ELEC 481 Notes:

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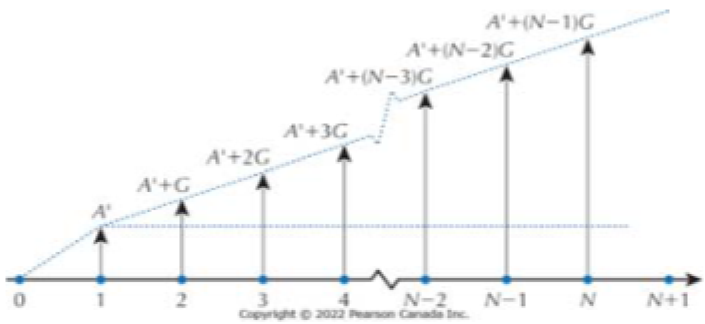
Interest rates:

- Nominal interest: (interest rate per compound period * number of periods in a year)
- Effective annual interest: $i_a = (1 + r/m)^m - 1$
  Where m is the number of compounding periods and r is the nominal annual interest rate

Discrete cash flow models:

- Cash flows and compounding occur at the *END* of defined periods
- Assumes all periods are of equal length
- Arithmetic gradient series: regular and uniformly growing payments (G).



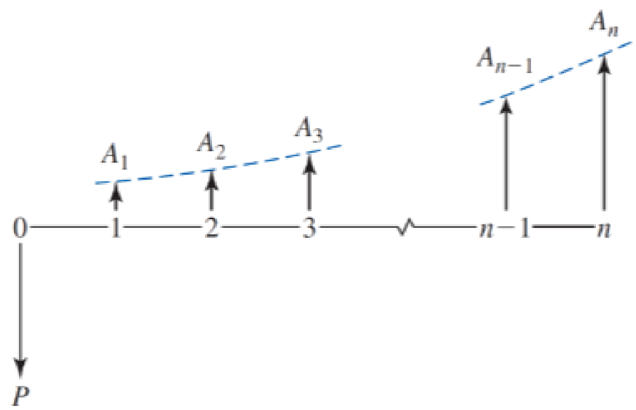
  - We break the formula up into the base annual amount (A') and the additional amount that's getting added over time

$$P' = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$P'' = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right]$$

  - The total present worth will be P=P'+P''

- Geometric gradient series: Regularly occurring deposits that increase with a growth rate (g) assuming a base disbursement (A1).



  - The present worth depends on if the interest is the same as the growth rate or not:

When i<>g:

$$P = A_1\left[\frac{1-(1+g)^n(1+i)^{-n}}{i-g}\right]$$

When i=g:

$$P = A_1\,[n(1+i)^{-1}]$$

- Capitalized cost: If we have a project that has an infinite lifetime (n=infty) and we know the annual expenses/earnings then the capitalized cost is

$$\text{Capitalized Cost} = P = A/i$$

Time value of money:

- Interest rates will mean that money in the future is worth less than the same amount now
- P is present value (year 0)
- F is future value (year n)
- A is annual worth (how much you need to pay over n years accounting for interest). Eg. how much do you need to pay per year on a mortgage accounting for both principal and interest to pay it off in n years
- Can convert with the following formulas:

$$F = A\left(\frac{F}{A}, i\%, n\right) = A\left[\frac{(1+i)^n - 1}{i}\right] \qquad A = F\left(\frac{A}{F}, i\%, n\right) = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$A = P\left(\frac{A}{P}, i\%, n\right) = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad P = A\left(\frac{P}{A}, i\%, n\right) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$F = P\left(\frac{F}{P}, i, n\right) = P(1+i)^n \qquad P = F\left(\frac{P}{F}, i, n\right) = F\frac{1}{(1+i)^n}$$

Salvage Value:

- One-time future cash flow at the end of the asset's life (can apply the basic F = P(1+i)^n)

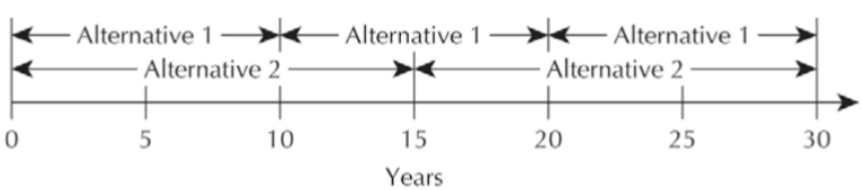
Equivalent Annual Cost:

$$\boxed{EAC = (P-S)\,(A/P, i, n) + S * i}$$

- Used to find the equivalent annual cost when we know the salvage value of an asset
  - Subtracts that value from the present cost

