CSCI-1200 Data Structures
Test 3 — Practice Problems

Note: This packet contains practice problems from three previous exams. Your exam will contain approximately one third as many problems.

1 Bitdiddle Post-Breadth Tree Traversal [/ 31]

1.1 Balanced Tree Example [/ 3]

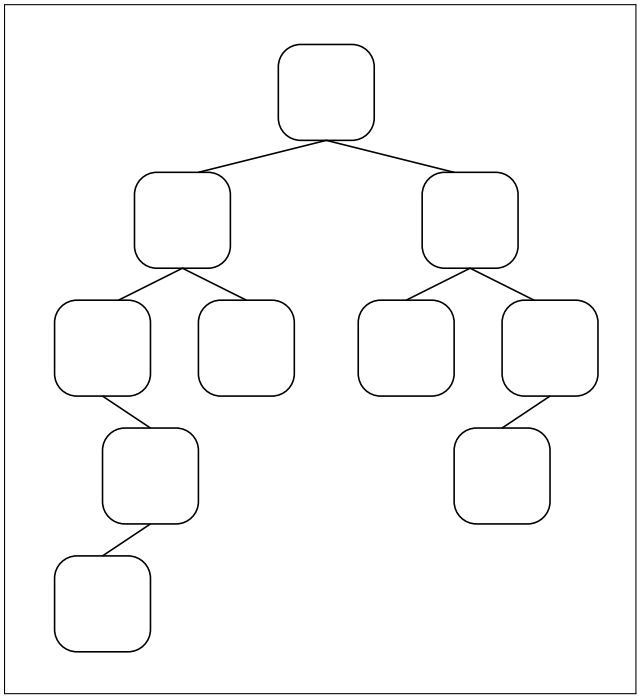
Ben Bitdiddle really wants to get his name on a traversal ordering. Even without a real world application for its use, he has invented what he calls the *post-breadth ordering*. His primary demonstration example is an exactly balanced, binary search tree with the numbers 1-15.

Your first task is to make a neat diagram of this tree in the box on the right.

For this example, Ben decrees that the `PrintPostBreadth` function should output:

```
LEVEL 0:  1 3 5 7 9 11 13 15
LEVEL 1:  2 6 10 14
LEVEL 2:  4 12
LEVEL 3:  8
```

1.2 Un-Balanced Tree Example [/ 3]



Alyssa P. Hacker rolls her eyes at Ben but agrees to help him with the implementation. However, before tackling the implementation she wants to make sure that Ben's idea is sound. She sketches the unbalanced tree shape on the left.

Your second task is to place the numbers 1-10 in this diagram so it is a proper binary search tree.

This unbalanced tree initially confuses Ben. But he thinks for a while and decides that for his new traversal ordering, level 0 is defined to be all of the leaves of the tree, level 1 is the parents of the leaves, level 2 is the grandparents, etc. So he decrees that for this second example, the output of the `PrintPostBreadth` function is:

```
LEVEL 0:  2 5 7 9
LEVEL 1:  3 4 8 10
LEVEL 2:  1 6
```

Alyssa studies Ben's sample output carefully and then asks Ben if the traversal ordering will ever contain repeated elements. Ben says no, each element in the structure should be output exactly once. Alyssa

suggests that they add a boolean *mark* member variable to the `Node` class since it will be helpful for an efficient implementation. This flag will help ensure the traversal ordering does not contain duplicates.

1.3 `CollectLeaves` Implementation [/ 11]

Alyssa's `Node` class is on the right.

She further suggests starting with the implementation of a helper function named `CollectLeaves`. This is a void recursive function that takes in two arguments: `ptr` is a pointer to a `Node` (initially the root of the tree), and `leaves` is an STL list of pointers to `Node`s (the list is initially empty) that will collect all of the leaves of the tree.

She also indicates that this function should initialize all of the `mark` variables. Only the leaf nodes should be marked `true`.

