CSCI-1200 Data Structures
Test 3 — Practice Problems

Note: This packet contains practice problems from three previous exams. Your exam will contain approximately one third as many problems.

1 Un-Occupied Erase [/ 39]

Ben Bitdiddle was overwhelmed during the Data Structures lecture that covered the implementation details of `erase` for binary search trees. Separately handling the cases where the node to be erased had zero, one, or two non-NULL child pointers and then moving data around within the tree and/or disconnecting and reconnecting pointers seemed pointlessly complex (pun intended). Ben's plan is to instead leave the overall tree structure unchanged, but mark a node as *unoccupied* when the node containing the value to be erased has one or more children.

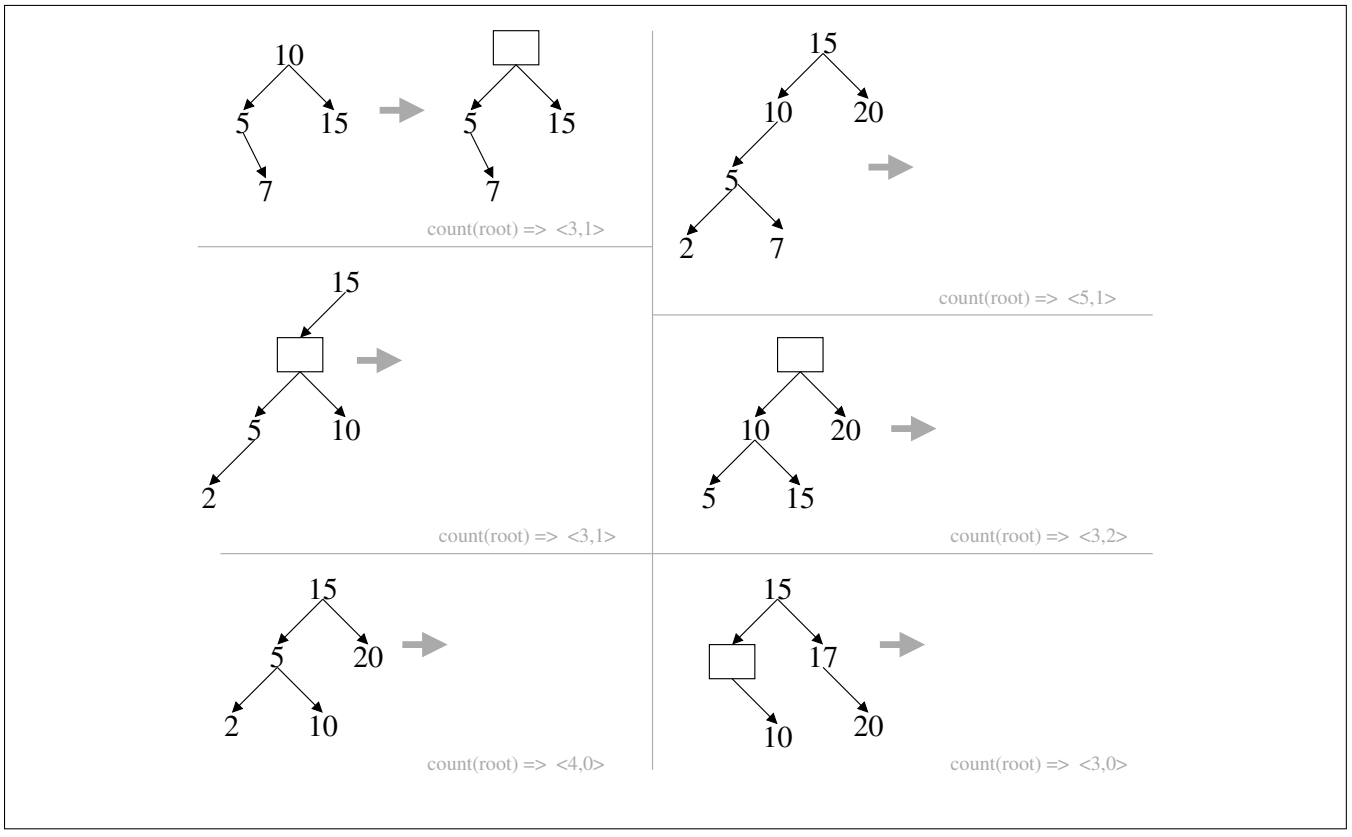
Ben's modified `Node` class is provided on the right.

```
template <class T>
class Node {
public:
  Node(const T& v) :
    occupied(true), value(v),
    left(NULL), right(NULL) {}
  bool occupied;
  T value;
  Node* left;
  Node* right;
};
```

1.1 Diagramming the Expected Output of `erase` [/ 6]

First, help Ben work through different test cases for the `erase` function. For each of the sample trees below, draw the tree after the call `erase(root,10)`. The first one has been done for you.

