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1.1 [...]

In any setting where we have a monoid M acting on a set S , and a subset $X \subset M$, we can form the graph $\Gamma_{\bullet}^{S/X}$ with

$$\Gamma_0^{S/X} := S \quad \text{and} \quad \Gamma_1^{S/X} := X \times S,$$

where the *source* and *target* maps

$$\partial_0^1, \partial_1^1 : \Gamma_1^{S/X} \longrightarrow \Gamma_0^{S/X}$$

that interpret each pair $(\chi, s) \in X \times S = \Gamma_1^{S/X}$ as the arrow

$$s \xrightarrow{\chi} \chi s.$$

In general, it's hard to "zoom out from" $\Gamma_{\bullet}^{S/X}$ to say anything substantive about the large-scale structure of this graph. That said, there are many special cases where we can say a lot about the structure of this graph. In the special case that - $S = \mathbb{Z}/\ell\mathbb{Z}$, the set of integers modulo some positive integer ℓ , - $M = \mathbb{Z}/\ell\mathbb{Z}$ equipped with its multiplicative structure,

we can say quite a bit about the structure of $\Gamma_{\bullet}^{S/X}$. The positive integer ℓ has unique prime factorization

$$\ell = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$$

The graph $\Gamma_{\bullet}^{S/X}$ contains a subgraph $\text{Div}(\ell)_{\bullet}$ with

$$\text{Div}(\ell)_0 := \{d \in \mathbb{Z}_{>0} : d \text{ divides } \ell\}$$

and

$$\text{Div}(\ell)_1 := \{d \xrightarrow{p} pd : pd \text{ divides } \ell, p \text{ prime}\}$$

[GR14] [Tym11] [Coh12]

References

- [Coh12] Richard Cohn. *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature*. Oxford Studies in Music Theory. Oxford University Press, January 2012.
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- [Tym11] Dmitri Tymoczko. *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice*. Oxford Studies in Music Theory. Oxford University Press, March 2011.