A Critical Examination of the Grammar of Graphics

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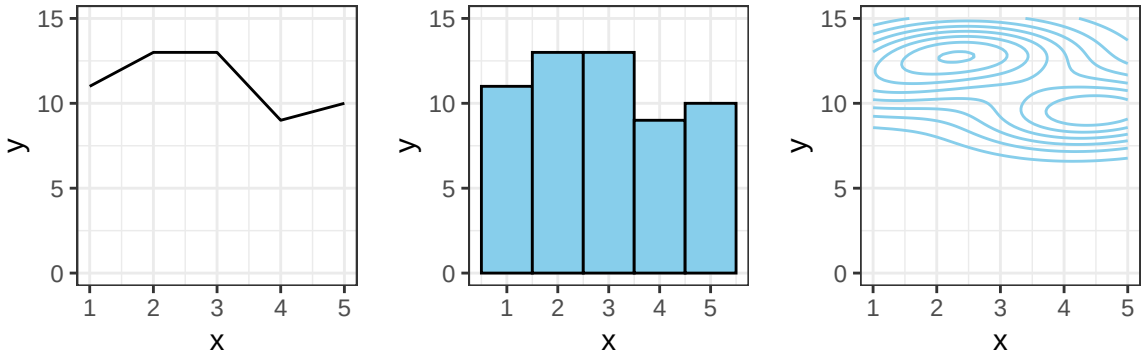
This paper explores the philosophy and applications of the Grammar of Graphics (Wilkinson 2010). The Grammar of Graphics uses seven orthogonal classes to deconstruct the process of creating statistical graphics. Through evidence-based reasoning, the classes can be reduced to a minimally sufficient set of features. I will then propose new additions to the minimum set of features for a more robust grammar for creating statistical graphics.

Introduction

Data visualizations are an essential method in conveying information (Tukey 1965). When presented with data, these graphics provide faster comprehension, albeit slightly less accurate, than data presented in a table or text format (Meyer, Shinar, and Leiser 1997; Prasad and Ojha 2012; Brewer et al. 2012). The computer boom in the later half of the twentieth century allowed for new innovations in graphics, along with the ability to create graphics faster and more easily (Tukey 1965; Beniger and Robyn 1978). The importance of data visualization is evident from its widespread use in science and media, which means that creating these graphics requires careful consideration to display meaningful information.

However, the mapping of data to the visual space inherently has creative freedom which results in nearly infinite representations. An example of this is produced in Figure 1, where the same bivariate data is represented in three different forms. Good practices have developed over the years for producing readable and effective graphics (Tufte 1983; Kosslyn 1989; Cleveland and McGill 1984; Heer and Bostock 2010b). A byproduct of graphics research is the recommendations for the forms for which graphics can take. Most modern graphical software packages, such as `ggplot2` (Wickham 2016) from R (R Core Team 2024), are able to create many types of graphics with minimal user effort. These packages inherently need to follow a well-designed process to produce statistical graphics.



(a) Line chart (b) Bar chart (c) Contour plot

Figure 1: Three graphical representations of identical data. The line chart in (a) is useful for showing a trend in y across x. When x is categorical, as in (b), y can be compared across the categories of x. Lastly, the countour plot in (c) shows the bivariate density of x and y. Each graphic conveys information from different contexts of the data.

Since the graphical representation of data can take many forms, it is important to establish a set of rules, or grammar, to define the process of creating data visualizations. One such proposal was the Grammar of Graphics by Wilkinson (2010). The Grammar of Graphics defines the process as a sequential set of steps, where each step equally contributes to the rendered graphic. This process includes the following steps: variables, algebra, scales, statistics, geometries, coordinates, and aesthetics. Taken together, these steps provide a system of creating graphics by reducing each step into its most basic form.

In this paper, I will discuss the Grammar of Graphics by Wilkinson (2010) and its applications in creating graphics. The grammar, while created with good intentions, has some philosophical flaws that I will address. These shortcomings are both in theory and application of the grammar, from which easy adaptations will allow it to functionally behave with existing graphics tools.

Grammar of Graphics

Wilkinson (2010) describes the Grammar of Graphics as a system of seven orthogonal classes. These classes produce an abstract sequential formulation for creating graphics. Wilkinson states nearly all statistical graphics can be created using the classes, but deviation from the classes or sequence can result in meaningless graphics. The Grammar of Graphics consist of the following seven classes: variables, algebra, scales, statistics, geometries, coordinates, and aesthetics.

Variables

Wilkinson (2010) states an important distinction between the definition of a *variable* and a *varset*. A *variable* is a mapping of objects to values, whereas a *varset* is a mapping of values to all possible ordered lists of objects. This distinction is claimed to simplify the process of algebra, although many graphics packages do not acknowledge the difference. The Graphics Production Language (GPL) is one such example of where the distinction is directly implemented (Wilkinson and Wills 2005), as well as at `graphic` package for Flutter ("Graphic | Flutter Package," n.d.). In the case of GPL, the distinction allows for data sources to take abstract forms that do not limit graph types to specific data structures (Wilkinson et al. 2000).

Algebra

The algebra class is a set of operations for producing combinations of variables given varsets. This step consists of three operations: cross (*), nest (/), and blend (+). Cross is the horizontal, or column, concatenation. Nest, while similar to cross, is essentially a left join that creates a nested structure. Blend is the vertical concatenation, like append, that should be used only when the varsets have matching or compatible units. These operations allow for a restructuring of the data, but the ending result resembles tidy data (Wickham 2014).