If a node is unoccupied, we draw it as an empty box. Below each result diagram we note the counts of occupied nodes and the number of unoccupied nodes within the tree. (We'll write the `count` function on the next page!) Note that an unoccupied node should always have at least one non-NULL child.



1.2 Counting Occupied & Unoccupied Nodes [/ 8]

Now let's write a recursive `count` function that takes a single argument, a pointer to the root of the tree, and returns an STL pair of integers. The first integer is the total number of *occupied* nodes in the tree and the second integer is the total number of *unoccupied* nodes in the tree. Refer to the diagrams on the previous page as examples.

sample solution: 10 line(s) of code

Alyssa P. Hacker stops by to see if Ben needs any help with his programming. She notes that when we insert a value into a tree, sometimes we will be able to re-use an unoccupied node, and other times we will have to create a new node and add it to the structure. She suggests a few helper functions that will be helpful in implementing the `insert` function for his binary search tree with unoccupied nodes:

```
template <class T>
const T& largest_value(Node<T>* p) {
  assert (p != NULL);
  if (p->right == NULL) {
    if (p->occupied)
      return p->value;
    else
      return largest_value(p->left);
  }
  return largest_value(p->right);
}
```

```
template <class T>
const T& smallest_value(Node<T>* p) {
  assert (p != NULL);
  if (p->left == NULL) {
    if (p->occupied)
      return p->value;
    else
      return smallest_value(p->right);
  }
  return smallest_value(p->left);
}
```

1.3 Implement `erase` for Trees with Unoccupied Nodes [/ 13]

Now implement the `erase` function for Ben's binary search tree with unoccupied nodes. This function takes in two arguments, a pointer to the root node and the value to erase, and returns true if the value was successfully erased or false if the value was not found in the tree.

sample solution: 28 line(s) of code

1.4 Implement `insert` for Trees with Unoccupied Nodes [/ 12]

Now implement the `insert` function for Ben's binary search tree with unoccupied nodes. This function takes in two arguments, a pointer to the root node and the value to insert, and returns true if the value was successfully inserted or false if the value was not inserted because it was a duplicate of a value already in the tree. Use the provided `smallest_value` and `largest_value` functions in your implementation.

sample solution: 25 line(s) of code

2 Classroom Scheduler Maps [/ 37]

Louis B. Reasoner has been hired to automate RPI's weekly classroom scheduling system. A big fan of the C++ STL `map` data structure, he decided that `map`s would be a great fit for this application. Here's a portion of the main function with an example of how his program works:

```
room_reservations rr;
add_room(rr,"DCC",308);
add_room(rr,"DCC",318);
add_room(rr,"Lally",102);
add_room(rr,"Lally",104);

bool success = make_reservation(rr, "DCC",   308, "Monday",    18, 2,  "DS Exam")    &&
               make_reservation(rr, "DCC",   318, "Monday",    18, 2,  "DS Exam")    &&
               make_reservation(rr, "DCC",   308, "Tuesday",   10, 2,  "DS Lecture") &&
               make_reservation(rr, "Lally", 102, "Wednesday", 10, 10, "DS Lab")     &&
               make_reservation(rr, "Lally", 104, "Wednesday", 10, 10, "DS Lab")     &&
               make_reservation(rr, "DCC",   308, "Friday",    10, 2,  "DS Lecture");
assert (success == true);
```

In the small example above, only 4 classrooms are schedulable. To make a reservation we specify the building and room number, the day of the week (the initial design only handles Monday-Friday), the start time (using military 24-hour time, where 18 = 6pm), the duration (in # of hours), and an STL `string` description of the event.

