

A Data-Driven Approach to Keyboard Optimization

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Abstract

The QWERTY layout, first developed in the 1870s for typewriters, is the most widely used keyboard layout today despite its inefficiencies. The development of an alternative layout faces two major challenges: quantifying the impact of ergonomic features and effectively navigating the vast search space of possible keyboard layouts. To address these challenges, this paper proposes a data-driven approach using simulated annealing with an objective function derived from empirical typing data. The two data sets used are the 136M Keystrokes data set from Aalto University, containing 8,228 hours of typing data from 168,000 participants, for a controlled analysis of four layouts (AZERTY, Dvorak, QWERTY, and QWERTZ), and the iWeb corpus, one of the largest collections of English text, for frequency analysis. A custom implementation of simulated annealing is employed, featuring a dynamic initial temperature and termination criterion that enables compatibility with different subsets of keys, corpora, and optimization criteria. An analysis reveals a number of factors that impact typing speed. Based on this analysis, a cost function is devised and fine-tuned using the Levenberg-Marquardt algorithm (MAE:12ms, R^2 :0.78 at 80 WPM). To estimate the typing efficiency of a candidate layout, the cost function evaluates the predicted typing time for each sequence in the corpus based on the given layout. The sum of these predicted times serves as the objective function for optimization via simulated annealing. The result is a layout estimated to be 6% faster for typing English compared to QWERTY.

1 Introduction

QWERTY is a keyboard layout first created for typewriters by Christopher Sholes in the early 1870s [14]. Today, 150 years since its invention, QWERTY is the most common keyboard layout and, by extension, the most common means of human-computer interaction, extending even to mobile devices. However, what was innovative then has become a source of concern in the modern era, as the design of QWERTY, conceived with little regard for ergonomics, has been linked to user discomfort and repetitive strain injuries [1].

One of the earliest and most influential advocates for alternative keyboard layouts was August Dvorak, who, along with his collaborator William Dealey, championed a more scientific approach to keyboard design. Their methodology involved motion studies, analysis of high-frequency letters and letter combinations, and a deep understanding of hand physiology, culminating in the creation of the Dvorak Simplified Keyboard in 1932 [9]. Dvorak's efforts in the 1930s marked a significant shift toward a more scientific approach to keyboard design and, at the very least, planted the idea that a more efficient alternative to the QWERTY layout was possible.

Despite these early endeavors, the field has continued to struggle to quantify the precise impact of ergonomic factors on typing efficiency and user comfort. This has led to a continued reliance on theoretical models informed by qualitative observations rather than empirical models derived from robust quantitative data. From these observations, several standard practices have emerged, such as minimizing the distance between keys, arranging columns based on perceived finger dexterity, limiting the use of the same finger twice in succession, and considering character frequency [12].

Beyond quantifying the impact of ergonomic features, finding an optimal layout presents a significant computational challenge. For n keys, there are $n!$ possible layouts, making the problem combinatorially complex. Even for the 26 letters of the English alphabet, this complexity renders brute-force solutions infeasible. This complexity places the problem among other well-known combinatorial problems, such as the Traveling Salesman Problem (TSP) and the Quadratic Assignment Problem (QAP). In the TSP, QAP, and keyboard layout optimization, the goal is to minimize a cost as it relates to distance: for the TSP, the distance is between cities; for the QAP, the distance is between facilities; and for keyboard optimization, the distance is between keys. The QAP, unlike the TSP, also accounts for flow, which measures the intensity of interactions (e.g., between related facilities) – analogous to character frequency in keyboard layout optimization. Given these similarities, researchers have applied a multitude of combinatorial optimization algorithms to address keyboard layout optimization, such as simulated annealing [12], ant colony optimization [6], swarm optimization [15], and genetic algorithms [11], among others.

This paper presents a novel, data-driven approach to keyboard layout optimization. We propose a quantitative objective function that evaluates keyboard layouts based on key placement, finger dexterity, and character frequency, informed by a comprehensive analysis of real-world typing data and a large English corpus. To optimize these layouts, we implement a custom version of the simulated annealing algorithm that dynamically adjusts the initial temperature and termination criterion. This approach allows for the generation of keyboard layouts tailored to specific typing speed ranges, corpora including other languages, and individual user needs.