Project Comparison Methods:

- **PW:** compares projects by looking at present worth of all cash flows → project with higher PW should be pursued
  - PW must be > 0 for the project to be acceptable
  - When projects have the same PW, choose project with highest intangible value
  - Projects need equal lives to be compared. If they don't have equal lives, can take repeated lives approach



- **AW:** same as PW but with annual worth of all cash flows

    - Note that when we have salvage we need to use EAC

      When there is an initial cost (P) followed by a salvage value (S) the equivalent uniform annual worth (AW) can be computed by:

      - $AW = P(A/P, i, n) - S(A/F, i, n)$

      Or, equivalently:

      - $AW = (P - S)(A/F, i, n) + Pi$

      Or, equivalently:

      - $AW = (P - S)(A/P, i, n) + Si$

- **<u>FW</u>**: same as PW but with future worth of all cash flows
    - These 3 methods always yield the same result!!!!
    - The other comparison methods don't always yield the same result…
- **<u>IRR:</u>** compares how high an interest rate you can borrow money and still break even (higher IRR is better)
    - <u>MARR (minimum acceptable rate of return):</u> a benchmark of how much you think you can earn from investments
        - Projects need to earn at least the MARR to be desirable
        - Typical MARR values are 12%-15%, but when a project has high risk, we usually increase the MARR
        - MARR is often used as the discount rate in economic analysis (a company may have a fixed MARR they use to analyze projects)
        - IRR must be higher than MARR to be desirable
    - <u>IRR (internal rate of return):</u> the rate of return on an investment, or the interest rate at which a project "breaks even"
        - Interest rate i* such that when all cash flows of a project are discounted with i*, PW = 0
            - E.g. if $100 invested today yields $110 in a year, the IRR is 10%
        - Solved using trial and error, complicated math equations or built-in excel IRR function
        - Projects must also have equal lives (if unequal, use same method as for PW)
    - Downsides:
        - If we have two options: invest $1 and get back $2 or invest $1000 and get back $1900, option 1 will have the higher IRR despite option 2 being more profitable
    - Incremental analysis: a way of comparing two options using IRR.
        - We determine the IRR of the annual difference in cost/revenues between option 1 and option 2
        - This IRR determines the rate of return in which option 1 and option 2 are equally good options. Above this rate, one option will be better and below that rate the other option will be better
    - Choosing between options:

        - First reject any projects that don't meet the MARR
        - Order the projects by highest initial cost and perform incremental analysis between projects
        - Construct a choice table to see at what interest rates will what project be most optimal
- **Benefit-Cost Analysis:**
    - Can be based on AW or PW values
    - Conventional benefit-cost ratio:

$$\mathbf{BCR} = \frac{\mathbf{PW(users' \ benefits)}}{\mathbf{PW(sponsors' \ costs)}}$$

        - Project is desirable is BCR > 1
    - Modified benefit-cost ratio:

$$\mathrm{BCRM} = \frac{\mathrm{PW(users' \ benefits)} - \mathrm{PW(sponsors' \ operating \ costs)}}{\mathrm{PW(sponsors' \ capital \ costs)}}$$

    - To perform benefit-cost analysis among mutually exclusive projects, perform incremental analysis
        - For mutually exclusive projects X and Y (that both have a BCR > 1), with benefits B_X and B_Y, and costs C_X and C_Y, where $C_X > C_Y$:

$$\mathbf{BCR(X-Y)} = \frac{B_X - B_Y}{C_X - C_Y}$$

        - If BCR(X-Y) > 1, X is preferred. Otherwise, Y is preferred.
        - If $C_X = C_Y$, choose project with greatest PW of benefits
        - If $B_X = B_Y$, choose project with lowest PW of costs
    - If there is a salvage value, subtract its PW from the PW(sponsors' costs)
- **Payback Period:** number of years it takes for an investment to be recovered

$$\textbf{Payback period} = \frac{\textbf{First cost}}{\textbf{Annual savings}}$$

