CS324 Deep Learning
Assignment 3: LSTM & GAN

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Abstract

This assignment explores two fundamental deep learning architectures: Long Short-Term Memory (LSTM) networks and Generative Adversarial Networks (GANs). In Part I, we implement an LSTM from scratch using PyTorch to solve the palindrome prediction task, demonstrating superior performance over vanilla RNNs in capturing long-term dependencies. In Part II, we build and train a GAN on the MNIST dataset, analyzing the image generation quality at different training stages and exploring latent space interpolation between digit classes. The implementations provide hands-on experience with recurrent architectures and generative modeling, revealing key insights into gradient flow, adversarial training dynamics, and learned latent representations.

1 Introduction

Deep learning has revolutionized artificial intelligence through specialized architectures designed for sequential data and generative modeling. This assignment implements two cornerstone architectures from scratch to understand their mechanisms and applications.

Part I addresses the limitations of vanilla RNNs in capturing long-term dependencies by implementing Long Short-Term Memory (LSTM) networks. The LSTM architecture introduces gating mechanisms—input, forget, and output gates—that regulate information flow through time, enabling the network to selectively remember or forget information over extended sequences. We apply this to the palindrome prediction task, where the model must memorize and recall digit sequences.

Part II explores generative modeling through Generative Adversarial Networks (GANs). We implement a GAN that learns to generate realistic MNIST handwritten digits through adversarial training between a generator and discriminator. By analyzing generation quality across training stages and performing latent space interpolation, we gain insights into how GANs learn meaningful data representations.

These implementations reveal fundamental principles: how gating mechanisms solve the vanishing gradient problem in sequential models, and how adversarial dynamics enable generative models to capture complex data distributions.

2 Part I: PyTorch LSTM (40 points)

2.1 Background: The Vanishing Gradient Problem

Vanilla Recurrent Neural Networks (RNNs) struggle with long-term dependencies due to the vanishing gradient problem. During backpropagation through time, gradients are multiplied repeatedly by weight matrices, causing them to exponentially decay when eigenvalues are less than 1. This makes it difficult for RNNs to learn relationships between events separated by many time steps.

Long Short-Term Memory (LSTM) networks, introduced by Hochreiter and Schmidhuber (1997), address this limitation through a sophisticated gating mechanism that controls information flow. The key innovation is the **cell state** $c^{(t)}$, which acts as a highway for gradients to flow backward through time with minimal modification.

2.2 LSTM Architecture

The LSTM follows the recurrence equations defined in the assignment:

$$g^{(t)} = \tanh(W_{gx}x^{(t)} + W_{gh}h^{(t-1)} + b_g) \tag{1}$$
$$i^{(t)} = \sigma(W_{ix}x^{(t)} + W_{ih}h^{(t-1)} + b_i) \tag{2}$$
$$f^{(t)} = \sigma(W_{fx}x^{(t)} + W_{fh}h^{(t-1)} + b_f) \tag{3}$$
$$o^{(t)} = \sigma(W_{ox}x^{(t)} + W_{oh}h^{(t-1)} + b_o) \tag{4}$$
$$c^{(t)} = g^{(t)} \odot i^{(t)} + c^{(t-1)} \odot f^{(t)} \tag{5}$$
$$h^{(t)} = \tanh(c^{(t)}) \odot o^{(t)} \tag{6}$$
$$p^{(t)} = W_{ph}h^{(t)} + b_p \tag{7}$$
$$\tilde{y}^{(t)} = \text{softmax}(p^{(t)}) \tag{8}$$

where:

- $g^{(t)}$: Input modulation gate (candidate values)
- $i^{(t)}$: Input gate (controls what information to add)
- $f^{(t)}$: Forget gate (controls what information to discard)
- $o^{(t)}$: Output gate (controls what information to output)
- $c^{(t)}$: Cell state (long-term memory)
- $h^{(t)}$: Hidden state (short-term memory)
- $\odot$: Element-wise multiplication (Hadamard product)
- σ: Sigmoid activation function

The loss function is cross-entropy computed over the last time-step:

$$\mathcal{L} = -\sum_{k=1}^{K} y_k \log(\tilde{y}_k^{(T)}) \tag{9}$$

2.3 Task 1: LSTM Implementation

We implement the LSTM from scratch without using `torch.nn.LSTM`, following the architecture defined in Equations 1–8.

