Assignment 10

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June 2022

Outline

Question

Show that central limit theorem does not hold if the random variables x_i have a Cauchy density.

Central Limit Theorem

Definition

If $X_1, X_2, X_3, ..., X_n$ are random samples drawn from a population with overall mean μ and finite variance σ^2, and $\bar{X}_n$ is sample mean of first n samples, then the limiting form of the distribution, $Z = \lim_{n\to\infty}\left(\frac{\bar{X}_n-\mu}{\frac{\sigma}{\sqrt{n}}}\right)$, is a standard normal distribution.

In other words, the central limit theorem (CLT) states that the distribution of sample means approximates a normal distribution as the sample size gets larger, regardless of the population's distribution.

Conditions for CLT

In order for Central Limit Theorem to hold good, the distribution needs to have finite mean (μ) and finite variance (σ^2).

Cauchy distribution

Let the random variable X_i have a Cauchy density

$$f_{X_i}(x) = \frac{c_i}{\pi(c_i^2 + x^2)} \tag{1}$$

where, c_i is the parameter.

Variance

$$\text{Variance} = \sigma^2 = \int_{-\infty}^{\infty} x^2 . f_{X_i}(x) dx \tag{2}$$

$$\sigma^2 = \frac{c_i}{\pi} \int_{-\infty}^{\infty} \frac{x^2}{c_i^2 + x^2} dx \tag{3}$$

$$\sigma^2 = \frac{c_i}{\pi} \int_{-\infty}^{\infty} \left(1 - \frac{c_i^2}{c_i^2 + x^2} \right) dx \tag{4}$$

$$\sigma^2 = \infty \tag{5}$$

Thus, variance of a Cauchy distribution is infinity, which is why, Central Limit Theorem does not hold good.