

High-speed data transmission with PAM4 signal

A short preparation summary

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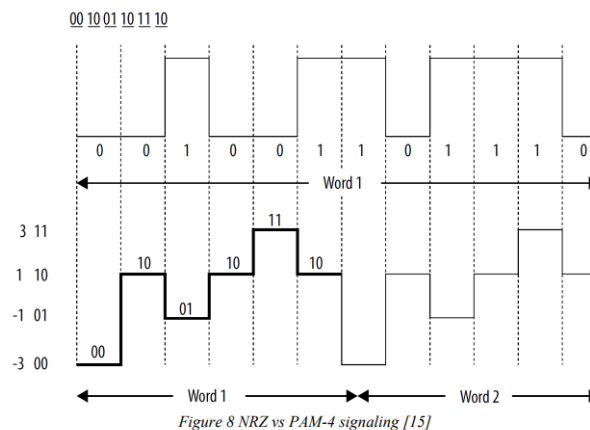
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1 Introduction

The frequency selective fading and the Inter-symbol interference pose great challenges to achieve high-speed data transmission. The current notes cover some basic topics related to the PAM4 signal, the channel, and the equalization algorithms. A comparison of error probabilities of those equalization algorithms and the AWGN channel is presented in Figure 18. The numerical results show that the equalization algorithms can significantly improve the accuracy of high-speed communication. Note that the simulations are built on discrete symbol-wise operations, not continuous signals.

2 PAM4 signal

- **Motivation:** to achieve a higher rate compared to NRZ, we introduce 4 levels of modulation, that is PAM4. Higher-level PAM is limited by the sensitivity of materials.
- **eg:** NRZ: 16GBaud \rightarrow 16Gb/s, PAM: 16GBaud \rightarrow 32 Gb/s
- **Modulation:** given a binary string, the NRZ and PAM modulation are shown below



Based on the above figure, PAM4 can reduce half the number of required symbols for data transmission. However, with more levels, PAM4 suffers a higher error probability.

- **Distinctive transitions:**
 - a) **NRZ:** contains only 1 rising edge and 1 falling edge
 - b) **PAM4:** includes 6 rising edges and 6 falling edges that result in 12 transitions

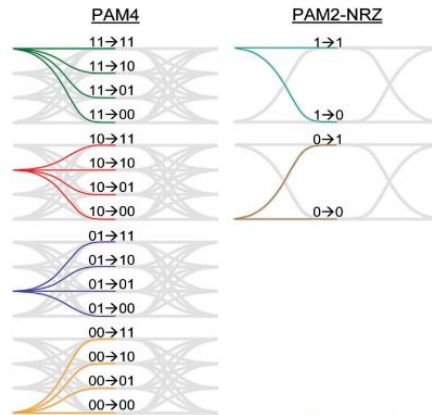


Figure 9 Distinctive transitions of PAM-4 vs NRZ

- **eye diagram:**

a) **NRZ:** two levels of voltage with a single eye

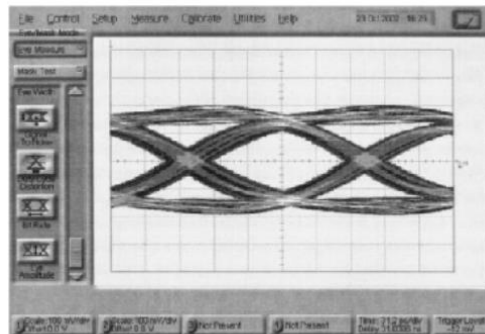


Figure 5 NRZ eye diagram [14]

b) **PAM4:** 4 levels of voltage with three eyes

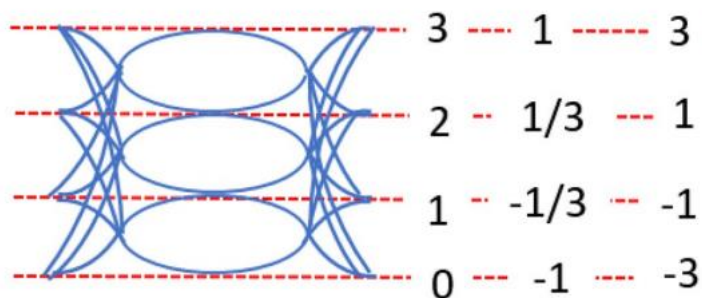
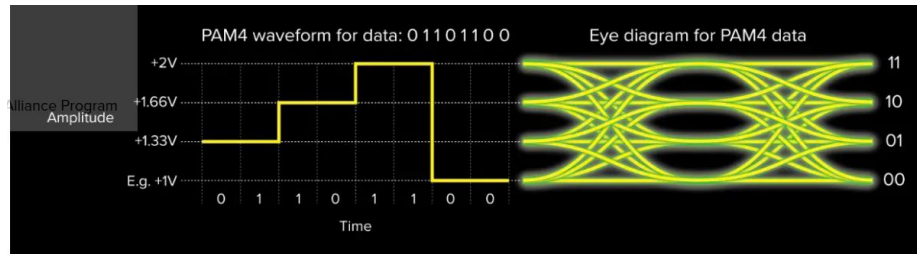


Figure 7 PAM-4 voltage levels and naming conventions

- **PAM encoder:** by using the Gary code, every two adjacent levels of voltage only differ by one bit, reducing the number of bits[1].



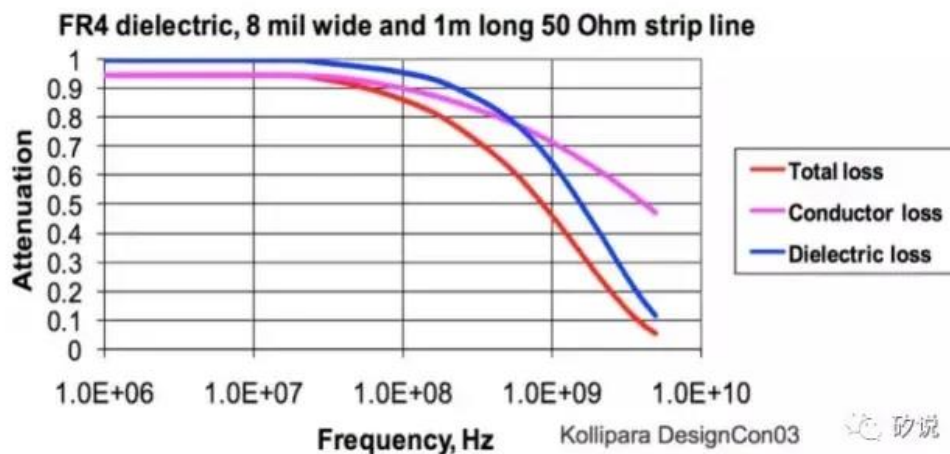
2.1 Up-sampling[9]

Two upsampling methods:

- **Expansion:** Create a sequence $x_L[n]$, comprising the original samples $x[n]$, separated by $L - 1$ zeros.
- **Interpolation:** Smooth out the discontinuities with a *lowpass filter*, which replaces the zeros.

3 Channel models[5][7][13][14][19]

- **Channel properties:** the channel is similar to a low pass filter (LPF). It leads to higher loss for high frequency signals.

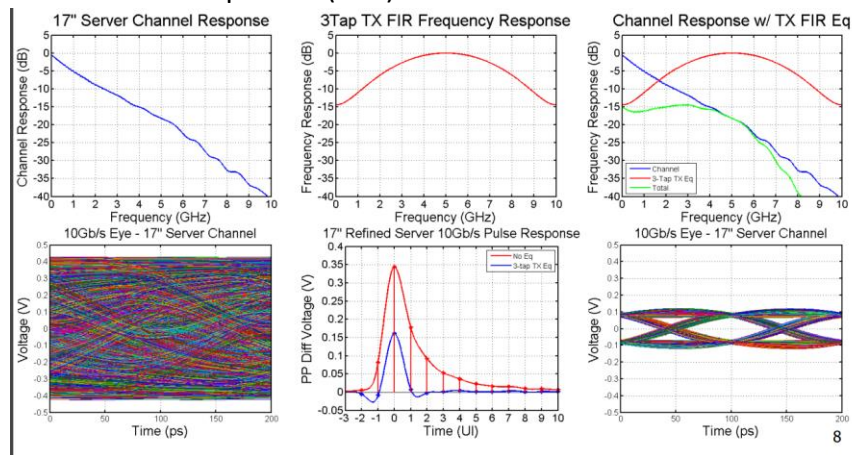


- **Inter-symbol interference (ISI):** The variance of loss with respect to frequency results ISI. More specifically, previous bits residual state can distort the current bit, resulting in ISI. ISI is caused by Reflections, Channel resonances, Channel loss (dispersion).