    - If the annual savings is not constant we can't use the above formula and instead deduct each year of savings from the first cost until the first cost is recovered
    - Disadvantages:
        - Discriminates against long-term projects
        - Ignores the time value of money
        - Ignores the expected service life

Opportunity Cost:
- Net cost/benefit (or IRR) of next best opportunity that was rejected

Return on Investment (ROI):
- Percentage increase or decrease in an investment over a set period

- $$\text{ROI} = \frac{F-P}{P}$$

Assumptions for Comparison Methods:

- Costs and benefits can be quantified / are measurable in terms of money
- Future cash flows are known with certainty

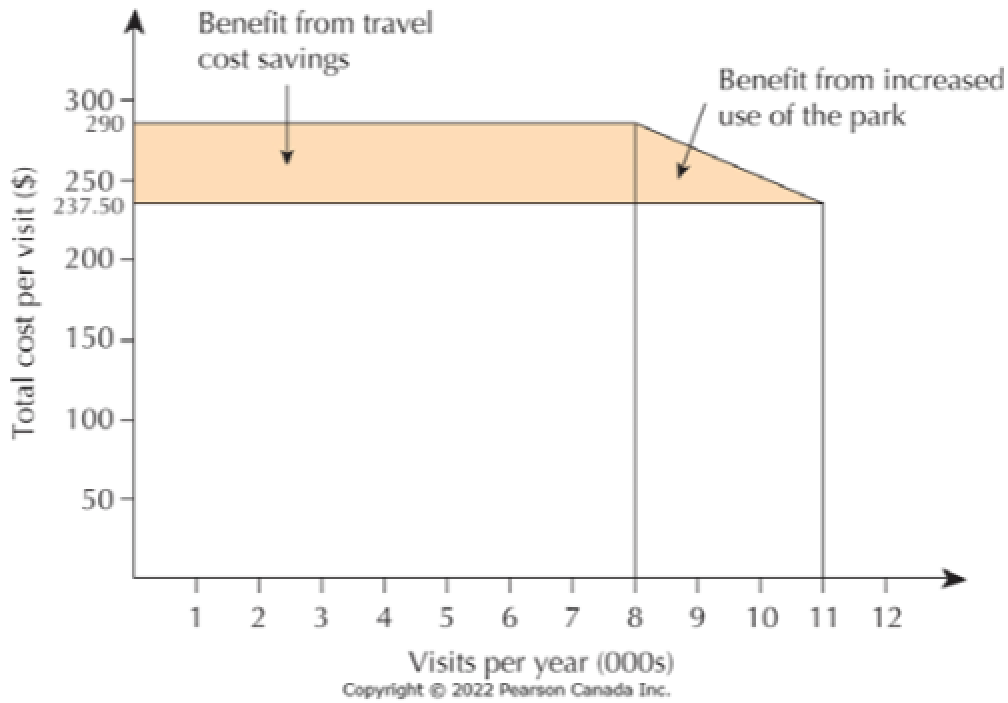
Public Sector Decision Making:

- Investment decisions are more difficult in the public sector vs. private sector due to:
  - Broader scope of analysis and impact
  - Implications of long-lived projects
  - Conflicting benefits
  - Financial caps
  - Politics :(
- **Market failure:** market fails to make decisions where resources are allocated efficiently (total costs higher than total benefits)
  - Remedies:
    - Formal: government set policies, ability to seek compensation in court, government provision of goods and services
    - Informal: boycotts
- **Financing:**
  - Evaluating cost/benefit and other comparison methods can be tricky as there are multiple parties involved that may not all benefit
  - Uses a lower MARR than private sector
  - MARR used should be the interest rate on capital borrowed to fund a project

Quantifying Benefits and Costs (Triple Bottom Line):

- **Triple Bottom Line:** Performing economic analysis (business case) along with environmental and social analysis
- **Costs:**
  - Direct costs: directly attributable to a project
  - Indirect costs: stimulated indirectly by project
  - If project generates savings, these are deducted from the costs to produce a net cost
- **Benefits:**
  - All goods/services that result from project
  - Social costs are negative effects, and they are subtracted from benefits to product a net benefit
  - Challenging to measure benefits because they may not be reflected in the cash flows of the project
- **Analysis:** Example: consider a bridge being built to give better access to a park