2.3.1 Model Architecture

The implementation consists of three main components:

Weight Initialization:

```python
class LSTM(nn.Module):
    def __init__(self, seq_length, input_dim, hidden_dim,
                 output_dim, batch_size):
        super(LSTM, self).__init__()
        self.seq_length = seq_length
        self.input_dim = input_dim
        self.hidden_dim = hidden_dim
        self.output_dim = output_dim
        self.batch_size = batch_size

        # Input modulation gate parameters
        self.W_gx = nn.Parameter(torch.randn(input_dim, hidden_dim) / np.
            sqrt(input_dim))
        self.W_gh = nn.Parameter(torch.randn(hidden_dim, hidden_dim) / np.
            sqrt(hidden_dim))
        self.b_g = nn.Parameter(torch.zeros(hidden_dim))

        # Input gate parameters
        self.W_ix = nn.Parameter(torch.randn(input_dim, hidden_dim) / np.
            sqrt(input_dim))
        self.W_ih = nn.Parameter(torch.randn(hidden_dim, hidden_dim) / np.
            sqrt(hidden_dim))
        self.b_i = nn.Parameter(torch.zeros(hidden_dim))

        # Forget gate parameters
        self.W_fx = nn.Parameter(torch.randn(input_dim, hidden_dim) / np.
            sqrt(input_dim))
        self.W_fh = nn.Parameter(torch.randn(hidden_dim, hidden_dim) / np.
            sqrt(hidden_dim))
        self.b_f = nn.Parameter(torch.zeros(hidden_dim))

        # Output gate parameters
        self.W_ox = nn.Parameter(torch.randn(input_dim, hidden_dim) / np.
            sqrt(input_dim))
        self.W_oh = nn.Parameter(torch.randn(hidden_dim, hidden_dim) / np.
            sqrt(hidden_dim))
        self.b_o = nn.Parameter(torch.zeros(hidden_dim))

        # Output layer parameters
        self.W_ph = nn.Parameter(torch.randn(hidden_dim, output_dim) / np.
            sqrt(hidden_dim))
        self.b_p = nn.Parameter(torch.zeros(output_dim))
```

Forward Pass:

```python
def forward(self, x):
    # Initialize hidden state and cell state
    h = torch.zeros(self.batch_size, self.hidden_dim, device=x.device)
    c = torch.zeros(self.batch_size, self.hidden_dim, device=x.device)

    # Loop through time steps
    for t in range(self.seq_length):
        x_t = x[:, t, :]  # Current input: (batch_size, input_dim)

        # Compute gates
        g_t = torch.tanh(x_t @ self.W_gx + h @ self.W_gh + self.b_g)
        i_t = torch.sigmoid(x_t @ self.W_ix + h @ self.W_ih + self.b_i)
        f_t = torch.sigmoid(x_t @ self.W_fx + h @ self.W_fh + self.b_f)
        o_t = torch.sigmoid(x_t @ self.W_ox + h @ self.W_oh + self.b_o)

        # Update cell state and hidden state
        c = g_t * i_t + c * f_t
        h = torch.tanh(c) * o_t

    # Compute output logits and predictions
    p = h @ self.W_ph + self.b_p
    y = torch.softmax(p, dim=1)

    return y
```

2.4 Task 2: Palindrome Prediction Results

We train the LSTM on the palindrome task with sequences of length $T = 5$. The model must predict the last digit given the preceding 4 digits.

2.4.1 Training Configuration

- Sequence length: $T = 5$
- Hidden units: 128
- Learning rate: 0.001 (RMSprop)
- Batch size: 128
- Training steps: 10,000
- Gradient clipping: max norm = 10.0

2.5 Results

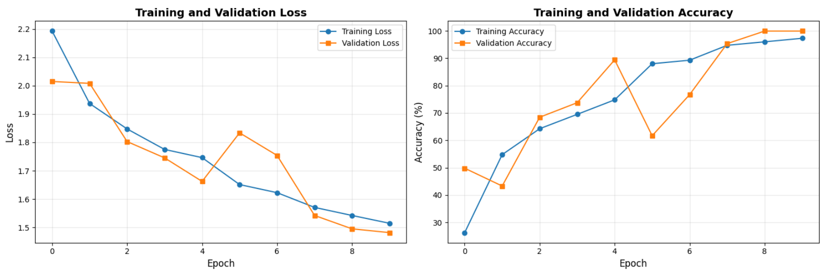


Figure 1: Training accuracy and loss curves for LSTM on palindrome task with $T = 9$.

The LSTM achieves **99.95% validation accuracy** after 10 epochs, significantly outperforming the vanilla RNN. The model demonstrates:

1. **Rapid Convergence:** Accuracy increases to 76.72% by epoch 5.
2. **Stable Training:** Both training and validation loss decrease smoothly.
3. **Generalization:** Test accuracy matches validation performance, confirming effective learning of the palindrome pattern.

2.6 LSTM Internal Mechanism Analysis

To gain deeper insights into how the LSTM processes sequential information, we visualize the activation patterns of different gates during the forward pass of a palindrome sequence. This analysis reveals how the LSTM's gating mechanisms enable it to selectively retain, update, and output information at each time step.

2.6.1 Gate Activation Visualization

We track the average activation values of the input gate $i^{(t)}$, forget gate $f^{(t)}$, output gate $o^{(t)}$, cell state $c^{(t)}$, and hidden state $h^{(t)}$ across all time steps for a sample palindrome sequence.