4 Equalizer[10][12][16][18]

- **Goal:** Equalization is to flatten the frequency response out to the Nyquist Frequency and remove time-domain ISI.
- **Explanation:** suppress frequency magnitude in the low-frequency domain and flat the frequency response[12]

- **Types:**
 - a) TX FIR: Feed Forward Equalizer (FFE)
 - b) RX Continuous Time Linear Equalizer (CTLE):
 - c) RX Decision Feedback Equalizer (DFE)



4.1 TX/RX FFE^[1]

- **Motivation:** pre-distorts transmitted pulse in order to invert channel distortion at the cost of attenuated transmit signal (de-emphasis).
- **Solution:** FFE is a Finite Impulse Response (FIR) filter to reduce the disparity between high and low-frequency channel loss. Because the channel is a low-pass filter, the high-frequency part will be suppressed after passing through the channel. **FFE increases the magnitude of the high-frequency signal.** The effect of the FFE is shown in Figure 1^[19].

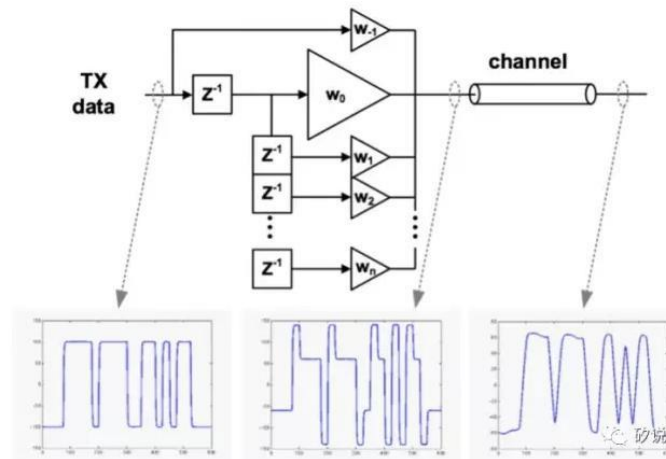


Figure 1 The FFE and its function in signal

- **Challenge:** how to choose coefficients? The best coefficients may be obtained by MMSE^[12]. However, it costs a lot of computation cost.
- **Common FFE:** The common implementation of FFE is a **3-tap FIR filter**. PCIe configuration comprises of 1 pre-cursor tap, 1 main-cursor tap, and 1 post-cursor tap

where the typical implementation places the taps 1 UI apart^[1]. The coefficient may be selected based on experience, e.g., $[-0.050 \ 0.750 \ -0.200]$ with normalized weights.

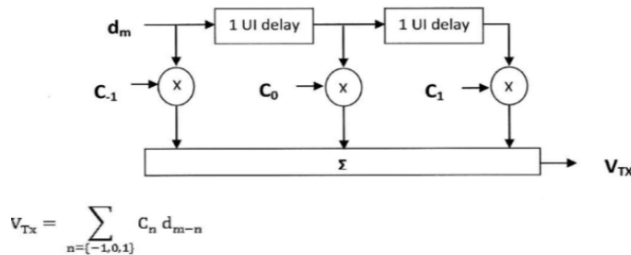


Figure 14 FIR filter [7]

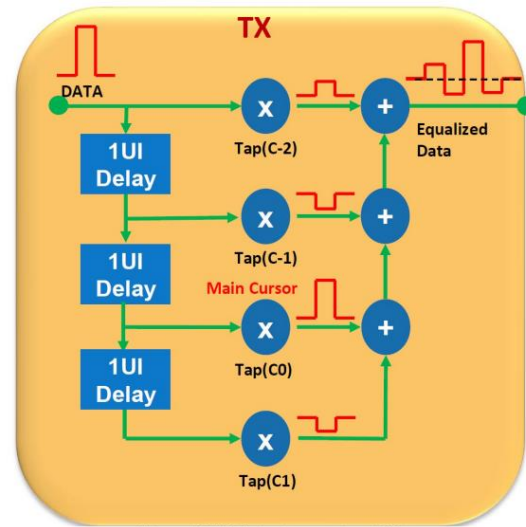
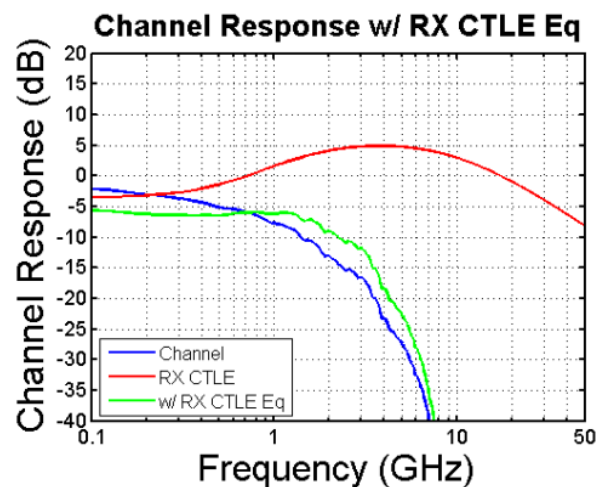


Figure 16 FFE design in transmitter [20]

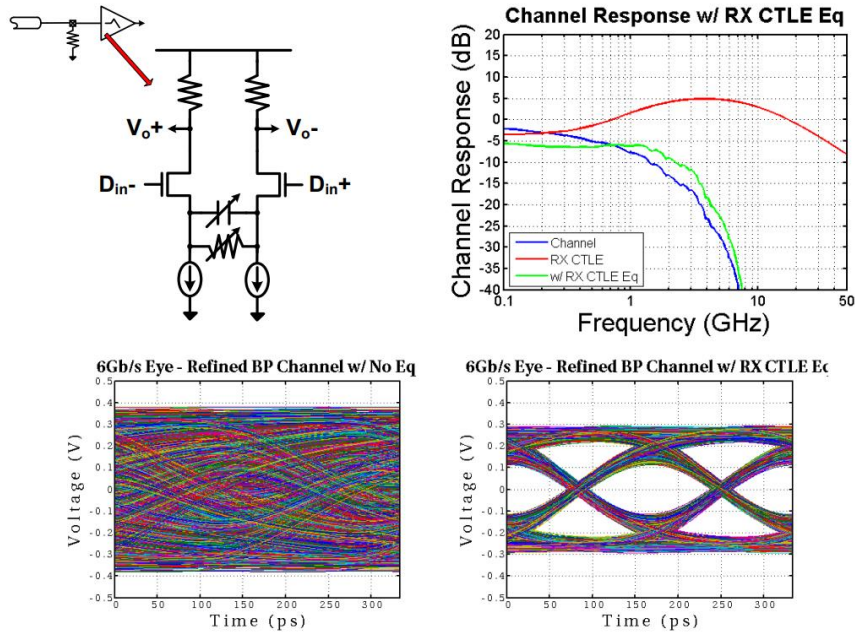
Figure 2 FFE and waveform before and after FFE^[1]

4.2 RX CTLE^{[1][12] [19]}

- **Motivation:** Because the channel is a low-pass filter, the CTLE implements an analogy high-pass transfer function to compensate for channel loss. More specifically, it suppresses the low-frequency amplitude to achieve a flat frequency response.



- **Time domain:** the CTLE can cancel both precursor and long-tail ISI. It can be purely passive or combined with an amplifier to provide gain.
- **Example to explain:** channel with CTLE equalization and NRZ signal



- **Real application:** in simulation setting, we may concatenate multiple stages of CTLE. Furthermore, the CTLE in general is coworked with AGC.
- **Challenges:** the CTLE may amplify both signal and noise at the same time. Furthermore, CTLE needs to support a wide bandwidth, supporting the Nyquist frequency.

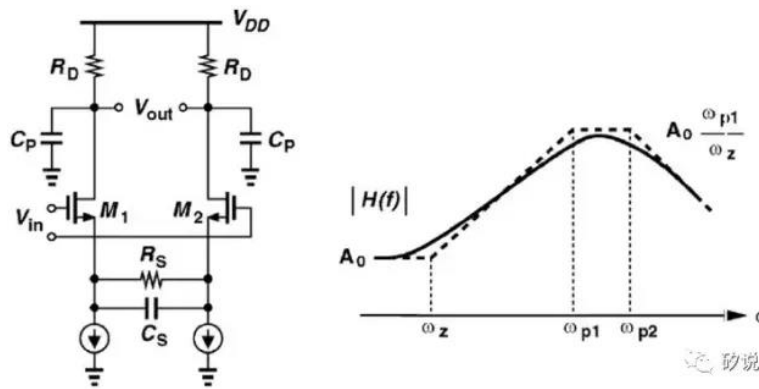


Figure 3 CTLE and transfer function $H(\omega)$ [JSSC 2007]^[19]

4.3 RX DFE^{[1][12][19]}

- **Motivation of Decision Feedback Equalizer (DFE):** further cancel the crosstalk after quantization.
- **Key properties[12]:**
 - a) DFE is not a linear equalizer, i.e., similar to IIR.
 - b) Slicer makes a symbol decision, i.e., quantizes input.
 - c) ISI is then directly subtracted from the incoming signal via a feedback FIR filter.
 - d) Most importantly, DFE can only cancel post-cursor ISI, not pre-cursor ISI.