Complete the implementation below.

```
class Node {
public:
  // CONSTRUCTOR
  Node(int v) : value(v), mark(false),
    left(NULL), right(NULL), parent(NULL) {}
  // REPRESENTATION
  int value;
  bool mark;
  Node* left;
  Node* right;
  Node* parent;
};
```

```
void CollectLeaves(                    ptr,                    leaves) {
```

sample solution: 9 line(s) of code

```
}
```

1.4 `PrintPostBreadth` Implementation [/ 14]

Now finish the implementation of the `PrintPostBreadth` function:

```
void PrintPostBreadth(Node* root) {
   std::list<Node*> current;
   CollectLeaves(root,current);
   int count = 0;
   while (current.size() > 0) {
      std::cout << "LEVEL " << count <<  ": ";
```

sample solution: 11 line(s) of code

```
      std::cout << std::endl;
   }
}
```

2 Genome Difference Maps [/ 36]

Louis B. Reasoner has taken a job at a genome sequencing startup working on algorithms to detect differences between the genomes of different species. He came up with the sketch of the data structure on the right and showed it to his manager and got approval to start implementation.

totals

dog	5
fruit fly	4
human	9

kmers

ACT	dog	2
	fruit fly	2
	human	3
CAT	fruit fly	1
GAG	dog	1
	fruit fly	1
	human	2
TAG	dog	2
	human	4

He's defined two typedefs named `count_t` and `kmer_t` to improve the readability of his code. Here's an example of how this data structure is constructed using the `Add` function:

```
kmer_t kmers;
count_t totals;
Add(totals,kmers,"human","ACT");     Add(totals,kmers,"human","ACT");
Add(totals,kmers,"human","ACT");     Add(totals,kmers,"human","GAG");
Add(totals,kmers,"human","GAG");     Add(totals,kmers,"human","TAG");
Add(totals,kmers,"human","TAG");     Add(totals,kmers,"human","TAG");
Add(totals,kmers,"human","TAG");
Add(totals,kmers,"dog","ACT");       Add(totals,kmers,"dog","ACT");
Add(totals,kmers,"dog","GAG");       Add(totals,kmers,"dog","TAG");
Add(totals,kmers,"dog","TAG");
Add(totals,kmers,"fruit fly","ACT"); Add(totals,kmers,"fruit fly","ACT");
Add(totals,kmers,"fruit fly","CAT"); Add(totals,kmers,"fruit fly","GAG");
```

Two of the key operations for this data structure are to query the number of matches of a given k-mer for a particular species and to find the most frequently occurring k-mer for a species. Here are several example usages of the `Query` and `MostCommon` functions:

```
assert (Query(kmers,"human","ACT") == 3);   assert (MostCommon(kmers,"human") == "TAG");
assert (Query(kmers,"human","CAT") == 0);   assert (MostCommon(kmers,"fruit fly") == "ACT");
assert (Query(kmers,"human","TAG") == 4);   assert (MostCommon(kmers,"cat") == "");
assert (Query(kmers,"cat","ACT") == 0);
assert (Query(kmers,"dog","GAG") == 1);
```

Finally, we can compute the difference between two species. The *k-mer fraction* is the percent of a species total k-mers that match the particular k-mer. The *k-mer difference* is the absolute value of the difference between the k-mer fractions for each of the species. And the overall difference between two species is the sum over all k-mers of the k-mer difference. Here is the math to calculate the difference between a `human` and a `dog`:

ACT:	abs(2/5 - 3/9)	=	0.067
CAT:		=	0.000
GAG:	abs(1/5 - 2/9)	=	0.022
TAG:	abs(2/5 - 4/9)	=	0.044
overall:		=	0.133

Here is code to call the `Difference` helper function:

```
std::cout << "Difference between human & dog       "
          << Difference(totals,kmers,"human","dog") << std::endl;
std::cout << "Difference between human & fruit fly "
          << Difference(totals,kmers,"human","fruit fly") << std::endl;
std::cout << "Difference between dog & fruit fly   "
          << Difference(totals,kmers,"dog","fruit fly") << std::endl;
```

And the resulting output:

```
Difference between human & dog       0.133
Difference between human & fruit fly 0.889
Difference between dog & fruit fly   0.800
```

2.1 The `typedefs` [/ 4]

First, fill in the `typedef` declarations for the two shorthand types used on the previous page.

```
typedef [                                        ] count_t;

typedef [                                        ] kmer_t;
```

2.2 `Add` Implementation [/ 7]

Next, finish the implementation of the `Add` function.

```
void Add( [            ] totals, [            ] kmers,
          [            ] species, [            ] kmer) {
```

sample solution: ≤ 4 line(s) of code

```
}
```

If the data structure contains s different species, and k unique k-mers, and each animal contains p total k-mers, what is the order notation for the running time of a single call to `Add`? Write 2-3 concise and well-written sentences justifying your answer.

