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Question 1

In [109]:

```
import numpy as np
from cvxopt import matrix
from cvxopt import solvers

P = np.array([[0,0,0],
              [0,1,0],
              [0,0,1]])

q = np.array([0,0,0]).T

g1 = np.array([-1,4,0])
g2 = np.array([-1,1,3])
g3 = np.array([-1,1,-1])
g4 = np.array([1,0,0])
g5 = np.array([1,2,-5])
g6 = np.array([1,2,3])
g7 = np.array([1,2,3])

G = -np.vstack((g1,g2,g3,g4,g5,g6,g7))

h = -np.ones((7,1))

P = matrix(P, tc='d')
q = matrix(q, tc='d')
G = matrix(G, tc='d')
h = matrix(h, tc='d')

print("-"*50)
print("Solved Optimum: \n", sol['x'], sep='')
```

```
--------------------------------------------------
Solved Optimum:
[ 4.32e-09]
[ 7.04e-01]
[ 7.04e-01]
[ 8.89e-01]
[ 2.59e-01]
[ 2.59e-01]
[ 5.27e-10]
```

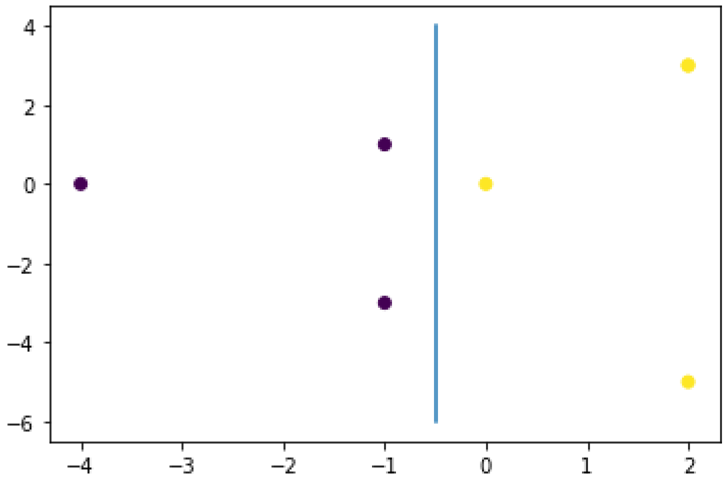
In [3]:

```
import matplotlib.pyplot as plt
x = [[1,0], [0,1], [0,-1], [-1,0], [0,2], [0,-2], [-2,0]]
y = [-1,-1,-1,1,1,1,1]

def zTrans(x):
    x1 = x[0]
    x2 = x[1]
    z1 = x2**2 - 2*x1 -2
    z2 = x1**2 - 2*x2 -1
    return [z1,z2]

z = [zTrans(xx) for xx in x]
z1, z2 = zip(*z)

plt.figure()
plt.scatter(z1, z2, c=y)
plt.plot([-0.5,-0.5], [-6,4])
plt.show()
```



By the solution given above, we have $b = 1$, $w_1 = 2$ and $w_2 \approx 0$.
Hence, the hyperplane can be written as $2z_1 + 1 = 0$,
that is, $2x_2^2 - 4x_1 - 3 = 0.(eq.1)$

Question 2

In [110]:

```
x = [[1,0], [0,1], [0,-1], [-1,0], [0,2], [0,-2], [-2,0]]
x = np.array(x)
y = [-1,-1,-1,1,1,1,1]

P = np.zeros([7,7])
for i in range(7):
    for j in range(7):
        P[i,j] = y[i]*y[j]*(1+sum(x[i]*x[j]))**2

q = -1*np.ones((7,1))

A = np.array(y).reshape(1,7)
c = 0

G = -np.identity(7)

h = np.zeros((7,1))

P = matrix(P, tc='d')
q = matrix(q, tc='d')
G = matrix(G, tc='d')
h = matrix(h, tc='d')
A = matrix(A, tc='d')
c = matrix(c, tc='d')

print("-"*50)
print("Solved Optimum: \n", sol['x'], sep='')
```

```
--------------------------------------------------
Solved Optimum:
[ 4.32e-09]
[ 7.04e-01]
[ 7.04e-01]
[ 8.89e-01]
[ 2.59e-01]
[ 2.59e-01]
[ 5.27e-10]
```