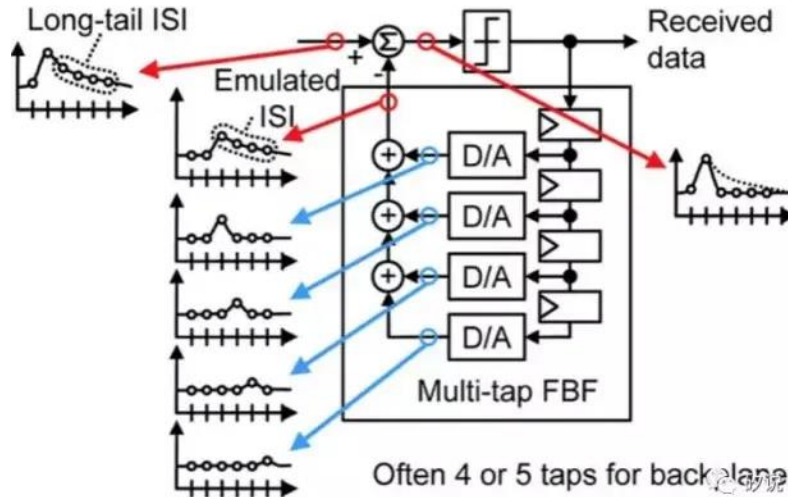


Figure 4 DFE and how to cancel post-cursor ISI ^[19]

- **Advantages:** As a digital filter, DFE can amplify the signal while suppressing the noise. From time domain, DFT reshapes the impulse response and cancels the long tails caused by the low-pass channels.
- **Challenges:** How to design DFE with low latency? How to set suitable coefficients? ^{[1][10][12]} In particular, the work in ^[10] are every helpful.
- **Setting in application:** in application, the system may apply a DFT with as many as 5 taps of feedback FIR, where the weights can be the post-cursor ISI of the channel.

4.4 RX MLSD Equalization^{[10][27]}

- **Motivation:** minimize the mean square error (MSE) of the received signal r_k and decoded signal y_k .
- **The received signal:** For the channel with ISI, the received signal y satisfies

$$y[k] = \sum_{i=0}^L x[k-i]h[i] + n[k]$$

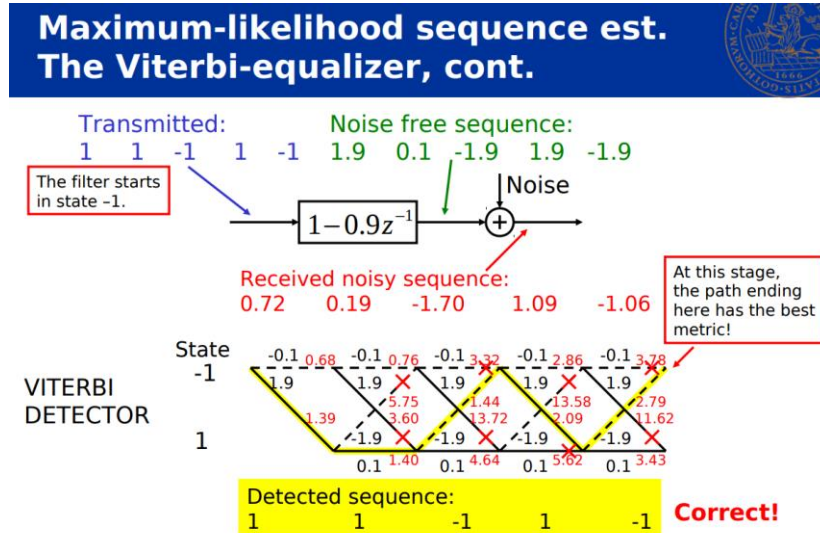
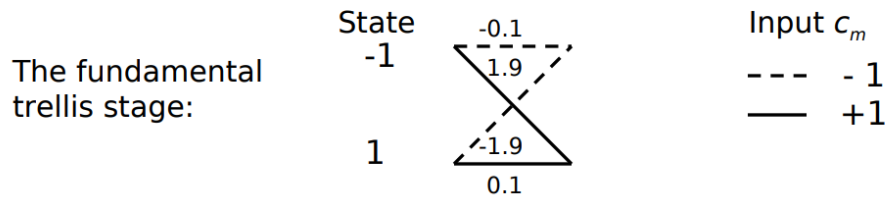
- **Maximum-likelihood sequence est. Principle:** Given the L -tap impulse response and a sequence of symbols x , the corresponding "noise free signal alternative is

$$\bar{y}[k] = \sum_{i=0}^L x[k-i]h[i],$$

Then the MLSE decision is then the sequence of symbols x minimizing this distance

$$\hat{x} = \underset{x}{\operatorname{argmin}} \sum_k |y[k] - \bar{y}[k]|^2$$

- **Solution:** Based on the trellis graph and the Viterbi equalization algorithm. An example is shown below, where $h(z) = 1 - 0.9z^{-1}$.

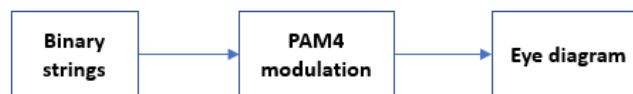


- **Key challenges:** one important requirement is to estimate the channel at the receiver. In particular, if FFE, DFE are applied with MLSD Equalizer, how can we estimate the channel?

5 Simulation

5.1 PAM4 signal

- **Matlab simulation:** generate PAM4 signal and view it based on the following diagram
- **Ideal PAM4:** the PAM4 symbols are directly viewed by eye diagram.



- a) **comm.PAMModulator:** modulate binary strings into PAM4 symbols in the range of $[-3, -1, 1, 3]$.

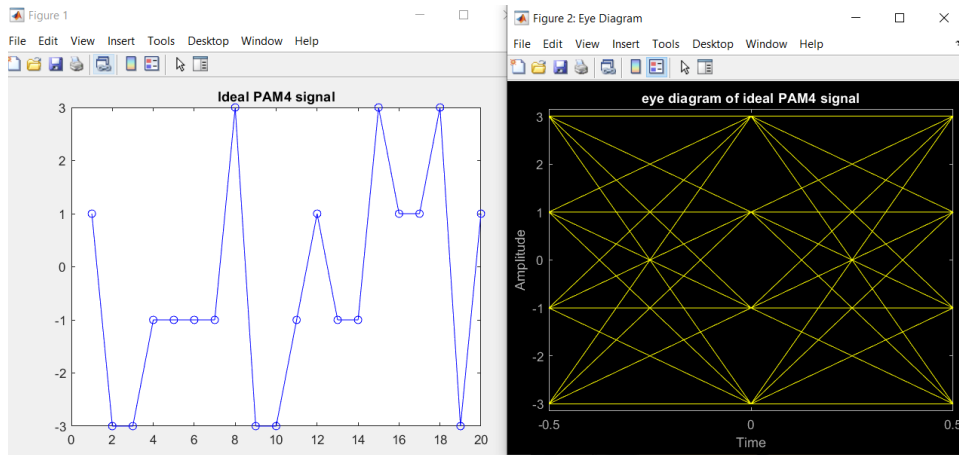
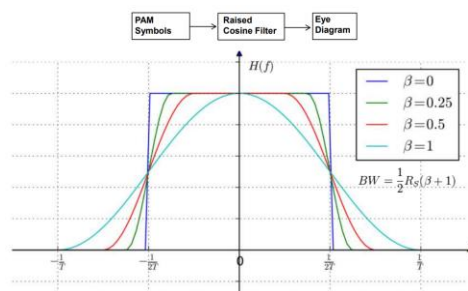


