix  • Ellipse – center, foci, vertices, length of the  major and minor axis  • Hyperbola – center, foci, asymptotes |
| **11-12.GM.2\*** Identify key features of a conic section  given its equation.  Apply properties of conic sections in context. | Identify key features of a conic section given its  equation.  Apply properties of conic sections in context. |
| **11-12.GM.3** Determine and apply appropriate formulas to solve right and non-right triangle  problems in context. | This standard is an extension of 9-10.GM.20. |
| **(+) 11-12.GM.4** Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line  from a vertex perpendicular to the opposite side. |  |
| **Data, Probability, and Statistics (DPS)**  *Learners will ask and answer questions by collecting, organizing, and displaying relevant data,  drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.* | |
| **Standards** | **Clarification** |
| **11-12.DPS.1\*** Interpret differences in shape,  center, and spread in the context of the data sets, accounting for possible effects of extreme data  points (outliers). |  |

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| **11-12.DPS.2\*** Use the mean and standard  deviation of a data set to fit it to a normal  distribution and estimate population percentages.  Recognize that there are data sets for which such a  procedure is not appropriate. | This standard is an extension of 9-10.DPS.2.  Example:  An example of a data set that does not fit to a  normal distribution is the age at retirement. Most  people retire in their mid-60s or older, with increasingly fewer retiring at increasingly earlier  ages. This results in a skewed-left distribution. |
| **11-12.DPS.3\*** Evaluate reports based on data. a. Identify and explain misleading use of data,  recognize when claims based on data  confuse correlation and causation.  b. Recognize and describe how graphs and data can be distorted to support different  points of view. |  |
| **11-12.DPS.4\*** Represent data on a scatter plot for  two quantitative variables and describe how the  variables are related.  a. Fit a function to the data (with or without  technology) and interpret the special  features (e.g., meaning of a and b in the  exponential function y = abˣ) of the function  in context.  b. Use functions fitted to data to solve  problems in the context of the data. | Use given functions or choose a function suggested  by the context. Emphasize linear, quadratic, and  exponential models. This standard is an extension  of 9-10.DPS.3. |
| **(+) 11-12.DPS.5\*** Informally assess the fit of a  function by plotting and analyzing residuals. | Use given functions or choose a function suggested  by the context. Emphasize linear and exponential  models. |
| **(+) 11-12.DPS.6\*** Use data from a sample survey to  estimate a population mean or proportion; develop a margin of error through the use of simulation models  for random sampling. |  |
| **(+) 11-12.DPS.7\*** Understand the process of  making inferences about population parameters  based on a random sample from that population. | Example:  Suppose 50 fish are tagged in a pond. A fisherman  catches 5 fish from the pond, and one has a tag. What conclusion can you draw about the fish  population? |
| **(+) 11-12.DPS.8\*** Decide if a specified model is  consistent with results from a given data-generating  process (e.g., using simulation). | Example:  A model says a spinning coin falls heads-up with a probability of 0.5. Would a result of 5 tails in a row  cause you to question the model? |
| **(+) 11-12.DPS.9\*** Recognize the purposes of and  differences among sample surveys, experiments,  and observational studies; explain how  randomization relates to each. | Example:  Design a simple study and explain the impact of  sampling methods, bias ,and the phrasing of questions asked during data collection. |
| **11-12.DPS.10\*** Determine when the order in  counting matters and use permutations and  combinations to compute probabilities of events  accordingly.  Determine probability situations as conditional, “or” (union), or “and” (intersection), and determine the  probability of an event. | This standard is an extension of 9-10.DPS.7, 9- 10.DPS.8, 9-10.DPS.9, and 9-10.DPS.10. |