By the solution given above,
we have $a_1 = a_7 \approx 0, a_2 = a_3 = 0.704, a_4 = 0.889, a_5 = a_6 = 0.259.$
Since the support vector's $a \neq 0,$
the support vectors are $x_2, x_3, x_4, x_5, x_6.$

Question 3

By the lectures,

$$g_{svm}(x) = sign(\sum_{sv} a_n y_n K(x_n, x) + b)$$
$$= sign(0.704 \times -1 \times (1 + 0x_1 + 1x_2)^2 +$$
$$0.704 \times -1 \times (1 + 0x_1 + -1x_2)^2 +$$
$$0.889 \times 1 \times (1 + -1x_1 + 0x_2)^2 +$$
$$0.259 \times 1 \times (1 + 0x_1 + 2x_2)^2 +$$
$$0.259 \times 1 \times (1 + 0x_1 + -2x_2)^2 +$$
$$1.666$$
$$= sign(0.664x_2^2 + 0.889x_1^2 - 1.778x_1 + 1.665)$$
$$= sign(eq.3))$$

Question 4

$$eq.1 = 2x_2^2 - 4x_1 - 3 = 0$$
$$eq.3 = 0.664x_2^2 + 0.889x_1^2 - 1.778x_1 - 1.665$$

By comparing the formulae, eq.1 and eq.3 are different.
Furthermore, they should not be the same as well, since they have different support vectors and different kernels.

Question 5

$$L((b, w, \epsilon), (\alpha, \beta)) = \frac{1}{2} \; ww^T + C \cdot \sum_{n=1}^{N} \epsilon_n \; + \; \sum_{n=1}^{N} \alpha_n \cdot (\rho_n - \epsilon_n - y_n(w^t z_n + b$$

Question 6

$$max_{\alpha_n \geq 0, \beta_n \geq 0}\{\; min_{\beta, w, \epsilon}\{\; L((b, w, \epsilon), (\alpha, \beta)) \;\}\;\}$$

Considering $\frac{\partial L}{\partial \epsilon} = 0$, we can solve the problem without lossing optimality
if solving with implicit constraint $\beta_n = C - \alpha_n$ and explicity constraint $0 \leq \alpha_n \leq C$.
We can simplify the Lagrange dual problem to:

$$max_{0 \leq \alpha_n \leq C, \beta_n = C - \alpha_n}\{\; min_{\beta, w, \epsilon}\{\; \frac{1}{2} w^T w \; + \sum_{n=1}^{N} \alpha_n \cdot (\rho_n - y_n(w^t z_n + b)) \;\}\;\}$$

For the inner problem

considering $\frac{\partial L}{\partial b} = 0$ and $\frac{\partial L}{\partial w} = 0$,
we can solve the problem without lossing optimality
if solving with implicit constraint $\sum_{n=1}^{N} \alpha_n y_n = 0$ and $w_i = \sum_{n=1}^{N} \alpha_n y_n z_n$

We can simplify the Lagrange dual problem to:

$max_{0 \le \alpha_n \le C, \beta_n = C - \alpha_n} \{ -\frac{1}{2} || \sum_{n=1}^{N} \alpha_n y_n z_n ||^2 + \sum_{n=1}^{N} \rho_n \alpha_n \}$,

which is
$min_{\alpha} \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_n \alpha_m y_n y_m z_n^T z_m - \sum_{n=1}^{N} \rho_n \alpha_n$

subject to
$\sum_{n=1}^{N} \alpha_n y_n = 0$;
$\alpha_n \ge 0$, for $n = 1, 2, \dots, N$

Question 7

This is the P_1^{'} SVM with $\rho_n = 0.5$

$(P_1^{'})\ min_{w^{'}, \beta^{'}, \epsilon^{'}}\ \frac{1}{2}\ w^{'} w^{'T} + C \cdot \sum_{n=1}^{N} \epsilon_n^{'}$
s.t. $y_n(w^{'T} x_n + b^{'}) \ge 0.5 - \epsilon_n^{'}$

By scaling the objective function by 4 and constraint by 2 and we can have:

$(P_1^{'})\ min_{w^{'}, \beta^{'}, \epsilon^{'}}\ \frac{1}{2}\ 4w^{'} w^{'T} + 2C \cdot \sum_{n=1}^{N} 2\epsilon_n^{'}$
s.t. $y_n(2w^{'T} x_n + 2b^{'}) \ge 1 - 2\epsilon_n^{'}$