Typing is a complex interaction between physical constraints, muscle memory, and cognitive processes [9]. An optimal design must not only account for these complex interactions, but also make decisions that, at times, may seem counterintuitive to balance them. This study focuses on optimizing one clear and measurable aspect of typing – typing speed. While typing comfort is equally important, it remains a multifaceted and subjective concept that is difficult to quantify. By focusing our efforts on typing speed, we aim to establish a scientifically sound foundation. Comfort, on the other hand, may be where human artistry and subjective preferences come into play, reflecting a more individualized aspect of keyboard design.

2 Methodology

2.1 Data Processing

Two data sets are used to estimate the typing time. The first data set, the iWeb corpus [4] is one of the largest available collections of English text and uses a systematic selection of websites to ensure high-quality data. This corpus facilitates the frequency analysis of characters and character sequences. The second, the 136M Keystrokes data set from Aalto University [5], consists of typing test performance data, totaling approximately 8,228 hours from 168,000 participants. Participant metadata enables a controlled analysis of four layouts: AZERTY, Dvorak, QWERTY, and QWERTZ.

To ensure a high-quality analysis of the keystroke data, several preprocessing steps are performed. First, the data is normalized by segmenting it into sessions and users, while correcting occasional errors in the source, leading to minimal data loss. Next, keystroke accuracy is assessed by applying approximate string matching between the target and typed text. This approach enables the identification and removal of any keystrokes caused by typing errors.

Three common typing errors are taken into account: insertion, deletion, and substitution [13], as illustrated in Table 1. A keystroke validity record is generated for each typing test session. This record is updated dynamically on a per-window basis, with each window being a section of text up to some error or key input that requires processing. When a user uses

Table 1. Types of Error Handled by Approximate String Matching

Typing Error	String typed
None	But thank you for the offer
Insertion	But thank _l you for the offer
Deletion	But tha _k you for the offer
Substitution	But thab _k you for the offer

arrow keys or backspaces, the correctness of the current window is assessed, added to the keystroke validity record, and a new window starts at the most recent navigation action. This procedure repeats until the last keystroke, at which point any remaining input data is included in the record. The validity record helps identify the location of errors for further processing. Although insertion and substitution errors are relatively straightforward to process, deletion errors require special handling because a keystroke resulting in a deletion error can be both correct and incorrect. For example, if a user intends to type "there" but types "tere," the "e" is incorrect in its position but effectively completes the remaining substring "ere." The analysis accounts for this duality, ensuring that the focus is solely on unaffected keystroke sequences.

Table 2. Top 5 Bigrams and Trigrams Extracted from the Corpus

Bigram	Occurences	Trigram	Occurences
th	9709171	the	6076523
he	8552661	ing	3227179
in	7913861	and	2998065
an	6389345	ion	1716878
er	6348583	ent	1519196

After preprocessing, a sliding window decomposes both the corpus and keystroke data. Character sequences from the corpus decomposition are referred to as ngrams. Bigrams and trigrams represent ngrams of lengths two and three, respectively. Analogously, sequences from the keystroke data are referred to as nstrokes, with bistrokes and tristrokes denoting lengths two and three. Each ngram encountered in the corpus is stored along with its frequency of occurrence. Each nstroke, excluding those made in errors, is recorded as a tuple comprising its characters and a positional vector indicating the keys used. These tuples serve as identifiers for storing the typing times of unique nstroke instances within the data set. For example, the identifier for the tristroke "the" on AZERTY, QWERTY, and QWERTZ keyboards would be ((-1, 3), (1, 2), (-3, 3), 'the'). This ensures that the typing durations of each unique pattern instance, regardless of the layout they occur on, are cataloged and merged. Outliers in recorded typing times are removed using the interquartile range (IQR), and the remaining times are averaged within

specific words per minute (WPM) ranges. These average typing times are later used to fine-tune the cost functions.