Scales

Scales are transformations on the units of the varsets. These are done before statistical calculations to avoid issues with meaningless results. This is easy to see in the case of averaging logarithmic-transformed data, where given a set of observations x_i for $i = 1, \dots, n$

$$\log(\bar{X}) = \log(\frac{1}{n}\sum_{i=1}^{n} x_i) \neq \frac{1}{n}\sum_{i=1}^{n}\log(x_i) \qquad x_i > 0$$

For example, the field of food science tends to use the logarithmic colony-forming units (log CFU) when measuring bacteria counts due to extremely large responses (Johnston et al. 2005; Abadias et al. 2008). The interest is in the log CFU units, so averages and other statistical calculations are performed on the log scale. Adjustments to the scales before calculating statistics helps to avoid misrepresented values.

Statistics

The statistics class takes a varset and outputs a summary varset. Some cases simply use the identity for its statistic, where each observation gets mapped. In other cases, summaries of the varsets can be calculated, such as counts, averages, predicted values from models (such as regression or clustering).

Geometries

The abstract mapping of data into the visual space is performed with the geometry class. This takes the output of the previous classes and arranges the form that their visual representation will take. This includes the geometric shapes of points, lines/paths, areas, bars/intervals, schemas (boxplots), polygons, contours, and edges. The geometry class produces geometric sets, but still requires further specification before it enters the visual space.

Coordinates

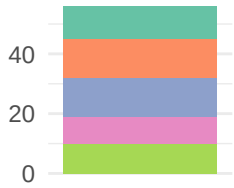
The coordinate class is how geometries are mapped into the visual space. The most common system is the Cartesian coordinate system, where up to three dimensions can be shown at a time. In most cases, only two dimensions are used since three dimensional representations provide less accurate comparisons (Croxton and Stein 1932; Cleveland and McGill 1984; Barfield and Robless 1989; Fischer 2000; Hughes, n.d.). Polar coordinates are another common coordinate system when circular geometries are desired, such as pie charts.

Aesthetics

The final class, aesthetics, is the actual mapping of geometries onto the coordinate space. This includes mapping varsets to the coordinates of the geometries and other components, such as color, shape, and size. Aesthetics can also include sound and motion, extending the Grammar of Graphics beyond a static visual space.

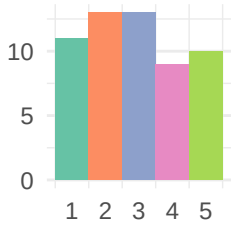
Philosophy

The purpose of the Grammar of Graphics (Wilkinson 2010) was to establish a mathematics-based foundation by using a series of mapping functions. Data is initially mapped to varsets before using set operations to join varsets together. These varsets then have direct numeric operations with the scales and statistics. Finally, the varsets are mapped into the visual space with geometries, coordinate systems, and aesthetics. The steps should compliment each other to produce other meaningful statistical graphics.

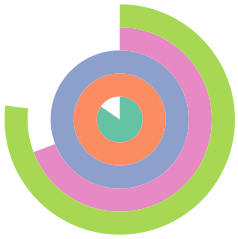


(a) Stacked bar chart in Cartesian coordinates.

(b) Pie chart in polar coordinates.



(c) Bar chart in Cartesian coordinates.



(d) Radial chart in polar coordinates.

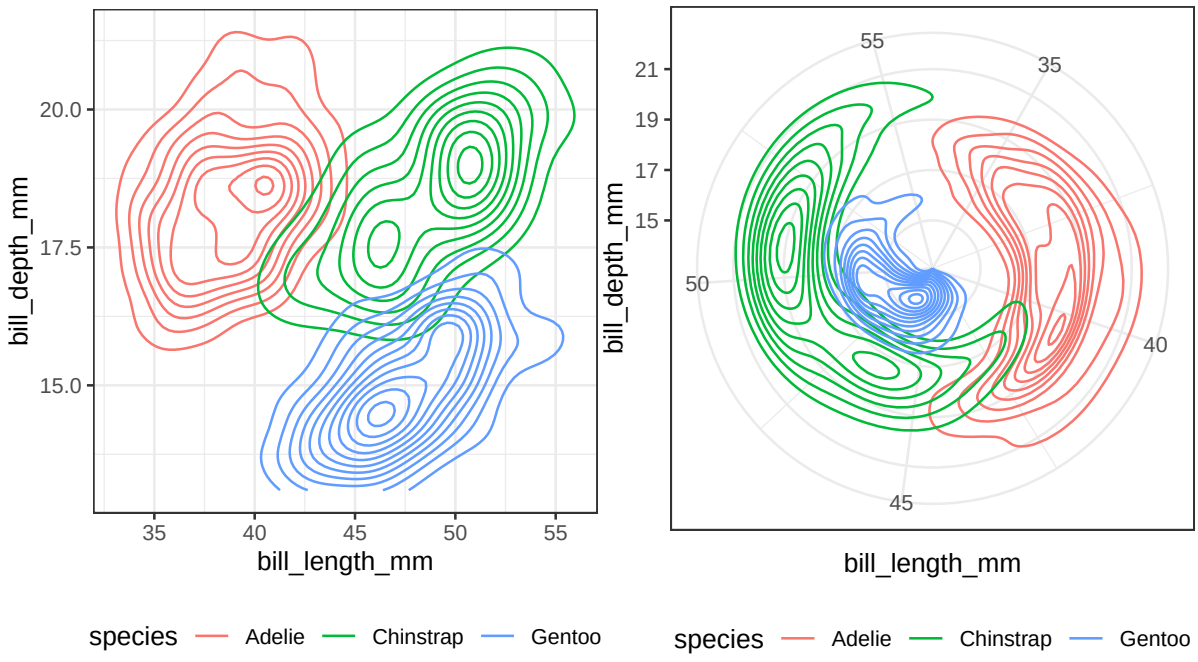
Figure 2: Two examples of bar charts in both Cartesian and polar coordinates. Figures (a) and (b) are identical with the exception of the coordinate system. Similarly for figures (c) and (d).