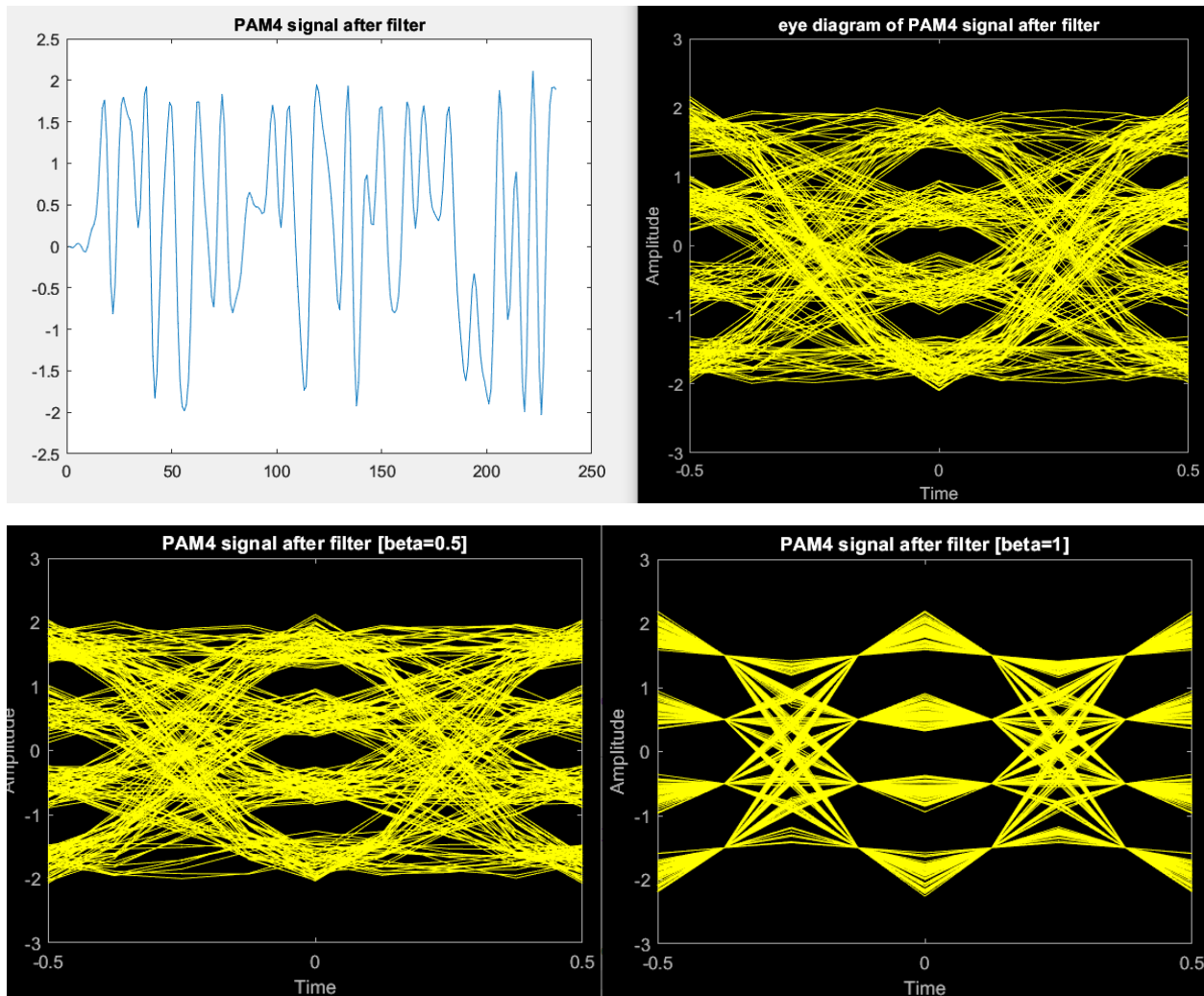
Figure 5 Ideal PAM4 signal and the eye diagram

- Passing PAM4 symbols through a raised cosine filter



- Transfer Binary strings to PAM symbols
- Pass through a raised cosine pass filter[5][6] with suitable *roll-off factor* β . Note that $\beta = 0$ represents an ideal low pass filter.
- comm.RaisedCosineTransmitFilter**: set $\beta = 0.3$ and the output sampling rate is 8 per symbol. The output signal is close to a wave signal.





PAM4 signals with Raised cosine filters

Question: The `comm.RaisedCosineTransmitFilter` function in the MATLAB is not good.

5.2 AWGN channel

- **AWGN channel models:** The ideal PAM4 signal is passed through a gaussian channel with AWGN. The gaussian channel model is shown in the following diagram.

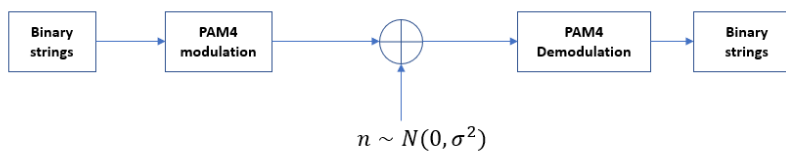


Figure 6 The AWGN channel with PAM4 signal

- The input and output signals:
 - a) The input PAM4 signal is $x \in [-3, -1, 1, 3]$. We normalize the signal by

$$x' = \frac{x}{\sqrt{5}}$$

b) Then the output signal y' can be represented as

$$y' = x' + n = \frac{x}{\sqrt{5}} + n,$$

Where the additional gaussian white noise n has the variance of noise as $\sigma^2 = \frac{1}{10^{SNR/10}}$.

Because the PAM4 demodulation still works for symbols in the range of $[-3, -1, 1, 3]$. We modify the output y' as

$$y = x + \sqrt{5}n = x + n'$$

Where $n' \sim N(0, 5\sigma^2)$.

- The **error probability**: for a specific x , e.g., 1, the signal $y \sim N(1, 5\sigma^2)$. Then $dec(y) \neq x$ if $y < 0$ or $y > 2$. Therefore, the error probability is $2\Phi(-\frac{1}{\sigma\sqrt{5}})$, where

$$\Phi(a) = P(x < a) = \frac{1}{\sqrt{2\pi}} \int^a \exp(-x^2) dx.$$

Furthermore, when $x \in [-3, 3]$, the error occurs only in one tail side. By assuming uniform distribution for $x \in [-3, -1, 1, 3]$, the **error probability of decoding PAM4 symbols (SER)** is

$$Pe = \frac{6}{4} \Phi\left(-\frac{1}{\sigma\sqrt{5}}\right), \quad (1)$$

Where $\sigma^2 = \frac{1}{10^{SNR/10}}$.

- **Simulation results**: the Matlab simulation model is built based on the diagram above.
 - a) Generate binary strings m and transform them as PAM4 symbols x
 - b) Add additional white additional noise $n' \sim N(0, 5\sigma^2)$.
 - c) Obtain the signal $y = x + n'$;
 - d) Demodulation y as m' ,
 - e) Analyze the error probability of the channel.
- Analyze simulation results:

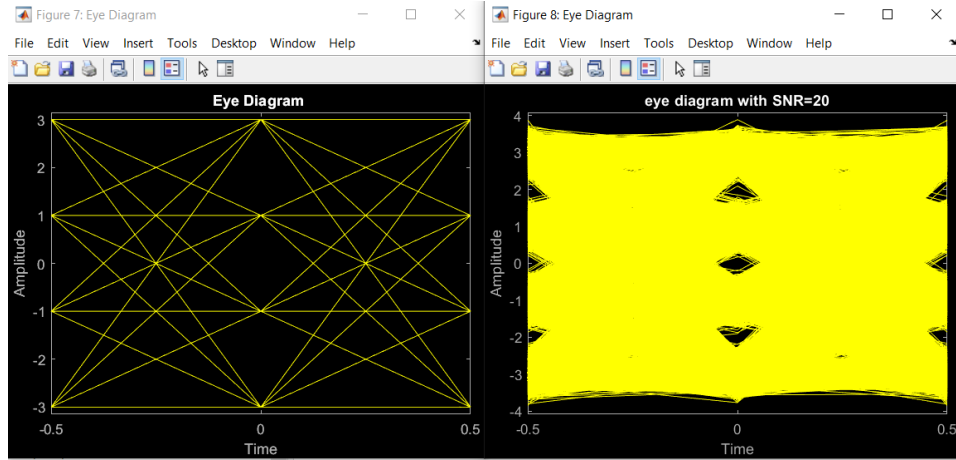


Figure 7 Comparison of the input and the output eye diagrams

Figure 7 compares the eye diagrams of the input and output signals of the AWGN channels. After adding additional white Gaussian noise, the eye diagram of the output signal closes obviously at $SNR = 20 \text{ dB}$.

