Digital signal processing

The fundamental concepts

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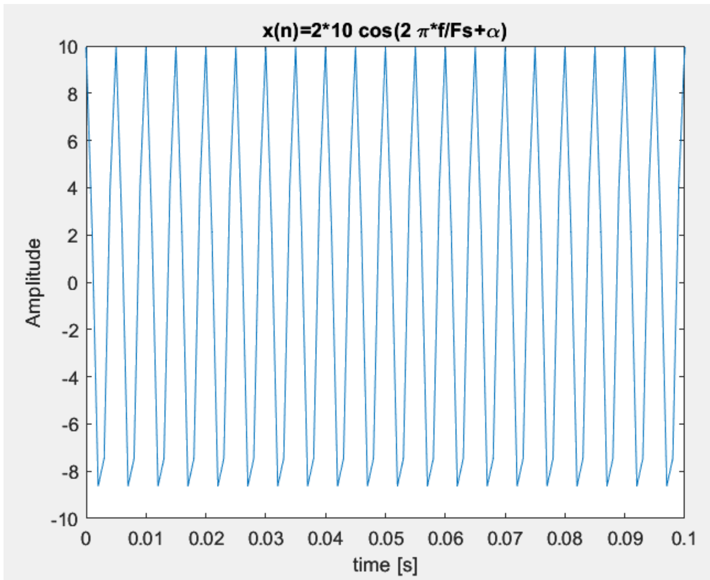
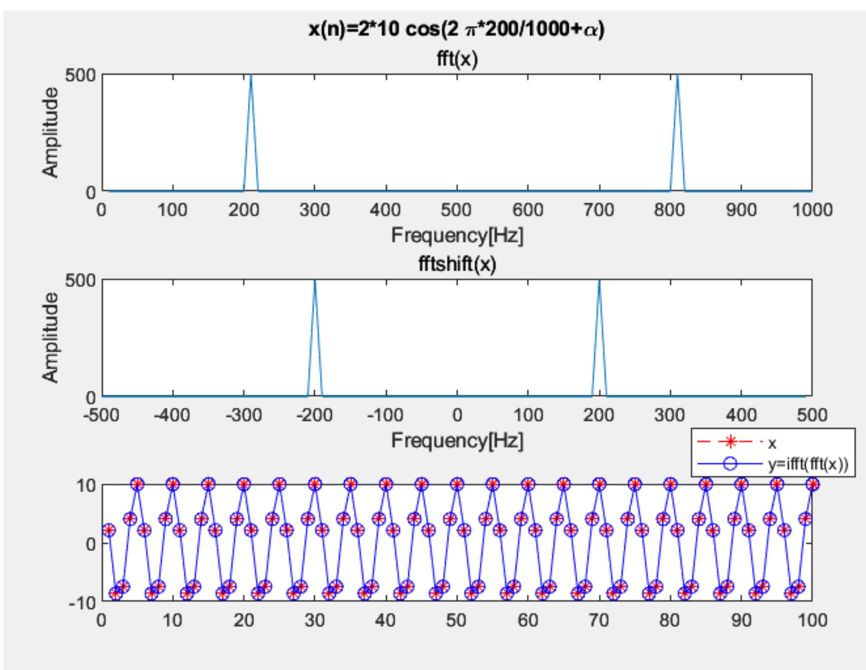
1 signal

$$x(n) = 2 * 10 \cos\left(2\pi \frac{f}{Fs} + \alpha\right)$$



2 FFT and FFTshift

$$x(n) = 2 * 10 \cos\left(2\pi \frac{200}{1000} + \alpha\right)$$



Modify the frequency range

$N = 100$ samples, sampling rate Fs=1000

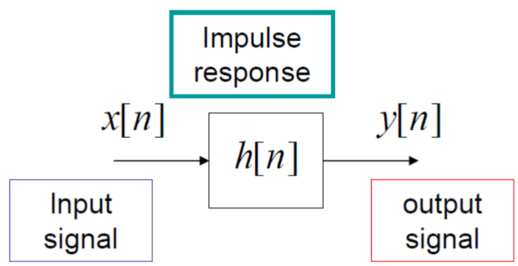
Frequency range:

1st plot: $F = (1{:}1{:}N) * Fs/N$

2nd Plot: $F = (-\frac{N}{2}{:}1{:}\frac{N}{2} - 1) * Fs/N$

y(n)=ifft(fft(x))

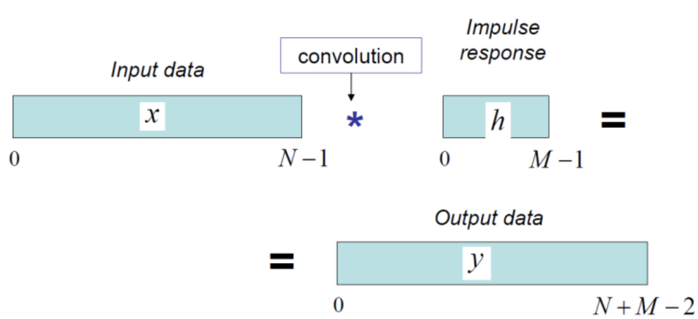
3 Convolution



$$y[n] = h[n] * x[n] = \sum_{\ell=-\infty}^{+\infty} h[\ell]x[n-\ell]$$

- Discrete convolution

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + \ldots + h[M-1]x[n-M+1]$$

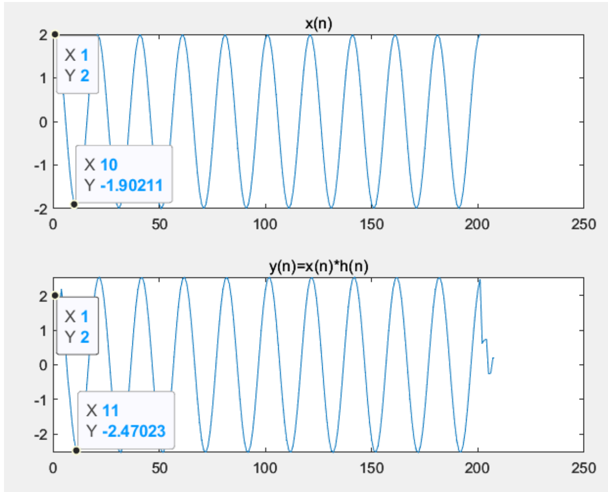


h=[1,0,0,0.5,0,-0.2,0,0.1]; % impulse response

n=0:200; x=2*cos(0.1*pi*n); % input signal

y=conv(x,h); % output signal

plot(y)



4 Auto Correlation

For a signal with zero mean, to see if the samples are "correlated" with each other.

Definition of Auto Correlation: $r_x[m] = \sum_{n=-\infty}^{+\infty} x[n]x^*[n-m]$

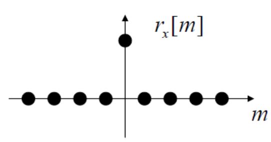
Again, you deal with signals of finite length

