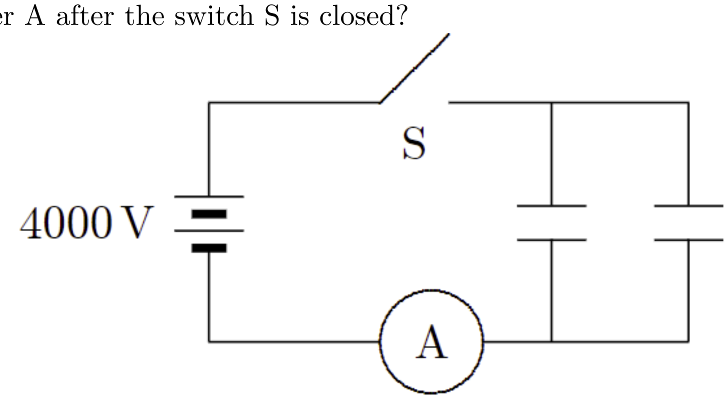
Physics 158 Circuits Problem Bank

Time Independent Circuits

Time Dependent Circuits

Problem 1

Each of the two $25\,\mu$F capacitors shown is initially uncharged. How many coulombs of charge pass through the ammeter A after the switch S is closed?



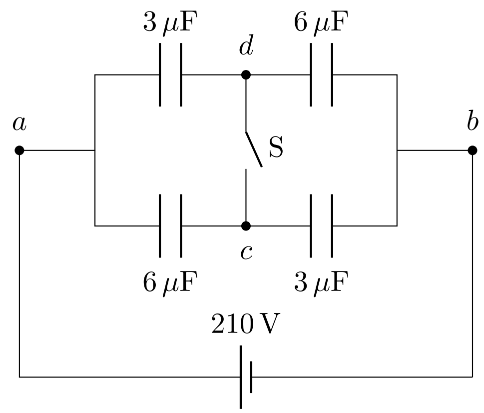
Solution:
When the switch is closed the two capacitors and power supply are all at 4000V.
The charge on each capacitor is $q = CV$ which in this case is

$$q = (25\,\mu\text{F})(4000\,\text{V}) = 0.1\,\text{C}$$

for each capacitor so a total of $\boxed{0.2\,\text{C}}$ flows through the ammeter

Problem 2

The capacitors in the figure are initially uncharged and are connected, as in the diagram, with the switch S open. The applied potential difference is $V_{ab} = 210\,\text{V}$



a) What is the potential difference V_{cd}

b) What is the potential difference across each capacitor after the switch S is closed?

c) How much charge flowed through the switch when it was closed?

Solution:
With the switch open each pair of $3.00\,\mu$F and $6\,\mu$F capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed each pair of $3\,\mu$F and $6\,\mu$F capacitors are in parallel with each other and the two pairs are in series.

a) With the switch open

$$C_{\text{eq}} = \frac{1}{\frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}} + \frac{1}{\frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}} = 4\,\mu\text{F}$$
$$Q_{\text{total}} = C_{\text{eq}}V = (4\,\mu\text{F})(210\,\text{V}) = 8.40 \cdot 10^{-4}\,\text{C}$$

By symmetry, each capacitor carries $4.20 \cdot 10^{-4}$ C. The voltages are then calculated via $V = \frac{Q}{C}$. This gives $V_{ad} = \frac{Q}{C_3} = 140\,\text{V}$ and $V_{ac} = \frac{Q}{C_6} = 70\,\text{V}$. We then get V_{cd} as

$$V_{cd} = V_{ad} - V_{ac} = \boxed{70\,\text{V}}$$

b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is

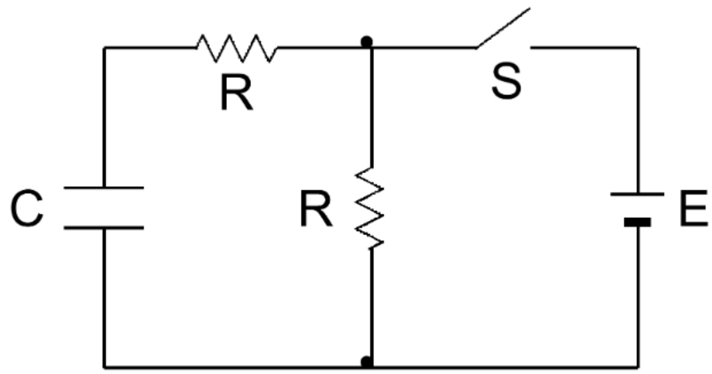
$$C_{\text{eq}} = \frac{1}{\frac{1}{(3+6)\,\mu\text{F}} + \frac{1}{(3+6)\,\mu\text{F}}} = 4.5\,\mu\text{F}$$
$$Q_{\text{total}} = C_{\text{eq}}V = (4.5\,\mu\text{F})(210\,\text{V}) = 9.5 \cdot 10^{-4}\,\text{C}$$

and each capacitor has the same potential difference of $\boxed{105\,\text{V}}$ (again, by symmetry).

c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\,\mu$C, so $\boxed{315\,\mu\text{C}}$ of charge flowed through the switch.
Note: It is better to compute the absolute charges on each plate before and after the switch is closed and then to follow the flow of the electrons through the switch.

Problem 3

In the circuit shown, both resistors have the same value R. Suppose switch S is initially closed for a very long time.



a) Find all currents initially.

b) Find all currents the instant after the switch is opened.

c) Find the time constants for both case switch open and switch closed.

Solution:

a) The switch has been closed for a long time so we can assume that the capacitor is fully charged. This will mean that it acts as a short-circuit so it will have no current flowing through it. So the current through branch 1 is $\boxed{I_1 = 0}$ and the current through branch 2 is $\boxed{\mathrm{I}_2 = \frac{\mathrm{E}}{\mathrm{R}}}$

b) The instant before the switch is opened we can use Kirchoff's loop law to state that the large outer loop must have a volatge that sums to 0. We know the current through the upper resistor is 0 from part a so then we can say that the voltage of the capacitor must equal that of the battery.
When the switch is opened, it removes the battery from the circuit so we just have the loop on the left. Using Kirchoff's law agian, we know that the voltage must sum to 0 so we can say that the voltage of the resistors must equal the voltage of the capacitor.

$$V_C = V_{R_1} + V_{R_2}$$

We know the resistors are the same and we know $V_C = E$ so we get $E = 2V_R = 2IR$. We can solve for the current to get that the current in the left loop is $\boxed{I = \frac{E}{2R}}$ and there is no current through the battery.

c) We can break this down into two cases:

- Charging the capacitor (after the switch is closed)
When the switch is closed, take the loop through the battery and C. This gives $\boxed{\tau = RC}$

- Discharging the capacitor (after the switch is opened)
  When the switch is open, only the loop not containing the battery has current which gives a time constant of $\boxed{\tau = 2RC}$ because both resistors are included.

RLC Circuits

AC Circuits