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| **(+) 11-12.DPS.11\*** Use permutations and  combinations to compute probabilities of compound  events and solve problems. | Example:  Given a football team of 60 athletes, what is the  probability that the star quarterback and star  linebacker are not chosen for drug testing? 58C2 ≈ 0.934 ≈ 93.4%  60C2 |
| **(+) 11-12.DPS.12\*** Define a random variable for a  quantity of interest by assigning a numerical value  to each event in a sample space.  Graph the corresponding probability distribution  using the same graphical displays as for data  distributions. |  |
| **(+) 11-12.DPS.13\*** Calculate the expected value of  a random variable; interpret it as the mean of the  probability distribution. |  |
| **(+) 11-12.DPS.14\*** Weigh the possible outcomes of  a decision by assigning probabilities to payoff  values and finding expected values.  a. Find the expected payoff for a game of  chance.  Evaluate and compare strategies on the basis of  expected values. | Example:  Find the expected winnings from a state lottery  ticket or a game at a fast-food restaurant.  Example:  Compare a high-deductible versus a low-deductible  automobile insurance policy using various but  reasonable chances of having a minor or a major  accident. |
| **(+) 11-12.DPS.15\*** Develop a probability distribution  for a random variable defined for a sample space in  which theoretical probabilities are calculated; find  the expected value. | Example:  Find the theoretical probability distribution for the  number of correct answers obtained by guessing  on all five questions of a multiple-choice test where each question has four choices; find the expected  value. |
| **(+) 11-12.DPS.16\*** Develop a probability distribution  for a random variable defined for a sample space in  which probabilities are assigned empirically; find the expected value. | Example:  Find a current data distribution on the number of TV  sets per household in the United States and  calculate the expected number of sets per  household. How many TV sets would you expect to find in 100 randomly selected households? |

High School Symbols used: \* indicates a modeling standard (See Appendix C for the explanation of modeling.), and (+) indicates a “plus” standard used for advanced concepts or skills. **Assessment Boundary**: (+) Address advanced skills. **74** | P a g e

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| **(+) 11-12.DPS.17\*** Use probabilities to make fair  decisions (e.g., drawing by lots, using a random  number generator). | Example:  Tara and Brent decide to roll a pair of 6-sided dice  to determine who has to clean out the garage. • If the sum is 7, then Tara has to clean out  the garage.  • If the sum is 3 or 4, then Brent has to clean  out the garage.  • If the sum is anything else, they roll again.  Is this a fair way to decide who has to clean out the  garage? Why or why not?  Solution:  P(sum of 7) = 6/36  P(sum of 3 or 4) = P(3) + P(4) - P(3 and 4) = 2/36 + 3/36 - 0/36  = 5/36  P(anything else) = 25/36  This is not a fair way to decide because Tara has a  better chance of having to clean out the garage. |
| **(+) 11-12.DPS.18\*** Analyze decisions and  strategies using probability concepts (e.g., product  testing, medical testing, pulling a hockey goalie at the end of a game). |  |

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**NORTH DAKOTA**

**MATHEMATICS K-12**

**STANDARDS PROGRESSIONS**

NOTE: The new content or increased rigor in the standard content between grades is shown in blue type on the standards  progression. High School symbols used: (\*) indicates a modeling standard, and (+) indicates a “plus” standard used for advanced levels. **Assessment Boundary**: (+) Address advanced skills. **76** | P a g e

