MECH 260 - Unofficial Formula Sheet

Average Stress and Strain

Normal Stress and Strain

Stress $\sigma = \frac{F}{A_\perp}$

Strain $\varepsilon = \frac{\delta}{l_0}$

Young's Modulus $E = \frac{\sigma}{\varepsilon}$

Deflection $\delta = \frac{Fl}{AE}$

Poisson's Ratio $\varepsilon_{x,y} = \frac{-\nu}{E}\sigma_z$

Normal Strain $\varepsilon_{x|y|z} = \frac{1}{E}\left(\sigma_\parallel - \nu(\sigma_{\perp 1} + \sigma_{\perp 2})\right)$

Normal Stress $\sigma_{x|y|z} = \left(\frac{E}{(1+\nu)(1-2\nu)}\right)\left((1-\nu)\varepsilon_\parallel + \nu(\varepsilon_{\perp 1} + \varepsilon_{\perp 2})\right)$

Matrix Form
$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu\\ -\nu & 1 & -\nu\\ -\nu & -\nu & 1\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{bmatrix}$$

Volumetric Stress and Strain

Volumetirc Strain $\varepsilon_V = \frac{\Delta V}{V_0} \approx \varepsilon_x + \varepsilon_y + \varepsilon_z = \left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y + \sigma_z)$

Bulk Modulus $K = \frac{E}{3(1-2\nu)}$

Shear Stress and Strain

Shear Stress $\tau = \frac{V}{A_\parallel}$

Shear Strain $\gamma = \frac{\delta}{l}$

Shear Modulus $G = \frac{\tau}{\gamma}$

Matrix Form
$$\begin{bmatrix}\tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix} = \begin{bmatrix}G & 0 & 0\\ 0 & G & 0\\ 0 & 0 & G\end{bmatrix}\begin{bmatrix}\gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

Thermal Stress and Strain

Thermal Strain $\varepsilon = \alpha_L \Delta T$

$\varepsilon_V = \alpha_V \Delta T \approx 3\alpha_L \Delta T$

Total Strain $\varepsilon_{x|y|z} = \varepsilon_{\text{normal}} + \alpha_L \Delta T$

Total Stress $\sigma_{x|y|z} = \sigma_{\text{normal}} - \left(\frac{E}{1-2\nu}\right)\alpha_L \Delta T$

Volumetric $\varepsilon_V = \left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y + \sigma_z) + 3\alpha_L \Delta T$

$\sigma_x + \sigma_y + \sigma_z = \left(\frac{E}{1-2\nu}\right)(\varepsilon_V - 3\alpha_L \Delta T)$

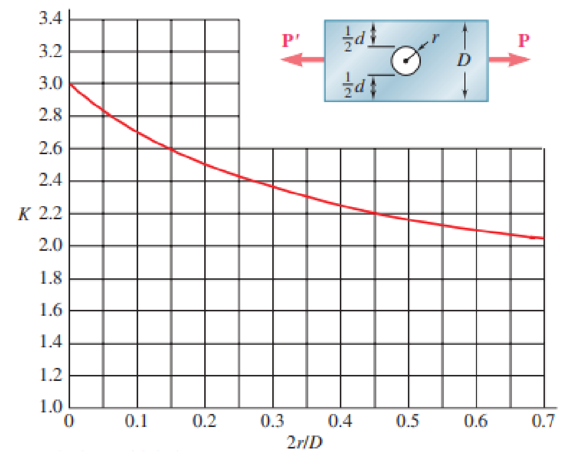
Nonuniform Stress and Strain

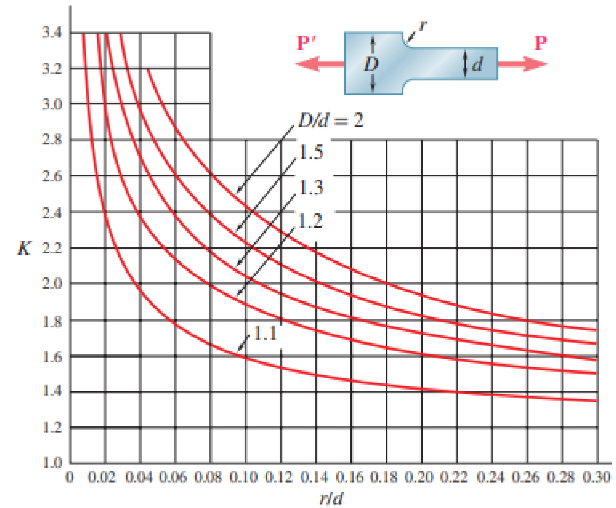
Nonuniform Stress

Safety Factor
$$f_s = \begin{cases}\frac{\sigma_Y}{|\sigma_{\text{nom}}|} & \text{(ductile)}\\ \frac{\sigma_{UT}}{|\sigma_{\max}|} & \text{(brittle, tension)}\\ \frac{|\sigma_{UC}|}{|\sigma_{\max}|} & \text{(brittle, compression)}\end{cases}$$

Nominal Stress $\sigma_{\text{nominal}} = \frac{F_{\text{avg}}}{A_{\min}}$

K Value $K = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$





Torsion and Bending

Torsion

Shear Strain $\gamma(r) = \frac{r}{l}\phi$

Shear Stress $\frac{\tau(r)}{r} = \frac{T}{J} = \frac{G}{l}\phi$

Polar Moment of Area
$$J = \begin{cases} \frac{\pi}{2} r_{\text{surf}}^4 & \text{(Solid Circular Shaft)} \\ \frac{\pi}{2}(r_{\text{surf}}^4 - r_{\text{in}}^4) & \text{(Hollow Circular Shaft)} \\ 2\pi t r_{\text{surf}}^3 & \text{(Hollow Circular, } t \ll r_{\text{surf}}) \end{cases}$$

$$J_{\text{rectangle}} = \frac{bh(b^2+h^2)}{12}$$

Bending

Bending $\frac{\sigma_x}{-y} = \frac{M}{I_z} = \frac{E}{\rho}$ (where ρ is radius of curvature)

Center of Area $y_C^* = \frac{1}{A_{\text{total}}} \sum y_i^* A_i$

Parallel Axis Theorem $(I_z)_O = (I_z)_C + A\Delta y^2$

Rectangular Moment of Area $(I_z)_{\text{rectangle}} = \frac{bh^3}{12}$

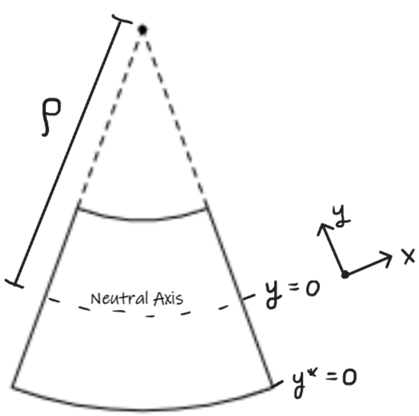
$(I_z)_{\text{circle}} = \frac{J}{2}$

Summation of I_z $I_z = \sum I_{z,i} + \sum A_i (y_{NA}^* - y_{NA,i}^*)^2$

Moment of Area $J = I_z + I_y$

Composite Cross Sections $b_{\text{transformed}} = b\left(\frac{E}{E_{\text{ref}}}\right)$

$$\frac{\sigma_x}{-y}\left(\frac{E_{\text{ref}}}{E}\right) = \frac{M}{I_z} = \frac{E_{\text{ref}}}{\rho}$$



$y = y^* - y_{NA}^*$

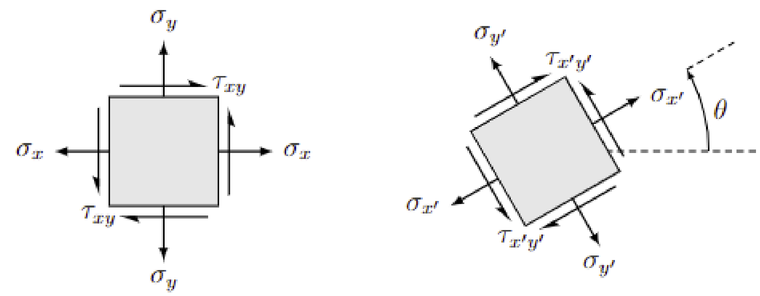
Mohr's Circle

Stress Transformation

$$\sigma_{x'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



Note that θ for the stress cube corresponds to 2θ in Mohr Circle space.

Principal Planes and Max Shear Stress

Center Point $C = \sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}$

Radius $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

Principal Stresses $\sigma_{p1,p2} = C \pm R$

Principal Angles $2\theta_{p1} = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$

$\theta_{p2} = \theta_{p1} + 90^\circ$

Max Shear Stress $\tau_{max} = R$

Critical Planes $\theta_{cp1,cp2} = \theta_{p1,p2} + 45^\circ$

Failure Criteria

Tresca $\sigma_{\max} - \sigma_{\min} = 2\tau_{\max} = \frac{\sigma_Y}{f_s^{\text{Tresca}}}$

von Mises $\frac{1}{2}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2\right) = \left(\frac{\sigma_Y}{f_s^{\text{vM}}}\right)^2$

Mohr $\frac{\sigma_{\max}}{\sigma_{UT}} - \frac{\sigma_{\min}}{|\sigma_{UC}|} = \frac{1}{f_s^{\text{Mohr}}}$ (for brittle materials)

Thin-Walled Pressure Vessels

Sphere $\sigma_\theta^{\text{sph}} = \frac{Pr}{2t}$

Cylinder $\sigma_{\text{ax}} = \frac{Pr}{2t}$ $\sigma_\theta^{\text{cyl}} = \frac{Pr}{t}$

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