The ability to create statistical graphics using the Grammar of Graphics does not guarantee that the all permutations of classes produce meaningful graphics. For example, consider Figure 3 with the penguins (Horst, Hill, and Gorman 2020) dataset. Here, the Cartesian coordinate display of a contour plot is much easier to understand than the same data represented in polar coordinates. While the Grammar of Graphics is capable of producing many types of charts, it does not provide guidance on what charts are useful for users.

Modern Visualization Tools

The classes contained within the Grammar of Graphics (Wilkinson 2010) are used in many modern graphics packages (Wickham 2016; Waskom 2021; Satyanarayan et al. 2017). These packages typically use similar terms found in the Grammar of Graphics, such as data, scales, geometry, coordinates, and aesthetics. Code examples are provided with Listing 1 and Listing 2. Using `ggplot2` with R, data is provided in the `ggplot()` function, and aesthetics are mapped using a dedicated `aes()` function. Here, the coordinate positions are included within the aesthetics along with color. Python's `seaborn` module initially calls the geometry with `os.scatterplot()`, using function arguments to identify data, coordinate positions, and color. The plot from Listing 1 is produced in Figure 4.

The Grammar of Graphics was built around GPL, and thus needs adaptations for general use. Consider Listing 2. To create a graphic, all that was minimally required was data and



(a) Cartesian coordinates

(b) Polar coordinates

Figure 3: Contour plots of `penguins` dataset from the palmerpenguins package in R. The left panel is in Cartesian coordinates and the right panel is in polar coordinates. The polar coordinates are more difficult to understand than the Cartesian coordinates when extracting information about the different species.

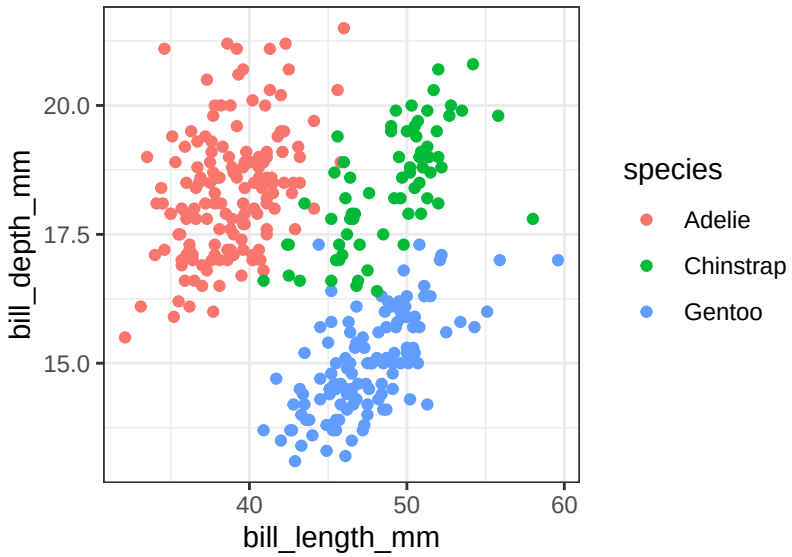


Figure 4: Scatterplot of `penguins` from Listing 1 using `ggplot2` in R.

Listing 1 Code to produce scatterplot of `penguins` data using `ggplot2` in R.

```
library(palmerpenguins)
data("penguins")
ggplot(data = penguins, mapping = aes(x = bill_length_mm,
                                      y = bill_depth_mm,
                                      color = species)) +
  geom_point()
```

Listing 2 Code to produce scatterplot of `penguins` data using `seaborn` in Python.

```
import seaborn as os
os.scatterplot(data=r.penguins, x="bill_length_mm",
  y="bill_depth_mm",
  hue="species")
```

a geometry mapping to coordinates x and y (hue was not necessary to create the plot). The same can be said for Listing 1, although the geometry step is more explicit as its own step. A commonality is that both figures were produced with tidy data (Wickham 2014). However, most classes from the Grammar of Graphics are either used inherently or by default settings. Therefore, an adaptation of the Grammar of Graphics can be considered by redefining classes and their practicality in modern data visualizations.

Evaluation of the Seven Classes

Establishing variables from data is an essential step in the Grammar of Graphics. This is obvious from the fact that statistical graphics are the visual mapping of data. The only requirement from the variable class is to create varsets, the mapping of values to all possible ordered lists of objects. From a practical point of view, a variable is simply a $n \times 1$ vector of values that share the same data type.

The algebra step is the first class from the Grammar of Graphics that is implicitly performed by the data structure. For a single data set that is already in a tidy format (Wickham 2014), the algebra step can be ignored since no crossing, nesting, or blending is needed. However, data often is stored across multiple sources, such as in a relational database (Harrington 2016). The joining of data results in another dataset, which can then become the new data source. Consider Listing 3 using the band_members and band_instruments from the `dplyr` package (Wickham et al. 2023). Here, data is stored separately and then joined to create a new data object. Now considering the joined data as the new data source, the algebra step is no longer required. This shows that algebra is only an intermittent step in producing data.

Listing 3 Full join of `band_members` and `band_instruments` from the `dplyr` package in R. The joined data is stored into `band_info` object.

```
band_info <- dplyr::band_members %>%
  full_join(dplyr::band_instruments, by = 'name')
```

The next two classes, scales and statistics, follow a similar logic to that of the algebra class. For scales, a transformation onto a variable can be used to create a new dataset with the newly defined variable. Statistics follow a similar process, where summaries and models can be calculated and arranged into a tidy format. Listing 4 shows an example of a scale transformation and averages, where the resulting data is saved into a new data object.