2.3 `Query` Implementation [/ 6]

```
int Query(                  kmers,                  species,                  kmer) {
```

sample solution: 7 line(s) of code

```
}
```

2.4 `MostCommon` Implementation [/ 7]

```
                MostCommon(                  kmers,                  species) {
    std::string answer = "";
    int count = -1;
```

sample solution: 8 line(s) of code

```
    return answer;
}
```

2.5 `Difference` Implementation [/ 12]

```
float Difference(                totals,                kmers,
                 speciesA,                speciesB) {
```

sample solution: 3 line(s) of code

```
    if (                                                ) {
        std::cerr << "ERROR!  One or both species are unknown" << std::endl;
        return -1;
    }
```

sample solution: 8 line(s) of code

```
}
```

If the data structure contains s different species, and k unique k-mers, and each animal contains p total k-mers, what is the order notation for the running time of a single call to `Difference`? Write 2-3 concise and well-written sentences justifying your answer.

3 Prescribed Pre-Ordering [/ 21]

In this problem we will create an algorithm to construct a binary search tree from the desired pre-order traversal order. The driver function (below) takes in this sequence as a STL `vector`. If the contents of the vector is not a valid pre-order traversal order of a binary search tree, the function should return NULL.

```
template <class T> class Node {
public:
  Node(T v) : value(v),left(NULL),right(NULL) {}
  T value;
  Node* left;
  Node* right;
};

template <class T> void destroy(Node<T>* root) {
  if (root == NULL) return;
  destroy(root->left);
  destroy(root->right);
  delete root;
}

// "driver" function (starts the recursive function that does the actual work)
template <class T> Node<T>* MakePreOrderTree(const std::vector<T>& values) {
  if (values.size() == 0) return NULL;
  return MakePreOrderTree(values,0,values.size()-1);
}
```

3.1 Test Cases [/ 7]

First, create 4 different test cases of input for this problem. Each input vector should contain the numbers 1-7. The first two should be valid pre-orderings for a binary search tree containing these 7 numbers. *Draw the corresponding tree for these cases.* The other two test case inputs should be invalid pre-orderings.

valid	valid

invalid	invalid

3.2 Finish the `MakePreOrderTree` Implementation [/ 14]

Note: If you discover the input sequence is an invalid pre-ordering for a binary search tree, make sure you do not leak any memory!

```
template <class T>
Node<T>* MakePreOrderTree(const std::vector<T>& values, int start, int end) {
   assert (start <= end);
   // find the split between the left & right branches
```

sample solution: 9 line(s) of code

```
   // make the new node
   Node<T>* answer = new Node<T>(values[start]);
   // recurse left and/or right as needed
```

sample solution: 14 line(s) of code

```
   return answer;
}
```

4 Un-Occupied Erase [/ 39]

Ben Bitdiddle was overwhelmed during the Data Structures lecture that covered the implementation details of `erase` for binary search trees. Separately handling the cases where the node to be erased had zero, one, or two non-NULL child pointers and then moving data around within the tree and/or disconnecting and reconnecting pointers seemed pointlessly complex (pun intended). Ben's plan is to instead leave the overall tree structure unchanged, but mark a node as *unoccupied* when the node containing the value to be erased has one or more children.