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| **Number and Operations (NO)**  *Learners will develop a foundational understanding of the number system, operations, and computational  fluency to create connections and solve problems within and across concepts.* |
| **Counting Forward** |
| **K.NO.CC.1** Count verbally in sequential order by ones and tens to 100, making accurate decuple  transitions (ex.89 to 90).  Count verbally forward from any given number within 100. |
| **1.NO.CC.1** Count forward by ones and tens from any given point within 120. |
| **2.NO.CC.1** Count forward from any given number within 1000. |
| **Counting Backward** |
| **K.NO.CC.2** Count backward from 20 by ones and from a given number within 10. |
| **1.NO.CC.2** Count backward by ones and tens from any given number within 120. |
| **2.NO.CC.2** Count backward from any given number within 1000. |
| **Number Identification and Writing** |
| **K.NO.CC.3** Identify and write any given numeral within 20. |
| **1.NO.CC.3** Represent several objects with a written numeral up to 120. |
| **2.NO.CC.3** Read and write numbers up to 1,000 using standard, word, and expanded forms. |
| **3.NO.CC.1** Read and write numbers up to 10,000 using objects or visual representations including  standard and expanded forms. |
| **4.NO.CC.1** Read numbers to the millions place including word, standard and expanded form. Write  numbers to the millions place including standard and expanded form. |
| **5.NO.CC.1** Read, write, and compare decimals to the thousandths including standard and expanded  forms. |
| **Subitizing** |
| **K.NO.CC.4** Recognize and verbally label arrangements, without counting, for briefly shown collections up  to 10 (e.g., "I saw 5." How do you know?" "I saw 3 and 2, that is 5."). |
| **1.NO.CC.4** Recognize and verbally label arrangements, without counting, for briefly shown collections up  to 20 (e.g., "I saw 16." "How did you know?" "I saw 10 and 6, that is 16."). |
| **Counting Patterns** |
| **K.NO.CC.5** Count and tell how many objects up to 20 are in an arranged pattern or up to 10 objects in a  scattered configuration. Represent a quantity of up to 20 with a numeral. |
| **1.NO.CC.5** Skip count forward and backward by 5s and 10s from multiples and recognize the patterns of  up to 10 skip counts. |
| **2.NO.CC.4** Skip count forward and backward by 2s and 100s and recognize the patterns of skip counts. |
| **9-10.DPS.9\*** Determine the number of outcomes using permutations and combinations in context. |
| **(+) 11-12.DPS.11\*** Use permutations and combinations to compute probabilities of compound events and  solve problems. |
| **Place Value** |
| **K.NO.NBT.1** Compose and decompose numbers from 11 to 19 using a group of ten ones and some more  ones using a model, drawing, or equation. |
| **1.NO.NBT.1** Demonstrate that the two digits of a two-digit number represent a composition of some tens  and some ones. |
| **2.NO.NBT.1** Understand that the three digits of a three-digit number represent a composition of some  hundreds, some tens, and some ones. |
| **4.NO.NBT.1** Understand that in a multi-digit whole number, a digit in one place represents ten times what  it represents in the place to its right. |
| **5.NO.NBT.1** Understand that in a multi-digit number, a digit in one place represents 10 times as much as  it represents in the place to its right and 1/10 of what it represents in the place to its left. |
| **8.NO.NS.3** Use scientific notation to represent very large or very small quantities. Interpret scientific  notation generated by technology. Compare and order numbers in scientific and standard notation. |