Table 3. Keyboard Character Mapping for QWERTY

Q	W	E	R	T	Y	U	I	O	P
A	S	D	F	G	H	J	K	L	;
Z	X	C	V	B	N	M	,	.	/
SPACE									
(-5,3)	(-4,3)	(-3,3)	(-2,3)	(-1,3)	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)
(-5,2)	(-4,2)	(-3,2)	(-2,2)	(-1,2)	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)
(-5,1)	(-4,1)	(-3,1)	(-2,1)	(-1,1)	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)
(0,0)									

2.2 Data Analysis

Our analysis was limited to data from participants who reported using 9 to 10 fingers to type, as these participants were more likely to adhere to touch typing practices. Touch typing is a standardized method in which typists use muscle memory to locate keys without looking at the keyboard, and it is the most common method of typing among proficient typists. Although users may deviate from perfect touch typing in practice, this approach enables a systematic examination of typing patterns while still reflecting real-world usage.

2.2.1 Bistroke Analysis

The bistroke analysis reveals that frequency is the most significant predictor of typing time. As illustrated in Figure 1, typing speed follows a logarithmic curve with frequency, indicating that muscle memory aids high-frequency bistrokes regardless of placement. Once the impact of frequency is accounted for, the effect of key placement and the positional relationships between keys can be examined. As expected, the analysis shows that finger dexterity plays a role in typing speed, with the pinkies being the slowest and the index and middle fingers being the fastest. Notably, the top row is predicted to be marginally faster than the home row. This could be a bias in the data, as the top is the most used row on QWERTY, QWERTZ, and AZERTY, three of the four layouts analyzed. Further analysis is required to determine whether the slight advantage of the top row arises from an inherent biomechanical ease of movement when the fingers operate closer to their natural midpoint of flexion or if it reflects learned behavior. Analyzing the positional relationships between keys reveals three major categories of bistrokes:

- **ALT (Alternating Bistroke):** a bigram typed by alternating hands. This is the fastest bistroke category as little coordination is needed to articulate the movement.
- **SHB (Single-hand Bistroke):** a bigram typed with the same hand. This category is typically slower due to the need for sequential finger movements on the same hand and, therefore, greater coordination.

- **SFB (Single-finger Bistroke):** A bigram typed using the same finger in succession. This is the slowest category, requiring both high coordination and inherent downtime. SFBs are especially slow at high words per minute.

The key finding of this analysis is that the impact of a feature on speed prediction depends on the WPM range of the data used. This is clearly demonstrated by the effect of the WPM range on SFBs, which become significantly worse at high WPMs. This observation underscores the need to tailor keyboard optimization to specific typing speed goals, which in turn complicates both decision-making in the layout creation process and the implementation of the simulated annealing algorithm.

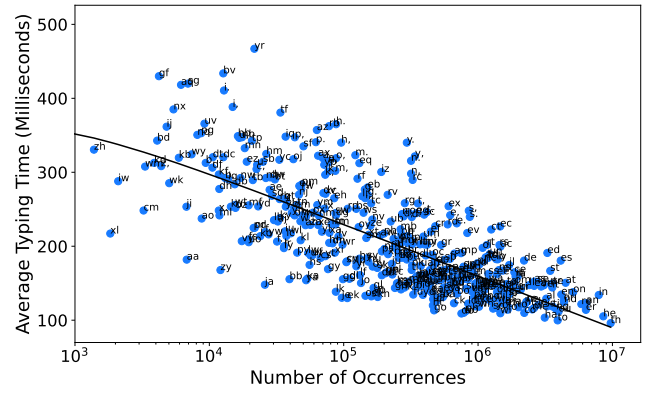


Figure 1. Relationship Between English Bigram Frequency and Average Typing Time on QWERTY

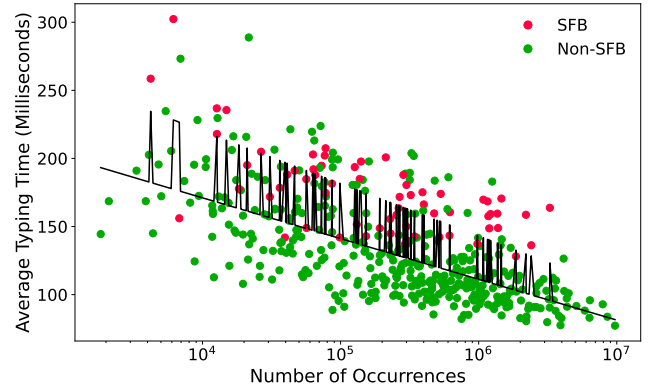


Figure 2. The Effect of SFBs and Frequency on Typing Time (> 80WPM)