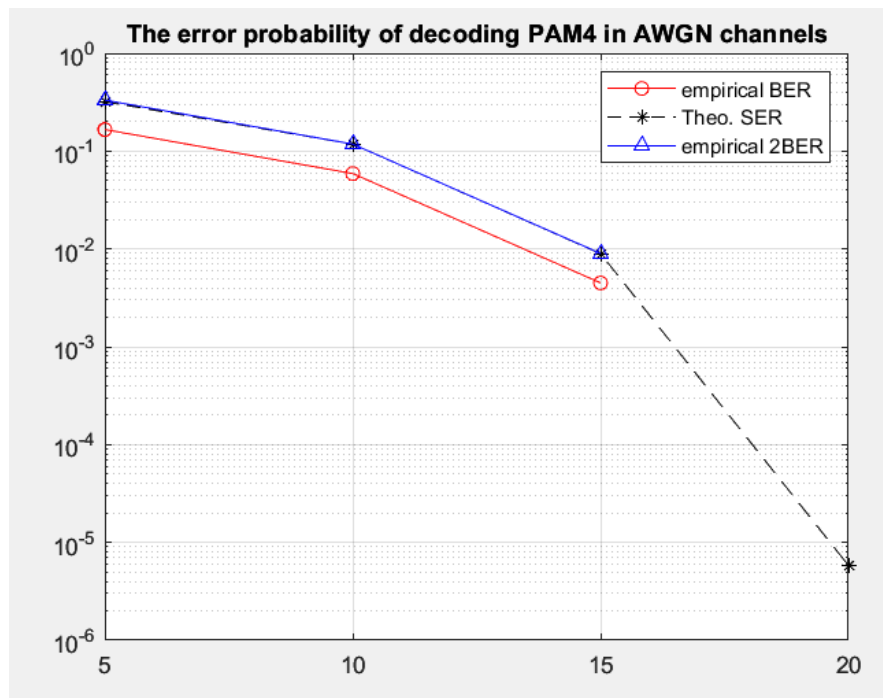


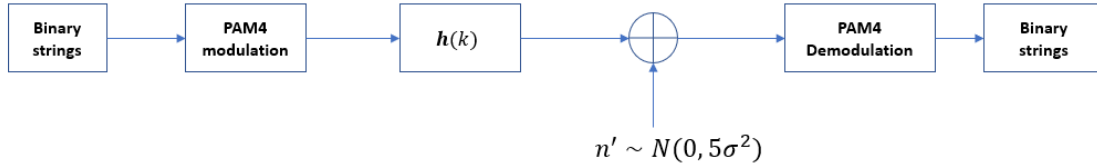
Figure 8 The error probability of decoding PAM4 symbols

Figure 8 analyzes the error probability of decoding PAM4 symbols with respect to SNR. The dashed curve represents the theoretical SER derived by **equation (1)**. The red solid curve represents the empirical bit error probability (BER) of message binary strings, and the blue solid curve represents 2 times the empirical BER. **Because the Gary code is applied in the PAM4 modulation and demodulation, the empirical BER (red-solid curve) should only be half of the**

theoretical SER (black-dashed curve). Therefore, our simulation results match the analytical-numerical results.

5.3 Exponential-AWGN channel

- **Channel Model:** In this subsection, we simulate an exponential channel (with AWGN), which is also a low pass filter. The system diagram is shown below, where $h(k)$ is an exponential channel.



- **Exponential channel:** we use an exponential channel, i.e., $h(t) = \exp(-at)$ with $a = 2$. Based on the **Fourier transform**, the frequency response is $H(f) = \frac{1}{a - j2\pi f}$. To simplify the simulation, I directly generate the discrete (symbol-wise) impulse response $h(k) = \exp(-a k)$ for $k \geq 0$ and $h(k) = 0$ for $k < 0$ (a casual channel).
- **Channel simulation:** Given $a = 2$, the impulse response and the frequency response of the exponential channel are shown below.

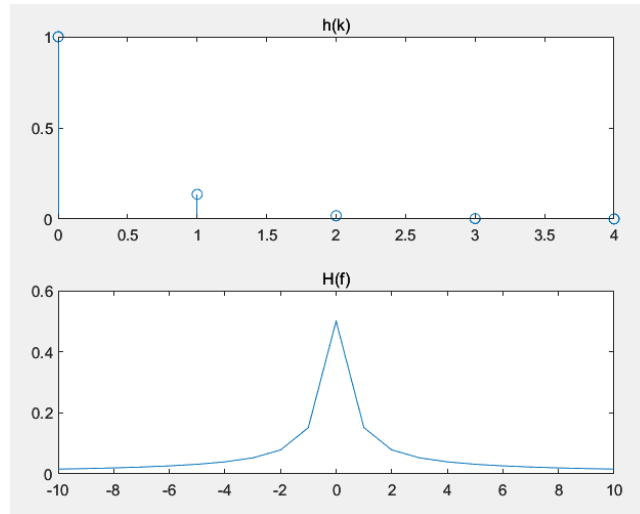


Figure 9 the impulse response and the frequency response of the exponential channel

- **Channel expression:** the output signal is represented as

$$y[k] = x[k] * h[k] + n[k] = \sum_{i=0}^k x[k-i]h[i] + n[k],$$

By setting a length- N binary string ($N/2$ PAM4 symbols) and $h[k], k \in \{0,1,2,3,4\}$, the output y contains $\frac{N}{2} + 5$ elements. We choose the first $\frac{N}{2}$ in the output.

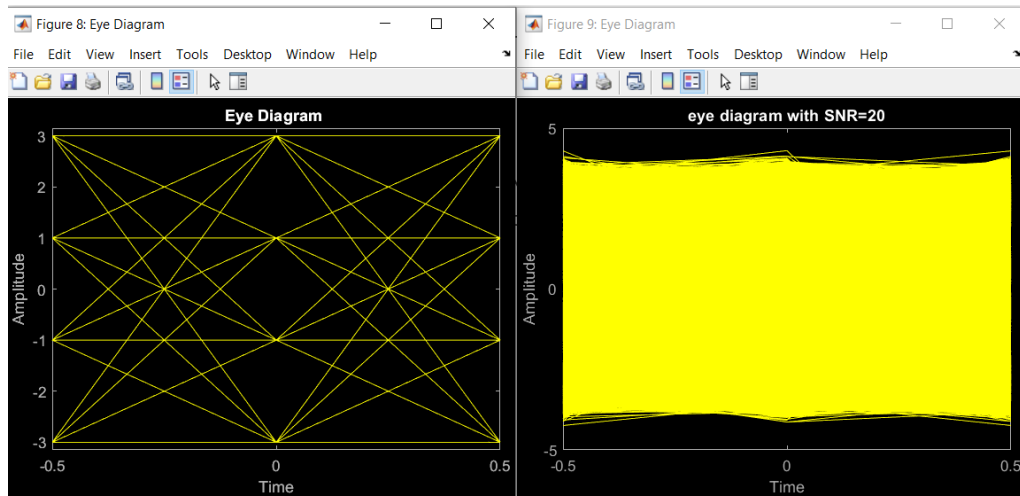


Figure 10 Comparison of eye diagrams of exponential-AWGN channel

- **Simulation results:** Figure 10 compares the eye diagrams of the input and the output signals when $SNR = 20 \text{ dB}$. Based on the observations, the ISI introduced by the exponential channel significantly increases the error probability of channel.

