Physics 158 Formula Sheet

Constants

Coulomb's Constant	$k = \frac{1}{4\pi\epsilon_0} \approx 8.988 \times 10^9\ \mathrm{Nm^2/C^2}$
Electric Constant	$\epsilon_0 = 8.854 \times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Elementary Charge	$e = -1.602 \times 10^{-19}\ \mathrm{C}$
Vacuum Permeability	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$
Speed of Light	$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8\ \mathrm{m/s}$

DC Circuits

Resistor Circuits

Ohm's Law	$V = IR$
Power Dissipated	$P = IV = I^2 R = \frac{V^2}{R}$
Resistors in Series	$R_{eq} = R_1 + R_2 + \cdots$
Resistors in Parallel	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

RC Circuits

Time Constant	$\tau = RC$
Charging	$q(t) = Q_{\max}(1 - e^{-t/\tau})$
Discharging	$q(t) = Q_{\max} e^{-t/\tau}$

RL Circuits

Time Constant	$\tau = \frac{L}{R}$
Charging	$i(t) = I_0(1 - e^{-t/\tau})$
Discharging	$i(t) = I_0 e^{-t/\tau}$

RLC Circuits

Time Constant	$\tau = \frac{2L}{R}$
Resonance Frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$
Frequency	$\omega = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$
Charge	$q(t) = Q_{\max} e^{-t/\tau} \cos(\omega t + \phi)$

AC Circuits

Reactance and Impedance

Capacitor Reactance	$X_C = \frac{1}{\omega C}$
Capacitor Voltage	$V_C = X_C I$
Inductor Reactance	$X_L = \omega L$
Inductor Voltage	$V_L = X_L I$
Impedance (in Series)	$\lvert Z\rvert^2 = R^2 + (X_L - X_C)^2$
Voltage	$V = IZ$

Phase Angles

Phase Angle	$\tan\phi = \frac{X_L - X_C}{R}$
	$\phi = \arg(v) - \arg(i)$
If $v(t) = V_0 \cos(\omega t)$	then $i(t) = I_{\max}\cos(\omega t - \phi)$

Power

Power Factor	$\cos\phi = \frac{R}{Z}$
Average Power	$P_{\text{avg}} = V_{\text{RMS}} I_{\text{RMS}} \cos\phi = I_{\text{RMS}}^2 R$
RMS Current	$I_{\text{RMS}} = \frac{I_{\max}}{\sqrt{2}}$

Capacitors

Capacitance	$C = \frac{Q}{V}$
Stored Energy	$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$
Capacitors in Series	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$
Capacitors in Parallel	$C_{eq} = C_1 + C_2 + \cdots$
Parallel Plate Capacitor	$C = \frac{\epsilon_0 A}{d}$
Dielectrics	$C_{\text{dielectric}} = \kappa C_{\text{vacuum}}$

Inductors

Self-Induced EMF	$\mathcal{E} = -L\frac{di}{dt}$
Stored Energy	$U = \frac{1}{2}LI^2$
Inductors in Series	$L_{eq} = L_1 + L_2 + \cdots$
Inductors in Parallel	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots$

Solenoids

Coil Density	$n = N/L$
Magnetic Field	$B = \mu_0 n I$
Inductance	$L = \frac{N\Phi_B}{I}$

Electrostatics

Electric Force

Coulomb's Law	$\lvert\vec{F}\rvert = k\frac{\lvert q_1 q_2\rvert}{r^2} = q\lvert\vec{E}\rvert$

Electric Field

Gauss's Law	$\oint\!\!\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$
$\vec{E}$ from Point Charge	$\vec{E} = \frac{kq}{r^2}\hat{r}$
$\vec{E}$ from Charged Rod	$E(h) = \frac{kQ}{h\sqrt{h^2 + a^2}}$
$\vec{E}$ from Charged Ring	$E(z) = \frac{kQz}{(R^2 + h^2)^{3/2}}$
$\vec{E}$ from Charged Disk	$E(z) = \frac{2Qk}{R^2}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$
$\vec{E}$ from Infinite Sheet	$E(z) = \frac{\sigma}{2\epsilon_0}\hat{n}$
Flux	$\Phi_E = \iint_S \vec{E} \cdot d\vec{A}$
Energy Density	$u_E = \frac{\epsilon_0}{2}E^2$