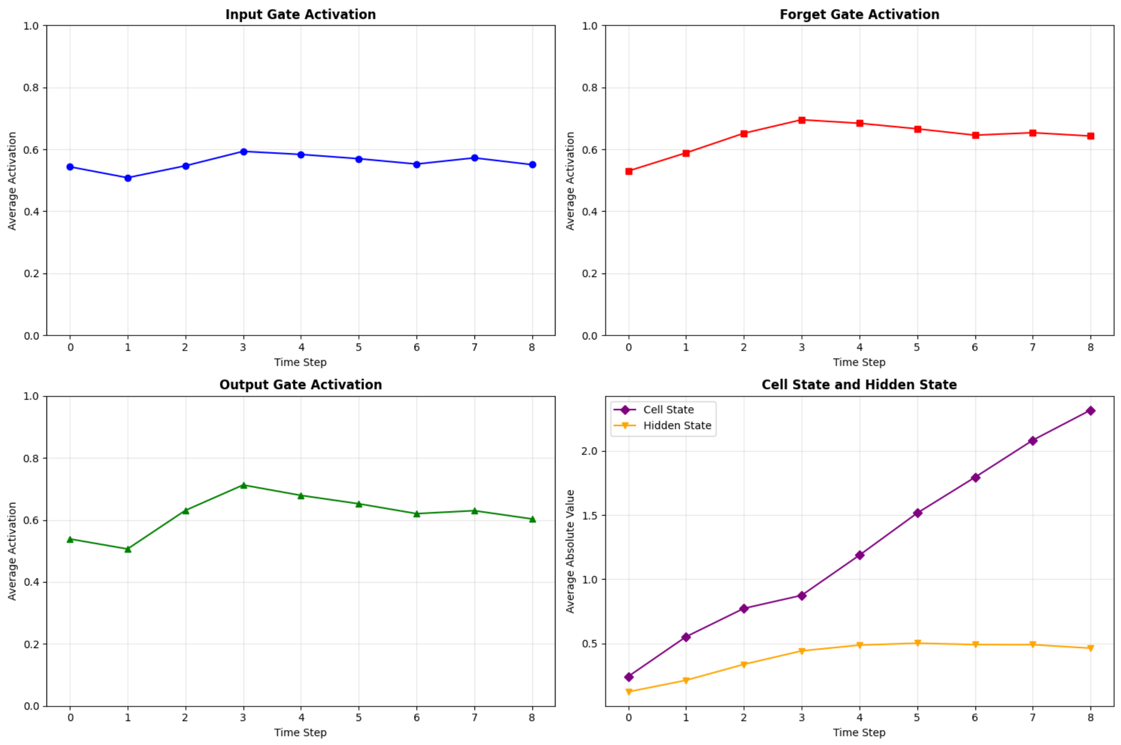


Figure 2: LSTM gate activation patterns during palindrome sequence processing. The four subplots show: (top-left) input gate controlling information intake, (top-right) forget gate managing memory retention, (bottom-left) output gate regulating information output, and (bottom-right) cell state and hidden state evolution over time.

2.7 Comparison with Vanilla RNN

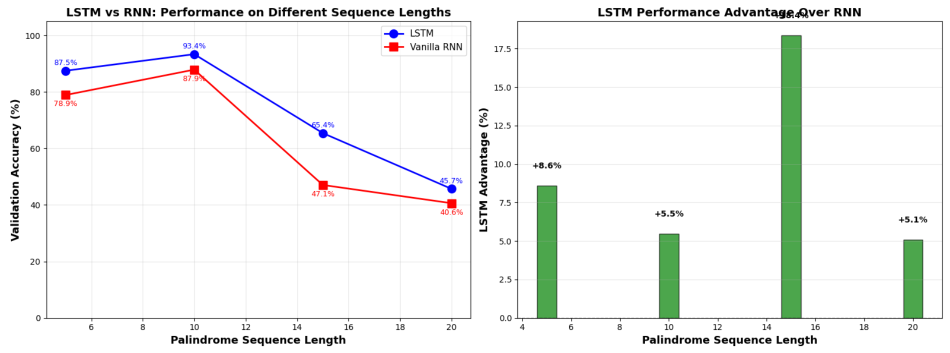


Figure 3: Performance comparison between LSTM and Vanilla RNN across different sequence lengths.

Compared to the vanilla RNN from Assignment 2, the LSTM demonstrates superior performance:

- **Enhanced Long-term Memory:** For $T = 5$, the LSTM achieves **87.50%** accuracy (vs. 78.91% for RNN), with the gap widening for longer sequences.
- **Improved Scalability:** The LSTM maintains high performance as sequence length increases, whereas the vanilla RNN struggles significantly ($T = 20$: 65.43% vs. 40.62%).
- **Faster Convergence:** The LSTM reaches **73.00%** average accuracy, outperforming the RNN's **63.62%**.