It can also be represent as the eqaution below:

$min_{w, \beta, \epsilon}\ \frac{1}{2}\ ww^T + 2C \cdot \sum_{n=1}^{N} \epsilon_n$
s.t. $y_n(w^T x + b) \ge 1 - \epsilon_n$

where $w = 2w^{'}$, $b_n = 2b^{'}$ and $\epsilon_n = 2\epsilon_n^{'}$

Hence, Assume that $(\beta_*^{'}, w_*^{'})$ is the optimal solution of solving $P_1^{'}$ with all $\rho_n = 0.5$.
The optimal solution of P_1 with $C_1 = 2C$ can be express as $(\beta_*, w_*) = (2\beta_*^{'}, 2w_*^{'})$.

Question 8

Soft-margin SVM dual is almost the same as hard-margin's, except that soft-margin SVM dual has a upper-bound C for each α_n. Hence, if $C \geq max_{1 \leq n \leq N} a_n^*$, we can add the constraint $0 \leq a_n \leq C$ to hard-margin SVM dual while not affecting the optimum result. As a result, this hard-margin SVM dual becomes a soft-margin SVM dual, having the same optimun $a*$.

Question 9

Let the GRAM matrix of K and K_1 be M and M_1 respectively.

(a)

$K = (1 - K_1(x, x^`))^1$ is not a valid kernel.
Here is a Counterexample:

Let $M_1 = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$, then $M = \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix}$.
Since $det(M) < 0$, M is not a p.s.d matrix, therefore K is not a valid kernel.

(b)

$K = (1 - K_1(x, x^`))^0$ is a valid kernel.
Since $M = J$, and J is a p.s.d. matrix.

(c)

$K = (1 - K_1(x, x^`))^{-1}$ is a valid kernel.

By Taylor expansion, $K = \sum_{i=0}^{\infty}(K_1(x, x^`))^i$

Let $K_n(x, x^`)$ and $K_m(x, x^`)$ be valid kernels

$$\begin{aligned} K_n(x, x^`) \times K_m(x, x^`) &= \phi_n(x)^T \phi_n(x^`)\phi_m(x)^T \phi_m(x) \\ &= \sum_{i=1}^{n} \phi_n^i(x)\phi_n^i(x^`) \sum_{i=1}^{n} \phi_m^j(x)\phi_m^j(x) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \phi_n^i(x)\phi_n^i(x^`)\phi_m^j(x)\phi_m^j(x) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} (\phi_n^i(x)\phi_m^j(x))(\phi_n^i(x^`)\phi_m^j(x)) \end{aligned}$$

Let $\Phi^i(x) = \sum_{j=1}^n \phi_1^i(x)\phi_2^j(x)$

$\Phi(x) = \sum_{i=1}^n \Phi^i(x)$
$(\Phi^i(x))^T(\Phi^i(x)) = \sum_{j=1}^n \phi_1^i(x)\phi_2^j(x)\phi_1^i(x)\phi_2^j(x)$

$$
\begin{aligned}
(\Phi(x))^T(\Phi(x)) &= \sum_{i=1}^n (\Phi^i(x))^T(\Phi^i(x)) \\
&= \sum_{i=1}^n \sum_{j=1}^n \phi_1^i(x)\phi_2^j(x)\phi_1^i(x)\phi_2^j(x) \\
&= K_n(x, x^`) \times K_m(x, x^`)
\end{aligned}
$$

Hence, $K_n(x, x^`) \times K_m(x, x^`)$ is a valid kernel.
Also, the sum of valid kernels is also a valid kernel since the sum of p.s.d. matrices is still a psd matix.

Hence, $K = \sum_{i=0}^\infty (K_1(x, x^`))^i$ is a valid kernel.

(d)

$K = (1 - K_1(x, x^`))^{-2}$ is a valid kernel.

Since $(1 - K_1(x, x^`))^{-1}$ is a valid kernel,
$(1 - K_1(x, x^`))^{-2} = (1 - K_1(x, x^`))^{-1} \times (1 - K_1(x, x^`))^{-1}$ is also a valid kernel.

Question 10

This is the SVM dual of kernel K and parameter C.

