Assignment 11

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Outline

Question

The process x(t) is W.S.S. (*Wide Sense Stationary*) with $R_{XX}(t) = 5\delta(t)$ and

$$y'(t) + 2y(t) = x(t) \tag{1}$$

Find $Ey^2(t), R_{xy}(t_1, t_2), R_{yy}(t_1, t_2)$ if

(a) (1) holds for all t

(b) $y(0) = 0$ and (1) holds for $t \geq 0$

Solution

$$y(t) = x(t) \times h(t) \tag{2}$$

$$h(t) = e^{-2t}U(t) \tag{3}$$

Part (a)

$$\implies E\{y^2(t)\} = 5 \times e^{-4t} U(t) \tag{4}$$

$$E\{y^2(t)\} = \frac{5}{4} \tag{5}$$

$$\implies R_{xy}(t_1, t_2) = 5\delta(t_1 - t_2) \times e^{-2t_2} U(t_2) \tag{6}$$

$$R_{xy}(t_1, t_2) = 5e^{-2(t_2 - t_1)} U(t_2 - t_1) \tag{7}$$

$$R_{xy}(\tau) = 5e^{-2\tau} U(\tau) \tag{8}$$

$$\implies R_{yy}(t_1, t_2) = 5e^{-2(t_2 - t_1)} U(t_2 - t_1) \times e^{-2t_1} U(t_1) \tag{9}$$

$$R_{yy}(t_1, t_2) = \frac{5}{4} e^{-2|t_2 - t_1|} \tag{10}$$

$$R_{yy}(\tau) = \frac{5}{4} e^{-2|\tau|} \tag{11}$$

Part (b)

For $t_1 < 0$ or $t_2 < 0$,

$$R_{xy}(t_1, t_2) = 0 \tag{12}$$

$$R_{yy}(t_1, t_2) = 0 \tag{13}$$

Part (b) - For $0 < t_1 < t_2$

$$\implies R_{xy}(t_1, t_2) = 5\delta(t_1 - t_2) \times e^{-2t_2} \tag{14}$$

$$R_{xy}(t_1, t_2) = 5e^{-2t_2} \tag{15}$$

$$\implies R_{yy}(t_1, t_2) = \int_0^{t_1} 5e^{-2(t_1-\tau)} e^{-2(t_1-\tau)} d\tau \tag{16}$$

$$R_{yy}(t_1, t_2) = \frac{5}{4} e^{-2(t_2-t_1)}(1 - e^{-4t_1}) \tag{17}$$

Part (b) - For $0 < t_2 < t_1$

$$\implies R_{xy}(t_1, t_2) = 5\delta(t_1 - t_2) \times e^{-2t_1} \tag{18}$$

$$R_{xy}(t_1, t_2) = 5e^{-2t_1} \tag{19}$$

$$\implies R_{yy}(t_1, t_2) = \int_0^{t_1} 5e^{-2(t_1-\tau)} e^{-2(t_1-\tau)} d\tau \tag{20}$$

$$R_{yy}(t_1, t_2) = \frac{5}{4} e^{-2(t_2-t_1)}(1 - e^{-4t_1}) \tag{21}$$