PHYS 250 - Unofficial Formula Sheet

Relativity

Lorentz Transformations

Beta $\beta = \frac{u}{c} = \frac{x'}{t'c}$

Gamma $\gamma = \frac{1}{\sqrt{1-(\frac{u}{c})^2}}$

Transforms $x' = \gamma(x - \beta ct)$

$ct' = \gamma(ct - \beta x)$

Inverse Transforms $x = \gamma(x' + \beta ct')$

$ct = \gamma(ct' + \beta x')$

Lorentz Contractions

Length Contraction $L_{\text{moving}} = \frac{L_{\text{rest}}}{\gamma}$

Time Dilation $T_{\text{moving}} = \gamma T_{\text{rest}}$

Doppler Effect $f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-\beta}{1+\beta}} = f_{\text{source}}\sqrt{\frac{c+u}{c-u}}$

$\Delta T_{obs} = \Delta T_{\text{source}}\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$

$\lambda_{\text{obs}} = \lambda_{\text{source}}\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$

$\beta > 0$ when source and observer are moving apart

Velocity Addition $v_{\text{rel}} = \frac{v'+u}{1+\frac{v'u}{c^2}}$, $\beta = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}$

Photons

Planck-Einstein $E = hf = \frac{hc}{\lambda}$

Einstein photoelectric $E_{max} = hf - \phi q$

X-ray tube spectrum $\lambda_{min} = \frac{hc}{E_{\text{electron}}} = \frac{hc}{\text{qV}}$

Stopping Potential $V_{stop} = \frac{hf}{q} - \phi_{work}$

Bragg's Law $2d\sin(\theta_{\text{surface}}) = n\lambda$ $d =$ spacing

Compton Scattering $\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$ $\frac{h}{mc} = 2.426$ pm

Photon Polarization $\text{Prob} = \cos^2\phi$ $\phi =$ angle between polarization

Blackbody Spectrum $dI = \frac{2hf^3}{c^2}\cdot\frac{1}{e^{\frac{hf}{kT}}-1}\cdot df$

Constants

$1\,\text{eV} = 1.602\times10^{-19}\,\text{J}$

$c = 2.998\times10^{8}\,\text{m}\,\text{s}^{-1}$

$h = 6.626\times10^{-34}\,\text{J}\,\text{s} = 4.135\times10^{-15}\,\text{eV}\,\text{s}$

$hc = 1240\,\text{eV}\,\text{nm} = 1.986\times10^{-25}\,\text{J}\,\text{m}$

$q = 1.602\times10^{-19}\,\text{C}$

$\hbar = \frac{h}{2\pi} = 1.055\times10^{-34}\,\text{J}\,\text{s}$

$\hbar = 6.5821\times10^{-16}\,\text{eV}\,\text{s}$

$m_e = 9.11\times10^{-31}\,\text{kg} = 5.11\times10^{5}\,\text{eV}/c^2$

$\epsilon_0 = 8.854\times10^{-12}\,\text{F}\,\text{m}^{-1}$

Relativistic Momentum

Spacetime 4-Vector and Invariance

Observers in different reference frames may disagree on order events, and/or their spatial relation, but the length of the spacetime 4-vector and its dot products is invariant (same for all observers):

Definition : $\underbrace{\underset{\rightarrow}{X}}_{\text{Space-time}} = \langle \underbrace{ct}_{\text{Observer's time}}, \underbrace{\vec{x}}_{\text{Distance from Observer}} \rangle$

Properties : $\underset{\rightarrow}{X}\cdot\underset{\rightarrow}{Y} = \underset{\rightarrow}{X}'\cdot\underset{\rightarrow}{Y}' = c^2 t_x\cdot t_y - \vec{x}\cdot\vec{y}$

Note that 'proper time' τ, the passage of time for the moving object's frame of reference, is also the same for all observers. Relativistic energy and momentum are intertwined through spacetime, it is their sum that is conserved:

Spacetime Energy-Momentum : $\underset{\rightarrow}{P} = m\frac{d}{d\tau}\underset{\rightarrow}{X} = \left\langle\frac{E_{rel}}{c}, \vec{p}\right\rangle$

Energy-Momentum Invariance : $\underset{\rightarrow}{P}\cdot\underset{\rightarrow}{P} = \underset{\rightarrow}{P}'\cdot\underset{\rightarrow}{P}' = \frac{1}{c^2}(E^2 - |\vec{p}|^2c^2)$