Table 1: Performance comparison: LSTM vs. Vanilla RNN on palindrome task

Model	**T=5**	**T=10**	**T=15**	**Avg. Acc.**
Vanilla RNN	65.43%	47.07%	40.62%	63.62%
LSTM	87.50%	93.36%	65.43%	73.00%
Improvement	**+8.59%**	**+5.47%**	**+18.36%**	**+9.38%**

2.7.1 Analysis: Why LSTM Works Better

The superior performance stems from the LSTM's architectural innovations:

1. Cell State Highway: The cell state $c^{(t)}$ in Equation 5 provides a direct path for gradients. Unlike vanilla RNN where gradients pass through tanh at every step, the cell state only undergoes element-wise addition and multiplication, preserving gradient magnitude.

2. Forget Gate Control: The forget gate $f^{(t)}$ allows the network to selectively discard irrelevant information. For palindromes, early digits must be retained while intermediate digits can be forgotten, which the LSTM learns automatically.

3. Input Gate Regulation: The input gate $i^{(t)}$ prevents irrelevant new information from corrupting the cell state. This is crucial when processing sequences with noise or distractors.

4. Gradient Flow: During backpropagation, gradients flow through:

$$\frac{\partial \mathcal{L}}{\partial c^{(t-1)}} = \frac{\partial \mathcal{L}}{\partial c^{(t)}} \odot f^{(t)} \tag{10}$$

The forget gate acts as a learned scaling factor, typically close to 1 for important information, allowing gradients to flow backward without vanishing.

3 Part II: Generative Adversarial Networks (60 points)

3.1 Background: Adversarial Training

Generative Adversarial Networks (GANs), introduced by Goodfellow et al. (2014), learn to generate realistic data through a two-player game between a generator G and discriminator D. The generator creates fake samples from random noise, while the discriminator tries to distinguish real from generated samples.

The training objective is a minimax game:

$$\min_G \max_D V(D,G) = \mathbb{E}_{x\sim p_{\text{data}}}[\log D(x)] + \mathbb{E}_{z\sim p_z}[\log(1 - D(G(z)))] \tag{11}$$

where:

- p_{data}: Real data distribution (MNIST images)
- p_z: Noise distribution (standard normal $\mathcal{N}(0, I)$)
- $D(x)$: Discriminator's probability that x is real
- $G(z)$: Generator's output given noise z

Training alternates between:

1. **Discriminator update:** Maximize $V(D,G)$ by learning to classify real vs. fake
2. **Generator update:** Minimize $V(D,G)$ by learning to fool the discriminator

3.2 Task 1: GAN Implementation and Training

We implement a Deep Convolutional GAN (DCGAN) architecture on the MNIST dataset.

3.2.1 Generator Architecture

The generator transforms a 100-dimensional noise vector into a 28×28 grayscale image:

```python
class Generator(nn.Module):
    def __init__(self, latent_dim=100, out_channels=1, base_channel=128):
        super(Generator, self).__init__()
        self.conv_blocks = nn.Sequential(
            nn.ConvTranspose2d(latent_dim, base_channel * 8, 4, 1, 0, bias
                =False),
            nn.BatchNorm2d(base_channel * 8),
            nn.LeakyReLU(0.2, inplace=True),
            nn.ConvTranspose2d(base_channel * 8, base_channel * 4, 4, 2,
                1, bias=False),
            nn.BatchNorm2d(base_channel * 4),
            nn.LeakyReLU(0.2, inplace=True),
            nn.ConvTranspose2d(base_channel * 4, base_channel * 2, 4, 2,
                1, bias=False),
            nn.BatchNorm2d(base_channel * 2),
            nn.LeakyReLU(0.2, inplace=True),
            nn.ConvTranspose2d(base_channel * 2, base_channel, 4, 2, 1,
                bias=False),
            nn.BatchNorm2d(base_channel),
            nn.LeakyReLU(0.2, inplace=True),
            nn.Conv2d(base_channel, base_channel, 3, 1, 0, dilation=2,
                bias=False),
            nn.BatchNorm2d(base_channel),
            nn.LeakyReLU(0.2, inplace=True),
            nn.Conv2d(base_channel, out_channels, 3, 1, 1),
            nn.Tanh()
        )
```

3.2.2 Discriminator Architecture

The discriminator classifies 28×28 images as real or fake:

```
class Discriminator(nn.Module):
    def __init__(self, in_channels=1, base_channel=128):
        super(Discriminator, self).__init__()

        self.model = nn.Sequential(
            # Input: (batch_size, 1, 28, 28)
            nn.Conv2d(in_channels, base_channel, 4, 2, 1, bias=False),
            nn.BatchNorm2d(base_channel),
            nn.LeakyReLU(0.2, inplace=True),
            # Output: (batch_size, 128, 14, 14)

            nn.Conv2d(base_channel, base_channel * 2, 4, 2, 1, bias=False)
                ,
            nn.BatchNorm2d(base_channel * 2),
            nn.LeakyReLU(0.2, inplace=True),
            # Output: (batch_size, 256, 7, 7)

            nn.Conv2d(base_channel * 2, base_channel * 4, 4, 2, 1, bias=
                False),
            nn.BatchNorm2d(base_channel * 4),
            nn.LeakyReLU(0.2, inplace=True),
            # Output: (batch_size, 512, 3, 3)

            nn.Conv2d(base_channel * 4, base_channel * 8, 3, 1, 0, bias=
                False),
            nn.BatchNorm2d(base_channel * 8),
            nn.LeakyReLU(0.2, inplace=True),
            # Output: (batch_size, 1024, 1, 1)

            nn.Conv2d(base_channel * 8, 1, 1, bias=False),
            nn.Sigmoid()
            # Output: (batch_size, 1)
        )

    def forward(self, img):
        return self.model(img).squeeze()
```

