# PHYS 158 - Unofficial Formula Sheet

## Beats and Standing Waves

### Oscillatory Motion

| | |
|---|---|
| Frequency | $f = \frac{1}{T}$ |
| Angular Frequency | $\omega = 2\pi f = \frac{2\pi}{T}$ |
| Wave Number | $k = \frac{2\pi}{\lambda}$ |
| Wave Speed | $v = \lambda f$ |
| Speed of Sound | $v_{sound} \approx 343 m/s$ |

### Standing Waves

| | |
|---|---|
| Equation | $y(x,t) = A_{SW}\sin(kx)\sin(\omega t)$ |
| Fundamentals | $f_m = m f_1$ |
| Two End Nodes | $l = \frac{m\lambda}{2}$ |
| One End Node | $l = \frac{m\lambda}{4}$ |

### Beats

| | |
|---|---|
| Equation | D(t)=2Acos$(\frac{1}{2}(\omega_1 - \omega_2)t)\sin(\frac{1}{2}(\omega_1 + \omega_2)t)$ |
| Average Freq | $\frac{1}{2}(f_1 + f_2)$ |
| Beat Freq | $f_{beat} = f_1 - f_2 = 2 f_{amp}$ |

## Thin Film and Interference

### Path Difference

| | |
|---|---|
| Constructive | $\Delta r = m\lambda_{med}$ |
| Destructive | $\Delta r = (m - \frac{1}{2})\lambda_{med}$ |
| Constructive | $\Delta$arg$= 2\pi m$ |
| Destructive | $\Delta$arg$= (2m-1)\pi$ |

### Change of Medium

| | |
|---|---|
| Speed of Light | $c = 2.998 \cdot 10^8 m/s$ |
| Index of Refraction | $n = \frac{c}{v_{med}} = \frac{\lambda}{\lambda_{med}}$ |
| Film Medium Interference | $\Delta r = s_1 n_1 - n_2 s_2$ |
| Snell's Law | $n_1 \sin\theta_1 = n_2 \sin\theta_2$ |

### Double Slit

$\Delta r \approx d \sin\theta$ for $d \ll R$

$\Delta r \approx \frac{d y_m}{R}$ for $d \ll R$ and $d \ll y_m$

### Thin Film

| | |
|---|---|
| Fast to Slow Medium | $\phi = \pi$ |
| Slow to Fast Medium | $\phi = 0$ |
| Interference | $\Delta$arg$= 2sk_f + \Delta\phi = \frac{4s\pi n_f}{\lambda} + \Delta\phi$ |

## General Circuits

### Resistors

| | |
|---|---|
| Ohm's Law | $V = IR$ |
| Power Dissipated | $P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R}$ |
| Series | $R_{eq} = R_1 + R_2 + R_3 + ....$ |
| Parallel | $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ....$ |

### Capacitors

| | |
|---|---|
| Capacitance | $C = \frac{Q}{V_{ab}}$ |
| Stored Energy | $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$ |
| Series | $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ....$ |
| Parallel | $C_{eq} = C_1 + C_2 + C_3 + ....$ |

### Inductors

| | |
|---|---|
| Self-induced emf | $\mathcal{E}_L = -L\frac{di}{dt}$ |
| Power | $dP = i(t)V = iL\frac{di}{dt}$ |
| Stored Energy | $U_L = \frac{1}{2}L \cdot (I_f^2 - I_0^2)$ |
| Series | $L_{eq} = L_1 + L_2 + L_3 + ....$ |
| Parallel | $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ....$ |

## Time Dependent RC

### RC Circuit Charging

1. Find a differential equation to relate the current $i = \frac{dq}{dt}$ and the charge $(q)$ using voltage loop law

| | |
|---|---|
| Charging Capacitor | $V - iR - \frac{q}{C} = 0$ |
| (derivative) | $-R\frac{di}{dt} - \frac{1}{C}\frac{dq}{dt} = 0$ |
| $(i = \frac{dq}{dt})$ | $-R\frac{di}{dt} - \frac{1}{C}i = 0$ |
| | $\frac{di}{dt} = -\frac{1}{RC}i$ |
| **Current Function**: | $i(t) = i_0 \cdot e^{-\frac{t}{R_{eq}C_{eq}}}$ |
| Substitution | $\frac{dq}{dt} = \frac{V}{R}e^{-\frac{t}{RC}}$ |
| **Charge Function:** | $q(t) = Q_f(1 - e^{-\frac{t}{RC}})$ |
| | $Q_f$= final charge |