Momentum and Energy

Relativistic Momentum $\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}} = \gamma m\vec{v}$

Kinetic Energy $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$

Net Energy $E = K + mc^2 = \gamma mc^2$

$E^2 = (mc^2)^2 + (pc)^2$

Along Particle Path $p_{decay} = \gamma p' + \beta\gamma E'$

Center of Mass

General Center of Mass $\vec{p}_1 + \vec{p}_2 = \vec{0}$

$\left(\frac{E_1+E_2}{c}\right)^2 = \left(\frac{E_{CM}}{c}\right)^2 = (m_1c)^2 + \underset{\rightarrow}{P}_1\cdot\underset{\rightarrow}{P}_2 + (m_2c)^2$

Beam and Particle at Rest $\left(\frac{E_{CM}}{c}\right)^2 = (m_1c)^2 + 2m_2E_{1\ \text{lab}} + (m_2c)^2$

Particle Decay $\left(\underset{\rightarrow}{P}_0\right)^2 = (m_0c)^2 = (\underset{\rightarrow}{P}_1 + \underset{\rightarrow}{P}_2)^2 = \langle\frac{E_1+E_2}{c}, \vec{p}_1 + \vec{p}_2\rangle^2$

Atoms and Bohr

Constants

$B = 364.5\,\text{nm}$ for Hydrogen

$R = \frac{4}{B} = 1.097 \times 10^7\,\text{m}^{-1}$

Equations

Angular Momentum	$L = n\hbar$
Diffraction	$sin\theta = n\frac{\lambda}{d}$, if $d \gg \lambda$ diffraction is invisible
Improved Balmer	$\lambda = B\frac{n^2}{n^2-m^2},\ n > m$
	$m=1$: Lyman (UV) $m=2$: Balmer (visible) $m=3$: Paschen (IR)
Improved Rydberg	$\frac{1}{\lambda} = RZ^2(\frac{1}{n_1^2} - \frac{1}{n_2^2})$ with $n_2 > n_1$
Photon Energy	$E = 13.6\,\text{eV} \cdot Z^2(\frac{1}{n_1^2} - \frac{1}{n_2^2})$
Rutherford Orbit Energy	$E_{\text{total}} = -\frac{qQ}{8\pi\epsilon_0 r} = -\frac{m}{2}\left(\frac{qQ}{4\pi\epsilon_0}\right)^2\frac{1}{L^2}$
Rutherford Atomic Decay	$t_{\text{seconds}} = 39.2\,\text{ns} \cdot \lvert E_{\text{eV}}\rvert^{-3}$
Bohr Atomic Energy	$E = -\frac{m}{2}\left(\frac{q^2}{4\pi\epsilon_0\hbar}\right)^2\frac{Z^2}{n^2} = -13.6\,\text{eV}(\frac{Z^2}{n^2})$
Bohr Model Radius	$r = \frac{4\pi\epsilon_0\hbar^2 n^2}{Zmq^2} = \frac{q^2 Z}{8\pi\epsilon_0 E} = 52.97\,\text{pm}(\frac{n^2}{Z})$
Bohr Orbit Velocity	$v = \frac{Zq^2}{L4\pi\epsilon_0} = \frac{Zq^2}{4\pi\epsilon_0 n\hbar}$
	$\beta = \frac{v}{c} = 7.293 \cdot 10^{-3}(\frac{Z}{n})$
Moseley's Law (X-Ray Emission)	$E = 13.6\,\text{eV}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot (Z-b)^2$
	Kα: $b=1, n_1=1, n_2=2$ Kβ: $b=1, n_1=1, n_2=3$ Lα: $b=7.4, n_1=2, n_2=3$
de Broglie Wavelength	$\lambda_{\text{electron}} = \frac{1.227\sqrt{\text{eV}}\cdot\text{nm}}{\sqrt{E_{\text{electron}}}}$
	$\lambda_{\text{matter}} = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2 E_{\text{kinetic}}}}$
Reduced Mass	$\mu = \frac{m_e m_X}{m_e + m_X}$, X is mass of nucleus

LAZERS!