Listing 4 Data aggregation using scales and statistics on the `penguins` dataset from the `palmerpenguins` package in R. Data is saved into a new data set called `penguins_summary`.

```
penguins_summary <- penguins %>%
  mutate(bill_length_cm = bill_length_mm/10,
         bill_depth_cm = bill_depth_mm/10) %>%
  group_by(species, island, sex, year) %>%
  summarise(avg_bill_length_cm = mean(bill_length_cm, na.rm = T),
            avg_bill_depth_cm = mean(bill_depth_cm, na.rm = T))
```

Thus far, the classes of algebra, scales, and statistics are argued to be non-essential in the Grammar of Graphics framework. The reasoning is that these classes are able to result in a new dataset that foregoes the necessity to reevaluate the data aggregation steps. In reality, the application of these classes is more fluid than having a strict ordering. For example, the magrittr package (Bache and Wickham 2022) in R allows for piping, a "this-then-that" approach, where data does not need to be saved into a new object. Additionally, many graphical packages allow for transformations withing the geometry class. Consider Figure 5 with the penguins dataset (Horst, Hill, and Gorman 2020) where the binning statistic is used within the geom_bar() function. The binning could have taken place before introducing the data into the ggplot() function, but including the transformation with the geometry increases readability and makes it easier to create the graphic. It is not always necessary to calculate relevant transformations and/or statistics as their own steps, so they can be considered non-essential classes of the Grammar of Graphics methodology.

```
ggplot(penguins, mapping = aes(x = bill_length_mm)) +
  geom_bar(stat = 'bin', fill = 'skyblue', color = 'black') +
  theme(aspect.ratio = 1/2)
```

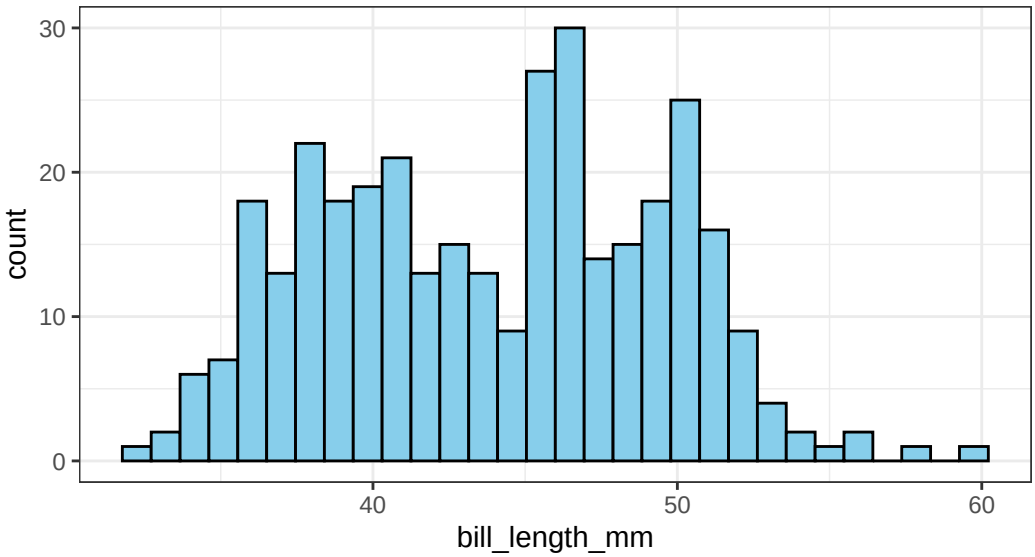
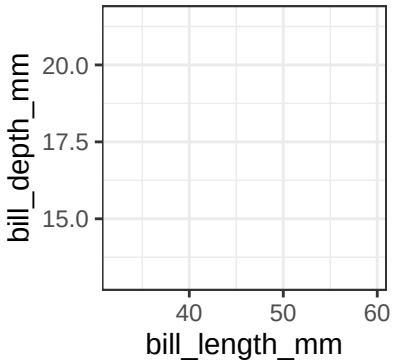


Figure 5: Histogram of bill length (mm) from the `penguin` dataset. Binning is performed inside the geometry of the bar chart.

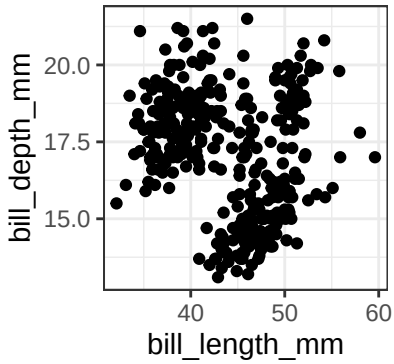
The mapping of data into an abstract geometric space includes the classes of geometry and coordinates. The geometry class is essential in defining the visual form of the data, which is the key point of making graphics. However, the placement of geometry requires a coordinate system, which makes the coordinate class inherently essential. Given the limitations of the physical world and computers, displays of dimensional data are restricted to a maximum of three dimensions. The standard Cartesian coordinates have been around since at least the 17th century, making them an ideal default system (Vince 2010, chap. 5). The geometry and coordinate classes work in tandem to create the form that graphic will use in its final product.

