Investigating the relationship between horizontal and vertical motion of a projectile

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INTRODUCTION

Hypothetical situation: A hunter with a blowgun encounters a monkey hanging in a tree, at the same level as the hunter's head. The hunter must shoot down the animal. However, it is here when a problem is revealed. Where should the hunter aim to hit the monkey falling from the tree? This question, in order to be answered, requires a thorough analysis and demonstration of Newton's second law of motion.

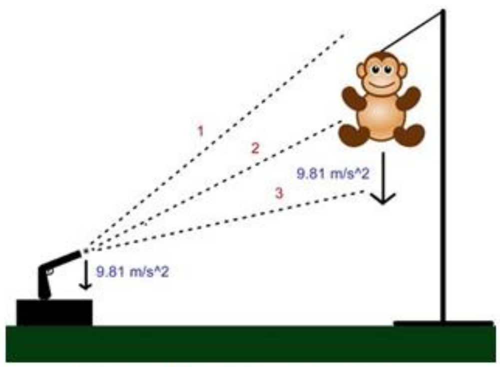

Figure 1: The monkey and the hunter

THE INVESTIGATION

Research Question: What is the relationship between the horizontal and vertical motion of a projectile?

My hypothesis is that, in order to hit the target, one may aim in three possible directions: above, below, or at the target itself.

Constants: Acceleration due to gravity; launch velocity of projectile

Controlled variables: Drop height of target

Materials: Backboard assembly with release mechanism and dart launcher; heavy books; monkey target

Figure 2: The experimental set consisting of the backboard, dart and spring

The initial step to this experimentation was to test the different aiming lines of the projectile. I decided to experiment with launching the trajectory directly at the paper monkey target cutout and record the results of the launch the dart and the monkey target were released at the same time. The drop height of the target is 24 centimeters. The aiming path measurements are shown in the table below.

x	2	4	6	8	10	12
y	4	8	12	16	20	24

Table 1: Measurements of the trajectory's aiming line. All measurements are in cm

What can be seen from the table above is that the given values are liable to produce a straight line when plotted in a graph. The measurements of the trajectory's actual path are shown below.

x	2	4	6	8	10	12
y	3.5	6.2	9.3	10.6	11	10.6

Table 2: Measurements of the trajectory's path

Thus, these values allow for a comprehensive graph diagram to help visualize the relationship between the two.

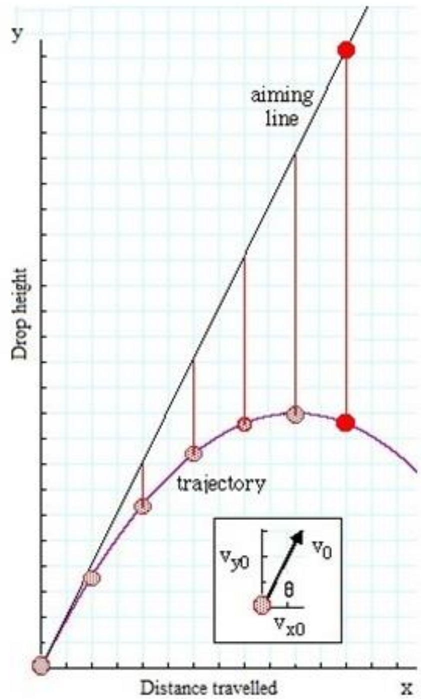

Figure 3: Distance-against-height graph, showing the aiming line and the actual trajectory path

From the graph above it can be seen that while the aiming line of the projectile is a straight line, the actual path that the dart undertakes is in the shape of a parabola. The lines between the path and the aiming line represent the amount the target has fallen below aiming line.

Figure 4: The interception of the dart and the target

With the actual experiment performed, the results show that the dart, when aimed directly at the monkey target, does indeed intercept it when the two are simultaneously released. This confirms the graph's validity that the two meet at a certain point.

While the result of the experiment was the desired interception of the projectile and the target, a technical factor in the demonstration kit may have affected the accuracy of the recorded outcome. To make the dart launch an elastic band was used; hence, the motion of the trajectory is not truly free fall but rather a push caused by elasticity. In addition, there may have been miscalculations in the measurements of the actual monkey target's position.

KINEMATICS

Given the previously performed experiments with projectile motion, it is still unknown why shooting directly at a falling target is a valid option. Therefore, we need to analyze the kinematics behind all types of motion involved in the trial.