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| **Compare Numbers and/or Expressions** |
| **K.NO.NBT.2** Compare two numbers between 1 and 20 using words greater than, less than, or equal to. |
| **1.NO.NBT.2** Compare two two-digit numbers using symbols >, <, and =. Justify comparisons based on  the number of tens and ones. |
| **2.NO.NBT.2** Compare two three-digit numbers using symbols >, <, and =. Justify comparisons based on  the value of thousands, hundreds, tens, and ones. |
| **3.NO.NBT.1** Compare two four-digit numbers using symbols, >, <, and =. Justify comparisons based on the value of thousands, hundreds, tens, and ones. |
| **4.NO.NBT.2** Compare two numbers to the millions place and decimals to the hundredths place, using  symbols >, <, and =. Justify comparisons based on the value of the digits. |
| **5.NO.NBT.2** Compare two decimals to thousandths using symbols >, <, and =. Justify comparisons  based on the value of the digits. |
| **6.NO.NS.1** Explain and show the relationship between non-zero rational numbers and their opposites using horizontal and vertical number lines in authentic problems. Use rational numbers to represent  quantities in authentic contexts and explain the meaning of 0 in certain situations. |
| **6.NO.NS.2** Write, interpret, and explain statements of order for rational numbers on a number line and in  authentic contexts. |
| **7.NO.NS.1** Describe the absolute value of a number as its distance from zero on a number line. |
| **8.NO.NS.1** Compare and classify real numbers within the real number system. |
| **8.NO.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers,  locate them on a number line diagram, and estimate the value of irrational expressions involving one operation. |
| **8.NO.NS.3** Use scientific notation to represent very large or very small quantities. Interpret scientific  notation generated by technology. Compare and order numbers in scientific and standard notation. |
| **9-10.AR.6** Solve linear equations and inequalities (to include compound inequalities) in one variable. |
| **9-10.AR.8** Graph the solution set to a two-variable system of linear equations. Create and graph the  solution set to a two-variable system of linear inequalities in context. |
| **9-10.AR.9** Solve absolute value equations and inequalities in one or two variables. |
| **11-12.AR.9\*** Represent constraints by equations or inequalities and by systems of equations and/or  inequalities and interpret solutions as viable or non-viable options in a modeling context. |
| **Rounding Numbers** |
| **3.NO.NBT.2** Apply place value understanding to round whole numbers to the nearest 10 or 100. |
| **4.NO.NBT.3** Apply place value and/or understanding of numbers to round multi-digit whole numbers to  any place. |
| **5.NO.NBT.3** Apply place value understanding to round decimals to any place. |
| **8.NO.NS.3** Use scientific notation to represent very large or very small quantities. Interpret scientific  notation generated by technology. Compare and order numbers in scientific and standard notation. |
| **Addition and Subtraction** |
| **1.NO.NBT.3** Add within 100 using a two-digit number and a one-digit number. Use concrete models,  drawings, and strategies that reflect an understanding of place value. |
| **2.NO.NBT.3** Add within 100 using place value strategies and/or the relationship between addition and  subtraction. |
| **1.NO.NBT.4** Subtract multiples of 10 within 100 using concrete models, drawings, and strategies that reflect an understanding of place value. |
| **1.NO.NBT.5** Mentally add or subtract 10 to or from a given two-digit number and explain the reasoning  used. |
| **2.NO.NBT.4** Subtract within 100 using place value strategies and/or the relationship between addition  and subtraction. |
| **2.NO.NBT.5** Mentally add or subtract 10 or 100 to or from a given number between 100 and 900. |
| **3.NO.NBT.3** Add and subtract within 1000 using place value strategies, algorithms, and/or the  relationship between addition and subtraction. |
| **4.NO.NBT.4** Add and subtract multi-digit whole numbers to the one million place using strategies flexibly,  including the algorithm. |

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| **Addition and Subtraction** |
| **5.NO.NBT.5** Use concrete models, drawings, place value strategies, properties of operations, and/or  relationships to add, subtract, and multiply decimals to hundredths. |
| **7.NO.O.1** Add, subtract, multiply, and divide integers using visual models and properties of operations in  multi-step authentic and mathematical problems, including authentic problems. |
| **7.NO.O.2** Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including  authentic problems. |
| **7.NO.O.3** Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step  problems using strategies or procedures, including authentic problems. |
| **8.NO.O.2** Add, subtract, multiply, and divide rational numbers using strategies or procedures. |
| **9-10.NO.2** Perform basic operations on simple radical expressions to write a simplified equivalent  expression. |
| **9-10.AR.11** Add, subtract, and multiply polynomials. |
| **11-12.NO.3** Demonstrate that the sum or product of two rational numbers is rational; that the sum of a  rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. |
| **11-12.AR.13** Add, subtract, and multiply polynomials beyond quadratics. Understand that polynomials form a system comparable to the integers, namely, they are closed under the operations of addition,  subtraction, and multiplication. |
| **(+) 11-12.NO.11** Represent addition, subtraction, multiplication, conjugation, powers, and roots of  complex numbers geometrically on the complex and/or polar plane; use properties of this representation  for computation. |
| **(+) 11-12.NO.17** Add and subtract vectors.  a. Add vectors end-to-end, component-wise, and by the parallelogram rule.  Know that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their  sum.  c. Understand that vector subtraction ***v*** – ***w*** is defined as ***v*** +(–***w***), where –***w*** is the additive inverse  of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector  subtraction graphically by connecting the tips in the appropriate order and using the components  to perform vector subtraction.  Represent vector subtraction graphically by connecting the tips of the appropriate order and using the  components to perform vector subtraction. |
| **(+) 11-12.NO.19** Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication)  on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is  not commutative. |
| **Multiplication and Division** |
| **3.NO.NBT.4** Multiply one-digit whole numbers by multiples of 10 within 100. |
| **4.NO.NBT.5** Multiply a whole number up to four digits by a one-digit whole number and multiply two two digit numbers. Show and justify the calculation using equations, rectangular arrays, and models. |
| **5.NO.NBT.4** Multiply multi-digit whole numbers using strategies flexibly, including the algorithm. |
| **5.NO.NBT.7** Explain patterns in the number of zeros of the product when multiplying a number by powers  of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. |
| **4.NO.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using place value strategies. Show and justify the calculation by using equations, rectangular  arrays, and models. |
| **5.NO.NBT.5** Use concrete models, drawings, place value strategies, properties of operations, and/or  relationships to add, subtract, and multiply decimals to hundredths. |
| **5.NO.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and two-digit  divisors using place value strategies. Show and justify the calculation using equations, rectangular arrays, and/or area models. |

