

Heat to Height: A study on the physical dimensionality of heat maps

Introduction

In the 20th century, advancements in technology made data visualizations increasingly more affordable and accessible to a broader population. The primary change in the construction of data visualizations was from hand-crafted charts to computer-rendered charts (Tukey, 1965), yet other technological advances have allowed data visualizations to enter other mediums. These include the ability to effectively use the three-dimensional (3D) world around us with the novel use of 3D-printed charts. This newer type of visualization has gained traction in recent years as a method of producing tangible charts.

Current literature on data visualization practices is mainly limited to 2D projections of charts, where comparisons of 2D and 3D charts show mixed results. In 2D projections, this is mainly attributed to the purpose of the depth cues in 3D charts. When the increased dimensionality does not include additional information, information extraction tends to be worse for 3D charts than for 2D charts (Zacks et al., 1998). However, the inclusion of information in the depth axis tends to lower error rates in numerical estimations and increase memorability of results (Barfield & Robless, 1989; Jeong et al., 2025). These discrepancies suggest that 3D charts have situational use cases where they can outperform other types of charts in delivering statistical information.

For continuous responses with two predictor variables, heat maps are a common type of data visualization display. Typical presentations of heat maps are in two-dimensional formats, relying on color to convey the response variable. When presented in three dimensions, heat maps use height to map the response, sometimes with the inclusion of color. Using 3D printing, we are able to explore how the physical dimensionality of charts affects the extraction of numerical information from statistical graphics. We hypothesize that 3D-printed heat maps will outperform 2D and digitally rendered 3D heat maps in numerical estimations of stimuli ratios.

Methods

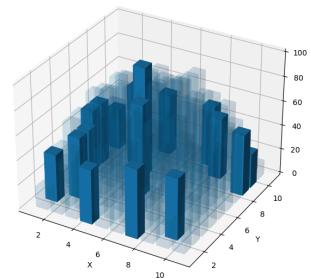
Our study is designed to evaluate and expand the literature on numerical estimation of ratios for 3D charts. To accommodate discrepancies with perceived magnitudes of heights and volumes (Cleveland & McGill, 1984; Stevens & Stevens, 1986), we used the method of constant stimuli for selecting values to be used for ratio estimations. All stimuli pairs are

placed onto a single heat maps to reduce the number of 3D-printed charts required in the study.

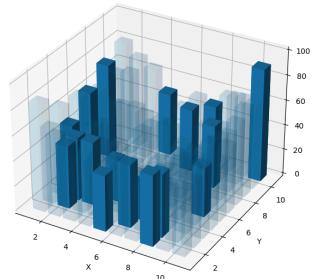
Selection of Stimuli Values

Stimuli values are chosen so that the constant stimuli is at 50 units. We set the maximum stimuli at 90 units and equally partition the ratios between $50/90 \approx 0.56$ and $50/50 = 1$ to get stimuli values between 50 units and 90 units, giving ratios of 0.56, 0.67, 0.78, and 0.89. We then set 50 units as the maximum value and use the same ratios to obtain stimuli values that are less than 50 units. Additionally, we include a stimuli pair where both values are the same, resulting in 9 total pairs of stimuli.

Each heat map consists of a 10×10 grid with values generated from the mixture of a uniform distribution and spherical functions. Two data sets were created to representing the top and bottom halves of a sphere centered on the coordinate grid. Stimuli pairs were placed via simulation to appear natural while maintaining appropriate spacing and distribution across the grid.



(a) Data set 1



(b) Data set 2

Figure 1: Placement of stimuli in the heat map data sets.
Transparent bars represent randomized values.

Chart types

We considered three types of charts in our experiment: 2D-digital (2dd), 3D-digital (3dd), and 3D-printed (3dp). We used the 3dp chart as a baseline for the construction of 2dd and 3dd charts. The 3dp chart was designed so that each tile is 1 cm^2 with a maximum height of 10 cm. We printed these charts with either solid or gradient filaments. The print files for 3dp charts were converted into WebGL displays (Murdoch & Adler, 2024), creating exact one-to-one digital displays of the 3dp charts. Without a direct mapping of color to height, we designed the 2dd chart to have a gradient fill that mirrored the lighting conditions of our initial 3dp charts, interpolated with the `scale_fill_gradient()` function from

the `ggplot2` package (Wickham, 2016) using #0C2841 and #66D9FF as the color range.