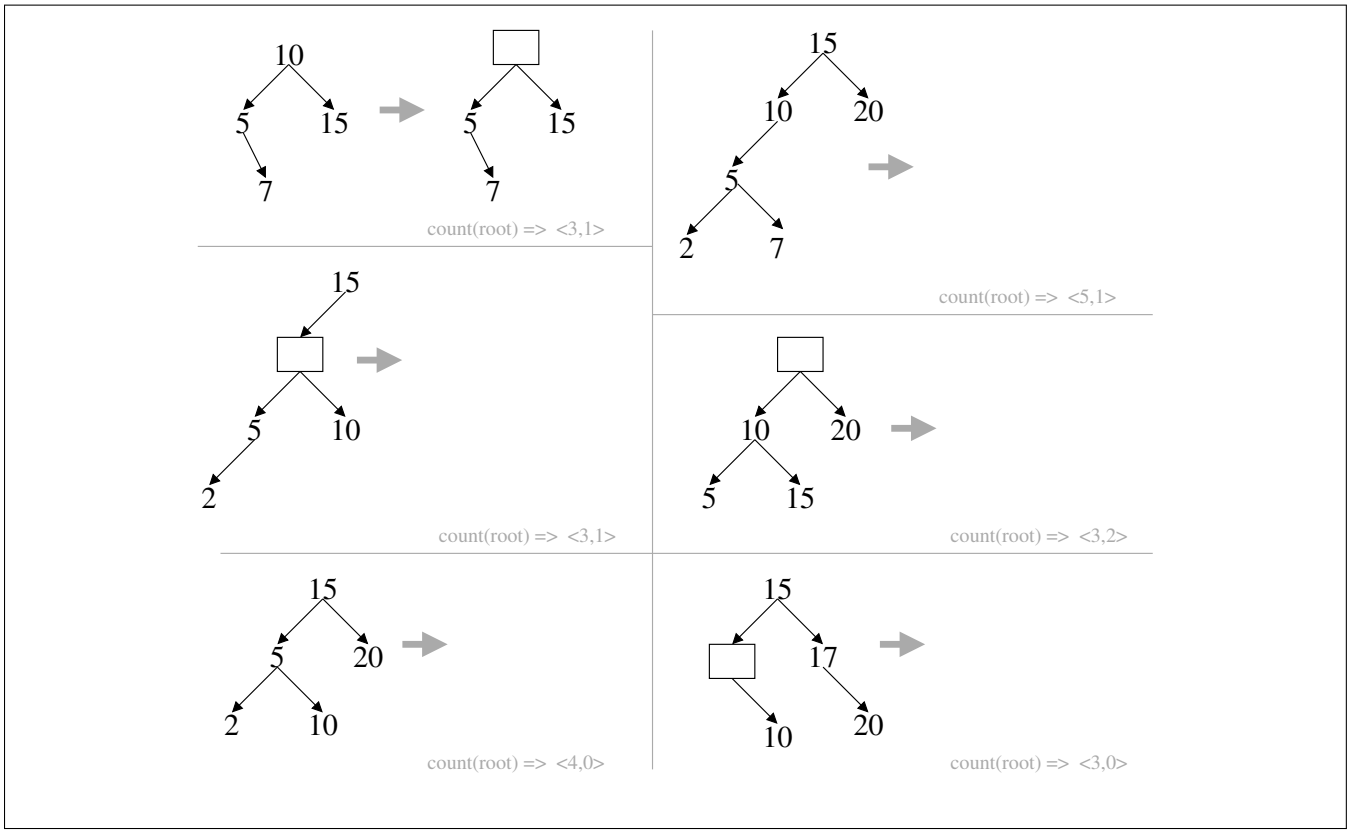
Ben's modified `Node` class is provided on the right.

```
template <class T>
class Node {
public:
  Node(const T& v) :
    occupied(true), value(v),
    left(NULL), right(NULL) {}
  bool occupied;
  T value;
  Node* left;
  Node* right;
};
```

4.1 Diagramming the Expected Output of `erase` [/ 6]

First, help Ben work through different test cases for the `erase` function. For each of the sample trees below, draw the tree after the call `erase(root,10)`. The first one has been done for you.

If a node is unoccupied, we draw it as an empty box. Below each result diagram we note the counts of occupied nodes and the number of unoccupied nodes within the tree. (We'll write the `count` function on the next page!) Note that an unoccupied node should always have at least one non-NULL child.



4.2 Counting Occupied & Unoccupied Nodes [/ 8]

Now let's write a recursive `count` function that takes a single argument, a pointer to the root of the tree, and returns an STL pair of integers. The first integer is the total number of *occupied* nodes in the tree and the second integer is the total number of *unoccupied* nodes in the tree. Refer to the diagrams on the previous page as examples.

sample solution: 10 line(s) of code

Alyssa P. Hacker stops by to see if Ben needs any help with his programming. She notes that when we insert a value into a tree, sometimes we will be able to re-use an unoccupied node, and other times we will have to create a new node and add it to the structure. She suggests a few helper functions that will be helpful in implementing the `insert` function for his binary search tree with unoccupied nodes:

```
template <class T>
const T& largest_value(Node<T>* p) {
  assert (p != NULL);
  if (p->right == NULL) {
    if (p->occupied)
      return p->value;
    else
      return largest_value(p->left);
  }
  return largest_value(p->right);
}
```

```
template <class T>
const T& smallest_value(Node<T>* p) {
  assert (p != NULL);
  if (p->left == NULL) {
    if (p->occupied)
      return p->value;
    else
      return smallest_value(p->right);
  }
  return smallest_value(p->left);
}
```

4.3 Implement `erase` for Trees with Unoccupied Nodes [/ 13]

Now implement the `erase` function for Ben's binary search tree with unoccupied nodes. This function takes in two arguments, a pointer to the root node and the value to erase, and returns true if the value was successfully erased or false if the value was not found in the tree.