Here are a few key functions Louis wrote:

```
bool operator< (const std::pair<std::string,int> &a, const std::pair<std::string,int> &b) {
  return (a.first < b.first || (a.first == b.first && a.second < b.second));
}

void add_room(room_reservations &rr, const std::string &building, int room) {
  week_schedule ws;
  std::vector<std::string> empty_day(24,"");
  ws[std::string("Monday")]    = empty_day;
  ws[std::string("Tuesday")]   = empty_day;
  ws[std::string("Wednesday")] = empty_day;
  ws[std::string("Thursday")]  = empty_day;
  ws[std::string("Friday")]    = empty_day;
  rr[std::make_pair(building,room)] = ws;
}
```

Unfortunately, due to hard disk crash, Louis has lost the details of the two `typedef`s and his implementation of the `make_reservation` function. Your task is to help him recreate the implementation.

He does have a few more test cases for you to examine. Given the current state of the reservation system, these attempted reservations will all fail:

```
success = make_reservation(rr, "DCC", 308, "Monday",   19, 3, "American Sniper") ||
          make_reservation(rr, "DCC", 307, "Monday",   19, 3, "American Sniper") ||
          make_reservation(rr, "DCC", 308, "Monday",   22, 3, "American Sniper") ||
          make_reservation(rr, "DCC", 308, "Saturday", 19, 3, "American Sniper");
assert (success == false);
```

With these explanatory messages printed to `std::cerr`:

```
ERROR! conflicts with prior event: DS Exam
ERROR! room DCC 307 does not exist
ERROR! invalid time range: 22-25
ERROR! invalid day: Saturday
```

2.1 The `typedefs` [/ 5]

First, fill in the `typedef` declarations for the two shorthand types used on the previous page.

`typedef` _____ `week_schedule;`

`typedef` _____ `room_reservations;`

2.2 Diagram of the data stored in `room_reservations rr` [/ 8]

Now, following the conventions from lecture for diagramming `map` data structures, draw the specific data stored in the `rr` variable after executing the instructions on the previous page. Yes, this is actually quite a big diagram, so don't attempt to draw *everything*, but be neat and draw enough detail to demonstrate that you understand how each component of the data structure is organized and fits together.

2.3 Implementing `make_reservation` [/ 16]

Next, implement the `make_reservation` function. Closely follow the samples shown on the first page of this problem to match the arguments, return type, and error checking.

sample solution: 28 line(s) of code

2.4 Performance and Memory Analysis [/ 8]

Now let's analyze the running time of the `make_reservation` function you just wrote. If RPI has b buildings, and each building has on average c classrooms, and we are storing schedule information for d days (in the sample code d=5 days of the week), and the resolution of the schedule contains t time slots (in the sample code $t = 24$ 1-hour time blocks), with a total of e different events, each lasting an average of s timeslots (data structures lecture lasts 2 1-hour time blocks), what is the order notation for the running time of this function? Write 2-3 concise and complete sentences explaining your answer.

Using the same variables, write a simple formula for the approximate upper bound on the memory required to store this data structure. Assume each int is 4 bytes and each string has at most 32 characters = 32 bytes per string. Omit the overhead for storing the underlying tree structure of nodes & pointers. Do not simplify the answer as we normally would for order notation analysis. Write 1-2 concise and complete sentences explaining your answer.

Finally, using the same variables, what would be the order notation for the running time of a function (we didn't ask you to write this function!) to find all currently available rooms for a specific day and time range? Write 1-2 concise and complete sentences explaining your answer.

3 Fashionable Sets [/ 14]

In this problem you will write a recursive function named `outfits` that takes as input two arguments: `items` and `colors`. `items` is an STL `list` of STL `string`s representing different types of clothing. `colors` is an STL `list` of STL `set`s of STL `string`s representing the different colors of each item of clothing. Your function should return an STL `vector` of STL `string`s describing each unique outfit (in any order) that can be created from these items of clothing.

Here is a small example:

```
items   =  { "hat", "shirt", "pants" }
colors  =  { { "red" },
             { "red", "green", "white" },
             { "blue", "black" } }
```

```
red hat &   red shirt &  blue pants
red hat & green shirt &  blue pants
red hat & white shirt &  blue pants
red hat &   red shirt & black pants
red hat & green shirt & black pants
red hat & white shirt & black pants
```

sample solution: 22 line(s) of code

4 Spicy Chronological Sets using Maps [/ 33]

Ben Bitdiddle is organizing his spice collection using an STL `set` but runs into a problem. He needs the fast `find`, `insert`, and `erase` of an STL set, but in addition to organizing his spices alphabetically, he also needs to print them out in chronological order (so he can replace the oldest spices).