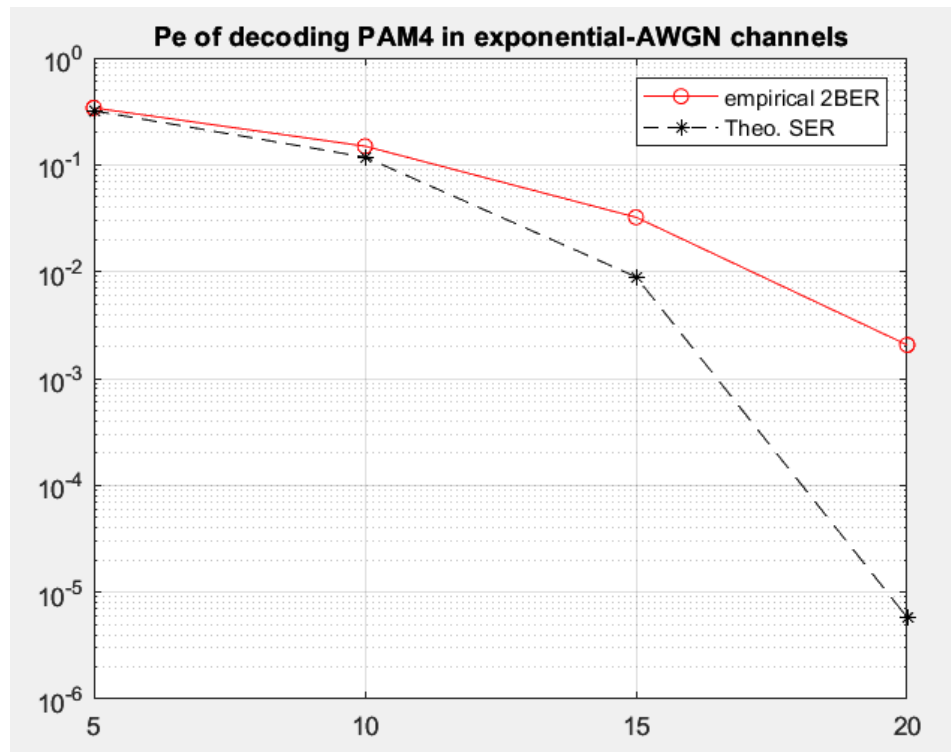
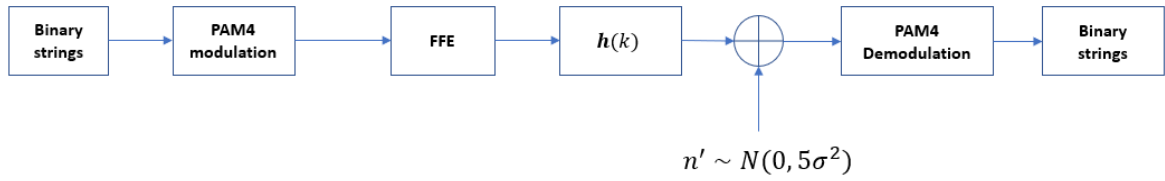


Figure 11 Error probability of decoding PAM4 symbols of the exponential-AWGN channel

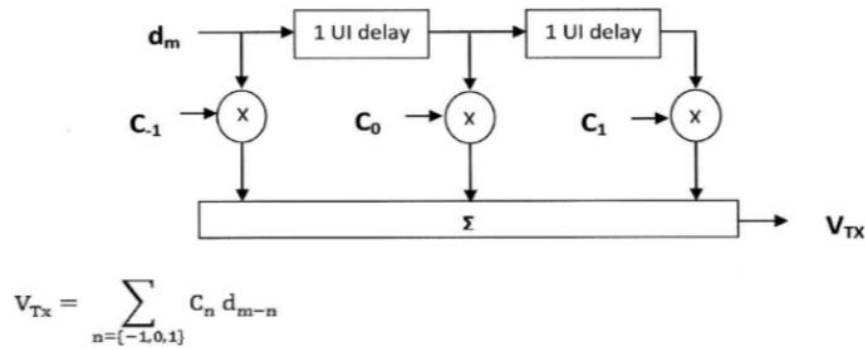
Figure 11 analyzes the error probability of decoding PAM4 symbols of the exponential-AWGN channel with respect to SNR. Compared to Figure 8, the exponential channel with ISI significantly increases the error probability (SER).

5.4 Exponential-AWGN channel with FFE equalization

- **Motivation:** Because the exponential channel is similar to a low pass filter, the FFE equalization (FIR) can compensate for the channel loss by suppressing the amplitude of the low-frequency signal.
- **Channel diagram:** The Exponential-AWGN simulation system with the TX FFE equalization is shown in the following diagram.



TX Feed Forward Equalizer (FFE): In this subsection, we use FFE as a **3-tap FIR filter**, similar to the one in [1]. The coefficient is set as $\mathbf{c} = [c_{-1}, c_0, c_1]$. Furthermore, **the coefficients in \mathbf{c} will be normalized to keep the SNR.**



- **The output signal:** Then the input signal after FFE equalization is

$$\bar{x}[k] = \sum_{j=-1}^1 x[k-j]c_j$$

Then the output signal y is shown as

$$y[k] = \bar{x}[k] * h[k] + n[k] = \sum_{i=0} \bar{x}[k-i]h[i] + n[k],$$

- **Simulation results:** in the simulation, we let $\mathbf{c} = [0, 0.95, -0.05]$ and then normalize the parameters. **Note that since $h[k]$ has no pre-cursor tap coefficient, we let $c_{-1} = 0$ to reduce the error probability.**

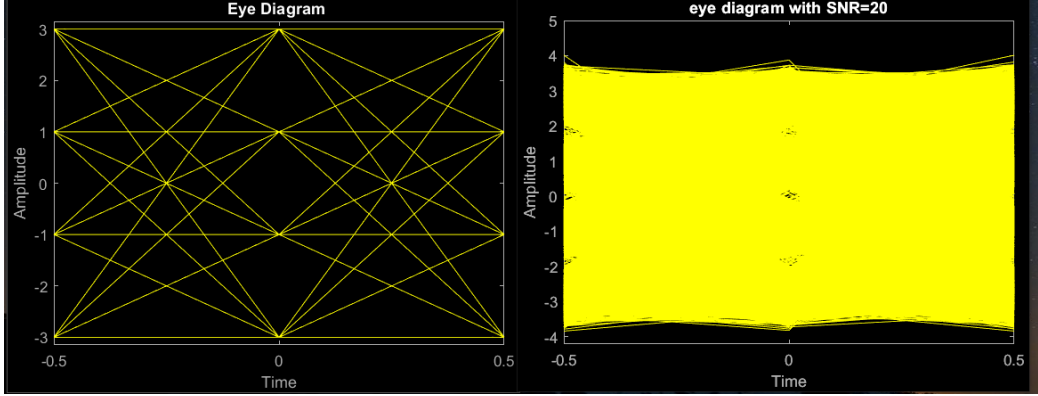


Figure 12 eye diagrams of FFE-Exp-AWGN channel

Figure 12 presents the eye diagrams before and after passing through the FFE-Exp-AWGN channel. Compared to Figure 10, a suitable FFE can open the eye of the eye diagram.

Figure 13 compares the error probability of three different systems as well as the theoretical value. The empirical results show that FFE equalization can decrease the error probability of demodulation PAM4, especially when the SNR is large.

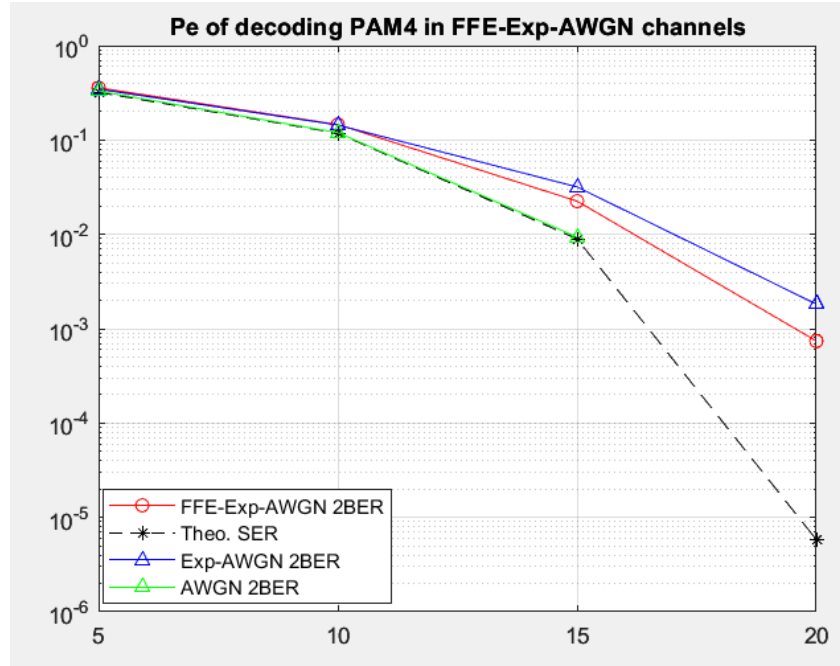
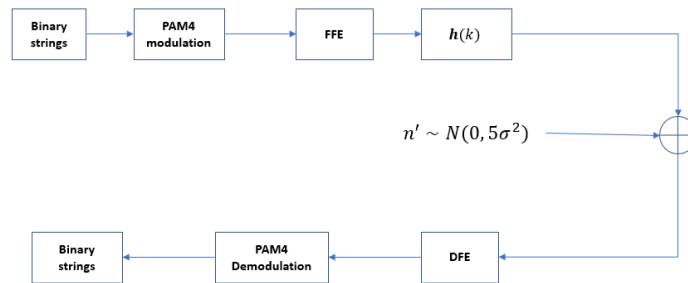


Figure 13 Pe of FFE-Exp-AWGN channel

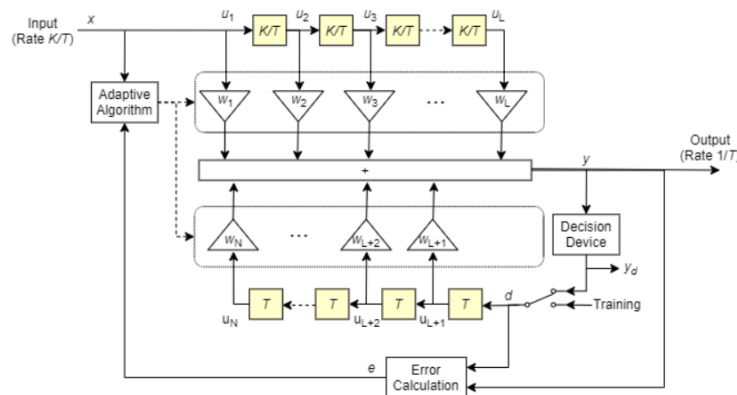
In summary, the FFE equalization can make the eye open and decrease the error probability. However, one question would be how to choose the best coefficients. The problem is open to future work.

