Assignment 9

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May 2022

Outline

Question

Suppose that the voltage v is a random variable given by $v = i(r + r_0)$, where $i = 0.01$ A, $r_0 = 1000\Omega$. If the resistance r is a random variable with uniform distribution between 900Ω and 1100Ω, what is the distribution of the voltage v?

Distribution of voltage v

Voltage is given by,

$$v = i(r + r_0) \tag{1}$$

As resistance r lies between 900Ω and 1100Ω, voltage v lies between $19V$ and $21V$.

Formula

The probability density function is given by

$$f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|} + \frac{f_X(x_2)}{|g'(x_2)|} + \frac{f_X(x_3)}{|g'(x_3)|} + ... + \frac{f_X(x_n)}{|g'(x_n)|}$$

where, n is the number of solutions.

Consider the equation $y = \frac{1}{x}$. It has a single solution $x = \frac{1}{y}$. Thus, we have,

$$f_Y(y) = \frac{1}{y^2} f_X(\frac{1}{y}) \tag{2}$$

Cauchy density

$$f_X(x) = \frac{\alpha/\pi}{x^2 + \alpha^2} \text{is a Cauchy density with parameter } \alpha \tag{3}$$

$$f_Y(y) = \frac{1/\alpha\pi}{y^2 + 1/\alpha^2} \text{is a Cauchy density with parameter } 1/\alpha \tag{4}$$

Conductance is given by,

$$g = \frac{1}{r} \tag{5}$$

Using (2), we get,

$$f_g(g) = \frac{1}{g^2} f_r(r) \tag{6}$$

Also, since $f_r(r) = \frac{1}{200}$, for r between 900 and 1100, it follows from (2) that,