Ben is sure he'll have to make a complicated custom data structure, until Alyssa P. Hacker shows up and says it can be done using an STL `map`. She quickly sketches the diagram below for Ben, but then has to dash off to an interview for a Google summer internship.

Alyssa's diagram consists of 3 variables. The first variable, containing most of the data, is defined by a `typedef`. Even though he's somewhat confused by Alyssa's diagram, Ben has pushed ahead and decided on the following interface for building his spice collection:

```
chrono_set cs;
std::string oldest = "";
std::string newest = "";
insert(cs,oldest,newest,"garlic");
insert(cs,oldest,newest,"oregano");
insert(cs,oldest,newest,"nutmeg");
insert(cs,oldest,newest,"cinnamon");
insert(cs,oldest,newest,"basil");
insert(cs,oldest,newest,"sage");
insert(cs,oldest,newest,"dill");
```

chrono_set cs:

"basil"	<"cinnamon", "sage">
"cinnamon"	<"nutmeg", "basil">
"dill"	<"sage", "">
"garlic"	<"","oregano">
"nutmeg"	<"oregano","cinnamon">
"oregano"	<"garlic", "nutmeg">
"sage"	<"basil", "dill">

std::string oldest: "garlic"

std::string newest: "dill"

Ben would like to output the spices in 3 ways:

```
ALPHA ORDER:        basil   cinnamon       dill     garlic     nutmeg    oregano       sage
OLDEST FIRST:      garlic    oregano     nutmeg   cinnamon      basil       sage       dill
NEWEST FIRST:        dill       sage      basil   cinnamon     nutmeg    oregano     garlic
```

If he buys more of a spice already in the collection, the old spice jar should be discarded and replaced. For example, after calling:

```
insert(cs,oldest,newest,"cinnamon");
```

The spice collection output should now be:

```
ALPHA ORDER:        basil   cinnamon       dill     garlic     nutmeg    oregano       sage
OLDEST FIRST:      garlic    oregano     nutmeg      basil       sage       dill   cinnamon
NEWEST FIRST:    cinnamon       dill       sage      basil     nutmeg    oregano     garlic
```

4.1 The `typedef` [/ 3]

First, help Ben by completing the definition of the typedef below:

```
typedef [                                                              ] chrono_set;
```

4.2 Printing out the spice collection [/ 8]

Next, write the code to output (to `std::cout`) Ben's spices in alphabetical and chronological order:

```
std::cout << "ALPHA ORDER:  ";
```

sample solution: 4 line(s) of code

```
std::cout << std::endl;
std::cout << "OLDEST FIRST: ";
```

sample solution: 5 line(s) of code

```
std::cout << std::endl;
```

4.3 Performance Analysis [/ 5]

Assuming Ben has n spices in his collection, what is the order notation for each operation? *Note: You may want to first complete the implementation of the* `insert` *operation on the next page.*

printing in alphabetical order:

printing in chronological order:

`insert`-ing a spice to the collection:

4.4 Implementing `insert` for the `chrono_set` [/ 17]

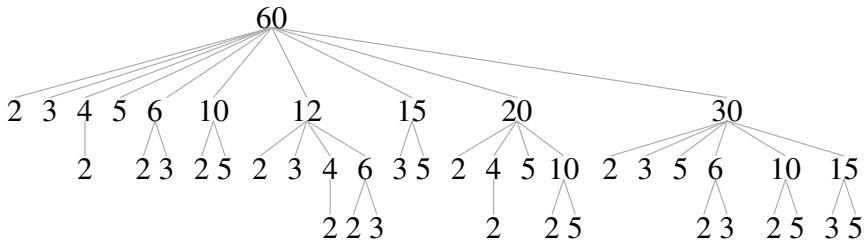
Finally, implement the `insert` function for Ben's spice collection. Make sure to handle all corner cases.

sample solution: 26 line(s) of code

5 Factor Tree [/ 13]

Write a recursive function named `factor_tree` that takes in a single argument of integer type and constructs the tree of the factors (and factors of each factor) of the input number. The function returns a pointer to the root of this tree. The example below illustrates the tree returned from the call `factor_tree(60)`.

```
class Node {
public:
  int value;
  std::vector<Node*> factors;
};
```