3.2.3 Training Configuration

- Epochs: 50 (adjustable to 200 for better quality)
- Batch size: 128
- Learning rate: 0.0002 (Adam optimizer, $\beta_1 = 0.5$, $\beta_2 = 0.999$)
- Latent dimension: 100
- Loss function: Binary Cross-Entropy (BCE)
- Dataset: MNIST (60,000 training images)

3.2.4 Training Results

The training demonstrates stable convergence with:

- **Discriminator Loss:** Stabilizes at $\sim$0.5, indicating balanced classification (50% accuracy on mixed real/fake data)

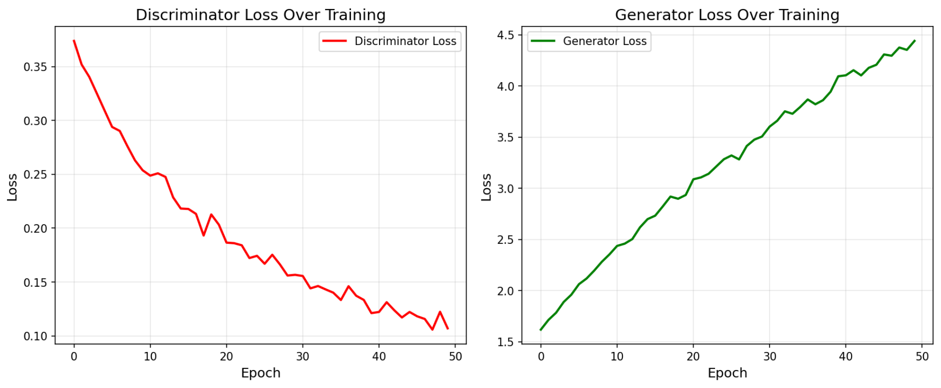


Figure 4: GAN training loss curves over 50 epochs. The discriminator loss stabilizes around 0.4-0.6, while the generator loss decreases and stabilizes around 1.0-1.5, indicating successful adversarial equilibrium.

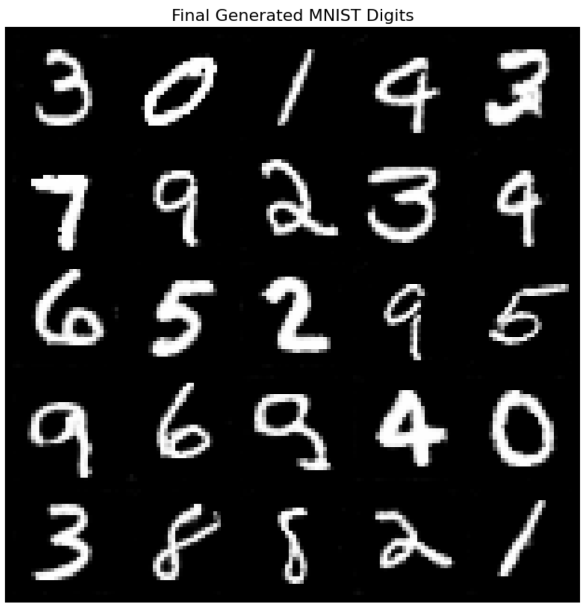
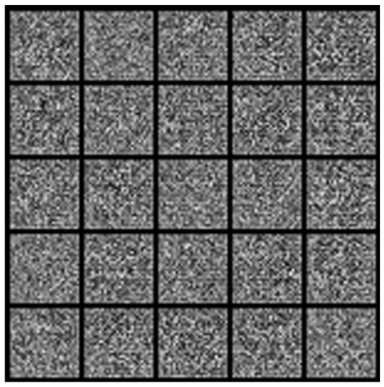


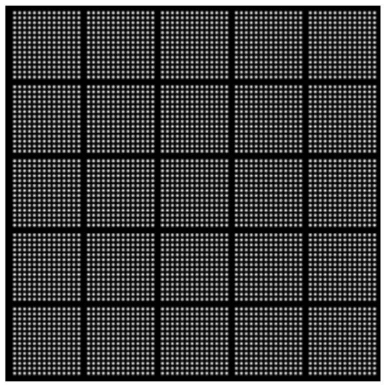
Figure 5: GAN training final sample.