$min_\alpha \ \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \alpha_n \alpha_m y_n y_m K(x_n, x_m) - \sum_{n=1}^N \alpha_n$
s.t.
$\sum_{n=1}^N \alpha_n y_n = 0;$
$0 \le \alpha_n \le C$, for $n = 1, 2, \ldots, N$

This is the SVM dual of kernel pK and parameter C/p.

$min_{\alpha^*} \ \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \alpha_n^* \alpha_m^* y_n y_m pK(x_n, x_m) - \sum_{n=1}^N \alpha_n^*$
s.t.

$\sum_{n=1}^{N} \alpha_n^* y_n = 0;$
$0 \leq \alpha_n^* \leq C/p$, for $n = 1, 2, \ldots, N$

The SVM dual of kernel pK and parameter C/p can also be written as:
$min_{\alpha^*} \frac{1}{2} \frac{1}{p} (\sum_{n=1}^{N} \sum_{m=1}^{N} p\alpha_n^* p\alpha_m^* y_n y_m K(x_n, x_m) - \sum_{n=1}^{N} p\alpha_n^*)$
$0 \leq p\alpha_n^* \leq C$, for $n = 1, 2, \ldots, N$

Let $\alpha^{**} = p\alpha^*$.

$min_{\alpha^{**}} \frac{1}{2} \frac{1}{p} (\sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_n^{**} \alpha_m^{**} y_n y_m K(x_n, x_m) - \sum_{n=1}^{N} \alpha_n^{**})$
s.t.
$\sum_{n=1}^{N} \alpha_n^{**} y_n = 0;$
$0 \leq \alpha_n^{**} \leq C$, for $n = 1, 2, \ldots, N$

Since p is a constant, we can ignore it for our objective function and find this dual problem is the same as that of kernel K with parameter C. Hence, the two dual problems share the same optimal solution, that is, $\alpha_{optim} = \alpha_{optim}^{**} = p\alpha_{optim}^*$.

We first get the solution of b^* by b

$$\begin{aligned} b^* &= y_m - \sum_{n=1}^{N} a_n^* y_n K^*(x_n, x) \\ &= y_m - \sum_{n=1}^{N} \frac{\alpha_n}{p} y_n p K^(x_n, x) \\ &= b \end{aligned}$$

(each y_m's corresponding $a_m \neq 0$)

We can show the two SVM has the same classifier.

$$\begin{aligned} g^*(x) &= sign(\sum_{n=1}^{N} a_n^* y_n K^*(x_n, x) + b*) \\ &= sign(\sum_{n=1}^{N} \frac{\alpha_n}{p} y_n p K(x_n, x) + b) \\ &= g(x) \end{aligned}$$

Question 11

Load in data...

In [5]:

```
import numpy as np
import csv
import matplotlib.pyplot as plt
import matplotlib
from libsvm.svmutil import *
file_train = "./features.train.txt"
file_test = "./features.test.txt"
x_train = []
y_test = []

with open(file_train, 'r', newline='\n') as f:
    rows = csv.reader(f, delimiter=' ')
    train = [[float(ele) for ele in row if ele!=''] for row in rows]

with open(file_test, 'r', newline='\n') as f:
    rows = csv.reader(f, delimiter=' ')
    test = [[float(ele) for ele in row if ele!=''] for row in rows]

train = np.array(train)
test = np.array(train)
x_train = train[:,1:]
x_test = test[:,1:]
```

In [6]:

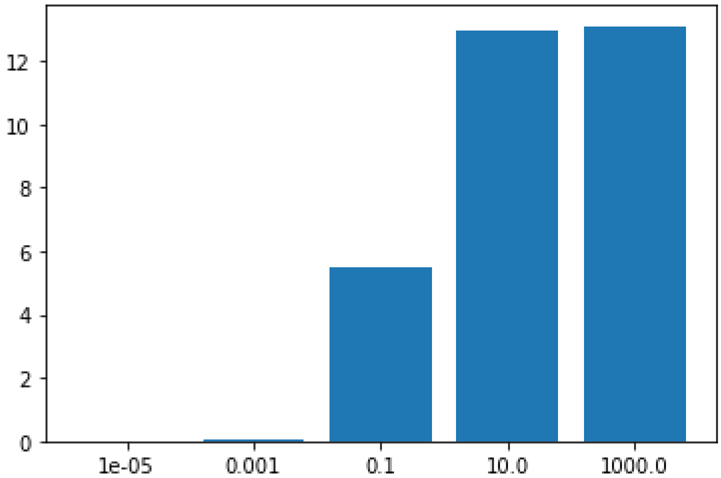
```
y_train_q11 = [1 if row[0]==0 else -1 for row in train]
y_test_q11 = [1 if row[0]==0 else -1 for row in test]
prob  = svm_problem(y_train_q11, x_train)

def getWeightNorm(c_str):

    param = svm_parameter(('-t 0 -c '+c_str))
    m = svm_train(prob, param)
    p_label, p_acc, p_val = svm_predict(y=[1,1,1,1], x=[[0.,0.,0.], [1.,0.,0.], [0.,
    w_q11 = np.array(p_val[1:])
    w_q11 = w_q11-p_val[0]
    return np.linalg.norm(w_q11)

C = [1e-5, 1e-3, 1e-1, 1e+1, 1e+3]
C_str = [str(c) for c in C]
w_norm = [getWeightNorm(c_str) for c_str in C_str]
plt.figure()
plt.bar(C_str, w_norm)
plt.show()
```

```
Accuracy = 0% (0/4) (classification)
Accuracy = 0% (0/4) (classification)
Accuracy = 25% (1/4) (classification)
Accuracy = 25% (1/4) (classification)
Accuracy = 25% (1/4) (classification)
```



As C getting greater in value, $||w||$ also increase.
It makes sense because when C is small, the objective function will mainly focus on minimizing 1/2$||w||^2$,
so the optimal $||w||$ would be small.

Question 12

In [7]:

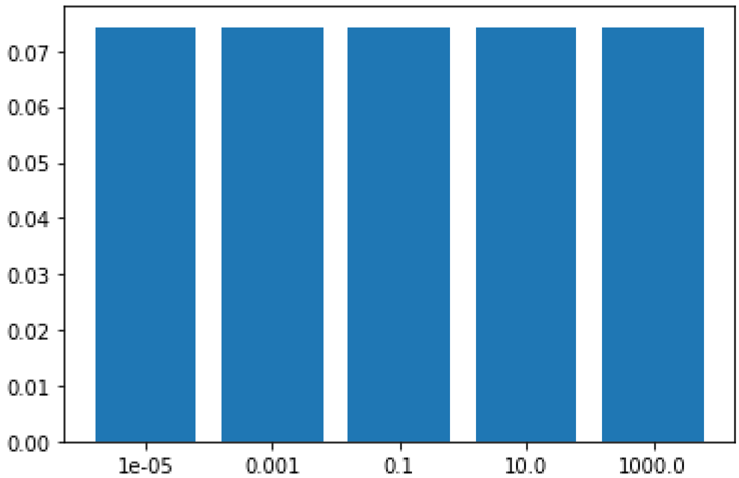
```
y_train_q12 = [1 if row[0]==8 else -1 for row in train]
y_test_q12 = [1 if row[0]==8 else -1 for row in test]

def getEin(c_str):
    prob  = svm_problem(y_train_q12, x_train)
    param = svm_parameter('-t 1 -d 2 -c '+c_str) #Set parameters
    m = svm_train(prob, param) #Training
    p_label, p_acc, p_val = svm_predict(y_train_q12, x_train, m) #Predict training s
    return (100-p_acc[0])/100 #Calculate E_in

C = [1e-5, 1e-3, 1e-1, 1e+1, 1e+3]
C_str = [str(c) for c in C]
Ein_q12 = [getEin(c_str) for c_str in C_str]

plt.figure()
plt.bar(C_str, Ein_q12)
plt.show()
```

```
Accuracy = 92.5662% (6749/7291) (classification)
Accuracy = 92.5662% (6749/7291) (classification)
Accuracy = 92.5662% (6749/7291) (classification)
Accuracy = 92.5662% (6749/7291) (classification)
Accuracy = 92.5662% (6749/7291) (classification)
```



E_{in} doesn't change with the value of C.