2.2.2 Tristroke Analysis

Several features unique to tristrokes were identified; however, they were excluded from the analysis due to their low statistical significance and the increased risk of overfitting

the model. The only exception to this exclusion is the single-finger skipstroke category. Single-finger skipstrokes, like single-finger bistrokes, involve two strokes made by the same finger, but with an intervening stroke between them. An example of this on the QWERTY keyboard is the word "fit," as both the 'f' and 't' are typed with the left index finger, leading to a coordinative delay. Single-finger skipstrokes are important to consider because, for fast typists, they function similarly to single-finger bistrokes with an added delay. Since single-finger bistrokes notably impact typing speed – especially at higher typing speeds – accounting for single-finger skipstrokes is imperative when targeting faster typing speeds.

2.3 Creating the Objective Function

The objective function we are optimizing for is the predicted time to type a corpus on a specific layout in a given WPM range. This prediction is achieved by evaluating the cost, or typing time, of each trigram within the corpus, multiplying it by the trigram's occurrence count, and summing the results, yielding the estimated time required to type the corpus on that particular layout.

The limited diversity of layouts in the dataset makes developing an accurate time prediction model challenging, as it has the potential to introduce significant bias. The averaging of typing times further reduces the effective sample size, amplifying the risk of overfitting to the specific nuances of these layouts. To mitigate this risk, a cost function is devised based on insights from our data analysis, and the number of adjustable parameters within the cost function is minimized. The adjustable parameters are later fine-tuned using the Levenberg-Marquardt algorithm. This approach, in contrast to more complex techniques such as neural networks, helps prevent overfitting by retaining only the most salient relationships observed in the data.

Calculation of the tristroke cost function relies on first calculating the cost of its constituent bistrokes. So we first define the bistroke cost function, $C(b)$, as the predicted time required to type a given bistroke b of category i , with base positional penalties P_x and P_y , categorical positional penalties $P_x^{(i)}$ and $P_y^{(i)}$, and a frequency penalty $f(b)$. The frequency penalty is modeled using a logarithmic function with free parameters p_1 , p_2 , and p_3 . The columnar penalties, P_x and $P_x^{(i)}$, are calculated based on the absolute value of the x-coordinate for the second keystroke in the bistroke. Taking the absolute value is done to limit any potential hand bias that could stem from adaptive behaviors to an uneven workload. A free parameter is assigned to each possible column value to weight its influence on typing speed.

The row penalties, P_y and $P_y^{(i)}$, are calculated based on the y-coordinate of the second key in the bistroke. Each possible y-coordinate is assigned a free parameter to weight its effect on typing speed. It is assumed that a row-staggered keyboard

is used, as it is the most common layout. On a row-staggered keyboard, the top row has a -0.25 key offset, and the bottom row has a 0.5 key offset. This offset must be accounted for in the single-finger bistroke's distance calculation, so Δ is introduced to represent the row-stagger-adjusted distance between keys. For any other bistroke, Δ is set to 0. A free parameter, p_4 , is added to Δ to establish a baseline penalty weight. This parameter determines the extent to which distance contributes to the penalty on top of the row-stagger adjustment and also prevents Δ from becoming zero, which would negate the placement penalties. The bistroke cost function, $C(b)$, is calculated as the product of the finger penalties and the frequency logarithm.

$$C(b) = (p_1 \log(f(b) + p_2) + p_3) \times \left(1 + P_x(b)P_y(b) + P_x^{(i)}(b)P_y^{(i)}(b)(\Delta + p_4)\right)$$

For the tristroke cost function $C(t)$, we take the sum of the cost of its constituent bistrokes $b_1(t)$ and $b_2(t)$, then add a small penalty for the trigram's associated skipstroke $s(t)$, if it is a single-finger skipstroke. The single-finger skipstroke penalty, like the single-finger bistroke penalty, considers positional penalties P_x and P_y , as well as a distance penalty $(\Delta + p_5)$ with a free parameter p_5 . It should be noted that the single-finger skipstroke penalty defaults to 0 if there is no single-finger skipstroke.

