Math Notes

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1 Introductory Logic

1.1 Arguments

1.1.1 Introduction to Arguments, Fallacies, and Logic

Familiar types of logic:

- Quarrels
- Legislative debates
- Labour negotiations
- Diplomatic discussions
- Legal arguments
- Mathematical proofs

- Scientific demonstrations

Arguments in the broad sense:

- attempt to build a case in favour of some claim
- involve the presentation of reasons or evidence (premises) in support of a conclusion
- are social exchanges often involving a series of speech acts uttered by two or more parties
- are governed by a set of rules or standards (sometimes implicit)
- attempt to advance knowledge by justifying or undermining a conclusion

Arguments in the narrow sense are sets of propositions composed of an argument's *premises* and *conclusion*.
Logic, understood broadly, studies arguments in the broad sense. Logic, understood narrowly, studies arguments in the narrow sense.

Placing Arguments in Standard Form Ex: Archimedes must be either a hero or a martyr. After all, anyone who dies in battle is one or the other. And, as we know, Archimedes perished during the capture of Syracuse.

Premise 1: Anyone who dies in battle is either a hero or a martyr
Premise 2: Archimedes perished during the capture of Syracuse

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Conclusion: Therefore, Archimedes must be either a hero or a martyr

How to place an argument in standard form:

1. Identify the premises and conclusion
2. Eliminate any unnecessary or redundant words or phrases
3. Clarify any ambiguities
4. Separate the premises from the conclusion with a horizontal line

How to Handle Sub-Arguments
You could display as one argument or display as separate arguments with the conclusion of the sub-argument becoming a premise of the main argument
Ex2: If Bill wants to live in Auckland, then he has to learn to sail. If Bill has to learn to sail, then he will have to learn navigation. It turns out that Bill does want to live in Auckland. So Bill will have to learn navigation.

Displayed as one argument:
Premise: If Bill wants to live in Auckland, then he has to learn to sail
Premise: If Bill has to learn to sail, then he has to learn navigation
Premise: Bill wants to live in Auckland

Conclusion: Therefore, Bill has to learn navigation.

Displayed as two arguments:

Premise: If Bill wants to live in Auckland, then he has to learn to sail
Premise: Bill wants to live in Auckland

Intermediate Conclusion: Bill has to learn to sail.

Premise: If Bill has to learn to sail, then he has to learn navigation
Premise: Bill has to learn to sail.

Conclusion: Therefore, Bill has to learn navigation.

Ex3: Geometry should not include lines that are strings, in that they are sometimes straight and sometimes curved, since the ratios between straight and curved lines are not known, and I believe cannot be discovered by human minds, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact. [Rene Descartes]

The strings are sometimes straight and sometimes curved
The ratios between straight and curved lines are not known and cannot be discovered by human minds

---

No conclusion based upon the ratios between straight and curved lines can be accepted as rigorous and exact

No conclusion based upon the ratios between straight and curved lines can be accepted as rigorous and exact

---

Geometry should not include lines that are strings

Identifying Propositions Propositions are

- bearers of truth (either true/false)
- distinct from questions (interrogatives), commands (imperatives), prayers, etc.
- usually expressed by statements (declarations), but sometimes also by other means, e.g. rhetorical questions

Ex: Sue is visiting Hong Kong.
is a proposition
Ex2: When is Bill's birthday?
not a proposition
Ex3: Who doesn't know London is a great city?
This is a rhetorical question stating that London is a great city
so it is a proposition

Indicator Words or Phrases for Conclusions

- thus
- hence
- therefore
- consequently
- which implies that

- so
- it follows that
- for this reason
- we may conclude that
- which proves that

Indicator Words or Phrases for Premises

- because
- as
- since
- moreover
- given that
- provided that
- assuming that
- on the grounds that

Evaluating Arguments The quality of an argument depends on

- the truth of the argument's premises
- the strength of the consequence relation that holds between the arguments premises and conclusion

Two main ways to criticize an argument are

- to show that the premises are (likely) not true
- to show that the consequence relation is (likely) not strong

Consequence Relations

- If the premises of an argument, when assumed to be true, provide good evidence in favour of that argument's conclusion, the argument has a strong consequence relation
- If the premises of an argument, when assumed to be true, provide poor evidence in favour of that argument's conclusion, the argument has a weak consequence relation
- A consequence relation that is neither strong nor weak is moderate

Ex: Strong
Socrates is a father

Therefore Socrates is male

Ex2: Moderate
Plato lives in Athens

Therefore Plato is Greek

Ex3: Weak
Aristotle is male

Therefore Aristotle is a father

Deductive Arguments
A *deductive argument* is one presented with the intention of being judged by *deductive standards.*
An argument meets the standard of deductive adequacy only when it is impossible for its conclusion to be false, assuming that its premises are all true.
The consequence relation is as strong as it can get.

Inductive Arguments
An *inductive argument* is one presented with the intention of being judged by *inductive standards.*
An argument meets the standard of inductive adequacy only when it is not deductively adequate and its premises, if true, increase the likelihood of its conclusion also being true.
Ex: Deductive vs. inductive
Deductive:
Russell had a brother

Russel had a sibling

Inductive:
Russell had a sibling

Russell had a brother

Logic vs. Rhetoric
Rhetoric:

- studies arguments that are *in fact* persuasive
- is defined as the science of persuasion

Logic:

- studies arguments that *ought to be* persuasive or that would be persuasive to an ideally rational agent.
- is defined as the science of reasoned (or rational) persuasion.

1.1.2 Types of Arguments

The Quarrel The "Yes, you did," "No, I didn't" Quarrel
Sue: I thought you told me that you were going to be at the library last Friday.
Bill: No. Don't you remember? I said I would be at the beach.
Sue: No. You said you would be at the library. Otherwise I would have wanted to go with you.
Bill: You're not remembering things correctly—that was the day I said I would be at the beach!

Bill and Sue disagree about what the facts are. They disagree what premises they share. They have *promissory instability.*

The"You don't love me anymore" Quarrel
Bill: You don't love me.
Sue: But I do! How could I call you a "mindless bore" without caring for you enough to risk giving offense, perhaps even losing you?
Bill: Sure, sure. How do I know that this isn't just what you have in mind—losing me, as you call it, or breaking up, as I would say?
Sue: Bill, you just don't understand. Be reasonable.

Bill and Sue agree about the facts but they disagree about what the facts imply. Bill and Sue disagree about what conclusions they should draw. They have *conclusional instability.*

Definition:
A *quarrel* is any argument in which:

- the disputants suffer from premissory or conclusional instability (or both)
- lack of a shared method for conflict resolution
- refure to agree to disagree

Fallacies A *fallacy* is:

- a bad argument or inference that has a propensity to appear good
- a common or seductive error in reasoning

Ex: The press should print all news that is in the public interest. The public interest in this murder case is intense. Therefore, the press should print news about this case.

Note that the argument involves an ambiguous phrase "public interest" (meaning "of benefit to the public" versus "of interest to the public"). This is called an *equivocation.*
The Ad Baculum An *ad baculum* argument occurs whenever a conclusion is drawn on the basis of an appeal to force, intimidation, or threat of bad consequences.
If the appeal is relevant, it is non-fallacious
If the appeal is irrelevant, it is fallacious

Ex: Our paper certainly deserves the support of every German. We shall continue to forward copies of it to you, and hope that you will not want to expose yourself to unfortunate consequences in the case of cancellation.

This does not recommend any explicit belief so it is not fallacious.

- If an argument is about what is sensible or prudent to *do*, the argument is not fallacious
- If an argument is about what is sensible or rational to *believe*, the argument is fallacious.

Ex:2 "Toyota Motor said will build a new plant in Baja, Mexico, to build Corolla cars for U.S. NO WAY! Build a plant in U.S. or pay big border tax." [Donald Trump, 2017]
Not fallacious.

Ex3: Zach: "My father owns the department store that gives your newspaper forty percent of your advertising revenue, and my understanding is that you don't want to publish any story of my arrest for spray painting the college."
Newspaper editor: "Yes, Zach, I see your point. The story doesn't seem to be newsworthy.

Whether or not the story deserves the attention of the public needs to be judged by some other criteria, not the ones given by Zach, thus the argument is fallacious.

Ex4: Zach: "My father owns the department store that gives your newspaper forty percent of your advertising revenue, and my understanding is that you don't want to publish any story of my arrest for spray painting the college."
Newspaper editor: "Yes, Zach, I see your point. Looks like we shouldn't do it lest we all lose our jobs, indeed.

As a consideration of what the newspaper should do in order to avoid the bad consequences, the appeal is highly relevant, this non-fallacious argument.

The Ad Hominem An ad hominem argument occurs whenever a conclusion is drawn on the basis of an appeal to some fact or alleged fact about one's opponent.
If the appeal is relevant, the argument is non-fallacious
If the appeal is irrelevant, the argument is fallacious

There are two main types of ad hominem

- the *abusive ad hominem*, in which an insulting or unwelcome allegation is advanced about one's opponent
- the *circumstantial ad hominem*, in which a non-abusive allegation is advanced about one's opponent, or about his or her circumstances

Ex: Abusive
No one should trust Sue's argument about hospital funding. After all, I heard that she was fired from her job in the acute care unit.

Ex2: Circumstantial ad hominem
No one should trust Bill's argument about school funding. After all. his sister is a teacher.

Ex3: Critic: "How can you derive pleasure from gunning down a helpless animal? Surely the killing of deer or trout for amusement is barbarous."
Sportsman: "If you're so concerned, why do you feed on the flesh of animals? Aren't you being inconsistent?"

What's so special about logic?

- Logic is Public. Unlike "feelings" or "intuitions"—which also motivate beliefs but which are essentially private—reasoned arguments provide us with a public mechanism for the resolution of disagreements
- Logic is Safe. Unlike physical means of conflict resolution—which range from personal intimidation to warfare—logic gives us a safe, non-harmful mechanism for the resolution of disagreements
- Logic is Effective. Careful, logical reasoning from acceptable premises to reliable conclusions is the method most likely to lead to accurate beliefs and, hence, the method most likely to help us improve our lives

1.1.3 Debates

Debates occur in parliaments, congress, diets, bunds, and other government houses, as well as in other contexts. They are well-governed contests of words between two or more sides, presented over by a speaker, referee, or chairperson. They are won and lost depending upon who wins the approval of "the house" or some other jury

The goal of a debate is not explicitly to discover the truth but, rather, to win the approval of the house. It is only through open debate, the public offering of conjecture and criticism, that large scale advances in human knowledge are possible. Because of their structure, debates allow for the type of public conjecture and criticism necessary for advances in human knowledge. Debates remain an effective and objective *route* to the truth.

Social institutions in which knowledge advances through debate includes:

- opposition parties in government
- trial by judge or jury in an adversarial legal system
- peer-reviewed scientific and scholarly journals
- the free press

Advantages:
Because a debate is rule-governed and presided over by a neutral speaker or chairperson, there are clear winners and losers.
Because a debate is resolved by vote of the house, or by some other similar mechanism, total premissory or conclusional agreement among the contending parties is not required.
As long as a debate opens each debater's views to a wide variety of criticism, it is likely to be an effective and objective method for advancing truth.

Disadvantages:
Because the ultimate goal is to win, factors such as *sophistry, insincerity,* and *ambiguity* may be invoked by the participants
Because of *ad populum* arguments and the *bandwagon effect*, group decision-making can be unstable and non-objective

Because many debates are highly regulated, they cease to be free markets for ideas.
Because even eyewitness reports and expert testimony can be unreliable, evidence offered during debates can sometimes be misleading
Because expert testimony can be non-objective, the fallacy of ad verecundiam may be involved
The Ad Populum An ad populum argument occurs whenever a conclusion is drawn, or invited to be drawn, on the basis of an appeal to popular belief.
If the appeal is relevant, the argument is non-fallacious
If the appeal is irrelevant, the argument is fallacious.
There are two main types of ad populum:

- **boosterism** arguments, in which appeal is made to the sentiments or prejudices of one's audience
- **popularity** arguments, in which appeal is made to "common knowledge" or popular belief more generally

Ex1: Boosterism:
"Your right to bear arms is as Canadian as maple syrup" [John Robson]
In this case, the author merely advances a view that appeals to his intended local audience, namely that being "free", or "having a right", to possess a firearm must be taken as an indispensable part of being a "true" Canadian.

Ex2:
Bill: If your position is that physicians should not be allowed to opt out of medicare, and must be bound entirely by standards and rates set by government, how can you defend the declining standard of health care that is resulting from the current emigration of the best qualified physicians from our community?
Sue: I was born and raised not far from here, and what's clear to me is that the people of this fine community have a right to medical care when family members are in desperate need. I know that, because people support this fundamental right, they also support medicare, regardless of the whims of a lot of fancy, overpaid specialists.

In this case, Sue merely advances a view that appeals to her local audience, namely that medicare is a good thing for their town.

Ex3: popularity:
"Those who say that astrology is not reliable are mistaken. The wisest men of history have all been interested in astrology; kings and queens of all ages have guided the affairs of nations by it."

An appeal to how many people throughout history held this belief
"Those who say ... are mistaken" makes it a fallacious argument

Ex4: popularity:
"The wisest men of history have all been interested in astrology; kings and queens of all ages have guided the affairs of nations by it. So, it probably makes sense to pay attention to what it says

and spend some time trying to figure out whether it's really reliable."

As a claim that the position in question (Astrology is reliable), based on the given information, deserves more attention/further investigation, it is totally legitimate, thus a non-fallacious argument

Enthymemes

An enthymeme is an argument in which one or more core propositions (a premise or conclusion) is not stated explicitly, but is merely assumed implicitly to be part of the argument.

Ex: Sue will make an excellent kindergarten teacher; everyone who loves children always does, you know.
Premise: Everyone who loves children will make an excellent kindergarten teacher
Premise (implied): [Sue loves children]
Conclusion: Sue will make an excellent kindergarten teacher

Ad populum arguments as enthymemes:
Everyone believes p
___
Therefore, p is true
Ex:
Everyone believes that water is wet
___
Therefore, water is wet
We can add the implied premise [Water's being wet is the best explanation of the fact that everyone believes that water is wet]

Evaluating arguments as enthymemes:
Before judging an argument to be fallacious, we must determine whether the argument is an enthymeme.
If it is an enthymeme, we must reconstruct the argument showing all unstated propositions
We should apply the *principle of clarity* when reconstructing enthymemes. (i.e. we should give people the benefit of the doubt, though not be so charitable that we add what was not intended)
The Ad Verecundiam An ad verecundiam argument occurs whenever a conclusion is drawn, or invited to be drawn, on the basis of an appeal to the expert opinion of an authority.
If the appeal is relevant, the argument is non-fallacious
If the appeal is irrelevant, the argument is fallacious

Ex: My favourite basketball player recommends using this brand of toothpaste; he's obviously successful, so I think I'll try it

This is fallacious because he's not an expert in the field of dentistry.

Ex2: My doctor tells me that I have pneumonia and that I need to take my medicine and stay in bed, so I think I better do what I'm told

This is non-fallacious as the expertise is relevant

Conditions for arguments from authority

- The authority must have special competence in an area, and not simply glamour, prestige, or popularity
- The judgment of the authority must be within his or her special field of competence
- The authority must be interpreted correctly
- Direct evidence must always be available, at least in principle
- The authority must not be hand-picked from among competing experts because of his or her opinion
- A consensus technique is required for adjusting disagreements among equally qualified authorities

Locke's ad verecundiam argument

Proposition p is endorsed by people who are experts on this matter

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Therefore, it is immodest of you, indeed it is a kind of insolence, to persist in your opposition to p.

The Port Royal ad verecundiam fallacy

Proposition p is endorsed by people who are superior social rank

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Therefore, it is appropriate to agree to p

De Facto vs. De Jure Authorities
A de facto authority is someone who is an expert in a given field
A de jure authority is someone who has particular abilities as a result of his or her title or office

Ex:
An expert forensic accountant who tells you that you are entitled to a payment of $10,000 from your employer
vs. a trial judge who tells you that you are entitled to a payment of $10,000 from your employer
The Ad Misericordiam Ad ad misericordiam argument occurs whenever a conclusion is drawn, or invited to be drawn, on the basis of an appeal to mercy or pity
If the appeal is relevant, the argument is non-fallacious
If the appeal is irrelevant, the argument is fallacious

Ex: Although the accused was rightly convicted of a serious crime, and although the sentence proposed by the prosecution would be just, the court is asked to show the accused mercy, owing to his bad health and advanced age.

Emotive fallacies
These occur whenever emotion interferes inappropriately with the ultimate goals of argument
Examples: ad hominem (abusive), ad populum (boosterism), ad misericordiam

1.1.4 Dialectic

Aristotle's Basic Rules of Dialectic A dialogue or dialogical argument involves a discussion or dialogue between two or more parties.
A dialectical argument is a specific type of dialogical argument. It involves questions and answers between two or more parties.

- *Examination arguments* are used to discover what prepositions two parties jointly hold, and what follows from these or other propositions.
- *Instruction (or Socratic) arguments* are used whenever a teacher directs a student to the right answer by asking a series of questions.
- *Refutation arguments* are arguments used to show that a respondent accepts contradictory propositions and to show that a respondent cannot consistently defend a thesis

Ex: Examination
Child: Why do I have to do what I'm told?
Parent: Because I want you to be safe. You want to be safe, too, don't you?
Child: Yes
Parent: Well, doing what you're told will help keep you safe
Child: Why?
Parent: Because I know what's safe and what isn't.
Child: So when I know what's safe and what isn't, then I won't have to do what I'm told.
Parent: Yes. By then you'll be able to choose what to do yourself.
Child: Okay.

Ex2: Instruction
Student: Why are there infinitely many prime numbers?
Teacher: Think about it this way: assume that there are only finitely many prime numbers. Now multiply them all together and then add one. Would this new number be prime?
Student: No, because it would be bigger than every prime number.
Teacher: And would it be composite (i.e. evenly divisible by some prime)?
Student: No, since it would always have one as a remainder.
Teacher: And must every number be either prime or composite?
Student: Yes.
Teacher: So it follows that ...
Student: ... that there can be no such number and the assumption that there are only finitely many primes must be false.

Ex3: Refutation
Lawyer: Where were you on the evening of November 15th?
Witness: At home watching television with my brother.
Lawyer: And why do you remember this so clearly?
Witness: Because we always watch Monday night football together.
Lawyer: So you'd remember if you weren't at home that night?
Witness: Yes.
Lawyer: But it turns out that you signed several credit card receipts for dinner and the movies on

the evening of the 15th.
Witness: I guess I did.
Lawyer: So it's not true that you remember watching football with your brother that night?
Witness: I guess not.

Possible results of a refutation argument:
Refutation in the *strong sense* is when a thesis or proposition is refuted in the strong sense when it is shown to be false
Refutation in the *weak sense* is when a thesis or proposition is refuted in the weak sense when it is shown that a respondent has insufficient grounds for holding it.
A *stalemate* occurs when all participants concede that the questioner is not going to succeed in refuting the respondent's thesis in either the strong or the weak sense.

Eight rules of dialectic:

1. **Selecting Participants:** are the participants equally matched?
2. **Defining a Goal:** is this a refutation? An instruction argument? An examination argument?
3. **Questioning the Respondent:** does the questioner ask clear and straightforward questions?
4. **Responding to the Questioner:** does the respondent reply truthfully and consistently?
5. **Dealing with Ignorance:** how does the respondent deal with ignorance?
6. **Postponing Answers:** are answers only postponed only by mutual consent?
7. **Terminating the Exchange:** how is the dialogue terminated?
8. **Changing Dialectic Roles:** are changes in dialectic roles made only by mutual consent?

The Ad Ignorantiam An ad ignorantiam argument occurs whenever a conclusion is drawn, or invited to be drawn, on the basis of an appeal to ignorance.
If the appeal is relevant, the argument is non-fallacious
If the appeal is irrelevant, the argument is fallacious

Ex:

A cure for cancer hasn't been found
Therefore, no cure for cancer can be found

This is fallacious
Ex2:

The existence of ghosts has not yet been proved
Therefore, ghosts do not exist

This is also fallacious

Ad ignoranciam fallacies confuse refutations in the strong sense with refutations in the weak sense

You are unable to provide proof of your thesis
Therefore, your thesis must be false

Autoepistemic Reasoning:
In contrast, some ad ignorantiam arguments are valid

If p were the case, I would know that p
But I don't know that p

Therefore, it is not the case that p

Ex:

If I am in the middle of a blizzard, then I would know that I am in the middle of a blizzard
But I do not know that I am in the middle of a blizzard

Therefore, it is not the case that I am in the middle of a blizzard

Another non-fallacious ad ignorantiam
Sometimes ignorance of the consequences of actions or policies also justifies caution in decision-making
Ex:

We are currently ignorant of the biological and social consequences of human cloning

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Therefore, human cloning ought not to be permitted at the present time

Logical Necessities
A proposition is *logically necessary* (or a logical truth) if it is true regardless of how the world might be
Ex: Socrates was born in Athens or it's not the case that Socrates was born in Athens

Logical impossibilities:
A proposition is logically impossible (or a logical falsehood) if it is false regardless of how the world might be
Ex: Socrates was born in Athens and it's not the case that Socrates was born in Athens

Logical Contingency:
A proposition is logically contingent if it is neither logically necessary nor logically impossible
Ex: Socrates was born in Athens
Ex: If Socrates was born in Athens then his father was Greek

The Fallacy of Complex Questions The fallacy of complex question occurs whenever

- a question contains a hidden, illicit, or unsupported assumption
- it involves two or more questions rolled into one assumption
- it is misleading because it makes it difficult for a respondent to counter false or unjustified presuppositions

Ex: have you stopped beating your dog?

Safe and risky questions:
Questions ask respondents to select between a series of alternative propositions. These alternative propositions are the question's *direct answers*. Any proposition implied by all of a question's direct answers is a *presupposition* of that question. A question is *safe* if all its presuppositions are logically necessary. A question is risky if it is not safe.
Safe questions cannot mislead us since none of their presuppositions can be false. A question is

moderately safe if all of its presuppositions are true. Moderately safe questions will not normally mislead us since none of their presuppositions are false.

Ex: Is the ambassador to Australia or is she the ambassador to New Zealand?
Direct answers:

- She is the ambassador to Australia
- She is the ambassador to New Zealand

Sample presupposition: She is the ambassador to Australia or she is the ambassador to New Zealand
Evaluation: This question is risky since some of its presuppositions are not necessarily true.

Ex2: Is she the ambassador to Australia or not?
Direct answers:

- She is the ambassador to Australia
- It is not the case that she is the ambassador to Australia

Sample presupposition: She is the ambassador to Australia or it is not the case that she is the ambassador to Australia.
Evaluation: This question is safe since all its presuppositions are necessarily true

Ex3: Is the king of the U.S.A. is coming to UBC next week?
Direct answers:

- Yes, the king of the US is coming to UBC next week
- No, the king of the US is not coming to UBC next week

A presupposition: At least, the king of the US exists now
Evaluation: question is not safe

Ex4: Is the queen of England coming to UBC next week?
Direct answers:

- Yes, the queen of England is coming to UBC next week
- No, the queen of England is not coming to UBC next week

A presupposition: Queen of England exists, but it is not necessarily true. (England may not have a queen at that time)
Evaluation: It is risky, not safe, yet it is moderately safe

1.2 Elementary Logic

1.2.1 Entailment

An argument is an *entailment* iff (if and only if) its conclusion conclusively follows from its premises
A set of premises *entails* a conclusion iff the conclusion conclusively follows from the premises.

Ex:
All beagles are dogs
All dogs are mammals

Therefore, all beagles are mammals

Validity:
An argument or inference is valid iff it is not possible for the premises to be (jointly) true and, at the same times, the conclusion to be false.

Ex:
All horses are mammals
All mammals are warm-blooded

Therefore, all horses are warm-blooded

Inference from the premises to the conclusion is good so the argument is valid.

Ex2:
All people are horses
All horses have 3 heads

Therefore, all people have 3 heads

Despite the premises being false, the inference from the premises to the conclusion is good so the argument is valid.

Ex3:
If today is Monday then tomorrow is Tuesday
Tomorrow is Tuesday

Therefore, today is Monday

The logical skeleton is displayed as the following:
If p then q
q

Therefore, p

An example of this is:
If you are Canadian, then you understand English or French
You understand English or French

Therefore, you are Canadian

Validity is the matter of the argument's logical form only

Other invalid argument forms:
If p then q
Not p

Therefore, not q

Either p or q
Not p

Therefore, not q

Valid argument:
Either p or q
Not p

Therefore, q

If p then not q
Either p or q

Therefore, either not p or not q

Soundness:
An argument or inference is *sound* iff it is valid and it has true premises.

Impossibility vs. Improbability:
Recall that a proposition is *logically necessary* if it is true regardless of how the world might be and it is *logically impossible* if it is false regardless of how the world might be. It is *logically contingent* if it is neither one or the other.

A proposition is *logically impossible* if it is inconsistent with the laws of logic
A proposition is *physically impossible* if it is inconsistent with the laws of nature.
A proposition is *improbable* if it is (not impossible, but) unlikely to be true.

Propositional connectives are words or phrases which, together with one or more propositions, can be used to create new propositions

- ...and
- ...or
- if...then
- ...because

A proposition with no connectives is *atomic*
A proposition with one or more connectives is *molecular* or *compound.*

Propositional constants and variables
Propositional constants are expressed as $A, B, C, \ldots$ which stand for specific propositions which are either true or false.
Propositional variables are expressed as $a, b, c, \ldots$ and stand for arbitrary propositions which are neither true nor false.
(think constants vs. variables in math)

1.2.2 Conjunction, Disjunction, and Negation

Truth-functional connectives
A *truth-functional connective* is any connective for which the truth values of its resulting molecular propositions are determined solely by the meaning of the connective together with the truth values of its component propositions
Examples:

- $\wedge$ which is used to abbreviate the word "and"

- $\vee$ which is used to abbreviate one use of the word "or"
- $\sim$ which is used to abbreviate the phrase "it is not the case that"

Conjunction, Disjunction, and Negation

Conjunction:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$\sim p$
T	F
F	T

Ex: $(p\&q) \vee r$ vs $p\&(q \vee r)$

p	q	r	$(p\&q) \vee r$	$p\&(q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	T	F
F	F	F	F	F

1.2.3 Conditionals and Biconditionals

Material conditional and biconditional
Material conditional: if p then q

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

Material biconditional: checks if p and q have the same truth value (analogous to NXOR)

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

When we combine multiple logical statements, the *major connective* is the logical statement which is applied last.
Ex: for $(p\&q) \vee (r\&s)$, $\vee$ is the major connective

$\veebar$ denotes the exclusive disjunction (exclusive or).

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

Nor:

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

Nand:

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

1.2.4 Testing Arguments for Validity

Tautology, Contingency, Contradictions
Ex: $p \supset (q \supset p)$

p	q	$p \supset (q \supset p)$
T	T	T
T	F	T
F	T	T
F	F	T

This is called a *tautology* (a logical truth)
More examples:
$p \vee (\sim p)$
$p \supset p$
$\sim (p \supset q) \equiv (p\& \sim q)$
$(p \equiv q) \equiv ((p\&q) \vee (\sim p\& \sim q))$

Ex2: $(\sim p \equiv \sim q) \equiv (\sim p \equiv q)$

p	q	$(\sim p \equiv \sim q) \equiv (\sim p \equiv q)$
T	T	F
T	F	F
F	T	F
F	F	F

This is called a *self-contradictory claim* (a logical impossibility)
Some examples:
$p \& \sim p$
$\sim (p \vee \sim p)$
$(p \equiv q) \equiv \sim (p \equiv q)$

Ex3: $p \supset (p \supset q)$

p	q	$p \supset (p \supset q)$
T	T	T
T	F	F
F	T	T
F	F	T

This is called a *contingency*

Truth-table methods for testing validity

$p \supset q$
p
$\therefore q$

For the argument to be valid, it requires that the conclusion cannot be false if premises are true
Joint truth-table for the whole argument:

p	q	$p \supset q$	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

There are no counterexamples so it is a valid argument (called Modus Ponens, M.P.)

Ex:
If Sue is busy, then she is doing well financially
She is doing well financially
Therefore, Sue is busy
P= Sue is busy
Q= Sue is doing well financially

$P \supset Q$
Q
P

Joint truth-table:

p	q	$p \supset q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Row 3 is a counterexample so it is an invalid argument

Steps for testing for validity via truth tables

1. Identify the premises and conclusion
2. Identify all atomic propositions
3. Identify all truth-functional connectives
4. Formalize the argument
5. Design a truth table
6. Complete the truth table
7. Test for truth-functional validity by looking for a counterexample

Ex2:
If Bill throws the fight, Sue will reject him
If Bill doesn't throw the fight, the mob will take him for a ride
Either Bill will throw the fight or he will not

Therefore, Sue will reject Bill or the mob will take him for a ride
$P=$ Bill throws the fight
$Q=$ Sue will reject Bill
$R=$ The mob will take Bill for a ride
$P \supset Q$
$\sim P \supset R$
$P \vee \sim P$

$Q \vee R$
Joint truth-table:

p	q	r	$p \supset q$	$\sim p \supset r$	$p \vee \sim p$	$q \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

No counterexample so the argument is valid

Three theories about the meaning of "unless"

- Bill's dog will not come unless it's called

- Bill's dog will not come if it is not called
- If Bill's dog is not called, it will not come

"p unless q" iff "p if not q"
$\rightarrow$ "if not q then p"
$\sim q \supset p$

- Mary will be fired unless she shapes up
- Either Mary will shape up or she will be fired

"p unless q" iff "q or p"
$\rightarrow p \vee q$

- Bill will be late unless Sue phones him
- If Sue doesn't phone him Bill will be late and if Sue does phone him Bill won't be late

"p unless q" iff "(if not q then p) and (if q then not p)"
$\rightarrow (\sim q \supset p)\&(q \supset\sim p)$
$\sim q \equiv p$

Valid arguments with invalid forms:
If Sue has gone for a walk, then she has gone for a walk on the beach
She has gone for a walk on the beach
Therefore, Sue has gone for a walk
Has argument form
$p \supset q$
q This is valid because the two claims are not independent (they are related)
p

Fallacies of relevance
Many arguments are invalid because they commit the fallacy of *ignoratio elenchi* (their premises are not related to their conclusions)
Ex:
Bill loves Sue
Therefore, 2+2=4

1.2.5 Formal and Informal Logic

Grammatical vs Logical form
The *grammatical form* of a proposition (or of an argument) is the structure of the proposition (or argument) as indicated by the surface grammar of its natural language
The *logical form* of a proposition (or of an argument) is the logically effective structure of the proposition (or argument) as indicated by the meanings of the logical terms it contains
Ex: "Tom, Dick and Harry lifted the box"

Grammatical form: (Tom, Dick, Harry) lifted the box
Potential logical forms:
(Tom, Dick, Harry) lifted the box
(Tom lifted the box) and (Dick lifted the box) and (Harry lifted the box)

Is validity always a function of an argument's logical form?
Formalists claim that all logical properties can be explained using logical form alone
Anti-formalists claim that not all logical properties can be explained using logical form alone
Ex: Socrates is a father, therefore, Socrates is male
vs. Socrates is a father [All fathers are male], therefore, Socrates is male

Uniform substitutions:
If you statement depends on a variable, such as x, the substitution must be uniform. i.e. if you have several occurrences of x in the same instance, you must substitute the same value for all occurrences of that variable.

Arithmetic:
Numbers: 1,2,3,...
Variables: $x, y, z, \ldots$
Logic:
Atomic Propositions: P, Q, R
Compound/Molecular Propositions: $(P\&Q), \sim R \supset \sim P, \ldots$
Propositional Variables: p, q, r
Propositional Forms: $(p\&q), \sim r \supset \sim p, \ldots$
Note: when doing substitutions, propositions must be well-formed (w.f.f.)
Ex: P is wff
Ex2: $\sim (P\&Q)$ is wff
Ex3: $(P\&Q$ is not wff
Ex4: $\sim \&P$ is not wff
Ex5: $(R \sim P) \vee Q))$ is not wff

Ex: Start with a propositional form: $(p\&q) \vee \sim p$
Substitution 1: $p \to P$ and $q \to Q$
gives $(P\&Q) \vee \sim P$
Substitution 2: $p \to \sim P$ and $q \to \sim Q$
gives $(\sim P\& \sim Q) \vee \sim\sim P$
Note the result is not $(\sim P\& \sim Q) \vee P$. We don't simplify
For any given proposition, there exist infinitely many logically equivalent formulas which are not identical to the original one.

Ex: consider the case P unless Q. It can be written as $P \vee Q$, $\sim Q \supset P$, or $\sim P \supset Q$

Formal logic studies the formal (or structural) attributes of propositions that affect validity and other logical properties and obtains a proposition's logical form by uniformly replacing its non-logical terms with variables

Informal logic studies the informal attributes of propositions that affect validity and other logical properties.

Begging the question:
This is a type of argument in the broad sense. It occurs whenever an arguer uses as a premise of his argument any proposition that his opponent presently rejects. (also called the fallacy of *petitio principii*
Moral: One does not defeat an opponent simply by mouthing propositions he already disagrees with

Ex: Student: You can't give me a C!
Prof: Oh, I thought your paper sort of suggested the opposite... Why do you think so?
Student: Why, I'm an A student

Arguing in a circle:
Circular arguments are a type of argument in the narrow sense. It occurs whenever an argument's conclusion simply repeats a premise, or asserts a proposition contained within or that is equivalent to, a premise.
Note: because an opponent is always likely to reject a premise that simply assumes (or presupposes) the very proposition that is supposed to be proved, arguing in a circle is one (main) way of begging the question.

Ex: Sue: Natural selection, roughly, is a theory that only the "fittest survive"
Bill: Yes, that's what I often hear. But I don't really understand what the predicate "fittest" mean. How do you define the individuals who are the "fittest"?
Sue: Well, clearly, these are the ones that leave the most offspring.
Bill: Hold on a second! Doesn't that "leave the most offspring" mean exactly the same thing as those who survive?
Sue's reply assumes that the fittest individuals leave the most offspring, but she defined the fittest individuals as those that leave the most offspring

Sextus' Puzzle:
Are all valid arguments circular?
Do they assume the very proposition that they are trying to prove? If not, how can they guarantee their conclusions?

The fallacy of equivocation:
The fallacy of equivocation occurs whenever an argument depends inappropriately on a semantic ambiguity or whenever a semantic ambiguity plays a significant but inappropriate role in an argument.

Ex: Criminal actions are illegal, and all murder trials are criminal actions, thus all murder trials are illegal.
Here the term "criminal actions" is used with two different meanings.

Ex2: The end of a thing is its perfection. Death is the end of life. Therefore, death is the perfection

of life
Here the equivocation on the word “end” (i.e. goal versus termination) making four possible interpretations.

1. The *goal* of a thing is its perfection (T)
   Death is the *goal* of life (F)
   Therefore, death is the perfection of life (F)

2. The *termination* of a thing is its perfection (F)
   Death is the *termination* of life (T)
   Therefore, death is the perfection of life (F)

3. The *goal* of a thing is its perfection (T)
   Death is the *termination* of life (T)
   Therefore, death is the perfection of life (F)

4. The *termination* of a thing is its perfection (F)
   Death is the *goal* of life (F)
   Therefore, death is the perfection of life (F)

The fallacy of Amphiboly
The fallacy of ampliboly occurs whenever an argument depends inappropriately on a grammatical, rather than a purely semantic ambiguity or whenever a grammatical ambiguity plays a significant but inappropriate role in an argument

Ex: Thrifty people save old cardboard boxes and waste paper
Therefore, thrifty people waste paper
Waste can have two meanings so it can have the structures
$\dfrac{p \wedge q}{q}$ or $\dfrac{p \wedge q}{r}$

1.3 Formal Deductive Systems

1.3.1 Classification of Syetem P and Formal Logic

Idea of a proof:
Given A and $A \supset (B\&C)$, can we prove C?

$$\begin{array}{l} A \\ A \supset (B\&C) \\ \hline C \end{array}$$

The first inference has this form:

$$\begin{array}{l} q \supset q \\ p \\ \hline \therefore q \end{array} \quad \text{with } p \to A \text{ and } q \to (B\&C)$$

This form of reasoning is called *Modus Ponens* (MP)
MP has no counterexamples in its truth table so it is a valid inference

Formal Logical System
A natural language (e.g. English):

The alphabet is the set $\{a, A, b, B, \ldots, z, Z\}$
Words: {apple, red,...}
An artificial language (Logical system P):
Vocabulary (alphabet) = {A,B,C,...,&, $\vee, \sim, \supset, \equiv,$ (,)}
Well-Formed-Formulas (wff) (words) = $\{A, \sim B, (A \supset B), \ldots\}$

3 Formation rules of system P:

1. $A, B, C, \ldots$ are wffs
2. If p, q are wffs then so are $\sim p, (p \& q), (p \vee q), (p \supset q), (p \equiv q)$
3. Np other formulas are wffs

So the set of all wffs of System P, $\sum$, is
$\sum = \{A, B, C, \ldots, \sim A, \sim B, \sim C, \ldots, (A \& B), (B \vee D), (A \equiv Z), \ldots, \sim (B \vee D), \ldots\}$

The *primitive basis* for a formal system (or logic system) has two parts:
An *object language*, defined by a vocabulary and a grammar (or formation rules)
A *logic* defined by a (possibly empty) set of axioms and a set of transformation rules (or rules of inference)
Propositions derived from the axioms by means of the rules of inference are called the theorems of the formal system.

The primitive basis for System P:
Vocabulary

- an infinite number of propositional constants: $A, B, C, \ldots$
- 5 propositional connectives: $\sim, \vee, \&, \supset, \equiv$
- 2 grouping indicators: ()

Grammar:

- three formation rules

Axioms:

- none

Transformation Rules:

- 10 conditional rules
- 10 biconditional rules
- 2 hypothetical rules
- truth-tables

Conditional Transformation Rules

- MP: (modus ponens)
  $$\frac{\begin{array}{l} p \supset q \\ p \end{array}}{\therefore q}$$
- MT:
  $$\frac{\begin{array}{l} p \supset q \\ \sim q \end{array}}{\therefore \sim p}$$
- Conjunction
  $$\frac{\begin{array}{l} p \\ q \end{array}}{\begin{array}{l} \therefore \\ p \& q \end{array}}$$
- Simplification
  $$\frac{p \& q}{\therefore p} \text{ or } \frac{p \& q}{\therefore q}$$
- Addition
  $$\frac{p}{p \vee q}$$
- Deductive Syllogism (DS)
  $$\frac{\begin{array}{l} p \vee q \\ \sim p \end{array}}{\therefore q} \text{ or } \frac{\begin{array}{l} p \vee q \\ \sim q \end{array}}{\therefore p}$$
- Hypothetical Syllogism (HS)
  $$\frac{\begin{array}{l} p \supset q \\ q \supset r \end{array}}{\therefore p \supset r}$$
- Repetition
  $$\frac{p}{\therefore p}$$
- Constructive Dilemma (CD)
  $$\frac{\begin{array}{l} p \supset r \\ q \supset s \\ p \vee q \end{array}}{\therefore r \vee s}$$
- Distructive Dilemma (DD)
  $$\frac{\begin{array}{l} p \supset r \\ q \supset s \\ \sim r \vee \sim s \end{array}}{\therefore \sim p \vee \sim q}$$

Ex: Prove that the following argument is valid:

$(A\& \sim B) \supset C$

A

$\sim B$

$A\&C$

step by step

1. $(A\& \sim B) \supset C$
2. A
3. $\sim B$

---

4. $A\& \sim B$ by conjunction of 2 and 3
5. C by MP of 1 and 4
6. $A\&C$ by conjunction of 2 and 5

Using a truth-table you can confirm that there are no counter examples, therefore, the inference is valid

Any argument can be proved to be valid in terms of truth tables iff there exists a proof of this argument in terms of rules of inference.

Ex2: Prove that $\begin{array}{l} A \\ A \supset B \\ B \supset C \\ \hline \therefore C \vee D \end{array}$ is valid

We can prove that this is valid using truth tables but this is time consuming. It is much quicker using transformation rules.

1. A
2. $A \supset B$
3. $B \supset C$

---

4. B from MP on 2,1
5. C from MP on 3,4
6. $C \vee D$ from addition on 5

For any valid argument there exist (no less than) infinitely many correct proofs using transformation rules

Ex3: prove $W \vee D$

1. A
2. $(A \vee B) \supset\sim C$
3. $\sim C \supset D$

---

4. $(A \vee B) \supset D$ by HS 2,3
5. $A \vee B$ by addition 1
6. D by MP 4,5
7. $W \vee D$ by addition 6

Ex4: prove D

1. $(A \vee B) \supset (C\&D)$
2. $C \supset E$
3. $A\& \sim E$

---

4. A by simp 3
5. $\sim E$ by simp 3
6. $\sim C$ by MT 2,5
7. $A \vee B$ by add 4
8. $C \vee D$ by MP 1,7
9. D by DS 8,6

Ex5: prove $\sim D$

1. $(\sim A\& \sim B) \supset (\sim C\vee \sim D)$
2. $(E\vee \sim F) \supset\sim A$
3. $\sim H \supset (B \supset J)$
4. $(\sim F\& \sim H) \supset (\sim\sim C\& \sim J)$
5. $\sim H\&(F \supset H)$

---

6. $\sim H$ simp 5
7. $F \supset H$ simp 5
8. $\sim F$ MT 7.6
9. $B \supset J$ MP 3,6
10. $\sim F\& \sim H$ conj 8,6
11. $\sim\sim C\& \sim J$ MP 4,10
12. $\sim\sim C$ simp 11
13. $\sim J$ simp 11
14. $\sim B$ MT 9,13
15. $E\vee \sim F$ add 8
16. $\sim A$ MP 2,15
17. $\sim A\& \sim B$ conj 16,14
18. $\sim C\vee \sim D$ MP 1,17
19. $\sim D$ DS 18,12

Conditional Rules guidelines:

- Allows us to introduce a new line in a proof on the basis of one or more previous lines
- Indicate the logical form of both the line being introduced and the previous lines used as justification
- Can be applied to the whole formula in a line, not to its smaller subformulas

Biconditional Rules:

- Double Negation (DN)
  $p ::\sim\sim p$
  This :: means that the left-hand-side is logically equivalent to the right-hand-side. This allows us to substitute one for the other in a proof
- Communication (Comm)
  $p \vee q :: q \vee p$ or $p\&q :: q\&p$

- Distribution
  $p\&(q \vee r) :: (p\&q) \vee (p\&r)$ or $p \vee (q\&r) :: (p \vee q)\&(p \vee r)$

- Contraposition
  $p \supset q :: \sim q \supset \sim p$

- Tautology
  $p :: p\&p$ or $p :: p \vee p$

- Implication
  $p \supset q :: \sim p \vee q$

- Association
  $p\&(q\&r) :: (p\&q)\&r$ or $p \vee (q \vee r) :: (p \vee q) \vee r$

- DeMorgan's Laws
  $\sim (p\&q) :: \sim p \vee \sim q$ or $\sim (p \vee q) :: \sim p \& \sim q$

- Exportation
  $(p\&q) \supset r :: p \supset (q \supset r)$

- Equivalence
  $p \equiv q :: (p \supset q)\&(q \supset p)$ or $p \equiv q :: (p\&q) \vee (\sim p \& \sim q)$

Ex: prove $\sim\sim C \vee A$

1.	$A \vee (B\&C)$
2.	$(A \vee B)\&(A \vee C)$
3.	$A \vee B$
4.	$A \vee C$
5.	$C \vee A$
6.	$\sim\sim C \vee A$

Ex2: Prove $\sim Q \supset P$

1.	$P \vee Q$
2.	$\sim\sim P \vee Q$ DN 1
3.	$\sim P \supset Q$ Impl 2
4.	$\sim P \supset \sim\sim Q$ DN 3
5.	$\sim Q \supset P$ Contra 4

$\phi \vDash \Psi$ iff the argument $\dfrac{\phi}{\therefore \Psi}$ is truth-functionally valid. (called ϕ entails Ψ)

$\phi \vdash \Psi$ is ϕ implies Ψ and is iff there exists a proof of the argument in terms of transformation rules.

$\phi \vDash \Psi$ iff $\phi \vdash \Psi$

Ex3: prove Q

1. $P \equiv\sim Q$
2. $\sim (P \vee S)$

---

3. $(P \supset\sim Q)\&(\sim Q \supset P)$ equiv 1
4. $P \supset\sim Q$ simp 3
5. $\sim Q \supset P$ simp 3
6. $\sim P\& \sim S$ DeM 2
7. $\sim P$ simp 6
8. $\sim S$ simp 6
9. $\sim\sim Q$ MT 5,7
10. Q DN 9

Ex4: prove U

1. $P \vee Q$
2. $Q \supset (R\&S)$
3. $P \supset (U \vee W)$
4. $\sim (S \vee W)$

---

5. $\sim S\& \sim W$ DeM 4
6. $\sim S$ simp 5
7. $\sim W$ simp 6
8. $\sim R\vee \sim S$ add 6
9. $\sim (R\&S)$ DeM 8
10. $\sim Q$ MT 2,9
11. P DS 1,10
12. $U \vee W$ MP 3,11
13. U DS 12,7

1.3.2 Conditional Proof

Ex: prove $A \supset C$

1. $(A \vee B) \supset (C\vee \sim D)$
2. D

---

3. $\sim (A \vee B) \vee (C\vee \sim D)$ imp 1
4. $(\sim A\& \sim B) \vee (C\vee \sim D)$ DeM 3
5. $(C\vee \sim D) \vee (\sim A\& \sim B)$ Comm 4
6. $((C\vee \sim D)\vee \sim A)\&((C\vee \sim D)\vee \sim B)$ Dist 5
7. $(C\vee \sim D)\vee \sim A$ simp 6
8. $(C\vee \sim D)\vee \sim B$ simp 6
9. $C \vee (\sim D\vee \sim A)$ assoc 7
10. $(\sim D\vee \sim A) \vee C$ comm 9
11. $\sim D \vee (\sim A \vee C)$ assoc 10
12. $\sim D \vee (A \supset C)$ imp 11
13. $\sim\sim D$ DN 2
14. $A \supset C$ DS 12,13

For arguments of $p \supset q$ we can assume p and analyze the assumption in the scope of assumption.
Ex: prove $A \supset C$

1. $(A \vee B) \supset (C \vee \sim D)$
2. D

---

3. A assp (CP)
4. $A \vee B$ add 3
5. $C \vee \sim D$ MP 1,4
6. $\sim\sim D$ DN 2
7. C DS 5,6
8. $A \supset C$ CP 3-7

Ex2: prove $(A \& E) \supset \sim K$

1. $(A \vee B) \supset \sim (C \vee D)$
2. $(\sim C \& E) \supset F$
3. $(F \vee H) \supset K$

---

4. $A \& E$ assp (CP)
5. A simp 4
6. E simp 4
7. $A \vee B$ add 5
8. $\sim (C \vee D)$ MP 1,7
9. $(\sim C \& \sim D)$ DeM 8
10. $\sim C$ simp 9
11. $\sim D$ simp 9
12. $\sim C \& E$ conj 10,6
13. F MP 2,12
14. $F \vee H$ add 13
15. $\sim K$ MP 3,14

Ex3: prove $(\sim B \supset \sim A) \supset (A \supset C)$

1. $A \supset (B \supset C)$

---

2. $\sim B \supset \sim A$ assp (CP)
3. A assp (CP)
4. $A \supset B$ contra 2
5. B MP 4,3
6. $B \supset C$ MP 1,3
7. C MP 6,5
8. $A \supset C$ CP 3-7
9. $(\sim B \supset \sim A) \supset (A \supset C)$ CP 2-8

Indirect Proof

Assume the opposite of the proof and prove a contradiction (i.e. $\phi \& \sim \phi$)

Ex: prove $\sim P$

1.	$P \supset Q$
2.	$(P\&Q) \supset R$
3.	$R \supset\sim Q$
4.	$\sim\sim P$ Assp (IP)
5.	P DN 4
6.	Q MP 1,5
7.	$P\&Q$ conj 5,6
8.	R MP 2,7
9.	$\sim Q$ MP 3.8
10.	$Q\& \sim Q$ Conj 6.9
11.	$\sim P$ IP 4-10

1.4 Non-Classical Logic

1.4.1 Non-Classical Propositional Logic

If an object x stands in the relation R to object y, we write $R(x, y)$, otherwise $\sim R(x, y)$

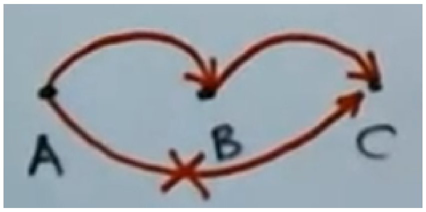
Def: R is reflexive iff for all x, $R(x, x)$
eg. $R(x, y)$ is "x is the father of y" is not reflexive
$R(x, y)$ is "x is identical to y" is reflexive

Def: R is symmetric iff for all pairs of objects x, y if $R(x, y)$ then $R(x, y)$
eg. R is "x is the father of y" is not symmetric
R is "x is identical to y" is symmetric

Def: R is transitive iff for all triangles of objects x, y, z if $R(x, y)\&R(y, z)$ then $R(x, z)$
eg. R is "x is the father of y" is not transitive
R is "x is to the left of y" is transitive
Def: R is equivalence iff R is reflexive, symmetric, and transitive.

Def: R is a partial order iff R is reflexive, transitive, and antisymmetric

System RP:
Relation R is written $R(p, q)$ and is read "p is related to q".
It holds between propositions that share a common subject matter
Ex: A="Plato was a student of Socrates"
B="Aristotle was a student of Plato"
C="Aristotle was a great logician"
Topics mentioned in A: {Plato, student, Socrates}
Topics mentioned in B: {Aristotle, student, Plato}
Topics mentioned in C: {Aristotle, logician}
$R(A, B)$ is true and $R(B, A)$ is true
$R(B, C)$ is true and $R(C, B)$ is true
Also $\sim R(A, C)$ and $\sim R(C, A)$



So the system in not transitive

p	q	$R(p,q)$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	F
F	F	T	T	T
T	T	F	F	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

p	$R(p,p)$	$\neg p$
T	T	F
F	T	T

p	q	$R(p,q)$	$p \wedge q$	$p \vee q$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	F
T	T	F	T	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

Ex: A=“Plato was a student of Socrates”
B=“Aristotle was a student of Plato”
C=“Aristotle was a great logician”
Is $\neg B \to (A \vee \neg C)$ true or false?
$R(A,B)$, $R(B,C)$, $\sim R(A,C)$
Topics in $\neg C$ are the same as topics in C so we have $\sim R(A, \neg C)$
So $A \vee \neg C$ is F
Topics in $A \vee \neg C$: topics in A + topics in $\neg C$ = {Plato, students, Socrates, Aristotle, logician}
$R(\neg B, (A \vee \neg C))$ is true
We have $\neg B$ is F and $A \vee \neg C$ is F so $\neg B \to (A \vee \neg C)$ is T

Ex2: Is the formula $(p \to q) \to (\neg p \vee q)$ a tautology, contingency, or contradiction?
This would be done with a truth table

Ex3:

$A \to C$
$B \to D$
$\neg C \vee \neg D$

$\therefore \neg A \vee \neg B$
$A \to C$ is T so $R(A, C)$ is T
$B \to D$ is T so $R(B, D)$ is T
$\neg C \vee \neg D$ is T so $R(C, D)$ is T
but $R(A, B)$ is F so $\neg A \vee \neg B$ is F

Aristotle's problem of future contingents:
Ex: "Athens will win the sea battle tomorrow"
We can analyze cases like this using Lucasiewicz's Logic
This introduces a third truth value, I, meaning indeterminate/unknown
Analyzing these cases,

p	q	$p\&q$	$p \vee q$	$p \supset q$	$p \equiv q$
T	I	I	T	I	I
F	I	F	I	T	I
I	T	I	T	T	I
I	F	F	I	I	I
I	I	I	I	T	T

Bochvor's System:
This deals with paradoxical systems and any operation involving and I will result in I.

1.4.2 Term Logic

Subject + Predicate
Ex: Victoria is the capital of BC
Victoria is the subject and "is the capital of BC" is the predicate
This refers to a single individual object
Called *singular sentence*

Ex2: Dogs are nice (creatures)
refers to a class/group/category// called *categorical sentence*
Def: A categorical sentence is one in which both subject and predicate are classes/sets/categories of objects that states an inclusion (or exclusion) relation between these two classes.

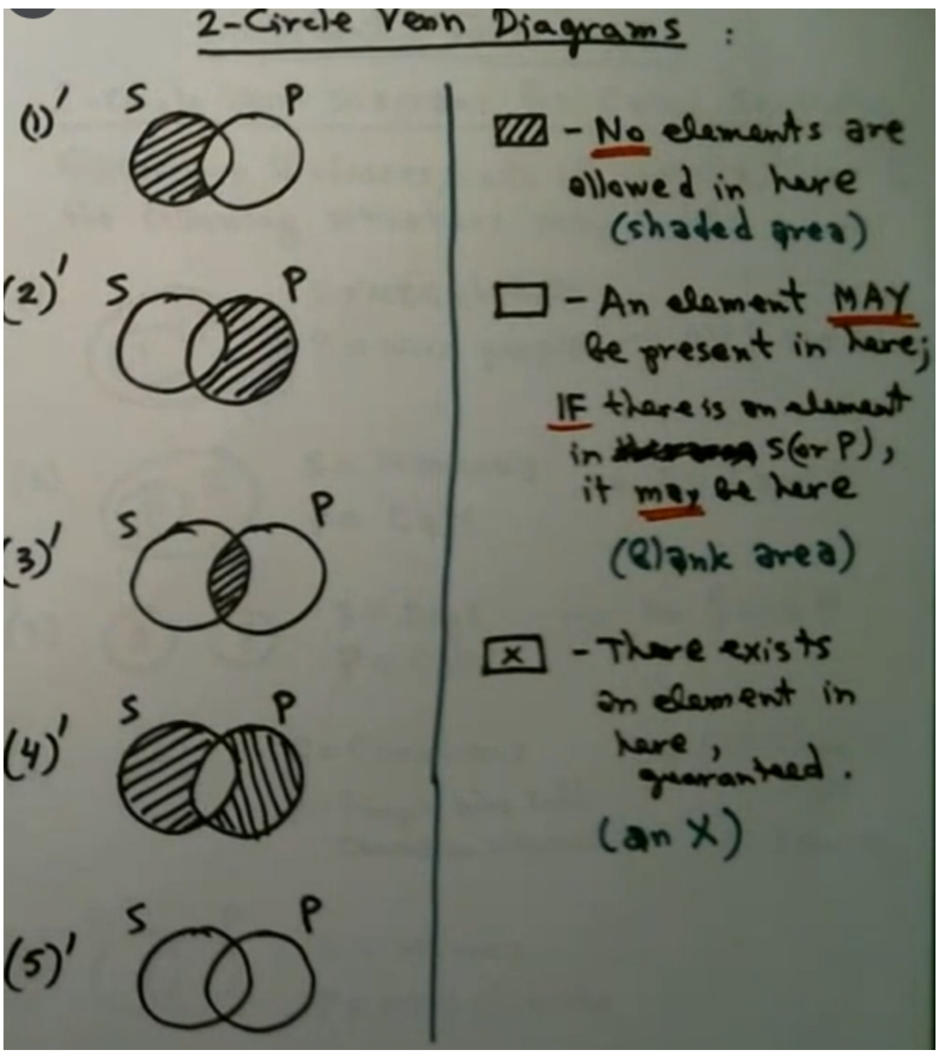
Term logic:
Capital letters stand for a description of a set/group of objects (with some property)
Ex: P="UBC students", Q="Dogs that like to chase cats", R="Nice creatures"

Ex: All dogs are mammals
dogs are the subject and "are" is called the *copula*. Mammals is the predicate class.
This is *total inclusion* (dogs is a subset of mammals) Ex2: No dogs are cats
This is *total exclusion* (the two sets have no overlap)
Ex3: Some dogs are US presidents
dogs is the subject, US presidents is the predicate class.

Some is defined as "There exists at least one"
This is *partial inclusion* (at least one object belongs to both sets)
Ex4: Some dogs are not UBC students
This is *partial exclusion* (at least one object does not belong to both sets)

Types of claims:

- A: All S are P (universal affirmative)
  If you're in S then you're in P. $(p \supset q)$
- E: No S are P (universal negative)
  If you're in S then you're not in P $(p \supset \sim q)$
- I: Some S are P (particular affirmative)
  There exists at least one S which is also in P
- O: Some S are not P (particular negative)



Def: The *converse* of a categorical proposition is obtained by interchanging the proposition's subject and predicate terms
E and I types have their converse equal to their original claims but this is not the case for A and O claims.

Immediate inference:
Ex:

$$\frac{\text{No senators are politicians}}{\text{No politicians are senators}} \text{ (valid, E-conversion)}$$

$$\frac{\text{Some books are valuable objects}}{\text{Some valuable objects are books}} \text{ (valid, I-conversion)}$$

$$\frac{\text{All students are hard workers}}{\text{All hard workers are students}} \text{ (invalid, A-conversion)}$$

$$\frac{\text{Some mammals are not dogs}}{\text{Some dogs are not mammals}} \text{ (invalid, O-conversion)}$$

The *contrapositive* of a categorical proposition is obtained by converting it and negating both of its two non-logical terms
A and O types have their contrapositives equal to their original claims but E and I claims do not.

Ex:

$$\frac{\text{All men are mortal creatures}}{\text{All non-mortal creatures are non-men}} \text{ (valid, A-contraposition)}$$

$$\frac{\text{Some people are not bankers}}{\text{Some non-bankers are not non-people}} \text{ (valid, O-contraposition)}$$

$$\frac{\text{No students are employees}}{\text{No non-employees are non-students}} \text{ (invalid, E-contrapostion)}$$

$$\frac{\text{Some objects are red things}}{\text{Some non-red things are non-objects}} \text{ (invalid, I-contraposition)}$$

The *obverse* of a categorical proposition is obtained by negating the predicate term and changing the proposition from affirmative to negative or from negative to affirmative.
i.e. All S are $P \to$ No S are non-P
Some S are $P \to$ Some S are not non-P
All A,E,I,O propositions are logically equivalent to their obverses

Contradiction
Given two contradictory propositions, at most one can be true and at most one can be false

Contrariety
Given two contrary propositions, at most one can be true, although both may be false

Subcontrariety
Given two subcontrary propositions, at most once can be false although both may be true

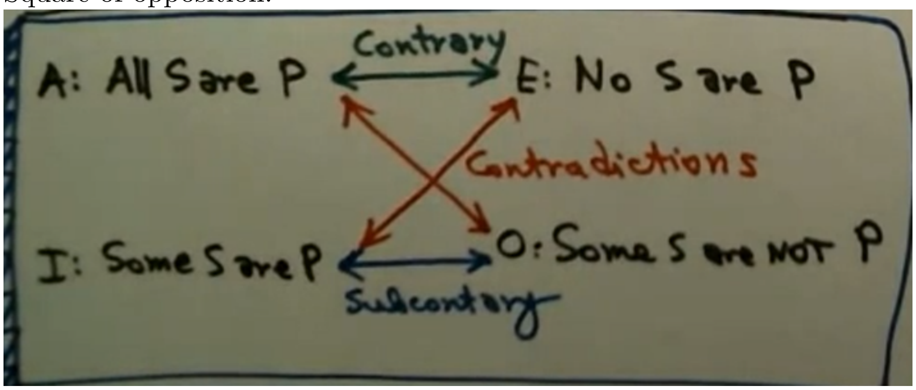
Ex: Contradiction

All human differences are determined by the environment
Not all human differences are determined by the environment

Ex: Contrariety
All human differences are determined by the environment
No human differences are determined by the environment

Ex: Subcontrariety
Some human differences are determined by the environment
Some human differences are not determined by the environment

Square of opposition:

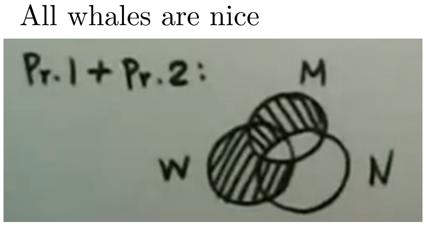


Ex: Not all people are cats
Not (all S are P)
= Some S are not P
Ex2: It's false that no people are students
Not (no S are P)
= Some S are P
Ex3: It's not true that all cats are not dogs
Not (no S are P)
= Some S are P

Syllogisms:
Def: A *syllogism* is an argument in which there are exactly 3 non-logical referring terms, there are exactly 3 categorical propositions, and each term appears in exactly 2 of these propositions.
Ex:

All whales are mammals
All mammals are nice

All whales are nice



Ex2:
All P are L
Some G are P

Some G are L
Def: the *major term* appears in the predicate of the conclusion
The *minor term* appears in the subject of the conclusion
The *middle term* appears in both of the premises but not in the conclusion

Def: A referring term is *distributed* iff that proposition says something about every member of that's term extension
A: All UBC students are people
So students would be distributed

In all universal claims (A,E) the subject is distributed
In all negative claims (E,O) the predicate is distributed

5 Rules for Validity of Syllogisms
All syllogisms must have the following:

- A middle term that is distributed at least once
- Major and minor terms that are distributed in their premises if they are distributed in the conclusion
- At least one affirmative premise
- A negative conclusion iff one of the premises is negative
  i.e a) A negative conclusion and exactly one negative premise or
  b) An affirmative conclusion and either 2 premises are negative or none of the premises are negative
  The total number of negations must not be odd
- A particular premise if the conclusion is particular