Question 13

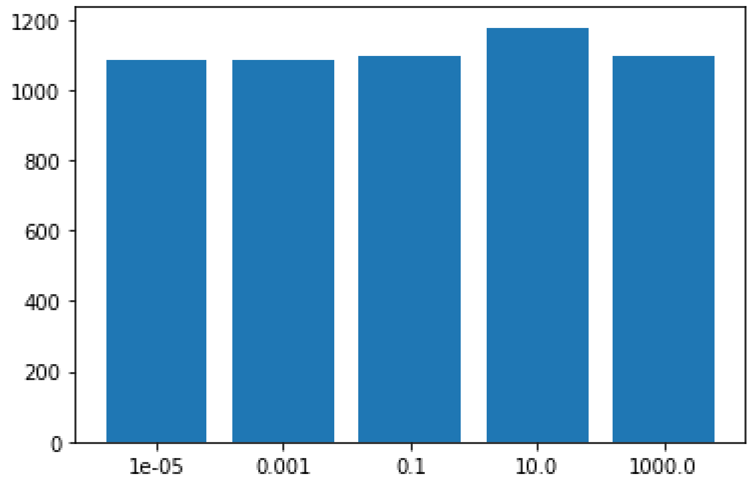
In [108]:

```
y_train_q12 = [1 if row[0]==8 else -1 for row in train]
y_test_q12 = [1 if row[0]==8 else -1 for row in test]

def getNumSV(c_str):
    prob  = svm_problem(y_train_q12, x_train)
    param = svm_parameter('-t 1 -d 2 -c '+c_str) #Set parameters
    m = svm_train(prob, param) #Training
    return(m.get_nr_sv()) #Get the number of SVs

C = [1e-5, 1e-3, 1e-1, 1e+1, 1e+3]
C_str = [str(c) for c in C]
nSV_q12 = [getNumSV(c_str) for c_str in C_str]

plt.figure()
plt.bar(C_str, nSV_q12)
plt.show()
```



Similar to E_{in}, the number of SV doesn't change much with the value of C.

Question 14

Much thanks to the classmates who discussed this question on the forum.

$$
\begin{aligned}
g(x) &= w^T z + b \\
&= \sum_{sv} \alpha_n y_n K(x_n, x) + b \\
&= \sum_{sv} \alpha_n y_n K(x_n, x) + y_{sv} - \sum_{sv} \alpha_n y_n K(x_n, x) \\
&= y_{sv}
\end{aligned}
$$

Hence, in the Z space, the distance from the hyperplane to a free-SV is

$$\frac{1}{||w||}|w^T z + b| = \frac{1}{||w||}|g(x)| = \frac{1}{||w||}$$

For calculating $||w||$, we know that $w = \sum_{sv} \alpha_i y_i z_i$, and we can have $||w||^2 = \sum_{sv} \sum_{sv} \alpha_i y_i z_i \alpha_j y_j z_j = \sum_{sv} \sum_{sv} \alpha_i y_i \alpha_j y_j K(x_i, x_j)$

In [96]:

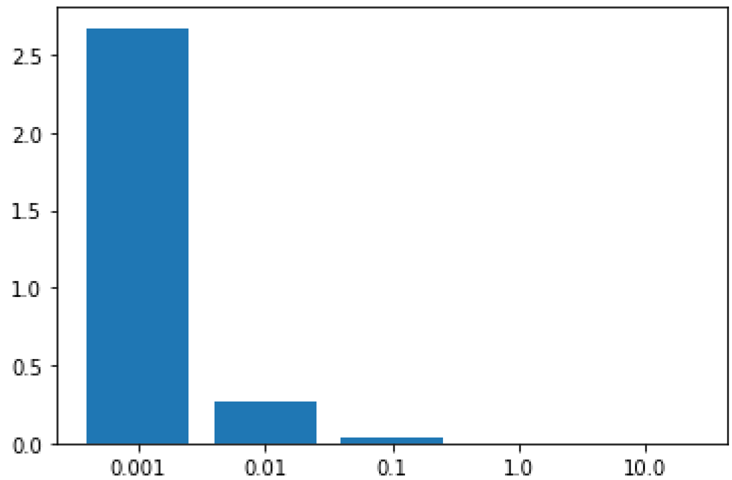
```
y_train_q14 = [1 if row[0]==0 else -1 for row in train]
y_test_q14 = [1 if row[0]==0 else -1 for row in test]

def K_rbf(x1, x2, g=80):
    return np.exp(-g*sum((x1-x2)**2))

def getFreeSVDist(c_str, y=y_train_q14):
    prob  = svm_problem(y, x_train)
    param = svm_parameter('-t 2 -g 80 -c '+c_str)
    m = svm_train(prob, param)
    SV = m.get_SV() #Get SVs
    SV = [np.array([sv[1], sv[2]]) for sv in SV] #Extract SV from dictionary to nump
    SV_idx = m.get_sv_indices() #Get the indices of SVs in the training data
    a = m.get_sv_coef() #Get coeficient a
    a = [aa[0] for aa in a] #Extract a from nested list to list
    nSV = len(SV) #Get the number of SVs
    # Calculate the ||w||^2
    w_norm2 = np.sum([a[i]*a[j]*y[SV_idx[i]-1]*y[SV_idx[j]-1]*K_rbf(SV[i], SV[j]) fo
    return(w_norm2**(-1/2)) #return 1/||w||

C = [1e-3, 1e-2, 1e-1, 1e+0, 1e+1]
C_str = [str(c) for c in C]
dist_q14 = [getFreeSVDist(c_str) for c_str in C_str]

plt.figure()
plt.bar(C_str, dist_q14)
plt.show()
```



