Tactile Heatmaps: A Novel Visualisation Technique for Data Analysis with Tactile Charts

Christin Engel*
TU Dresden
Dresden, Saxony, Germany
christin.engel@tu-dresden.de

Emma Franziska Müller*
TU Dresden
Dresden, Saxony, Germany
emma_franziska.mueller@tu-dresden.de

Gerhard Weber
TU Dresden
Dresden, Saxony, Germany
gerhard.weber@tu-dresden.de

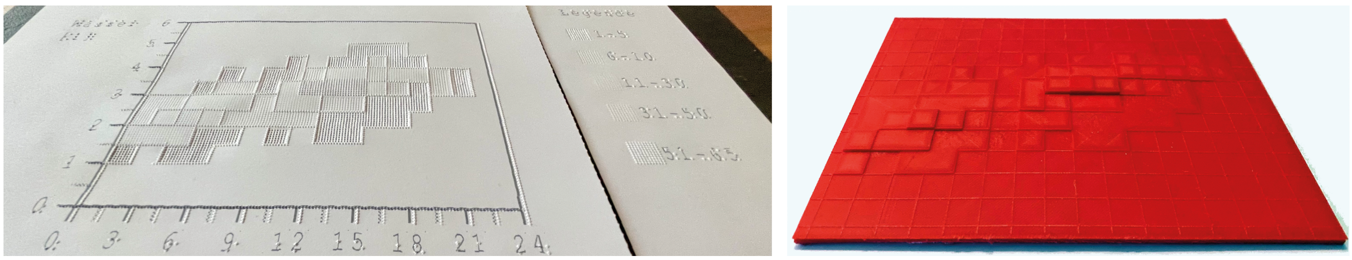

Figure 1: Tactile printed heatmaps. Left: embossed heatmap and legend. Right: 3D printed Heatmap

ABSTRACT

Analysing large data sets for various purposes is a growing requirement for many professions. Tactile charts are suitable to enable people with visual impairment and blindness performing data analysis tasks. However, only a few approaches focus on the development of tactile charts for data analysis purposes. Concepts are needed to represent a sufficient amount of data with tactile charts and address arising challenges, such as information overload. In this paper, we first discuss and analyse the scalability of data represented by tactile charts using tactile scatterplots. We further address the data size limitations and present methods to identify critical, tactile representation with limited readability respecting the analysis task. Moreover, we propose methods to increase the amount of data represented in tactile scatterplots. We further introduce tactile heatmaps as an innovative and new concept for haptic data representation that utilises different elevation levels. We evaluated our design concept as well as the feasibility of varying elevation levels with 11 blind and visually impaired people. We compared four design conditions for embossed tactile heatmaps as well as the suitability of 3D-printed heatmaps. The results show that tactile heatmaps are suitable for representing more data than previously known tactile representation methods. They support obtaining an overview of a high amount of data and can be applied for data analysis purposes.

*Both authors contributed equally to this research.

PETRA 2021, June 29-July 2, 2021, Corfu, Greece

ACM ISBN 978-1-4503-8792-7/21/06…$15.00
https://doi.org/10.1145/3453892.3458045

CCS CONCEPTS

• **Human-centered computing** → **Accessibility theory, concepts and paradigms**; **Heat maps**; *Accessibility design and evaluation methods.*

KEYWORDS

accessible charts, tactile data analysis, tactile charts, heatmap, tactile information visualisation

ACM Reference Format:

Christin Engel, Emma Franziska Müller, and Gerhard Weber. 2021. Tactile Heatmaps: A Novel Visualisation Technique for Data Analysis with Tactile Charts. In *The 14th PErvasive Technologies Related to Assistive Environments Conference (PETRA 2021), June 29-July 2, 2021, Corfu, Greece.* ACM, New York, NY, USA, 10 pages. https://doi.org/10.1145/3453892.3458045

1 INTRODUCTION

The analysis of more and more data for several purposes is a growing requirement for many professions. Information Visualisations (InfoVis) are common to analyse a high amount of data. They take advantages of the visual sense, such as the fast detection of patterns and outliers and the number and distribution of data points. InfoVis are suitable to gain knowledge from big data and to search for relations and insights. Information Overload (IO) as well as overplotting are major challenges, limiting the size of data that can be analysed effectively. Much research in this area focuses on effective design for data analysis or the development of new visualisation types, as well as on specific user tasks for static and especially dynamic InfoVis [6, 33] to address these challenges. The "Information seeking Mantra", proposed by Ben Shneiderman [29], is the most popular framework to deal with IO in InfoVis.

However, people with blindness or visual impairment (PBVI) cannot analyse data visually. Tactile charts (TCs) are suitable to provide alternative access to visual graphics. They consist of raised lines, symbols and textures as well as Braille labels and can be explored by the sense of touch. TCs have to be explored sequentially, element by element, which makes it hard to obtain an overview of the data. Moreover, the design of tactile and visual charts differ because TCs require specific design guidelines, e.g. tactile elements require minimum sizes and distances to be recognised and distinguished. IO can be identified as a major challenge for TCs. The research field of TCs lacks investigations on how to address the IO to make tactile data analysis more effective.

Most previous research tends to focus on specific design guidelines for TCs and the distinguishability of the chart's elements, rather than investigating how to increase the amount of data represented. Consequently, current TCs typically represent a small amount of data and have been used primarily to access a visual chart or read out specific values [10]. Furthermore, the development of TCs is limited to few chart types, such as bar, line and pie charts, and scatterplots. Other chart types were rarely discussed. Numerous approaches aim to reduce the IO by developing multimodal interaction concepts for TCs (e.g. [9, 14]) which requires effective tactile chart design. Our main goal is to enable PBVI analysing data independently and effectively with TCs. Therefore, we investigated approaches to increase the amount of data represented through TCs. In this work, on the one hand, we discuss the scalability of the data, i.e. when data-dependent representation problems occur in TCs and how they can be detected. Knowledge of this is a necessary prerequisite to automate the creation of qualitative and effectively usable TCs, allowing PBVI to create tactile graphics also independently. On the other hand, we propose a workflow for authors to deal with IO in TCs. As a meaningful contribution, we present a novel concept for tactile heatmaps, which has never been addressed before. Moreover, we compared four design alternatives for tactile heatmap and their readability and comprehensibility with 11 PBVI.

2 RELATED WORK

Guidelines for accessibility, such as the Web Content Accessibility Guidelines (WCAG), recommend developing a textual image description to enable PBVI accessing graphical content. Image descriptions are not suitable for complex graphics, such as mathematical graphics, schemes or charts, because their creation and use are time-consuming, authors need knowledge in developing image descriptions for PBVI, and they do not provide the same information as graphical representations. Moreover, the spatial arrangement of the elements and their meaning can be better perceived with TCs than with image descriptions, which content strongly depends on the author [31].

The design of TCs requires specific guidelines to ensure the legibility and distinguishability of chart elements. As the elements have to be explored by hand, one after the other, the design should guide the user through the graphic. Existing design guidelines for tactile graphics (e.g. [5, 25]) lack specific aspects for the design of TCs and do not address effective chart design. In contrast, it is often recommended to base the design of TCs on the design of its visual counterpart. This recommendation assumes that the tactile chart is created as a transcription of a visual chart. However, this is not always the case, especially in scenarios where TCs are made explicitly for tactile data analysis purposes. Designing TCs to perform data analysis is rarely discussed.

A wide range of research examined specific design criteria for TCs, such as the design and readability of grid lines [1, 2, 21], textures [7, 26] or axes in charts [1]. Fewer studies investigated in effects of tactile variables for TCs, such as the elevation level (EL). In particular, Barth [2] investigated the readability of an incised grid in combination with an embossed data curve for line charts and found that this design condition can improve chart reading performance. In contrast, Schiff et al. [28] used different ELs with vacuum-formed plastic to increase the reading accuracy of tactile bar charts. In this work, the authors redundantly coded the data values using the height of the bars and the different ELs, which decreases the error rate. Aldrich et al.[1] used three different ELs to support the discrimination of area, line and point symbols in tactile line graphs, which in some cases provided better results. ELs were also used here for redundant coding of information. Vasconcellos [30] first presented a mapping from visual variables, proposed by Bertin [3], to tactile variables by adding elevation. It is important to distinguish between elevation that raises prints and differences in height [8]. According to Dinar et al., the latter works as an ordinal variable. Jehoel et al. [17] investigated the minimum elevation at which tactile features of symbols are readable in the context of map reading tasks. They determine 0.16 mm as the minimum EL at which performance is maximised regarding the reading time. According to the authors, the EL cannot be controlled accurately, depending on the production method. They produced the tactile materials in this study with an specific inkjet printer that builds multiple polymer ink layers. In a follow-up study, a minimum elevation of 0.2 mm was determined for map-reading tasks [18]. Besides, a recent study found evidence of the usefulness and effectiveness of using varying ELs especially for charts representing numerical or spatial data [16]. The authors state that the information must be spatially related and that there must be adjacent points for relative comparisons. Götzelmann [15] used five different ELs with 3D-printed maps but did not explicitly evaluate the recognisability of the like. For his work, the maximum EL to encode map features have to be below 1 mm to ensure that proximity touches can be recognised. Previous research has shown that elevation can be used to encode information, but its recognisability is highly dependent on the context and the production method. There are no reliable data on whether and how ELs can be distinguished and understood for different production methods and contexts.

However, only a few approaches (e.g. [12]) examine the effectiveness of certain design approaches and consider them depending on the chart type and the analysis task influencing design requirements. Furthermore, the current research and design guidelines only cover a small set of established chart types, primarily line, bar and pie charts were supported. Most design studies were performed with small and simple data sets. In particular, scatterplots, which are suitable for representing more data than bar or pie charts, have been rarely addressed [10]. It is needed - especially for data analysis purposes - to develop specific representations for the haptic domain.

All in all, TCs are well-known but have been used mainly in education to teach concepts of visual charts or access a visual one [10]. Analysing data with TCs and approaches to overcome typical challenges have been less explored. Furthermore, there is a lack of development of rich haptic representations that are suitable for data analysis. Moreover, the scalability of the data represented by TCs has not yet been sufficiently considered.

In contrast, much work has been done in the area of InfoVis to improve the data analysis process by addressing common visualisation challenges (e.g. overplotting, distinguishability of elements). In particular, the process of visual data analysis and the analysis tasks and characteristics of data have been much discussed (e.g. [4, 29, 33]). Multivariate visualisation types have been developed to analyse complex data sets (e.g. star plots, parallel coordinates, heatmaps or treemaps [13, 29]). Although many of these approaches based on interaction with the visualisation (e.g. [29, 33]), they are applicable to static visualisations, too.

The **encode** technique changes the design of the visualisation to avoid overplotting of data points, e.g. in scatterplots by adding transparency [24]. The use of transparencies helps to recognise overlapping data points and better perceive the distribution of the data, as the transparency value of overlapping data points is accumulated, making them appear darker. Another method is to adjust the point size or shape [22, 32]. Encoding can also imply changing the visualisation type suitable for a larger data set or a different analysis task [33].

Filtering techniques are commonly used in the data preparation step and are suitable for reducing the amount of data [29, 33]. For static TCs, filtering is mostly realised by splitting the data into multiple charts [5]. For this purpose, quality metrics or a calculation of clusters are meaningful (e.g. [4, 23]). Furthermore, several reduction techniques based on statistical sampling are also applicable for filtering.

Data modification techniques can be applied to overcome visualisation challenges arising from overplotting, fluctuations or outliers. In particular, *jittering* slightly shifts similar or identical data points to reduce overplotting [20]. *Smoothing* techniques are applicable to reduce noise in the data, resulting in a better perception of the visualisation, especially for data with high fluctuations [27]. *Data aggregation* methods reduce the total amount of data, e.g. by summing or averaging of data chunks (e.g. [13, 19]). Fisher [13] applies aggregation methods to scatterplots to create an equivalent heatmap. The heatmap consists of a grid with coloured squares that have been assigned to specific ranges of values to represent the amount of data in each square area. Furthermore, Fisher suggests aggregating time data to overcome IO, e.g. the aggregation of daily measured values into monthly ones presented in line charts [13].

3 IDENTIFY AND IMPROVE CRITICAL TACTILE REPRESENTATIONS

Although previous work shows the suitability of TCs for data analysis, there is no research yet on how to create TCs depending on the data and how to deal with data leading to critical representations, where readability is very limited. However, TCs were often created with few data points. Since scatterplots are suitable to represent a high amount of data and they have already been considered for the tactile use [12], we will discuss the related research questions regarding the scalability of data using scatterplots:

- How many data can be represented and understood with TCs?
- Which representation problems can occur with TCs?
- How to deal with representation problems in TCs?

We then propose basic practices to help TCs developers decide on the best tactile representation method concerning the data and the intended analysis task.

3.1 Scalability of Data and Limitations of Tactile Charts

Limitations of the scalability of data in TCs depend on three main factors: the chart type, the data to be represented and the analysis task, which is further influenced by the production method of the TCs. Tactile representations can be considered critical if the analysis task cannot be fulfilled or the entire graphic or parts are not readable. Poor readability can occur if required sizes for elements are not maintained, elements overlap (overplotting) or too many elements are represented (IO), and consequently, they can no longer be distinguished from each other or recognised. Until now, critical tactile representations have been identified mainly manually or in user tests. Although it does not replace user studies, knowing under which conditions TCs are not legible can help to identify them automatically, which can generally improve the quality of automatically generated TCs.

In addition to the number of data points, the number of different distinguishable textures, symbols and line styles is also limited. The majority of researchers indicate a maximum of five per symbol class. In tactile scatterplots, different point symbols are used to distinguish different data sets. Another criterion is compliance with minimum sizes or distances, which is necessary to ensure recognisability. For example, if point symbols are too small, they can no longer be perceived as data or confused with Braille letters. The degree of overplotting can be seen as another essential factor for misleading TCs, especially for scatterplots. Three important measures were identified that can provide sufficient information about the readability of a tactile scatterplot:

1. Covered Area (*CA*): The proportion of the total area of the chart that is completely covered by the data points.
2. Degree of Overplotting (*O*): The total number of overlaps in the chart.
3. Limitations for multiple data sets (*M*): The number of different data sets is limited in terms of the number of different, distinguishable point symbols.

Overplotting in scatterplots leads primarily to critical representations when:

One data set: Too many data points in the same range will cause the data points to merge into one range, making it hard to detect individual data points, estimate the distribution in the range, or estimate the total number of data points. Trends and clusters may remain identifiable but cause cluttering and are less accurate than optimal representations. A measurement for this could be *CA*, which can only indicate whether a visualisation is likely to be critical, not the reverse. In contrast, the total number of data points is less suitable due to the dependency on the graphic's design. Representation problems can also occur if a large number of data points present similar values, which leads to a high degree of *O*, regardless of *CA*. This highly depends on the scale of the chart and is strongly influenced by outliers. For this reason, a metric that indicates the degree of intersection of data points could also be feasible to determine the readability of the chart.

Multiple data sets: CA, as well as *O*, can also be measurements for multiple data sets. Overplotting leads to misleading single symbols or the distribution of different data sets. Recognition of single values and the comparison of multiple data sets is substantially more difficult or impossible. Moreover, the challenges for one data set can occur individually for each data set. Furthermore, the number of different data sets is limited to a maximum of five (*M*), although it is unclear whether really five different point symbols are distinguishable at all in scatterplots.

The fulfilment of all three measurements is a sufficient but not necessary condition for TCs not to be readable. Thresholds for all conditions highly depends on the envisaged analysis tasks (e.g. focus on overview, precise values, clusters or comparisons). For example, if needed to read out precise values from the chart, then even a low value of *O* is a hindering factor for carrying out the analysis task. In contrast, if data points cover a large area (high value for *CA*) without any overlap, representation remains readable overall. Thresholds can be determined for each measure, beyond which the visualisation is highly unlikely to be readable.

CA can be determined by dividing the area covered by data points by the total area available for the representation. Depending on the data points' size, a threshold can then be determined for each visualisation, above which an overlap-free presentation is no longer possible. This parameter can be used to calculate the approximate maximum number of data points N_{max} that can be represented with the scatterplot depending on the size of point symbols (see Eq. 1). This is done by calculating the area that the data points would cover if they were *evenly distributed* over the chart area. To avoid data points being displayed outside the chart area, the radius respectively half the edge length of the bounding box l_{point} of data points and the value for the minimum distance to other elements d_{min} must first be subtracted from the width w_{chart} and height h_{chart} of the available area. Then the remaining area can be divided by the area each data point needs (assuming that data points have equal heights and widths) increased by half of the required distance to other elements, depending on the output production method.

$$N_{max} = \frac{(w_{chart} - 0.5 * l_{point} - d_{min}) * (h_{chart} - 0.5 * l_{point} - d_{min})}{(l_{point} + 0.5 * d_{min})^2} \quad (1)$$

If the calculated limit for data points is exceeded, it can be assumed that the representation is misleading. In practice, however, this will already occur with a smaller number of data points, as the calculation assumes an equal distribution of data points. However, the value indicates the suitability of a particular presentation method for a given data set, which can be used as an indicator when charts were generated automatically. For example, in a scatterplot embossed on an A4 sized sheet (chart area 120 mm x 240 mm), 115 equally distributed point symbols with a diameter of 10 mm are possible. Practical evaluations are needed to determine more accurate thresholds.

O can be calculated by counting the points that overlap others. It is not possible to give an exact value for the number of overlaps above which a critical visualisation can be assumed. This threshold can be determined by further empirical studies with PBVI. However, a maximum value can be set above which individual data points can no longer be recognised and distinguished. This case certainly occurs when more overlapping points than data points exist, i.e. when *O* is greater or equal to the total number of points represented with the chart. Here applies, too: in practice, representation problems will occur below this value. Whereby smaller overlays may only slightly interfere with the recognition, depending on the point symbol, when just one point symbol is used. We assume that overplotting is more critical when multiple data sets are present because it is more difficult to distinguish between different point symbols than distinguishing between two identical symbols. *O* may be higher to be still able to identify identical symbols [12]. Furthermore, it could be appropriate to include the number of intersections per data point as well as the degree of overlap for each data point in the calculation. Further research is necessary to prove this. In general, due to the embossers' rasterisation, the calculations are just an approximation and do not correspond exactly to the final embossed result.

However, when multiple data points represent the same values, the chart's readability is primarily limited. In these cases, the user cannot determine that several data points lie on top of each other. While even a small number of identical values can be challenging for comparative tasks and detailed analysis, many identical values can be disturbing to obtain an overview.

The following section describes how to handle data causing critical representations.

3.2 Handle Critical Tactile Representations

In the following, we discuss known concepts for dealing with critical representations in visual charts in terms of their applicability in TCs. In addition to the data, the analysis task and the chart type are two major factors influencing how the data can be mapped.

Previous work identified several approaches to increase the amount of data represented with InfoVis. Although most approaches focus on interactive visualisations, we analyse these approaches for static TCs. This practice does not exclude that these approaches can also be applied for interactive, tactile representations. However, interaction concepts are not considered here. In this section, we will discuss the applicability of the following three key concepts we identified:

(1) **Encode:** Change the design of the visualisation
(2) **Filter:** Show only a part of the data at once
(3) **Data manipulation:** Change the data to improve the visualisation

As a first step, authors of TCs need to decide which chart type best fits into accomplishing the envisaged analysis tasks. Chart types support different analysis tasks and are limited in the type of data they represent (e.g. numerical, categorical, ordinal). While bar charts are suitable for categorical values, scatterplots are best suited to detect trends and outliers in bivariate data with numerical values. Line charts are suitable to analyse trends over time, and pie charts highlight proportions. Furthermore, chart types differ in the amount of data they can represent effectively. Tactile bar charts can represent generally less data than scatterplots, which can be determined using the approaches proposed in section 3.1.

The methods apply to all chart types, but they need to be considered individually and their steps examined for feasibility.

3.2.1 Encode. Changing design properties implies the slightest changes to the representation, so its applicability should always be tested first. The following changes can be made to save space in scatterplots:

Change format or layout: If it is feasible and does not conflict with the intention or use case, the layout of the graphic should be adapted. For example, a larger format can be chosen for the output, whereby charts should not be embossed larger than the DIN A3 format. Switching between portrait and landscape format can also be advantageous in terms of space utilisation, depending on the data. Furthermore, care should be taken to ensure that the entire chart area is used effectively. Otherwise, the scaling of the axes could be adjusted.

Decrease the size of data points: This allows distances between the points and other elements to be increased. Minimum sizes should be maintained so that legibility is not affected. Point symbols (depending on the used symbol shape) should not be smaller than 6 mm when using just one symbol for the scatterplot. The smaller the symbols, the more difficult it is to recognise overlapping.

Shortening or omitting of Braille labels: Braille labels, especially labelling the axes, need much space in the chart. They are needed to understand the graphic. Using abbreviations referenced in the legend to save space meets applicable standards (e.g. [5]). Furthermore, the labelling of numerical values on the axes can also be transferred to the legend. If possible, the best way is to support another modality, e.g. audio output, which replaces Braille labels (e.g., using a digital pen [9] or a touch-sensitive device).

Changes in the design may provide a little more space on the chart but will not significantly affect the data amount to be represented. However, it may increase readability depending on the underlying data.

3.2.2 Filter the Data. Representing just a part of the data while hiding the rest of it (filtering) is a ubiquitous method, especially interactive, but also useful for static representations. For this purpose, they can be divided into multiple sheets. This method can be applied differently for data with one or multiple data sets:

Multiple data sets: The chart can be split, so each sub chart represents one data set or only parts of each data set. Headlines should point out the relation between multiple charts. Splitting the data sets could reduce the comparability between them. Therefore, splitting multiple data sets should be omitted if their comparison is the chart's primary analysis task. Each sub chart should represent the same layout and design as well as the same scale for both axes to ensure comparability between data sets. If this splitting method still results in critical visualisations, then one data set methods can be applied.

One data set: Splitting the data of one data set highly relates to the represented data and chart type. The first step should be to determine whether individual outliers are causing large empty areas, while other areas have a high density of data points that negatively affect the readability. If only a few outliers are present (estimated up to 5), they can be removed from the chart area and listed separately in the legend. The chart axes' scale can then be adjusted so that more space is available for the remaining data, and empty spaces were reduced. Otherwise, the whole data set must be split into multiple charts according to the value range. It depends on the specific analysis task into which value ranges the chart should be divided. Optimisation algorithms can be applied to obtain an ideal partition depending on the distribution of the data. In this way, data values with defined value ranges can be compared with each other. In contrast, scatterplots could, if data allows, be divided regarding the represented clusters of points. Clusters can be separated into different charts with same scale, which makes it difficult to compare data from different clusters. Several research for the computation of clusters as well as other quality metrics, especially for high-dimensional data, exists for this purpose (e.g. [4, 23]).

In general, the data is not changed when filtering, but only a part of the data can be presented and analysed at a time. Which method should be applied highly depends on the objective of the analysis. All in all, this concept enables the analysis of larger amounts of data, although comparability decreases as the number of different charts increases. It is also challenging to obtain an overview of all data, although detailed analyses are encouraged. Further studies are needed to determine the extent to which data presentation on different sheets affects comparability.

3.2.3 Data Manipulation. The modification of the data to increase the readability of the resulted visualisation is linked to a wide range of research and methods. Their feasibility and effectiveness for TCs still need to be investigated. Therefore, promising approaches are briefly described at this point but not discussed in detail. Modifications of the data should be carefully applied because they may change insights into the data.

Jittering: It is a common method to address overplotting issues. It requires the positions of the data points to be shifted minimally in order to minimise overlaps. However, jittering can only slightly increase the readability of individual, overlapping data points, which is especially helpful when a few data points of different symbols overlap. Due to embossing inaccuracies, a shift of at least 2 mm should be implemented depending on the degree of overlap.

Smoothing: Smoothing algorithms aim to prevent large fluctuations in the data by computing an approximating function to capture important patterns and reduce noising in the data. It can be helpful, especially for better traceability of lines in line charts. Smoothing can also lead to errors in detailed data analysis. Therefore, it should only be used to gain an overview of the data.

Aggregation: Another approach is to aggregate data based on a specific criterion that finally leads to a loss of information. The challenge is to aggregate the data in such a way that their general trend remains preserved. Aggregations in scatterplots can highlight data distribution or clusters, improving the overview of a large amount of data by aggregating the entire data set or parts (e.g. clusters). The latter requires identifying data points from the original data set and data points that result from the aggregation process. In contrast, all data points can be aggregated depending on their position. For this purpose, aggregated data points indicate the amount of data within a defined area. Pixel-based visualisation techniques (e.g. heatmaps) make use of aggregation concepts, with the advantage that the available space is used effectively and large amounts of data can be displayed. Otherwise, a detailed analysis of individual data values is made difficult. It can be applied to give an overview and identify anomalies as well as trends. Cells in a heatmap are usually colour-coded. There are no investigations in the feasibility and the design of tactile heatmaps so far.

In summary, approaches that manipulate the data can significantly affect the analysis results and should only be used with careful consideration. Jittering can be applied for charts with less overplotting to increase individual data points' recognisability, especially when multiple symbol types are used. Smoothing and aggregation techniques can be applied to large data sets and can improve tactile data analysis. They can be integrated into a workflow where representations generated in this way provide an overview, and detailed analyses are then implemented using the other proposed techniques. Whether approaches are suitable for the application in TCs and how they affect the scalability of the data and the analysis, is the subject of further studies. We investigated a novel concept where we applied aggregation methods on tactile scatterplots. In the following, we present the concept for tactile heatmaps, making it possible to analyse a high amount of data.

4 TACTILE HEATMAPS TO INCREASE THE REPRESENTED AMOUNT OF DATA

Heatmaps are common to represent the distribution of bivariate, numerical data. Colours typically represent cell values. We introduce a concept for tactile heatmaps, where we examined the tactile distinguishability of the cells by use of different ELs: the higher the cell, the higher the amount of data.

Heatmaps avoid overplotting, and the data distribution is easily recognisable. Identical data values are taken into account even if they cannot be identified directly. Heatmaps can be created based on scatterplots, especially if the recognition of distributions is difficult with a tactile scatterplot. Distributions, trends and clusters can be identified well by comparing the EL of cells. However, the creation of heatmaps is accompanied by a loss of information due to data aggregation, making detailed data analysis difficult. Furthermore, heatmaps are limited to single data sets. The comparison of different data sets is not possible with one chart. Besides, exact single values cannot be assumed. Heatmaps are, therefore, suitable when detailed data values are not the focus of interest. In order to support information seeking, we propose to use the tactile heatmap in addition to a tactile scatterplot so that both - the overview and detailed analysis - can be applied. To compare different data sets, we suggest creating a heatmap for each data set. The distributions and clusters can be first analysed with the heatmap to obtain an overview of the data and find interesting areas. Next, a detailed analysis of the data and intersections of different data sets for interesting areas can be carried out with another suitable chart type (see 3.1).

Heatmaps consist of cells representing the number of data points in the range the cell covers on the horizontal and vertical axis with a limited number of intervals. They can be seen as a transformation of scatterplots where every data point from the scatterplot is assigned exactly to one cell in the corresponding heatmap. The following definition can be applied: A point P(x_p,y_p) is located in an interval from x_i to x_j and y_i to y_j exactly when: $x_i \leq x_p < x_j$ and $y_i \leq y_p < y_j$. The number of points within each cell can be mapped to a linear ordinal scale. Tactile heatmaps require a predefined number of feasible ELs, as this makes it easier to distinguish and reference them than if they could be assigned continuously. The meaning of different ELs should be provided in the legend. The number of meaningful intervals is limited by the production method as well as by their tactile recognisability. Besides, the number of cells and the cells' size play an essential role in designing tactile heatmaps. The cells should be uniformly distributed and square in shape to ensure comparability of the areas. The resulting cells require a minimum size to be recognisable by touch. It is also unclear how many ELs are distinguishable under specific design conditions, which is why we propose feasible ELs and cell sizes for the evaluation with PBVI. The number of ELs depends on the output device, which is highly limited, especially with conventional TCs, e.g. graphic embossers and swell paper. Every EL corresponds to an interval of data points. The highest cell's value must be divided among the available ELs to find a suitable mapping for the ELs. The intervals must be explained in the legend. We recommend using linear steps for intervals to provide an intuitive understanding of the levels as they all have the same distance to each other. However, this can lead to a heatmap where not all predefined ELs appear, e.g. if no cell represents the interval with the fewest data points.

4.1 Production Methods for Tactile Heatmaps

The concept for tactile heatmaps requires a production method that allows at least more than one EL. Braille printers are, therefore, usually not suitable for this purpose. We tried to produce different ELs with swell paper and evaluated the results blind-folded by ourselves. Swell paper is inaccurate and elevations cannot be controlled, even if uniform heating of the paper can be ensured throughout. We could only distinguish two different levels, and very bright cells were not swollen at all, confirming related research findings. Therefore, swell paper is not suitable for different ELs. In contrast, graphic embossers and 3D printers are much more promising. We investigate in the ELs with the tactile graphic printer VP-Elite from ViewPlus (resolution: 20 dpi) and a standard 3D printer (Raised3D Pro2 3D printer). ViewPlus states that seven ELs can be created with the graphic embosser. Initial tests with the graphic embosser through blindfolding indicated that only three levels could be well distinguished (RGB values: Black (0,0,0), Grey (191,191,191), Light Grey (220,220,220)). Furthermore, we provide two more incised levels by embossing on the sheet's backside with two different levels (black and grey). We assumed that all five levels (two incised and three raised heights) could be distinguishable. Incised levels represent the lowest values, whereby the values increase with increasing heights.

In contrast, a 3D printer can overcome inaccuracies of embossing because of its high resolution. We, therefore, developed a 3D printed heatmap in addition to the embossed one to ensure that we could evaluate our concept, even if ELs could not be recognised with the embossed print. To ensure comparability with the 2.5D representations produced with graphic embossers, we evaluated the feasibility of eight different levels with a step distance based on previous research of 0.3 mm. It would also have to be investigated how many level are meaningful for tactile heatmaps and can be referenced in the legend.

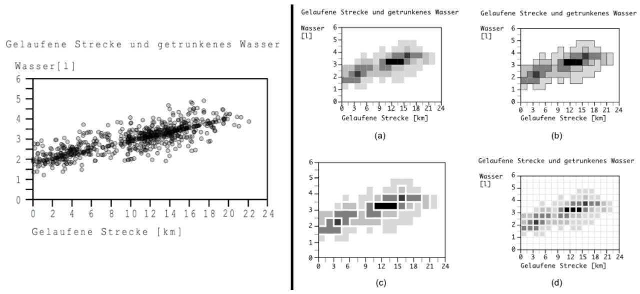

Figure 2: Left: Overplotted scatterplot. Right: Design conditions A-D of tactile heatmaps of the same data shown in the scatterplot. The darker the cell the higher the ELs.

4.2 Design of Tactile Heatmaps

For an effective design of tactile heatmaps, the perceptibility of ELs is the main aspect to recognise distributions in the data. Cell boundaries must also be easily perceptible to be able to identify data values. In general, a border around the chart area can help with orientation. We further recommend showing double-sided axes to facilitate the reading of values on the axes, as it is also helpful in other chart types [1, 11]. We identified further questions regarding the readability and design of the heatmap: How can the design support the reader in recognising different ELs of neighbouring cells? What is the best way to identify cluster boundaries? How can the design support the analysis process without distraction? Existing guidelines can inspire design suggestions. To ensure distinguishability, the recommended minimum distance between chart elements is 3 mm, areas should be at least 6 mm^2 to be perceived [5]. It is not clear whether these values can be used for tactile heatmaps and are applicable for different ELs. Therefore, we evaluated the readability of three cell sizes in a pilot study. Concerning the cell design, we assume that the cell's distinguishability and distributions can be influenced by the following design parameters: the size of the cells, border, space between cells as well as the use of grid lines. To proof this, we developed four design conditions (see Figure 2 right).

Design A: No outline, no spacing. The first design condition includes cells arranged directly next to each other, as shown in Figure 2 (a). In this way, the finger reaches one EL after the other without interruptions while exploring. We assume that cell level changes can be perceived better with this design, supporting trends' recognition. However, due to the lack of haptic indicators between cells, it may be difficult to distinguish ELs from each other.

Design B: Contour around cells of the same EL. Second condition adds contours to areas with same EL, as shown in Figure 2 (b) and Figure 1 left. This facilitates the identification of neighbouring cells with the same EL during exploration. We assume that ELs are better distinguishable in this condition.

Design C: Distance between cells of different ELs. The third design adds distances (no embossing) of 6 mm between cells of different ELs instead of a contour. With this design, we compare whether distances or contours better support adjacent cells' distinguishability and elevations. In contrast to Design B, the chart contains less embossed elements, which could reduce cluttering effects. That is why we estimate that distance leads to a better distinguishability than contours. However, as seen in Figure 2 (c), this condition requires more space overall.

Design D: Grid between all cells. Design D adds grid lines to all cells (see Fig. 2 (d)). This design has a cell size of 7 x 7 mm, grid lines with an edge length of 10 mm and a line width of 1 mm. grid lines were embossed with a lower EL than other chart elements to be at least significant. Cells have a distance of 1 mm to the grid lines and 3 mm to each other. We assume that this design condition leads to better recognition of cells and cluster boundaries. Furthermore, grid lines may support the orientation on the graphic as well as the reading of values. It is also feasible that grid lines decrease the data's readability because of the IO.

The legibility of heatmaps is highly dependent on the distinguishability of the ELs. To ensure that the concept of heatmaps could be evaluated regarding the usefulness, we also produced a tactile heatmap using a 3D printer (see Fig. 1 right) with much higher accuracy. In addition to the design features described in section 4.1, the cells have an edge length of 10 mm and are directly next to each other. Cells without data points are separated from each other with guiding lines.
We compared the presented design conditions in a pilot study with PBVI. We also evaluated the suitability of heatmaps for cluster and trend analyses of bivariate data.

5 PILOT STUDY ON TACTILE HEATMAPS

First, we evaluated the design of cell size and cell spacing in relation to the recognisability of cells and ELs. Second, we compared the four design conditions for tactile heatmaps with 11 PBVI.

5.1 Participants

11 PBVI (7 female, 4 males) took part in the pilot study, which was organised in the form of a workshop. The participants consist of nine students (aged between 14 and 18 years) and two adults (about 40 years). All of the seven participants with blindness have good to very good braille reading skills and are already familiar with TCs whereby four participants with blindness use TCs weekly. Most participants knew line charts and bar charts. The four participants with visual impairment (1 highly visually impaired, three visually impaired) have neither experiences with TCs, nor knowledge with reading Braille. Four participants were familiar with scatterplots (two blind, two visually impaired), while only the participant with most vision knew heatmaps.

5.2 Procedure and Materials

The workshop was held before the Covid-19 pandemic as part of an orientation week for blind and visually impaired people interested in studying. The procedure took about two hours. It was structured in five parts.

The first part includes introducing tactile graphics where general reading strategies where explained and example graphics were shown. Participants could try out exploring strategies on their own by guessing tactile animals. All materials used for this study (embossed graphics and 3D-prints) were embossed with graphic printer VP-Elite from ViewPlus as described in 4.1. Participants with visual impairment got materials including black printed letters whereby the graphical elements were just embossed, as in Figure 1 left.

For the second part, the participants were divided into three groups, whereby each group performed the rest of the study with two supervisors in separated rooms. A training phase followed, in which the structure and relationship of scatterplots and heatmaps were explained by handing each participant a simple tactile scatterplot with 20 data points and the corresponding embossed tactile heatmap with three ELs. Participants could explore the charts on their own or with the supervisor's help and could ask questions.

In the third part, we evaluated the recognisability of cells depending on their size and distances. We used cells with an edge length of 13 mm, 10 mm and 5 mm and the distances between the cells with 1 mm, 3 mm and 6 mm. In practice, due to the inaccuracies in embossing, 1 mm distances can often not be perceived as such. This part of the study addresses two main objectives: Which conditions are best for recognising individual cells? Which conditions are the best to distinguish ELs of different cells? The nine design variations were represented on two sheets (a combination of three edge lengths and three distances). Each variation consists of multiple squares with two or three ELs. Squares with the same edge length were arranged directly among each other in two rows and several columns. In this way, the horizontal and vertical discriminability of cells can be examined for different levels. First, participants explored squares with the edge length of 10 mm and varying distances of 6 mm, following 3 mm distance and 1 mm last. This order is the same for all square size conditions. Participants were asked to freely explore squares of the same size using the thinking-aloud technique. After exploring all squares of the same size, the participants should rate the squares' recognisability on a scale ranging from 1 (very good) to 6 (not recognisable). They should also rate the most appropriate distances to distinguish between different ELs. This part was performed just by participants with blindness.

The fourth part's objective was to evaluate the comprehensibility and suitability of heatmaps for data analysis tasks. Participants got a scatterplot with a high degree of overplotting representing 727 data points (see Figure 2 left). The distribution of data points indicates an increasing trend with a gap between 9 and 10 for horizontal values. The data points' density increases from the value 10 on the horizontal axis, represented by transparency in Figure 2 left. Two main clusters can be estimated. The point symbols feel like a flat surface due to the high overplotting, causing a critical representation of the described concept's meaning. First, the participants could explore the scatterplot on their own. Afterwards, they have to identify general trends and clusters as well as the range with the highest point density represented with the chart. Finally, participants should rate the chart's readability from 1 (very good) to 6 (very poor).

After that, the participants got embossed heatmaps showing the four design conditions in a random order to avoid learning effects (see Figure 2 right). All heatmaps had the same legend and represented the same data as the previously analysed scatterplot. They consist of five different ELs whereby two levels were incised. All squares have an edge length of 10 mm. At first, participants could explore the first design condition on their own without having the legend. Afterwards, they were asked to name general clusters, trends and the range with the highest point density. The participants should then identify the number of different ELs they can feel on the heatmap. After that, they were given the legend to explore all five available ELs. Now they were asked to try referencing the levels showing in the legend within the heatmap. The participants used this knowledge to examine the remaining three design alternatives for the heatmap. Finally, the participants got the 3D-printed heatmap, described in section 4.2. Axes and labels have been omitted as they are the same as in the embossed heatmap, and they could be confusing due to unfamiliarity with 3D prints. Participants were asked to compare both representations for heatmaps (embossed vs 3D-printed). Furthermore, they could give suggestions for improvements. Finally, the participants discussed the advantages and disadvantages of the design conditions and named their preferred design.

5.3 Results

Due to the study's workshop format, few quantitative data, such as error rates or completion times, were collected. Instead, much valuable qualitative data - either from observations or the participants' comments and responses - was included. Based on the results, we derived basic design guidelines for tactile heatmaps.

Evaluation of Cell Design: Smaller cells were rated better regarding the recognisability than large cells (5 mm: 4-times mark 1, 1-time mark 2; 10 mm: 4-times mark 1, 1-time mark 3; 13 mm: 3-times mark 1, 1-time mark 2, 1-time mark 5). These ratings depended strongly on the ELs and the cell spacing. Individual cells with a higher EL could be detected better than cells with a lower EL. Larger distances lead to better recognition of individual cells and borders. The 6 mm distances archived best ratings across all cell size conditions (62 %) whereby two participants (3 of 21 ratings) could not decide on a preferred distance. In contrast, participants reported that smaller distances lead to better distinguishability of ELs. As a result, four participants recommended an edge length of 10 mm with a cell spacing of 3 mm as compromising for both - recognition of cells and levels.

Tactile Scatterplot: The tactile scatterplot was rated as poorly legible by all participants (4-times mark 6). Individual elements were difficult to recognise due to the high degree of overplotting. No correlation could be found between the answers and the age of a participant. However, the blind participants with high experiences in TCs were able to determine trends and correlations correctly. It was also difficult to give precise values due to the lack of a grid. Four blind participants identified the gap in the data as well as clusters before and after the gap. Just one blind participant could name the interval with the highest point density correctly. Some visually impaired participants indicated that they would prefer a large print with black lettering and semi-transparent data points.

Evaluation of Tactile Heatmaps: Overall, the heatmap representation was rated much better than the scatterplot. All participants could recognise correlations with the tactile heatmap in all design conditions what was feasible with the scatterplot for only four participants. Moreover, all participants were able to name the area with the highest point density using the heatmap. Most of the participants could only distinguish and assign three ELs in embossed heatmaps (with and without the legend). The incised levels were often not recognised. This result could be explained by the fact that incised elements do not correspond to the usual tactile reading

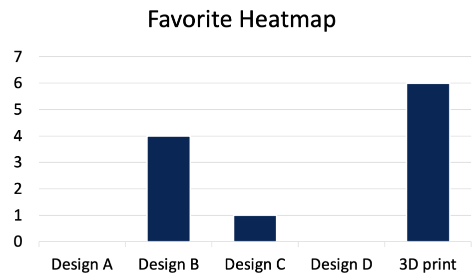

Figure 3: Rating of the preferred heatmap design by the participants. Number of participants on the vertical axis.

experience and are therefore ignored. Besides, participants reported that they perceive the ELs as differing in the paper's roughness. The distinctiveness of incised and raised ELs with the same grey value (height) was complicated. Here, no correlation could be found between the answers and the age of participants.

Design A was rated as the worst design condition by all participants. One participant could only recognise different levels when the legend was given, whereby the participant started with this condition. In contrast, two participants who also started with this condition were able to identify cells with the highest values.

Design B was the most preferred embossed design (4 ratings), as shown in Figure 3. Three blind participants rated the distinguishability of the ELs as good as the 3D print. Most participants were able to identify three ELs reliably. One participant could even recognise all five ELs. Another participant reported that the contours would bring order to the graphic. One participant noticed that this condition enables a better assignment of cell ELs to the legend.

Although most participants could understand the heatmap's content with Design C, this design is generally not preferred in terms of readability and recognition of different levels. Two blind participants do not recognise the distances between cells independently. Contrary to our assumption, this presentation seemed confusing for two participants. Besides, one participant recommended increasing the distances between cells.

Design D was rated as the second-best embossed design by two participants. According to our assumption, this heatmap was the only one where participants could reference precise values without finger displacements. One participant could best distinguish the ELs in this condition, whereby this person could distinguish all ELs. On the one hand, three participants perceived this design as structured. On the other hand, for one participant, this design was harder to read than the scatterplot, and another participant noted that the grid caused cluttering.

The 3D-printed heatmap was rated best by the participants (6 ratings). Overall, the heatmap concept was much easier to understand with the 3D print than with the embossed ones. One of the blind participants had to leave earlier, so the 3D printed heatmap was only explored by 10 participants. All of them were able to recognise the ELs, distribution and clusters correctly. The six blind participants agreed that the 3D model is most suitable for distinguishing relative ELs. However, five of them noted that the absolute level differences were difficult to determine and recommended using different surface textures additionally. One blind participant recommended obtaining a better overview of the data with the 3D heatmap than with the embossed ones. Several participants with visual impairment are interested in a coloured 3D print of the heatmap.

In the final discussion, participants stated that the heatmap's suitability as an alternative for a scatterplot strongly depends on the use case and the analysis task (detailed analysis vs getting an overview). Two participants said they could not see any advantage in using both the scatterplot and the heatmap for analysis in parallel because the given scatterplot does not allow a more detailed analysis of the data. The participants also mentioned suggestions for improvement as well as design alternatives: less ELs in the embossed prints; use embossed textures for cells instead of ELs; usage of guiding lines (as in 3D print) instead of continuous grid lines; colouring cells in 3D-printed heatmaps for people with visual impairments; same axes ranges of scatterplot and heatmap for better comparability. All in all, participants could better determine trends and clusters with the heatmap (all blind participants) than with the scatterplot (four blind participants) used for the study, whereby both represent the same data. Although none of the participants had been familiar with the concept of tactile heatmaps, most of them understood it and considered it useful. Distinguishing different ELs were also much easier with the 3D-printed heatmap. Although not all levels could be reliably detected with embossed heatmaps, tactile prints may still be suitable for this purpose. At least three different levels are reliable. The design of the EL should be evaluated with different roughnesses for incised and embossed elements. When incised levels are used, users should be made aware of them. Otherwise, they may not notice them. Two participants were able to distinguish all five ELs (1x Design B, 1x Design D). We summarise our findings of the study by deriving some basic design guidelines for tactile heatmaps to be used for data distribution analysis:

- Provide well distinguishable ELs with different roughnesses
- 10 mm edge length for cells is recommended
- Provide a contour around areas with the same EL for better orientation
- grid lines should be present for cluster analysis tasks
- If a scatterplot and a heatmap are used simultaneously for the analysis, they should have the same axis range
- 10 mm edge length of cells and a distance of at least 6 mm is recommended to optimise both - recognition of cells and elevations
- Adjacent cells in the heatmap should not be placed directly next to each other without contours or distances

Behind these guidelines, the design should be adapted to individual users' preferences to facilitate individual data analysis.

6 CONCLUSION AND FUTURE WORK

This paper discussed the scalability of data in tactile scatterplots and identified parameters that may significantly affect the readability. Furthermore, we propose a method to identify critical tactile representation, which forms the basis for automatically recognising critical TCs. Furthermore, we provide approaches to overcome critical TCs. Further studies with PBVI need to determine thresholds that are reliable to detect critical visualisations for different production methods. The applicability of common methods, such as smoothing or aggregations, should also be investigated for TCs more in detail. It should be further investigated to what extent these methods change the tactile perception of the data.

We further show the usefulness of tactile heatmaps for data analysis purposes, particularly for getting an overview and distribution tasks for a high amount of data, to overcome overplotting in tactile scatterplots and to increase the amount of data which can be analysed with TCs. In this context, we have shown that three different ELs are distinguishable with embossed graphics, using incised and raised elements. The influence of the roughness of the ELs on the distinguishability should be investigated more closely. We assume that it might be possible to distinguish five ELs in embossed heatmaps in this way. Our proof-of-concept study with a 3D printed heatmap, in which the EL could be well distinguished, shows that the concept of tactile heatmaps is a promising approach to enable tactile data analysis. Further studies need to investigate the effectiveness of the analysis of distributions and clusters with tactile heatmaps compared to other tactile presentations. Further research is needed to see how the whole data analysis process (overview and detailed analysis) can be effectively established with heatmaps (e.g., using multiple chart types for different analysis tasks). More research on the design of novel visualisation techniques for the tactile domain is needed to provide effective data analysis for PBVI. Furthermore, specific use cases that may be relevant, e.g. in a professional context, should also be considered to ensure that people benefit from the possibility of data analysis.

ACKNOWLEDGMENTS

Special thanks go to Meinhardt Branig for creating the 3D model of the heatmap and his active support in planning and conducting the pilot study. Furthermore, we thank the student assistants, who supported us in conducting the pilot study. Finally, we would like to thank all the participants in the pilot study for their participation.

REFERENCES

[1] Frances K. Aldrich and Alan J. Parkin. 1987. Tangible Line Graphs: An Experimental Investigation of Three Formats Using Capsule Paper. *Human Factors* 29, 3 (1987), 301–309. https://doi.org/10.1177/001872088702900304 arXiv:https://doi.org/10.1177/001872088702900304 PMID: 3623565.

[2] John L. Barth. 1984. Incised Grids: Enhancing the Readability of Tangible Graphs for the Blind. *Human Factors* 26, 1 (1984), 61–70. https://doi.org/10.1177/001872088402600106 arXiv:https://doi.org/10.1177/001872088402600106 PMID: 6735408.

[3] Jacques Bertin. 1974. Grafische Semiologie. *Diagramme–Netze–Karten.* (1974).

[4] E. Bertini, A. Tatu, and D. Keim. 2011. Quality Metrics in High-Dimensional Data Visualization: An Overview and Systematization. *IEEE Transactions on Visualization and Computer Graphics* 17, 12 (2011), 2203–2212. https://doi.org/10.1109/TVCG.2011.229

[5] Braille Authority of North America and Canadian Braille Authority. 2010. *Guidelines and standards for tactile graphics.* http://www.brailleauthority.org/tg.

[6] Ed Huai-hsin Chi. 2000. A taxonomy of visualization techniques using the data state reference model. In *IEEE Symposium on Information Visualization 2000. INFOVIS 2000. Proceedings.* IEEE, 69–75.

[7] Sidney S Culbert and William T Stellwagen. 1963. Tactual discrimination of textures. *Perceptual and Motor Skills* 16, 2 (1963), 545–552.

[8] Snir Dinar, Jonathan Rowell, and Don McCallum. 2005. The uniqueness of symbol profile as a design variable in tactile cartography. In *International Cartographic Conference 2005.*

[9] Christin Engel, Nadja Konrad, and Gerhard Weber. 2020. TouchPen: Rich Interaction Technique for Audio-Tactile Charts by Means of Digital Pens. In *International Conference on Computers Helping People with Special Needs.* Springer, 446–455.

[10] Christin Engel and Gerhard Weber. 2017. Improve the accessibility of tactile charts. In *IFIP Conference on Human-Computer Interaction.* Springer, 187–195.

[11] Christin Engel and Gerhard Weber. 2018. A user study to evaluate tactile charts with blind and visually impaired people. In *International Conference on Computers Helping People with Special Needs.* Springer, 177–184.

[12] Christin Engel and Gerhard Weber. 2019. User Study: A Detailed View on the Effectiveness and Design of Tactile Charts. In *IFIP Conference on Human-Computer Interaction.* Springer, 63–82.

[13] Danyel Fisher. 2016. Big data exploration requires collaboration between visualization and data infrastructures. In *Proceedings of the Workshop on Human-In-the-Loop Data Analytics.* 1–5.

[14] John A Gardner and Vladimir Bulatov. 2006. Scientific Diagrams Made Easy with IVEO ™. (2006), 1243–1250.

[15] Timo Götzelmann. 2016. LucentMaps: 3D Printed Audiovisual Tactile Maps for Blind and Visually Impaired People. In *Proceedings of the 18th International ACM SIGACCESS Conference on Computers and Accessibility* (Reno, Nevada, USA) *(ASSETS '16).* Association for Computing Machinery, New York, NY, USA, 81–90. https://doi.org/10.1145/2982142.2982163

[16] R. Gupta, P. V. M. Rao, M. Balakrishnan, and S. Mannheimer. 2019. Evaluating the Use of Variable Height in Tactile Graphics. In *2019 IEEE World Haptics Conference (WHC).* 121–126. https://doi.org/10.1109/WHC.2019.8816083

[17] Sandra Jehoel, Snir Dinar, Don McCallum, Jonathan Rowell, and Simon Ungar. 2005. A scientific approach to tactile map design: Minimum elevation of tactile map symbols. In *Proceedings of XXII International Cartographic Conference A Coruρa, Spain.*

[18] Sandra Jehoel, Don McCallum, Jonathan Rowell, and Simon Ungar. 2006. An empirical approach on the design of tactile maps and diagrams: The cognitive tactualization approach. *British Journal of Visual Impairment* 24, 2 (2006), 67–75.

[19] Jimmy Johansson and Matthew Cooper. 2008. A screen space quality method for data abstraction. In *Computer Graphics Forum*, Vol. 27. Wiley Online Library, 1039–1046.

[20] D. A. Keim and A. Herrmann. 1998. The Gridfit algorithm: an efficient and effective approach to visualizing large amounts of spatial data. In *Proceedings Visualization '98 (Cat. No.98CB36276).* 181–188. https://doi.org/10.1109/VISUAL.1998.745301

[21] Susan J. Lederman and Jamie I. Campbell. 1982. Tangible Graphs for the Blind. *Human Factors* 24, 1 (1982), 85–100. https://doi.org/10.1177/001872088202400109 arXiv:https://doi.org/10.1177/001872088202400109 PMID: 7068153.

[22] Jing Li, Jean-Bernard Martens, and Jarke J van Wijk. 2010. A model of symbol size discrimination in scatterplots. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems.* 2553–2562.

[23] H. Liao, Y. Wu, L. Chen, and W. Chen. 2018. Cluster-Based Visual Abstraction for Multivariate Scatterplots. *IEEE Transactions on Visualization and Computer Graphics* 24, 9 (2018), 2531–2545. https://doi.org/10.1109/TVCG.2017.2754480

[24] Justin Matejka, Fraser Anderson, and George Fitzmaurice. 2015. Dynamic opacity optimization for scatter plots. In *Proceedings of the 33rd Annual ACM Conference on Human Factors in Computing Systems.* 2707–2710.

[25] Round Table on Information Access for People with Print Disabilities Inc. 2005. Guidelines on Conveying Visual Information. (2005).

[26] Denise Prescher, Jens Bornschein, and Gerhard Weber. 2017. Consistency of a Tactile Pattern Set. 10, 2 (2017), 1–29.

[27] P. Rosen and G. J. Quadri. 2020. LineSmooth: An Analytical Framework for Evaluating the Effectiveness of Smoothing Techniques on Line Charts. *IEEE Transactions on Visualization and Computer Graphics* (2020), 1–1. https://doi.org/10.1109/TVCG.2020.3030421

[28] William Schiff and Herbert Isikow. 1966. Stimulus redundancy in the tactile perception of histograms. *International Journal for the Education of the Blind* (1966).

[29] Ben Shneiderman. 2003. The eyes have it: A task by data type taxonomy for information visualizations. In *The craft of information visualization.* Elsevier, 364–371.

[30] R Vasconcellos. 1991. Knowing the Amazon through tactual graphics. In *Proceedings 15th International.*

[31] Tetsuya Watanabe, Toshimitsu Yamaguchi, and Masaki Nakagawa. 2012. Development of software for automatic creation of embossed graphs: Comparison of non-visual data presentation methods and development up-to-date. *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)* 7382 LNCS, PART 1 (2012), 174–181. https://doi.org/10.1007/978-3-642-31522-0_25

[32] Allison Woodruff, James Landay, and Michael Stonebraker. 1998. Constant density visualizations of non-uniform distributions of data. In *Proceedings of the 11th annual ACM symposium on User interface software and technology.* 19–28.

[33] J. S. Yi, Y. a. Kang, J. Stasko, and J. A. Jacko. 2007. Toward a Deeper Understanding of the Role of Interaction in Information Visualization. *IEEE Transactions on Visualization and Computer Graphics* 13, 6 (2007), 1224–1231. https://doi.org/10.1109/TVCG.2007.70515