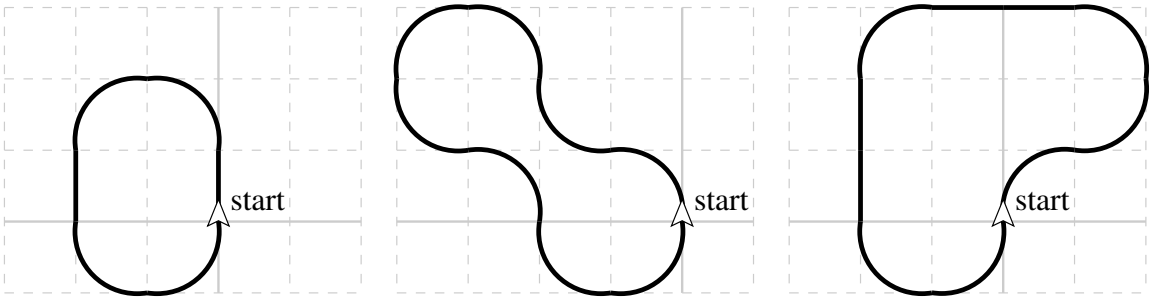


sample solution: 10 line(s) of code

6 Driving in Circles [/ 18]

In this problem you will write a recursive function named `driving` that outputs to `std::cout` all *closed loop* paths of driving instructions on a rectangular grid less than or equal to a specified maximum path length. The car begins at (0,0) pointing north and at each step can go *straight*, *left*, or *right*. A path is said to "close the loop" if it is finishes where it started, pointing in the same direction. For example, here are three sample closed loop paths (also illustrated below):

```
closed loop:  straight left left straight left left
closed loop:  left right left left left right left left
closed loop:  right left left straight straight left straight straight left left
```



We provide the `Car` class and several helper functions:

```
class Car {
public:
  Car(int x_,int y_,std::string dir_) : x(x_),y(y_),dir(dir_) {}
  int x;
  int y;
  std::string dir;
};
bool operator==(const Car &a, const Car &b) {
  return (a.x == b.x && a.y == b.y && a.dir == b.dir);
}
Car go_straight(const Car &c) {
  if      (c.dir == "north") { return Car(c.x  ,c.y+1,c.dir); }
  else if (c.dir == "east")  { return Car(c.x+1,c.y  ,c.dir); }
  else if (c.dir == "south") { return Car(c.x  ,c.y-1,c.dir); }
  else                       { return Car(c.x-1,c.y  ,c.dir); }
}
Car turn_left(const Car &c) {
  if      (c.dir == "north") { return Car(c.x-1,c.y+1,"west");  }
  else if (c.dir == "east")  { return Car(c.x+1,c.y+1,"north"); }
  else if (c.dir == "south") { return Car(c.x+1,c.y-1,"east");  }
  else                       { return Car(c.x-1,c.y-1,"south"); }
}
Car turn_right(const Car &c) {
  if      (c.dir == "north") { return Car(c.x+1,c.y+1,"east");  }
  else if (c.dir == "east")  { return Car(c.x+1,c.y-1,"south"); }
  else if (c.dir == "south") { return Car(c.x-1,c.y-1,"west");  }
  else                       { return Car(c.x-1,c.y+1,"north"); }
}
```

Your function should take in 3 arguments: the path constructed so far, the current car position & direction, and the maximum number of steps/instructions allowed. For example:

```
std::vector<std::string> path;
Car car(0,0,"north");
int max_steps = 10;
driving (path,car,max_steps);
```

Now implement the recursive `driving` function.

sample solution: 25 line(s) of code

7 Maps of Sets of Factors [/ 32]

In this problem we will use STL `map` and STL `set` to store and access a collection of integers and their factors. Below are the commands we use to initialize the two data structures diagrammed on the right.

```
factor_type factors;
add_factors(factors,6);
add_factors(factors,14);
add_factors(factors,5);
add_factors(factors,8);
add_factors(factors,10);
add_factors(factors,9);
add_factors(factors,13);
add_factors(factors,15);
add_factors(factors,12);
add_factors(factors,21);
factor_type is_factor_of = reverse(factors);
```

factors

5	
6	2 3
8	2 4
9	3
10	2 5
12	2 3 4 6
13	
14	2 7
15	3 5
21	3 7

is_factor_of

2	6 8 10 12 14
3	6 9 12 15 21
4	8 12
5	10 15
6	12
7	14 21

7.1 The `typedef` [/ 2]

First, complete the definition of the typedef below:

`typedef` [ ] `factor_type;`

7.2 Implementing `add_factors` [/ 8]

Now, implement the `add_factors` function. Note that this function only initializes the `factors` table.

sample solution: 8 line(s) of code

If we are storing the factors of n different numbers in the `factors` structure, f different factors will eventually be stored in the `is_factor_of` structure, each number has on average (or at most) j factors, and each factor is a factor of on average (or at most) k numbers, what is the order notation for the running time of your `add_factors` function to add the number x and the factors of x?