The distance from the hyperplane to a free-SV decreases drastically while C increase. It is probably because that when C is small, the SVM has more tolerence to error terms, so the optimizer can come up with a larger margin.

Question 15

In [31]:

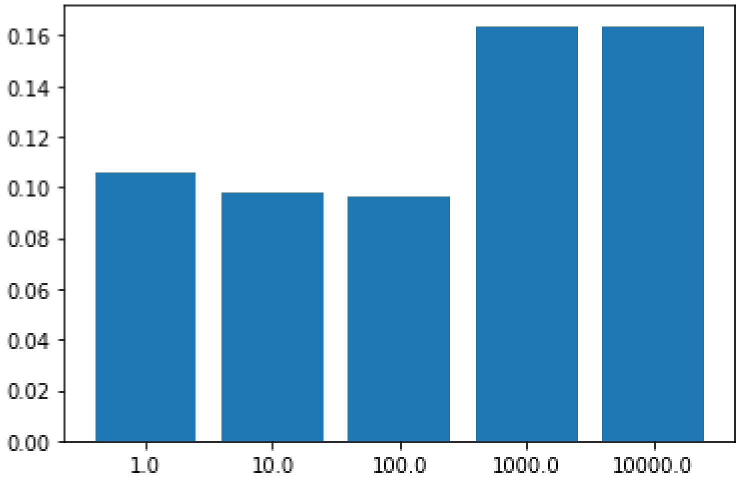
```
prob  = svm_problem(y_train_q14, x_train)
param = svm_parameter('-t 2 -g 80 -c 0.1')
m = svm_train(prob, param)

def getEout(g_str):
    prob  = svm_problem(y_train_q14, x_train)
    param = svm_parameter('-t 2 -c 0.1 -g '+g_str) #Ser parameters
    m = svm_train(prob, param) #Training
    p_label, p_acc, p_val = svm_predict(y_test_q14, x_test, m) #Predict testing set
    return (100-p_acc[0])/100 #Calculate E_out

G = [1e0, 1e+1, 1e+2, 1e+3, 1e+4]
G_str = [str(g) for g in G]
Eout_q15 = [getEout(g_str) for g_str in G_str]

plt.figure()
plt.bar(G_str, Eout_q15)
plt.show()
```

```
Accuracy = 89.439% (6521/7291) (classification)
Accuracy = 90.2208% (6578/7291) (classification)
Accuracy = 90.3991% (6591/7291) (classification)
Accuracy = 83.6236% (6097/7291) (classification)
Accuracy = 83.6236% (6097/7291) (classification)
```



The result shows that it is important to find proper γ while testing the model.
The model is not always better with the big parameter, and vice versa.

Question 16

In [98]:

```
import random
idx = np.arange(len(x_train)) #Set indices from sampling
x_train = np.array(x_train)
y_train_q14 = np.array(y_train_q14)

# Function to calculate E_out with assigned gamma value
def getEval(g_str, x_train, y_train, x_val, y_val):
    prob  = svm_problem(y_train, x_train)
    param = svm_parameter('-t 2 -c 0.1 -g '+g_str)
    m = svm_train(prob, param)
    p_label, p_acc, p_val = svm_predict(y_val, x_val, m)
    return (100-p_acc[0])/100

def getBestG():
    random.shuffle(idx) #Shuffle the indices
    x_val_q16 = x_train[idx[:1000], ] #Get the first 1000 indices for validating se
    y_val_q16 = y_train_q14[idx[:1000]] #Get the first 1000 indices for validating s
    x_train_q16 = x_train[idx[1000:], ] #Get the rest of the indices for training se
    y_train_q16 = y_train_q14[idx[1000:]] # Get the rest of the indices for testing

    G = [1e-1, 1e0, 1e+1, 1e+2, 1e+3]
    G_str = [str(g) for g in G]
    #Calculate E_out for each gamma in G_str
    Eout_q15 = [getEval(g_str, x_train_q16, y_train_q16, x_val_q16, y_val_q16) for g
    return G_str[np.argmin(Eout_q15)] #Return the gamma with the minimal E_out
```