$$C(t) = C(b_1(t)) + C(b_2(t)) + P_x(s(t))P_y(s(t))(\Delta + p_5)$$

Finally, the data is limited to the desired WPM range. The bistroke cost function is then fit to the bistroke data, and the tristroke cost function is fit to the tristroke data, using the Levenberg-Marquardt algorithm. The MAE of the fit changes depending on the target WPM, but the R^2 metric remains constant. For 80 WPM, this results in $C(b)$ having an MAE of 12 milliseconds and an R^2 of 0.78. $C(t)$ is less accurate with an MAE of 26 milliseconds and an R^2 of 0.42. The cumulative cost $C(l)$ for a layout l is the sum of each tristroke's predicted typing time multiplied by its number of occurrences, effectively estimating the typing time for a given layout across the corpus.

$$C(l) = \sum_{t \in T} (C(t) \times f(t))$$

This is the objective function passed onto the simulated annealing algorithm for optimization.

2.4 Simulated Annealing

Simulated annealing is a metaheuristic optimization technique inspired by the annealing process in metallurgy, where metals are cooled gradually to achieve a stable state with minimized energy [10]. In the context of keyboard optimization, it is used as an iterative algorithm that explores different keyboard layouts by gradually accepting key swaps that reduce the overall energy (i.e., predicted typing time). Unlike gradient descent and other methods that always aim for an

absolute minimum, simulated annealing allows for the occasional acceptance of higher-cost solutions to explore a broader search space, preventing the algorithm from getting stuck in local optima. Eventually, simulated annealing converges toward a stable, low-cost configuration – the global minimum or an approximation of it.

Selecting the parameters in simulated annealing for convergence can be challenging, usually requiring experimentation or domain knowledge [3]. We outline a dynamic approach to setting the initial temperature and termination criterion to address this. This ensures layouts can be explored across a variety of data modifications. For example, layouts may be optimized for different WPM ranges, languages, or a specific subset of keys. In these cases, the cost function may yield drastically different values, and the number of swaps required for convergence may vary, necessitating adjustments to the parameters.

2.4.1 Solution Space and Transitions

A crucial step in implementing simulated annealing is defining the solution space and transition process. This allows the algorithm to explore potential solutions effectively. Changes to the solution should occur gradually to ensure smooth traversal of the solution landscape, preventing abrupt jumps that overlook promising solutions [3].

For keyboard optimization, a solution is defined as a mapping between characters and their positions on a matrix, as illustrated in Table 3. The chosen coordinate system mirrors real-world keyboard usage by grouping fingers based on absolute values, differentiating hands by sign, and enabling easy distance calculations. This encoding strategy allows the algorithm to effectively evaluate the ergonomic implications of different key arrangements. A transition is defined as the swapping of two keys on the keyboard matrix.

The acceptance of candidate solutions in simulated annealing is determined by a probabilistic criterion derived from the Boltzmann distribution function. Originating from statistical mechanics, this function describes the probability of particles occupying different energy states within a system [8]. This criterion is mathematically represented as follows:

$$A(i) = \begin{cases} \exp\left(-\frac{\Delta E}{T}\right), & \text{if } \Delta E > 0 \\ 1, & \text{otherwise} \end{cases}$$

Here, ΔE represents the change in energy within the system or, in this case, in the estimated typing time between the current layout and the candidate layout, and T denotes the temperature, a parameter that controls the amount of randomness in the system. If the change in typing time is negative ($\Delta E < 0$), indicating an improvement in estimated typing time, the candidate layout is accepted with certainty. However, if the change in estimated typing time is positive, indicating a worsening layout, the candidate layout is accepted with a probability determined by the Boltzmann

factor, which is more likely to accept slight decreases in performance over larger ones.

This choice of probabilistic criterion allows the simulated annealing algorithm to converge on an optimal or near-optimal solution through gradual refinement [7]. Initially, the temperature is high, allowing the algorithm to explore the solution space and escape local minima by accepting suboptimal moves. As the algorithm progresses and the temperature decreases, the probability of accepting suboptimal moves diminishes, refining the search to more promising regions. The reduction in temperature is controlled by a cooling schedule, which dictates how fast and effectively the algorithm converges to a solution. We adopt a monotonically decreasing geometric cooling schedule defined as follows:

$$T_i = \alpha^i T_0$$

where T_0 is the initial-temperature and α is the cooling-rate, $0 < \alpha < 1$.