5.5 Exp-AWGN channel with both FFE and DFE equalization[23][26]

- **Motivation:** After quantization, the receiver applies the FFE equalization to reduce the ISI. However, the DFE can only cancel the post-cursor ISI.
- **Channel diagram:** The system diagram of the FFE-Exp-AWGN-DFE channel model is shown in the following diagram.



- **RX Decision Feedback Equalizer (DFE):** a nonlinear equalizer that reduces ISI but will not enhance the the noise.
- **Matlab comm.DecisionFeedbackEqualizer:** the function contains both the forward filter and the backward filter. We set L forward taps and $N - L$ backward taps. Furthermore, we may select suitable adaptive Algorithms (i.e., Least Mean Square (LMS) Algorithm, Recursive Least Square (RLS) Algorithm, and Constant Modulus Algorithm (CMA)) and whether train the taps.



- **Simulation results:** in the simulation, I use **comm.DecisionFeedbackEqualizer**, and let $N = 6, L = 1$, the algorithm as LMS. Furthermore, I also add 1000 symbols at the beginning to train the coefficients of DFE.

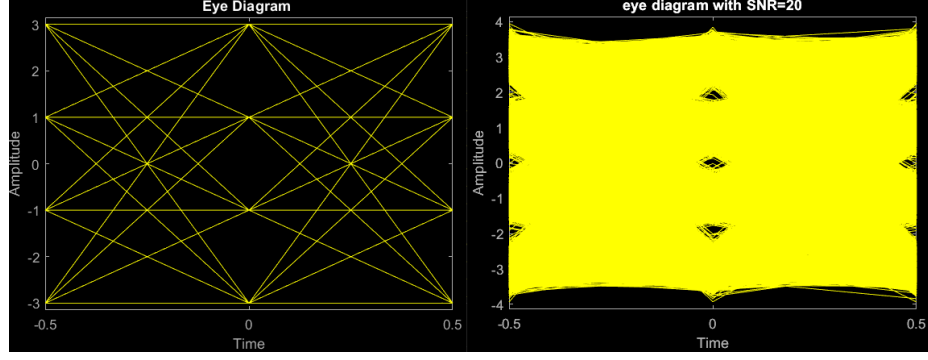


Figure 14 Comparison of eye diagrams of the FFE-Exp-AWGN-DFE system

Figure 14 compares the eye diagrams of the input and the output signals of the FFE-Exp-AWGN-DFE system. When SNR=20, compared to the eye diagram in Figure 12, the DFE can open eyes of the eye diagram.

Figure 15 analyzes the error probability of demodulating PAM4 symbols with respect to SNR. The numerical results show that training of DFE can further decrease the error probability and reduce the ISI when the SNR at the receiver is higher, e.g., SNR=20. However, when SNR is low, the DFE will obviously increase the error probability. Therefore, the DFE will not enhance the noise.

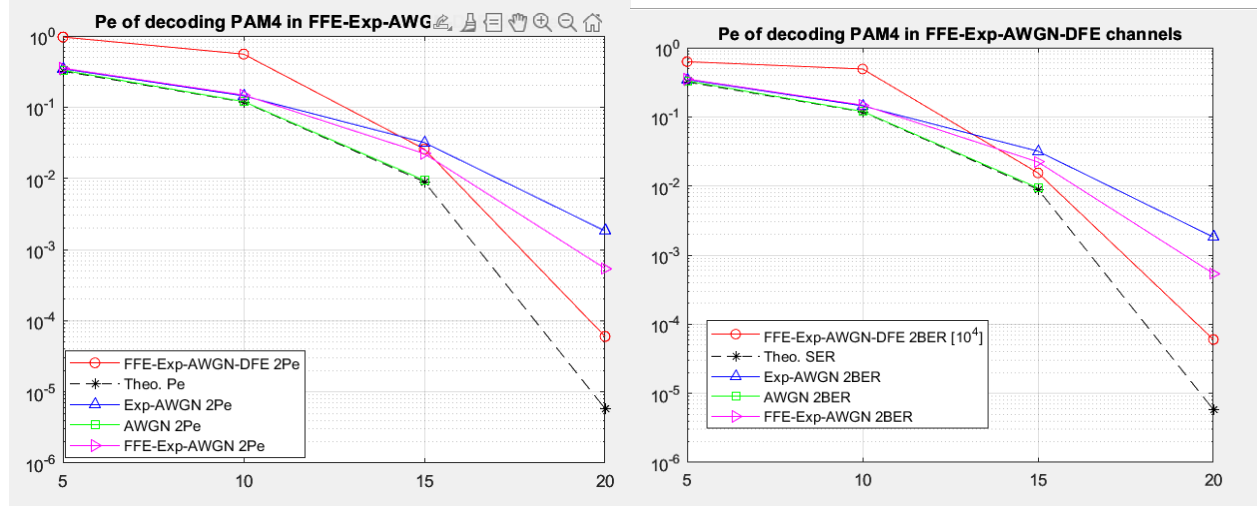


Figure 15 Pe of FFE-Exp-AWGN-DFE model

Question: how to deal with the case when SNR is very low? To solve this problem, we write a DFE function with fixed feedback coefficients only.

- **Output signal of DFE:** given a set of coefficients $w = [w_1, w_2, \dots, w_m]$ and the signal y , the output signal is

$$r[k] = Q \left(y[k] - \sum_{i=1}^m r[k-i]w_i \right), \quad (2)$$

Where $Q(x) \in [-3, -1, 1, 3]$ is a quantizer function to get a value in the set $[-3, -1, 1, 3]$, i.e., quantizing based on the thresholds. The DFE coefficients $w = [w_1, w_2, \dots, w_m]$ are chosen (to be smaller) based on the post-cursor ISI of the channel.

- **MATLAB simulation:** I wrote a function $RX_DFE(y, w)$ with $w = [0.1, 0.02]$ based on (2). Then the eye diagram and the error probability of demodulation are shown below.

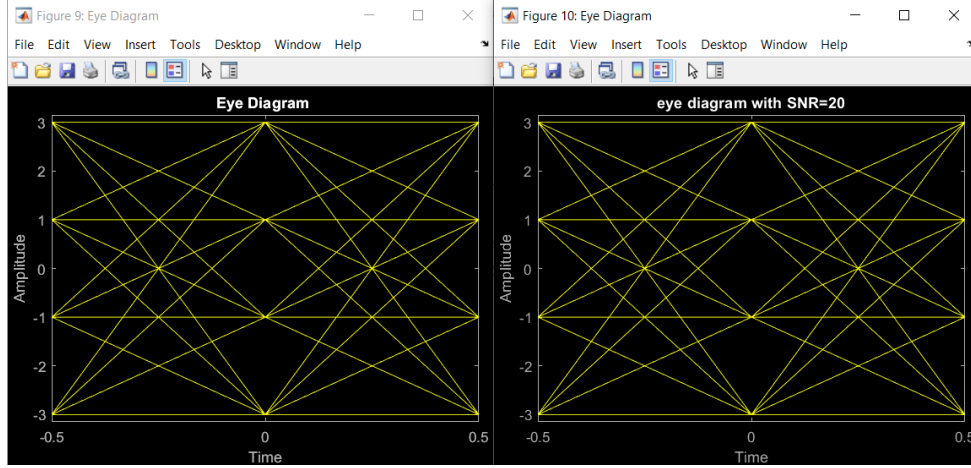


Figure 16 eye diagram with FFE-Exp-AWGN-DFE model (own DFE algorithm)