DART'S MOTION

Assume that the elastic band's force acts over a very short duration of time so that the dart is launched from the ground ($y_i = 0$) toward the monkey in the tree ($y = h$) with an initial velocity, v, and at an angle, θ, with respect to the ground. The horizontal and vertical components of the dart's velocity vector are therefore $v_x = v \cos \theta$; $v_y = v \sin \theta$, respectively. The time it takes for the dart to travel the horizontal distance d is equal to: $t = d/v_x$. In the same time, $t = d/v_x$, the dart's vertical position (y_f) will change as a function of the initial vertical velocity component (v_y) and of the constant acceleration due to gravity, -g (the negative sign designates downward acceleration).

$$y_f = \frac{1}{2}at^2 + v_y t + y_i$$

$$y_f = -\frac{1}{2}g(d/v_x)^2 + v_y(d/v_x) + 0$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + dv\sin\theta/v\cos\theta$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + d\tan\theta$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + dh/d$$

MONKEY'S MOTION

The monkey is initially at rest and then only moves in the vertical direction as a result of the force of gravity pulling it down. The monkey's vertical position after the same flight time of the dart (d/v_X) is shown in the equations below.

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + v_y t + h$$
$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + h$$

Equations 6 and 8 are identical, meaning the final height of the dart and monkey target will be the same after the dart has traveled the horizontal distance, d. As long as the hunter aims the dart directly at the monkey, and the dart and monkey are released simultaneously, the dart will hit the monkey on the way down.

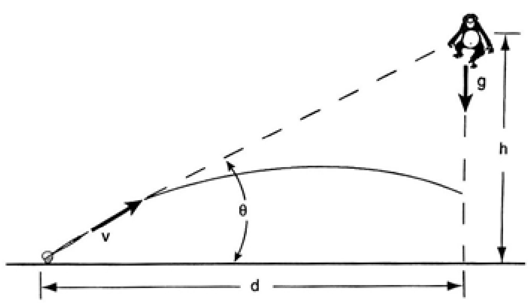

Figure 3: Distance of the projectile against the drop height of the falling target

DISCUSSION

Physicist Galileo Galilei proposed that all objects fall at the same increasing rate (in a vacuum), something that Isaac Newton verified. Regardless of their mass all objects will accelerate toward the Earth equally. For instance, in a vacuum, when there is no drag friction due to air, a heavy hammer will fall at exactly the same rate as a light feather. This was demonstrated during the Apollo 15 moon landing when David Scott dropped a hammer and a feather at the same time and watched them hit the lunar surface at the same time, announcing that Galileo was correct. At the surface of the Earth, the acceleration toward the center of the Earth experienced by all objects is measured to be (on average) 9.8 m/s^2.

Newton also demonstrated that forces can be separated into horizontal and vertical components that are independent of each other. Thus, for a force that pushes a ball up at an angle with respect to the ground, the force is said to have one force component that is vertical and one that is horizontal. Both force components depend on the total force and on the angle of the force with respect to the ground, but they are independent of each other. A vertical force will have no effect in the magnitude of the horizontal force component, and vice versa.

In the Shoot the Monkey demonstration, both the constant acceleration of gravity and the independence of component forces can be observed and studied. The two objects (the dart and monkey target) are released from the same height at the same time. Because they are both acted on by the force of gravity that pulls them toward the Earth with the same acceleration, the dart and target will fall the same distance in a given amount of time. This means that as long as they are released at the same time, they will always be at the same relative height in a given time.

The component forces acting on the dart are a horizontal force from the pull of the stretched elastic band launching it toward the backboard, and a vertical force of gravity pulling down. The target is initially at rest with no vertical or horizontal movement. The target will only be acted on by the pull of gravity. When the dart is released, it is launched horizontally and it begins to fall with a constant acceleration of 9.8 m/s^2. The target drops at the same moment the dart is released and also begins to fall with an acceleration of 9.8 m/s^2. The dart and target will always be at the same relative height as they fall, but the distance between them quickly decreases as the dart travels toward the board. No other vertical forces act on the dart or the target besides gravity so each object will continue to fall at the same rate until the horizontally traveling dart hits the target. The monkey target's path is just a straight vertical line. The dart's path is not straight - it is in the shape of a parabola.

CONCLUSION

In my conclusion I will relate the results I obtained in the experiment with what I expected to get. My experiment investigated the relationship between the launch angle of a projectile and the drop height of a falling object. From both the theoretical equations and the experiments, I was able to conclude that a projectile's path at an angle is not straight but in the shape of a parabola. I initially expected the dart to travel in a straight line until it hit a surface; however, my graph showed otherwise. Regardless of the angle for the aiming line, the launched trajectory will not continuously travel in a straight line but gradually descend much like a parabola. Therefore, regardless of the drop height, the dart will always hit the monkey if it is being fired directly at the target and has sufficient velocity. Hence my original interpretation of the relationship between horizontal and vertical components of motion was invalid. The most significant problem in my experimentation was the data measurements and their uncertainties. To improve the quality of data and produce more accurate results I would consider the following things.

I would try to obtain a projectile-launching device, such as a toy gun, instead of employing a tedious and complex mechanism that involves tying a number of strings to a single dart and using an elastic band which jeopardizes the accuracy of data.

In addition, I would try to use a faux model of an actual monkey in order to bring more realism and produce accurate results. In the experiment I used a paper cutout of a monkey which is vastly different from an object with the mass and size of an actual animal.

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