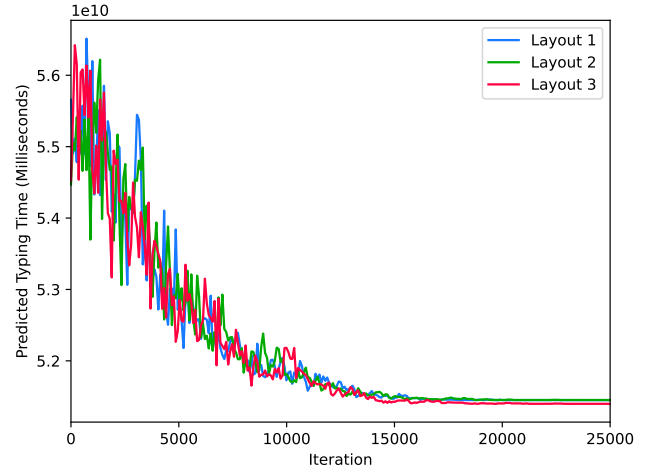


Figure 3. Convergence of Estimated Typing Time during Simulated Annealing

2.4.2 Setting the Initial Temperature

Before any transitions are performed, an initial temperature must be set for the cooling schedule to iterate on. A common approach to setting the initial temperature is to manually estimate it based on the problem domain and its characteristics. This initial temperature is frequently chosen so that the acceptance probability at the beginning of simulated annealing is approximately equal to some value. Lacking relevant literature for our problem domain, we resort to the approach outlined in Ben-Ameur [2]. We set a variable χ_0 to be the desired initial acceptance probability of suboptimal transitions at the beginning of the algorithm. An iterative method is employed to compute the initial temperature T_0 such that the acceptance probability approaches χ_0 . To compute the estimated acceptance probability $\hat{\chi}(T)$ for a given temperature T , first, we use the set S of possible positive

transitions (i.e., transitions where $E_{before} < E_{after}$), where each transition s represents a swap of two keys. Finally, we take the ratio between the sum of probabilities of generating and accepting each positive transition s and the sum of the probabilities of generating each positive transition s :

$$\hat{\chi}(T_n) = \frac{\sum_{s \in S} \exp\left(-\frac{E_{after}(s)}{T_n}\right)}{\sum_{s \in S} \exp\left(-\frac{E_{before}(s)}{T_n}\right)}$$

This can now be used to find T_0 . First, we define an initial guess temperature T_1 , then recursively apply this formula:

$$T_{n+1} = T_n \left(\frac{\ln(\hat{\chi}(T_n))}{\ln(\chi_0)} \right)$$

Once $|\hat{\chi}(T_n) - \chi_0| \leq \epsilon$ for some error ϵ , we have found our initial temperature T_0 .

2.4.3 Termination Criterion

Finally, we establish a termination criterion for the simulated annealing process to be the point at which the number of iterations lacking improvement reaches the probabilistic threshold that all potential swaps have been evaluated. We do this to ensure the simulated annealing algorithm has settled on some minima while allowing suboptimal swaps to be heuristically accepted, potentially escaping local minima.

To find the termination criterion, let S be the number of iterations to evaluate all potential swaps. Let k be the number of keys; for k keys, the number of swappable key pairs $n = \binom{k}{2}$. We consider $S = \sum_{i=1}^n s_i$ where s_i is the number of iterations required to evaluate the i -th pair after $i-1$ pairs have been evaluated.