sample solution: 28 line(s) of code

4.4 Implement `insert` for Trees with Unoccupied Nodes [/ 12]

Now implement the `insert` function for Ben's binary search tree with unoccupied nodes. This function takes in two arguments, a pointer to the root node and the value to insert, and returns true if the value was successfully inserted or false if the value was not inserted because it was a duplicate of a value already in the tree. Use the provided `smallest_value` and `largest_value` functions in your implementation.

sample solution: 25 line(s) of code

5 Classroom Scheduler Maps [/ 37]

Louis B. Reasoner has been hired to automate RPI's weekly classroom scheduling system. A big fan of the C++ STL `map` data structure, he decided that `map`s would be a great fit for this application. Here's a portion of the main function with an example of how his program works:

```
room_reservations rr;
add_room(rr,"DCC",308);
add_room(rr,"DCC",318);
add_room(rr,"Lally",102);
add_room(rr,"Lally",104);

bool success = make_reservation(rr, "DCC",   308, "Monday",    18, 2,  "DS Exam")    &&
               make_reservation(rr, "DCC",   318, "Monday",    18, 2,  "DS Exam")    &&
               make_reservation(rr, "DCC",   308, "Tuesday",   10, 2,  "DS Lecture") &&
               make_reservation(rr, "Lally", 102, "Wednesday", 10, 10, "DS Lab")     &&
               make_reservation(rr, "Lally", 104, "Wednesday", 10, 10, "DS Lab")     &&
               make_reservation(rr, "DCC",   308, "Friday",    10, 2,  "DS Lecture");
assert (success == true);
```

In the small example above, only 4 classrooms are schedulable. To make a reservation we specify the building and room number, the day of the week (the initial design only handles Monday-Friday), the start time (using military 24-hour time, where 18 = 6pm), the duration (in # of hours), and an STL `string` description of the event.

Here are a few key functions Louis wrote:

```
bool operator< (const std::pair<std::string,int> &a, const std::pair<std::string,int> &b) {
  return (a.first < b.first || (a.first == b.first && a.second < b.second));
}

void add_room(room_reservations &rr, const std::string &building, int room) {
  week_schedule ws;
  std::vector<std::string> empty_day(24,"");
  ws[std::string("Monday")]    = empty_day;
  ws[std::string("Tuesday")]   = empty_day;
  ws[std::string("Wednesday")] = empty_day;
  ws[std::string("Thursday")]  = empty_day;
  ws[std::string("Friday")]    = empty_day;
  rr[std::make_pair(building,room)] = ws;
}
```

Unfortunately, due to hard disk crash, Louis has lost the details of the two `typedef`s and his implementation of the `make_reservation` function. Your task is to help him recreate the implementation.

He does have a few more test cases for you to examine. Given the current state of the reservation system, these attempted reservations will all fail:

```
success = make_reservation(rr, "DCC", 308, "Monday",   19, 3, "American Sniper") ||
          make_reservation(rr, "DCC", 307, "Monday",   19, 3, "American Sniper") ||
          make_reservation(rr, "DCC", 308, "Monday",   22, 3, "American Sniper") ||
          make_reservation(rr, "DCC", 308, "Saturday", 19, 3, "American Sniper");
assert (success == false);
```

With these explanatory messages printed to `std::cerr`:

```
ERROR! conflicts with prior event: DS Exam
ERROR! room DCC 307 does not exist
ERROR! invalid time range: 22-25
ERROR! invalid day: Saturday
```