We love lasers

Spontaneous emissions per second	$\frac{N}{\tau}$ where τ is the lifetime
Law of collisions	$\rho\sigma\lambda = 1$ where σ is collision cross section per atom
Number of photons after travelling distance	$N(x) = N_0 e^{-\frac{x}{\lambda}}$

Basics of Quantum Mechanics

Wave Equation

Schrodinger Equation	$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi$
Angular Frequency	$\omega = \frac{2\pi}{T} = 2\pi f$
Energy	$E_K = hf = \hbar\omega = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Momentum	$p = \hbar k$
Free Particle Solutions	$\Psi(x,t) = A\sin(\pm kx - \omega t + \phi),\ k = \frac{2\pi}{\lambda}$
	$\Psi(x,t) = e^{i(kx-\omega t)} = e^{i\frac{px-Et}{\hbar}}$

Operators

Momentum Operator	$\hat{p}\Psi = -i\hbar\frac{\partial}{\partial x}\Psi$
Hamiltonian Operator	$\hat{H}\Psi = \frac{\hat{p}^2}{2m}\Psi + V\Psi = -\frac{\hbar^2}{2m}\Psi + V\Psi$

Probability Density

Probability Density	$\rho(x) = \Psi^*\Psi$	$\int_{-\infty}^{\infty}\Psi^*\Psi dx = 1$
Normalized Gaussian	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$	
Transformed Gaussian	$\sigma_x\sigma_k = 1$	
Heisenberg Uncertainty Principle	$\sigma_x\sigma_p \geq \frac{\hbar}{2}$	

Schrodinger PDEs

TISE	$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = E\psi$
Time Dependent Solution	$\varphi(t) = e^{-\frac{iE_n}{\hbar}t}$
Solution to Schrodinger	$\Psi_n(x,t) = \psi_n(x)\varphi_n(t) = \psi_n(x)e^{-\frac{iE_n}{\hbar}}$

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Step Potentials, Barriers, and Wells

Step Potentials

Wavefunctions $\begin{cases} \psi_I = e^{ikx} & \text{(Incident)} \\ \psi_R = Re^{-ikx} & \text{(Reflected)} \\ \psi_T = Te^{ik'x} & \text{(Transmitted)} \end{cases}$

Wavenumbers	$k = \frac{\sqrt{2mE}}{\hbar}$	$k' = \frac{\sqrt{2m(E-V)}}{\hbar}$
Amplitudes	$R = \frac{k-k'}{k+k'}$	$T = \frac{2k}{k+k'}$
Flux	$\Phi = \text{density} \cdot \text{velocity}$	
Incident Flux	$\Phi_I = \frac{\hbar k}{m}$	$\Phi_I = \Phi_R + \Phi_T$
Reflected Flux	$\Phi_R = \frac{\hbar k}{m} R^2 = \frac{\hbar k}{m}\left(\frac{k-k'}{k+k'}\right)^2$	
Transmitted Flux	$\Phi_T = \frac{\hbar k'}{m} T^2 = \frac{\hbar k'}{m}\left(\frac{2k}{k+k'}\right)^2$	
Probability	$P(R) = \frac{\Phi_R}{\Phi_I} = R^2$	$P(T) = \frac{\Phi_T}{\Phi_I} = \frac{k'}{k} T^2$

Potential Barriers

Wavefunctions $\begin{cases} \psi_I = e^{ikx} & \text{(Incident)} \\ \psi_R = Re^{-ikx} & \text{(Reflected)} \\ \psi_T = Te^{ikx} & \text{(Transmitted)} \\ \psi_F = Fe^{ik'x} & \text{(Forward)} \\ \psi_B = Be^{-ik'x} & \text{(Backward)} \end{cases}$

Amplitudes $\begin{bmatrix} -1 & 1 & 1 & 0 \\ k & k' & -k' & 0 \\ 0 & e^{ik'w} & e^{-ik'w} & -e^{ikw} \\ 0 & k'e^{ik'w} & -k'e^{-ik'w} & -ke^{ikw} \end{bmatrix} \begin{bmatrix} R \\ F \\ B \\ T \end{bmatrix} = \begin{bmatrix} 1 \\ k \\ 0 \\ 0 \end{bmatrix}$

Approximate Tunneling $P(T) \approx 16\frac{E}{V}\left(1 - \frac{E}{V}\right)e^{-2\kappa l},\ \kappa = \frac{\sqrt{2m(V-E)}}{\hbar}$, l is length

Infinite Square Well

Energy	$E_n = -\frac{\hbar^2\pi^2 n^2}{2mw^2}$, w is width
Inside Wavefunction	$\psi_n(x) = \sqrt{\frac{2}{w}}\sin(k_n x),\ k_n = \frac{n\pi}{w}$
Outside Wavefunction	$\psi_n(x) = e^{\pm k'x},\ k'_n = \frac{n\pi}{w}$

Finite Potential Wells

Energy	$E_n < \frac{\hbar^2\pi^2 n^2}{2mw^2} - V < 0,\ E > 0$
Wavefunction Inside	$\psi_n(x) = A\sin(kx) + B\cos(kx),\ k = \sqrt{\frac{2mE}{\hbar^2}}$
Wavefunction Outside	$\psi_n(x) = Ce^{k'x} + De^{-k'x},\ k' = \sqrt{\frac{2m(V-E)}{\hbar^2}}$,

Real Potentials

Linear Potentials $V(x) = Fx$

Airy Equation	$\frac{\partial^2 y}{\partial x^2} = xy,\ x > 0 \Rightarrow \text{Ai}(x) \approx 0.35503e^{-x^{1.25}}$
	$x < 0 \Rightarrow \text{Ai}(x) \approx 0.56417\lvert x\rvert^{-0.25}\sin\left(\frac{\lvert x\rvert^{1.5}}{1.5} + \frac{\pi}{4}\right)$
	$x \to x + E/F$ so $\frac{\partial^2\psi}{\partial x^2} = \frac{Fx}{\hbar^2/2m}\psi$
Ai(x) roots	$\lvert\text{roots}\rvert = -2.33811, -4.08795, -5.52056, -6.78671$
	$\lvert\text{roots}\rvert \approx -((n - \frac{1}{4})1.5\pi)^{2/3}$
Energy	$E = (\frac{\hbar^2 F^2}{2m})^{1/3} \cdot \lvert\text{roots}\rvert$

Quantum Harmonic Oscillator $V(x) = \frac{1}{2}kx^2$

Energy	$E_n = \frac{2n+1}{2}\hbar\sqrt{\frac{k}{m}} = \frac{2n+1}{2}\hbar\omega_{\text{classical}}$
Wavefunction	$\psi_n(x) = H_n\left(\frac{x}{b}\right)e^{-\frac{x^2}{2b^2}}$
Terms	$b^2 = (2\sigma)^2 = \frac{\hbar}{\sqrt{km}},\ \omega_{\text{classical}} = \sqrt{\frac{k}{m}}$

Hermite Polynomials $\begin{cases} H_0(x) = 1 \\ H_1(x) = 2x \\ H_2(x) = 4x^2 - 2 \\ H_3(x) = 8x^3 - 12x \\ H_4(x) = 16x^4 - 48x^2 + 12 \\ H_5(x) = 32x^5 - 160x^3 + 120x \\ H_6(x) = 64x^6 - 480x^4 + 720x^2 - 120 \\ H_7(x) = 128x^7 - 1344x^5 + 3360x^3 - 1680 \\ H_8(x) = 256x^8 - 3584x^6 + 13440x^4 - 13440x^2 + 1680 \end{cases}$

Schrodinger in 3D

Free Particle

Wavefunction	$\Psi(\vec{x}, t) = e^{i(\vec{k}\cdot\vec{x} - \omega t)}$
Energy	$E = \frac{\vec{p}^2}{2m} = \frac{(\hbar^2\vec{k})^2}{2m} = \hbar\omega$

Particle in a Box $(0 < x < a,\ 0 < y < b,\ 0 < z < c)$

Wavefunction	$\psi_{n_x,n_y,n_z}(\vec{x}) = \sin\left(\frac{n_x\pi x}{a}\right)\sin\left(\frac{n_y\pi y}{b}\right)\sin\left(\frac{n_z\pi z}{c}\right)$
Energy	$E_{n_x,n_y,n_z} = \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)\frac{\hbar^2\pi^2}{2m},\ n > 0$
Energy in a Cube	$E_{n_x,n_y,n_z} = \left(n_x^2 + n_y^2 + n_z^2\right)\frac{\hbar^2\pi^2}{2mw^2},\ n > 0$

Spherical Potentials

General Spherical Relationships

Spherical Laplacian $\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}$

Separable Functions $\Psi(r,\theta,\phi) = F(r)G(\theta)H(\phi)$

Equations
$$\begin{cases} \frac{\partial^2 H(\phi)}{\partial\phi^2} = -\mu H(\phi) \\ \sin(\theta)\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial G(\theta)}{\partial\theta}\right) = \left(\mu - \lambda sin^2\theta\right)G(\theta) \\ -\frac{\hbar^2}{2M}\frac{\partial^2 U(r)}{\partial r^2} + [V(r) + \frac{\hbar^2\lambda}{2Mr^2}]U(r) = E\cdot U(r) \end{cases}$$

ϕ dependent $H(\phi) = e^{im\phi},\ m\in\mathbb{Z}$

θ dependent $G(\theta) = P_\ell^m(\theta)$ (the associated Legendre functions)

r dependent $F(r) = \frac{1}{r}U_{k\ell}(r)$

Spherical Harmonics $Y_\ell^m(\theta,\phi) = P_\ell^m(\theta)e^{im\phi}$

	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
$m=4$					$+\sqrt{\frac{315}{512\pi}}\sin^4\theta\, e^{4i\phi}$
$m=3$				$-\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{3i\phi}$	$-\sqrt{\frac{315}{64\pi}}\sin^3\theta\cos\theta\, e^{3i\phi}$
$m=2$			$+\sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{2i\phi}$	$+\sqrt{\frac{35}{64\pi}}\sin^2\theta\cos\theta\, e^{2i\phi}$	$+\sqrt{\frac{45}{128\pi}}\sin^2\theta\left(7\cos^2\theta-1\right)e^{2i\phi}$
$m=1$		$-\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{i\phi}$	$-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$	$-\sqrt{\frac{21}{64\pi}}\sin\theta\left(5\cos^2\theta-1\right)e^{i\phi}$	$-\sqrt{\frac{45}{64\pi}}\sin\theta\left(7\cos^3\theta-3\cos\theta\right)e^{i\phi}$
$m=0$	$\sqrt{\frac{1}{4\pi}}$	$+\sqrt{\frac{3}{4\pi}}\cos\theta$	$+\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta-1\right)$	$+\sqrt{\frac{7}{16\pi}}\left(5\cos^3\theta-3\cos\theta\right)$	$+\sqrt{\frac{9}{256\pi}}\left(35\cos^4\theta-30\cos^2\theta+3\right)$
$m=-1$		$+\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{-i\phi}$	$+\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{-i\phi}$	$+\sqrt{\frac{21}{64\pi}}\sin\theta\left(5\cos^2\theta-1\right)e^{-i\phi}$	$+\sqrt{\frac{45}{64\pi}}\sin\theta\left(7\cos^3\theta-3\cos\theta\right)e^{-i\phi}$
$m=-2$			$+\sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{-2i\phi}$	$+\sqrt{\frac{35}{64\pi}}\sin^2\theta\cos\theta\, e^{-2i\phi}$	$+\sqrt{\frac{45}{128\pi}}\sin^2\theta\left(7\cos^2\theta-1\right)e^{-2i\phi}$
$m=-3$				$+\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{-3i\phi}$	$+\sqrt{\frac{315}{64\pi}}\sin^3\theta\cos\theta\, e^{-3i\phi}$
$m=-4$					$+\sqrt{\frac{315}{512\pi}}\sin^4\theta\, e^{-4i\phi}$

$m = \{\cdots,-2,-1,0,1,2,\cdots\}$ $\quad \ell = \{0,1,2,\cdots\}$ $\quad |m|\le\ell$

$|m|$ is the power of $\sin\theta$ and ℓ is the power of $\sin\theta$ plus the highest power of $\cos\theta$

General Solution $\psi_{k\ell m}(r,\theta,\phi) = \frac{1}{r}U_{k\ell}(r)Y_\ell^m(\theta,\phi)$

Infinite Spherical Square Well

Energy $E_{k0} = k^2\frac{\hbar^2\pi^2}{2MR^2}$

Wavefunction $\Psi_{k00} = \frac{1}{r}\sin\left(\frac{k\pi r}{R}\right)Y_0^0(\theta,\phi) = \frac{1}{r}\sin\left(\frac{k\pi r}{R}\right)$

Coulomb Potential $V(r) = -\frac{q^2}{4\pi\epsilon_0}$

Wavefunction $(n=\ell+1)$ $U_{n\ell}(r) = r^n e^{-\frac{r}{na_0}},\ a_0 = r_{\text{Bohr}} = 52.97\,\text{nm}$

Wavefunction $(n=\ell+2)$ $U_{n\ell}(r) = \left(r^n - n(n-1)a_0 r^{n-1}\right)e^{-\frac{r}{na_0}}$

Energy $E = -\frac{E_{\text{Bohr}}}{n^2},\ E_{\text{Bohr}} = 13.6\,\text{eV}$

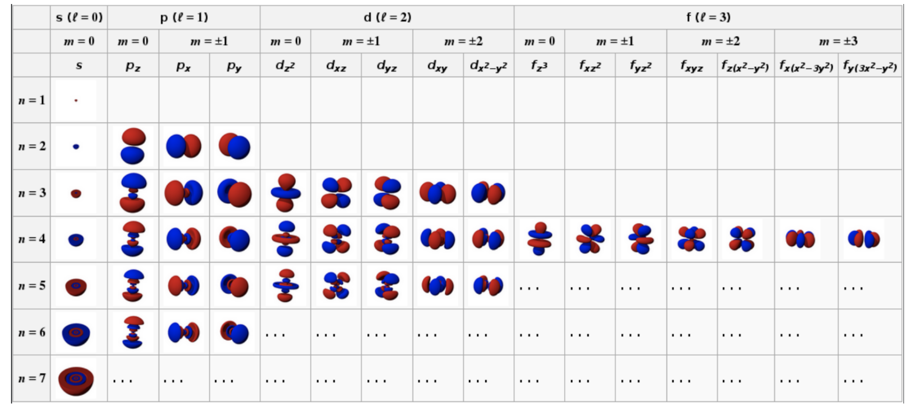
Hydrogen Wavefunction

Hydrogen Wavefunction

Quantum Numbers $n = k+\ell$

General Wavefunction $\phi_{n\ell m}(r,\theta,\phi) = \sqrt{\left(\frac{2}{ma_0}\right)^3\frac{(n-\ell-1)!}{2n(\ell-1)!}}e^{-\frac{\rho}{2}}L_{n-\ell-1}^{2\ell+1}(\rho)Y_\ell^m(\theta,\phi)$

$\rho = \frac{2r}{na_0}$



The n value appears in the denominator of the exponential

The n value is the highest power of r in the exponential plus 1

3D Quantum Harmonic Oscillator

Cartesian Harmonic Oscillator

Wavefunction $\psi_{n_x,n_y,n_z}(x,y,z) = H_{n_x}\left(\frac{x}{b}\right)e^{-\frac{x^2}{2b^2}}H_{n_y}\left(\frac{y}{b}\right)e^{-\frac{y^2}{2b^2}}H_{n_z}\left(\frac{z}{b}\right)e^{-\frac{z^2}{2b^2}}$

Energy $E_{n_x,n_y,n_z} = \left(n_x+n_y+n_z+\frac{3}{2}\right)\hbar\sqrt{\frac{k}{m}},\ n\ge 0$

Spherical Harmonic Oscillator

Dimensionless Variables $E = \eta\hbar\sqrt{\frac{k}{M}}$ $\qquad r = b\rho$

Energy $E_n = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{M}}$ $\qquad n > 0$

Wavefunction $\psi_{n\ell m}(r,\theta,\phi) = \frac{U_{n\ell}(r)}{r}Y_\ell^m(\theta,\phi)$

Solutions of $u_{\eta\ell}(\rho)$

	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
$\eta=4+\frac{3}{2}$	$\left(\rho^5-5\rho^3+\frac{15}{4}\rho\right)e^{-\frac{\rho^2}{2}}$		$\left(\rho^5-5\rho^3\right)e^{-\frac{\rho^2}{2}}$		$\rho^5 e^{-\frac{\rho^2}{2}}$
$\eta=3+\frac{3}{2}$		$\left(\rho^4-3\rho^2\right)e^{-\frac{\rho^2}{2}}$		$\rho^4 e^{-\frac{\rho^2}{2}}$	
$\eta=2+\frac{3}{2}$	$\left(\rho^3-\frac{3}{2}\rho\right)e^{-\frac{\rho^2}{2}}$		$\rho^3 e^{-\frac{\rho^2}{2}}$		
$\eta=1+\frac{3}{2}$		$\rho^2 e^{-\frac{\rho^2}{2}}$			
$\eta=0+\frac{3}{2}$	$\rho e^{-\frac{\rho^2}{2}}$				