- We consider the benefits to be the savings for those that already visit the park plus the benefit from increased use of the park (we consider only half of this benefit as it’s an approximation so we take the average)



Methods for Measuring Benefits:

- **Different types of value:**
  - Option value: the value of having the option available should you want to do it
  - Bequest value: the value of having the option available should someone else want to do it
  - Existence value: the value of having it available for the sake of it being in existence
- **Travel cost:** a proxy of value that uses the amount of money people spend to visit a place (eg. how much money do people spend to see a national park)
- **Hedonic price (property value):** the value of parts of the environment using aspects such as available view, location, etc
- **Averting behavior:** expenses spent on avoiding stuff (eg. money spent on bottled water, buying a safe car to reduce injury)
- **Wage:** value associated with performing risky work (eg. more money paid to clean up radioactive waste, workers settle for lower pay levels to live in a big city vs. places with fewer amenities)
- **Value of a statistical life:** A way to estimate the average value of risk reduction
- **Relocation cost:** monetary cost of relocating a facility that would be damaged by a change in environmental quality
- **Replacement cost:** cost to replace a damaged asset
- **Dose-response:** eg. how does air pollution affect the cost of crop production
- **Contingent valuation method:**

    - Hypothetical value of what certain things are worth to people
        - Eg. how much would you value air quality
    - Can consider WTP (willingness to pay) and WTA (willingness to accept)
    - Need to consider that people may not be 100% truthful in surveys and that surveys can be subject to sample bias

Inflation:

- Increase in average price paid for goods and services over time
- Price index: measures inflation by logging the price of a good every year in a table
    - Can measure the rate of increase using:
    - $$\% \; increase, n = \frac{Index\ (n) - Index(n-1)}{Index(n-1)} \times 100\%$$
    - Can measure average rate of increase (inflation rate) using the simple P to F formula: F = P(1+f)^n, where f is the inflation rate
- Consumer Price Index (CPI): measures price changes in food, shelter, medical care, transportation, apparel, etc
    - Can use average historical percentage increases from indexes to estimate costs and benefits
- Real interest rate i': interest rate that would yield the same number of real dollars in the absence of inflation as the current rate yields in the presence of inflation
    - Measures "real" growth of money in absence of inflation
- $$i' = \frac{1+i}{1+f} - 1$$
- Actual (current) vs. real vs. dollars:
    - Actual (also called current dollars): the number you see on your bill
    - Real: constant dollars that are inflation-free (represent purchasing power of a base year)
        - Eg. if something costs \$2200 one year from now, that is its actual cost. If inflation is supposed to increase by 10% over that year, it costs \$2000 in real dollars (with the current year being the base year)
    - Conversion of current dollars in year N (C_N) to real dollars in year N (R_N):
        - $$R_N = \frac{C_N}{(1+f)^N}$$
- Actual (current) vs. real MARR:
    - Actual: MARR when cash flows are expressed in actual dollars
    - Real: MARR when cash flows are expressed in real, or constant, dollars
    - Conversion: $\mathbf{MARR_C = MARR_R + f + MARR_R \times f}$
        - Can use same equation on IRR_C and IRR_R
    - Current IRR will be the real IRR plus an upward adjustment that reflects inflation

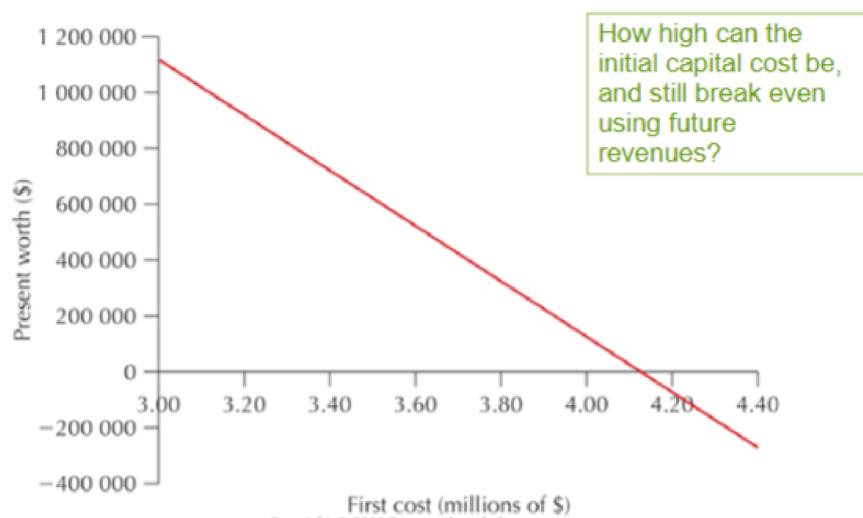
- Need to work with real cash flow and real MARR or actual cash flows and actual MARR
    - If they are different, can convert one to the other for analysis