$$f_g(g) = \frac{1}{200g^2} \tag{7}$$

Graphs

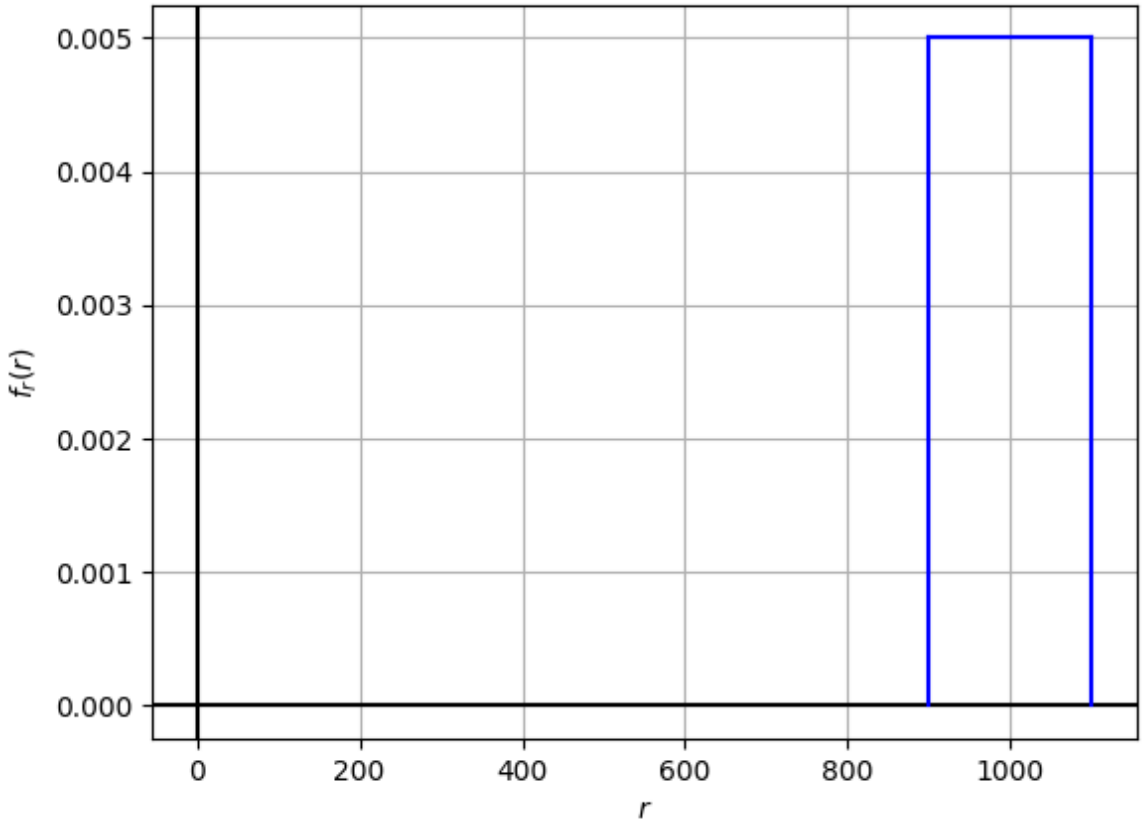

Figure 0: P.D.F. of resistance r

Graphs

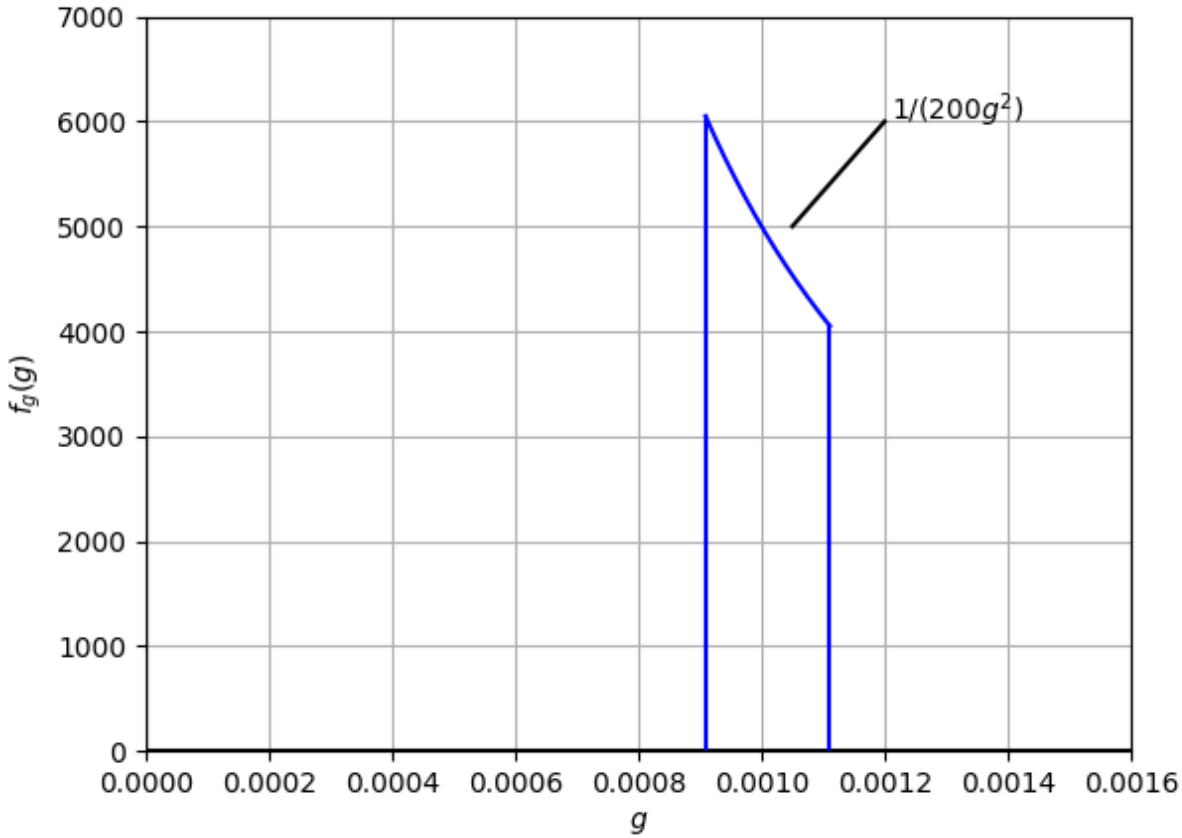

Figure 0: P.D.F. of conductance g