Physics 158 Electric Fields Problem Bank

Problem 1

Created by Tyler Wilson 2023

A 16 cm long wire of charge $Q = 40\,\mu\text{C}$ is bent into a square.

a) Find the electric field strength 20 cm vertically above the center of the square in the xy-plane.

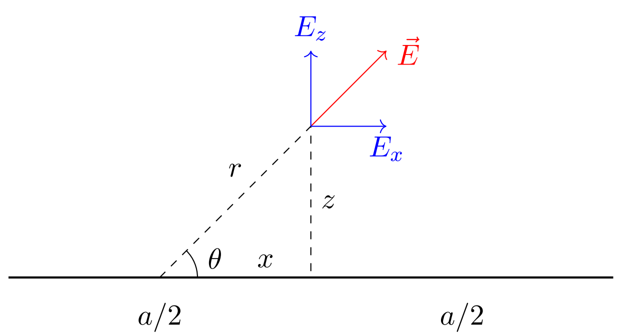
A point charge of mass $m = 100\,\text{g}$ and charge $-5\,\mu\text{C}$ is now placed 20 cm above the center of the square.

b) Find the magnitude and direction of the force acting on the point charge initially when $z = 20\,\text{cm}$.

c) Plot the acceleration of the point charge as a function of it's distance z above the center of the square.

Solution:

a) We can break this problem down into the simpler problem of computing the electric field due to a straight piece of wire. We can then use the superposition principle to find the electric field due to the square.
Electric field due to a straight piece of wire of length a:



$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

$$dq = \lambda dx$$

$$r = \sqrt{x^2 + z^2}$$

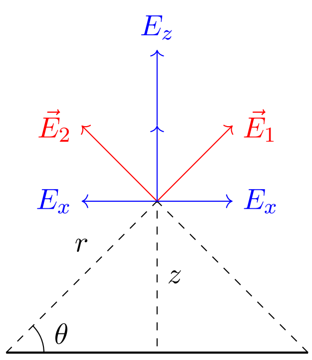
$$\vec{E} = \int d\vec{E} = \int_{-a/2}^{a/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{x^2 + z^2}\hat{r}$$

$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0} \int_{-a/2}^{a/2} \frac{dx}{x^2 + z^2} \hat{r}$$

Note that because of symmetry on each side the electric field will only have a vertical component.

$$\hat{r} = \cos(\theta)\hat{x} + \sin(\theta)\hat{z}$$
$$\sin\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2 + z^2}}$$
$$E_z = \frac{\lambda}{4\pi\epsilon_0} \int_{-a/2}^{a/2} \frac{dx}{(x^2 + z^2)^{3/2}}$$
$$E_z = \frac{\lambda}{4\pi\epsilon_0} \left[\frac{x}{z\sqrt{x^2 + z^2}}\right]_{x=-a/2}^{x=a/2}$$
$$E_z = \frac{\lambda}{4\pi\epsilon_0} \left[\frac{a}{z\sqrt{a^2/4 + z^2}} - \frac{-a}{z\sqrt{a^2/4 + z^2}}\right]$$
$$E_z = \frac{\lambda}{4\pi\epsilon_0} \frac{2a}{z\sqrt{a^2/4 + z^2}}$$
$$\vec{E} = E_z\hat{z} = \frac{\lambda}{2\pi\epsilon_0} \frac{a}{z\sqrt{a^2/4 + z^2}} \hat{z} = \frac{2k\lambda a}{z\sqrt{a^2/4 + z^2}} \hat{z}$$

Let us consider the length of the wire to be $L = 16\,\text{cm}$. We can break this down to four straight pieces of wire of length $a = L/4 = 4\,\text{cm}$. The electric field due to each piece of wire will be the same magnitude and direction due to the symmetry of the problem. The electric field due to the square will be the sum of the electric fields due to each piece of wire.