Uncertainty:

- **Sensitivity analysis:** assess economic performance (net PW or IRR) by changing key parameters (such as initial cost, discount rate, salvage) one at a time
    - Ex:

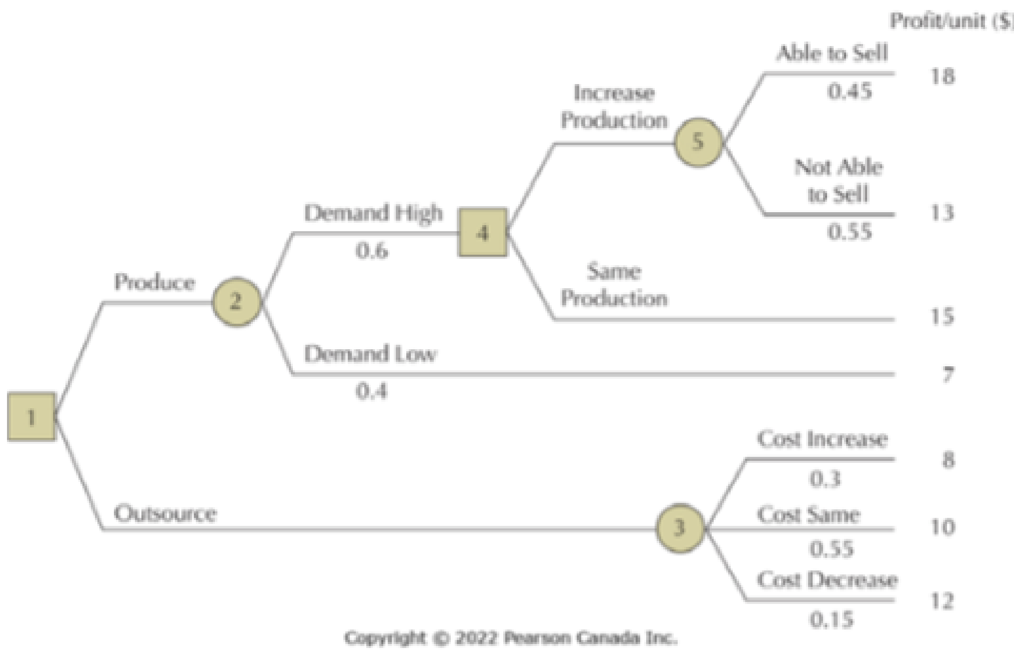
Category	−10%	−5%	Base Case	+5%	+10%
Initial investment	$1 426 343	$1 276 343	$1 126 343	$ 976 343	$ 826 343
Annual chemical, operations, and maintenance costs	1 174 894	1 150 619	1 126 343	1 102 067	1 077 792
Cooling tower overhaul (after 10 years)	1 126 890	1 126 617	1 126 343	1 126 069	1 125 796
Turbogenerator overhauls (after 4, 8, 12, and 16 years)	1 131 450	1 128 897	1 126 343	1 123 789	1 121 236
Annual wood costs	1 406 447	1 266 395	1 126 343	986 291	846 239
Savings in electricity costs	379 399	752 871	1 126 343	1 499 815	1 873 287
MARR	1 456 693	1 286 224	1 126 343	976 224	835 115

- **Break-even analysis:** analyze key parameters to see what we require to break even
    - One example of this is IRR which is how high the MARR can be and still allow us to break even
    - Another example:

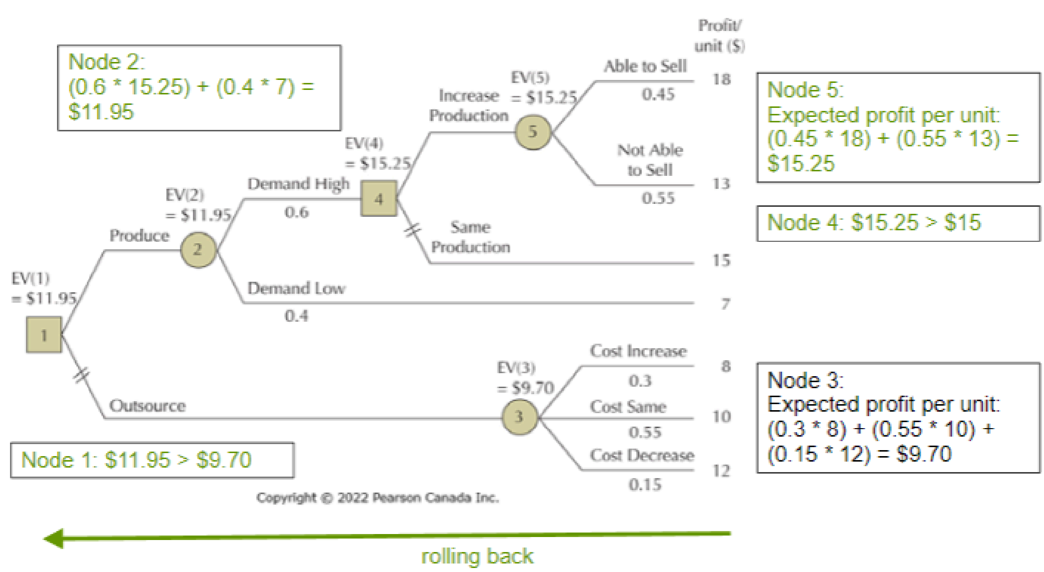


- **Decision Trees:** a graphical structure of the decision-making process between options

- First, sketch out the decision tree and label the options and probabilities associated with various outcomes:



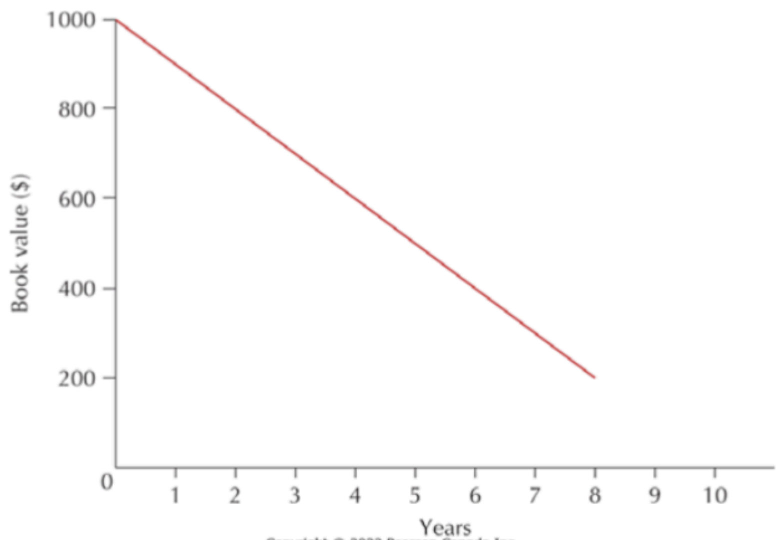
- Then compute the expected value for each node and select the branch that would be the most profitable



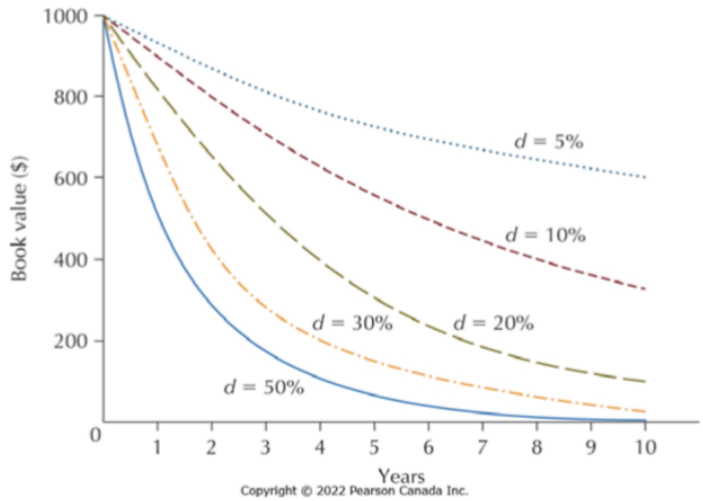
Accounting:

- Liabilities: debts owed
- Owner's equity: Assets - Liabilities
  - What the owner actually owns

Depreciation:

- Assets lose value, or depreciate, over time
    - It is a non-cash cost (doesn't appear in cash flows) but has a real cash impact because it affects profits and taxes
- Market value: actual value an asset can be sold for in open market
- Book value: value of an asset calculated from a depreciation model for accounting purposes
- Scrap value: actual or estimated value at the end of an asset's life when broken up for material value of its parts
- **<u>Straight-line depreciation:</u>**
    - Book value of asset determined by drawing straight line between its first cost and its salvage or cost value
    - 

        - Advantage: easy to calculate and understand
        - Disadvantage: not very realistic, most assets don't depreciate linearly
- **<u>Declining-balance depreciation:</u>**
    - Models loss in value of an asset over a period as a constant fraction of the asset's current book value



- Book value at the end of n periods:
  - $$BV_{db(n)} = P(1 - d)^n$$
    - P is the purchase price and d is the depreciation rate
  - Depreciation in period n:
  - $$D_{db}(n) = BV_{db}(n-1)\ x\ d$$
    - $D_{db}(n)$ is the depreciation change in period n
    - $BV_{db}(n)$ is the book value at the end of period n
- **CCA rate:** a declining-balance rate that is mandated for use in calculating the depreciation expenses for capital assets

Taxes:

- Governments tax individuals and businesses to support processes
  - Reduce the benefits of a successful project
  - Reduce the costs of an unsuccessful project
- For a individual, taxes are based on income
- For a corporation, taxes are based on gross income minus expenses (company making no profit pays no taxes)
  - Gross Profit: profit before taxes, (net sales - cost of goods sold)
  - Net Profit: profit after all expenses, including taxes
- Before and after taxes MARR (or IRR):
  - $$\mathbf{MARR}_{\text{after-tax}} \approx \mathbf{MARR}_{\text{before-tax}} \times (1 - t)$$
  - Where t is the corporate tax rate

- For a present worth comparison, each of the component cash flows is converted to a present worth and summed.
  - First cost:
    - Add the PW of tax savings due to the depreciation expense to the first cost
  - Savings or expenses:
    - Multiply by $(1 - t)$ and convert to PW
  - Salvage or scrap value:
    - Multiply by $(1 - t)$ and convert to PW
    - Can be more complicated as will be seen when we look at the Canadian tax system
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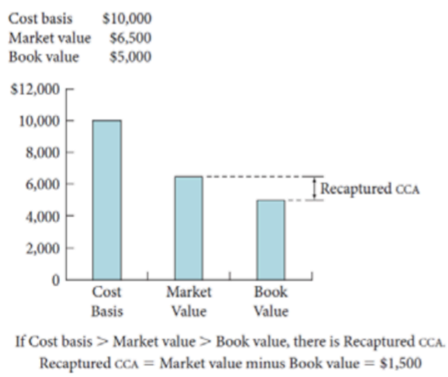
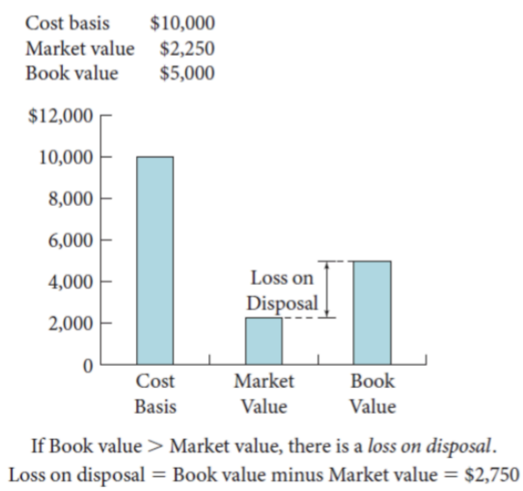
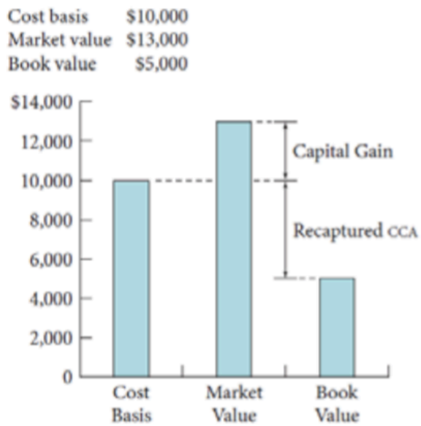
- Same calculations can be done on annual worth
- Recaptured CCA: when an asset sold for more than the book value
    - 