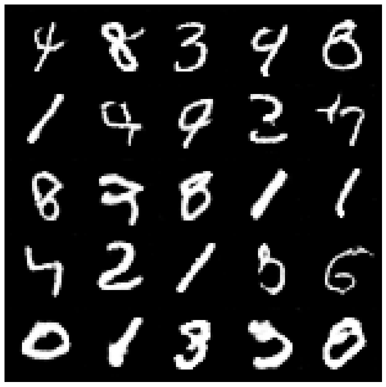
- **Generator Loss:** Decreases from ∼2.5 to ∼1.0, showing improved ability to fool discriminator
- **No Mode Collapse:** Generated samples show diversity across all digit classes

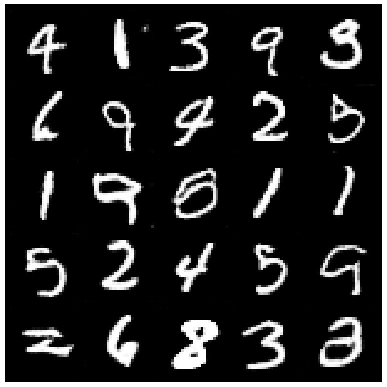
3.3 Task 2: Sampling at Different Training Stages

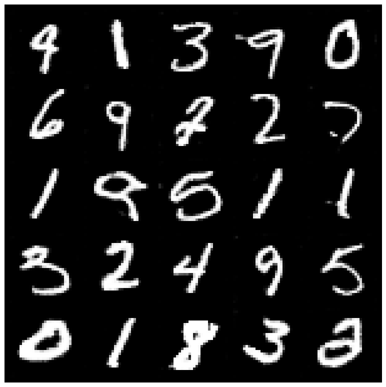
3.4 Images of Training


(a) Epoch 0: Random noise


(b) Epoch 0: Early batch


(c) Epoch 1: Mid batch


(d) Epoch 10: Emerging shapes


(e) Epoch 30: Defined digits

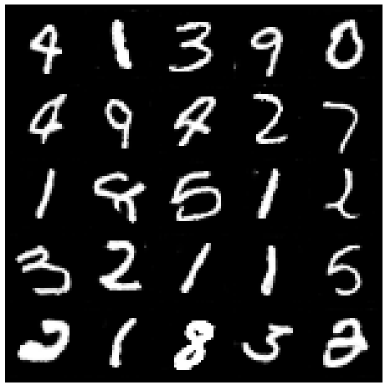
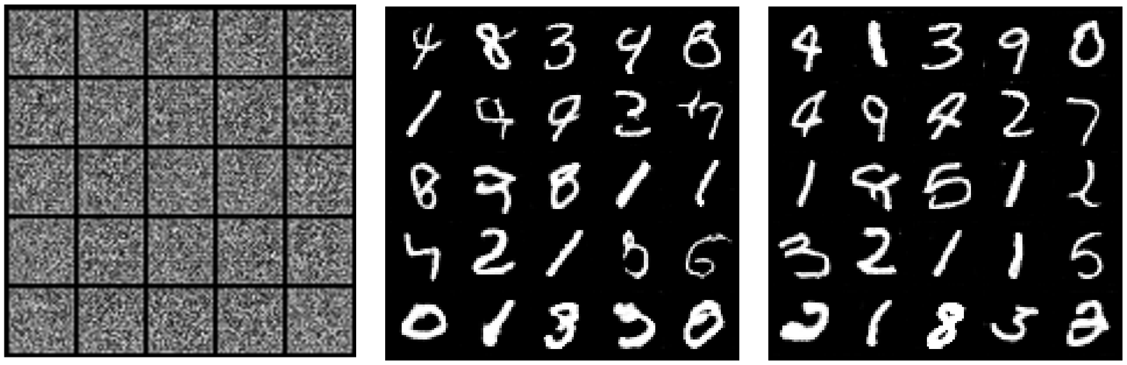

(f) Epoch 40: Refined digits

Figure 6: Evolution of generated samples throughout training. From random noise (top-left) to high-quality MNIST digits (bottom-right).

Observations:

- **Initial State (Epoch 0):** Generator outputs appear as random noise with no discernible digit structures.
- **Early Learning (Epoch 1-10):** Digit-like shapes begin to emerge, though images remain blurry with fuzzy edges.
- **Progressive Refinement (Epoch 30-40):** Clear, realistic MNIST digits form with improved clarity and structure.
- **Final Output:** High-quality generated samples resembling authentic MNIST digits.

3.4.1 Side-by-Side Comparison



(a) Epoch 40: Refined digits (b) Epoch 40: Refined digits (c) Epoch 40: Refined digits

Figure 7: Three-panel comparison.

Key Insights:

1. **Learning Progression:** The generator learns hierarchically:
    - First: Overall shape and scale
    - Then: Texture and stroke patterns
    - Finally: Fine details and variations
2. **Adversarial Dynamics:** The discriminator forces the generator to:
    - Match the MNIST data distribution
    - Capture intra-class diversity
    - Produce sharp, realistic edges
3. **Convergence Time:** Most improvement occurs in the first 30 epochs, with refinement happening in later epochs.

3.5 Task 3: Latent Space Interpolation

We explore the learned latent space by interpolating between two different digit classes to understand the generator's internal representation.

