UNIVERSITÁ DEGLI STUDI DI GENOVA

DIBRIS

Robotics engineering

Computer vision

Image filtering and Fourier transform

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Introduction

The main goal of this laboratory is to apply various filters to different images and understand how they affect them. Afterwards, an analysis of the Fourier transform is conducted to examine its impact on both images and filters. All the operations were performed using *MATLAB R2023a*.

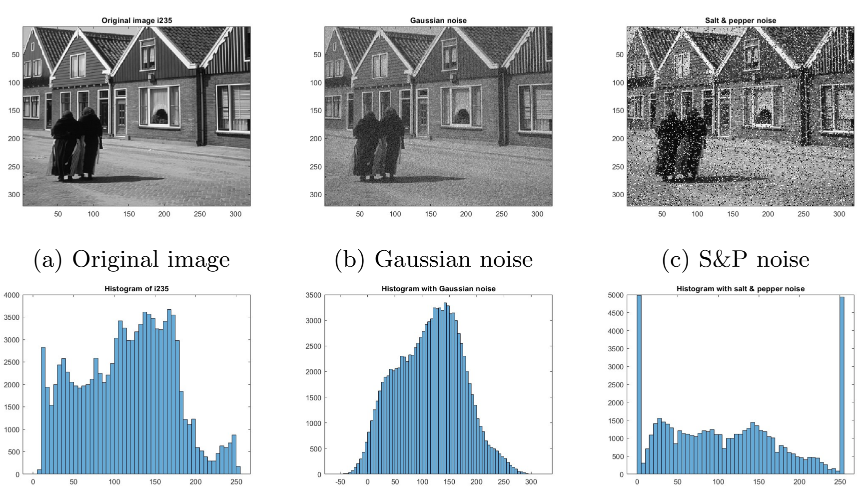
1 Noise

Real-world images inevitably contain noise. For this laboratory, two types of noise are considered: *low-pass Gaussian* and *salt & pepper*. Gaussian noise causes a variation in pixel intensity, so the image appears blurry. The salt & pepper noise introduces some random white and black pixels.

In the first part of the laboratory, it was decided to add additional noise to the images in order to better see the effects of the filters.

Gaussian noise (figure 1b) with a standard deviation of 20 was applied, and also salt & pepper noise (figure 1c) with a density of 20%.

From the histogram, it is possible to see the distribution of pixels based on their intensity of brightness.



(a) Original image (b) Gaussian noise (c) S&P noise

(d) Histogram original (e) Histogram Gaussian (f) Histogram S&P

Figure 1: Noise and their histogram

From the histogram 1e, it is possible to see that the brightness of the pixels is redistributed as a bell-shaped curve around a mean value.

The histogram in figure 1f shows a high concentration of pixels at both the maximum and minimum intensity values.

2 Filtering

There are various techniques to filter an image with different types of filters. In this laboratory, three types of filters were used: moving average filter, Gaussian filter, and median filter.

2.1 Moving average filter

The idea behind this filter is to compute a mean value of the intensity of a pixel with its neighbours, since noise is independent from pixel to pixel.

It is possible to choose how many neighbours we want to consider, by changing the size of the kernel. In the laboratory, it was applied both 3×3 and 7×7 size filters.

The filter 3×3 used to filter the images was defined as follows (the filter with the size of 7×7 had the same structure):

$$H_3 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The filter was applied to the images with both Gaussian noise and salt & pepper noise.

For the first type of noise, the 3×3 filter performs effectively, reducing the visibility of the noise; however, the edges appear less sharp (figure 2).

The 7×7 filter removes most of the noise; however, the resulting image appears excessively blurred.

Figure 2: Moving average filter with Gaussian noise

With the images affected by salt & pepper noise, the moving average filter doesn't perform well, either with the size of 3×3 or 7×7.

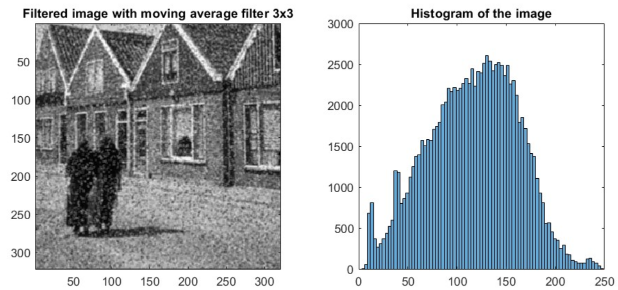


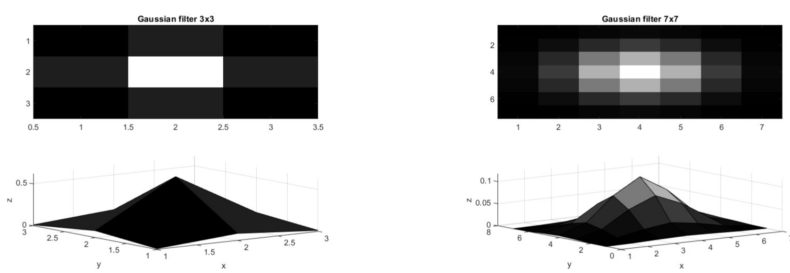
Figure 3: Moving average filter with S&P noise