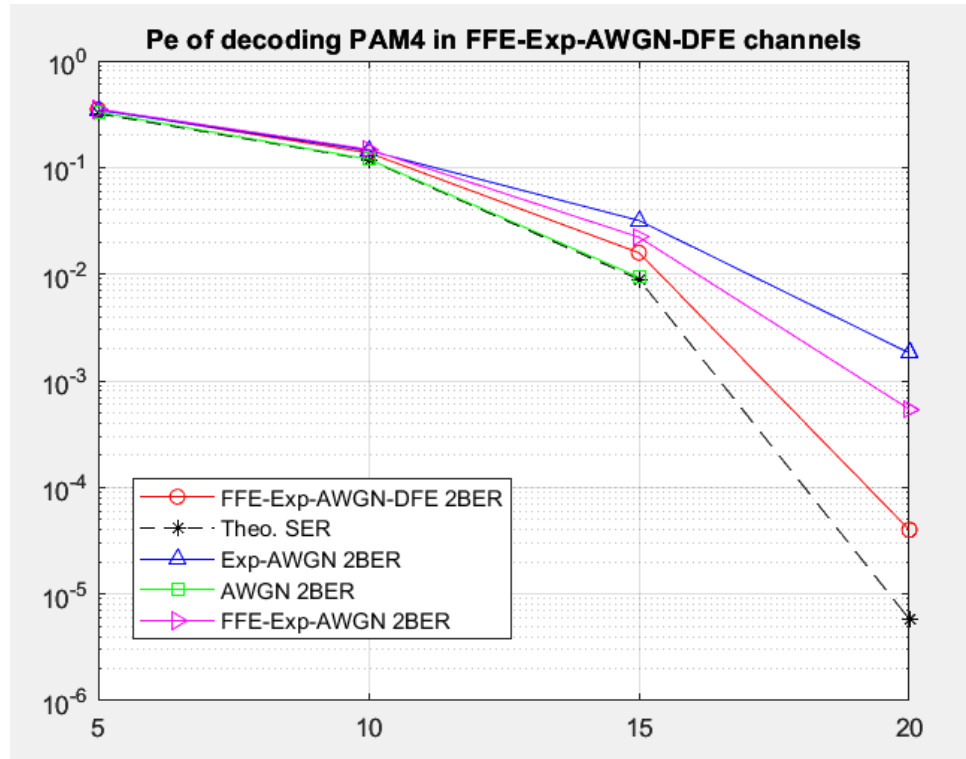


Figure 17 Pe of FFE-Exp-AWGN-DFE model (fixed coefficients)

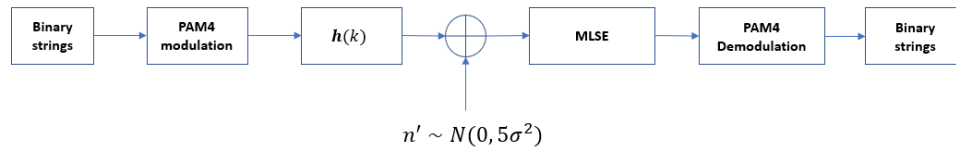
Figure 17 presents the error probability of demodulation PAM4 symbols by applying DFE in equation (2). Compared to Figure 15, by choosing suitable fixed coefficients, the DFE can

decrease the error probability on both the lower and higher SNR range. Furthermore, the error probability approximates the theoretical error probability.

- **Challenges:** how to choose the suitable number of taps and the corresponding coefficients?

5.6 Exp-AWGN channel with MLSD equalization

- **Motivation:** The optimal equalizer, in the sense that it with the highest probability correctly detects the transmitted sequence is the maximum-likelihood sequence estimator (MLSE).
- **Channel model:** The diagram of the channel model is shown below.



- **The received signal:** For the channel with ISI, the received signal y satisfies

$$y[k] = \sum_{i=0}^L x[k-i]h[i] + n[k]$$

- **Maximum-likelihood sequence est. Principle:** Given the L -tap impulse response and a sequence of symbols x , the corresponding “noise free signal alternative is

$$\bar{y}[k] = \sum_{i=0}^L x[k-i]h[i],$$

Where the operation is similar to a convolution operation with rate 1.

Then the MLSE decision is then the sequence of symbols x minimizing this distance

$$\hat{x} = \underset{x}{\operatorname{argmin}} \sum_k |y[k] - \bar{y}[k]|^2$$

- **Solution:** Based on the trellis graph and the **Viterbi equalization algorithm**.
- **MATLAB simulation:** we use the function `comm.MLSEequalizer` function to equalize the signal.

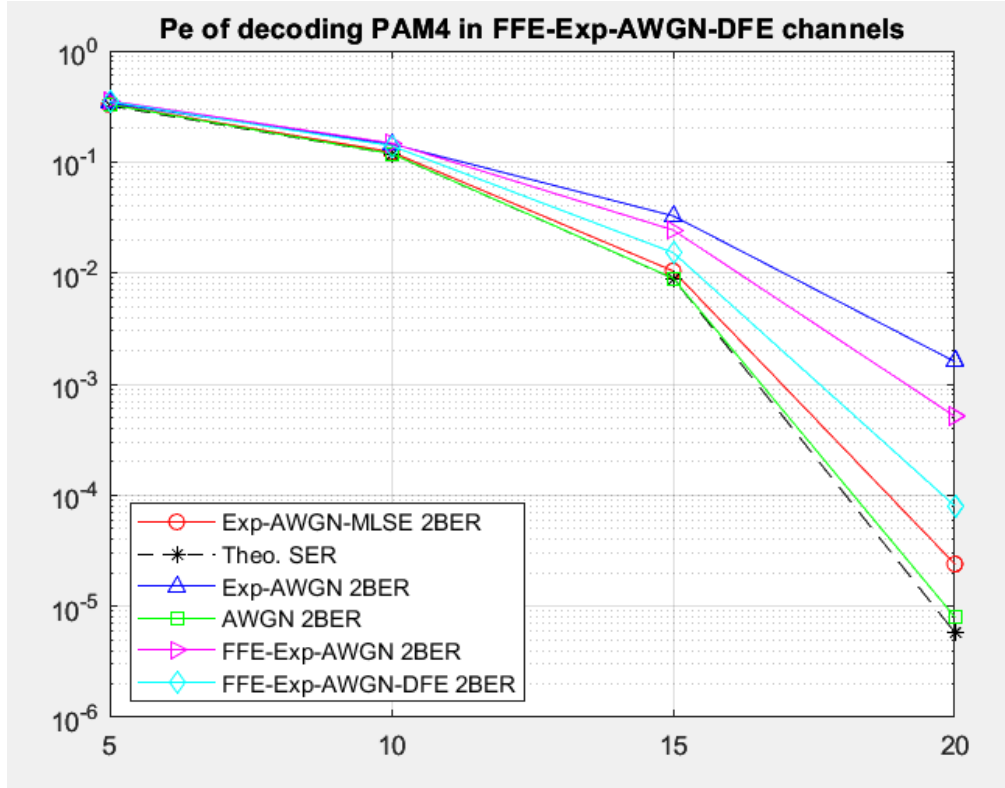


Figure 18 P_e of exp-AWGN channel with multiple equalization algorithms

Figure 18 explores the error probability of demodulating PAM4 symbols of different equalization algorithms. Suppose the channel coefficients are accurately estimated, the decreasing order of equalization methods based on the performance of error probability is MLSE, FFE+DFE, DFE, and finally no equalization. Furthermore, the error probability of the MLSD equalizer is very close to the theoretical error probability of the AWGN channel.

- **Limitation:** compared to other equalization methods, the MLSE is more complicated since the Viterbi algorithm is applied.

6 Summary and future work

We start by summarizing the main work in the notes.

- We reviewed the properties of the PAM4 signal and the channels, including the eye diagrams and the ISI introduced by the frequency-selective fading.
- To mitigate the ISI, the notes reviewed the frequently used equalization algorithms, including the DFE, FFE, MLSE, and CTLE.
- By applying the MATLAB, we simulated a simple exponential-AWGN channel with different equalization algorithms, FFE, DFE, MLSE, and compared their SERs. The numerical results showed that the numerical SER of MLSD Equalizer approximates the optimal analytical SER of the AWGN channel.

To future enhance my understanding about the system, I may continue explore other topics in the future, including:

- a) Understand TDEC for PAM4 ('TDECQ')[28]
- b) Simulate the system based on continuous signals.
- c) Explore good channel estimation algorithms for frequency selective fading channels since the equalizations rely on those coefficients.

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Important overview

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