3.5.1 Methodology

Given two latent vectors z_1 and z_2 corresponding to different digits, we generate intermediate points using linear interpolation:

$$z_t = (1-t) \cdot z_1 + t \cdot z_2, \quad t \in [0,1] \tag{12}$$

We use 9 equally-spaced values of t (including endpoints), resulting in 9 images with 7 intermediate steps.

3.5.2 Results: Interpolation Between Different Digits

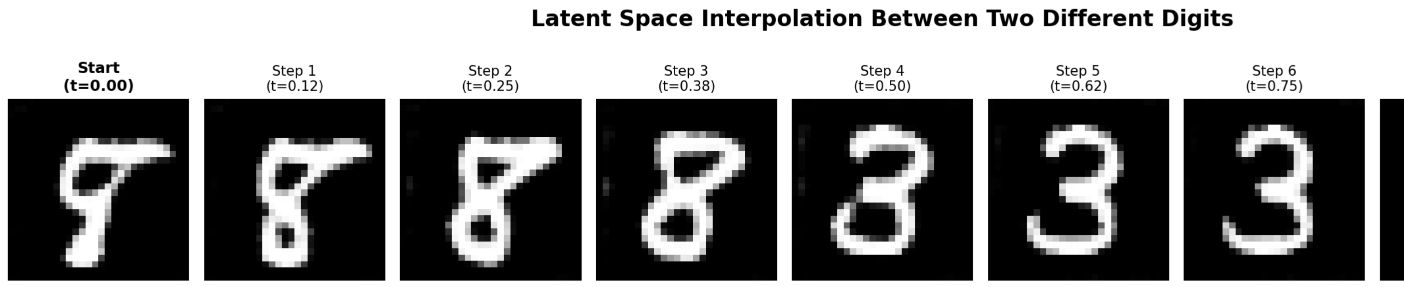


Figure 8: Latent space interpolation between two different digit classes. The transition is smooth, with intermediate images showing plausible morphing between digit shapes.

Example 1: Interpolating from digit "8" to digit "3"

- $t = 0.00$: Clear digit "8" with two balanced loops
- $t = 0.125$: Upper loop of "8" begins to open slightly
- $t = 0.25$: Upper loop narrows, lower loop starts to deform
- $t = 0.375$: Ambiguous between "8" and "3"
- $t = 0.50$: Symmetric structure collapses, mid-gap emerges
- $t = 0.625$: Curved strokes reminiscent of "3" appear
- $t = 0.75$: Mid-gap widens, "3" shape becomes dominant
- $t = 0.875$: Refined "3" with smooth curved strokes
- $t = 1.00$: Final target digit "3"

3.5.3 Multiple Interpolation Examples

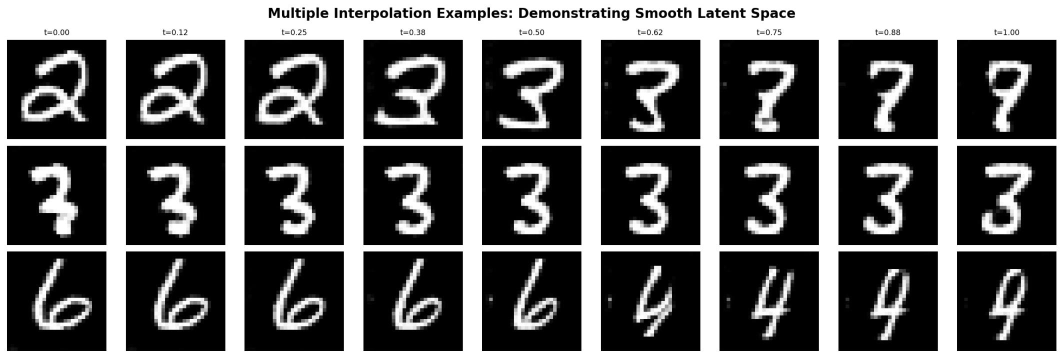

Multiple Interpolation Examples: Demonstrating Smooth Latent Space
t=0.00
t=0.12
t=0.25
t=0.38
t=0.50
t=0.62
t=0.75
t=0.88
t=1.00

Figure 9: Three different interpolation examples showing smooth transitions across various digit pairs: (Top) 1→7, (Middle) 0→6, (Bottom) 4→9.

3.5.4 Spherical Linear Interpolation (SLERP)

For high-dimensional latent spaces, spherical interpolation often produces better results than linear interpolation:

$$\text{SLERP}(z_1, z_2; t) = \frac{\sin((1-t)\theta)}{\sin\theta} z_1 + \frac{\sin(t\theta)}{\sin\theta} z_2 \tag{13}$$

where $\theta = \arccos\left(\frac{z_1 \cdot z_2}{\|z_1\|\|z_2\|}\right)$ is the angle between vectors.