5.1 The `typedefs` [/ 5]

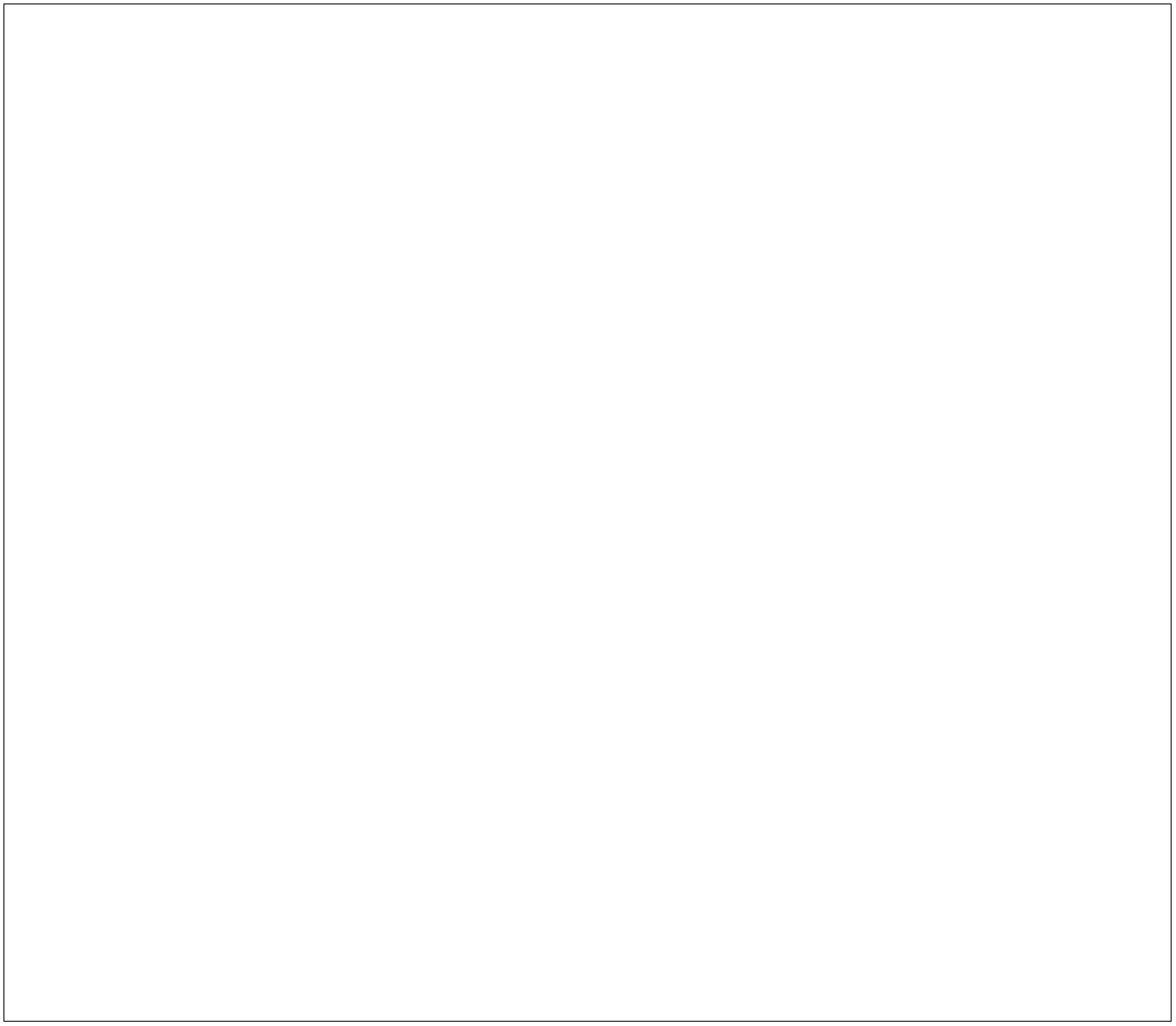
First, fill in the `typedef` declarations for the two shorthand types used on the previous page.

`typedef` [ ] `week_schedule;`

`typedef` [ ] `room_reservations;`

5.2 Diagram of the data stored in `room_reservations rr` [/ 8]

Now, following the conventions from lecture for diagramming `map` data structures, draw the specific data stored in the `rr` variable after executing the instructions on the previous page. Yes, this is actually quite a big diagram, so don't attempt to draw *everything*, but be neat and draw enough detail to demonstrate that you understand how each component of the data structure is organized and fits together.



5.3 Implementing `make_reservation` [/ 16]

Next, implement the `make_reservation` function. Closely follow the samples shown on the first page of this problem to match the arguments, return type, and error checking.

sample solution: 28 line(s) of code

5.4 Performance and Memory Analysis [/ 8]

Now let's analyze the running time of the `make_reservation` function you just wrote. If RPI has b buildings, and each building has on average c classrooms, and we are storing schedule information for d days (in the sample code d=5 days of the week), and the resolution of the schedule contains t time slots (in the sample code $t = 24$ 1-hour time blocks), with a total of e different events, each lasting an average of s timeslots (data structures lecture lasts 2 1-hour time blocks), what is the order notation for the running time of this function? Write 2-3 concise and complete sentences explaining your answer.

Using the same variables, write a simple formula for the approximate upper bound on the memory required to store this data structure. Assume each int is 4 bytes and each string has at most 32 characters = 32 bytes per string. Omit the overhead for storing the underlying tree structure of nodes & pointers. Do not simplify the answer as we normally would for order notation analysis. Write 1-2 concise and complete sentences explaining your answer.

Finally, using the same variables, what would be the order notation for the running time of a function (we didn't ask you to write this function!) to find all currently available rooms for a specific day and time range? Write 1-2 concise and complete sentences explaining your answer.