The final class is aesthetics. By technical definitions, mapping data to aesthetics is not necessary in producing charts, with the exception of the coordinates of the geometries. Figure 5 is one such example where only geometries are mapped to locations. However, it is often useful to encode additional information through one or more aesthetics (Wilkinson and Wills 2005). Figure 6 shows separate tasks from aesthetics, where only geometry mapping is essential in creating a graphic. Aesthetics are a semi-required class, noting that only some aesthetic mappings are necessary to produce a visual graphic.

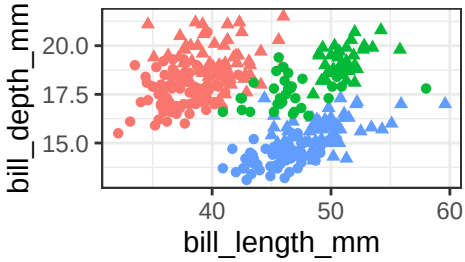
To summarize, the only required classes are variables, geometries, coordinates, and aesthetics. Variables are the link to the data. Geometry is the representation of the variables and is mapped into a space using a coordinate system. Lastly, aesthetics bring the graph into the visual space. These features form the set of the minimally required classes to produce statistical graphics.



(a) Cartesian coordinates



(b) Aesthetic: coordinate locations



(c) Aesthetic: color + shape

Figure 6: Demonstration of each visual class from the Grammar of Graphics. In (a), a coordinate grid is created based on the identify statistic of bill length (mm) and bill depth (mm). Panel (b) uses the point geometry to map the variable pairs onto the coordinate space. Panel (d) map species to color and sex to shape. Only (b) was required to create a graphic from data, but (c) adds additional context.

Two Applications of Essential Classes

In this section, I will evaluate the grammar of ggplot2 in R (Wickham 2010) and VegaLite in JavaScript (Satyanarayan et al. 2017). The ggplot2 package was created from the Grammar of Graphics philosophy, using a layering approach navigate through the classes. VegaLite was similarly created using the Grammar of Graphics, but opted to use differing terminology and incorporate interactive elements. Each graphics package has its own implementation of the Grammar of Graphics and the minimally required classes.

ggplot2

Many of the examples in this paper were created with `ggplot2` (Wickham 2016), a package inspired by the Grammar of Graphics. To implement the required classes, data is first supplied to the data argument in the `ggplot()` function. Then, aesthetics are supplied into the `mapping` argument through the `aes()` function. Lastly, a geometry is chosen by adding a `geom_function()` function. Alternatively, variables can be directly supplied into the `aes()` function and/or within the `geom_function()` function. An example of the minimally required features is supplied in Listing 5.

Listing 5 `ggplot2` code demonostrating the minimum required classes from the Grammar of Graphics.

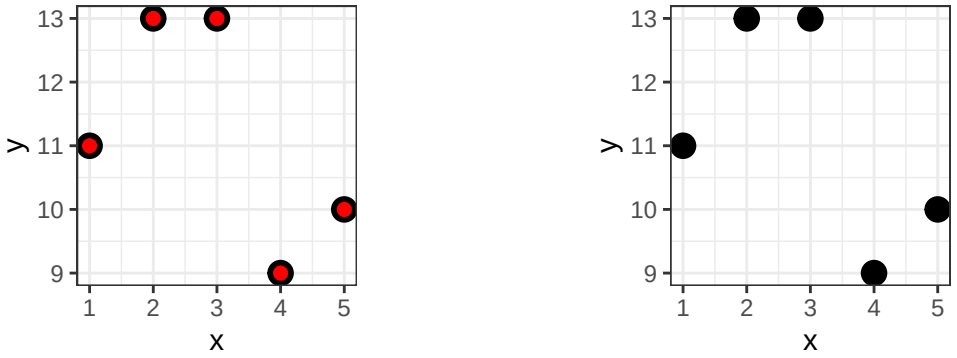
```
ggplot(data = data, mapping = aes(x = x, y = y)) +
  geom_function()
```

When making graphics with ggplot2, the order is important to consider. The evaluation of steps is in the presented order of layers. For example, two geometries can be mapped onto the coordinate space, but the latter geometry will overlap the former (Figure 7). This shows that `ggplot2` uses looped sequence through the Grammar of Graphics, where graphics are layered sequentially after each evaluation of the grammar.

VegaLite

VegaLite (Satyanarayan et al. 2017) uses *units* to define graphics, which consists of three required classes: data, mark-types, and encodings. Data follows the same form as tidy data (Wickham 2014). Mark-types define the geometry of the data before using the visual space. Lastly, encodings are the visual space of geometry location and other features, such as color and shape.

The method of implementing VegaLite is through a JSON syntax. Each class is supplied into its corresponding JSON field, which are called *data*, *mark*, and *encoding*. In the data field, the



(a) Black points mapped before red points.

(b) Red points mapped before black points.

Figure 7: Two scatterplots with different orders of layering `geom_point()` objects. When the red points are layered second (left panel), they appear on top of the black points. When the red points are layered first (second panel), they disappear behind the black points.

file path and format type need to be specified. The mark field specifies the geometry. Within the encoding field, the x and y coordinates are supplied as their own fields, along with the variable and variable type. Overlapping geometries are created by duplicating the individual grammars as objects supplied to a parent `layers` field.

Comparing ggplot2 and VegaLite

The primary factor separating `ggplot2` and VegaLite is the syntax. `ggplot2` uses layers, which are added onto previous steps. Arguments supplied to the `ggplot()` function are applied globally and followed by the addition of extra layers. On the contrary, VegaLite compiles its layers through the JSON syntax. The JSON syntax only allows for one geometry to be applied at a time, with the exception of geometries created under different encoding

Another difference between ggplot2 and VegaLite is the terminology. `ggplot2` mainly uses the same terminology as the Grammar of Graphics (Wilkinson 2010), whereas VegaLite tends to copy the terminology from Heer and Shneiderman (2012). What VegaLite calls "mark" and "encoding" are equivalent to `ggplot2`'s "geom(etry)" and "aes(thethic)", respectively. The difference in terminology between these two graphics packages is mostly semantic.

