Comparison the slope and growth trend between different functions（Monotonicity & Concavity）

Priority is given to developing modules to explore the images, derivatives, and convexity of functions.

Support basic functions (power function, trigonometric function, exponent, logarithm, etc.) image drawing

Add a first-order derivative image to mark the increment or subtraction of the interval.

Add a second-order derivative image to mark the uneven and convex intervals and inflection points.

Segmented function support (handling symbol interruption/definition domain problems)

You can select the input function expression and automatically generate the image + property report.

First step: define its first derivative(slope)

For the definition of the slope of tangent to a function:

$$tangent = lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

```
def f(x):
    return function
h=0.000000001 #h→0
```

```
def d(f,x):
    return (f(x+h)-f(x))/h
```

Second step: Find the overall monotony of a function in its domain.

The definition of Monotonicity

- **Strictly increasing function:**

  A function f(x) is **increasing** on an interval

  For any

$$x1, x2 \in I,$$

$$x1 < x2 \implies f(x1) < f(x2)$$

For calculus (find the derivative of a function)

$$f\prime(x) > 0 \iff increasing$$

- **Strictly decreasing function:**

  A function f(x) is **decreasing** on an interval

  For any:

$$x1, x2 \in I,$$

$$x1 < x2 \implies f(x1) > f(x2)$$

$$f\prime(x) < 0 \iff decreasing$$

Third step: find the second derivative of the function (growth trend)

We all know:

$$f\prime(x) = lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

The same reason：

$$f\prime\prime(x) = lim_{h \to 0} \frac{f\prime(x) - f\prime(x-h)}{h}$$

$$= \frac{\frac{f(x+h)-f(x)}{h} - \frac{f(x)-f(x-h)}{h}}{h}$$

$$f\prime\prime(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

```
def checksecondderivative():
second=(f(x+h)-2*f(x)+f(x-h))/h**2
positive=0
negative=0
if second>0:
positive=positive+1
elif second<0:
negative=negative+1
if positive>0 and negative==0:
#explain the definition of the increasing function
print("the function is concave up")
elif negative>0 and positive==0:
#explain the definition of the decreasing function
print("the function is concave down")
else:
print("the function is neither concave up or concave down")
```

Final framework

```
def linspace(start,stop,num):
    step=(stop-start)/(num-1)
    return[start+i*step for i in range(num)]
    domain=linspace(0.1,10,1000)
def f(x):
    return #enter a function
h=0.0001
def checkmonotony():
    positive_count=0
    negative_count=0
```

```
    for x in domain:
        df=(f(x+h)-f(x))/h
        if df>0:
            positive_count=positive_count+1
        elif df<0:
            negative_count=negative_count+1
        if positive_count>0 and negative_count=0:
            print("the function is increasing")
        elif negative_count>0 and positive_count=0:
            print("the function is decreasing")
        else:
            print("the function is neither increasing nor decreasing")
checkmonotony()

def checksecondderivative():
    positive=0
    negative=0
    for x in domain:
        second=(f(x+h)-2*f(x)+f(x-h))/h**2
        if second>0:
            positive=positive+1
        elif second<0:
            negative=negative+1
        if positive>0 and negative==0:
            print("the function is concave up")
        elif negative>0 and positive==0:
            print("the function is concave down")
        else:
            print("the function is neither concave up nor concave down")
checksecondderivative()
```

Result

$$y = e^x$$

```
python3 mai...

increasing
concave up
```

$$y = sin(x)$$

```
python3 mai...

Environment updated. Reloading shell...
the function is neither increasing nor decreasing
neither concave up nor concave down
```

$$y = In(x)$$

```
python3 mai...

Environment updated. Reloading shell...
the function is increasing
concave down
```

$$y = x^3 + 3x^2 + 2x + 1$$

```
python3 mai...

Environment updated. Reloading shell...
increasing
concave up
```