Investigating the Effect of Gravity on Projectile Motion

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1 Introduction

Hypothetical situation: A hunter with a blowgun encounters a monkey hanging in a tree, at the same level as the hunter's head. The hunter must shoot down the animal. However, it is here when a problem is revealed. Where should the hunter aim to hit the monkey falling from the tree? This question, in order to be answered, requires a thorough analysis and demonstration of Newton's second law of motion.

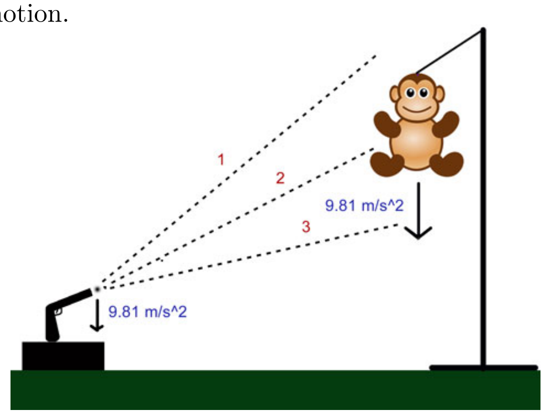

Figure 1.1 The monkey and the hunter

2 The Investigation

Research Question: How does gravity influence the interception of a projectile with a falling object?
Constants: Acceleration due to gravity; Projectile motion; Falling objects
Materials: Backboard assembly with release mechanism and dart launcher; Heavy books; Monkey target
Independent variable: dropping height

3 Dart's motion

Assume that the elastic band's force acts over a very short duration of time so that the dart is launched from the ground ($y_i = 0$)toward the monkey in the tree (y = h) with an initial velocity, v, and at an angle, θ, with respect to the ground. The horizontal and vertical components of the dart's velocity vector are therefore $v_x = v\cos\theta; v_y = v\sin\theta$, respectively. The time it takes for the dart to travel the horizontal distance d is equal to: t = d/v_x.
In the same time, t = d/v_x,the dart's vertical position (y_f) will change as a function of the initial vertical velocity component (v_y) and of the constant acceleration due to gravity, -g (the negative sign designates downward acceleration).

$$y_f = \frac{1}{2}at^2 + v_y t + y_i$$

$$y_f = -\frac{1}{2}g(d/v_x)^2 + v_y(d/v_x) + 0$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + dv\sin\theta/v\cos\theta$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + d\tan\theta$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + dh/d$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + h$$

4 Monkey's motion

The monkey is initially at rest and then only moves in the vertical direction as a result of the force of gravity pulling it down. The monkey's vertical position after the same flight time of the dart (d/v_x) is shown in the equations below.

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + v_y t + h$$

$$y_f = -\frac{1}{2}(d/v\cos\theta)^2 + h$$

Equations 6 and 8 are identical, meaning the final height of the dart and monkey target will be the same after the dart has traveled the horizontal distance, d. As long as the hunter aims the dart directly at the monkey, and the dart and monkey are released simultaneously, the dart will hit the monkey on the way down.

5 Discussion

What Galileo proposed and Newton verified is that all objects fall at the same increasing rate (in a vacuum). That is, all objects will accelerate toward the Earth equally, regardless of their mass. In a vacuum, when there is no drag friction due to air, a heavy hammer will fall at exactly the same rate as a light feather. This was demonstrated during the Apollo 15 moon landing when David Scott dropped a hammer and a feather at the same time and watched them hit the lunar surface at the same time, proudly announcing that Galilelo was correct. At the surface of the Earth, the acceleration toward the center of the Earth experienced by all objects is measured to be (on average) 9.8 m/s^2. Newton also demonstrated that forces can be separated into horizontal and vertical components that are independent of each other. Thus, for a force that pushes a ball up at an angle with respect to the ground, the force is said to have one force component that is vertical and one that is horizontal. Both force components depend on the total force and on the angle of the force with respect to the ground, but they are independent of each other. A vertical force will have no effect in the magnitude of the horizontal force component, and vice versa.

In the Shoot the Monkey demonstration, both the constant acceleration of gravity and the independence of component forces can be observed and studied. The two objects (the dart and monkey target) are released from the same height at the same time. Because they are both acted on by the force of gravity that pulls them toward the Earth with the same acceleration, the dart and target will fall the same distance in a given amount of time. This means that as long as they are released at the same time, they will always be at the same relative height in a given time.

The component forces acting on the dart are a horizontal force from the pull of the stretched elastic band launching it toward the backboard, and a vertical force of gravity pulling down. The target is initially at rest with no vertical or horizontal movement. The target will only be acted on by the pull of gravity. When the dart is released, it is launched horizontally and it begins to fall with a constant acceleration of 9.8 m/s^2.The target drops at the same moment the dart is released and also begins to fall with an acceleration of 9.8 m/s^2.The dart and target will always be at the same relative height as they fall, but the distance between them quickly decreases as the dart travels toward the board. No other vertical forces act on the dart or the target besides gravity so each object will continue to fall at the same rate until the horizontally traveling dart hits the target. The monkey target's path is just a straight vertical line. The dart's path is not straight - it is in the shape of a parabola.

6 Conclusion

7 References

http://www.physicscentral.com/explore/action/monkey-hunter.cfm