2.2 Low-pass Gaussian filter

This filter performs a weighted average between the intensity of a pixel and its neighbours. The weights follow a 2D Gaussian shape, in order to have the peripheral pixels less important than the central ones (figure 4).

In order to obtain a good Gaussian shape for the filter, the standard deviation was set based on the formula

$$\sigma = \frac{Size\ of\ the\ filter}{6}$$

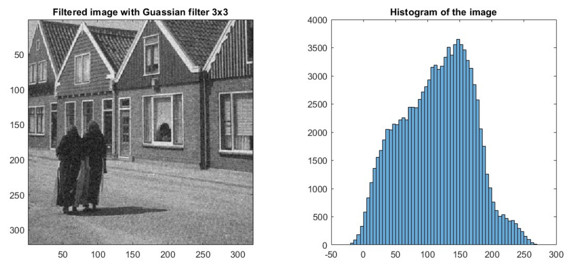


(a) 3×3 (b) 7×7

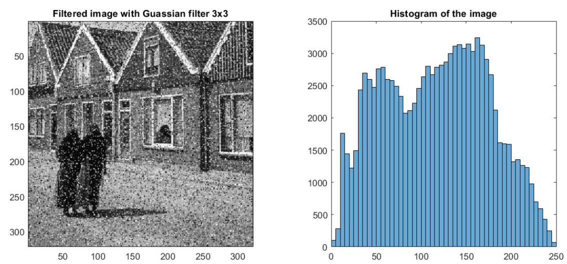
Figure 4: Gaussian filters

When applied to the images with the Gaussian noise, the filter with the size of 3×3 is more effective; the noise is less visible, but the edges are still sharp (figure 5a). The filtered images appear slightly degraded with respect to the original ones. With the size filter of 7×7 the noise is completely removed, but the image results a little bit blurred.

When the Gaussian filter is applied to the images with salt & pepper noise, the results are not satisfactory (figure 5b). The noise is still visible, and with the size of 7×7 the images result blurred too.



(a) Gaussian filter on Gaussian noise



(b) Gaussian filter on S&P

Figure 5: Gaussian filter results

2.3 Median filter

The median filter is a non-linear operator that replaces each pixel with the median value of its neighbourhood.

Although the median filter results in a certain loss of information, it effectively preserves image edges. Its performance is limited in the presence of Gaussian noise, but it provides satisfactory results when dealing with salt & pepper noise.

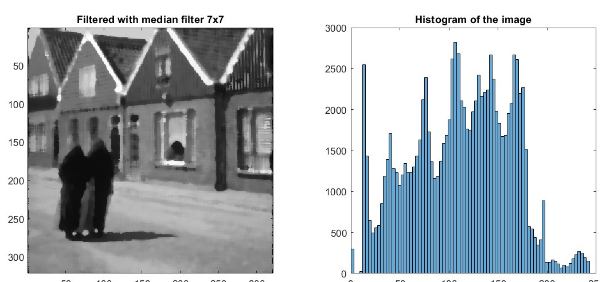


Figure 6: Median filter with S&P noise

In Figure 6, a kernel size of 7×7 was used. While the noise is almost entirely removed, this results in a substantial loss of detail. For instance, the bricks on the house wall are no longer visible. Using a smaller kernel of 3×3 preserves more detail, but some noise remains apparent. Hence, the kernel size must be chosen as a trade-off between noise suppression and detail preservation.

3 Practice with linear filters

This section of the laboratory work is dedicated to analysing the effects of linear filters on images. All filters discussed here were implemented with a kernel size of 7×7. The images used were the original one, without additional noise.

3.1 Impulse

Applying a kernel composed of zeros with a single one at the centre leaves the input image unchanged.

3.2 Shift

When the position of the one is shifted away from the centre, the output changes accordingly. For example, if the one is moved three positions to the right, the resulting image remains identical to the original but is shifted three pixels to the right.