7.3 Implementing `reverse` [/ 10]

Next, implement the `reverse` function to build the `is_factor_of` table from the completed `factors` table.

sample solution: 9 line(s) of code

Using the variables n, f, j, and k as defined above, what is the order notation for the running time of your `reverse` function?

7.4 Implementing `remove` [/ 12]

Finally, we would like to remove data from the tables. The `remove` function will remove a given number's row from the `factors` table **and** remove the number from each of its factors in the `is_factor_of` table. For example, the call below results in the tables to the right.

```
remove(12,factors,is_factor_of);
```

Your task is to efficiently implement the `remove` function. Using the variables defined above, you should assume that $n \geq f \geq k \geq j$.

factors

5	
6	2 3
8	2 4
9	3
10	2 5
13	
14	2 7
15	3 5
21	3 7

is_factor_of

2	6 8 10 14
3	6 9 15 21
4	8
5	10 15
7	14 21

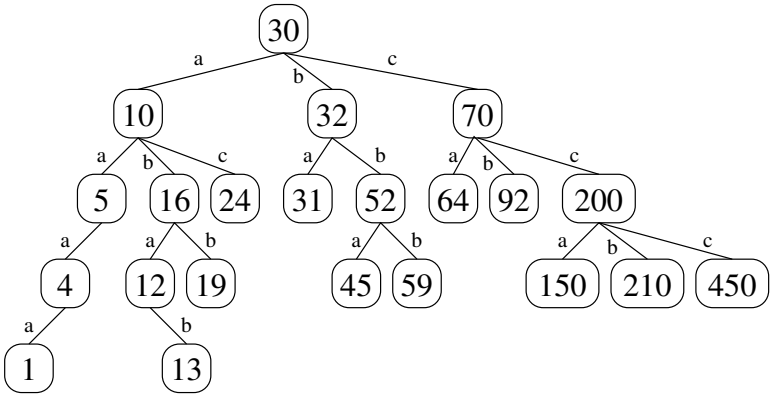
sample solution: 14 line(s) of code

Using the variables n, f, j, and k as defined above, what is the order notation for the running time of your `remove` function?

8 Double Tries [/ 30]

```
class Node {
public:
  int value;
  Node *a;
  Node *b;
  Node *c;
  Node *parent;
};
```

Ben Bitdiddle thinks he's come up with a fantastic enhancement for binary search trees of integers he calls the Double Trie. Each `Node` will have up to 3 children. The *a* branch will store all elements less than the current node. The *b* branch will store all elements greater than the current node, but less than or equal to twice the current node. And the *c* branch will store all elements greater than twice the current node.



Ben suggests we start the implementation by writing a recursive `insert` function that takes in a pointer to node (initially the *root* of the Double Trie), the value to insert, and a pointer to the parent node (initially NULL). The function returns true if the value was successfully inserted and false if the value is already in the structure.

8.1 Implementing `insert` [/ 10]

sample solution: 19 line(s) of code

Ben's project partner Alyssa P. Hacker isn't thrilled with the design. (She's not sure it will significantly reduce the tree height because this structure is difficult to keep balanced.) However, they have a deadline, so this is a make-it-work moment and she tackles the challenge of *reverse iteration* over this structure. Specifically if the `root` variable points to the top of the diagram on the previous page, she would like this fragment of code:

```
Node *tmp = find_largest(root);
while (tmp != NULL) {
  std::cout << tmp->value << " ";
  tmp = find_previous(tmp);
}
std::cout << std::endl;
```

to print all of the data in the tree *in reverse order*:

```
450 210 200 150 92 70 64 59 52 45 32 31 30 24 19 16 13 12 10 5 4 1
```

8.2 Implementing `find_largest` [/ 6]

Next, Alyssa implements the `find_largest` function:

sample solution: 8 line(s) of code

Given a reasonably balanced Double Trie with n elements, what is the order notation of the running time of the `find_largest` function? Write one or two sentences explaining your answer.