6 Fashionable Sets [/ 14]

In this problem you will write a recursive function named `outfits` that takes as input two arguments: `items` and `colors`. `items` is an STL `list` of STL `string`s representing different types of clothing. `colors` is an STL `list` of STL `set`s of STL `string`s representing the different colors of each item of clothing. Your function should return an STL `vector` of STL `string`s describing each unique outfit (in any order) that can be created from these items of clothing.

Here is a small example:

```
items   =  { "hat", "shirt", "pants" }
colors  =  { { "red" },
             { "red", "green", "white" },
             { "blue", "black" } }
```

```
red hat &   red shirt &  blue pants
red hat & green shirt &  blue pants
red hat & white shirt &  blue pants
red hat &   red shirt & black pants
red hat & green shirt & black pants
red hat & white shirt & black pants
```

sample solution: 22 line(s) of code

7 Spicy Chronological Sets using Maps [/ 33]

Ben Bitdiddle is organizing his spice collection using an STL `set` but runs into a problem. He needs the fast `find`, `insert`, and `erase` of an STL set, but in addition to organizing his spices alphabetically, he also needs to print them out in chronological order (so he can replace the oldest spices).

Ben is sure he'll have to make a complicated custom data structure, until Alyssa P. Hacker shows up and says it can be done using an STL `map`. She quickly sketches the diagram below for Ben, but then has to dash off to an interview for a Google summer internship.

Alyssa's diagram consists of 3 variables. The first variable, containing most of the data, is defined by a `typedef`. Even though he's somewhat confused by Alyssa's diagram, Ben has pushed ahead and decided on the following interface for building his spice collection:

```
chrono_set cs;
std::string oldest = "";
std::string newest = "";
insert(cs,oldest,newest,"garlic");
insert(cs,oldest,newest,"oregano");
insert(cs,oldest,newest,"nutmeg");
insert(cs,oldest,newest,"cinnamon");
insert(cs,oldest,newest,"basil");
insert(cs,oldest,newest,"sage");
insert(cs,oldest,newest,"dill");
```

chrono_set cs:

"basil"	<"cinnamon", "sage">
"cinnamon"	<"nutmeg", "basil">
"dill"	<"sage", "">
"garlic"	<"","oregano">
"nutmeg"	<"oregano","cinnamon">
"oregano"	<"garlic", "nutmeg">
"sage"	<"basil", "dill">

std::string oldest: "garlic"

std::string newest: "dill"

Ben would like to output the spices in 3 ways:

```
ALPHA ORDER:        basil   cinnamon       dill     garlic     nutmeg    oregano       sage
OLDEST FIRST:      garlic    oregano     nutmeg   cinnamon      basil       sage       dill
NEWEST FIRST:        dill       sage      basil   cinnamon     nutmeg    oregano     garlic
```

If he buys more of a spice already in the collection, the old spice jar should be discarded and replaced. For example, after calling:

```
insert(cs,oldest,newest,"cinnamon");
```

The spice collection output should now be:

```
ALPHA ORDER:        basil   cinnamon       dill     garlic     nutmeg    oregano       sage
OLDEST FIRST:      garlic    oregano     nutmeg      basil       sage       dill   cinnamon
NEWEST FIRST:    cinnamon       dill       sage      basil     nutmeg    oregano     garlic
```

7.1 The `typedef` [/ 3]

First, help Ben by completing the definition of the typedef below:

```
typedef [                                                              ] chrono_set;
```

7.2 Printing out the spice collection [/ 8]

Next, write the code to output (to `std::cout`) Ben's spices in alphabetical and chronological order:

```
std::cout << "ALPHA ORDER:  ";
```

sample solution: 4 line(s) of code

```
std::cout << std::endl;
std::cout << "OLDEST FIRST: ";
```

sample solution: 5 line(s) of code

```
std::cout << std::endl;
```

7.3 Performance Analysis [/ 5]

Assuming Ben has n spices in his collection, what is the order notation for each operation? *Note: You may want to first complete the implementation of the* `insert` *operation on the next page.*

printing in alphabetical order:

printing in chronological order:

`insert`-ing a spice to the collection:

7.4 Implementing `insert` for the `chrono_set` [/ 17]

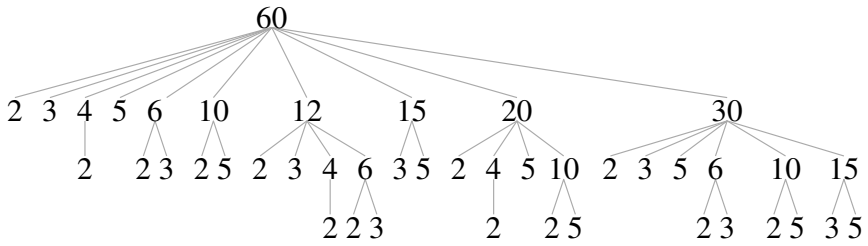
Finally, implement the `insert` function for Ben's spice collection. Make sure to handle all corner cases.

sample solution: 26 line(s) of code

8 Factor Tree [/ 13]

Write a recursive function named `factor_tree` that takes in a single argument of integer type and constructs the tree of the factors (and factors of each factor) of the input number. The function returns a pointer to the root of this tree. The example below illustrates the tree returned from the call `factor_tree(60)`.

```
class Node {
public:
  int value;
  std::vector<Node*> factors;
};
```

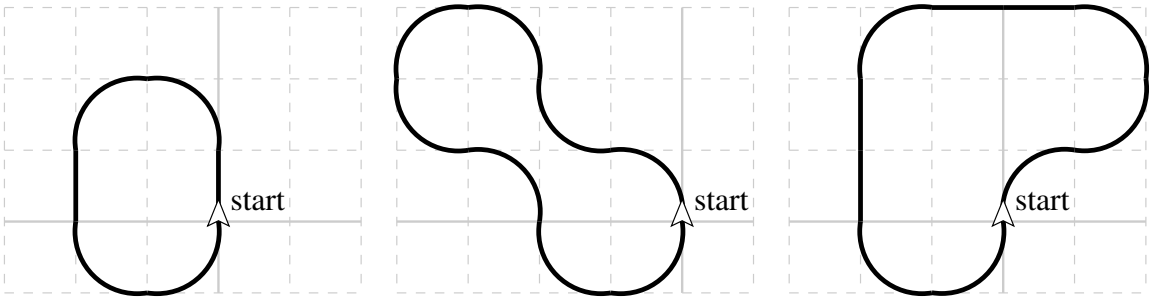


sample solution: 10 line(s) of code

9 Driving in Circles [/ 18]

In this problem you will write a recursive function named `driving` that outputs to `std::cout` all *closed loop* paths of driving instructions on a rectangular grid less than or equal to a specified maximum path length. The car begins at (0,0) pointing north and at each step can go *straight*, *left*, or *right*. A path is said to "close the loop" if it is finishes where it started, pointing in the same direction. For example, here are three sample closed loop paths (also illustrated below):

```
closed loop:  straight left left straight left left
closed loop:  left right left left left right left left
closed loop:  right left left straight straight left straight straight left left
```



We provide the `Car` class and several helper functions:

```
class Car {
public:
  Car(int x_,int y_,std::string dir_) : x(x_),y(y_),dir(dir_) {}
  int x;
  int y;
  std::string dir;
};
bool operator==(const Car &a, const Car &b) {
  return (a.x == b.x && a.y == b.y && a.dir == b.dir);
}
Car go_straight(const Car &c) {
  if      (c.dir == "north") { return Car(c.x  ,c.y+1,c.dir); }
  else if (c.dir == "east")  { return Car(c.x+1,c.y  ,c.dir); }
  else if (c.dir == "south") { return Car(c.x  ,c.y-1,c.dir); }
  else                       { return Car(c.x-1,c.y  ,c.dir); }
}
Car turn_left(const Car &c) {
  if      (c.dir == "north") { return Car(c.x-1,c.y+1,"west");  }
  else if (c.dir == "east")  { return Car(c.x+1,c.y+1,"north"); }
  else if (c.dir == "south") { return Car(c.x+1,c.y-1,"east");  }
  else                       { return Car(c.x-1,c.y-1,"south"); }
}
Car turn_right(const Car &c) {
  if      (c.dir == "north") { return Car(c.x+1,c.y+1,"east");  }
  else if (c.dir == "east")  { return Car(c.x+1,c.y-1,"south"); }
  else if (c.dir == "south") { return Car(c.x-1,c.y-1,"west");  }
  else                       { return Car(c.x-1,c.y+1,"north"); }
}
```

Your function should take in 3 arguments: the path constructed so far, the current car position & direction, and the maximum number of steps/instructions allowed. For example:

```
std::vector<std::string> path;
Car car(0,0,"north");
int max_steps = 10;
driving (path,car,max_steps);
```

Now implement the recursive `driving` function.

sample solution: 25 line(s) of code