Electric Potential

Potential

Difference Notation	$V_{ba} = V_b - V_a$
V from Point Charge	$V = \frac{kq}{r} + \text{Constant}$
Potential Difference from $\vec{E}$	$V = -\int_a^b \vec{E} \cdot d\vec{l}$
Electric Field from V	$\vec{E} = -\nabla V$
	$E_x = -\frac{dV}{dx}$

Potential Energy

Work	$W_{a\to b} = U_a - U_b = -\Delta U$
Potential Energy from V	$U = qV$
Between Point Charges	$U = \frac{kq_1q_2}{r}$

Magnetostatics

Magnetic Force

Lorentz Force	$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$
Force on Current	$\vec{F} = I\vec{L} \times \vec{B}$
Force Between Wires	$\frac{F}{L}F = \frac{\mu_0 I_1 I_2}{2\pi d}$

Magnetic Fields

Biot-Savart Law	$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$
Ampere's Law	$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$
Loop of Current	$\vec{B} = \frac{\mu_0 I R^2}{2(h^2+R^2)^{3/2}}\hat{n}$
Straight Wire	$B = \frac{\mu_0 I}{4\pi r}\sin\theta\Big\vert_{\theta_L}^{\theta_R} = \frac{\mu_0 I x}{2\pi r\sqrt{x^2+r^2}}\Big\vert_{x_L}^{x_R}$
Flux	$\Phi_B = \iint_S \vec{B} \cdot d\vec{A}$
Energy Density	$\frac{1}{2\mu_0}B^2$

Torque on Current Loop

Torque Vector	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Magnetic Dipole Moment	$\vec{\mu} = NI\vec{A}$
Potential Energy	$U = -\vec{\mu} \cdot \vec{B}$

Maxwell's Equations

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Electromagnetic Induction

Induced EMF	$\mathcal{E} = -\frac{d\Phi_B}{dt}$
Motional EMF	$\mathcal{E} = \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{l}$

Mechanics

Kinematics

Linear Motion	$x = x_0 + \frac{1}{2}(v_0 + v)t$
	$x = x_0 + vt + \frac{1}{2}at^2$
	$v = v_0 + at$
	$v^2 = v_0^2 + 2a(x - x_0)$
Circular Motion	$a_c = \frac{v^2}{r}$

Forces

Newton's Second Law	$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$
Spring Force	$\vec{F} = -kx\hat{x}$
Friction Force	$F = \mu N$
Damping Force	$\vec{F} = -b\vec{v}$
Bouyant Force	$\vec{F} = \rho V g$

Work and Energy

Work	$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta\vec{r}$
Kinetic Energy	$K = \frac{1}{2}mv^2$
Gravitational Potential	$\Delta U_g = mgy$
Spring Potential Energy	$\Delta U_s = \frac{1}{2}kx^2$
Conservative Forces	$\vec{F} = -\nabla U$
Power	$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

Mathematics

Area and Volume

Volume of a Sphere	$V = \frac{4}{3}\pi r^3$
Volume of a Cylinder	$V = \pi r^2 L$
Area of a Sphere	$A = 4\pi r^2$
Area of a Cylinder	$A = 2\pi r L$
Area of a Circle	$A = \pi r^2$
Circumference of a Circle	$C = 2\pi r$

Trigonometry

Pythagorean Theorem	$a^2 + b^2 = c^2$
Arc Length	$s = r\theta$
Pythagorean Identity	$\sin^2\theta + \cos^2\theta = 1$
Double Angle	$\sin(2\theta) = 2\sin\theta\cos\theta$
	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
Half Angle	$\sin^2(\frac{\theta}{2}) = \frac{1-\cos\theta}{2}$
	$\cos^2(\frac{\theta}{2}) = \frac{1+\cos\theta}{2}$

Integrals

$$\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} + \text{Constant} & n \neq -1 \\ \ln|x| + \text{Constant} & n = -1 \end{cases}$$

Vectors

Dot Product	$\vec{a} \cdot \vec{b} = ab\cos\theta$
Cross Product	$\lVert\vec{a} \times \vec{b}\rVert = ab\sin\theta$
	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

Right Hand Rule