The probability P_i of selecting a new pair to swap is $P_i = \frac{n-(i-1)}{n} = \frac{n-i+1}{n}$. Consequently, s_i follows a geometric distribution with an expectation of $E(s_i) = \frac{n}{n-i+1}$. By the linearity of expectations, we derive:

$$E(S) = n \sum_{i=1}^n \frac{1}{i}$$

To increase performance, we can use the approximation:

$$E(S) = n \log n + \gamma n + \frac{1}{2} + O\left(\frac{1}{n}\right)$$

where $\gamma \approx 0.5772156649$ is the Euler-Mascheroni constant. Since the number of iterations must be a positive integer, we set our final stopping point to be:

$$\left\lceil \binom{k}{2} \log \binom{k}{2} + \gamma \binom{k}{2} + \frac{1}{2} \right\rceil$$

3 Results

For the final layout, we choose to use the 30 most common characters in our corpus, which is the standard English alphabet, plus the addition of four special characters: the comma, period, hyphen, and apostrophe. This layout differs by 2 characters from the QWERTY layout, so to compare

the final layout with QWERTY requires we only predict the time for strings in the corpus that can be typed on both layouts. Although the average typing speed was 47 WPM, we set the target WPM to ≥ 80 as a litmus test of proficiency. This results in single-finger bistrokes having greater impact and frequency having less. Optimizing for an above-average WPM like this is desirable if the goal is to raise the upper threshold of potential typing speed. Using the methodology in this paper, one could also aim to optimize for the average user, improving the mean speed. The resulting layout can be seen in Table 4.

Table 4. Generated Layout for Typing Speeds ≥ 80 WPM

M	R	T	C	W	,	K	A	E	'
L	N	D	S	V	Y	U	O	I	G
H	X	Z	B	F	.	-	Q	J	P
SPACE									

The predicted time to type the iWeb corpus on QWERTY is 54,934,582 seconds, Dvorak is 52,565,249 seconds, a speed up of 4%, and the layout produced in this paper is predicted to take 51,429,827 seconds, a speed up of 6%. Our analysis revealed positional categories of bistroke that played a significant role in the prediction of typing speed. Namely, ALTs which are shown to be faster and SFBs which are shown to be slower. For QWERTY, 18.3% of all bistrokes are ALTs and 5.7% of them are SFBs, dvorak has 33.6% ALTs and 2.8% SFBs, and the layout produced in this paper has the best result of 33.6% ALTs and 1.4% SFBs. The fact that speed optimization yields only marginal improvements in typing efficiency is not unexpected but remains a valuable take away, as it suggests prioritizing theoretical features over solely focusing on speed optimization may offer substantial benefits.

4 Future Work

The overrepresentation of the QWERTY, AZERTY, and QWERTZ layouts in the current data set likely biases the model, as evidenced by the observed preference for the top row. The sparsity of data may impair the model's ability to generalize accurately. Although the proposed cost function is simple and generalizable, it does not fully capture the complexities of human typing behavior. Future studies should prioritize the collection of data from a wider variety of keyboard layouts and typing speeds to mitigate this limitation. To facilitate this, an open-source tool, Kiakl, was developed to crowdsource data from alternative keyboard layouts. However, due to time constraints and the limited volume of data gathered, Kiakl was not used in this study. A more comprehensive data set would enable the exploration of more sophisticated models, such as transformers or recurrent neural networks, offering the potential for a more nuanced and accurate evaluation of keyboard layouts.

The incorporation of previously excluded tristroke and bistroke features also warrants consideration. These features include lateral stretches, redirects, rolls, and scissors. A lateral stretch occurs when two adjacent fingers are pulled apart horizontally, complicating the articulation of a bistroke. These occur almost exclusively from keystrokes in the inner index columns. Scissors occur when one finger of a hand reaches towards the top row, while another finger on the same hand contracts to press a key on the bottom row. Redirects occur when a tristroke, typed with the same hand, changes direction, frequently resulting in typos. An example of this in the QWERTY layout is the tristroke "sad," as the "sa" is typed outward while "ad" is typed inward. A roll occurs when a tristroke is typed with the same hand and maintains a consistent direction, which may be faster to articulate. Modeling these movements may increase predictive accuracy, offering opportunities for further optimization.

Further improvements to the keyboard optimization process could involve refining the simulated annealing algorithm itself. Although geometric cooling was effective in this study, exploring alternative cooling schedules could improve performance. Modifying the cooling rate over time or experimenting with alternative patterns, such as polynomial or logarithmic decays, may enhance the algorithm's ability to find global optima. Finally, incorporating formal mathematical validation, such as estimating the lower bound of optimization, would help verify and guide future results.

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