- Loss on disposal: when is sold for less than the book value
    - 

- Capital gains: occurs when an asset is sold for more than the original cost

- 

If Market value > Cost basis, there is a *capital gain* plus *recaptured depreciation.*
Capital gain = Market value minus Cost basis = $3,000
Recaptured depreciation = Cost basis minus Book value = $5,000

WACC (Weighted Average Cost of Capital):

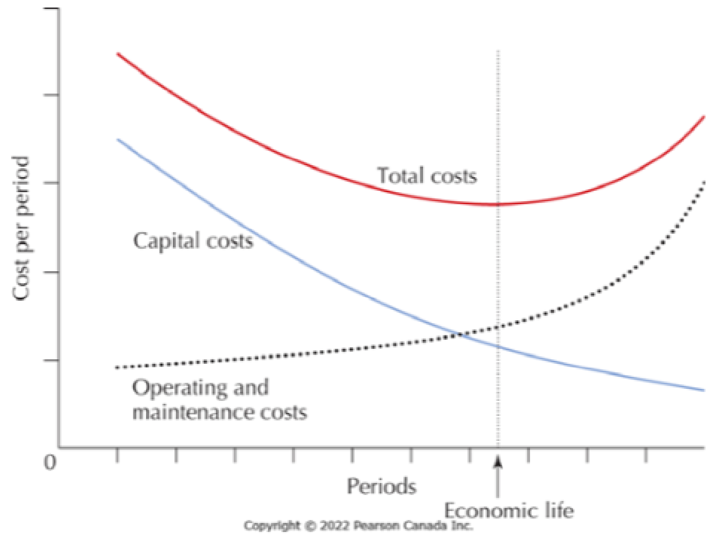
- Two ways of raising capital from investors:
  - Debt (from lenders)
  - Equity (from owners)
  - Equity is more risky than debt because reliance on debt is limited both by the company (for fear of not being able to meet obligations from the lender) and by the lenders (for fear of not having their money returned)
- Different sources of capital may have different weights in terms of the <u>amount</u> of capital and different <u>rates of return</u>.
- The WACC is the sum of (fraction of total capital)(ROR) for each source of capital
- WACC is a coarse measure of how burdened a firm is by borrowing costs
- WACC can be used as a MARR
- Ex: the WACC can be computed as the expected value of the interest rate

Source of Capital	Amount Raised	Interest Rate
Milan Bank	$15M	9.0%
Chemical Bank	$18M	8.5%
Stock	$57M	7.8%
Bonds	$60M	6.0% (effective annual yield to maturity)

Asset Replacement:

- Costs:
  - Capital costs (P): difference between price paid and what it can be sold for later (usually expressed as EAC)
  - Installation costs (I)
  - Operating and maintenance costs (O&M)
- When to replace:

- As the life of the asset goes on the operating and maintenance costs are expected to increase so we want to determine the optimal time to replace the asset.



- Can do this by computing the AW analysis for if we replace it after 1 year, 2 years, 3 years, etc. We can compare the AW for each option and select the replacement time that gives the smallest AW

- Defender vs. challenger
    - When looking to replace, see what the costs would be for keeping the defending asset for one more year and compare that to the annual cost of the challenging asset. If the defender costs more then replace it

Project Management:

- Work breakdown structure: categorizes all the work to be done on a project into smaller units
- Gantt charts: depict timing and sequence of project activities
- Critical path: set of activities that must be completed exactly as planned to keep project on schedule
    - To find critical path need two passes through the network
    - First pass: find the earliest start/finish times for each activity
    - Second pass: Find the latest time each activity can finish and have the project still be on schedule
- Slack: the difference between the latest start time and earliest start time of an activity
    - Activities that lie in the critical path will have no slack