Although interactivity is not considered in the Grammar of Graphics (Wilkinson 2010), VegaLite incorporates additional grammar for *selection*. Selection consists of the following components: name, type, predicate, domain|range, event, init, transforms, and resolve. This grammar class allows users to interact with values in the graph to create new graphics. The default `ggplot2` package does not support *selection*, but the `Plotly` package (Sievert 2020) is an extension that allows `ggplot2` users to interact with graphics.

A New Grammar

- Algebra, scales, and statistics all can form new data
- Example with exp(normal) data, use model to get data from model before plotting
- Geometry, coordinates, and aesthetics all used together
- Need to have readable graph, so use

We have seen that data, geometry, coordinates, and aesthetics are all essential classes to the Grammar of Graphics (Wilkinson 2010). Without these classes, it would not be possible create graphics. However, these essential classes only allow for basic visualizations to be created. In this section, I propose a new grammar that adapts the Grammar of Graphics to a practical setting in the modern era.

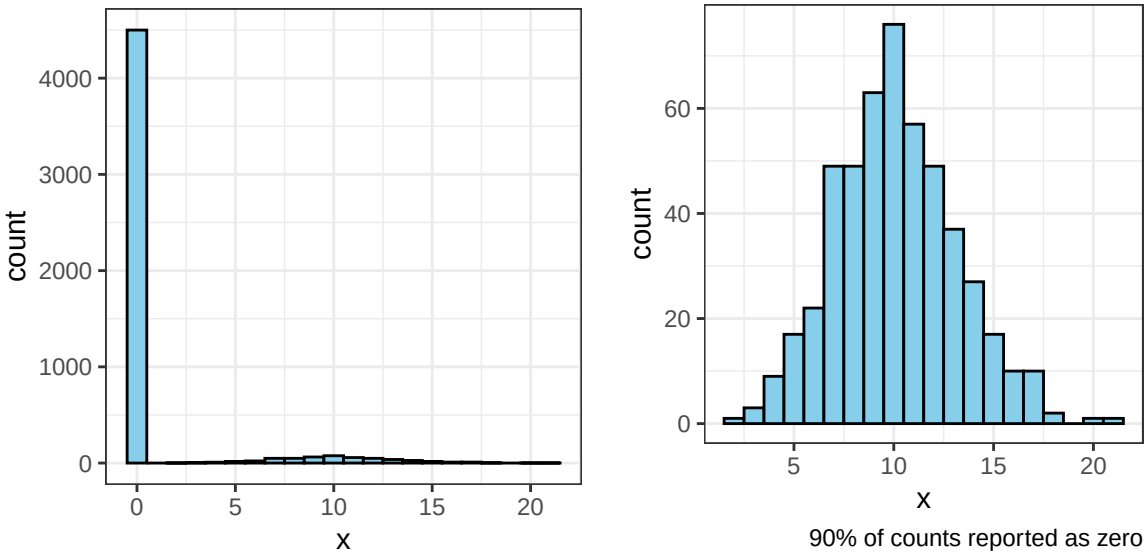
The first class in my proposed grammar is *data summary*. All statistical graphics start from a realization of data, where the data can take many structural forms. Computer software for creating graphics mostly rely on tidy data (Wickham 2014), which is where each row is an observation and each column is a variable. What the *data summary* class does is supply a data set after combining variables, transforming scales, and calculating statistics. However, some data sources cannot be coerced into a tidy format (Filonik et al. 2019), which can be mitigated by using the same aggregation steps to the data strucutre. By replacing the first four Grammar of Graphics classes with a unified *data summary* class, the data source is prepared for both graphics and other tasks, like tables or output files. The *data summary* class simplifies the grammar by assuming the data is already prepared for graphing.

My next proposed class is *visual mapping*, a class that combines geometries, coordinates, and aesthetics. Graphics software generally combine geometries and aesthetics into a single step. For example, `ggplot2` can accomplish the combined classes with `geom_function(mapping = aes(x = x, y = y, color = color_var, shape = shape_var, ...)`. The distinction between geometry and aesthetics is primarily performed behind the scenes when rendering the graphic, and thus not truly differentiated for the user. Graphics software also tends to default to Cartesian coordinates, resulting in alternative coordinate themes only being used when necessary. Combining geometry, coordinates, and aesthetics into a unified *visual mapping* class makes it so that the user only considers the components necessary to produce their desired graph.

Before introducing new classes, there is an unmentioned benefit to simplifying the classes into *data summary* and *visual mapping*. The strict sequencing of the Grammar of Graphics made it difficult to overlay multiple geometries. This was evident with VegaLite (Satyanarayan et al. 2017), where each iteration of the grammar was used as its own instance within "layers". Combining the classes for each stage into data and visual mappings allows for complicated graphics to be produced without adding further restrictions.

The final class I propose is *graph readability*, a new class that evaluates the effectiveness of statistical graphics. This class is designed to ensure that the graph is readable given the nature of the data. Similarly to how statistical models can produce nonsensical results if not used correctly, the effectiveness of a graph sometimes hinges on the nature of the data. In addition to the structure of the data, research shows that some graphs will outperform other graphs simply by changing the comparison method (Cleveland and McGill 1984; Heer and Bostock 2010a; Fischer, Dewulf, and Hill 2005). Therefore, creating graphics is not the end result and using *graph readability* will help in a finalized product.

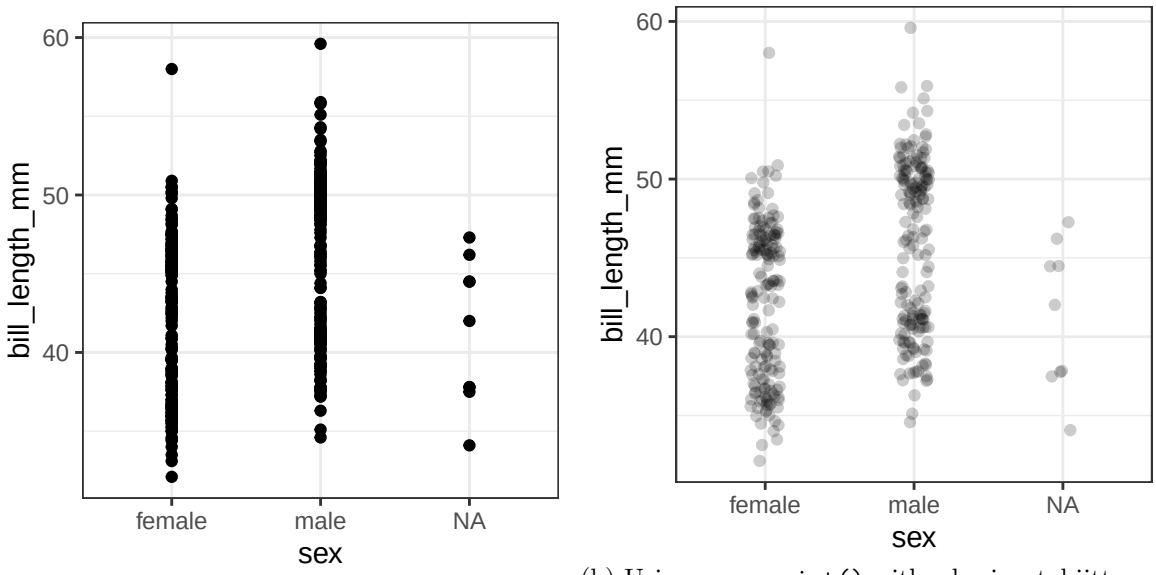
Consider Figure 8 with zero-inflated Poisson count data. When the proportion of zeros is high, including the zeros in the histogram results in difficulties differentiating values for non-zero counts. One possible solution is summarize the proportion of zeros as text. Another instance shown in Figure 9 where a graph is made with points. The overlapping points hide the distributions, but adding a small amount of random noise to the placement and adjusting the opacity helps to make the graph more readable. In both examples, the graph are constructed "correctly" from a Grammar of Graphics framework (Wilkinson 2010). However, taking the extra step to in the final design creates more meaningful graphics.



(a) Zero counts included in histogram.

(b) Zero counts summarized in caption.

Figure 8: Two histograms of zero-inflated Poisson count data. In the right panel, the percentage of zeros is reported in the caption so that the distribution of counts



(a) Only using `geom_point()`

(b) Using `geom_point()` with a horizontal jitter adjustment.

Figure 9: Two plots using points to measure bill length (mm). The left panel does not make any adjustments to point locations, resulting in overlap and difficulties determining the distribution. The right panel adds a small amount of horizontal jitter and adjusts the opacity of the points to help distinguish the density of the points.

Conclusion

Throughout this paper, I discussed the Grammar of Graphics (Wilkinson 2010) and its application for creating data visualizations. The Grammar of Graphics developed a mathematical framework for creating graphics, where the final result is produced through the final aesthetic class. While the framework technically works in creating graphics, the practical application needs adaptations.

References

Abadias, M., J. Usall, M. Anguera, C. Solsona, and I. Viñas. 2008. "Microbiological Quality of Fresh, Minimally-Processed Fruit and Vegetables, and Sprouts from Retail Establishments." *International Journal of Food Microbiology* 123 (1-2): 121–29. https://doi.org/10.1016/j.ijfoodmicro.2007.12.013.

Bache, Stefan Milton, and Hadley Wickham. 2022. *Magrittr: A Forward-Pipe Operator for r*. https://CRAN.R-project.org/package=magrittr.

Barfield, Woodrow, and Robert Robless. 1989. "The Effects of Two- or Three-Dimensional Graphics on the Problem-Solving Performance of Experienced and Novice Decision Makers." *Behaviour & Information Technology* 8 (5): 369–85. https://doi.org/10.1080/01449298908914567.

Beniger, James R., and Dorothy L. Robyn. 1978. "Quantitative Graphics in Statistics: A Brief History." *The American Statistician* 32 (1): 1–11. https://doi.org/10.2307/2683467.

Brewer, Noel T., Melissa B. Gilkey, Sarah E. Lillie, Bradford W. Hesse, and Stacey L. Sheridan. 2012. "Tables or Bar Graphs? Presenting Test Results in Electronic Medical Records." *Medical Decision Making* 32 (4): 545–53. https://doi.org/10.1177/0272989X12441395.

Cleveland, William S., and Robert McGill. 1984. "Graphical Perception: Theory, Experimentation, and Application to the Development of Graphical Methods." *Journal of the American Statistical Association* 79 (387): 531–54. https://doi.org/10.1080/01621459.1984.10478080.

Croxton, Frederick E., and Harold Stein. 1932. "Graphic Comparisons by Bars, Squares, Circles, and Cubes." *Journal of the American Statistical Association* 27 (177): 54–60. https://doi.org/10.1080/01621459.1932.10503227.

Filonik, Daniel, Markus Rittenbruch, Marcus Foth, and Tomasz Bednarz. 2019. "2019 23rd International Conference in Information Visualization – Part II." In, 18–23. https://doi.org/10.1109/IV-2.2019.00013.

Fischer, Martin H. 2000. "Do Irrelevant Depth Cues Affect the Comprehension of Bar Graphs?" *Applied Cognitive Psychology* 14 (2): 151–62. https://doi.org/10.1002/(SICI)1099-0720(200003/04)14:2%3C151::AID-ACP629%3E3.0.CO;2-Z.

Fischer, Martin H., Nele Dewulf, and Robin L. Hill. 2005. "Designing Bar Graphs: Orientation Matters." *Applied Cognitive Psychology* 19 (7): 953–62. https://doi.org/10.1002/acp.1105.

"Graphic | Flutter Package." n.d. https://pub.dev/packages/graphic.

Harrington, Jan L. 2016. *Relational Database Design and Implementation*. 4th ed. Amsterdam, [Netherlands: Morgan Kaufmann.

Heer, Jeffrey, and Michael Bostock. 2010a. "Crowdsourcing Graphical Perception: Using Mechanical Turk to Assess Visualization Design." In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, 203–12. Atlanta Georgia USA: ACM. https://doi.org/10.1145/1753326.1753357.

———. 2010b. "CHI '10: CHI Conference on Human Factors in Computing Systems." In, 203–12. Atlanta Georgia USA: ACM. https://doi.org/10.1145/1753326.1753357.

Heer, Jeffrey, and Ben Shneiderman. 2012. "Interactive Dynamics for Visual Analysis." *Com-*

munications of the ACM 55 (4): 45–54. https://doi.org/10.1145/2133806.2133821.

Horst, Allison Marie, Alison Presmanes Hill, and Kristen B Gorman. 2020. *Palmerpenguins: Palmer Archipelago (Antarctica) Penguin Data*. https://doi.org/10.5281/zenodo.3960218.

Hughes, Brian M. n.d. "Just Noticeable Differences in 2D and 3D Bar Charts: A Psychophysical Analysis of Chart Readability."

Johnston, Lynette M., Lee-Ann Jaykus, Deborah Moll, Martha C. Martinez, Juan Anciso, Brenda Mora, and Christine L. Moe. 2005. "A Field Study of the Microbiological Quality of Fresh Produce." *Journal of Food Protection* 68 (9): 1840–47. https://doi.org/10.4315/0362-028X-68.9.1840.

Kosslyn, Stephen M. 1989. "Understanding Charts and Graphs." *Applied Cognitive Psychology* 3 (3): 185225. https://doi.org/10.1002/acp.2350030302.

Meyer, Joachim, David Shinar, and David Leiser. 1997. "Multiple Factors That Determine Performance with Tables and Graphs." *Human Factors: The Journal of the Human Factors and Ergonomics Society* 39 (2): 268–86. https://doi.org/10.1518/001872097778543921.

Prasad, Gollapudi V. R. J. Sai, and Amitash Ojha. 2012. "2012 IEEE Fourth International Conference on Technology for Education (T4E)." In, 126–31. Hyderabad, India: IEEE. https://doi.org/10.1109/T4E.2012.18.

R Core Team. 2024. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.

Satyanarayan, Arvind, Dominik Moritz, Kanit Wongsuphasawat, and Jeffrey Heer. 2017. "Vega-Lite: A Grammar of Interactive Graphics." *IEEE Transactions on Visualization and Computer Graphics* 23 (1): 341–50. https://doi.org/10.1109/TVCG.2016.2599030.

Sievert, Carson. 2020. *Interactive Web-Based Data Visualization with r, Plotly, and Shiny*. Chapman; Hall/CRC. https://plotly-r.com.

Tufte, Edward R. 1983. *The Visual Display of Quantitative Information*. Cheshire, Conn. (Box 430, Cheshire 06410): Graphics Press.

Tukey, John W. 1965. "The Technical Tools of Statistics." *The American Statistician* 19 (2): 23–28. https://doi.org/10.2307/2682374.

Vince, John. 2010. *Mathematics for Computer Graphics*. Undergraduate Topics in Computer Science. London: Springer London. https://doi.org/10.1007/978-1-84996-023-6.

Waskom, Michael L. 2021. "Seaborn: Statistical Data Visualization." *Journal of Open Source Software* 6 (60): 3021. https://doi.org/10.21105/joss.03021.

Wickham, Hadley. 2010. "A Layered Grammar of Graphics." *Journal of Computational and Graphical Statistics* 19 (1): 3–28. https://www.jstor.org/stable/25651297.

———. 2014. "Tidy Data." *Journal of Statistical Software* 59 (10): 123. https://doi.org/10.18637/jss.v059.i10.

———. 2016. *Ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag New York. https://ggplot2.tidyverse.org.

Wickham, Hadley, Romain François, Lionel Henry, Kirill Müller, and Davis Vaughan. 2023. *Dplyr: A Grammar of Data Manipulation*. https://CRAN.R-project.org/package=dplyr.

Wilkinson, Leland. 2010. "The Grammar of Graphics." *WIREs Computational Statistics* 2 (6): 673–77. https://doi.org/10.1002/wics.118.

Wilkinson, Leland, Daniel J. Rope, Daniel B. Carr, and Matthew A. Rubin. 2000. "The

Language of Graphics." *Journal of Computational and Graphical Statistics* 9 (3): 530–43. https://doi.org/10.1080/10618600.2000.10474897.

Wilkinson, Leland, and Graham Wills. 2005. *The Grammar of Graphics*. 2nd ed. Statistics and Computing. New York: Springer.