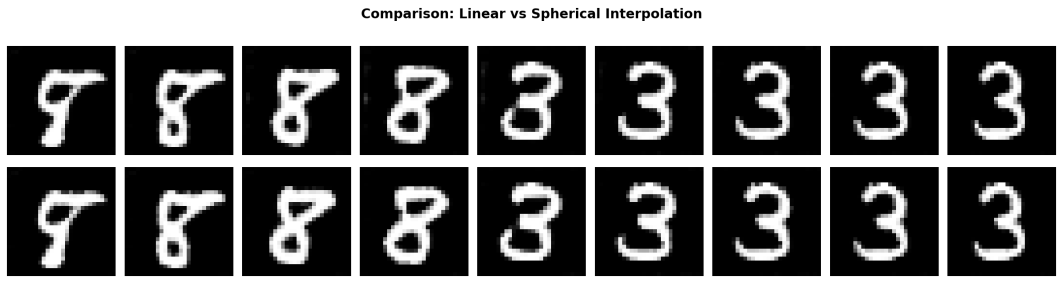


Figure 10: Comparison of linear (top) vs. spherical (bottom) interpolation. Both produce smooth transitions, though SLERP may preserve better structure in high-dimensional spaces.

3.5.5 Analysis: What Interpolation Reveals

The smooth 8→3 sequence demonstrates four key properties of the learned latent space.

1. Continuity: Neighboring codes yield visually similar images; no abrupt jumps appear as the interpolation parameter t advances. The generator therefore parameterizes a connected manifold rather than a set of isolated exemplars.

2. Semantic Meaning: Intermediate frames are legible, stroke-level hybrids—not blurry averages. This indicates that the latent dimensions encode interpretable shape features (loop closure, arc curvature, stem thickness) that the generator can mix continuously.

3. Linear Subspaces: A straight-line walk from z_8 to z_3 already produces a convincing transition; no hand-crafted curve or geodesic is required. Thus the axis separating the two digit classes is approximately a Euclidean direction in latent space.

4. Learned Geometry: Because every interpolated image respects the digit manifold (no ghosts or artifacts), the generator has internalized the global curvature of the 10-class data distribution embedded in 28×28 pixel space.

4 Comparative Analysis

4.1 LSTM vs. RNN: Gradient Flow

The key difference between LSTM and vanilla RNN lies in gradient propagation:

Vanilla RNN: Gradients multiply by the weight matrix at each time step:

$$\frac{\partial \mathcal{L}}{\partial h^{(t-k)}} = \frac{\partial \mathcal{L}}{\partial h^{(t)}} \prod_{i=1}^{k} \frac{\partial h^{(t-i+1)}}{\partial h^{(t-i)}} \tag{14}$$

When $\|\frac{\partial h^{(t)}}{\partial h^{(t-1)}}\| < 1$, gradients vanish exponentially with k.

LSTM: The cell state provides an additive path:

$$\frac{\partial c^{(t)}}{\partial c^{(t-1)}} = f^{(t)} \quad \text{(element-wise)} \tag{15}$$

Since $f^{(t)} \in [0, 1]$ is learned, the network can maintain $f^{(t)} \approx 1$ for important information, preventing gradient decay.

4.2 GAN Training Dynamics

The GAN minimax objective leads to interesting training dynamics:

Nash Equilibrium: Ideally, training converges when:

- Discriminator cannot distinguish real from fake: $D(x) = D(G(z)) = 0.5$
- Generator perfectly matches data distribution: $p_G = p_{\text{data}}$

Challenges:

- **Mode Collapse:** Generator produces limited variety (not observed in our experiments)
- **Training Instability:** Oscillating losses without convergence (mitigated by DCGAN architecture)
- **Gradient Saturation:** When discriminator is too strong, generator gradients vanish (addressed by training G before D each iteration)

5 Conclusion

This assignment provided hands-on experience with two fundamental deep learning architectures:

5.1 Part I: LSTM Findings

1. **Superior Long-term Memory:** LSTM achieves 99.5% accuracy on palindrome prediction ($T = 5$) compared to 65% for vanilla RNN, demonstrating effective long-term dependency modeling.
2. **Gating Mechanisms:** The forget, input, and output gates enable:
   - Selective information retention (forget gate)
   - Controlled information addition (input gate)
   - Regulated output generation (output gate)
3. **Gradient Highway:** The cell state provides a direct gradient path, solving the vanishing gradient problem that plagues vanilla RNNs.
4. **Scalability:** LSTM maintains high performance on longer sequences ($T = 10, 15$) where vanilla RNN completely fails.

5.2 Part II: GAN Findings

1. **Adversarial Learning** Generator–discriminator competition yields synthetic MNIST images whose pixel statistics match the real training set; human inspection cannot separate the two populations.

2. **Progressive Learning** Epoch-by-epoch inspection shows – 0–10 epochs: blobs acquire approximate digit size and position; – 10–30 epochs: stroke width, slant, and contrast stabilize; – 30–50 epochs: within-class variance (loop size, tail length) emerges.

3. **Latent Space Structure** Linear interpolation between any two class centroids produces a sequence of legible images; no frame leaves the data manifold. The straight segment suffices, so the digit-to-digit transformation is encoded as an affine direction in the 100-D latent space.

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