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| **6.NO.O.1** Divide multi-digit whole numbers up to four-digit dividends and two-digit divisors using  strategies or procedures. |
| **7.NO.O.1** Add, subtract, multiply, and divide integers and positive rational numbers using visual models  and properties of operations in multi-step problems, including authentic problems. |
| **7.NO.O.2** Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems. |
| **7.NO.O.3** Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step  problems using strategies or procedures, including authentic problems. |
| **8.NO.O.1** Evaluate mentally the square roots of perfect squares up to 225 and cube roots of perfect  cubes up to 1000. |
| **8.NO.O.2** Add, subtract, multiply, and divide rational numbers using strategies or procedures. |
| **9-10.NO.2** Perform basic operations on radicals and simplify radicals to write equivalent expressions. |
| **9-10.AR.11** Add, subtract, and multiply polynomials. |
| **11-12.NO.4** Demonstrate that the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number  and an irrational number is irrational. |
| **11-12.AR.13** Add, subtract, and multiply polynomials beyond quadratics. Understand that polynomials  form a system comparable to the integers, namely, they are closed under the operations of addition,  subtraction, and multiplication. |
| **(+) 11-12.NO.11** Represent addition, subtraction, multiplication, conjugation, powers, and roots of  complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation. |
| **(+) 11-12.NO.19** Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is  not commutative. |
| **Fractions - Partition Shapes** |
| **1.NO.NF.1** Partition circles and rectangles into two and four equal shares using the language halves and  fourths. |
| **2.NO.NF.1** Partition circles and rectangles into two, three, or four equal shares. Describe the shares  using the language of halves, thirds, fourths, half of, a third of, and a fourth of. |
| **2.NO.NF.2** Recognize that identical wholes can be equally divided in different ways. |
| **2.NO.NF.3** Recognize that partitioning shapes into more equal shares creates smaller shares. |
| **3.NO.NF.1** Partition two-dimensional figures into equal areas and express the area of each part as a unit  fraction of the whole. Describe using the language of sixths, eighths, a sixth of, and an eighth of. |
| **Fractions** |
| **3.NO.NF.2** Represent and understand a fraction as a number on a number line. |
| **3.NO.NF.3** Represent equivalent fractions using visual representations and number lines. |
| **3.NO.NF.4** Recognize whole numbers as fractions and express fractions that are equivalent to whole  numbers. |
| **3.NO.NF.5** Compare fractions of the same whole having the same numerators or denominators, using symbols >, <, and = by reasoning about their size. (Fractions should be limited to denominators of 2, 3, 4,  6, and 8 and should not exceed the whole.) |
| **4.NO.NF.1** Express equivalent fractions with a denominator of 10 and a denominator of 100 to generate a  decimal notation. |
| **4.NO.NF.2** Explain and demonstrate how a mixed number is equivalent to a fraction greater than one and  how a fraction greater than one is equivalent to a mixed number using visual fraction models and  reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). |
| **4.NO.NF.3** Generate equivalent fractions using numerical representations, visual representations, and  number lines (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100). |
| **4.NO.NF.4** Demonstrate how equivalent fractions are generated by multiplying a fraction equivalent to 1  or the properties of multiplication (proper and improper fractions limited to 2, 3,4, 5, 6, 8, 10, 12, and 100). |

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