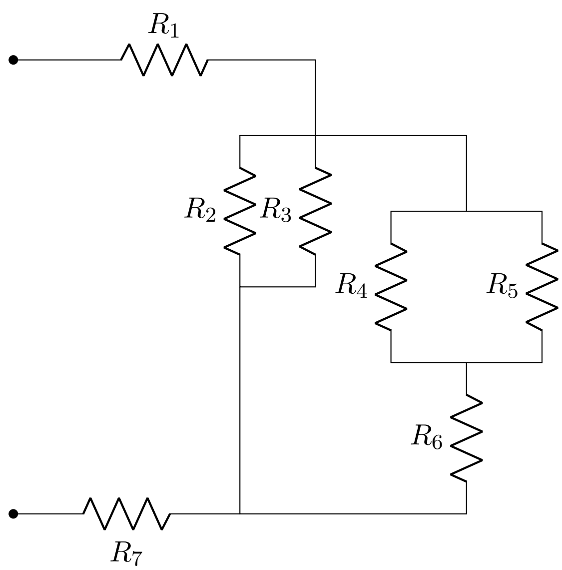
Physics 158 Circuits Problem Bank

Time Independent Circuits

Problem 15

For the following resistor configuration, find the equivalent resistance.
$R_1 = 2\,\Omega$, $R_2 = 8\,\Omega$, $R_3 = 3\,\Omega$, $R_4 = 10\,\Omega$, $R_5 = 12\,\Omega$, $R_6 = 7\,\Omega$, $R_7 = 4\,\Omega$.



Solution:
We can recognize that R_2 and R_3 are in parallel and so we can compute the equivalent as

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{8} + \frac{1}{3} = \frac{11}{24} \Rightarrow R_{23} = \frac{24}{11}$$

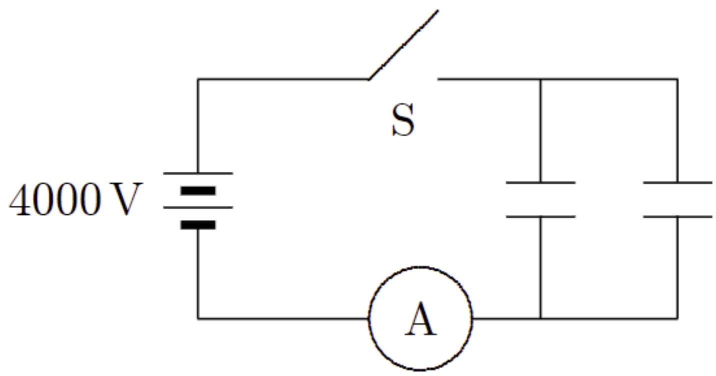
R_4 and R_5 are also in parallel and can be combined

$$\frac{1}{R_{45}} = \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{10} + \frac{1}{12} = \frac{11}{60} \Rightarrow R_{54} = \frac{60}{11}$$

Time Dependent Circuits

Problem 1

Each of the two $25\,\mu$F capacitors shown is initially uncharged. How many coulombs of charge pass through the ammeter A after the switch S is closed?



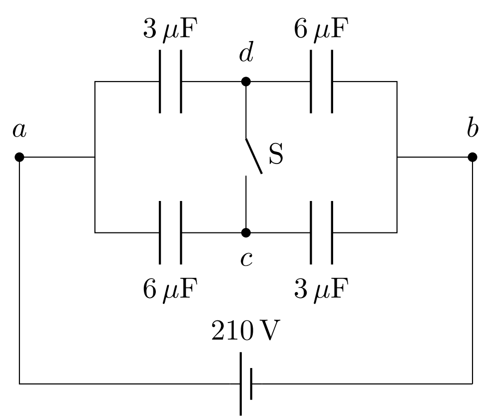
Solution:
When the switch is closed the two capacitors and power supply are all at 4000V.
The charge on each capacitor is $q = CV$ which in this case is

$$q = (25\,\mu\text{F})(4000\,\text{V}) = 0.1\,\text{C}$$

for each capacitor so a total of $\boxed{0.2\,\text{C}}$ flows through the ammeter

Problem 2

The capacitors in the figure are initially uncharged and are connected, as in the diagram, with the switch S open. The applied potential difference is $V_{ab} = 210\,\text{V}$



a) What is the potential difference V_{cd}

b) What is the potential difference across each capacitor after the switch S is closed?

c) How much charge flowed through the switch when it was closed?

Solution:
With the switch open each pair of $3.00\,\mu$F and $6\,\mu$F capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed each pair of $3\,\mu$F and $6\,\mu$F capacitors are in parallel with each other and the two pairs are in series.

a) With the switch open

$$C_{\text{eq}} = \frac{1}{\frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}} + \frac{1}{\frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}} = 4\,\mu\text{F}$$
$$Q_{\text{total}} = C_{\text{eq}} V = (4\,\mu\text{F})(210\,\text{V}) = 8.40 \cdot 10^{-4}\,\text{C}$$

By symmetry, each capacitor carries $4.20{\cdot}10^{-4}$ C. The voltages are then calculated via $V = \frac{Q}{C}$. This gives $V_{ad} = \frac{Q}{C_3} = 140\,\text{V}$ and $V_{ac} = \frac{Q}{C_6} = 70\,\text{V}$. We then get V_{cd} as

$$V_{cd} = V_{ad} - V_{ac} = \boxed{70\,\text{V}}$$

b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is

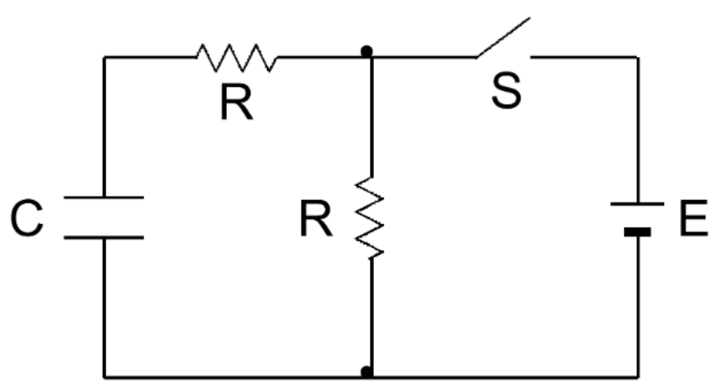
$$C_{\text{eq}} = \frac{1}{\frac{1}{(3{+}6)\,\mu\text{F}} + \frac{1}{(3{+}6)\,\mu\text{F}}} = 4.5\,\mu\text{F}$$
$$Q_{\text{total}} = C_{\text{eq}} V = (4.5\,\mu\text{F})(210\,\text{V}) = 9.5 \cdot 10^{-4}\,\text{C}$$

and each capacitor has the same potential difference of $\boxed{105\,\text{V}}$ (again, by symmetry).

c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\,\mu\text{C}$, so $\boxed{315\,\mu\text{C}}$ of charge flowed through the switch.
Note: It is better to compute the absolute charges on each plate before and after the switch is closed and then to follow the flow of the electrons through the switch.

Problem 3

In the circuit shown, both resistors have the same value R. Suppose switch S is initially closed for a very long time.



a) Find all currents initially.

b) Find all currents the instant after the switch is opened.

c) Find the time constants for both case switch open and switch closed.

Solution:

a) The switch has been closed for a long time so we can assume that the capacitor is fully charged. This will mean that it acts as a short-circuit so it will have no current flowing through it. So the current through branch 1 is $\boxed{I_1 = 0}$ and the current through branch 2 is $\boxed{\mathrm{I}_2 = \frac{\mathrm{E}}{\mathrm{R}}}$

b) The instant before the switch is opened we can use Kirchoff's loop law to state that the large outer loop must have a volatge that sums to 0. We know the current through the upper resistor is 0 from part a so then we can say that the voltage of the capacitor must equal that of the battery.
When the switch is opened, it removes the battery from the circuit so we just have the loop on the left. Using Kirchoff's law agian, we know that the voltage must sum to 0 so we can say that the voltage of the resistors must equal the voltage of the capacitor.

$$V_C = V_{R_1} + V_{R_2}$$

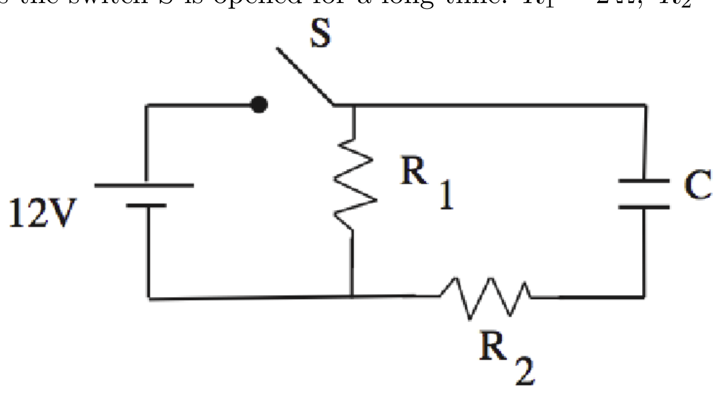
We know the resistors are the same and we know $V_C = E$ so we get $E = 2V_R = 2IR$. We can solve for the current to get that the current in the left loop is $\boxed{I = \frac{E}{2R}}$ and there is no current through the battery.

c) We can break this down into two cases:

- Charging the capacitor (after the switch is closed)
When the switch is closed, take the loop through the battery and C. This gives $\boxed{\tau = RC}$
- Discharging the capacitor (after the switch is opened)
When the switch is open, only the loop not containing the battery has current which gives a time constant of $\boxed{\tau = 2RC}$ because both resistors are included.

Problem 4

The circuit below has the switch S is opened for a long time. $R_1 = 2\,\Omega,\ R_2 = 4\,\Omega,\ C = 2\,\mathrm{F}$



a) The switch S is now closed. Find all currents just after the switch is closed.

b) Find all currents after the switch has been closed for a very long time.

c) After the switch was closed for a very long time it is opened again find the current through R_2 as a function of time.

Solution:

a) The capacitor will want to initially act as a wire so we can analyze the circuit as two resistors in parallel. Due to Kirchoff's loop law, we can say that each resistor must have a voltage drop of 12 V and we can get the current of each from Ohm's law:

$$I_1 = \frac{\varepsilon}{R_1} = \frac{12}{2} = \boxed{6\,\mathrm{A}}$$
$$I_2 = \frac{\varepsilon}{R_2} = \frac{12}{4} = \boxed{3\,\mathrm{A}}$$

b) After the switch has been closed for a long time, the capacitor will be fully charged and act as a short circuit. The circuit can then be analyzed as the loop going through the battery and R_1

$$I_1 = \frac{\varepsilon}{R_1} = \frac{12}{2} = \boxed{6\,\mathrm{A}}$$
$$\boxed{I_2 = 0\,\mathrm{A}}$$

c) After the switch is opened the current will flow through the loop containing R_1, R_2, and C. We can write the voltage loop equation as

$$0 = V_C + V_{R_1} + V_{R_2}$$

$$0 = \frac{q}{C} + iR_1 + iR_2$$

We know that $i = \frac{dq}{dt}$ and can take the derivative of both sides to get a 1st order differential equation and solve for $i(t)$

$$0 = \frac{i}{C} + \frac{di}{dt}(R_1 + R_2)$$
$$\frac{di}{dt} = -\frac{i}{(R_1 + R_2)C}$$
$$\frac{di}{i} = -\frac{dt}{(R_1 + R_2)C}$$
$$\int \frac{di}{i} = -\int \frac{dt}{(R_1 + R_2)C}$$
$$\ln|i| = -\frac{t}{(R_1 + R_2)C} + \text{Constant}$$
$$i = i_0 e^{-\frac{t}{(R_1+R_2)C}}$$

We can solve for the initial current by using our same voltage loop equation and knowing that the initial voltage across the capacitor is 12 V from the charge stored on it. The capacitor will be discharging so the potential in the equation can be thought of as negative.

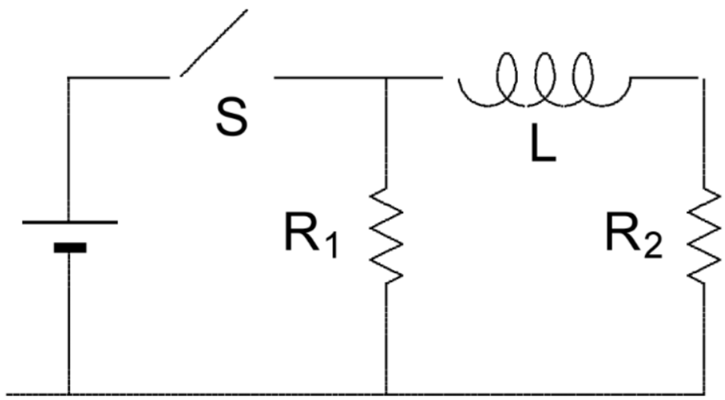
$$0 = V_C + i_0(R_1 + R_2)$$
$$0 = -12 + 6i_0 \Rightarrow i_0 = 2\,\mathrm{A}$$

Plugging this all in we get,

$$\boxed{i(t) = 2e^{-\frac{t}{12}}\ \text{Amps}}$$

Problem 5

When the switch S in the circuit shown is closed, the time constant for the growth of current in R_2 is?



Solution:
To find the time constant for the current through R_2, we can write out the voltage equation for a loop containing R_2. Let's choose the loop that contains the battery, L, and R_2. The equation will be

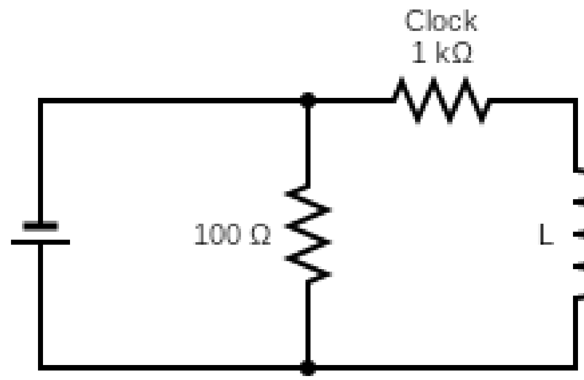
$$\varepsilon = -\frac{di}{dt}L + iR_2$$

If we isolate the $\frac{di}{dt}$ term then the coefficient of the i term will be $\frac{1}{\tau}$

$$\frac{di}{dt} = -\frac{\varepsilon}{L} + i\frac{R_2}{L}$$
$$\frac{R_2}{L} = \frac{1}{\tau}$$
$$\Rightarrow \boxed{\tau = \frac{L}{R_2}}$$

Problem 6

Using their newfound knowledge of LR circuits, a Phys 158 student came up with a clever idea for a prank. They want to design an alarm clock that will continue to play for 10 seconds after the battery is removed. The alarm clock can be thought of as a $1\,\mathrm{k\Omega}$ resistor which requires at least 1 Watt to operate. They designed the following circuit to achieve this.



a) What value should the battery be such that the power supplied to the clock does not exceed 3 Watts?

b) What value of inductor should they use so that the alarm clock remains on for 10 seconds after the battery is disconnected?

Solution:

a)

$$P_C = 3\,\mathrm{W}$$
$$P_C = i_C^2 R_C \Rightarrow i_C = \sqrt{\frac{P_C}{R_C}}$$
$$\varepsilon = V_C = i_C R_C = \sqrt{\frac{P_C}{R_C}} \cdot R_C = \sqrt{P_C R_C} = \sqrt{(3\,\mathrm{W})(1000\,\Omega)} = 54.8\,\mathrm{V}$$

b) We can start by writing out Kirchoff's loop voltage law for the circuit and then solving the resulting ODE to get an expression for the current as a function of time:

$$iR + iR_C + L\frac{di}{dt} = 0$$
$$L\frac{di}{dt} = -i(R + R_C)$$
$$\frac{di}{dt} = -\frac{R + R_C}{L} i$$
$$\frac{di}{i} = -\frac{R + R_C}{L} dt$$
$$\int \frac{di}{i} = -\frac{R + R_C}{L} \int dt$$

$$\ln|i| = -\frac{R+R_C}{L}t + \text{Constant}$$
$$i(t) = e^{-\frac{R+R_C}{L}t+\text{Constant}} = i_0 e^{-\frac{R+R_C}{L}t}$$

Alternatively, we can get the same expression by thinking about it conceptually and computing the time constant.
We know that the current will initially want to stay the same because of the inductor and will slowly decay to 0 so we can determine that the equation of the current should look like exponential decay and be of the form

$$i(t) = i_0 e^{-t/\tau}$$

We can then compute the time constant for an RL circuit as

$$\tau = \frac{L}{R_{eq}} = \frac{L}{R+R_C}$$

Plugging this in will yield the same expression as above.
The initial current, i_0, will be the current that was initially flowing through the inductor. We computed this in part (a) to be

$$i_0 = i_C = \sqrt{\frac{P_C}{R_C}} = \sqrt{\frac{3\,\mathrm{W}}{1000\,\Omega}} = 54.8\,\mathrm{mA}$$

Now we have a complete expression for the current as a function of time. We can get the power as a function of time as

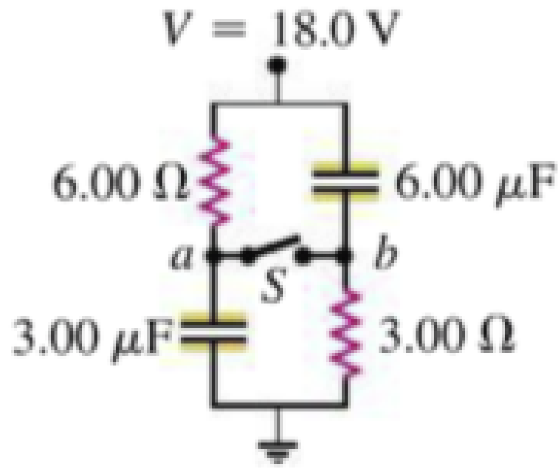
$$P(t) = i^2 R_C = i_0^2 R_C e^{-\frac{2(R+R_C)}{L}t}$$
$$i_0^2 R_C = \frac{P_C}{R_C} \cdot R_C = P_C$$
$$P(t) = P_C e^{-\frac{2(R+R_C)}{L}t}$$

We are told that the clock must have a minimum of 1 Watt and we want it to last for 10 seconds so we can set $P = 1$ and $t = 10$ and solve for L.

$$\frac{P}{P_C} = e^{-\frac{2(R+R_C)}{L}t}$$
$$\ln\left(\frac{P}{P_C}\right) = -\frac{2(R+R_C)}{L}t$$
$$L = -\frac{2(R+R_C)t}{\ln\left(\frac{P}{P_C}\right)} = -\frac{2(100\,\Omega+1000\,\Omega)(10\,\mathrm{s})}{\ln\left(\frac{1\,\mathrm{W}}{3\,\mathrm{W}}\right)} = 20,025\,\mathrm{H}$$

Problem 7

The switch has been open a very long time and the 18V power supply has been attached for a very long time.



a) Find the potential from a to b when the switch S is open.

b) Find the final charge on the two capacitors.

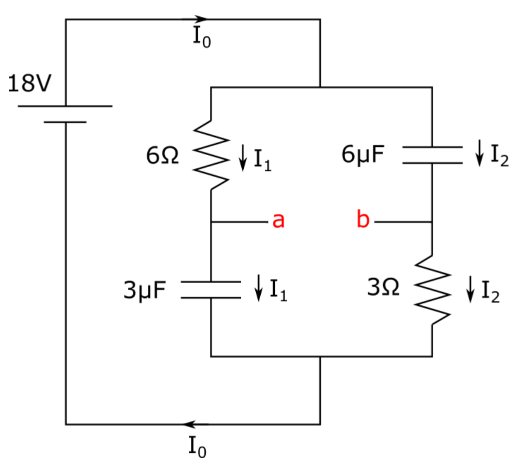
The switch is now closed

c) After a long time calculate the final charge on the capacitors.

d) How much charge flows through the switch in 300 seconds?

Solution:
Open Switch Case

$$I_0 = I_1 + I_2$$



a) after a long time both Capacitors are fully charged

$$\therefore I_1 = I_2 = 0$$

$$\therefore V_a = 18V, V_b = 0 \implies V_{ab} = V_a - V_b = 18V$$

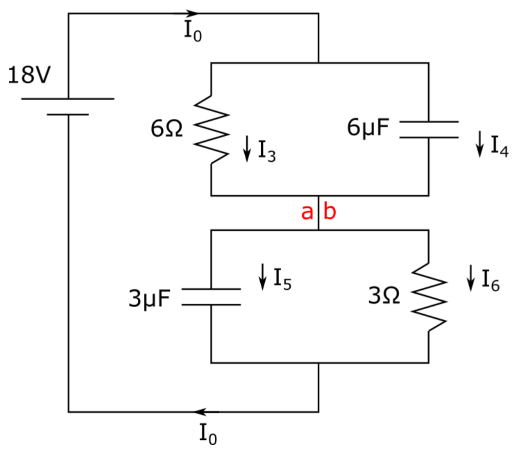
b) Hence

$$Q_{3\mu F} = C_1 V_{3\mu F} = (3\mu F)(18V) = 54\mu C$$

$$Q_{6\mu F} = C_2 V_{6\mu F} = 6\mu F(18V) = 108\mu C$$

Closed Switch Case ($\Longrightarrow$ $\underline{V_{ab} = 0}$)

$$I_0 = I_3 + I_4 = I_5 + I_6$$



c) after a long time Capacitors are charged

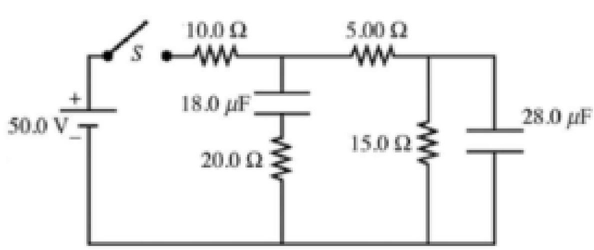
$$\Longrightarrow I_4 = I_5 \to 0, \therefore I_0 = I_3 = I_6$$
$$18V = I_3(6\Omega) + I_6(3\Omega) = I_3(9\Omega)$$
$$\Longrightarrow I_3 = I_6 = I_0 = 2\text{Amps}$$
$$\Longrightarrow V_6 = 2(6) = 12V$$
$$V_3 = 2(3) = 6V$$
$$\therefore V_{6\mu F} = V_{6\Omega} = 12V = \frac{Q_{6\mu F}}{6\mu F} \Longrightarrow \underline{Q_{6\mu F} = 72\mu C}$$
$$V_{3\mu F} = V_{3\Omega} = 6V = \frac{Q_{3\mu F}}{3\mu F} \Longrightarrow \underline{Q_{3\mu F} = 18\mu C}$$

d) the power supply (18V) is connected to the circuit all the time it can deliver current (and charge) forever.

$$Q_{Tot} = \int_0^{300} I(t)dt = (2A)(300s) = 600\text{Coulombs}$$

Problem 8

The circuit shown below initially has no charge on the capacitors and the switch S is originally open.



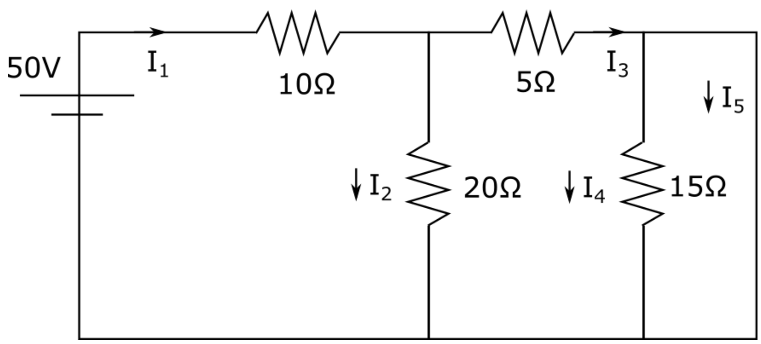
a) Just after closing the switch S, find all the currents.

b) After the switch has been closed for a very long time, find all the currents.

c) After the switch S has been closed for a very long time, find the potential difference across the 28.0 μF capacitor.

Solution:

a) just after we close the switch there is NO charge on the Capacitors. Hence we have the following circuit:



Note the Capacitors act like a wire when uncharged

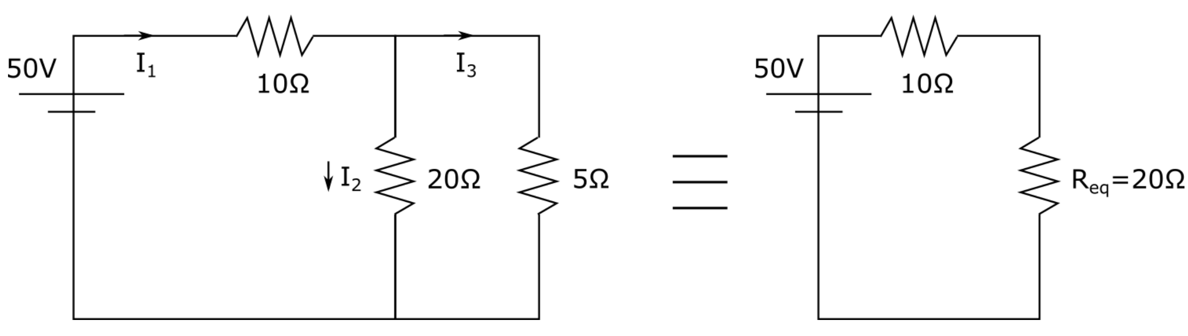
$K_1 \implies$ Hence $\underline{I_4 = 0}, I + 1 = I_2 + I_3, I_3 = I_4 + I_5 = I_5$

$K_2 \implies 50 - 10I_1 - 20I_2 = 0$

$0 = -5I_3 + 20I_2$

You now have 3 equations and 3 unknowns. Solve

EQUIVALENT Circuit



$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{5} = \frac{1}{20} + \frac{4}{20} = \frac{5}{20} = \frac{1}{4}$$

$$\rightarrow R_{eq} = 4\Omega$$

Hence $I_1 = \frac{50V}{14\Omega}$, Since $I_1 = I_2 + I_3$ and $20 I_2 = 5 I_3$ we have

$$\underline{I_1 = \frac{50}{14}\text{ Amps}} = 3.57\,A = I_2 + 4I_2 = 5I_2$$

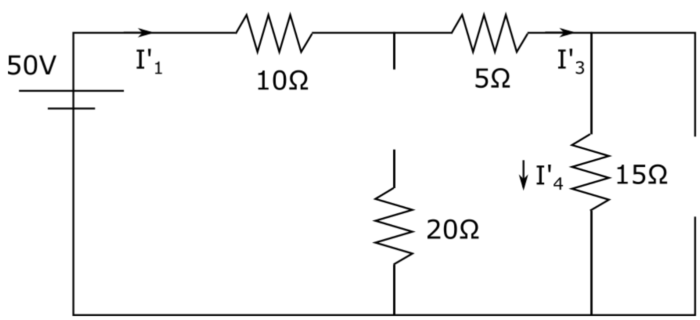
$$\therefore \underline{I_2 = \frac{10}{14}\text{ Amps}} = 0.714\,A$$

and finally

$$I_3 = 4I_2 = \underline{\frac{40}{14}\text{ Amps}} = 2.86\,A$$

b) after a long time the Capacitors are fully charged

$$\therefore I_2' = 0, I_5' = 0 \implies I_1' = I_3' = I_4'$$

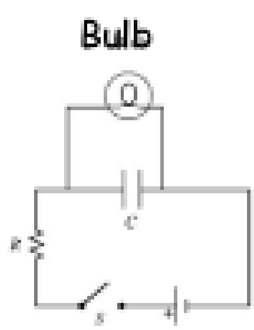


$$\therefore \underline{\frac{50V}{30\Omega} = I_1' = \frac{5}{3}\text{Amp}}$$

c) $V_{28\mu F} = V_{15\Omega} = I_4'(15\Omega) = \frac{5}{3}(15) = \underline{25V}$

Problem 9

A light bulb is connected in the circuit shown below – the switch S is initially open and the capacitor is is uncharged. The battery has no appreciable internal resistance and a voltage = 12 V. The resistance of the light bulb is 1000 Ω, $R = 50\,\Omega$ and $C = 100\,\mu$F . The switch is closed at $t = 0$.



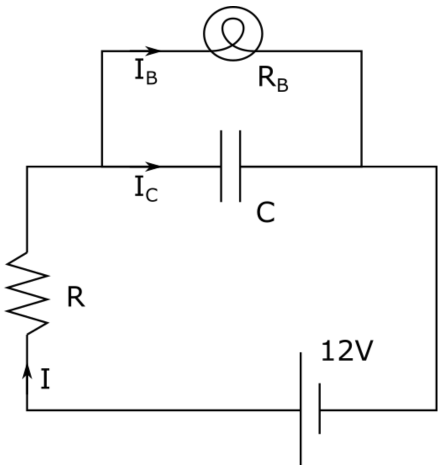
a) Find the all of the currents at $t = 0^+$.

b) Find all of the currents after a very long time.

c) Find the current in the bulb as a function of time.

d) Sketch the brightness of the bulb as a function of time.

Solution:
Light Bulb circuit:



$V = 12$Volts, $R_B = 1000\Omega$, $R = 50\Omega$, $C = 100\mu F$

$$K_1 \implies I = I_B + I_C$$
$$K_2 \implies V - IR - \frac{q_C}{C} = 0$$
$$V - IR - I_B R_B = 0$$

a) At $t = 0$ $q_C = 0$ so $V_C = 0$ and $V_C = V_B = 0 = I_B R_B$. Hence $I_B = 0$ and $\underline{I = I_C = \frac{V}{R} = \frac{12}{50}\text{Amps}}$.

b) As $t \to \infty$, $I_C \to 0$ so $I = I_B = \frac{V}{R+R_B} = \underline{\frac{12}{1050}A}$.

c) Now we study the time dependence of I, I_B, I_C. Consider the following equation from earlier $V - IR - q_C/C = 0$

$$\frac{d}{dr}\left(V - IR - \frac{1}{C}q_C\right) = 0 = 0 - R\frac{dI}{dt} - \frac{1}{C}\frac{dq}{dt}.$$

Hence

$$\frac{dq_C}{dt} = \underline{I_C = -RC\frac{dI}{dt}}.$$

Consider the following equation from earlier $I_C = I - I_B$ so $-RC\frac{dI}{dt} = I - I_B$

and finally use the equation $I_B = \frac{V-IR}{R_B}$ so that

$$-RC\frac{dI}{dt} = I - \frac{1}{R_B}(V - IR) = \frac{-V}{R_B} + I(1 + \frac{R}{R_B}).$$

Simplifying $\implies -RR_BC\frac{dI}{dt} = -V + I(R_B + R)$ or

$$\frac{dI}{dt} = \frac{V}{RR_BC} - \frac{I}{C}\left(\frac{R_B + R}{R_BR}\right)$$

$$\underline{\frac{dI}{dt} = \frac{V}{RR_BC} - \frac{I}{C}\left(\frac{1}{R} + \frac{1}{R_B}\right)}$$

this has a solution $I(t) = A\exp(-t/\tau) + B$ so

$$\frac{dI}{dt} = -\frac{A}{\tau}\exp(-t/\tau)$$

substituting and solving for A and B we find (using) $I(0) = \frac{V}{R}$

$$\implies A = \left(\frac{V}{R} - \frac{V}{R + R_B}\right),\ B = \left(\frac{V}{R + R_B}\right)$$

$$\therefore I(t) = V\left(\frac{1}{R} - \frac{1}{R + R_B}\right)\exp(-t/\tau) + \frac{V}{R + R_B}$$

$$\underline{I(t) = \frac{VR_B\exp(-t/\tau)}{R(R + R_B)} + \frac{V}{R + R_B}}$$

$$\rightarrow \tau = \frac{RR_BC}{R + R_B} \text{ from found equation for rate of change of current.}$$

Finally

$$I_C(t) = I(t) - I_B(t),\ \text{but}\ I_B(t) = \frac{V - IR}{R_B}.$$

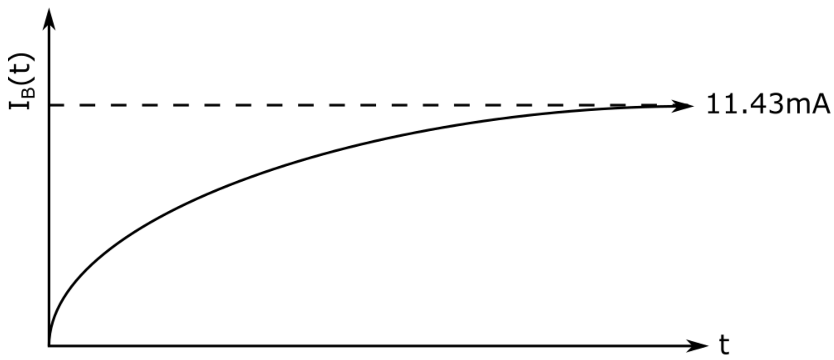
$$\underline{I_B(t) = \frac{V}{R_B} - \frac{R}{R_B}\left(\frac{V}{R + R_B}(1 + R_B\exp(-t/\tau)\right)}$$

Inserting the values $\tau = 4.76ms$, $\frac{1}{\tau} = 210s^{-1}$

$I_C(t) = 240\exp(-210t)mA$

$I_B(t) = 11.43mA(1 - \exp(-210t))$

d) See sketch below

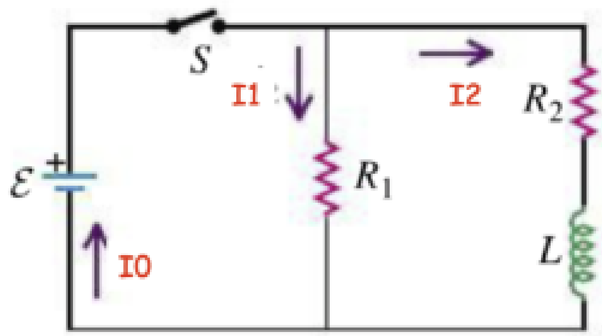


$P_B = I_B^2 R_B$

This problem is too difficult for any EXAM!!

Problem 10

The circuit below has been open for a very long time and then the switch is closed at $t = 0$.

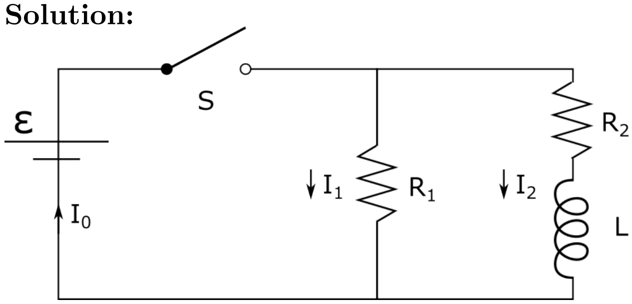


a) Find all of the currents at $t = 0^+$.

b) Find all of the currents after a very long time.

c) Find all of the currents as a function of time

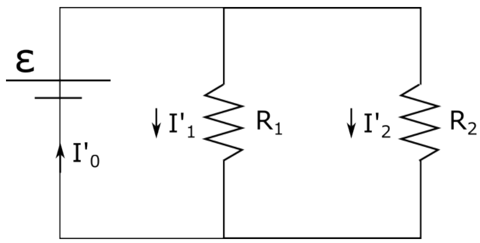
Solution:



a) Close switch at $t = 0$, then $I_2(0) = 0$ and the Inductor acts like a Battery
$I_0(0) = I_1(0)$

$$\mathcal{E} - I_1 R_1 = 0 \implies \underline{I_1 = \frac{\mathcal{E}}{R_1} = I_0}$$

b) As $t \to \infty$ the Inductor voltage $\to 0$ (becomes a wire)



Now $I'_0 = I'_1 + I'_2$:

$$I'_1 R_1 = I'_2 R_2 = \mathcal{E} = I$$

$$\therefore \underline{I'_1 = \frac{\mathcal{E}}{R_1},\ I'_2 = \frac{\mathcal{E}}{R_2},\ I'_0 = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

c) At any time t we have
$K_1 \implies I_0(t) = I_1(t) + I_2(t)$

$$K_2 \implies \begin{cases} \mathcal{E} - I_1 R_1 = 0 \\ \mathcal{E} - I_2 R_2 - L\frac{dI_2}{dt} = 0 \end{cases}$$

This last equation looks like a regular RL circuit, so its solution is $I_2(t) = A\exp(-t/\tau) + B$ with $\tau =$ time constant

$$I_2(\infty) \to \frac{\mathcal{E}}{R_2} = B,\ I_2(0) = A + B = 0 \implies A = \frac{\mathcal{E}}{R_2}$$

so $I_2(t) = -\frac{\mathcal{E}}{R_2}\exp(-t/\tau) + \frac{\mathcal{E}}{R_2} = \frac{\mathcal{E}}{R_2}(1 - \exp(t/\tau)$.

To find τ we use the aforementioned equation that looks like a regular RL circuit

$$\mathcal{E} - R_2\left(\frac{\mathcal{E}}{R_2}(1 - \exp(-t/\tau)\right) - \frac{L\mathcal{E}}{R_2}\left(\frac{1}{\tau}\exp(-t/\tau)\right) = 0.$$

$$\therefore \mathcal{E} - \mathcal{E} + \mathcal{E}\exp(-t/\tau) - \frac{L\mathcal{E}}{R_2}\frac{1}{\tau}\exp(-t/\tau) = 0,$$

and all $\mathcal{E}$ can be divide out, resulting in

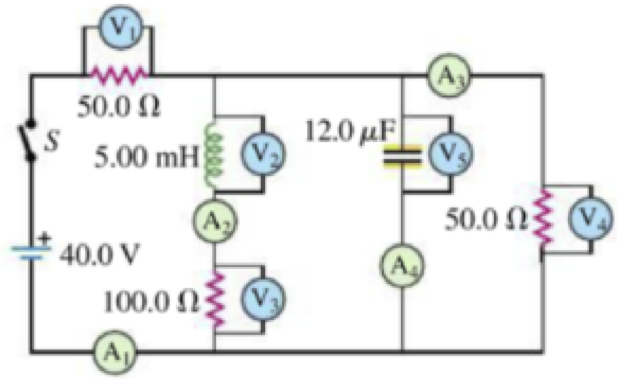
$$\left(1 - \frac{L}{R_2}\frac{1}{\tau}\right) = 0 \implies \tau = \frac{L}{R_2}$$

$$\implies \underline{I_2 = \frac{\mathcal{E}}{R_2}(1 - \exp(-tR_2/\tau))}$$

$$\underline{I_1 = \frac{\mathcal{E}}{R_1} \; I_0 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2}(1 - \exp(-tR_2/L))}$$

Problem 11

The switch S in has been open a very long time and then it is closed at $t = 0$. $Q_12(0) = 0$.

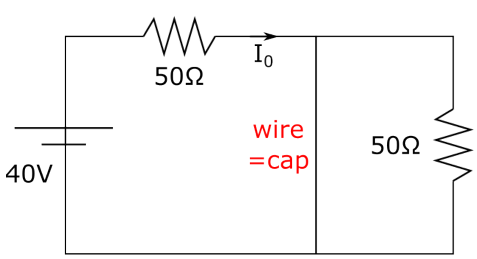


a) Calculate all the voltage readings and all of the currents at $t = 0^+$.

b) After the switch has been closed for a very long time calculate all the voltage and current values.

(The circles represent voltmeters and ammeters.)
Solution:

a) At $t = 0$, $V_C = 0$, $I_{ind} = 0 \implies$ Equivalent Circuit



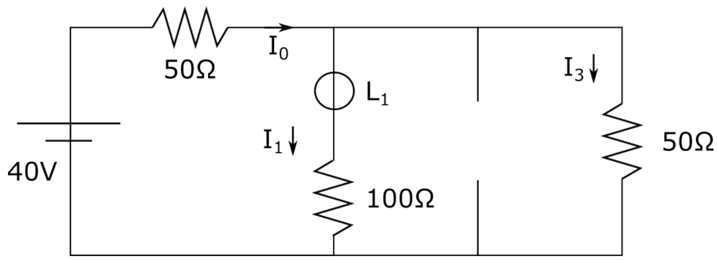
$12.0\mu F$ capacitor shorts the second 50Ω resistor on RHS

$40V = I_0(50\Omega) \implies \underline{I_0 = 0.8\text{Amps}}$

Hence $A_1 = 0.8A = A_4,\ A_2 = A_3 = 0,\ V_1 = 40V,\ V_2 = 0 = V_3 = V_4$

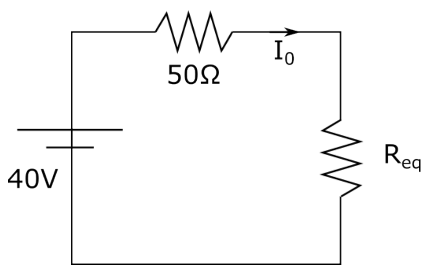
b) as $t \to \infty$ Capacitors are fully charged $I_C \to 0$ and the Inductor becomes a WIRE

Equivalent Circuit $\Longrightarrow$



$$\frac{1}{R_{eq}} = \frac{1}{100} + \frac{1}{50} = \frac{3}{100}$$

$$\to R_{eq} = 33.3\Omega$$

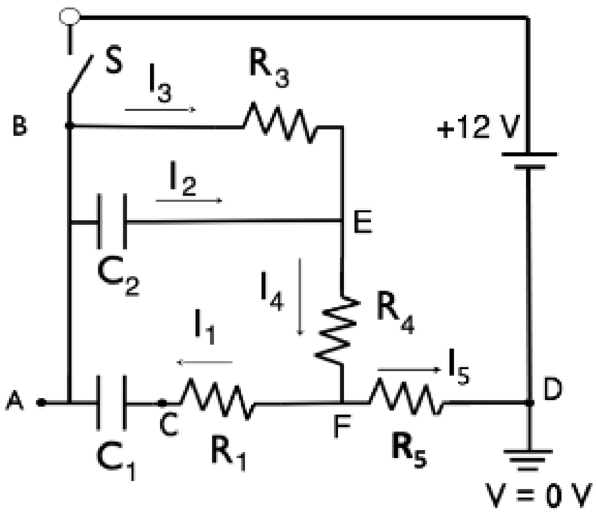


$I_0 = \frac{40}{83.3} = \underline{0.48\text{Amps}}$
$I_0 = I_1 + I_2 \qquad I_2 = I_3$
$100I = 50I_3 \implies I_3 = 2I_1$
$I_0 = I_1 + I_3 = 3I_1 = 0.48A$
$\therefore I_1 = 0.16A$ and $I_3 = 0.32A$

$V_1 = I_0(50) = 24V$
$V_2 = 0$
$V_3 = 0.16(100) = 16V$
$V_4 = 0.32(50) = 16V$
$\underline{V_5 = 16V}$

Problem 12

The circuit shown below initially has no charge on the capacitors and the switch S is originally open. Use $R_1 = 4\,\Omega$, $R_3 = 8\,\Omega$, $R_4 = 8\,\Omega$, $R_5 = 6\,\Omega$ $C_1 = 2\,\mu$F, and $C_2 = 6\,\mu$F.



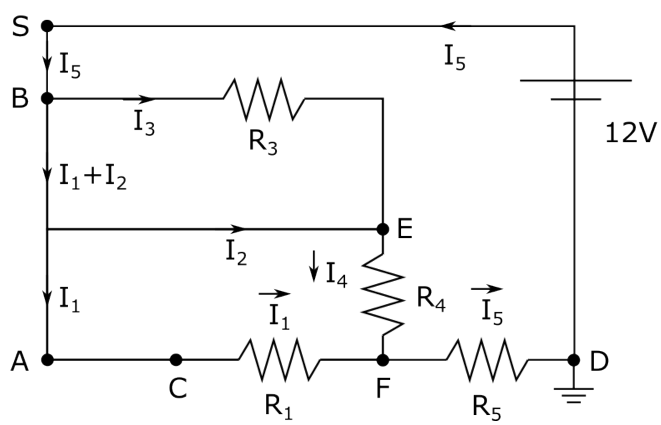
a) Just after closing the switch S, find the currents I_1, I_2, I_3, I_4, I_5.

b) After the switch has been closed for a very long time, find the currents I_1, I_2, I_3, I_4, I_5.

c) After the switch S has been closed for a very long time, find the potential at points A, B, C, D, E, & F.

(Note that the directions of the arrows don't necessarily mean that the current will be flowing in that direction.)
Solution:
$R_1 = 4\Omega,\ R_3 = 8\Omega,\ R_4 = 8\Omega,\ R_5 = 6\Omega$. $C_1 = 2\mu F,\ C_2 = 6\mu F$ at $t = 0$: $q_C(0) = 0$ for C_1 and C_2

a) At $t = 0$, $V_{C_1} = 0$ and $V_{C_2} = 0$ since $q_i(0) = 0 \implies$ both capacitors can be replaced by wires.



Note $V_A = V_E = V_B$ (as they are connected by wires), R_1 and R_4 are in parallel

$$\implies \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_4} = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

.

Hence $R_{eq} = \frac{8}{3} = 2.67A$, also $I_5 = I_1 + I - 2 + I_3$ but $\underline{I_3 = 0}$ (shorted)

Note that $I_4 = I_2 + I_3$ and $I_5 = I_1 + I_4$, $V_{EF} = V_{AC}$ so $I_4 R_4 = I_1 R_1$

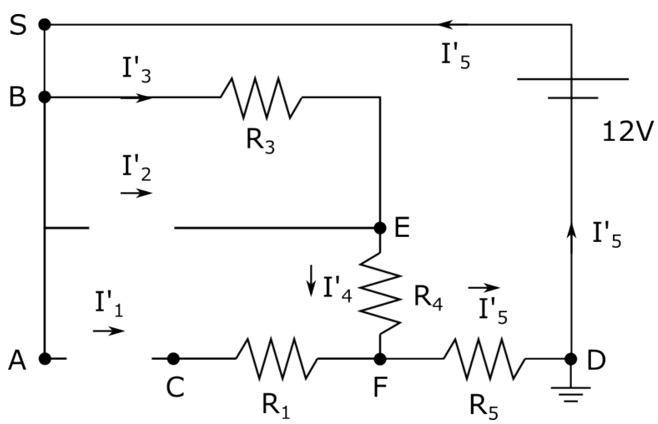
This means that $12V = I_4 R_4 + I_5 R_5 = I_1 R_1 + I_5 R_5$

Hence $12V = 8I_4 + 6(2I_4 + I_4) = I_4(8 + 12 + 6) = 26I_4$

$\implies I_4 = 0.46A$ (down)
$\implies I_1 = 0.92A$ (to right)
$\implies I_2 = I_4 = 0.46A$ (to right)
$\implies I_5 = 1.38A$ (to right)

b) After a long time $I_C \to 0$, $Q_{C_1} = V_{C_1}(C_1)$, and $Q_{C_2} = V_{C_2}(C_2)$. Additionally, the Capacitors will no longer allow charge to increase so $I_{C_1} = I_{C_2} = 0 \to$ an open circuit.

Redraw the circuit $\implies$



Note $\underline{I_2' = I_1' = 0}$ and $\underline{I_5' = I_3' = I_4'}$

Hence:

$$12V = I_5'(R_3 + R_4 + R_5) = I_5'(8 + 8 + 6) = 22I_5'$$

$$\underline{I_5' = \frac{12}{22}A = 0.545A}$$

c) $\underline{V_A = V_B = 12\text{Volts}}$, and we have:

$$12 - 0.545(8) = 12 - 0.436 = 7.64V$$

$$V_E = 12 - I_3' R_3 = 12 - 0.55(8) = 12 - 4.4 = 7.6V$$

$$V_E = 0 + I_5' R_5 + I_4' R_4 = I_5'(R_4 + R_5) = 0.545(6 + 8) = 0.545(14) = \underline{V_E = 7.63V}$$

$$V_C = V_F = I_5' R_5 = 0.545(6) = \underline{3.27V = V_C = V_F}$$

RLC Circuits

AC Circuits

Problem 13

Mystery RLC circuit: You are given an RLC circuit with elements connected in series. Values of R, L and C are unknown. You have at your disposal a source of AC voltage with $V_{\text{RMS}} = 8\,\text{V}$ and a tunable frequency ω. You also have an Ammeter which measures the RMS current I_{RMS} and the power factor $\cos\phi$.
Suppose you measured I_{RMS} as a function of frequency and found that the maximum RMS current occurs at $\omega_0 = 12.5\,\text{kHz}$ and is equal to $40\,\text{mA}$.

a) What is the resistance, R? What does this tell you about L and C?

b) What is the power factor at $\omega = \omega_0$?

c) In addition you find that at $\omega_1 = 17\,\text{kHz}$ the power factor is 0.5. Based on this information, what are the values of L and C?

Solution:
The key formulas that will help solve this problem are

$$I_{\text{RMS}} = \frac{V_{\text{RMS}}}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}$$

and

$$\tan\phi = \frac{L\omega - \frac{1}{C\omega}}{R}$$

a) At resonance frequency, we will have $L\omega_0 = \frac{1}{C\omega_0}$. This simplifies our above equation for the current to be $I_{\text{RMS}} = \frac{V_{\text{RMS}}}{R}$. Rearranging we can solve for R to be

$$R = \frac{V_{\text{RMS}}}{I_{\text{RMS}}} = \frac{8\,\text{V}}{40\,\text{mA}} = \boxed{200\,\Omega}$$

b) At resonance, the circuit is purely resistive so $\phi = 0$. Therefore, $\boxed{\cos\phi = 1}$

c) At $\omega_1 = 17\,\text{kHz}$, $\cos\phi = 0.5$ so $\phi = 60°$.

$$\tan 60° = \sqrt{3} = \frac{L\omega_1 - \frac{1}{\omega_1 C}}{R}$$
$$L\omega_1 - \frac{1}{C\omega_1} = R\sqrt{3}$$

and from part a we have $\frac{1}{C\omega_0} = L\omega_0$
Solve for L:

$$L\left(\omega_1 - \frac{\omega_0^2}{\omega_1}\right) = R\sqrt{3} \Rightarrow L = \frac{R\sqrt{3}}{\omega_1 - \frac{\omega_0^2}{\omega_1}} = 44.3\,\text{mH}$$
$$C = \frac{1}{L\omega_0^2} = 0.144\,\text{pF}$$

Problem 14

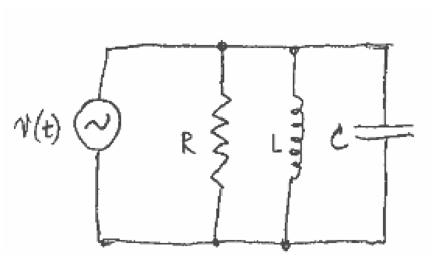
A $100\,\Omega$ resistor, $0.2\,\mu\text{F}$ capacitor and $50\,\text{mH}$ inductor are connected in parallel to an AC voltage source that provides instantaneous voltage according to $v(t) = V_{\text{peak}}\sin(\omega t)$. Draw the circuit, then answer the following questions:

a) Find the instantaneous voltages $v_R(t)$, $v_L(t)$, and $v_C(t)$ on the three elements.

b) Find the instantaneous currents $i_R(t)$, $i_L(t)$, and $i_C(t)$ flowing through the three elements.

c) Given that $V_{\text{peak}} = 100\,\text{V}$ and $\omega = 50\,\text{kHz}$ what is the total RMS current supplied by the source? What is the phase angle ϕ?

Solution:

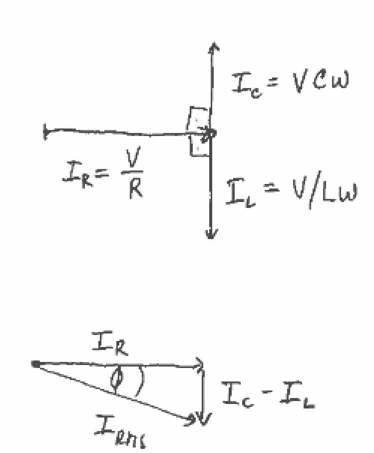


a) Because they are all connected to the source by a wire,

$$v_R(t) = v_L(t) = v_C(t) = V_{\text{peak}}\sin(\omega t)$$

b) $i_R(t) = \frac{V_{\text{peak}}}{R}\sin(\omega t)$ (in phase)
$i_L(t) = \frac{V_{\text{peak}}}{L\omega}\sin(\omega t - \frac{\pi}{2})$ (lags behind)
$i_C = V_{\text{peak}}C\omega\sin(\omega t + \frac{\pi}{2})$ (leads)

c) The RMS current is determined from the impedence triangle



$$I_{\mathrm{RMS}} = \sqrt{I_R^2 + (I_C - I_L)^2} = V_{\mathrm{RMS}}\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}$$

$$\tan\phi = \frac{I_C - I_L}{I_R} = R\left(C\omega - \frac{1}{L\omega}\right)$$