### RC Circuit Discharge

| | |
|---|---|
| Discharging Capacitor | $\frac{q(t)}{C} - iR = 0$ |
| (rearranging) | $-i = \frac{dq}{dt} = -\frac{1}{RC} \cdot q(t)$ |
| **Charge Function:** | $q(t) = q_0 \cdot e^{-\frac{t}{RC}}$ |
| $(q_0 = CV)$+derivative | $i(t) = i_0 e^{-\frac{t}{RC}}$ |
| **Current Function**: | $i(t) = i_0 \cdot e^{-\frac{t}{RC}}$ |

## Time Dependent RL

### RL Circuit Decay

1. Find a differential equation to relate the current $i$ and the change in current $\frac{di}{dt}$ using voltage loop law

| | |
|---|---|
| Discharging Inductor | $\mathcal{E}_L - iR = 0$ |
| | $-L\frac{di}{dt} - iR = 0$ |
| | $\frac{di}{dt} = -\frac{R}{L}i$ |
| **Current Function:** | $i(t) = i_0 \cdot e^{-\frac{R}{L}t}$ |

### RL Circuit Increasing

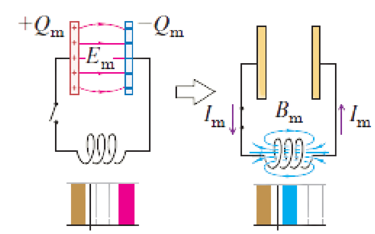
1. Follow the same procedure to solve differential equation as shown in RC Charging Circuit for Charge Function

| | |
|---|---|
| Voltage Law | $\mathcal{E} - iR - L\frac{di}{dt} = 0$ |
| **Current Function** | $i(t) = I_{final}(1 - e^{-\frac{R}{L}t})$ |

## LC Circuits

### LC Circuit Oscillations

| | |
|---|---|
| Angular Frequency | $\omega = \sqrt{\frac{1}{LC}}$ |
| Capacitor Charge | $q(t) = Q_{max} \cdot cos(\omega t + \phi)$ |
| $-\frac{dq}{dt}$ (Current) | $i(t) = \omega Q_{max} \cdot sin(\omega t + \phi)$ |
| Energy | $\frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} = \frac{Q_{max}^2}{2C}$ |




## RLC Circuits

### Damped Oscillations

Find the damped $\omega'$ similar to LC Circuit but damped

| | |
|---|---|
| Damped Omega | $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ |
| Damped Charge | $q(t) = Ae^{-\frac{R}{2L}t} \cdot cos(\omega' t + \phi)$ |
| Underdamped if | $\omega' > 0$ |


Compiled by Simon Ghyselincks and Tyler Wilson 2021

## AC Series Current and Voltage

1. Given $V(t) = V_{max} \cdot sin(\omega t + \phi_0)$
2. Solve for I(t) (Note, same phase across L, R, C)

**Inductive and Capacitive Resistance**

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

**Total Impedence Z:**

$|Z|^2 = R^2 + (X_L - X_C)^2$

**Phase Angle:**

$\tan\phi = \frac{X_L - X_C}{R}$

$\arg(v) - \arg(i) = \phi$

**Current as a function of time:**

$I(t) = \frac{V_{max}}{Z} \cdot sin(\omega t + \phi_0 - \phi)$

**Individual Voltages:**

$V_R(t) = R \cdot \frac{V_{max}}{Z} \cdot sin(\omega t + \phi_0 - \phi)$

$V_L(t) = X_L \cdot \frac{V_{max}}{Z} \cdot sin(\omega t + \phi_0 - \phi + \frac{\pi}{2})$

$V_C(t) = X_C \cdot \frac{V_{max}}{Z} \cdot sin(\omega t + \phi_0 - \phi - \frac{\pi}{2})$

**Relation of Max to RMS Voltage:**

$V_{rms} = \frac{1}{\sqrt{2}} \cdot V_{max}$

**Relation of Max to RMS Current:**

$I_{rms} = \frac{1}{\sqrt{2}} \cdot I_{max}$

## AC Circuit Power and Resonance

1. Find $\phi$, then $\cos(\phi)$ is the power factor:

**Average Power Function:**

$P_{av} = \frac{1}{2} V_{max} I_{max} \cdot \cos(\phi) = V_{rms} I_{rms} \cdot \cos(\phi)$

$P_{av} = I_{rms}^2 R$

$cos(\phi) = \frac{R}{Z}$

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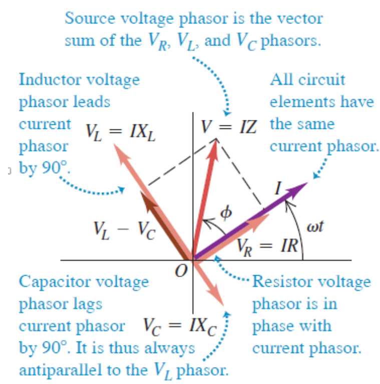
For Resonance $X_C = X_L$, peak current and $\phi = 0$

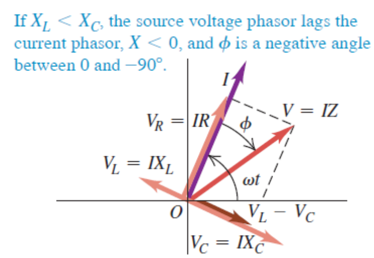
**Resonance**

$\omega_0 = \frac{1}{\sqrt{LC}}$

## Phasor Diagrams




(c) Phasor diagram for the case $X_L < X_C$




## AC Parallel Current and Voltage

Same voltages but currents are out of phase

**Inductive and Capacitive Resistance**

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

**Total Impedence Z:**

$\left|\frac{1}{Z}\right|^2 = \frac{1}{R^2} + (\frac{1}{X_L} - \frac{1}{X_C})^2$

**Phase Angle:**

$\tan\phi = R \cdot (\frac{1}{X_C} - \frac{1}{X_L})$

$\arg(i) - \arg(v) = \phi$

**Current as a function of time:**

$I(t) = \frac{V_{max}}{Z} \cdot \sin(\omega t + \phi_0 + \phi)$

*Note the angle is phi is added

## Electrostatics

### Constants

| | |
|---|---|
| Electric Constant | $\varepsilon_0 = 8.854 \cdot 10^{-12} C^2/N \cdot m^{-2}$ |
| Coulomb's Constant | $\frac{1}{4\pi\varepsilon_0} = 8.988 \cdot 10^9 N \cdot m^2/C^2$ |
| Elementary Charge | $e = 1.60217662 \cdot 10^{-19} C$ |

### Laws

| | |
|---|---|
| Coulombs Law | $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lvert q_1 q_2 \rvert}{r^2}\hat{r} = q\vec{E}$ |
| Electric Field | $\vec{E} = \frac{\vec{F_0}}{q_0} = \frac{kq}{r^2}\hat{r}$ |

### Charge Densities and Distributions

| | |
|---|---|
| Linear Charge Density | $\lambda = \frac{Q}{L}$ |
| Surface Charge density | $\sigma = \frac{Q}{A}$ |
| Charge Density | $\rho = \frac{Q}{V}$ |
| Non Uniform | $dQ = \lambda ds = \sigma dA = \rho dV$ |

### Electric Dipole

$\vec{p} = qd$

$\vec{\tau} = \vec{F}\vec{d}$=pE$\cdot \sin\varphi$

## Gauss's Law

### Flux Equations

| | |
|---|---|
| Gauss's Law | $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ |
| Uniform E | $\Phi_E = \vec{E} \cdot \vec{A} = EA \cdot \cos(\phi)$ |

### Electric Fields of Symmetric Objects

| | |
|---|---|
| Point Charge | $\vec{E} = k\frac{Q}{r^2}\hat{r}$ |
| Charged Rod | $\vec{E} = \frac{k\lambda l}{r\sqrt{r^2 + \frac{l^2}{4}}}\hat{r}$ |
| Charged Ring | $\vec{E} = \frac{kQh}{(R^2+h^2)^{3/2}}\hat{n}$ |
| Charged Disk | $\vec{E} = 2\pi k\sigma(\frac{1}{h} - \frac{1}{\sqrt{R^2+h^2}})\hat{n}$ |
| Infinite Plane | $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$ |
| Infinite Wire | $\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{r}$ |
| Infinite Slab | $\vec{E} = \frac{\rho s}{2\epsilon_0}\hat{n}$ for $0 \le s \le S$ |
| Infinite Cylinder | $\vec{E} = \frac{\rho r}{2\epsilon_0}\hat{r}$ for $0 \le r \le R$ |
| Sphere Interior | $\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}$ for $0 \le r \le R$ |

Slabs at large distances emulate a plane

Spheres at large distances emulate a point charge

Cylinders at large distances emulate a wire

## Electric Potential

Generally for Work:

$W_{a\to b} = U_a - U_b = -\Delta U$

$U = 0$ is at infinite charge separation

### Point Charges

| | |
|---|---|
| Two charges | $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r}$ |
| Multiple Charges | $U = \frac{q_0}{4\pi\varepsilon_0} \cdot (\frac{q_1}{r_1} + \frac{q_2}{r_2} + ... \frac{q_n}{r_n})$ |
| Sum of system | $U_{net} = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{i<j} \frac{q_i q_j}{r_{ij}}$ |

### Voltage and Potential

Generally, $V(\infty) = 0$ when not specified

Difference Notation is $V_{ab} = V_a - V_b$

Means $V_a$ with respect to $V_b$

| | |
|---|---|
| Voltage | $\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = V_a - V_b$ |
| Point Charge | $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$ |
| Multiple Charges | $V = \frac{1}{4\pi\varepsilon_0} \cdot (\frac{q_1}{r_1} + \frac{q_2}{r_2} + ... \frac{q_n}{r_n})$ |
| Distributed Charge | $V = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{dq}{r}$ |

| | |
|---|---|
| **V function of E** | $-\frac{\Delta U}{q_0} = V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$ |
| Infinitesimals | $dV = -\vec{E} \cdot d\vec{l}$ |
| **E function of V** | $\vec{E} = \vec{\nabla} V$ |
| Expanded | $\vec{E} = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$ |
| Radial | $\vec{E} = \frac{dV}{dr}$ |

## Capacitance and Dielectrics

### Capacitance and Energy

Parallel Plates: $C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$

Energy density: $u = \frac{1}{2}\epsilon_0 E^2$ and $U = \int u \cdot dV$

Stored Energy $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

### Dielectrics

| | |
|---|---|
| Dielectric Constant | $\kappa = \frac{C}{C_0}$ |
| V for constant Q | $V = \frac{V_0}{\kappa}$ |
| E for constant Q | $E = \frac{E_0}{\kappa}$ |
| Induced Charge | $\sigma_i = \sigma(1 - \frac{1}{\kappa})$ |
| Dielectric Gauss | $\oint \kappa \vec{E} \cdot d\vec{A} = \frac{Q_{conductor}}{\varepsilon_0}$ |

## Magnetism

### Cross Product

Use right hand rule for direction

$\vec{v} \times \vec{B} = vB \cdot sin(\phi)$

### Magnetic Force

| | |
|---|---|
| Magnetic Force | $\vec{F} = q\vec{v} \times \vec{B}$ |
| **Lorentz Force Equation** | |
| Net Elec & Magnetic F | $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ |

### Magnetic Flux and Gauss

Magnetic Flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$

$= \int B * cos(\phi) dA$

**Gauss Law** $\oiint \vec{B} \cdot d\vec{A} = 0$

### Cyclotron Motion

| | |
|---|---|
| Force from field | $F = qv_\perp B = \frac{mv_\perp^2}{R}$ |
| Radius | $R = \frac{mv_\perp}{qB}$ |
| Frequency | $\omega = \frac{v_\perp}{R} = \frac{qB}{m}$ |

## Current Motors Magnets

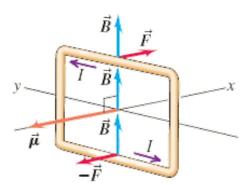
$$\boxed{\mu_0 = 4\pi \cdot 10^{-7} N/A^2}$$

### Current through wire

| | |
|---|---|
| Straight Wire | $\vec{F} = I \cdot \vec{l} \times \vec{B}$ |
| Wire segment | $d\vec{F} = Id\vec{l} \times \vec{B}$ |
| Mag Field Produced | $B = \frac{\mu_0 I}{2\pi r}$ |
| Two Wires Attraction | $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ |

### Dipole and Torque

| | |
|---|---|
| Dipole Moment | $\vec{\mu} = NI\vec{A}$ |
| Wire Loop Torque | $\tau = \mu B \sin(\phi)$ |
| Torque Vector | $\vec{\tau} = \vec{\mu} \times \vec{B}$ |
| Dipole Potential | $U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi$ |




## Maxwell's Equations

$\mu_0 \epsilon_0 = \frac{1}{c^2}$

### Integral Form

$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

$\oiint \vec{B} \cdot d\vec{A} = 0$

$\oint \vec{E} \cdot d\vec{s} = -\frac{\Phi_B}{dt}$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

### Differential Form

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

### Multivariable Calculus Formulas

| | |
|---|---|
| Stokes Theorem | $\iint \vec{\nabla} \times \vec{v} \cdot dA = \oint \vec{v} \cdot d\vec{l}$ |
| Divergence Theorem | $\iiint \vec{\nabla} \cdot \vec{v} d^3 r = \oiint \vec{v} \cdot d\vec{A}$ |

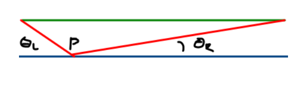
## Magnetic Fields

### Magnetic Field Equations

| | |
|---|---|
| Biot-Savart Law | $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{l} \times \hat{r}}{r^2}$ |
| Ampere's Law | $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$ |
| Moving Particle | $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$ |
| Loop of Current | $\vec{B} = \frac{\mu_0 I R^2}{2(h^2+R^2)^{3/2}} \hat{n}$ |
| Straight Wire | $B = \frac{\mu_0 I}{4\pi r} \sin\theta \vert_{\theta_L}^{\theta_R} = \frac{\mu_0 I x}{4\pi r\sqrt{x^2+r^2}} \vert_{x_L}^{x_R}$ |
| Straight Wire | $B = \frac{\mu_0 I l}{4\pi r\sqrt{r^2+\frac{l^2}{4}}}$ at center only |
| Infinite Wire | $B = \frac{\mu_0 I}{2\pi r}$ for $l \gg r$ |

### Solenoids

| | |
|---|---|
| Coil Density | $n = N/L$ |
| Toroid | $B = \frac{\mu_0 IN}{2\pi r}$ |
| Infinite Solenoid | $B = \mu_0 In$ |
| Finite Solenoid | $B = \frac{\mu_0 Inx}{2\sqrt{x^2+R^2}} \vert_{x_L}^{x_R}$ |
| | $B = \frac{\mu_0}{2} I \cdot n(\cos\theta_R - \cos\theta_L)$ |



## Electromagnetic Induction

### Faradays's Law

| | |
|---|---|
| Induced Emf | $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ |
| Solenoid Emf | $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$ |
| Motional Emf | $\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ |
| For moving Bar | $\mathcal{E} = vBL$ |

The direction of any magnetic induction effect is such as to oppose the cause of the effect. Induced current direction in a loop opposes the change in flux.

### Ampere-Maxwell Law

| | |
|---|---|
| Displacement Current | $i_{disp} = \epsilon_0 \frac{d\Psi_E}{dt}$ |
| Ampere-Maxwell Law | $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$ |

### Electromagnetic Waves (not on final)

| | |
|---|---|
| Proportionality | $E = cB$ |
| Speed of Light | $\mu_0 \epsilon_0 = \frac{1}{c^2}$ |
| Solar Wind | $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ |
| Intensity | $S_{avg} = I = \frac{E_{max} B_{max}}{2\mu_0}$ |
| Radiation Pressure | $P_{rad} = \frac{I}{c} = \frac{S_{avg}}{c}$ |