$$\vec{E}_{\text{total}} = 4E_z\hat{z}$$
$$E_z = \frac{2k\lambda a}{r\sqrt{a^2/4 + r^2}} \sin\theta$$
$$r = \sqrt{a^2/4 + z^2}$$

$$\sin\theta = \frac{z}{r}$$
$$E_z = \frac{2k\lambda a z}{r^2\sqrt{a^2/4 + r^2}}$$
$$r^2 = \frac{a^2}{4} + z^2$$
$$E_z = \frac{2k\lambda a z}{(a^2/4 + z^2)\sqrt{a^2/2 + z^2}}$$
$$a = \frac{L}{4}, \quad \lambda = \frac{Q}{L}$$
$$E_z = \frac{\frac{1}{2}kQz}{(L^2/64 + z^2)\sqrt{L^2/32 + z^2}}$$
$$\boxed{\vec{E}_{\text{total}} = 4E_z\hat{z} = \frac{2kQz}{(L^2/64 + z^2)\sqrt{L^2/32 + z^2}}\hat{z}}$$

b) The force can be computed as $\vec{F} = q\vec{E}$ and the direction will be toward the center of the square for a negative point charge.

$$\vec{F} = q\vec{E} = -\frac{2kQqz}{(L^2/64 + z^2)\sqrt{L^2/32 + z^2}}\hat{z}$$

c) We can get an expression for the acceleration from the force

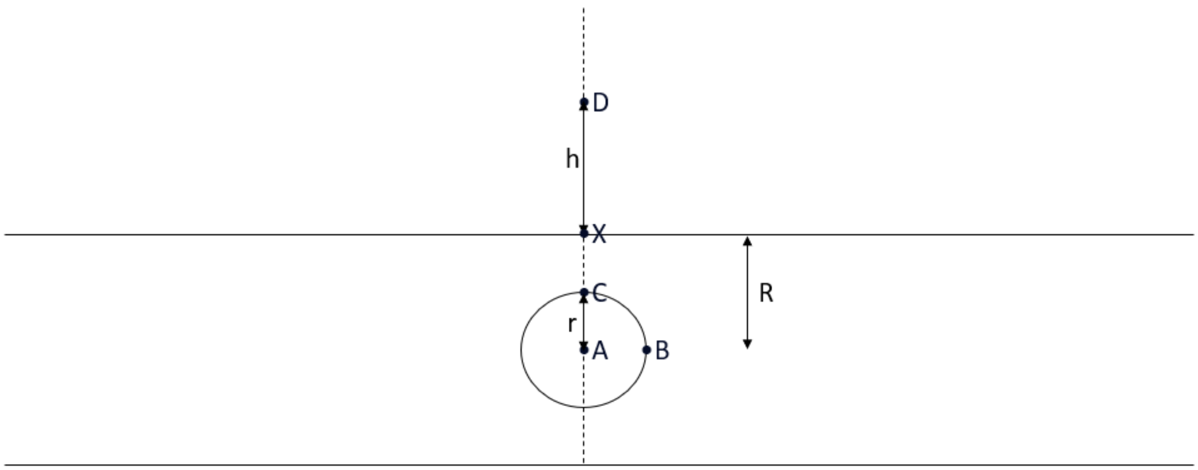
$$\vec{F} = m\vec{a}$$
$$\vec{a} = -\frac{2kQqz}{m(L^2/64 + z^2)\sqrt{L^2/32 + z^2}}\hat{z}$$

Problem 2

Created by Tyler Wilson 2023
An insulating cylinder of radius $R = 5\,\text{cm}$ and effectively infinite length in the z-direction contains a uniform charge density of $10\,\text{C/m}^3$.

a) Find the electric field everywhere in space

If there is now a hollow spherical cavity of radius $a = 1\,\text{cm}$ located at the center of the cylinder,



b) Find the electric field at the center of the sphere at point A.

c) Find the electric field just outside the sphere at point B.

d) Find the electric field just outside the sphere at point C.

e) If $h = 10\,\text{cm}$, find the electric field outside both objects at point D.

If the potential at the point X is 0V,

f) What is the potential at point A?

Solutions:

a) We can use Gauss's Law to find the electric field everywhere in space. We will use a Gaussian cylinder of radius r and length L.