In [99]:

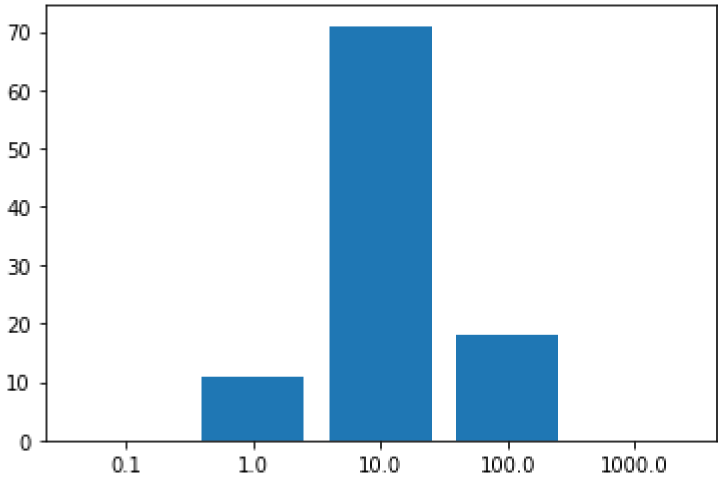
```
resBestG = [getBestG() for _ in range(100)]
```

```
Accuracy = 88.2% (882/1000) (classification)
Accuracy = 88.5% (885/1000) (classification)
Accuracy = 87.9% (879/1000) (classification)
Accuracy = 82.1% (821/1000) (classification)
Accuracy = 84.5% (845/1000) (classification)
Accuracy = 89.6% (896/1000) (classification)
Accuracy = 90.3% (903/1000) (classification)
Accuracy = 90.3% (903/1000) (classification)
Accuracy = 84.5% (845/1000) (classification)
Accuracy = 82.4% (824/1000) (classification)
Accuracy = 88.6% (886/1000) (classification)
Accuracy = 89.3% (893/1000) (classification)
Accuracy = 89.1% (891/1000) (classification)
Accuracy = 82.4% (824/1000) (classification)
Accuracy = 83.4% (834/1000) (classification)
Accuracy = 89.6% (896/1000) (classification)
Accuracy = 90% (900/1000) (classification)
Accuracy = 90% (900/1000) (classification)
Accuracy = 83.4% (834/1000) (classification)
Accuracy = 83.3% (833/1000) (classification)
```

In [100]:

```
from collections import Counter
resCount = dict(Counter(resBestG)) #Count duplicates of each gamma in the result
G = [1e-1, 1e0, 1e+1, 1e+2, 1e+3]
G_str = [str(g) for g in G]
#Fill out the list of gamma's duplicate
G_choice = [resCount.get(g_str) if resCount.get(g_str) else 0 for g_str in G_str]

plt.figure()
plt.bar(G_str, G_choice)
plt.show()
```



From the result of 100 times of vlidations, we can infer that model is suitable with $\gamma = 10$ while $C = 0.1$

Question 17

Let z_n be the transformed data that start we constant 1

$$
\begin{aligned}
w &= \sum_{sv} \alpha_n y_n z_n \\
&= \sum_{sv} \alpha_n y_n \phi(x_n) \\
&= \sum_{sv} \alpha_n y_n (1 + \ldots)
\end{aligned}
$$

We can seperate the constant term as w_i
and since $\alpha_{not\,SV} = 0$

$w_i = \sum_{sv} \alpha_n y_n (1) = \sum_{n=1}^{N} \alpha_n y_n$

Since the optimal weight w observes constraint $\sum_{n=1}^{N} a_n y_n = 0$
We can know that $w_i = 0$

In [ ]: