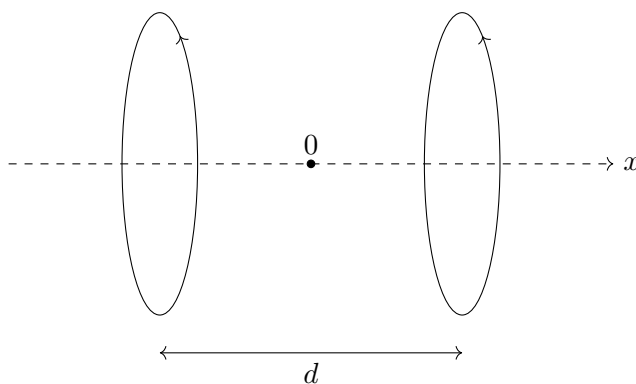


# Physics 158 Magnetism Problem Bank

## Problem 1

*Created by Tyler Wilson 2023*

Two current carrying loops, each with current  $I$  and radius  $R$ , are placed a distance  $d$  apart from each other, centered at the points  $(-\frac{d}{2}, 0, 0)$  and  $(\frac{d}{2}, 0, 0)$

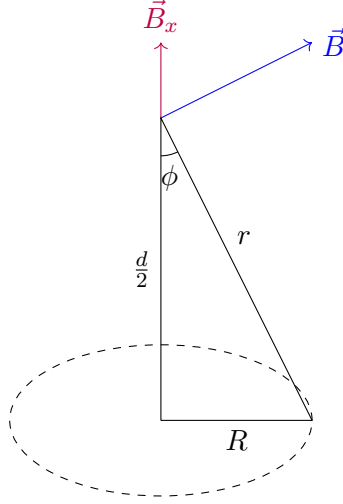


- Compute the magnitude of the magnetic field at the origin  $(0, 0, 0)$  due to this configuration.
- Compute the magnitude of the magnetic field for all points on the  $x$ -axis,  $|\vec{B}(x)|$ .
- Determine the optimal distance  $d$  between the two loops such that the magnetic field along the axis of symmetry is as uniform as possible.  
*Hint: This can be done by choosing  $d$  to make as many derivatives of  $|\vec{B}(x)|$  equal to zero at  $x = 0$  as possible.*

## Solution:

- We can start by writing the Biot-Savart law for the magnetic field due to a single current carrying loop.

$$B = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$



We can see that the magnetic field will only have a component in the  $x$ -direction, so we can write the magnetic field as

$$r = \sqrt{R^2 + \frac{d^2}{4}}$$

$$l = R\theta \Rightarrow dl = R d\theta$$

$$B_x = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \cdot \hat{i} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R d\theta}{R^2 + \frac{d^2}{4}} \sin \phi = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R d\theta}{R^2 + \frac{d^2}{4}} \frac{R}{\sqrt{R^2 + \frac{d^2}{4}}}$$

$$B_x = \frac{\mu_0 I R^2}{4\pi} \int_0^{2\pi} \frac{d\theta}{\left(R^2 + \frac{d^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{4\pi} \frac{2\pi}{\left(R^2 + \frac{d^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{2 \left(R^2 + \frac{d^2}{4}\right)^{\frac{3}{2}}}$$

This gives us the field for one wire. Using the right-hand-rule we can see that the two fields will add so the total field at the origin is double what we calculated.

$$|\vec{B}| = \frac{\mu_0 I R^2}{\left(R^2 + \frac{d^2}{4}\right)^{\frac{3}{2}}}$$

- b) We can use the same method as above to find the field at any point on the  $x$ -axis. If we use the same derivation as above but replace  $\frac{d}{2}$  with  $x + \frac{d}{2}$  for the left ring and  $x - \frac{d}{2}$  for the right ring then we get a general expression for the field at any point on the  $x$ -axis.

$$|\vec{B}|(x) = \frac{\mu_0 I R^2}{2} \left( \frac{1}{\left(R^2 + \left(x + \frac{d}{2}\right)^2\right)^{3/2}} + \frac{1}{\left(R^2 + \left(x - \frac{d}{2}\right)^2\right)^{3/2}} \right)$$

- c) We can find the optimal distance by finding the points where the derivative of the field is zero at  $z = 0$ .

$$\left. \frac{dB}{dx} \right|_{x=0} = -\frac{\mu_0 I R^2}{2} \frac{3}{2} \left( \frac{2\left(x + \frac{d}{2}\right)}{\left(R^2 + \left(x + \frac{d}{2}\right)^2\right)^{5/2}} + \frac{2\left(x - \frac{d}{2}\right)}{\left(R^2 + \left(x - \frac{d}{2}\right)^2\right)^{5/2}} \right) \Big|_{x=0}$$

$$= \frac{3\mu_0 I R^2}{2} \left( \frac{\frac{d}{2}}{\left(R^2 + \frac{d^2}{4}\right)^{5/2}} - \frac{\frac{d}{2}}{\left(R^2 + \frac{d^2}{4}\right)^{5/2}} \right) = 0$$

The first derivative didn't tell us anything so now we can look at the second derivative.

$$\begin{aligned} \frac{d^2 B}{dx^2} \Big|_{x=0} &= -\frac{3\mu_0 I R^2}{2} \frac{d}{dx} \left( \frac{x + \frac{d}{2}}{\left(R^2 + (x + \frac{d}{2})^2\right)^{5/2}} + \frac{x - \frac{d}{2}}{\left(R^2 + (x - \frac{d}{2})^2\right)^{5/2}} \right) \Big|_{x=0} = 0 \\ &= \frac{\left(R^2 + (x + \frac{d}{2})^2\right)^{5/2} - (x + \frac{d}{2})\left(\frac{5}{2}\right)\left(R^2 + (x + \frac{d}{2})^2\right)^{3/2}(2)\left(x + \frac{d}{2}\right)}{\left(R^2 + (x + \frac{d}{2})^2\right)^5} \Big|_{x=0} \\ &\quad + \frac{\left(R^2 + (x - \frac{d}{2})^2\right)^{5/2} - (x - \frac{d}{2})\left(\frac{5}{2}\right)\left(R^2 + (x - \frac{d}{2})^2\right)^{3/2}(2)\left(x - \frac{d}{2}\right)}{\left(R^2 + (x - \frac{d}{2})^2\right)^5} \Big|_{x=0} = 0 \\ &= \frac{2\left(R^2 + \frac{d^2}{4}\right)^{5/2} - \frac{5d^2}{2}\left(R^2 + \frac{d^2}{4}\right)^{3/2}}{\left(R^2 + \frac{d^2}{4}\right)^5} = 0 \\ &= 2\left(R^2 + \frac{d^2}{4}\right) - \frac{5d^2}{2} = 2R^2 - 2d^2 = 0 \end{aligned}$$

$$\boxed{d = R}$$