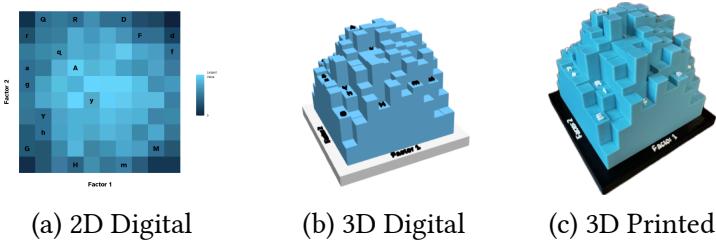


Figure 2: Chart types representing heat map data set 1.

Experimental Design

With a completely randomized design, participants would need to complete 54 trials (3 chart types \times 9 stimuli pairs \times 2 data sets). Instead, we opted for a balanced incomplete block design to reduce the effects contributed by fatigue, using 4 of the 9 stimuli pairs for each participant to construct the blocks. This results in 24 trials per participant, which should improve the overall quality of the responses. Trials were presented in a randomized order of chart type \times data set, where all four stimuli pairs were presented in a randomized order before moving to the next chart type \times data set combination.

Following the seminal work of Cleveland & McGill (1984) in data visualization practices, we asked participants two modified questions for each trial. The first question asked participants which value in the identified stimuli pair is larger. This question served as a basic attention check to determine whether the participant was correctly following the instructions for their ratio estimation. The second question asked participants to use a slider to estimate the quantity of the smaller value if the larger value in the stimuli pair represented 100 units. For the second question, the phrasing is designed to have participants estimate A/B , where $A \leq B$.

Results

196 students completed the experiment as part of a project in the curriculum of an introductory statistics course. Of 4072 submitted entries, 3218 were correct in identifying the larger value in a given stimuli pair (79%).

Similar to Cleveland & McGill (1984), we compute error as follows:

$$\log_2(|\text{judged percent} - \text{true percent}|)$$

Excluding the stimuli pair where both values were the same, a linear mixed model was fit to the log error, accounting for random participant and set \times media ordering. The effect of chart type ($p\text{-value} < .001$) and stimuli pair ($p\text{-value} = 0.0434$) had evidence of a statistically significant effect. For chart types, 2dd charts had larger errors than 3dd and 3dp charts, where no difference was detected between 3dd and 3dp

charts. Only one difference of errors between stimuli pairs was statistically significant ($p\text{-value} = 0.009$), which was the difference between the ratios of (33.3, 50) and (50, 56.2).

Discussion

In this abstract, we described an experiment that explored the relationship between physical dimensionality and the rendering of heat maps. While 2D displays of 2D and 3D data visualizations have been widely explored, our work contributes to the broader understanding of the physical dimensionality of 2D and 3D charts. We found that 3D heat maps are better than 2D heat maps for ratio estimations, but we did not detect a difference between digital and physical 3D heat maps. At the conclusion of the study, we will release a repository to encourage replication and sharing of experimental results.

Bibliography

- Barfield, W., & Robless, R. (1989). The Effects of Two- or Three-Dimensional Graphics on the Problem-Solving Performance of Experienced and Novice Decision Makers. *Behaviour & Information Technology*, 8(5), 369–385. <https://doi.org/10.1080/01449298908914567>
- Cleveland, W. S., & McGill, R. (1984). Graphical Perception: Theory, Experimentation, and Application to the Development of Graphical Methods. *Journal of the American Statistical Association*, 79(387), 531–554. <https://doi.org/10.1080/01621459.1984.10478080>
- Jeong, J., Choi, A., Kang, D., & Lee, Y. K. (2025). A Comparative Study of 2D vs. 3D Chart Visualizations in Virtual Reality. *Journal of Visualization*, 28(1), 239–253. <https://doi.org/10.1007/s12650-024-01033-6>
- Murdoch, D., & Adler, D. (2024). *rgl: 3D Visualization Using OpenGL*. <https://cran.r-project.org/package=rgl>
- Stevens, S. S., & Stevens, G. (1986). *Psychophysics: introduction to its perceptual, neural, and social prospects*. Transaction Books.
- Tukey, J. W. (1965). The Technical Tools of Statistics. *The American Statistician*, 19(2), 23–28. <https://doi.org/10.2307/2682374>
- Wickham, H. (2016). *ggplot2: Elegant Graphics for Data Analysis*. <https://ggplot2.tidyverse.org/>
- Zacks, J., Levy, E., Tversky, B., & Schiano, D. J. (1998). Reading bar graphs: Effects of extraneous depth cues and graphical context. *Journal of Experimental Psychology: Applied*, 4(2), 119–138. <https://doi.org/10.1037/1076-898X.4.2.119>