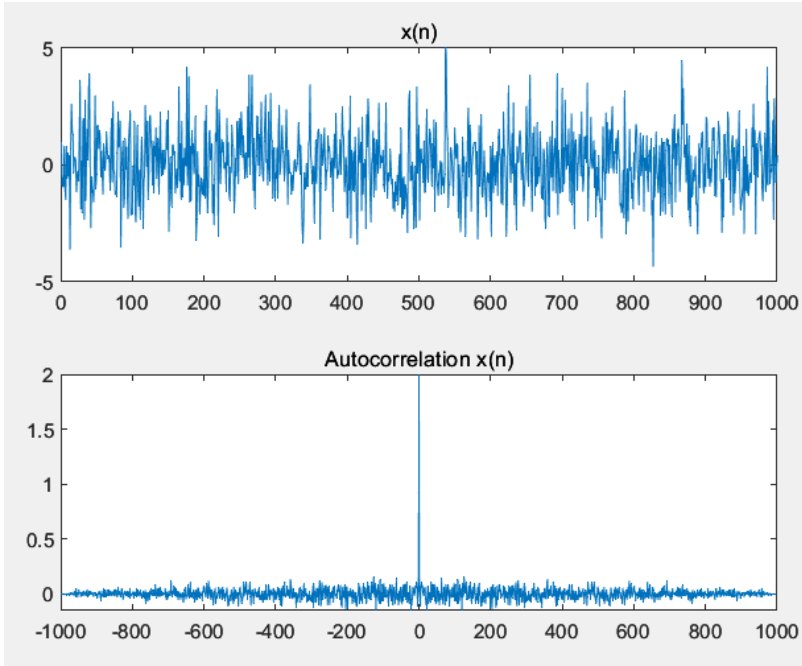
$$x[n],\ n = 0, \ldots, N-1$$

$$\boxed{r_x[m] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]x^*[n-m], \quad m = -N+1, \ldots, N-1}$$

Example: White Noise with standard deviation σ_x

$$r_x[m] = \begin{cases} \sigma_x^2 = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2, & \text{if } m = 0 \\ \approx 0, & \text{otherwise} \end{cases}$$