8.3 Implementing `find_previous` [/ 14]

Finally, Alyssa implements the `find_previous` function:

sample solution: 17 line(s) of code

9 Re-Truthization [/ 14]

The statements below are false. Make a small change to correct each statement, ensuring that it remains interesting and informative.

Binary Search Tree Iterators [/2] The average number of child or parent links that must be traversed when moving from one node to the next node in an in-order traversal is $O(\log n)$, where n is the number of elements in the tree.

Incomplete type [/2] In HW8 Friendly Recursion, many students encountered the compiler message "`error: invalid use of incomplete type 'class Message'`", which should be solved by implementing all custom class member functions in the class declaration `.h` file.

Breadth-First Search [/3] Executing a breadth-first search for the shortest path from root to leaf on a binary search tree will often be faster and require less additional memory than a depth-first search on the same tree.

Been Here Before? [/2] To optimize the HW6 Ricochet Robots solver, the search tree for a board with three robots can be pruned (and the forward search from that point terminated) if any one of the robots reaches a board location that it has previously occupied.

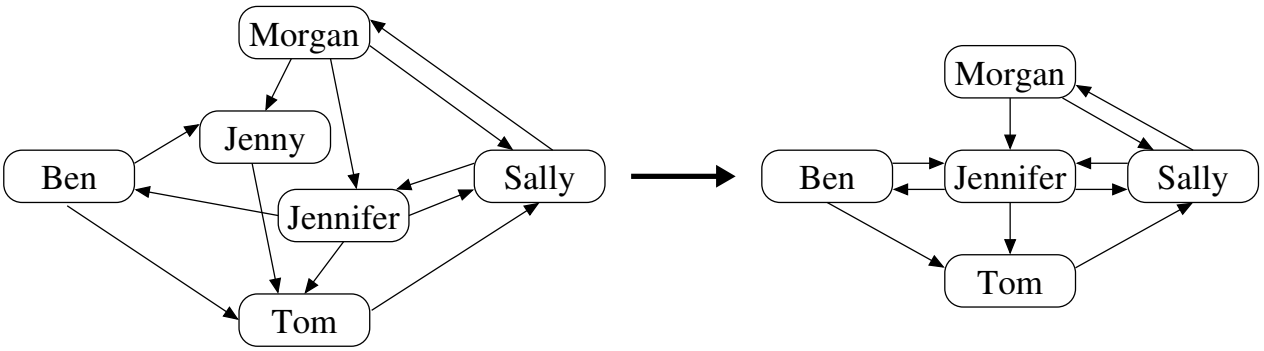
Hash Function Performance [/2] A hash function should run in $O(1)$ time, to ensure that the hash table will achieve $O(\log n)$ query time, where n is the number of elements in the hash table.

Red-Black Property [/3] Maintaining the Red-Black property for a hash table ensures that the data remains balanced and elements can be accessed in $O(\log n)$ time.

10 Friend Graphs [/ 21]

In this problem you will work with a simplified version of the Twitter or Google+ friendship/follower directed graph from HW8. Your task is to write a `merge_account` function that takes in a `Graph` object and the STL `string` names of two accounts and then modifies the graph to merge the two corresponding `Person` objects into a single object. Other people in the graph who were connected to either of the original accounts by a connection of either direction will be updated to link to the merged account. For example, let's start with the graph connectivity on the left below. We want to merge the accounts for "Jennifer" and "Jenny", preserving the name "Jennifer" on the merged account. We execute the following statement, resulting in the picture on the right.

```
merge_accounts(graph, "Jennifer", "Jenny");
```




```
class Person {
public:
  std::string name;
  std::set<Person*> friends;
};

class Graph {
public:
  std::vector<Person*> people;
};
```

10.1 Corner Cases for `merge_accounts` [/ 6]

Think carefully about a typical use case for merging accounts, and also about corner cases for the `merge_accounts` function. What different test cases will you need to write to ensure that your implementation is fully debugged and will work when attempting to join two arbitrary `Person` objects in a large graph? Write three or four concise and well written sentences describing sample input and the expected output.

10.2 Implementation of `merge_accounts` [/ 15]

Now, implement the `merge_accounts` function. Make sure that your function does not lead to memory errors or memory leaks.

sample solution: 33 line(s) of code