3.3 Box filter

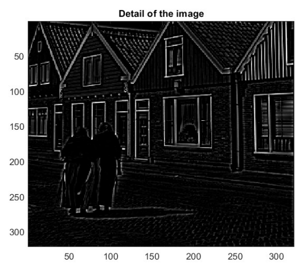
The kernel is defined as a matrix of ones normalized by the square of the filter size (in this case, 49). This filter calculates the average value of each pixel and its surrounding neighbours, producing a smoothed version of the original image.

3.4 Sharpening filter

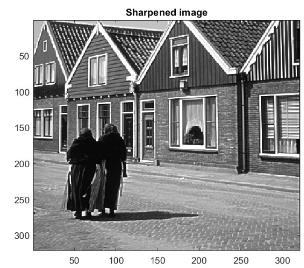
By subtracting the smoothed image obtained with the box filter from the original image, an image containing only the fine details is obtained (figure 7a). Adding these details back to the original image enhances its sharpness, resulting in a more defined appearance (figure 7b). This type of filter is a high-pass filter.

4 Fourier transform

The final part of the laboratory work was dedicated to the study of the Fourier transform, applied to both images and filters, with a specific focus on analysing the magnitude component.



(a) Details of the photo



(b) Sharpened image

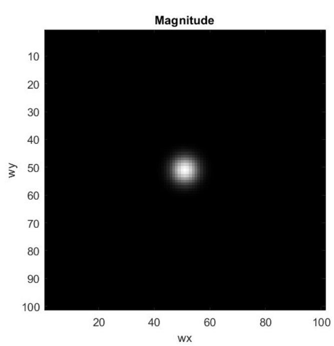
Figure 7

4.1 Image analysis

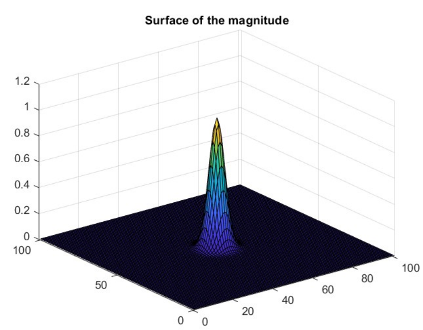
The magnitude of the FFT provides insight into the frequency content of an image. High-magnitude values near the centre correspond to low-frequency components, which represent the smoother regions of the image. In contrast, high-frequency components, associated with edges and fine details, are located toward the periphery of the transformed image.

4.2 Filters analysis

The Gaussian filter produces a central peak (figure 8). This is because the Gaussian filter is a low-pass filter and attenuates high-frequency components.



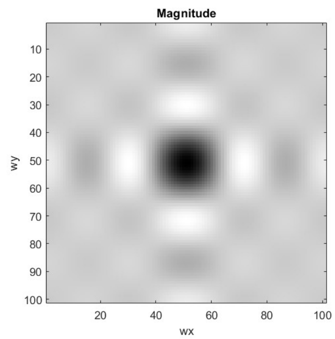
(a) Magnitude



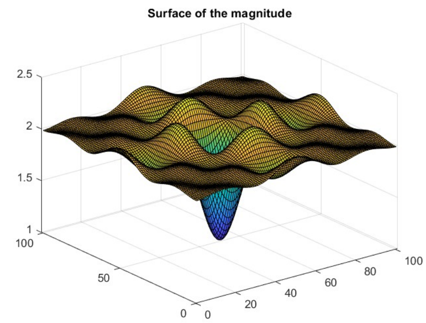
(b) Surface of the magnitude

Figure 8: Transformed Gaussian

The sharpening filter is a high-pass filter. In the Fourier domain, edges correspond to high-frequency components, as pixel intensity changes rapidly in these regions. The magnitude spectrum of the sharpening filter is essentially the opposite of that of a Gaussian filter, exhibiting high values far from the centre and low values near the centre (figure 9).



(a) Magnitude



(b) Surface of the magnitude

Figure 9: Fourier transform of the sharpening filter

5 Conclusion

In this laboratory, several types of filters were applied to noisy images to analyse their effects on image quality and detail preservation. Based on the analysis, it is possible to draw the following conclusions regarding their effectiveness and the choice of kernel size:

- for images with a prevalence of Gaussian noise, the low-pass Gaussian filter with a size of 3×3 is the best: most of the noise is removed but the image is not blurry and the edges are still sharp.
- for images with a prevalence of salt & pepper noise, the median filter with a size of the kernel of 7×7 is the most suitable: the noise is removed and the edges are still sharp. However a lot of detail are lost.

Choosing an appropriate kernel size is crucial to achieving a balance between noise suppression and the preservation of sharp edges.
The analysis of linear filters demonstrated how convolution operations can smooth or sharpen images depending on the kernel design.
Finally, the Fourier transform allowed a deeper understanding of how filters operate in the frequency domain, revealing the relationship between spatial and frequency characteristics.