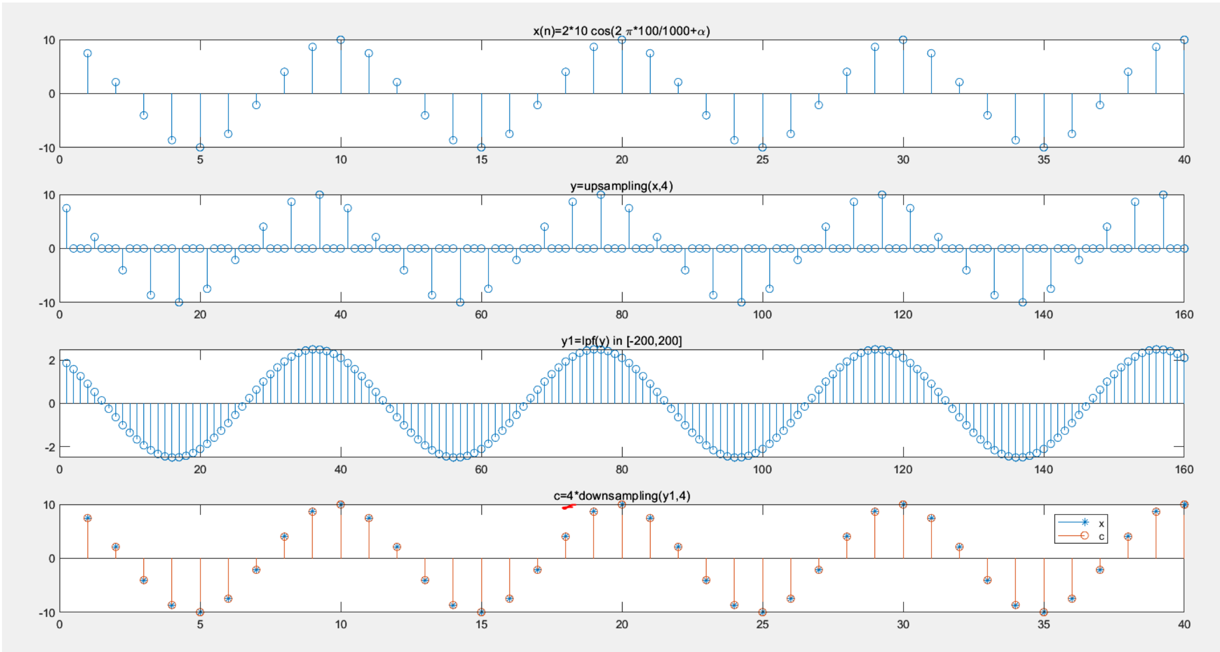


5 Upsampling and downsampling

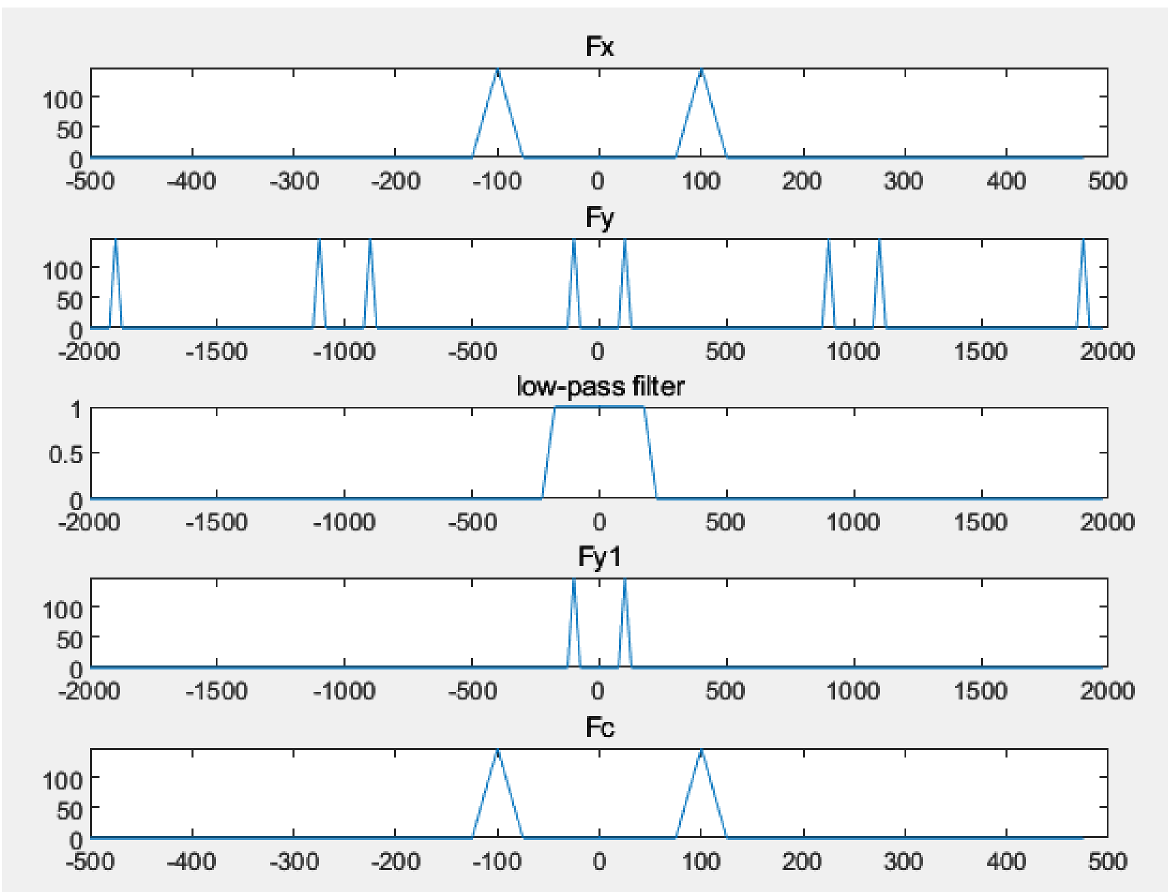
- Upsampling: including 2 steps:
  i) Upsampling
  ii) low-pass filter

- Downsampling
  i) downsampling with scaling in the amplitude.

- Analysis in the frequency domain
  i) Pay attention to the amplitude
  ii) The periodic frequency after upsampling
  iii) The amplitude after downsampling



Time domain



Frequency domain

Reference

[1]. rectangularPulse, https://www.mathworks.com/help/symbolic/sym.rectangularpulse.html#btke0hr-47

[2]. Upsampling and downsampling, https://www.geeksforgeeks.org/what-is-upsampling-in-matlab/

[3]. Roberto Cristi, Wireless Communications with Matlab and Simulink: IEEE802.16 (WiMax) Physical Layer, https://faculty.nps.edu/rcristi/WiMax/2009-08-20-WiMax.pdf

[4].