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

We will need to break this into two parts: the inside of the cylinder and the outside of the cylinder.
For the inside of the cylinder,

$$\oiint \vec{E}_{\text{in}} \cdot d\vec{A} = E_{\text{in}} A = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{enc}} = \rho V_{\text{enc}} = \rho \pi r^2 L$$

$$E_{\text{in}} A = \frac{\rho \pi r^2 L}{\epsilon_0}$$

$$2\pi r L E_{\text{in}} = \frac{\rho \pi r^2 L}{\epsilon_0}$$

$$E_{\text{in}} = \frac{\rho r}{2\epsilon_0}$$

$$\boxed{\vec{E}_{\text{in}} = \frac{\rho r}{2\epsilon_0}\hat{r}}$$

For the outside of the cylinder,

$$\oiint \vec{E}_{\text{out}} \cdot d\vec{A} = E_{\text{out}} A = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{enc}} = \rho V_{\text{enc}} = \rho\pi R^2 L$$

$$E_{\text{out}} A = \frac{\rho\pi R^2 L}{\epsilon_0}$$

$$2\pi r L E_{\text{out}} = \frac{\rho\pi R^2 L}{\epsilon_0}$$

$$E_{\text{out}} = \frac{\rho R^2}{2\epsilon_0 r}$$

$$\boxed{\vec{E}_{\text{out}} = \frac{\rho R^2}{2\epsilon_0 r}\hat{r}}$$

b)

Problem 3

Created by Tyler Wilson 2023

The potential above some charged Gaussian surface is given by the equation

$$V = \frac{k\sigma}{3y} \text{ Volts}$$

a) What is the equation for the electric field?

b) If $\sigma = 6\,\mu\text{C}$, find the electric field strength and direction at $y = 2\,\text{m}$.

c) What can you say about the electric field in the x-direction?

Hint:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

Solution:

a) The electric field is the negative gradient of the potential. This was defined in the hint above. The electric field would then be

$$\vec{E} = -\frac{\partial}{\partial y}\left(\frac{k\sigma}{3y}\right)\hat{j} = \boxed{\frac{k\sigma}{3y^2}\hat{j}}$$

b) Plugging in these values we would get

$$\vec{E}(y=2) = \frac{k(6 \cdot 10^{-6})}{3(2)^2}\hat{j} = \boxed{4495\hat{j}\text{ N/C}}$$

c)

$$\vec{E}_x = -\frac{\partial}{\partial x}\left(\frac{k\sigma}{3y}\right) = 0$$

Therefore, the electric field in the x-direction is 0.

Problem 4

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A rectangular box has dimensions $2\,\mathrm{m} \times 4\,\mathrm{m} \times 8\,\mathrm{m}$. At each corner of the box sits a point charge of value Q.

a) What is the electric field at the center of the box?

b) What is the potential at the center of the box?

c) What is the total energy of the system?

One of the point charges on the corners is removed.

d) What is the new electric field at the center of the box?

e) What is the new potential at the center of the box?

f) What is the work done on the system in removing that point charge?

g) What is the new total energy of the system?

Problem 5

Created by Tyler Wilson 2023
A parallel plate capacitor is hooked up to a battery in series. At time t, a dielectric is inserted into the capacitor and the volatge source is doubled. If the charge on the capacitor remains the same, find the dielectric constant.

Problem 6

Created by Tyler Wilson 2023
A resistor ($R = 2\,\Omega$) and a capacitor ($C = 16\,\mu\mathrm{F}$) are connected in parallel to a $10\,\mathrm{V}$ battery.

a) What is the current flowing through the resistor after a long period of time?

At time $t = t_0$ a dielectric of $\kappa = 4$ is inserted into the capacitor.

a) What is the new current flowing through the resistor at the instant just after the dielectric is inserted (at $t = t_0$)?

b) What is the new current flowing through the resistor after a long time has passed?

Problem 7

Created by Tyler Wilson 2023

A resistor ($R = 6\,\Omega$) and a capacitor of unknown value are connected in series to a $12\,\mathrm{V}$ battery at time $t = 0$. The current through the resistor 0.1 seconds after the battery is connected is measured to be $1\,\mathrm{A}$.

A dielectric of unknown value κ is then inserted in the capacitor. and the current through the resistor after 0.1 seconds is measured to be $0.75\,\mathrm{A}$.

What are the values of both the capacitor and the dielectric?